Chapter 3

Differentiation

Contents

Issac Newton

In the history of mathematics two names are prominent to share the credit for inventing calculus, Issac Newton (1642-1727) *and* G.W. Leibinitz (1646-1717). *Both of them independently invented calculus around the seventeenth century. After the advent of calculus many mathematicians contributed for further development of calculus. The rigorous concept is mainly attributed to the great mathematicians,* A.L. Cauchy, J.L., Lagrange *and* Karl Weierstrass.

Before 1900, *it was thought that calculus is quite difficult to teach. So calculus became beyond the reach of youngsters. But just in* 1900, *John Perry and others in England started propagating the view that essential ideas and methods of calculus were simple and could be taught even in schools.* F.L. Griffin, *pioneered the teaching of calculus to first year students. This was regarded as one of the most daring act in those days.*

Today not only the mathematics but many other subjects such as Physics, Chemistry, Economics and Biological Sciences are enjoying the fruits of calculus.

Differentiation

Introduction

The rate of change of one quantity with respect to some another quantity has a great importance. For example, the rate of change of displacement of a particle with respect to time is called its velocity and the rate of change of velocity is called its acceleration.

The rate of change of a quantity 'y' with respect to another quantity 'x' is called the derivative or differential coefficient of y with respect to x.

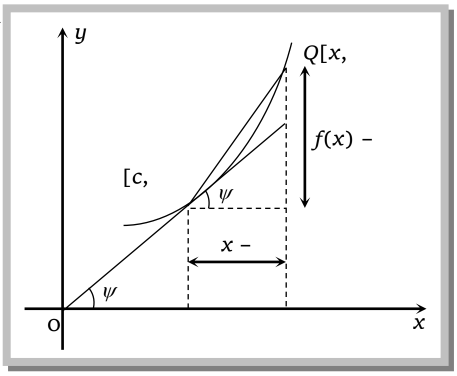
3.1 Derivative at a Point

The derivative of a function at a point $x = a$ is defined by $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ (provided the limit exists and is finite)

The above definition of derivative is also called derivative by first principle.

(1) **Geometrical meaning of derivatives at a point:** Consider the curve $y = f(x)$. Let $f(x)$ be differentiable at $x = c$. Let $P(c, f(c))$ be a point on the curve and $Q(x, f(x))$ be a neighbouring point on the curve. Then,

Slope of the chord $PQ = \frac{f(x) - f(c)}{x - c}$. Taking limit as $Q \to P$, *i.e.*, $x \to$



we get $\lim_{Q \to P}$ (slope of the chord PQ) $= \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ ......(i)

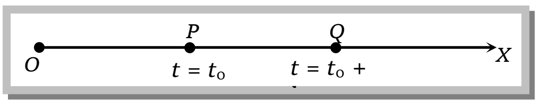
As $Q \to P$, chord PQ becomes tangent at P.

Therefore from (i), we have

Slope of the tangent at $P = \lim_{x \to c} \frac{f(x) - f(c)}{x - c} = \left(\frac{df(x)}{dx}\right)_{x = c}$.

Note : ❑ Thus, the derivatives of a function at a point $x = c$ is the slope of the tangent to curve, $y = f(x)$ at point $(c, f(c))$.

(2) **Physical interpretation at a point :** Let a particle moves in a straight line OX starting from O towards X. Clearly, the position of the particle at any instant would depend upon the time elapsed. In other words, the distance of the particle from O will be some function f of time t.



Let at any time $t = t_0$, the particle be at *P* and after a further time *h*, it is at *Q* so that *OP* = $f(t_0)$ and $OQ = f(t_0 + h)$. Hence, the average speed of the particle during the journey from *P* to *Q* is $\frac{PQ}{h}$, *i.e.,* $\frac{f(t_0+h) - f(t_0)}{h} = f(t_0, h)$. Taking the limit of $f(t_0, h)$ as $h \to 0$, we get its instantaneous speed to be $\lim_{h\to 0} \frac{f(t_0+h)-f(t_0)}{h}$, which is simply $f'(t_0)$. Thus, if $f(t)$ gives the distance of a moving particle at time *t*, then the derivative of *f* at $t = t_0$ represents the instantaneous speed of the particle at the point *P*, *i.e.,* at time $t = t_0$.

Important Tips

☞ $\frac{dy}{dx}$ *is* $\frac{d}{dx}(y)$ *in which* $\frac{d}{dx}$ *is simply a symbol of operation and not 'd' divided by dx.*

☞ *If* $f'(x_0) = \infty$, *the function is said to have an infinite derivative at the point* x_0. *In this case the line tangent to the curve of y = f(x) at the point* x_0 *is perpendicular to the x-axis*

Example: 1 If $f(2) = 4,\ \ f'(2) = 1,$ then $\lim_{x\to 2} \frac{xf(2) - 2f(x)}{x-2} =$ **[Rajasthan PET 1995, 2000]**

(a) 1 (b) 2 (c) 3 (d) – 2

Solution: (b) Given $f(2) = 4, f'(2) = 1$

$$\therefore\ \lim_{x\to 2} \frac{xf(2) - 2f(x)}{x-2} = \lim_{x\to 2} \frac{xf(2) - 2f(2) + 2f(2) - 2f(x)}{x-2} = \lim_{x\to 2} \frac{(x-2)f(2)}{x-2} - \lim_{x\to 2} \frac{2f(x) - 2f(2)}{x-2}$$

$$= f(2) - 2\lim_{x\to 2} \frac{f(x) - f(2)}{x-2} = f(2) - 2f'(2) = 4 - 2(1) = 4 - 2 = 2$$

Trick : Applying L-Hospital rule, we get $\lim_{x\to 2} \frac{f(2) - 2f'(2)}{1} = 2$.

Example: 2 If $f(x+y) = f(x).f(y)$ for all *x* and *y* and $f(5) = 2,\ \ f'(0) = 3,$ then $f'(5)$ will be

[IIT 1981; Karnataka CET 2000; UPSEAT 2002; MP PET 2002; AIEEE 2002]

(a) 2 (b) 4 (c) 6 (d) 8

Solution: (c) Let $x = 5, y = 0 \Rightarrow f(5+0) = f(5).f(0)$

$\Rightarrow f(5) = f(5)f(0) \Rightarrow f(0) = 1$

Therefore, $f'(5) = \lim_{h\to 0} \frac{f(5+h) - f(5)}{h} = \lim_{h\to 0} \frac{f(5)f(h) - f(5)}{h} = \lim_{h\to 0} 2\left[\frac{f(h) - 1}{h}\right]$ $\{\because f(5) = 2\}$

$$= 2\lim_{h\to 0}\left[\frac{f(h) - f(0)}{h}\right] = 2 \times f'(0) = 2 \times 3 = 6.$$

Example: 3 If $f(a) = 3, f'(a) = -2, g(a) = -1, g'(a) = 4,$ then $\lim_{x\to a} \frac{g(x)f(a) - g(a)f(x)}{x-a} =$ **[MP PET 1997]**

(a) – 5 (b) 10 (c) – 10 (d) 5

Solution: (b) $\lim_{x\to a}\frac{g(x)f(a)-g(a)f(x)}{x-a}$. We add and subtract $g(a)f(a)$ in numerator

$= \lim_{x\to a}\frac{g(x)f(a)-g(a)f(a)+g(a)f(a)-g(a)f(x)}{x-a} = \lim_{x\to a} f(a)\left[\frac{g(x)-g(a)}{x-a}\right] - \lim_{x\to a} g(a)\left[\frac{f(x)-f(a)}{x-a}\right]$

$= f(a)\lim_{x\to a}\left[\frac{g(x)-g(a)}{x-a}\right] - g(a)\lim_{x\to a}\left[\frac{f(x)-f(a)}{x-a}\right] = f(a)g'(a)-g(a)f'(a)$ [by using first principle formula]

= 3.4 – (–1)(–2) = 12 – 2 = 10

Trick : $\lim_{x\to a}\frac{g(x)f(a)-g(a)f(x)}{x-a}$

Using L–Hospital's rule, Limit = $\lim_{x\to a}\frac{g'(x)f(a)-g(a)f'(x)}{1}$;

Limit = $g'(a)\ f(a)-g(a)f'(a) = (4)(3)-(-1)(-2)$ = 12 – 2 = 10.

Example: 4 If $5f(x)+3f\left(\frac{1}{x}\right) = x+2$ and $y = xf(x)$ then $\left(\frac{dy}{dx}\right)_{x=1}$ is equal to

(a) 14 (b) $\frac{7}{8}$ (c) 1 (d) None of these

Solution: (b) $\because 5f(x)+3f\left(\frac{1}{x}\right) = x+2$ ......(i)

Replacing x by $\frac{1}{x}$ in (i), $5f\left(\frac{1}{x}\right)+3f(x) = \frac{1}{x}+2$ ......(ii)

On solving equation (i) and (ii), we get, $16f(x) = 5x-\frac{3}{x}+4$, $\therefore\ 16f'(x) = 5+\frac{3}{x^2}$

$\because\ y = xf(x) \Rightarrow \frac{dy}{dx} = f(x)+xf'(x) = \frac{1}{16}(5x-\frac{3}{x}+4)+x.\frac{1}{16}(5+\frac{3}{x^2})$

at $x=1$, $\frac{dy}{dx} = \frac{1}{16}(5-3+4)+\frac{1}{16}(5+3) = \frac{7}{8}$.

3.2 Some Standard Differentiation

(1) **Differentiation of algebraic functions**

(i) $\frac{d}{dx}x^n = nx^{n-1}, x\in R, n\in R, x>0$ (ii) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$ (iii) $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$

(2) **Differentiation of trigonometric functions :** The following formulae can be applied directly while differentiating trigonometric functions

(i) $\frac{d}{dx}\sin x = \cos x$ (ii) $\frac{d}{dx}\cos x = -\sin x$ (iii) $\frac{d}{dx}\tan x = \sec^2 x$

(iv) $\frac{d}{dx}\sec x = \sec x\tan x$ (v) $\frac{d}{dx}\text{cosec } x = -\text{cosec } x\cot x$ (vi) $\frac{d}{dx}\cot x = -\text{cosec}^2 x$

(3) **Differentiation of logarithmic and exponential functions :** The following formulae can be applied directly when differentiating logarithmic and exponential functions

(i) $\frac{d}{dx}\log x = \frac{1}{x}$, for $x > 0$ (ii) $\frac{d}{dx}e^x = e^x$

(iii) $\frac{d}{dx}a^x = a^x \log a$, for $a > 0$

(iv) $\frac{d}{dx}\log_a x = \frac{1}{x \log a}$, for $x > 0$, $a > 0$, $a \neq 1$

(4) **Differentiation of inverse trigonometrical functions :** The following formulae can be applied directly while differentiating inverse trigonometrical functions

(i) $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$, for $-1 < x < 1$

(ii) $\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$, for $-1 < x < 1$

(iii) $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$, for $|x| > 1$

(iv) $\frac{d}{dx}\text{cosec}^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}$, for $|x| > 1$

(v) $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$, for $x \in R$

(vi) $\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$, for $x \in R$

(5) **Differentiation of hyperbolic functions :**

(i) $\frac{d}{dx}\sin h\, x = \cos h\, x$

(ii) $\frac{d}{dx}\cos h\, x = \sin h\, x$

(iii) $\frac{d}{dx}\tan h\, x = \sec h^2 x$

(iv) $\frac{d}{dx}\cot h\, x = -\text{cosec}\ h^2 x$

(v) $\frac{d}{dx}\sec h\, x = -\sec h\, x \tan h\ x$

(vi) $\frac{d}{dx}\text{cosec}\, h\, x = -\text{cosec}\, h\, x \cot h\, x$

(vii) $\frac{d}{dx}\sin h^{-1} x = 1/\sqrt{(1+x^2)}$

(viii) $\frac{d}{dx}\cos h^{-1} x = 1/\sqrt{(x^2-1)}$

(ix) $\frac{d}{dx}\tan h^{-1} x = 1/(x^2-1)$

(x) $\frac{d}{dx}\cot h^{-1} x = 1/(1-x^2)$

(xi) $\frac{d}{dx}\sec h^{-1} x = -1/x\sqrt{(1-x^2)}$

(xii) $\frac{d}{dx}\text{cosec}\, h^{-1} x = -1/x\sqrt{(1+x^2)}$

(6) **Differentiation by inverse trigonometrical substitution:** For trigonometrical substitutions following formulae and substitution should be remembered

(i) $\sin^{-1} x + \cos^{-1} x = \pi/2$

(ii) $\tan^{-1} x + \cot^{-1} x = \pi/2$

(iii) $\sec^{-1} x + co\sec^{-1} x = \pi/2$

(iv) $\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left[x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right]$

(v) $\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left[xy \mp \sqrt{(1-x^2)(1-y^2)}\right]$

(vi) $\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left[\frac{x \pm y}{1 \mp xy}\right]$

(vii) $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$

(viii) $2\cos^{-1} x = \cos^{-1}(2x^2-1)$

(ix) $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

(x) $3\sin^{-1} x = \sin^{-1}(3x-4x^3)$

(xi) $3\cos^{-1} x = \cos^{-1}(4x^3-3x)$

(xii) $3\tan^{-1} x = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

(xiii) $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

(xiv) $\sin^{-1}(-x) = -\sin^{-1} x$

(xv) $\cos^{-1}(-x) = \pi - \cos^{-1} x$

(xvi) $\tan^{-1}(-x) = -\tan^{-1} x$ or $\pi - \tan^{-1} x$

(xvii) $\frac{\pi}{4} - \tan^{-1} x = \tan^{-1}\left(\frac{1-x}{1+x}\right)$

(7) **Some suitable substitutions**

S. N.	Function	Substitution	S. N.	Function	Substitution
(i)	$\sqrt{a^2 - x^2}$	$x = a\sin\theta$ or $a\cos\theta$	(ii)	$\sqrt{x^2 + a^2}$	$x = a\tan\theta$ or $a\cot\theta$
(iii)	$\sqrt{x^2 - a^2}$	$x = a\sec\theta$ or $a\,\text{cosec}\,\theta$	(iv)	$\sqrt{\frac{a-x}{a+x}}$	$x = a\cos 2\theta$
(v)	$\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}$	$x^2 = a^2\cos 2\theta$	(vi)	$\sqrt{ax - x^2}$	$x = a\sin^2\theta$
(vii)	$\sqrt{\frac{x}{a+x}}$	$x = a\tan^2\theta$	(viii)	$\sqrt{\frac{x}{a-x}}$	$x = a\sin^2\theta$
(ix)	$\sqrt{(x-a)(x-b)}$	$x = a\sec^2\theta - b\tan^2\theta$	(x)	$\sqrt{(x-a)(b-x)}$	$x = a\cos^2\theta + b\sin^2\theta$

3.3 Theorems for Differentiation

Let $f(x), g(x)$ and $u(x)$ be differentiable functions

(1) If at all points of a certain interval. $f'(x) = 0$, then the function $f(x)$ has a constant value within this interval.

(2) **Chain rule**

(i) **Case I :** If y is a function of u and u is a function of x, then derivative of y with respect to x is $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ or $y = f(u) \Rightarrow \frac{dy}{dx} = f'(u)\frac{du}{dx}$

(ii) **Case II :** If y and x both are expressed in terms of t, y and x both are differentiable with respect to t then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

(3) **Sum and difference rule :** Using linear property $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$

(4) **Product rule :** (i) $\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$ (ii) $\frac{d}{dx}(u.v.w.) = u.v.\frac{dw}{dx} + v.w.\frac{du}{dx} + u.w.\frac{dv}{dx}$

(5) **Scalar multiple rule :** $\frac{d}{dx}(k\,f(x)) = k\frac{d}{dx}f(x)$

(6) **Quotient rule :** $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$, provided $g(x) \neq 0$

Example: 5 The derivative of $f(x) = |x|^3$ at $x = 0$ is **[Rajasthan PET 2001; Haryana CEE 2002]**

(a) 0 (b) 1 (c) –1 (d) Not defined

Solution: (a) $f(x)=\begin{cases} x^3 & , x\geq 0 \\ -x^3 & , x<0 \end{cases}$ and $f'(x)=\begin{cases} 3x^2 & , x\geq 0 \\ -3x^2 & , x<0 \end{cases}$

$f'(0^+)=f'(0^-)=0$

Example: 6 The first derivative of the function $(\sin 2x\cos 2x\cos 3x+\log_2 2^{x+3})$ with respect to x at $x=\pi$ is

(a) 2 (b) –1 (c) $-2+2^{\pi}\log_e 2$ (d) $-2+\log_e 2$

Solution: (b) $f(x)=\sin 2x.\cos 2x.\cos 3x+\log_2 2^{x+3}$, $f(x)=\frac{1}{2}\sin 4x\cos 3x+(x+3)\log_2 2$, $f(x)=\frac{1}{4}[\sin 7x+\sin x]+x+3$

Differentiate *w.r.t.* x,

$f'(x)=\frac{1}{4}[7\cos 7x+\cos x]+1$, $f'(x)=\frac{1}{4}7\cos 7x+\frac{1}{4}\cos x+1$, $f'(\pi)=-2+1=-1$.

Example: 7 If $y=|\cos x|+|\sin x|$ then $\frac{dy}{dx}$ at $x=\frac{2\pi}{3}$ is

(a) $\frac{1-\sqrt{3}}{2}$ (b) 0 (c) $\frac{1}{2}(\sqrt{3}-1)$ (d) None of these

Solution: (c) Around $x=\frac{2\pi}{3}$, $|\cos x|=-\cos x$ and $|\sin x|=\sin x$

$\therefore\ y=-\cos x+\sin x\ \therefore\ \frac{dy}{dx}=\sin x+\cos x$

At $x=\frac{2\pi}{3}$, $\frac{dy}{dx}=\sin\frac{2\pi}{3}+\cos\frac{2\pi}{3}=\frac{\sqrt{3}}{2}-\frac{1}{2}=\frac{1}{2}(\sqrt{3}-1)$.

Example: 8 If $f(x)=\log_x(\log x)$, then $f'(x)$ at $x=e$ is **[IIT 1985; Rajasthan PET 2000; MP PET 2000; Karnataka CET 2002]**

(a) e (b) $1/e$ (c) 1 (d) None of these

Solution: (b) $f(x)=\log_x(\log x)=\frac{\log(\log x)}{\log x}\Rightarrow f'(x)=\frac{\frac{1}{x}-\frac{1}{x}\log(\log x)}{(\log x)^2}\Rightarrow f'(e)=\frac{\frac{1}{e}-0}{1}=\frac{1}{e}$

Example: 9 If $f(x)=|\log x|$, then for $x\neq 1, f'(x)$ equals

(a) $\frac{1}{x}$ (b) $\frac{1}{|x|}$ (c) $\frac{-1}{x}$ (d) None of these

Solution: (d) $f(x)=|\log x|=\begin{cases} -\log x, & \text{if } 0<x<1 \\ \log x, & \text{if } x\geq 1 \end{cases}\Rightarrow f'(x)=\begin{cases} -\frac{1}{x}, & \text{if } 0<x<1 \\ \frac{1}{x}, & \text{if } x>1 \end{cases}$.

Clearly $f'(1^-)=-1$ and $f'(1^+)=1$, $\therefore$ $f'(x)$ does not exist at $x=1$

Example: 10 $\frac{d}{dx}\left[\log\left\{e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right\}\right]$ equals to

(a) 1 (b) $\frac{x^2+1}{x^2-4}$ (c) $\frac{x^2-1}{x^2-4}$ (d) $e^x\frac{x^2-1}{x^2-4}$

Solution: (c) Let $y = \left[\log\left\{e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right\}\right] = \log e^x + \log\left(\frac{x-2}{x+2}\right)^{3/4}$

$\Rightarrow y = x + \frac{3}{4}[\log(x-2) - \log(x+2)] \Rightarrow \frac{dy}{dx} = 1 + \frac{3}{4}\left[\frac{1}{x-2} - \frac{1}{x+2}\right] = 1 + \frac{3}{(x^2-4)}$

$\Rightarrow \frac{dy}{dx} = \frac{x^2-1}{x^2-4}$.

Example: 11 If $x = \exp\left\{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right\}$ then $\frac{dy}{dx}$ equals **[MP PET 2002]**

(a) $2x[1+\tan(\log x)] + x\sec^2(\log x)$
(b) $x[1+\tan(\log x)] + \sec^2(\log x)$
(c) $2x[1+\tan(\log x)] + x^2 \sec^2(\log x)$
(d) $2x[1+\tan(\log x)] + \sec^2(\log x)$

Solution: (a) $x = \exp\left\{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right\} \Rightarrow \log x = \tan^{-1}\left(\frac{y-x^2}{x^2}\right)$

$\Rightarrow \frac{y-x^2}{x^2} = \tan(\log x) \Rightarrow y = x^2\tan(\log x) + x^2 \Rightarrow \frac{dy}{dx} = 2x.\tan(\log x) + x^2.\frac{\sec^2(\log x)}{x} + 2x$

$\Rightarrow \frac{dy}{dx} = 2x\tan(\log x) + x\sec^2(\log x) + 2x \Rightarrow \frac{dy}{dx} = 2x[1+\tan(\log x)] + x\sec^2(\log x)$.

Example: 12 If $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$, then $\frac{dy}{dx} =$ **[UPSEAT 1999; AMU 2002]**

(a) 0
(b) $\frac{1}{\sqrt{x}+1}$
(c) 1
(d) None of these

Solution: (a) $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) = \cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$ $\left\{\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right\}$

Example: 13 $\frac{d}{dx}\tan^{-1}\left[\frac{\cos x - \sin x}{\cos x + \sin x}\right]$ **[AISSE 1985, 87; DSSE 1982, 84; MNR 1985; Karnataka CET 2002; Rajasthan PET 2002, 03]**

(a) $\frac{1}{2(1+x^2)}$
(b) $\frac{1}{1+x^2}$
(c) 1
(d) – 1

Solution: (d) $\frac{d}{dx}\tan^{-1}\left[\frac{\cos x - \sin x}{\cos x + \sin x}\right] = \frac{d}{dx}\tan^{-1}\left[\tan\left(\frac{\pi}{4} - x\right)\right] = -1$.

Example: 14 $\frac{d}{dx}\left[\sin^2\cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}\right]$ equals **[MP PET 2002; EAMCET 1996]**

(a) -1
(b) $\frac{1}{2}$
(c) $-\frac{1}{2}$
(d) 1

Solution: (b) Let $y = \sin^2\cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

$\Rightarrow y = \sin^2\cot^{-1}\left\{\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right\} = \sin^2\cot^{-1}\left(\tan\frac{\theta}{2}\right) \Rightarrow y = \sin^2\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \cos^2\frac{\theta}{2} = \frac{1}{2}(1+\cos\theta) = \frac{1}{2}(1+x)$

$\therefore \frac{dy}{dx} = \frac{1}{2}$

Example: 15 If $y = \cos^{-1}\left(\frac{5\cos x - 12\sin x}{13}\right)$, $x \in \left(0, \frac{\pi}{2}\right)$, then $\frac{dy}{dx}$ is equal to

(a) 1 (b) – 1 (c) 0 (d) None of these

Solution: (a) Let $\cos\alpha = \frac{5}{13}$. Then $\sin\alpha = \frac{12}{13}$. So, $y = \cos^{-1}\{\cos\alpha.\cos x - \sin\alpha.\sin x\}$

$\therefore y = \cos^{-1}\{\cos(x+\alpha)\} = x + \alpha$ ($\because$ $x+\alpha$ is in the first or the second quadrant)

$\therefore \frac{dy}{dx} = 1$.

Example: 16 $\frac{d}{dx}\cosh^{-1}(\sec x) =$ **[Rajasthan PET 1997]**

(a) $\sec x$ (b) $\sin x$ (c) $\tan x$ (d) $\text{cosec}\, x$

Solution: (a) We know that $\frac{d}{dx}\cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$, $\frac{d}{dx}\cosh^{-1}(\sec x) = \frac{1}{\sqrt{\sec^2 x - 1}}\sec x\tan x = \frac{\sec x\tan x}{\tan x} = \sec x$.

Example: 17 $\frac{d}{dx}\left[\left(\frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x\tan^2 x}\right)\cot 3x\right]$ **[AMU 2000]**

(a) $\tan 2x\tan x$ (b) $\tan 3x\tan x$ (c) $\sec^2 x$ (d) $\sec x\tan x$

Solution: (c) Let $y = \frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x\tan^2 x} = \frac{(\tan 2x - \tan x)}{(1+\tan 2x\tan x)}\frac{(\tan 2x + \tan x)}{(1 - \tan 2x\tan x)} = \tan(2x-x)\tan(2x+x) = \tan x\tan 3x$.

$\therefore \frac{d}{dx}[y.\cot 3x] = \frac{d}{dx}[\tan x] = \sec^2 x$.

Example: 18 If $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$, then $f'(1)$ is equal to **[Rajasthan PET 2000]**

(a) – 1 (b) 1 (c) $\log 2$ (d) $-\log 2$

Solution: (a) $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$

Put $x^x = \tan\theta$, $\therefore y = f(x) = \cot^{-1}\left(\frac{\tan^2\theta - 1}{2\tan\theta}\right) = \cot^{-1}(-\cot 2\theta) = \pi - \cot^{-1}(\cot 2\theta)$

$\Rightarrow y = \pi - 2\theta = \pi - 2\tan^{-1}(x^x) \Rightarrow \frac{dy}{dx} = \frac{-2}{1+x^{2x}}.x^x(1+\log x) \Rightarrow f'(1) = -1$.

Example: 19 If $y = (1+x)(1+x^2)(1+x^4)\ldots\ldots(1+x^{2^n})$ then $\frac{dy}{dx}$ at $x = 0$ is

(a) 1 (b) – 1 (c) 0 (d) None of these

Solution: (a) $y = \frac{(1-x)(1+x)(1+x^2)\ldots..(1+x^{2^n})}{1-x} = \frac{1-x^{2^{n+1}}}{1-x}$

$\therefore \frac{dy}{dx} = \frac{-2^{n+1}.x^{2^{n+1}-1}(1-x) + 1 - x^{2^{n+1}}}{(1-x)^2}$, $\therefore$ At $x = 0$, $\frac{dy}{dx} = \frac{-2^{n+1}\,0.1 + 1 - 0}{1^2} = 1$.

Example: 20 If $f(x) = \cos x.\cos 2x.\cos 4x.\cos 8x.\cos 16x$ then $f'\left(\frac{\pi}{4}\right)$ is

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) None of these

Solution: (a) $f(x) = \frac{2\sin x \,.\cos x \,.\cos 2x \,.\cos 4x \,.\cos 8x \,.\cos 16x}{2\sin x} = \frac{\sin 32x}{2^5 \sin x}$

$\therefore \; f'(x) = \frac{1}{32} \cdot \frac{32\cos 32x \,.\sin x - \cos x \,.\sin 32x}{\sin^2 x}$

$\therefore \; f'\left(\frac{\pi}{4}\right) = \frac{32 \cdot \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \cdot 0}{32 \cdot \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{2}$.

3.4 Relation between *dy/dx* and *dx/dy*

Let x and y be two variables connected by a relation of the form $f(x, y) = 0$. Let Δx be a small change in x and let Δy be the corresponding change in y. Then $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ and $\frac{dx}{dy} = \lim_{\Delta y \to 0} \frac{\Delta x}{\Delta y}$.

Now, $\frac{\Delta y}{\Delta x} \cdot \frac{\Delta x}{\Delta y} = 1 \Rightarrow \lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x} \cdot \frac{\Delta x}{\Delta y} \right) = 1$

$\Rightarrow \; \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \cdot \lim_{\Delta y \to 0} \frac{\Delta x}{\Delta y} = 1 \; [\because \Delta x \to 0 \Leftrightarrow \Delta y \to 0\,] \Rightarrow \frac{dy}{dx} \cdot \frac{dx}{dy} = 1$. So, $\frac{dy}{dx} = \frac{1}{dx/dy}$.

3.5 Methods of Differentiation

(1) **Differentiation of implicit functions :** If y is expressed entirely in terms of x, then we say that y is an explicit function of x. For example $y = \sin x$, $y = e^x$, $y = x^2 + x + 1$ etc. If y is related to x but can not be conveniently expressed in the form of $y = f(x)$ but can be expressed in the form $f(x, y) = 0$, then we say that y is an implicit function of x.

(i) *Working rule* 1 : (a) Differentiate each term of $f(x, y) = 0$ with respect to x.

(b) Collect the terms containing dy/dx on one side and the terms not involving dy/dx on the other side.

(c) Express dy/dx as a function of x or y or both.

Note : ❑ In case of implicit differentiation, dy/dx may contain both x and y.

(ii) *Working rule* 2 : If $f(x, y)$= constant, then $\frac{dy}{dx} = -\frac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)}$

where $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial x}$ are partial differential coefficients of $f(x, y)$ with respect to x and y respectively.

Note : ❑ Partial differential coefficient of $f(x,y)$ with respect to x means the ordinary differential coefficient of $f(x,y)$ with respect to x keeping y constant.

Example: 21 If $xe^{xy} = y + \sin^2 x$, then at $x = 0$, $\frac{dy}{dx} =$ **[IIT 1996]**

(a) – 1 (b) – 2 (c) 1 (d) 2

Solution: (c) We are given that $xe^{xy} = y + \sin^2 x$

When $x = 0$, we get $y = 0$

Differentiating both sides *w.r.t.* x, we get, $e^{xy} + xe^{xy}\left[x\frac{dy}{dx} + y\right] = \frac{dy}{dx} + 2\sin x\cos x$

Putting, $x = 0$, $y = 0$, we get $\frac{dy}{dx} = 1$.

Example: 22 If $\sin(x+y) = \log(x+y)$, then $\frac{dy}{dx} =$ **[Karnataka CET 1993; Rajasthan PET 1989, 1992; Roorkee 2000]**

(a) 2 (b) – 2 (c) 1 (d) – 1

Solution: (d) $\sin(x+y) = \log(x+y)$

Differentiating with respect to x, $\cos(x+y)\left[1 + \frac{dy}{dx}\right] = \frac{1}{x+y}\left[1 + \frac{dy}{dx}\right]$

$\left[\cos(x+y) - \frac{1}{x+y}\right]\left[1 + \frac{dy}{dx}\right] = 0$

$\because$ $\cos(x+y) \neq \frac{1}{x+y}$ for any x and y. So, $1 + \frac{dy}{dx} = 0$, $\frac{dy}{dx} = -1$.

Trick: It is an implicit function, so $\frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y} = -\frac{\cos(x+y) - \frac{1}{x+y}}{\cos(x+y) - \frac{1}{x+y}} = -1$.

Example: 23 If $\ln(x+y) = 2xy$, then $y'(0) =$ **[IIT Screening 2004]**

(a) 1 (b) – 1 (c) 2 (d) 0

Solution: (a) $\ln(x+y) = 2xy \Rightarrow \frac{(1 + dy/dx)}{(x+y)} = 2\left(x\frac{dy}{dx} + y\right) \Rightarrow \frac{dy}{dx} = \frac{1 - 2xy - 2y^2}{2x^2 + 2xy - 1} \Rightarrow y'(0) = \frac{1-2}{-1} = 1$, at $x = 0,\ y = 1$.

(2) **Logarithmic differentiation :** If differentiation of an expression or an equation is done after taking log on both sides, then it is called logarithmic differentiation. This method is useful for the function having following forms.

(i) $y = [f(x)]^{g(x)}$

(ii) $y = \frac{f_1(x).f_2(x)........}{g_1(x).g_2(x).......}$ where $g_i(x) \neq 0$ (where i = 1, 2, 3,.....), $f_i(x)$ and $g_i(x)$ both are differentiable

(i) **Case I :** $y = [f(x)]^{g(x)}$ where $f(x)$ and $g(x)$ are functions of x. To find the derivative of this type of functions we proceed as follows:

Let $y = [f(x)]^{g(x)}$. Taking logarithm of both the sides, we have $\log y = g(x).\log f(x)$

Differentiating with respect to x, we get $\frac{1}{y}\frac{dy}{dx} = g(x).\frac{1}{f(x)}\frac{df(x)}{dx} + \log\{f(x)\}.\frac{dg(x)}{dx}$

$$\therefore \frac{dy}{dx} = y\left[\frac{g(x)}{f(x)}.\frac{df(x)}{dx} + \log[f(x)].\frac{dg(x)}{dx}\right] = [f(x)]^{g(x)}\left[\frac{g(x)}{f(x)}\frac{df(x)}{dx} + \log[f(x)]\frac{dg(x)}{dx}\right]$$

(ii) **Case II :** $y = \frac{f_1(x).f_2(x)}{g_1(x).g_2(x)}$

Taking logarithm of both the sides, we have $\log y = \log[f_1(x)] + \log[f_2(x)] - \log[g_1(x)] - \log[g_2(x)]$

Differentiating with respect to x, we get $\frac{1}{y}\frac{dy}{dx} = \frac{f_1'(x)}{f_1(x)} + \frac{f_2'(x)}{f_2(x)} - \frac{g_1'(x)}{g_2(x)} - \frac{g_2'(x)}{g_2(x)}$

$$\frac{dy}{dx} = y\left[\frac{f_1'(x)}{f_1(x)} + \frac{f_2'(x)}{f_2(x)} - \frac{g_1'(x)}{g_1(x)} - \frac{g_2'(x)}{g_2(x)}\right] = \frac{f_1(x).f_2(x)}{g_1(x).g_2(x)}\left[\frac{f_1'(x)}{f_1(x)} + \frac{f_2'(x)}{f_2(x)} - \frac{g_1'(x)}{g_1(x)} - \frac{g_2'(x)}{g_2(x)}\right]$$

Working rule : (a) To take logarithm of the function (b) To differentiate the function

Example: 24 If $x^m y^n = 2(x+y)^{m+n}$, the value of $\frac{dy}{dx}$ is **[MP PET 2003]**

(a) $x+y$ (b) $\frac{x}{y}$ (c) $\frac{y}{x}$ (d) $x-y$

Solution: (c) $x^m y^n = 2(x+y)^{m+n} \Rightarrow m\log x + n\log y = \log 2 + (m+n)\log(x+y)$

Differentiating *w.r.t.* x both sides

$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y}\left[1 + \frac{dy}{dx}\right] \Rightarrow \frac{dy}{dx} = \frac{y}{x}$.

Example: 25 If $y = (\sin x)^{\tan x}$, then $\frac{dy}{dx}$ is equal to **[IIT 1994; Rajasthan PET 1996]**

(a) $(\sin x)^{\tan x}.(1 + \sec^2 x.\log\sin x)$ (b) $\tan x.(\sin x)^{\tan x - 1}.\cos x$

(c) $(\sin x)^{\tan x}.,\sec^2 x\log\sin x$ (d) $\tan x.(\sin x)^{\tan x - 1}$

Solution: (a) Given $y = (\sin x)^{\tan x}$

$\log y = \tan x.\log\sin x$

Differentiating *w.r.t.* x, $\frac{1}{y}.\frac{dy}{dx} = \tan x.\cot x + \log\sin x.\sec^2 x$

$\frac{dy}{dx} = (\sin x)^{\tan x}[1 + \log\sin x.\sec^2 x]$.

(3) **Differentiation of parametric functions :** Sometimes x and y are given as functions of a single variable, *e.g.*, $x = \phi(t)$, $y = \psi(t)$ are two functions and t is a variable. In such a case x and y are called parametric functions or parametric equations and t is called the parameter. To find $\frac{dy}{dx}$ in case of parametric functions, we first obtain the relationship between x and y by eliminating the parameter t and then we differentiate it with respect to x. But every time it is

not convenient to eliminate the parameter. Therefore $\frac{dy}{dx}$ can also be obtained by the following formula $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

To prove it, let Δx and Δy be the changes in x and y respectively corresponding to a small change Δt in t.

Since $\frac{\Delta y}{\Delta x} = \frac{\Delta y / \Delta t}{\Delta x / \Delta t}$, $\quad \therefore \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{\lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}}{\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\Psi'(t)}{\phi'(t)}$

Example: 26 If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, $\frac{dy}{dx} =$ **[DCE 1999]**

(a) $\cos\theta$ (b) $\tan\theta$ (c) $\sec\theta$ (d) cosecθ

Solution: (b) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a[\cos\theta - \theta(-\sin\theta) - \cos\theta]}{a[-\sin\theta + \theta\cos\theta + \sin\theta]} = \frac{\theta\sin\theta}{\theta\cos\theta} = \tan\theta$.

Example: 27 If $\cos x = \frac{1}{\sqrt{1+t^2}}$ and $\sin y = \frac{t}{\sqrt{1+t^2}}$, then $\frac{dy}{dx} =$ **[MP PET 1994]**

(a) – 1 (b) $\frac{1-t}{1+t^2}$ (c) $\frac{1}{1+t^2}$ (d) 1

Solution: (d) Obviously $x = \cos^{-1}\frac{1}{\sqrt{1+t^2}}$ and $y = \sin^{-1}\frac{t}{\sqrt{1+t^2}}$

$\Rightarrow x = \tan^{-1} t$ and $y = \tan^{-1} t \Rightarrow y = x \Rightarrow \frac{dy}{dx} = 1$.

Example: 28 If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$, then $\frac{dy}{dx} =$ **[Karnataka CET 2000]**

(a) $\frac{-y}{x}$ (b) $\frac{y}{x}$ (c) $\frac{-x}{y}$ (d) $\frac{x}{y}$

Solution: (c) $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$

Put $t = \tan\theta$ in both the equations, we get $x = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$ and $y = \frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$.

Differentiating both the equations, we get $\frac{dx}{d\theta} = -2\sin 2\theta$ and $\frac{dy}{d\theta} = 2\cos 2\theta$.

Therefore $\frac{dy}{dx} = -\frac{\cos 2\theta}{\sin 2\theta} = -\frac{x}{y}$.

(4) **Differentiation of infinite series :** If y is given in the form of infinite series of x and we have to find out $\frac{dy}{dx}$ then we remove one or more terms, it does not affect the series

(i) If $y = \sqrt{f(x) + \sqrt{f(x) + \sqrt{f(x) + ...\infty}}}$, then $y = \sqrt{f(x) + y} \Rightarrow y^2 = f(x) + y$

$$2y\frac{dy}{dx} = f'(x) + \frac{dy}{dx}, \quad \therefore \frac{dy}{dx} = \frac{f'(x)}{2y-1}$$

(ii) If $y = f(x)^{f(x)^{f(x)^{f(x)\ldots\infty}}}$ then $y = f(x)^y$

$\therefore \log y = y \log f(x)$

$$\frac{1}{y}\frac{dy}{dx} = \frac{y.f'(x)}{f(x)} + \log f(x).\frac{dy}{dx}, \quad \therefore \frac{dy}{dx} = \frac{y^2 f'(x)}{f(x)[1 - y\log f(x)]}$$

(iii) If $y = f(x) + \cfrac{1}{f(x) + \cfrac{1}{f(x) + \ldots\infty}}$ then $\dfrac{dy}{dx} = \dfrac{yf'(x)}{2y - f(x)}$

Example: 29 If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots \text{ to } \infty}}}$ then $\dfrac{dy}{dx} =$ **[Rajasthan PET 2002]**

(a) $\dfrac{x}{2y-1}$ (b) $\dfrac{2}{2y-1}$ (c) $\dfrac{-1}{2y-1}$ (d) $\dfrac{1}{2y-1}$

Solution: (d) $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots \text{ to } \infty}}} \Rightarrow y = \sqrt{x+y} \Rightarrow y^2 = x + y \Rightarrow 2y\dfrac{dy}{dx} = 1 + \dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx}(2y-1) = 1 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2y-1}$

Example: 30 If $y = x^{x^{x\ldots\infty}}$, then $x(1 - y\log_e x)\dfrac{dy}{dx}$ is **[DCE 2000]**

(a) x^2 (b) y^2 (c) xy^2 (d) None of these

Solution: (b) $y = x^{x^{x\ldots\infty}} \Rightarrow y = x^y \Rightarrow \log_e y = y\log_e x \Rightarrow \dfrac{1}{y}\cdot\dfrac{dy}{dx} = \dfrac{y}{x} + \log_e x\dfrac{dy}{dx} \Rightarrow \left(\dfrac{1}{y} - \log_e x\right)\dfrac{dy}{dx} = \dfrac{y}{x} \Rightarrow$

$x(1 - y\log_e x)\dfrac{dy}{dx} = y^2$

Example: 31 If $y = x^2 + \cfrac{1}{x^2 + \cfrac{1}{x^2 + \cfrac{1}{x^2 + \ldots\ldots\infty}}}$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{2xy}{2y - x^2}$ (b) $\dfrac{xy}{y + x^2}$ (c) $\dfrac{xy}{y - x^2}$ (d) $\dfrac{2x}{2 + \dfrac{x^2}{y}}$

Solution: (a) $y = x^2 + \dfrac{1}{y} \Rightarrow y^2 = x^2 y + 1 \Rightarrow 2y\dfrac{dy}{dx} = y.2x + x^2\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{2xy}{2y - x^2}$

Example: 32 If $x = e^{y + e^{y + \ldots\ldots \text{to } \infty}}$, then $\dfrac{dy}{dx}$ is

(a) $\dfrac{1+x}{x}$ (b) $\dfrac{1}{x}$ (c) $\dfrac{1-x}{x}$ (d) $\dfrac{x}{1+x}$

Solution: (c) $x = e^{y+x}$

Taking log both sides, $\log x = (y + x)\log e = y + x \Rightarrow y + x = \log x \Rightarrow \dfrac{dy}{dx} + 1 = \dfrac{1}{x} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x} - 1 = \dfrac{1-x}{x}$

(5) **Differentiation of composite function :** Suppose function is given in form of $fog(x)$ or $f[g(x)]$

Working rule : Differentiate applying chain rule $\frac{d}{dx}f[g(x)] = f'[g(x)].g'(x)$

Example: 33 If $f(x) = |x-2|$ and $g(x) = f(f(x))$, then for $x>20$, $g'(x)$ equals

(a) –1 (b) 1 (c) 0 (d) None of these

Solution: (b) For $x > 20$, we have

$f(x) = |x-2| = x-2$ and, $g(x) = f(f(x)) = f(x-2) = x-2-2 = x-4$

$\therefore\ g'(x) = 1$

Example: 34 If g is inverse of f and $f'(x) = \frac{1}{1+x^n}$, then $g'(x)$ equals

(a) $1+x^n$ (b) $1+[f(x)]^n$ (c) $1+[g(x)]^n$ (d) None of these

Solution: (c) Since g is inverse of f. Therefore,

$fog(x) = x$ for all $x \Rightarrow \frac{d}{dx}\{fog(x)\} = 1$ for all x

$\Rightarrow f'(g(x)).g'(x) = 1 \Rightarrow f'\{g(x)\} = \frac{1}{g'(x)} \Rightarrow \frac{1}{1+[g(x)]^n} = \frac{1}{g'(x)}$ $\left[\because f'(x) = \frac{1}{1+x^n}\right]$

$\Rightarrow g'(x) = 1+[g(x)]^n$

3.6 Differentiation of a Function with Respect to Another Function

In this section we will discuss derivative of a function with respect to another function. Let $u = f(x)$ and $v = g(x)$ be two functions of x. Then, to find the derivative of $f(x)$ *w.r.t.* $g(x)$ i.e., to find $\frac{du}{dv}$ we use the following formula $\frac{du}{dv} = \frac{du/dx}{dv/dx}$

Thus, to find the derivative of $f(x)$ *w.r.t.* $g(x)$ we first differentiate both *w.r.t.* x and then divide the derivative of $f(x)$ *w.r.t.* x by the derivative of $g(x)$ *w.r.t.* x.

Example: 35 The differential coefficient of $\tan^{-1}\frac{2x}{1-x^2}$ *w.r.t.* $\sin^{-1}\frac{2x}{1+x^2}$ is

[Roorkee 1966; BIT Mesra 1996; Karnataka CET 1994; MP PET 1999; UPSEAT 1999, 2001]

(a) 1 (b) –1 (c) 0 (d) None of these

Solution: (a) Let $y_1 = \tan^{-1}\frac{2x}{1-x^2}$ and $y_2 = \sin^{-1}\frac{2x}{1+x^2}$

Putting $x = \tan\theta$

$\therefore\ y_1 = \tan^{-1}\tan 2\theta = 2\theta = 2\tan^{-1}x$ and $y_2 = \sin^{-1}\sin 2\theta = 2\tan^{-1}x$

Again $\frac{dy_1}{dx} = \frac{d}{dx}[2\tan^{-1}x] = \frac{2}{1+x^2}$ ....(i)

and $\frac{dy_2}{dx} = \frac{d}{dx}[2\tan^{-1}x] = \frac{2}{1+x^2}$ ....(ii)

Hence $\frac{dy_1}{dy_2} = 1$

Example: 36 The first derivative of the function $\left[\cos^{-1}\left(\sin\frac{\sqrt{1+x}}{2}\right) + x^x\right]$ with respect to x at $x = 1$ is

(a) $\frac{3}{4}$ (b) 0 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

Solution: (a) $f(x) = \cos^{-1}\left[\cos\left(\frac{\pi}{2} - \sqrt{\frac{1+x}{2}}\right)\right] + x^x = \frac{\pi}{2} - \sqrt{\frac{1+x}{2}} + x^x$

$\therefore\ f'(x) = -\frac{1}{\sqrt{2}} \cdot \frac{1}{2\sqrt{1+x}} + x^x(1+\log x) \Rightarrow f'(1) = -\frac{1}{4} + 1 = \frac{3}{4}$

3.7 Successive Differentiation or Higher Order Derivatives

(1) **Definition and notation :** If y is a function of x and is differentiable with respect to x, then its derivative $\frac{dy}{dx}$ can be found which is known as derivative of first order. If the first derivative $\frac{dy}{dx}$ is also a differentiable. function, then it can be further differentiated with respect to x and this derivative is denoted by d^2y/dx^2 which is called the second derivative of y with respect to x further if $\frac{d^2y}{dx^2}$ is also differentiable then its derivative is called third derivative of y which is denoted by $\frac{d^3y}{dx^3}$. Similarly n^{th} derivative of y is denoted by $\frac{d^ny}{dx^n}$. All these derivatives are called as successive derivative and this process is known as successive differentiation. We also use the following symbols for the successive derivatives of $y = f(x)$:

$y_1,\ y_2,\ y_{3,}.........,y_n,......$ $\qquad$ $y^I,\ y^{II},\ y^{III}.........,y^n,......$

$Dy,\ \ D^2y,\ \ D^3y.........,D^ny,......$ (where $D = \frac{d}{dx}$) $\qquad$ $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3},......., \frac{d^ny}{dx^n},..........$

$f'(x),\ \ f''(x),\ \ f'''(x),........,f^n(x),......$

If $y = f(x)$, then the value of the n^{th} order derivative at $x = a$ is usually denoted by

$\left(\frac{d^ny}{dx^n}\right)_{x=a}$ or $(y_n)_{x=a}$ or $(y^n)_{x=a}$ or $f^n(a)$

(2) n^{th} Derivatives of some standard functions :

(i) (a) $\frac{d^n}{dx^n}\sin(ax+b) = a^n \sin\left(\frac{n\pi}{2} + ax + b\right)$ $\qquad$ (b) $\frac{d^n}{dx^n}\cos(ax+b) = a^n \cos\left(\frac{n\pi}{2} + ax + b\right)$

(ii) $\frac{d^n}{dx^n}(ax+b)^m = \frac{m!}{(m-n)!}a^n(ax+b)^{m-n}$, where $m > n$

Particular cases :

(i) (a) When $m = n$

$D^n\{(ax+b)^n\} = a^n.n!$

(b) When $m < n, D^n\{(ax+b)^m\} = 0$

(ii) When $a = 1, b = 0$, then $y = x^n$

$\therefore\ D^n(x^m) = m(m-1).......(m-n+1)x^{m-n} = \frac{m!}{(m-n)!}x^{m-n}$

(iii) When $a = 1$, $b = 0$ and $m = n$,

(iv) When $m = -1$, $y = \frac{1}{(ax+b)}$

then $y = x^n$

$\therefore D^n(x^n) = n!$

$D^n(y) = a^n(-1)(-2)(-3)........(-n)(ax+b)^{-1-n}$

$= a^n(-1)^n(1.2.3......n)(ax+b)^{-1-n} = \dfrac{a^n(-1)^n n!}{(ax+b)^{n+1}}$

(3) $\dfrac{d^n}{dx^n}\log(ax+b) = \dfrac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$

(4) $\dfrac{d^n}{dx^n}(e^{ax}) = a^n e^{ax}$

(5) $\dfrac{d^n(a^x)}{dx^n} = a^x(\log a)^n$

(6) (i) $\dfrac{d^n}{dx^n}e^{ax}\sin(bx+c) = r^n e^{ax}\sin(bx+c+n\phi)$

where $r = \sqrt{a^2+b^2}$; $\phi = \tan^{-1}\dfrac{b}{a}$,

$y = e^{ax}\sin(bx+c)$

(ii) $\dfrac{d^n}{dx^n}e^{ax}\cos(bx+c) = r^n e^{ax}\cos(bx+c+n\phi)$

Example: 37 If $y = \left(x+\sqrt{1+x^2}\right)^n$, then $(1+x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx}$ is **[AIEEE 2002]**

(a) n^2y (b) $-n^2y$ (c) $-y$ (d) $2x^2y$

Solution: (a) $y = (x+\sqrt{1+x^2})^n \Rightarrow \dfrac{dy}{dx} = n(x+\sqrt{1+x^2})^{n-1}\left(1+\dfrac{x}{\sqrt{1+x^2}}\right) \Rightarrow \dfrac{dy}{dx} = \dfrac{n(x+\sqrt{1+x^2})^n}{\sqrt{1+x^2}} \Rightarrow (\sqrt{1+x^2})\dfrac{dy}{dx} = n\left(x+\sqrt{1+x^2}\right)^n$

$\Rightarrow \dfrac{d^2y}{dx^2}.\sqrt{1+x^2} + \dfrac{dy}{dx}\left(\dfrac{x}{\sqrt{1+x^2}}\right) = n^2\left(x+\sqrt{1+x^2}\right)^{n-1}\left(1+\dfrac{x}{\sqrt{1+x^2}}\right)$

$\Rightarrow (1+x^2).\dfrac{d^2y}{dx^2} + x.\dfrac{dy}{dx} = n^2(x+\sqrt{1+x^2})^n \Rightarrow (1+x^2)\dfrac{d^2y}{dx^2} + x.\dfrac{dy}{dx} = n^2y$.

Example: 38 If $f(x) = x^n$, then the value of $f(1) - \dfrac{f'(1)}{1!} + \dfrac{f''(1)}{2!} - \dfrac{f'''(1)}{3!} + ... + \dfrac{(-1)^n f^n(1)}{n!}$ is **[AIEEE 2003]**

(a) 2^n (b) 2^{n-1} (c) 0 (d) 1

Solution: (c) $f(x) = x^n \Rightarrow f(1) = 1$, $f'(x) = nx^{n-1} \Rightarrow f'(1) = n$

$f''(x) = n(n-1)x^{n-2} \Rightarrow f''(1) = n(n-1)$ .....

$f^n(x) = n! \Rightarrow f^n(1) = n!$, $\therefore f(1) - \dfrac{f'(1)}{1!} + \dfrac{f''(1)}{2!} ...... + \dfrac{(-1)^n f^n(1)}{n!}$

$= 1 - \dfrac{n}{1!} + \dfrac{n(n-1)}{2!} - \dfrac{n(n-1)(n-2)}{3!} + ... + (-1)^n\dfrac{n!}{n!} = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + ... + (-1)^n\ {}^nC_n = 0$.

Example: 39 If $f(x) = \tan^{-1}\left\{\dfrac{\log\left(\dfrac{e}{x^2}\right)}{\log(ex^2)}\right\} + \tan^{-1}\left(\dfrac{3+2\log x}{1-6\log x}\right)$, then $\dfrac{d^ny}{dx^n}$ is $(n \ge 1)$

(a) $\tan^{-1}\{(\log x)^n\}$ (b) 0 (c) 1/2 (d) None of these

Solution: (b) We have $y = \tan^{-1}\left(\dfrac{\log e - \log x^2}{\log e + \log x^2}\right) + \tan^{-1}\left(\dfrac{3+2\log x}{1-6\log x}\right) = \tan^{-1}\left(\dfrac{1-2\log x}{1+2\log x}\right) + \tan^{-1}\left(\dfrac{3+2\log x}{1-6\log x}\right)$

$= \tan^{-1}1 - \tan^{-1}(2\log x) + \tan^{-1}3 + \tan^{-1}(2\log x) \Rightarrow y = \tan^{-1}1 + \tan^{-1}3 \Rightarrow \dfrac{dy}{dx} = 0 \Rightarrow \dfrac{d^ny}{dx^n} = 0$.

Example: 40 If $f(x) = (\cos x + i\sin x)(\cos 3x + i\sin 3x)..... (\cos(2n-1)x + i\sin(2n-1)x)$, then $f''(x)$ is equal to

(a) $n^2 f(x)$ (b) $-n^4 f(x)$ (c) $-n^2 f(x)$ (d) $n^4 f(x)$

Solution: (b) We have, $f(x) = \cos(x+3x+....+(2n-1)x) + i\sin(x+3x+5x+....+(2n-1)x) = \cos n^2x + i\sin n^2x$

$\Rightarrow f'(x) = -n^2(\sin n^2x) + n^2(i\cos n^2x) \Rightarrow f''(x) = -n^4\cos n^2x - n^4 i\sin n^2x$

$\Rightarrow f''(x) = -n^4(\cos n^2x + i\sin n^2x) \Rightarrow f''(x) = -n^4 f(x)$

3.8 n^{th} Derivative using Partial fractions

For finding n^{th} derivative of fractional expressions whose numerator and denominator are rational algebraic expression, firstly we resolve them into partial fractions and then we find n^{th} derivative by using the formula giving the n^{th} derivative of $\dfrac{1}{ax+b}$.

Example: 41 If $y = \dfrac{x^4}{x^2-3x+2}$, then for $n > 2$ the value of y_n is equal to

(a) $(-1)^n n![16(x-2)^{-n-1} - (x-1)^{-n-1}]$ (b) $(-1)^n n![16(x-2)^{-n-1} + (x-1)^{-n-1}]$

(c) $n![16(x-2)^{-n-1} + (x-1)^{-n-1}]$ (d) None of these

Solution: (a) $y = \dfrac{x^4}{x^2-3x+2} = x^2+3x+7+\dfrac{15x-14}{(x-1)(x-2)} = x^2+3x+7-\dfrac{1}{(x-1)}+\dfrac{16}{(x-2)}$

$\therefore y_n = D^n(x^2) + D^n(3x) + D^n(7) - D^n[(x-1)^{-1}] + 16D^n[(x-2)^{-1}]$

$= (-1)^n n![-(x-1)^{-n-1} + 16(x-2)^{-n-1}] = (-1)^n n![16(x-2)^{-n-1} - (x-1)^{-n-1}]$.

3.9 Differentiation of Determinants

Let $\Delta(x) = \begin{vmatrix} a_1(x) & b_1(x) \\ a_2(x) & b_2(x) \end{vmatrix}$. Then $\Delta'(x) = \begin{vmatrix} a_1'(x) & b_1'(x) \\ a_2(x) & b_2(x) \end{vmatrix} + \begin{vmatrix} a_1(x) & b_1(x) \\ a_2'(x) & b_2'(x) \end{vmatrix}$

If we write $\Delta(x) = |C_1 C_2 C_3|$. Then $\Delta'(x) = |C_1' C_2 C_3| + |C_1 C_2' C_3| + |C_1 C_2 C_3'|$

Similarly, if $\Delta(x) = \begin{vmatrix} R_1 \\ R_2 \\ R_3 \end{vmatrix}$, then $\Delta'(x) = \begin{vmatrix} R_1' \\ R_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2' \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2 \\ R_3' \end{vmatrix}$

Thus, to differentiate a determinant, we differentiate one row (or column) at a time, keeping others unchanged.

Example: 42 If $f_r(x), g_r(x), h_r(x), r = 1,2,3$ are polynomials in x such that $f_r(a) = g_r(a) = h_r(a), r = 1,2,3$ and

$F(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$, then find $F'(x)$ at $x = a$ **[IIT 1985]**

(a) 0 (b) $f_1(a)g_2(a)h_3(a)$ (c) 1 (d) None of these

Solution: (a) $F'(x) = \begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1'(x) & g_2'(x) & g_3'(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1'(x) & h_2'(x) & h_3'(x) \end{vmatrix}$

$$\therefore\ F'(a)=\begin{vmatrix} f_1'(a) & f_2'(a) & f_3'(a)\\ g_1(a) & g_2(a) & g_3(a)\\ h_1(a) & h_2(a) & h_3(a)\end{vmatrix}+\begin{vmatrix} f_1(a) & f_2(a) & f_3(a)\\ g_1'(a) & g_2'(a) & g_3'(a)\\ h_1(a) & h_2(a) & h_3(a)\end{vmatrix}+\begin{vmatrix} f_1(a) & f_2(a) & f_3(a)\\ g_1(a) & g_2(a) & g_3(a)\\ h_1'(a) & h_2'(a) & h_3'(a)\end{vmatrix}$$

$= 0+0+0=0$ $[\because f_r(a)=g_r(a)=h_r(a), r=1,2,3]$

Example: 43 Let $f(x)=\begin{vmatrix} x^3 & \sin x & \cos x\\ 6 & -1 & 0\\ 1 & p^2 & p^3\end{vmatrix}$ where p is a constant. Then $\frac{d^3}{dx^3}[f(x)]$ at $x=0$ is **[IIT 1997]**

(a) p (b) $p + p^2$ (c) $p + p^3$ (d) Independent of p

Solution: (d) Given $f(x)=\begin{vmatrix} x^3 & \sin x & \cos x\\ 6 & -1 & 0\\ 1 & p^2 & p^3\end{vmatrix}$, 2nd and 3rd rows are constant, so only 1st row will take part in differentiation

$$\therefore \frac{d^3}{dx^3}f(x)=\begin{vmatrix} \frac{d^3}{dx^3}x^3 & \frac{d^3}{dx^3}\sin x & \frac{d^3}{dx^3}\cos x\\ 6 & -1 & 0\\ 1 & p^2 & p^3\end{vmatrix}$$

We know that $\frac{d^n}{dx^n}x^n=n!, \frac{d^n}{dx^n}\sin x=\sin(x+\frac{n\pi}{2})$ and $\frac{d^n}{dx^n}\cos x=\cos(x+\frac{n\pi}{2})$

Using these results, $\frac{d^3}{dx^3}f(x)=\begin{vmatrix} 3! & \sin\left(x+\frac{3\pi}{2}\right) & \cos\left(x+\frac{3\pi}{2}\right)\\ 6 & -1 & 0\\ 1 & p^2 & p^3\end{vmatrix}$

$\left.\frac{d^3}{dx^3}f(x)\right|_{\text{at } x=0}=\begin{vmatrix} 6 & -1 & 0\\ 6 & -1 & 0\\ 1 & p^2 & p^3\end{vmatrix}$ = 0 *i.e.*, independent of p.

3.10 Differentiation of Integral Function

If $g_1(x)$ and $g_2(x)$ both functions are defined on $[a, b]$ and differentiable at a point $x\in(a,b)$ and $f(t)$ is continuous for $g_1(a)\le f(t)\le g_2(b)$

Then $\frac{d}{dx}\int_{g_1(x)}^{g_2(x)} f(t)dt = f[g_2(x)]g_2'(x)-f[g_1(x)]g_1'(x) = f[g_2(x)]\frac{d}{dx}g_2(x)-f[g_1(x)]\frac{d}{dx}g_1(x)$.

Example: 44 If $F(x)=\int_{x^2}^{x^3}\log t\,dt\,(x>0)$, then $F'(x)=$ **[MP PET 2001]**

(a) $(9x^2-4x)\log x$ (b) $(4x-9x^2)\log x$ (c) $(9x^2+4x)\log x$ (d) None of these

Solution: (a) Applying formula we get $F'(x)=(\log x^3)3x^2-(\log x^2)2x$

$=(3\log x)3x^2-2x(2\log x) = 9x^2\log x-4x\log x = (9x^2-4x)\log x$.

Example: 45 If $x=\int_0^y \frac{1}{\sqrt{1+4t^2}}dt$, then $\frac{d^2y}{dx^2}$ is

(a) $2y$ (b) $4y$ (c) $8y$ (d) $6\ y$

Solution: (b) $x = \int_0^y \frac{1}{\sqrt{1+4t^2}} dt \Rightarrow \frac{dx}{dy} = \frac{1}{\sqrt{1+4y^2}} \Rightarrow \frac{dy}{dx} = \sqrt{1+4y^2} \Rightarrow \frac{d^2y}{dx^2} = \frac{4y}{\sqrt{1+4y^2}}\frac{dy}{dx} \Rightarrow \frac{d^2y}{dx^2} = \frac{4y}{\sqrt{1+4y^2}} \cdot \sqrt{1+4y^2} = 4y$

3.11 Leibnitz's Theorem

G.W. Leibnitz, a German mathematician gave a method for evaluating the nth differential coefficient of the product of two functions. This method is known as Leibnitz's theorem.

Statement of the theorem – If u and v are two functions of x such that their nth derivative exist then $D^n(u.v.) = {}^nC_0(D^n u)v + {}^nC_1 D^{n-1}u.Dv + {}^nC_2 D^{n-2}u.D^2v + ... + {}^nC_r D^{n-r}u.D^r v + ... + u.(D^n v)$.

Note : ❒ The success in finding the nth derivative by this theorem lies in the proper selection of first and second function. Here first function should be selected whose nth derivative can be found by standard formulae. Second function should be such that on successive differentiation, at some stage, it becomes zero so that we need not to write further terms.

Example: 46 If $y = x^2 e^x$, then value of y_n is

(a) $\{x^2 - 2nx + n(n-1)\}e^x$ (b) $\{x^2 + 2nx + n(n-1)\}e^x$

(c) $\{x^2 + 2nx - n(n-1)\}e^x$ (d) None of these

Solution: (b) Applying Leibnitz's theorem by taking x^2 as second function. We get , $D^n y = D^n(e^x.x^2)$

$$= {}^nC_0 D^n(e^x)x^2 + {}^nC_1 D^{n-1}(e^x).D(x^2) + {}^nC_2 D^{n-2}(e^x).D^2(x^2) + ... = e^x.x^2 + ne^x.2x + \frac{n(n-1)}{2!}e^x.2 + 0 + 0 + ...$$

$y_n = \{x^2 + 2nx + n(n-1)\}e^x$.

Example: 47 If $y = x^2 \log x$, then value of y_n is

(a) $\frac{(-1)^{n-1}(n-3)!}{x^{n-2}}$ (b) $\frac{(-1)^{n-1}(n-3)!}{x^{n-2}}.2$ (c) $\frac{(-1)^{n-1}(n-2)!}{x^{n-2}}$ (d) None of these

Solution: (b) Applying Leibnitz's theorem by taking x^2 as second function, we get, $D^n y = D^n(\log x.x^2)$

$$= {}^nC_0 D^n(\log x).x^2 + {}^nC_1 D^{n-1}(\log x).D(x^2) + {}^nC_2 D^{n-2}(\log x)D^2(x^2) + ...$$

$$= \frac{(-1)^{n-1}(n-1)!}{x^n}.x^2 + n.\frac{(-1)^{n-2}(n-2)!}{x^{n-1}}.2x + \frac{n(n-1)}{2!}\frac{(-1)^{n-3}(n-3)!}{x^{n-2}}.2 + 0 + 0.........$$

$$= \frac{(-1)^{n-1}(n-1)!}{x^{n-2}} + \frac{2n(-1)^{n-2}(n-2)!}{x^{n-2}} + \frac{n(n-1)(-1)^{n-3}(n-3)!}{x^{n-2}}$$

$$= \frac{(-1)^{n-1}(n-3)!}{x^{n-2}} \times \{(n-1)(n-2) - 2n(n-2) + n(n-1)\} = . \frac{(-1)^{n-1}(n-3)!}{x^{n-2}}.2$$

Assignment

Derivative at a Point

Basic Level

1. If $f(x) = |x|$, then $f'(0) =$ **[MNR 1982]**

(a) 0 (b) 1 (c) x (d) None of these

2. If $f(x) = \begin{cases} 1, & x < 0 \\ 1 + \sin x, & 0 \le x < \frac{\pi}{2} \end{cases}$ then $f'(0) =$ **[MP PET 1994]**

(a) 1 (b) 0 (c) ∞ (d) Does not exist

3. If $f(x) = \begin{cases} ax^2 + b; & x \le 0 \\ x^2; & x > 0 \end{cases}$ possesses derivative at $x = 0$, then

(a) $a = 0, b = 0$ (b) $a > 0, b = 0$ (c) $a \in R, b = 0$ (d) None of these

4. The derivative of $f(x) = 3|2 + x|$ at the point $x_0 = -3$ is **[Orissa JEE 2002]**

(a) 3 (b) – 3 (c) 0 (d) Does not exist

5. The derivative of $y = 1 - |x|$ at $x = 0$ is **[SCRA 1996]**

(a) 0 (b) 1 (c) – 1 (d) Does not exist

6. The derivative of $f(x) = |x^2 - x|$ at $x = 2$ is **[AMU 1999]**

(a) – 3 (b) 0 (c) 3 (d) Not defined

7. The value of $\frac{d}{dx}[|x-1| + |x-5|]$ at $x = 3$ is **[MP PET 2000]**

(a) – 2 (b) 0 (c) 2 (d) 4

8. If $f(x)$ has a derivative at $x = a$, then $\lim_{x \to a} \frac{xf(a) - af(x)}{x - a}$ is equal to

(a) $f(a) - af'(a)$ (b) $af(a) - f'(a)$ (c) $f(a) + f'(a)$ (d) $af(a) + f'(a)$

9. If $f(x) = x + 2$, then $f'(f(x))$ at $x = 4$ is **[DCE 2001]**

(a) 8 (b) 1 (c) 4 (d) 5

10. Let $3f(x) - 2f(1/x) = x$, then $f'(2)$ is equal to **[MP PET 2000]**

(a) 2/7 (b) 1/2 (c) 2 (d) 7/2

11. If $f(x)$ is a differentiable function, then $\lim_{x \to a} \frac{af(x) - xf(a)}{x - a}$ is

(a) $af'(a) - f(a)$ (b) $af(a) - f'(a)$ (c) $af'(a) + f(a)$ (d) $af(a) + f'(a)$

12. The differential coefficient of the function $|x - 1| + |x - 3|$ at the point $x = 2$ is **[Rajasthan PET 2002]**

(a) – 2 (b) 0 (c) 2 (d) Undefined

13. If $f(x) = |x - 3|$, then $f'(3) =$

(a) 0 (b) 1 (c) –1 (d) Does not exist

Advance Level

14. If $y = \cot^{-1}(\cos 2x)^{1/2}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ will be

(a) $\left(\frac{2}{3}\right)^{1/2}$ (b) $\left(\frac{1}{3}\right)^{1/2}$ (c) $(3)^{1/2}$ (d) $(6)^{1/2}$

15. The values of x, at which the first derivative of the function $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ *w.r.t.* x is $\frac{3}{4}$, are

(a) ± 2 (b) $\pm\frac{1}{2}$ (c) $\pm\frac{\sqrt{3}}{2}$ (d) $\pm\frac{2}{\sqrt{3}}$

16. The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval (0, 2), is **[MNR 1995]**

(a) 1 (b) 2 (c) 3 (d) 4

17. The set of all those points, where the function $f(x) = \frac{x}{1 + |x|}$ is differentiable, is

(a) $(-\infty, \infty)$ (b) $[0, \infty)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(0, \infty)$

18. Let $f(x+y) = f(x)f(y)$ and $f(x) = 1 + xg(x)G(x)$ where $\lim_{x \to 0} g(x) = a$ and $\lim_{x \to 0} G(x) = b$ then $f'(x)$ is equal to

(a) 1+ ab (b) ab (c) a/b (d) None of these

19. $f(x)$ is a function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$ and $h(x)$ is a function such that $h(x) = [f(x)]^2 + [g(x)]^2$ and $h(5) = 11$, then the value of $h(10)$ is

(a) 0 (b) 1 (c) 10 (d) None of these

20. Let $f(x+y) = f(x)f(y)$ for all x and y. Suppose that $f(3) = 3$ and $f'(0) = 11$, then $f'(3)$ is given by

(a) 22 (b) 33 (c) 28 (d) None of these

Some Standard Differentiation

Basic Level

21. If $y = \frac{(1-x)^2}{x^2}$, then $\frac{dy}{dx}$ is **[MP PET 1999]**

(a) $\frac{2}{x^2} + \frac{2}{x^3}$ (b) $-\frac{2}{x^2} + \frac{2}{x^3}$ (c) $-\frac{2}{x^2} - \frac{2}{x^3}$ (d) $-\frac{2}{x^3} + \frac{2}{x^2}$

22. If $2t = v^2$, then $\frac{dv}{dt}$ is equal to **[MP PET 1992]**

(a) 0 (b) 1/4 (c) 1/2 (d) $1/v$

23. If $x = y\sqrt{1-y^2}$, then $\frac{dy}{dx} =$ **[MP PET 2001]**

(a) 0 (b) x (c) $\frac{\sqrt{1-y^2}}{1-2y^2}$ (d) $\frac{\sqrt{1-y^2}}{1+2y^2}$

24. If $pv = 81$, then $\frac{dp}{dv}$ is at $v = 9$ equal to **[MP PET 1999]**

(a) 1 (b) –1 (c) 2 (d) None of these

25. If $y = \sqrt{\frac{1+x}{1-x}}$, then $\frac{dy}{dx} =$ **[AISSE 1981; Rajasthan PET 1995]**

(a) $\frac{2}{(1+x)^{1/2}(1-x)^{3/2}}$ (b) $\frac{1}{(1+x)^{1/2}(1-x)^{3/2}}$ (c) $\frac{1}{2(1+x)^{1/2}(1-x)^{3/2}}$ (d) $\frac{2}{(1+x)^{3/2}(1-x)^{1/2}}$

26. The derivative of $f(x) = x|x|$ is

(a) $2x$ (b) $-2x$ (c) $2x^2$ (d) $2|x|$

27. The derivative of $F[f\{\phi(x)\}]$ is **[AMU 2001]**

(a) $F'[f\{\phi(x)\}]$ (b) $F'[f\{\phi(x)\}]f'\{\phi(x)\}$ (c) $F'[f\{\phi(x)\}]f'\{\phi(x)\}$ (d) $F'[f\{\phi(x)\}]f'\{\phi(x)\}\phi'(x)$

28. $\frac{d}{dx}(\sin 2x^2)$ equals **[Rajasthan PET 1996]**

(a) $4x\cos(2x^2)$ (b) $2\sin x^2.\cos x^2$ (c) $4x\sin(x^2)$ (d) $4x\sin(x^2).\cos(x^2)$

29. If $y = \sec x^0$, then $\frac{dy}{dx} =$ **[MP PET 1997]**

(a) $\sec x \tan x$ (b) $\sec x^0 \tan x^0$ (c) $\frac{\pi}{180}\sec x^0 \tan x^0$ (d) $\frac{180}{\pi}\sec x^0 \tan x^0$

30. If $\sin y + e^{-x\cos y} = e$, then $\frac{dy}{dx}$ at $(1, \pi)$ is **[Kerala (Engg.) 2002]**

(a) $\sin y$ (b) $-x\cos y$ (c) e (d) $\sin y - x\cos y$

31. If $y = a\sin x + b\cos x$, then $y^2 + \left(\frac{dy}{dx}\right)^2$ is a

(a) Function of x (b) Function of y (c) Function of x and y (d) Constant

32. $\frac{d}{dx}[\cos(1-x^2)^2] =$ **[AISSE 1981; AI CBSE 1979]**

(a) $-2x(1-x^2)\sin(1-x^2)^2$ (b) $-4x(1-x^2)\sin(1-x^2)^2$ (c) $4x(1-x^2)\sin(1-x^2)^2$ (d) $-2(1-x^2)\sin(1-x^2)^2$

33. If $y = \cos(\sin x^2)$, then at $x = \sqrt{\frac{\pi}{2}}, \frac{dy}{dx} =$

(a) -2 (b) 2 (c) $-2\sqrt{\frac{\pi}{2}}$ (d) 0

34. $\frac{d}{dx}[\sin^n x \cos nx] =$

(a) $n\sin^{n-1}x\cos(n+1)x$ (b) $n\sin^{n-1}x\cos nx$ (c) $n\sin^{n-1}x\cos(n-1)x$ (d) $n\sin^{n-1}x\sin(n+1)x$

35. $\frac{d}{dx}\cos(\sin x^2) =$ **[DSSE 1979]**

(a) $\sin(\sin x^2).\cos x^2.2x$ (b) $-\sin(\sin x^2).\cos x^2.2x$ (c) $-\sin(\sin x^2).\cos^2 x.2x$ (d) None of these

36. If $y = \sin(\sqrt{\sin x + \cos x})$, then $\frac{dy}{dx} =$ **[DSSE 1987]**

(a) $\frac{1}{2}\frac{\cos\sqrt{\sin x + \cos x}}{\sqrt{\sin x + \cos x}}$ (b) $\frac{\cos\sqrt{\sin x + \cos x}}{\sqrt{\sin x + \cos x}}$

(c) $\frac{1}{2}\frac{\cos\sqrt{\sin x + \cos x}}{\sqrt{\sin x + \cos x}}(\cos x - \sin x)$ (d) None of these

37. If $y = \sin\left(\frac{1+x^2}{1-x^2}\right)$, then $\frac{dy}{dx} =$ **[AISSE 1987]**

(a) $\frac{4x}{1-x^2}.\cos\left(\frac{1+x^2}{1-x^2}\right)$ (b) $\frac{x}{(1-x^2)^2}.\cos\left(\frac{1+x^2}{1-x^2}\right)$ (c) $\frac{x}{(1-x^2)}.\cos\left(\frac{1+x^2}{1-x^2}\right)$ (d) $\frac{4x}{(1-x^2)^2}.\cos\left(\frac{1+x^2}{1-x^2}\right)$

38. $\frac{d}{dx}(x^2+\cos x)^4 =$ **[DSSE 1987]**

(a) $4(x^2+\cos x)(2x-\sin x)$ (b) $4(x^2-\cos x)(2x-\sin x)$ (c) $4(x^2+\cos x)^3(2x-\sin x)$ (d) $4(x^2+\cos x)^3(2x+\sin x)$

39. $\frac{d}{dx}\left(\frac{\cot^2 x-1}{\cot^2 x+1}\right) =$

(a) – sin 2x (b) 2 sin 2x (c) 2 cos 2x (d) – 2 sin 2x

40. $\frac{d}{dx}\sqrt{\frac{1-\sin 2x}{1+\sin 2x}} =$ **[AISSE 1985; DSSE 1986]**

(a) $\sec^2 x$ (b) $-\sec^2\left(\frac{\pi}{4}-x\right)$ (c) $\sec^2\left(\frac{\pi}{4}+x\right)$ (d) $\sec^2\left(\frac{\pi}{4}-x\right)$

41. If $y=\frac{\tan x+\cot x}{\tan x-\cot x}$, then $\frac{dy}{dx} =$

(a) 2 tan 2x sec 2x (b) tan 2x sec 2x (c) – tan 2x sec 2x (d) – 2 tan 2x sec 2x

42. $\frac{d}{dx}\sqrt{\sec^2 x+\cos ec^2 x} =$ **[DSSE 1981]**

(a) 4 cosec 2x . cot 2x (b) – 4 cosec 2x . cot 2x (c) – 4 cosec x . cot 2x (d) None of these

43. If $y=\frac{5x}{\sqrt[3]{(1-x)^2}}+\cos^2(2x+1)$, then $\frac{dy}{dx} =$ **[IIT 1980]**

(a) $\frac{5(3-x)}{3(1-x)^{5/3}}-2\sin(4x+2)$ (b) $\frac{5(3-x)}{3(1-x)^{2/3}}-2\sin(4x+4)$ (c) $\frac{5(3-x)}{3(1-x)^{2/3}}-2\sin(2x+1)$ (d) None of these

44. If $y=\sqrt{\sin x+y}$, then $\frac{dy}{dx}$ equals to **[Rajasthan PET 2001]**

(a) $\frac{\sin x}{2y-1}$ (b) $\frac{\cos x}{2y-1}$ (c) $\frac{\sin x}{2y+1}$ (d) $\frac{\cos x}{2y+1}$

45. $\frac{d}{dx}\log|x| = ...(x \ne 0)$

(a) $\frac{1}{x}$ (b) $-\frac{1}{x}$ (c) x (d) – x

46. $\frac{d}{dx}\log_{\sqrt{x}}(1/x)$ is equal to **[AMU 1999]**

(a) $-\frac{1}{2\sqrt{x}}$ (b) – 2 (c) $-\frac{1}{x^2\sqrt{x}}$ (d) 0

47. $\frac{d}{dx}\log(\log x) =$ **[IIT 1985]**

(a) $\frac{x}{\log x}$ (b) $\frac{\log x}{x}$ (c) $(x\log x)^{-1}$ (d) None of these

48. $\frac{d}{dx}(\log\tan x) =$ **[MNR 1986]**

(a) 2 sec 2x (b) 2 cosec 2x (c) sec 2x (d) cosec 2x

49. If $y=\log x^x$, then $\frac{dy}{dx} =$ **[MNR 1978]**

(a) $x^x(1+\log x)$ (b) log (ex) (c) $\log\left(\frac{e}{x}\right)$ (d) None of these

50. Derivative of the function $f(x)=\log_5(\log_7 x), x>7$ is **[Orissa JEE 2002]**

(a) $\frac{1}{x(\ln 5)(\ln 7)(\log_7 x)}$ (b) $\frac{1}{x(\ln 5)(\ln 7)}$ (c) $\frac{1}{x(\ln x)}$ (d) None of these

51. The differential coefficient of $f[\log(x)]$ when $f(x)=\log x$ is **[Kurukshetra CEE 1998; DCE 2000]**

(a) $x\log x$ (b) $\frac{x}{\log x}$ (c) $\frac{1}{x\log x}$ (d) $\frac{\log x}{x}$

52. If $y=\log\frac{1+\sqrt{x}}{1-\sqrt{x}}$, then $\frac{dy}{dx}=$

(a) $\frac{\sqrt{x}}{1-x}$ (b) $\frac{1}{\sqrt{x}(1-x)}$ (c) $\frac{\sqrt{x}}{1+x}$ (d) $\frac{1}{\sqrt{x}(1+x)}$

53. If $y=x^2\log x+\frac{2}{\sqrt{x}}$, then $\frac{dy}{dx}=$

(a) $x+2x\log x-\frac{1}{\sqrt{x}}$ (b) $x+2x\log x-\frac{1}{x^{3/2}}$ (c) $x+2x\log x-\frac{2}{x^{3/2}}$ (d) None of these

54. $\frac{d}{dx}\left[\log\sqrt{\frac{1-\cos x}{1+\cos x}}\right]=$ **[BIT Ranchi 1990]**

(a) sec x (b) cosec x (c) cosec $\frac{x}{2}$ (d) $\sec\frac{x}{2}$

55. $\frac{d}{dx}\left\{\log\left(\frac{e^x}{1+e^x}\right)\right\}=$

(a) $\frac{1}{1-e^x}$ (b) $-\frac{1}{1+e^x}$ (c) $-\frac{1}{1-e^x}$ (d) None of these

56. $\frac{d}{dx}\{\log(\sec x+\tan x)\}=$ **[AISSE 1982]**

(a) cos x (b) sec x (c) tan x (d) cot x

57. $\frac{d}{dx}\left[\log\left(x+\frac{1}{x}\right)\right]=$ **[MP PET 1995]**

(a) $\left(x+\frac{1}{x}\right)$ (b) $\frac{\left(1+\frac{1}{x^2}\right)}{\left(1+\frac{1}{x}\right)}$ (c) $\frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)}$ (d) $\left(1+\frac{1}{x}\right)$

58. $\frac{d}{dx}\log(x^{10})=$ **[Rajasthan PET 1992]**

(a) x^{-10} (b) $10x$ (c) $10/x$ (d) $10x^9$

59. If $y=\log\left\{\frac{x+\sqrt{(a^2+x^2)}}{a}\right\}$, then the value of $\frac{dy}{dx}$ is

(a) $\sqrt{a^2-x^2}$ (b) $a\sqrt{a^2+x^2}$ (c) $\frac{1}{\sqrt{a^2+x^2}}$ (d) $x\sqrt{a^2+x^2}$

60. If $y=\log(\sin^{-1}x)$, then $\frac{dy}{dx}$ equals

(a) $\frac{1}{\sin^{-1}x\sqrt{1-x^2}}$ (b) $-\frac{1}{\sin^{-1}x\sqrt{1-x^2}}$ (c) $\frac{-2x}{\sin^{-1}x\sqrt{1-x^2}}$ (d) None of these

61. If $y=e^{(1+\log_e x)}$, then the value of $\frac{dy}{dx}=$ **[MP PET 1996]**

(a) e (b) 1 (c) 0 (d) $\log_e xe^{\log_e ex}$

62. If $y=e^{\sqrt{x}}$, then $\frac{dy}{dx}$ equals

(a) $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$ (b) $\frac{\sqrt{x}}{e^{\sqrt{x}}}$ (c) $\frac{x}{e^{\sqrt{x}}}$ (d) $\frac{2\sqrt{x}}{e^{\sqrt{x}}}$

63. The derivative of $y=x^{\ln x}$ is

(a) $x^{\ln x}\ln x$ (b) $x^{\ln x-1}\ln x$ (c) $2x^{\ln x-1}\ln x$ (d) $x^{\ln x-2}$

64. Derivative of x^6+6^x with respect to x is **[Kerala (Engg.) 2002]**

(a) $12x$ (b) $x+4$ (c) $6x^5+6^x\log 6$ (d) $6x^5+x6^{x-1}$

65. $\frac{d}{dx}(e^x\log\sin 2x)=$ **[AI CBSE 1985]**

(a) $e^x(\log\sin 2x+2\cot 2x)$ (b) $e^x(\log\cos 2x+2\cot 2x)$ (c) $e^x(\log\cos 2x+\cot 2x)$ (d) None of these

66. If $y=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+......\infty$, then $\frac{dy}{dx}=$ **[Karnataka CET 1999]**

(a) y (b) $y-1$ (c) $y+1$ (d) None of these

67. $\frac{d}{dx}e^{x\sin x}=$ **[DSSE 1979]**

(a) $e^{x\sin x}(x\cos x+\sin x)$ (b) $e^{x\sin x}(\cos x+x\sin x)$ (c) $e^{x\sin x}(\cos x+\sin x)$ (d) None of these

68. $\frac{d}{dx}(xe^{x^2})=$ **[DSSE 1981]**

(a) $2x^2e^{x^2}+e^{x^2}$ (b) $x^2e^{x^2}+e^{x^2}$ (c) $e^x.2x^2+e^{x^2}$ (d) None of these

69. If $y=x^2+x^{\log x}$, then $\frac{dy}{dx}=$

(a) $\frac{x^2+\log x.x^{\log x}}{x}$ (b) $x^2+\log x.x^{\log x}$ (c) $\frac{2(x^2+\log x.x^{\log x})}{x}$ (d) None of these

70. $\frac{d}{dx}\{e^{-ax^2}\log(\sin x)\}=$ **[AI CBSE 1984]**

(a) $e^{-ax^2}(\cot x+2ax\log\sin x)$ (b) $e^{-ax^2}(\cot x+ax\log\sin x)$

(c) $e^{-ax^2}(\cot x-2ax\log\sin x)$ (d) None of these

71. If $y=\sqrt{\frac{1+e^x}{1-e^x}}$, then $\frac{dy}{dx}=$ **[AI CBSE 1987]**

(a) $\frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$ (b) $\frac{e^x}{(1-e^x)\sqrt{1-e^x}}$ (c) $\frac{e^x}{(1-e^x)\sqrt{1+e^{2x}}}$ (d) $\frac{e^x}{(1-e^x)\sqrt{1+e^x}}$

72. $\frac{d}{dx}\{e^x \log(1+x^2)\} =$ **[AI CBSE 1987]**

(a) $e^x\left[\log(1+x^2)+\frac{2x}{1+x^2}\right]$ (b) $e^x\left[\log(1+x^2)-\frac{2x}{1+x^2}\right]$ (c) $e^x\left[\log(1+x^2)+\frac{x}{1+x^2}\right]$ (d) $e^x\left[\log(1+x^2)-\frac{x}{1+x^2}\right]$

73. If $y=\frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$, then $\frac{dy}{dx}=$

(a) $\frac{-8}{(e^{2x}-e^{-2x})^2}$ (b) $\frac{8}{(e^{2x}-e^{-2x})^2}$ (c) $\frac{-4}{(e^{2x}-e^{-2x})^2}$ (d) $\frac{4}{(e^{2x}-e^{-2x})^2}$

74. $\frac{d}{dx}(e^{x^3})$ is equal to **[Rajasthan PET 1995]**

(a) $3xe^{x^3}$ (b) $3x^2e^{x^3}$ (c) $3x(e^{x^3})^2$ (d) $2x^3e^{x^3}$

75. $\frac{d}{dx}[e^{ax}\cos(bx+c)]=$ **[AISSE 1989]**

(a) $e^{ax}[a\cos(bx+c)-b\sin(bx+c)]$ (b) $e^{ax}[a\sin(bx+c)-b\cos(bx+c)]$

(c) $e^{ax}[\cos(bx+c)-\sin(bx+c)]$ (d) None of these

76. If $y=e^x\log x$, then $\frac{dy}{dx}$ is **[SCRA 1996]**

(a) $\frac{e^x}{x}$ (b) $e^x\left(\frac{1}{x}+x\log x\right)$ (c) $e^x\left(\frac{1}{x}+\log x\right)$ (d) $\frac{e^x}{\log x}$

77. If $f(1)=3$, $f'(1)=2$, then $\frac{d}{dx}\{\log f(e^x+2x)\}$ at $x = 0$ is **[AMU 1999]**

(a) 2/3 (b) 3/2 (c) 2 (d) 0

78. $\frac{d}{dx}(\sin^{-1}x)$ is equal to **[Rajasthan PET 1995]**

(a) $\frac{1}{\sqrt{1-x^2}}$ (b) $-\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{1}{\sqrt{x^2-1}}$

79. If $y=\sin^{-1}\sqrt{x}$, then $\frac{dy}{dx}=$ **[MP PET 1995]**

(a) $\frac{2}{\sqrt{x}\sqrt{1-x}}$ (b) $\frac{-2}{\sqrt{x}\sqrt{1-x}}$ (c) $\frac{1}{2\sqrt{x}\sqrt{1-x}}$ (d) $\frac{1}{\sqrt{1-x}}$

80. If $y=\sin^{-1}\sqrt{1-x^2}$, then $\frac{dy}{dx}=$ **[AISSE 1987]**

(a) $\frac{1}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1+x^2}}$ (c) $-\frac{1}{\sqrt{1-x^2}}$ (d) $-\frac{1}{\sqrt{x^2-1}}$

81. $\frac{d}{dx}\tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}=$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) – 1

82. If $y=\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right)$, then $y'(1)$ is **[AMU 2000]**

(a) 0 (b) $\frac{1}{2}$ (c) – 1 (d) $-\frac{1}{4}$

83. Differential coefficient of $\sec^{-1} x$ is

(a) $\frac{1}{x\sqrt{1-x^2}}$ (b) $-\frac{1}{x\sqrt{1-x^2}}$ (c) $\frac{1}{x\sqrt{x^2-1}}$ (d) $-\frac{1}{x\sqrt{x^2-1}}$

84. If $y=\cot^{-1}\left(\frac{1+x}{1-x}\right)$, then $\frac{dy}{dx}=$ **[DSSE 1984]**

(a) $\frac{1}{1+x^2}$ (b) $-\frac{1}{1+x^2}$ (c) $\frac{2}{1+x^2}$ (d) $-\frac{2}{1+x^2}$

85. If $y=\tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}$, then $\frac{dy}{dx}$ is equal to **[Roorkee 1995]**

(a) 0 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 1

86. If $f(x)=\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$, then $f'\left(\frac{\pi}{3}\right)=$ **[BIT Ranchi 1988]**

(a) $\frac{1}{2(1+\cos x)}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) None of these

87. If $y=\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$, then $\frac{dy}{dx}=$ **[MNR 1984]**

(a) 0 (b) 1 (c) 2 (d) 3

88. If $y=\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$, then $\frac{dy}{dx}=$

(a) $-\frac{1}{\sqrt{1-x^2}}$ (b) $\frac{x}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1-x^2}}$ (d) $\frac{\sqrt{1-x^2}}{x}$

89. If $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ equals **[Rajasthan PET 1996; EAMCET 1991]**

(a) $\frac{2}{1-x^2}$ (b) $\frac{1}{1+x^2}$ (c) $\pm\frac{2}{1+x^2}$ (d) $-\frac{2}{1+x^2}$

90. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{a-x}{1+ax}\right)\right]=$ **[Karnataka CET 2001]**

(a) $-\frac{1}{1+x^2}$ (b) $\frac{1}{1+a^2}-\frac{1}{1+x^2}$ (c) $\frac{1}{1+\left(\frac{a-x}{1+ax}\right)^2}$ (d) $\frac{-1}{\sqrt{1-\left(\frac{a-x}{1+ax}\right)^2}}$

91. If $y=\tan^{-1}\left[\frac{\sin x+\cos x}{\cos x-\sin x}\right]$, then $\frac{dy}{dx}$ is **[UPSEAT 2001]**

(a) 1/2 (b) $\pi/4$ (c) 0 (d) 1

92. $\frac{d}{dx}\left(\tan^{-1}\frac{\cos x}{1+\sin x}\right)=$ **[AISSE 1984, 85; MNR 1983; Rajasthan PET 1994, 96]**

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) – 1 (d) 1

93. If $y=\sin^{-1}\frac{2x}{1+x^2}+\sec^{-1}\frac{1+x^2}{1-x^2}$, then $\frac{dy}{dx}=$ **[Rajasthan PET 1996]**

(a) $\frac{4}{1-x^2}$ (b) $\frac{1}{1+x^2}$ (c) $\frac{4}{1+x^2}$ (d) $\frac{-4}{1+x^2}$

94. If $y=\tan^{-1}\frac{4x}{1+5x^2}+\tan^{-1}\frac{2+3x}{3-2x}$, then $\frac{dy}{dx}=$

(a) $\frac{1}{1+25x^2}+\frac{2}{1+x^2}$ (b) $\frac{5}{1+25x^2}+\frac{2}{1+x^2}$ (c) $\frac{5}{1+25x^2}$ (d) $\frac{1}{1+25x^2}$

95. $\frac{d}{dx}\sin^{-1}\left(\frac{x^2}{\sqrt{x^4+a^4}}\right)=$

(a) $\frac{2a^4x}{a^4+x^4}$ (b) $\frac{2a^2x}{a^4+x^4}$ (c) $\frac{2a^3x}{a^4+x^4}$ (d) $\frac{-2a^2x}{a^4+x^4}$

96. If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, then

(a) $(1-x^2)\frac{dy}{dx}-xy-1=0$ (b) $(1-x^2)\frac{dy}{dx}+xy-1=0$ (c) $(1-x^2)\frac{dy}{dx}+\frac{1}{2}xy-1=0$ (d) None of these

97. $\frac{d}{dx}\sin^{-1}(3x-4x^3)=$ **[Rajasthan PET 2003]**

(a) $\frac{3}{\sqrt{1-x^2}}$ (b) $\frac{-3}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1-x^2}}$ (d) $\frac{-1}{\sqrt{1-x^2}}$

98. $\frac{d}{dx}\sin^{-1}(2ax\sqrt{1-a^2x^2})=$

(a) $\frac{2a}{\sqrt{a^2-x^2}}$ (b) $\frac{a}{\sqrt{a^2-x^2}}$ (c) $\frac{2a}{\sqrt{1-a^2x^2}}$ (d) $\frac{a}{\sqrt{1-a^2x^2}}$

99. $\frac{d}{dx}\left[\sin^{-1}\left(\frac{3x}{2}-\frac{x^3}{2}\right)\right]$ equals

(a) $\frac{3}{\sqrt{4-x^2}}$ (b) $\frac{-3}{\sqrt{4-x^2}}$ (c) $\frac{1}{\sqrt{4-x^2}}$ (d) $\frac{-1}{\sqrt{4-x^2}}$

100. If $y=\sin^{-1}\left\{\frac{\sin x+\cos x}{\sqrt{2}}\right\}$, then $\frac{dy}{dx}=$

(a) 1 (b) – 1 (c) 0 (d) None of these

101. $\frac{d}{dx}\cos^{-1}\frac{x-x^{-1}}{x+x^{-1}}=$ **[DSSE 1985; Roorkee 1963]**

(a) $\frac{1}{1+x^2}$ (b) $-\frac{1}{1+x^2}$ (c) $\frac{2}{1+x^2}$ (d) $\frac{-2}{1+x^2}$

102. $\frac{d}{dx}\left\{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}=$ **[AISSE 1984]**

(a) $\frac{1}{1+x^2}$ (b) $-\frac{1}{1+x^2}$ (c) $-\frac{2}{1+x^2}$ (d) $\frac{2}{1+x^2}$

103. $\frac{d}{dx}\cos^{-1}\sqrt{\frac{1+x^2}{2}}=$ **[AI CBSE 1988]**

(a) $\frac{-1}{2\sqrt{1-x^4}}$ (b) $\frac{1}{2\sqrt{1-x^4}}$ (c) $\frac{-x}{\sqrt{1-x^4}}$ (d) $\frac{x}{\sqrt{1-x^4}}$

104. If $y = \tan^{-1}\left(\frac{\sqrt{a}-\sqrt{x}}{1+\sqrt{ax}}\right)$, then $\frac{dy}{dx} =$ **[AI CBSE 1988]**

(a) $\frac{1}{2(1+x)\sqrt{x}}$ (b) $\frac{1}{(1+x)\sqrt{x}}$ (c) $-\frac{1}{2(1+x)\sqrt{x}}$ (d) None of these

105. $\frac{d}{dx}\left[\tan^{-1}\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] =$ **[Roorkee 1980]**

(a) $\frac{-x}{\sqrt{1-x^4}}$ (b) $\frac{x}{\sqrt{1-x^4}}$ (c) $\frac{-1}{2\sqrt{1-x^4}}$ (d) $\frac{1}{2\sqrt{1-x^4}}$

106. If $y = (1+x^2)\tan^{-1}x - x$, then $\frac{dy}{dx} =$ **[Karnataka CET 2001]**

(a) $\tan^{-1}x$ (b) $2x\tan^{-1}x$ (c) $2x\tan^{-1}x - 1$ (d) $\frac{2x}{\tan^{-1}x}$

107. If $f(x) = (x+1)\tan^{-1}(e^{-2x})$, then $f'(0)$ equals

(a) $\frac{\pi}{6}+5$ (b) $\frac{\pi}{2}+1$ (c) $\frac{\pi}{4}-1$ (d) None of these

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108. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ is **[DCE 2002; Haryana CEE 2001]**

(a) $\frac{x}{\sqrt{1+x^2}}$ (b) $\frac{-x}{\sqrt{1+x^2}}$ (c) $\frac{x}{\sqrt{1-x^2}}$ (d) None of these

109. If $y = (1+x^{1/4})(1+x^{1/2})(1-x^{1/4})$, then $\frac{dy}{dx} =$ **[MP PET 1994]**

(a) 1 (b) – 1 (c) x (d) $\sqrt{x}$

110. If $f(x) = \sqrt{ax} + \frac{a^2}{\sqrt{ax}}$, then $f'(a) =$

(a) – 1 (b) 1 (c) 0 (d) a

111. $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then $\frac{dy}{dx} =$

(a) $1+x$ (b) $(1+x)^{-2}$ (c) $-(1+x)^{-1}$ (d) $-(1+x)^{-2}$

112. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then $\frac{dy}{dx} =$ **[MNR 1983; ISM Dhanbad 1987; Rajasthan PET 1991]**

(a) $\sqrt{\frac{1-x^2}{1-y^2}}$ (b) $\sqrt{\frac{1-y^2}{1-x^2}}$ (c) $\sqrt{\frac{x^2-1}{1-y^2}}$ (d) $\sqrt{\frac{y^2-1}{1-x^2}}$

113. Function $y = (x+\sqrt{x^2+1})^k$ satisfies **[IIT Screening]**

(a) $(x^2+1)y' = k^2y$ (b) $\sqrt{(x^2+1)}y' = ky$ (c) $(1+x^2)y'' + ky' - xy = 0$ (d) $(1+x^2)y'' + k^2 + xy' = 0$

114. The derivative of $\sqrt{\sqrt{x}+1}$ is

(a) $\frac{1}{\sqrt{x}(\sqrt{x}+1)}$ (b) $\frac{-1}{\sqrt{x}\sqrt{x+1}}$ (c) $\frac{4}{\sqrt{x(\sqrt{x}+1)}}$ (d) $\frac{1}{4\sqrt{x(\sqrt{x}+1)}}$

115. If $f(x)=\frac{1}{\sqrt{x^2+a^2}+\sqrt{x^2+b^2}}$, then $f'(x)$ is equal to **[Kurukshetra CEE 1998]**

(a) $\frac{x}{(a^2-b^2)}\left[\frac{1}{\sqrt{x^2+a^2}}-\frac{1}{\sqrt{x^2+b^2}}\right]$

(b) $\frac{x}{(a^2+b^2)}\left[\frac{1}{\sqrt{x^2+a^2}}-\frac{1}{\sqrt{x^2+b^2}}\right]$

(c) $\frac{x}{(a^2-b^2)}\left[\frac{1}{\sqrt{x^2+a^2}}+\frac{1}{\sqrt{x^2+b^2}}\right]$

(d) $(a^2-b^2)\left[\frac{1}{\sqrt{x^2+a^2}}-\frac{2}{\sqrt{x^2+b^2}}\right]$

116. If $\sqrt{(1-x^6)}+\sqrt{(1-y^6)}=a^3(x^3-y^3)$, then $\frac{dy}{dx}=$ **[Roorkee 1994]**

(a) $\frac{x^2}{y^2}\sqrt{\frac{1-x^6}{1-y^6}}$ (b) $\frac{y^2}{x^2}\sqrt{\frac{1-y^6}{1-x^6}}$ (c) $\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$ (d) None of these

117. If $y=\sqrt{x+\sqrt{x}}$, then $y\frac{dy}{dx}$ equals

(a) $\frac{2\sqrt{x}+1}{4\sqrt{x}}$ (b) $\frac{\sqrt{x}+1}{2\sqrt{x}}$ (c) $\frac{\sqrt{x}+1}{4x}$ (d) $\frac{x+1}{2\sqrt{x}}$

118. If $y=\sqrt{\sin\sqrt{x}}$, then $\frac{dy}{dx}=$ **[MP PET 1997]**

(a) $\frac{1}{2\sqrt{\cos\sqrt{x}}}$ (b) $\frac{\sqrt{\cos\sqrt{x}}}{2x}$ (c) $\frac{\cos\sqrt{x}}{4\sqrt{x}\sqrt{\sin\sqrt{x}}}$ (d) $\frac{1}{2\sqrt{\sin x}}$

119. $\frac{d}{dx}\sqrt{x\sin x}=$ **[AISSE 1985]**

(a) $\frac{\sin x+x\cos x}{2\sqrt{x\sin x}}$ (b) $\frac{\sin x+x\cos x}{\sqrt{x\sin x}}$ (c) $\frac{x\sin x+\cos x}{\sqrt{2\sin x}}$ (d) $\frac{x\sin x+\cos x}{\sqrt{2x\sin x}}$

120. If $y=f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x)=\sin x^2$, then $\frac{dy}{dx}=$ **[IIT 1982]**

(a) $\frac{6x^2-2x+2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$

(b) $\frac{6x^2-2x+2}{(x^2+1)^2}\sin^2\left(\frac{2x-1}{x^2+1}\right)$

(c) $\frac{-2x^2+2x+2}{(x^2+1)^2}\sin^2\left(\frac{2x-1}{x^2+1}\right)$

(d) $\frac{-2x^2+2x+2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$

121. $\frac{d}{dx}\left[\frac{e^{ax}}{\sin(bx+c)}\right]=$ **[AI CBSE 1983]**

(a) $\frac{e^{ax}[a\sin(bx+c)+b\cos(bx+c)]}{\sin^2(bx+c)}$

(b) $\frac{e^{ax}[a\sin(bx+c)-b\cos(bx+c)]}{\sin^2(bx+c)}$

(c) $\frac{e^{ax}[a\sin(bx+c)-b\cos(bx+c)]}{\sin^2(bx+c)}$

(d) None of these

122. If $y=b\cos\log\left(\frac{x}{n}\right)^n$, then $\frac{dy}{dx}=$

(a) $-n\ b\sin\log\left(\frac{x}{n}\right)^b$ (b) $n\ b\sin\log\left(\frac{x}{n}\right)^n$ (c) $\frac{-nb}{x}\sin\log\left(\frac{x}{n}\right)^n$ (d) None of these

123. If $y=f\left(\frac{5x+1}{10x^2-3}\right)$ and $f'(x)=\cos x$, then $\frac{dy}{dx}=$ **[MP PET 1987]**

(a) $\cos\left(\frac{5x+1}{10x^2-3}\right)\frac{d}{dx}\left(\frac{5x+1}{10x^2-3}\right)$ (b) $\frac{5x+1}{10x^2-3}\cos\left(\frac{5x+1}{10x^2-3}\right)$

(c) $\cos\left(\frac{5x+1}{10x^2-3}\right)$ (d) None of these

124. $\frac{d}{dx}\left(x^3\tan^2\frac{x}{2}\right)=$ **[AISSE 1979]**

(a) $x^3\tan\frac{x}{2}.\sec^2\frac{x}{2}+3x\tan^2\frac{x}{2}$ (b) $x^3\tan\frac{x}{2}.\sec^2\frac{x}{2}+3x^2\tan^2\frac{x}{2}$

(c) $x^2\tan^2\frac{x}{2}.\sec^2\frac{x}{2}+3x^2\tan^2\frac{x}{2}$ (d) None of these

125. $\frac{d}{dx}(\tan a^{1/x})=$

(a) $\sec^2(a^{1/x})\cdot\frac{(a^{1/x}.\log a)}{x^2}$ (b) $\sec^2(a^{1/x}).(a^{1/x}.\log a)$ (c) $\frac{\sec x.\log a}{x^2}$ (d) $-\frac{\sec^2(a^{1/x}).(a^{1/x}.\log_e a)}{x^2}$

126. If $y=\sqrt{\frac{1+\tan x}{1-\tan x}}$, then $\frac{dy}{dx}=$

(a) $\frac{1}{2}\sqrt{\frac{1-\tan x}{1+\tan x}}\cdot\sec^2\left(\frac{\pi}{4}+x\right)$ (b) $\sqrt{\frac{1-\tan x}{1+\tan x}}\cdot\sec^2\left(\frac{\pi}{4}+x\right)$ (c) $\frac{1}{2}\sqrt{\frac{1-\tan x}{1+\tan x}}\cdot\sec\left(\frac{\pi}{4}+x\right)$ (d)

127. If $A=\frac{2^x\cot x}{\sqrt{x}}$, then $\frac{dA}{dx}=$

(a) $\frac{2^{x-1}\left\{-2x\,\text{cosec}^2x+\cot x.\log\left(\frac{4^x}{e}\right)\right\}}{x^{3/2}}$ (b) $\frac{2^{x-1}\left\{-2x\,\text{cosec}^2x+\cot x.\log\left(\frac{4^x}{e}\right)\right\}}{x}$

(c) $\frac{2^{x}\left\{-2x\,\text{cosec}^2x+\cot x.\log\left(\frac{4^x}{e}\right)\right\}}{x^{3/2}}$ (d) None of these

128. Differential coefficient of $\sqrt{\sec\sqrt{x}}$ is **[MP PET 1996]**

(a) $\frac{1}{4\sqrt{x}}(\sec\sqrt{x})^{3/2}\sin\sqrt{x}$ (b) $\frac{1}{4\sqrt{x}}\sec\sqrt{x}\sin\sqrt{x}$ (c) $\frac{1}{2}\sqrt{x}(\sec\sqrt{x})^{3/2}\sin\sqrt{x}$ (d) $\frac{1}{2}\sqrt{x}\sec\sqrt{x}\sin\sqrt{x}$

129. $\frac{d}{dx}\left(\frac{\sec x+\tan x}{\sec x-\tan x}\right)=$ **[DSSE 1979, 81; CBSE 1981]**

(a) $\frac{2\cos x}{(1-\sin x)^2}$ (b) $\frac{\cos x}{(1-\sin x)^2}$ (c) $\frac{2\cos x}{1-\sin x}$ (d) None of these

130. If $x=f(m)\cos m-f'(m)\sin m$ and $y=f(m)\sin m+f'(m)\cos m$, then $\left(\frac{dy}{dm}\right)^2+\left(\frac{dx}{dm}\right)^2$ equals **[AMU 1997]**

(a) $[f(m)+f''(m)]^2$ (b) $[f(m)-f''(m)]^2$ (c) $\{f(m)\}^2+\{f'(m)\}^2$ (d) $\frac{\{f(m)\}^2}{\{f'(m)\}^2}$

131. If $x=\sec\theta-\cos\theta$ and $y=\sec^n\theta-\cos^n\theta$, then **[IIT 1989]**

(a) $(x^2+4)\left(\frac{dy}{dx}\right)^2=n^2(y^2+4)$ (b) $(x^2+4)\left(\frac{dy}{dx}\right)^2=x^2(y^2+4)$ (c) $(x^2+4)\left(\frac{dy}{dx}\right)^2=(y^2+4)$ (d)

132. If $y=\log_{\sin x}(\tan x)$, then $\left(\frac{dy}{dx}\right)_{\pi/4}=$

(a) $\frac{4}{\log 2}$ (b) $-4\log 2$ (c) $\frac{-4}{\log 2}$ (d) None of these

133. If $u(x, y) = y\log x + x\log y$, then $u_x u_y - u_x \log x - u_y \log y + \log x \log y =$ **[EAMCET 2003]**

(a) 0 (b) – 1 (c) 1 (d) 2

134. If $y = \log x . e^{(\tan x + x^2)}$, then $\frac{dy}{dx} =$

(a) $e^{(\tan x+x^2)}\left[\frac{1}{x}+(\sec^2 x + x)\log x\right]$ (b) $e^{(\tan x+x^2)}\left[\frac{1}{x}+(\sec^2 x - x)\log x\right]$

(c) $e^{(\tan x+x^2)}\left[\frac{1}{x}+(\sec^2 x + 2x)\log x\right]$ (d) $e^{(\tan x+x^2)}\left[\frac{1}{x}+(\sec^2 x - 2x)\log x\right]$

135. $\frac{d}{dx}\left[\log\sqrt{\sin\sqrt{e^x}}\right] =$

(a) $\frac{1}{4}e^{x/2}\cot(e^{x/2})$ (b) $e^{x/2}\cot(e^{x/2})$ (c) $\frac{1}{4}e^{x}\cot(e^{x})$ (d) $\frac{1}{2}e^{x/2}\cot(e^{x/2})$

136. If $y\sqrt{x^2+1} = \log\{\sqrt{x^2+1} - x\}$, then $(x^2+1)\frac{dy}{dx} + xy + 1 =$ **[Roorkee 1978; Kurukshetra CEE 1998]**

(a) 0 (b) 1 (c) 2 (d) None of these

137. If $y = \log_{\cos x}\sin x$, then $\frac{dy}{dx}$ is equal to

(a) $(\cot x \log\cos x + \tan x \log\sin x)/(\log\cos x)^2$ (b) $(\tan x \log\cos x + \cot x \log\sin x)/(\log\cos x)^2$

(c) $(\cot x \log\cos x + \tan x \log\sin x)/(\log\sin x)^2$ (d) None of these

138. If $y = \log(x + e^{\sqrt{x}})$ then $\frac{dy}{dx} =$

(a) $\frac{2\sqrt{x}+e^{\sqrt{x}}}{2\sqrt{x}(x+e^{\sqrt{x}})}$ (b) $\frac{2\sqrt{x}+e^{\sqrt{x}}}{2\sqrt{x}(x-e^{\sqrt{x}})}$ (c) $\frac{2\sqrt{x}-e^{\sqrt{x}}}{2\sqrt{x}(x+e^{\sqrt{x}})}$ (d) $\frac{2\sqrt{x}-e^{\sqrt{x}}}{2\sqrt{x}(x-e^{\sqrt{x}})}$

139. $\frac{d}{dx}(a^{\log_{10}\operatorname{cosec}^{-1}x}) =$ **[Roorkee 1990]**

(a) $a^{\log_{10}\operatorname{cosec}^{-1}x}\cdot\frac{1}{\operatorname{cosec}^{-1}x}\cdot\frac{1}{x\sqrt{x^2-1}}\cdot\log_{10}a$ (b) $-a^{\log_{10}\operatorname{cosec}^{-1}x}\cdot\frac{1}{\operatorname{cosec}^{-1}x}\cdot\frac{1}{|x|\sqrt{x^2-1}}\cdot\log_{10}a$

(c) $a^{\log_{10}\operatorname{cosec}^{-1}x}\cdot\frac{1}{\operatorname{cosec}^{-1}x}\cdot\frac{1}{|x|\sqrt{x^2-1}}\cdot\log_{10}a$ (d) $-a^{\log_{10}\operatorname{cosec}^{-1}x}\cdot\frac{1}{\operatorname{cosec}^{-1}x}\cdot\frac{1}{x\sqrt{x^2-1}}\cdot\log_{10}a$

140. $\frac{d}{dx}(e^{\sqrt{1-x^2}}.\tan x) =$ **[AI CBSE 1979]**

(a) $e^{\sqrt{1-x^2}}\left[\sec^2 x + \frac{x\tan x}{\sqrt{1-x^2}}\right]$ (b) $e^{\sqrt{1-x^2}}\left[\sec^2 x - \frac{x\tan x}{\sqrt{1-x^2}}\right]$ (c) $e^{\sqrt{1-x^2}}\left[\sec^2 x + \frac{\tan x}{\sqrt{1-x^2}}\right]$ (d)

141. $10^{-x\tan x}\left[\frac{d}{dx}(10^{x\tan x})\right]$ is equal to

(a) $\tan x + x\sec^2 x$ (b) $\ln 10(\tan x + x\sec^2 x)$

(c) $\ln 10\left(\tan x + \frac{x}{\cos^2 x} + \tan x\sec x\right)$ (d) $x\tan x\ln 10$

142. If $y=\sin\left[2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right]$, then $\frac{dy}{dx}=$

(a) $\frac{1}{2\sqrt{1-x^2}}$ (b) $\frac{-2x}{\sqrt{1-x^2}}$ (c) $\frac{-x}{\sqrt{1-x^2}}$ (d) $\frac{x}{\sqrt{1-x^2}}$

143. $\frac{d}{dx}\left(\cos^{-1}\sqrt{\frac{1+\cos x}{2}}\right)=$ **[AI CBSE 1982]**

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) None of these

144. If $f(x)=\cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right]$, then the value of $f'(e)=$ **[Karnataka CET 1999]**

(a) 1 (b) $\frac{1}{e}$ (c) $\frac{2}{e}$ (d) $\frac{2}{e^2}$

145. If $y=\frac{1}{\sqrt{a^2-b^2}}\cos^{-1}\left[\frac{a\cos(x-\alpha)+b}{\theta}\right]$ where $\theta=a+b\cos(x-\alpha)$, then $\frac{dy}{dx}=$ **[Orissa JEE 2003]**

(a) $\frac{1}{\theta}$ (b) $\frac{2}{\theta}$ (c) $\frac{1}{\theta^2}$ (d) $\frac{2}{\theta^2}$

146. $\frac{d}{dx}\tan^{-1}(\sec x+\tan x)=$ **[AISSE 1985, 87; DSSE 1982, 84]**

(a) 1 (b) 1/2 (c) $\cos x$ (d) $\sec x$

147. $\frac{d}{dx}\left[\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\right]=$ **[Ranchi BIT 1989; Roorkee 1989; Rajasthan PET 1996]**

(a) $-\frac{1}{2}$ (b) 0 (c) $\frac{1}{2}$ (d) 1

148. If $y=\tan^{-1}\left(\frac{x^{1/3}+a^{1/3}}{1-x^{1/3}a^{1/3}}\right)$, then $\frac{dy}{dx}=$ **[DSSE 1986]**

(a) $\frac{1}{3x^{2/3}(1+x^{2/3})}$ (b) $\frac{a}{3x^{2/3}(1+x^{2/3})}$ (c) $-\frac{1}{3x^{2/3}(1+x^{2/3})}$ (d) $-\frac{a}{3x^{2/3}(1+x^{2/3})}$

149. The differential coefficient of $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ is **[MP PET 2003]**

(a) $\sqrt{1-x^2}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{2\sqrt{1-x^2}}$ (d) x

150. $\frac{d}{dx}\left[\sin^2\cot^{-1}\frac{1}{\sqrt{\frac{1+x}{1-x}}}\right]$ equals **[EAMCET 1996]**

(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

151. If $f'(x)=\sin(\log x)$ and $y=f\left(\frac{2x+3}{3-2x}\right)$ then $\frac{dy}{dx}$ equals

(a) $\sin(\log x).\frac{1}{x\log x}$ (b) $\frac{12}{(3-2x)^2}\sin\left(\log\left(\frac{2x+3}{3-2x}\right)\right)$ (c) $\sin\left(\log\left(\frac{2x+3}{3-2x}\right)\right)$ (d) None of these

152. If $y=\tan^{-1}\left\{\frac{3a^2x-x^3}{a(a^2-3x^2)}\right\}$ then $\frac{dy}{dx}$ equals **[IIT 1969; Rajasthan PET 1998]**

(a) $\frac{3}{a^2+x^2}$ (b) $\frac{a}{a^2+x^2}$ (c) $\frac{3a}{a^2+x^2}$ (d) $\frac{3x}{a^2+x^2}$

153. If $y=\sinh^{-1}(\tan x)$, then the value of $\frac{dy}{dx}$ is

(a) $\sin x$ (b) $\cos x$ (c) $\sec x$ (d) $\text{cosec}\, x$

154. $\frac{d}{dx}[\sinh^{-1}x]^x$ equals

(a) $\frac{x}{\sqrt{1+x^2}.\sinh^{-1}x}+\log(\sinh^{-1}x)$ (b) $(\sinh^{-1}x)^{x-1}\frac{1}{\sqrt{1+x^2}}$

(c) $(\sinh^{-1}x)^x\left[\frac{x}{\sqrt{1+x^2}.\sinh^{-1}x}+\log(\sinh^{-1}x)\right]$ (d) None of these

155. If $y=\sin^{-1}(x\sqrt{1-x}+\sqrt{x}\sqrt{1-x^2})$, then $\frac{dy}{dx}=$ **[Roorkee 1981]**

(a) $\frac{-2x}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{x-x^2}}$ (b) $\frac{-1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$ (c) $\frac{1}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{x-x^2}}$ (d) None of these

156. If $y=\tan^{-1}\frac{x}{1+\sqrt{1-x^2}}+\sin\left\{2\tan^{-1}\sqrt{\left(\frac{1-x}{1+x}\right)}\right\}$, then $\frac{dy}{dx}=$

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1-2x}{\sqrt{1-x^2}}$ (c) $\frac{1-2x}{2\sqrt{1-x^2}}$ (d) $\frac{1}{1+x^2}$

157. If $y=\cot^{-1}\left[\frac{\sqrt{1+x^2}+1}{x}\right]$ then $\frac{dy}{dx}=$

(a) $\frac{1}{2}.\frac{1}{1+x^2}$ (b) $\frac{1}{2}.\frac{1}{1-x^2}$ (c) $\frac{2}{1+x^2}$ (d) $\frac{2}{1-x^2}$

158. If $y=\tan^{-1}\left(\frac{2^{x+1}}{1-4^x}\right)$, then $\frac{dy}{dx}=$

(a) $\frac{2^{x+1}\log_e 2}{4^x}$ (b) $\frac{2^{x+1}\log_e 2}{1+4^x}$ (c) $\frac{2^{x+1}\log_e 2}{1-4^x}$ (d) $\frac{2^{x+1}\log_2 e}{1-4^x}$

159. If $y=\tan^{-1}\left(\frac{2.a^x}{1-a^{2x}}\right)$, then $\frac{dy}{dx}=$

(a) $\frac{2.a^x\log a}{1-a^{2x}}$ (b) $\frac{2.a^x\log a}{1+a^{2x}}$ (c) $2.a^x\log a$ (d) $\frac{2.a^x\log a}{a^{2x}-1}$

160. If $y=x.e^{\cos^{-1}x}+\sec(2x-1)$, then $\frac{dy}{dx}$ equals **[Rajasthan PET 1986]**

(a) $e^{\cos^{-1}x}\left(1-\dfrac{x}{\sqrt{1-x^2}}\right)+\sec(2x-1).\tan(2x-1)$

(b) $e^{\cos^{-1}x}\left(1-\dfrac{x}{\sqrt{1-x^2}}\right)-\sec(2x-1).\tan(2x-1)$

(c) $e^{\cos^{-1}x}\left(1-\dfrac{x}{\sqrt{1-x^2}}\right)+2\sec(2x-1).\tan(2x-1)$

(d) None of these

161. If $y=\tan^{-1}\left(\dfrac{a+b\tan x}{b-a\tan x}\right)$, then $\dfrac{dy}{dx}=$

(a) 1 (b) – 1 (c) x (d) $\dfrac{1}{1+x^2}$

Methods of differentiation

Basic Level

162. If $x^3+8xy+y^3=64$, then $\dfrac{dy}{dx}=$

(a) $-\dfrac{3x^2+8y}{8x+3y^2}$ (b) $\dfrac{3x^2+8y}{8x+3y^2}$ (c) $\dfrac{3x+8y^2}{8x^2+3y}$ (d) None of these

163. If $\sin^2 x+2\cos y+xy=0$, then $\dfrac{dy}{dx}=$ **[AI CBSE 1980]**

(a) $\dfrac{y+2\sin x}{2\sin y+x}$ (b) $\dfrac{y+\sin 2x}{2\sin y-x}$ (c) $\dfrac{y+2\sin x}{\sin y+x}$ (d) None of these

164. If $y\sec x+\tan x+x^2y=0$, then $\dfrac{dy}{dx}=$ **[DSSE 1981; CBSE 1981]**

(a) $\dfrac{2xy+\sec^2 x+y\sec x\tan x}{x^2+\sec x}$ (b) $-\dfrac{2xy+\sec^2 x+\sec x\tan x}{x^2+\sec x}$

(c) $-\dfrac{2xy+\sec^2 x+y\sec x\tan x}{x^2+\sec x}$ (d) None of these

165. If $\sin(xy)+\dfrac{x}{y}=x^2-y$, then $\dfrac{dy}{dx}=$

(a) $\dfrac{y[2xy-y^2\cos(xy)-1]}{xy^2\cos(xy)+y^2-x}$ (b) $\dfrac{[2xy-y^2\cos(xy)-1]}{xy^2\cos(xy)+y^2-x}$ (c) $-\dfrac{y[2xy-y^2\cos(xy)-1]}{xy^2\cos(xy)+y^2-x}$ (d) None of these

166. If $3\sin(xy)+4\cos(xy)=5$, then $\dfrac{dy}{dx}=$ **[EAMCET 1994]**

(a) $-\dfrac{y}{x}$ (b) $\dfrac{3\sin(xy)+4\cos(xy)}{3\cos(xy)-4\sin(xy)}$ (c) $\dfrac{3\cos(xy)+4\sin(xy)}{4\cos(xy)-3\sin(xy)}$ (d) None of these

167. If $x^2e^y+2xye^x+13=0$, then $\dfrac{dy}{dx}=$ **[Rajasthan PET 1987]**

(a) $\dfrac{2xe^{y-x}+2y(x+1)}{x(xe^{y-x}+2)}$ (b) $\dfrac{2xe^{x-y}+2y(x+1)}{x(xe^{y-x}+2)}$ (c) $-\dfrac{2xe^{y-x}+2y(x+1)}{x(xe^{y-x}+2)}$ (d) None of these

168. If $\sin y = x\sin(a+y)$, then $\frac{dy}{dx} =$ **[Karnataka CET 2000; UPSEAT 2001]**

(a) $\frac{\sin^2(a+y)}{\sin(a+2y)}$ (b) $\frac{\sin^2(a+y)}{\cos(a+2y)}$ (c) $\frac{\sin^2(a+y)}{\sin a}$ (d) $\frac{\sin^2(a+y)}{\cos a}$

169. If $y = x^x$, then $\frac{dy}{dx} =$ **[AISSE 1984; DSSE 1982; MNR 1979; SCRA 1996; Rajasthan PET 1996; Kerala (Engg.) 2002]**

(a) $x^x \log ex$ (b) $x^x\left(1+\frac{1}{x}\right)$ (c) $(1 + \log x)$ (d) $x^x \log x$

170. If $y^x + x^y = a^b$, then $\frac{dy}{dx} =$

(a) $-\frac{y\,x^{y-1} + y^x \log y}{x\,y^{x-1} + x^y \log x}$ (b) $\frac{y\,x^{y-1} + y^x \log y}{x\,y^{x-1} + x^y \log x}$ (c) $-\frac{y\,x^{y-1} + y^x}{x\,y^{x-1} + x^y}$ (d) $\frac{y\,x^{y-1} + y^x}{x\,y^{x-1} + x^y}$

171. If $y = \sqrt{\frac{(x-a)(x-b)}{(x-c)(x-d)}}$, then $\frac{dy}{dx} =$

(a) $\frac{y}{2}\left[\frac{1}{x-a}+\frac{1}{x-b}-\frac{1}{x-c}-\frac{1}{x-d}\right]$ (b) $y\left[\frac{1}{x-a}+\frac{1}{x-b}-\frac{1}{x-c}-\frac{1}{x-d}\right]$

(c) $\frac{1}{2}\left[\frac{1}{x-a}+\frac{1}{x-b}-\frac{1}{x-c}-\frac{1}{x-d}\right]$ (d) None of these

172. $\frac{d}{dx}(x^{\log_e x}) =$ **[MP PET 1993]**

(a) $2x^{(\log_e x-1)}.\log_e x$ (b) $x^{(\log_e x-1)}$ (c) $\frac{2}{x}\log_e x$ (d) $x^{(\log_e x-1)}.\log_e x$

173. If $x^y = y^x$, then $\frac{dy}{dx} =$ **[DSSE 1986; MP PET 1997]**

(a) $\frac{y(x\log_e y + y)}{x(y\log_e x + x)}$ (b) $\frac{y(x\log_e y - y)}{x(y\log_e x - x)}$ (c) $\frac{x(x\log_e y - y)}{y(y\log_e x - x)}$ (d) $\frac{x(x\log_e y + y)}{y(y\log_e x + x)}$

174. If $y = x^{\sin x}$, then $\frac{dy}{dx} =$ **[DSSE 1983, 84]**

(a) $\frac{x\cos x.\log x + \sin x}{x}.x^{\sin x}$ (b) $\frac{y[x\cos x.\log x + \cos x]}{x}$

(c) $y[x\sin x.\log x + \cos x]$ (d) None of these

175. $\frac{d}{dx}\{(\sin x)^x\} =$ **[DSSE 1985, 87; AISSE 1983]**

(a) $\left[\frac{x\cos x + \sin x\log\sin x}{\sin x}\right]$ (b) $(\sin x)^x\left[\frac{x\cos x + \sin x\log\sin x}{\sin x}\right]$

(c) $(\sin x)^x\left[\frac{x\sin x + \sin x\log\sin x}{\sin x}\right]$ (d) None of these

176. If $2^x + 2^y = 2^{x+y}$, then the value of $\frac{dy}{dx}$ at $x = y = 1$ is **[Karnataka CET 2000]**

(a) 0 (b) – 1 (c) 1 (d) 2

177. If $x^y = e^{x-y}$, then $\frac{dy}{dx} =$ **[MP PET 1987; MNR 1984; Roorkee 1954; Ranchi BIT 1991; Rajasthan PET 2000]**

(a) $\log x.[\log(ex)]^{-2}$ (b) $\log x[\log(ex)]^2$ (c) $\log x.(\log x)^2$ (d) None of these

178. $\frac{d}{dx}\{(\sin x)^{\log x}\} =$ **[DSSE 1984]**

(a) $(\sin x)^{\log x}\left[\frac{1}{x}\log\sin x + \cot x\right]$ (b) $(\sin x)^{\log x}\left[\frac{1}{x}\log\sin x + \cot x\log x\right]$

(c) $(\sin x)^{\log x}\left[\frac{1}{x}\log\sin x + \cot x\right]$ (d) None of these

179. $y = (\tan x)^{(\tan x)^{\tan x}}$, then at $x = \frac{\pi}{4}$, the value of $\frac{dy}{dx} =$ **[West Bengal JEE 1990]**

(a) 0 (b) 1 (c) 2 (d) None of these

180. If $x^p y^q = (x+y)^{p+q}$, then $\frac{dy}{dx} =$ **[Rajasthan PET 1999; UPSEAT 2001]**

(a) $\frac{y}{x}$ (b) $-\frac{y}{x}$ (c) $\frac{x}{y}$ (d) $-\frac{x}{y}$

181. If $y = (\tan x)^{\cot x}$, then $\frac{dy}{dx} =$

(a) $y\,\text{cosec}^2 x(1 - \log\tan x)$ (b) $y\,\text{cosec}^2 x(1 + \log\tan x)$ (c) $y\,\text{cosec}^2 x(\log\tan x)$ (d) None of these

182. If $y = \frac{e^x \log x}{x^2}$, then $\frac{dy}{dx} =$ **[AI CBSE 1982]**

(a) $\frac{e^x[1+(x+2)\log x]}{x^3}$ (b) $\frac{e^x[1-(x-2)\log x]}{x^4}$ (c) $\frac{e^x[1-(x-2)\log x]}{x^3}$ (d) $\frac{e^x[1+(x-2)\log x]}{x^3}$

183. If $y = \frac{e^{2x}\cos x}{x\sin x}$, then $\frac{dy}{dx} =$

(a) $\frac{e^{2x}[(2x-1)\cot x - x\,\text{cosec}^2 x]}{x^2}$ (b) $\frac{e^{2x}[(2x+1)\cot x - x\,\text{cosec}^2 x]}{x^2}$

(c) $\frac{e^{2x}[(2x-1)\cot x + x\,\text{cosec}^2 x]}{x^2}$ (d) None of these

184. If $y = \frac{\sqrt{x}(2x+3)^2}{\sqrt{x+1}}$, then $\frac{dy}{dx} =$ **[AISSE 1986]**

(a) $y\left[\frac{1}{2x} + \frac{4}{2x+3} - \frac{1}{2(x+1)}\right]$ (b) $y\left[\frac{1}{3x} + \frac{4}{2x+3} + \frac{1}{2(x+1)}\right]$ (c) $y\left[\frac{1}{3x} + \frac{4}{2x+3} + \frac{1}{(x+1)}\right]$ (d)

185. If $y = \frac{2(x-\sin x)^{3/2}}{\sqrt{x}}$, then $\frac{dy}{dx} =$

(a) $\frac{2(x-\sin x)^{3/2}}{\sqrt{x}}\left[\frac{3}{2}.\frac{1-\cos x}{1-\sin x} - \frac{1}{2x}\right]$ (b) $\frac{2(x-\sin x)^{3/2}}{\sqrt{x}}\left[\frac{3}{2}.\frac{1-\cos x}{x-\sin x} - \frac{1}{2x}\right]$

(c) $\frac{2(x-\sin x)^{1/2}}{\sqrt{x}}\left[\frac{3}{2}.\frac{1-\cos x}{x-\sin x} - \frac{1}{2x}\right]$ (d) None of these

186. $\frac{d}{dx}[(x-2)^x] =$ **[Rajasthan PET 1992]**

(a) $(x-2)^x[x + \log(x-2)]$ (b) $(x-2)^{x-1}[(x-2)\log(x-2) + x]$ (c) $(x-2)^{x-1}[x + \log(x-2)]$ (d) None of these

187. The derivative of x^{a^x} is

(a) $x^{a^x}\left[\frac{a^x}{x}+a^x\log a\log x\right]$ (b) $x^{a^x}[a^x+xa^x\log x]$ (c) $x^{a^x}[xa^x+a^x\log x]$ (d) None of these

188. If $x=a\sin 2\theta(1+\cos 2\theta), y=b\cos 2\theta(1-\cos 2\theta)$, then $\frac{dy}{dx}=$ **[Kurukshetra CEE 1998]**

(a) $\frac{b\tan\theta}{a}$ (b) $\frac{a\tan\theta}{b}$ (c) $\frac{a}{b\tan\theta}$ (d) $\frac{b}{a\tan\theta}$

189. If $x=a\left(\cos t+\log\tan\frac{t}{2}\right)$, $y=a\sin t$, then $\frac{dy}{dx}=$ **[Rajasthan PET 1997; MP PET 2001]**

(a) $\tan t$ (b) $-\tan t$ (c) $\cot t$ (d) $-\cot t$

190. If $x=\sin^{-1}(3t-4t^3)$ and $y=\cos^{-1}(\sqrt{1-t^2})$, then $\frac{dy}{dx}$ is equal to

(a) 1/2 (b) 2/5 (c) 3/2 (d) 1/3

191. If $x=\frac{2t}{1+t^2}$, $y=\frac{1-t^2}{1+t^2}$, then $\frac{dy}{dx}$ equals **[Rajasthan PET 1999]**

(a) $\frac{2t}{t^2+1}$ (b) $\frac{2t}{t^2-1}$ (c) $\frac{2t}{1-t^2}$ (d) None of these

192. If $x=\frac{3at}{1+t^3}$, $y=\frac{3at^2}{1+t^3}$, then $\frac{dy}{dx}=$

(a) $\frac{t(2+t^3)}{1-2t^3}$ (b) $\frac{t(2-t^3)}{1-2t^3}$ (c) $\frac{t(2+t^3)}{1+2t^3}$ (d) $\frac{t(2-t^3)}{1+2t^3}$

193. If $x = a\,(t+\sin t)$ and $y = a(1-\cos t)$, then $\frac{dy}{dx}$ equals **[Rajasthan PET 1996; MP PET 2002]**

(a) $\tan(t/2)$ (b) $\cot(t/2)$ (c) $\tan 2t$ (d) $\tan t$

194. If $x=a\cos^4\theta$, $y=a\sin^4\theta$, then $\frac{dy}{dx}$ at $\theta=\frac{3\pi}{4}$ **[Kerala (Engg.) 2002]**

(a) – 1 (b) 1 (c) $-a^2$ (d) a^2

195. If $x=2\cos t-\cos 2t$, $y=2\sin t-\sin 2t$, then at $t=\frac{\pi}{4}, \frac{dy}{dx}=$

(a) $\sqrt{2}+1$ (b) $\sqrt{2+1}$ (c) $\frac{\sqrt{2+1}}{2}$ (d) None of these

196. If $\tan y=\frac{2t}{1-t^2}$ and $\sin x=\frac{2t}{1+t^2}$, then $\frac{dy}{dx}=$ **[Rajasthan PET 1994]**

(a) $\frac{2}{1+t^2}$ (b) $\frac{1}{1+t^2}$ (c) 1 (d) 2

197. If $x=at^2$, $y = 2at$ then $\frac{dy}{dx}$ at $t = 2$ **[Rajasthan PET 1992]**

(a) 2 (b) 4 (c) 1/2 (d) 1/4

198. If $x=t^2+t+1$ and $y=\sin\frac{\pi}{2}t+\cos\frac{\pi}{2}t$ then at $t=1\,\frac{dy}{dx}$ equals

(a) – π/6 (b) π/2 (c) – π/4 (d) π/3

199. If $y=e^{x+e^{x+e^{x+\ldots\infty}}}$, then $\frac{dy}{dx}=$

(a) $\frac{y}{1-y}$ (b) $\frac{1}{1-y}$ (c) $\frac{y}{1+y}$ (d) $\frac{y}{y-1}$

200. If $y=(\sin x)^{(\sin x)^{(\sin x)......\infty}}$, then $\frac{dy}{dx}=$

(a) $\frac{y^2 \cot x}{1-y\log \sin x}$ (b) $\frac{y^2 \cot x}{1+y\log \sin x}$ (c) $\frac{y \cot x}{1-y\log \sin x}$ (d) $\frac{y \cot x}{1+y\log \sin x}$

201. The differential equation satisfied by the function $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+......\infty}}}$, is **[MP PET 1998]**

(a) $(2y-1)\frac{dy}{dx}-\sin x=0$ (b) $(2y-1)\cos x+\frac{dy}{dx}=0$

(c) $(2y-1)\cos x-\frac{dy}{dx}-=0$ (d) $(2y-1)\frac{dy}{dx}-\cos x=0$

202. If $y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+......\infty}}}$, then $\frac{dy}{dx}=$

(a) $\frac{x}{2y-1}$ (b) $\frac{x}{2y+1}$ (c) $\frac{1}{x(2y-1)}$ (d) $\frac{1}{x(1-2y)}$

203. If $y=\sqrt{f(x)+\sqrt{f(x)+\sqrt{f(x)+......}}}$, then the value of $(2y-1)\frac{dy}{dx}$ is

(a) $f(x)$ (b) $f'(x)$ (c) $2f'(x)$ (d) None of these

Advance Level

204. If $x^2+y^2=t-\frac{1}{t}$, $x^4+y^4=t^2+\frac{1}{t^2}$, then $\frac{dy}{dx}$ equals **[Rajasthan PET 1999]**

(a) $1/xy^3$ (b) $1/x^3y$ (c) $-1/x^3y$ (d) $-1/xy^3$

205. If $f'(x)=\sin(\log x)$ and $y=f\left(\frac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx}=$ **[BIT Ranchi 1986]**

(a) $\frac{9\cos(\log x)}{x(3-2x)^2}$ (b) $\frac{9\cos\left(\log\frac{2x+3}{3-2x}\right)}{x(3-2x)^2}$ (c) $\frac{9\sin\left(\log\frac{2x+3}{3-2x^2}\right)}{(3-2x)^2}$ (d) None of these

206. $\frac{dy}{dx}$ of $\log(xy)=x^2+y^2$ is

(a) $\frac{y(2x^2-1)}{x(1-2y^2)}$ (b) $\frac{y(2x^2+1)}{x(1+2y^2)}$ (c) $\frac{x(2x^2-1)}{y(2y^2-1)}$ (d) $\frac{y(2x^2-1)}{x(2y^2-1)}$

207. $(x-y)e^{x/(x-y)}=k$, then

(a) $(y-2x)\frac{dy}{dx}+3x-2y=0$ (b) $y\frac{dy}{dx}+x-2y=0$ (c) $a\left(y\frac{dy}{dx}+x-2y\right)=1$ (d) None of these

208. If $y=(x^x)^x$, then $\frac{dy}{dx}=$

(a) $(x^x)^x(1+2\log x)$ (b) $(x^x)^x(1+\log x)$ (c) $x(x^x)^x(1+2\log x)$ (d) $x(x^x)^x(1+\log x)$

209. If $y=(x\log x)^{\log\log x}$, then $\frac{dy}{dx}=$ **[Roorkee 1981]**

(a) $(x\log x)^{\log\log x}\left\{\frac{1}{x\log x}(\log x+\log\log x)+(\log\log x)\left(\frac{1}{x}+\frac{1}{x\log x}\right)\right\}$

(b) $(x\log x)^{x\log x}\log\log x\left[\frac{2}{\log x}+\frac{1}{x}\right]$

(c) $(x\log x)^{x\log x}\frac{\log\log x}{x}\left[\frac{1}{\log x}+1\right]$

(d) None of these

210. If $y=\left(1+\frac{1}{x}\right)^x$, then $\frac{dy}{dx}=$ **[BIT Ranchi 1992]**

(a) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)-\frac{1}{1+x}\right]$ (b) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)\right]$

(c) $\left(x+\frac{1}{x}\right)^x\left[\log(x-1)-\frac{x}{1+x}\right]$ (d) $\left(x+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)+\frac{1}{1+x}\right]$

211. If $y=x^{(x^x)}$, then $\frac{dy}{dx}=$ **[AISSE 1989]**

(a) $y[x^x(\log ex).\log x+x^x]$ (b) $y[x^x(\log ex).\log x+x]$ (c) $y[x^x(\log ex).\log x+x^{x-1}]$ (d) $y[x^x(\log_e x).\log x+x^{x-1}]$

212. If $y=\frac{a^{\cos^{-1}x}}{1+a^{\cos^{-1}x}}$ and $z=a^{\cos^{-1}x}$, then $\frac{dy}{dz}=$ **[MP PET 1994]**

(a) $\frac{1}{1+a^{\cos^{-1}x}}$ (b) $-\frac{1}{1+a^{\cos^{-1}x}}$ (c) $\frac{1}{(1+a^{\cos^{-1}x})^2}$ (d) None of these

213. Let the function $y=f(x)$ be given by $x=t^5-5t^3-20t+7$ and $y=4t^3-3t^2-18t+3$, where $t\in(-2,2)$. Then $f'(x)$ at $t=1$ is

(a) $\frac{5}{2}$ (b) $\frac{2}{5}$ (c) $\frac{7}{5}$ (d) None of these

214. If $y=\sqrt{x}^{\sqrt{x}^{\sqrt{x}......\infty}}$, then $\frac{dy}{dx}=$

(a) $\frac{y^2}{2x-2y\log x}$ (b) $\frac{y^2}{2x+\log x}$ (c) $\frac{y^2}{2x+2y\log x}$ (d) None of these

215. If $y=x+\cfrac{1}{x+\cfrac{1}{x+\cfrac{1}{x+.....}}}$, then $\frac{dy}{dx}$ equals

(a) $\frac{y}{2y-x}$ (b) $\frac{y}{2y+x}$ (c) $\frac{y}{y-2x}$ (d) $\frac{y}{y+2x}$

216. If $y=\cfrac{x}{a+\cfrac{x}{b+\cfrac{x}{a+\cfrac{x}{b+.........}}}}$, then $\frac{dy}{dx}$ equals

(a) $\frac{b}{a(b+2y)}$ (b) $\frac{b}{b+2y}$ (c) $\frac{a}{b(b+2y)}$ (d) None of these

217. If $y=\cfrac{\sin x}{1+\cfrac{\cos x}{1+\cfrac{\sin x}{1+\cos x.......\infty}}}$, then $\frac{dy}{dx}=$

(a) $\frac{(1+y)\cos x+\sin x}{1+2y+\cos x-\sin x}$ (b) $\frac{(1+y)\cos x-\sin x}{1+2y+\cos x+\sin x}$ (c) $\frac{(1+y)\cos x+\sin x}{1+2y+\cos x+\sin x}$ (d) None of these

218. If $f(x)=\frac{1}{1-x}$, then the derivative of the composite function $f[f\{f(x)\}]$ is equal to **[Orissa JEE 2003]**

(a) 0 (b) 1/2 (c) 1 (d) 2

219. If $u=f(x^3), v=g(x^2), f'(x)=\cos x$ and $g'(x)=\sin x$ then $\frac{du}{dv}$ is

(a) $\frac{3}{2}x.\cos x^3.\text{cosec}\, x^2$ (b) $\frac{2}{3}\sin x^3.\sec x^2$ (c) tan x (d) None of these

220. Let $f(x)=e^x, g(x)=\sin^{-1}x$ and $h(x)=f(g(x))$, then $h'(x)/h(x)=$

(a) $e^{\sin^{-1}x}$ (b) $1/\sqrt{1-x^2}$ (c) $\sin^{-1}x$ (d) $1/(1-x^2)$

Differentiation of a Function with Respect to Another Function

Basic Level

221. The derivative of $\sin^2 x$ with respect to $\cos^2 x$ is **[DCE 2002]**

(a) $\tan^2 x$ (b) tan x (c) –tan x (d) None of these

222. The differential of e^{x^3} with respect to log$x$ is **[KCET 2002]**

(a) e^{x^3} (b) $3x^2e^{x^3}$ (c) $3x^3e^{x^3}$ (d) $3x^2e^{x^3}+3x^2$

223. The differential coefficient of x^6 with respect to x^3 is **[EAMCET 1988; UPSEAT 2000]**

(a) $5x^2$ (b) $3x^3$ (c) $5x^5$ (d) $2x^3$

224. The rate of change of $\sqrt{x^2+16}$ with respect to $\frac{x}{x-1}$ at x = 3, will be **[MP PET 1987]**

(a) $-\frac{24}{5}$ (b) $\frac{24}{5}$ (c) $\frac{12}{5}$ (d) $-\frac{12}{5}$

225. Differential coefficient of $\sin^{-1}\frac{1-x}{1+x}$ *w.r.t.* $\sqrt{x}$ is **[Roorkee 1984]**

(a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{\sqrt{x}}{\sqrt{1-x}}$ (c) 1 (d) None of these

226. Differential coefficient of $\sec^{-1}\frac{1}{2x^2-1}$ *w.r.t.* $\sqrt{1-x^2}$ at $x=\frac{1}{2}$ is

(a) 2 (b) 4 (c) 6 (d) 1

227. Differential coefficient of $\sin^{-1}x$ *w.r.t.* $\cos^{-1}\sqrt{1-x^2}$ is **[MNR 1983; AMU 2002]**

(a) 1 (b) $\frac{1}{1+x^2}$ (c) 2 (d) None of these

228. The differential coefficient of $\tan^{-1}\sqrt{x}$ with respect to $\sqrt{x}$ is

(a) $\frac{1}{\sqrt{1+x}}$ (b) $\frac{1}{2x\sqrt{1+x}}$ (c) $\frac{1}{2\sqrt{x(1+x)}}$ (d) $\frac{1}{1+x}$

229. Derivative of $\sec^{-1}\left\{\frac{1}{2x^2-1}\right\}$ *w.r.t.* $\sqrt{1+3x}$ at $x=-\frac{1}{3}$ is **[EAMCET 1991]**

(a) 0 (b) 1/2 (c) 1/3 (d) None of these

230. Differential coefficient of $\cos^{-1}(\sqrt{x})$ with respect to $\sqrt{(1-x)}$ is

(a) $\sqrt{x}$ (b) $-\sqrt{x}$ (c) $\frac{1}{\sqrt{x}}$ (d) $-\frac{1}{\sqrt{x}}$

231. Differential coefficient of $\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$ *w.r.t.* $\cos^{-1}(x^2)$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) 0

232. If $u=\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$ and $v=2\tan^{-1}x$, then $\frac{du}{dv}$ is equal to

(a) 4 (b) 1 (c) 1/4 (d) – 1/4

233. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ *w.r.t.* $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is **[Karnataka CET 2000]**

(a) – 1 (b) 1 (c) 2 (d) 4

234. Differential coefficient of $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$ *w.r.t.* $\sin^{-1}x$, is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $\frac{3}{2}$

235. The derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ *w.r.t.* $\cot^{-1}\left(\frac{1-3x^2}{3x-x^2}\right)$ is **[Karnataka CET 2003]**

(a) 1 (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

236. The differential coefficient of $e^{\sin^{-1}x}$ with respect to $\sin^{-1}x$ is

(a) $\cos^{-1}x$ (b) $e^{\cos^{-1}x}$ (c) $e^{\sin^{-1}x}$ (d) $\sin^{-1}x$

Advance Level

237. Differential coefficient of $\frac{\tan^{-1}x}{1+\tan^{-1}x}$ *w.r.t.* $\tan^{-1}x$ is

(a) $\frac{1}{1+\tan^{-1}x}$ (b) $\frac{-1}{1+\tan^{-1}x}$ (c) $\frac{1}{(1+\tan^{-1}x)^2}$ (d) $\frac{-1}{2(1+\tan^{-1}x)^2}$

238. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = 0$, is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

239. Differentiation of $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}(2x\sqrt{1-x^2})$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) – 1

240. Differentiation of $\sin^{-1}(2ax\sqrt{1-a^2x^2})$ with respect to $\sqrt{1-a^2x^2}$ is

(a) 2 (b) ax (c) $\frac{2}{ax}$ (d) $-\frac{2}{ax}$

241. Differentiation of $\tan^{-1}\left(\frac{1+ax}{1-ax}\right)$ with respect to $\sqrt{1+a^2x^2}$ is

(a) $\frac{1}{ax\sqrt{1+ax}}$ (b) $\frac{1}{\sqrt{1+ax}}$ (c) $\frac{1}{ax\sqrt{1+a^2x^2}}$ (d) $\frac{1}{ax\sqrt{1-a^2x^2}}$

242. The value of derivative of $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ *w.r.t.* to $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ at $x=\frac{1}{2}$ equals

(a) 1 (b) – 1 (c) 0 (d) None of these

Successive Differentiation or Higher Order Derivatives

Basic Level

243. If $y=(x^2-1)^m$, then the $(2m)^{th}$ differential coefficient of y is

(a) m (b) $(2m)!$ (c) $2m$ (d) $m!$

244. The n^{th} derivative of xe^x vanishes when **[AMU 1999]**

(a) $x = 0$ (b) $x = -1$ (c) $x = -n$ (d) $x = n$

245. If $x^py^q=(x+y)^{p+q}$, then $\frac{d^2y}{dx^2}=$ **[West Bengal JEE 1992]**

(a) 0 (b) 1 (c) 2 (d) None of these

246. If $y=A\cos nx+B\sin nx$, then $\frac{d^2y}{dx^2}=$ **[Karnataka CET 1996]**

(a) n^2y (b) $-y$ (c) $-n^2y$ (d) None of these

247. If $x = a\sin\theta$ and $y = b\cos\theta$, then $\frac{d^2y}{dx^2}$ is **[UPSEAT 2002]**

(a) $\frac{a}{b^2}\sec^2\theta$ (b) $\frac{-b}{a}\sec^2\theta$ (c) $\frac{-b}{a^2}\sec^3\theta$ (d) $\frac{-b}{a^2\sec^3\theta}$

248. If $y=a\cos(\log x)+\sin(\log x)$, then

(a) $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$ (b) $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-y=0$ (c) $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$ (d) $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$

249. If $e^y+xy=e$, then the value of $\frac{d^2y}{dx^2}$ for $x = 0$, is **[Kurukshetra CEE 2002]**

(a) $\frac{1}{e}$ (b) $\frac{1}{e^2}$ (c) $\frac{1}{e^3}$ (d) None of these

250. If $y=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-......$, then $\frac{d^2y}{dx^2}=$ **[Karnataka CET 2003]**

(a) x (b) $-x$ (c) $-y$ (d) y

251. If $y=ax^{n+1}+bx^{-n}$, then $x^2\frac{d^2y}{dx^2}=$

(a) $n(n-1)y$ (b) $n(n+1)y$ (c) ny (d) n^2y

252. If $y = a + bx^2$; a, b arbitrary constants, then **[EAMCET 1994]**

(a) $\frac{d^2y}{dx^2} = 2xy$ (b) $x\frac{d^2y}{dx^2} = \frac{dy}{dx}$ (c) $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0$ (d) $x\frac{d^2y}{dx^2} = 2xy$

253. If $y = x\log\left(\frac{x}{a+bx}\right)$, then $x^3\frac{d^2y}{dx^2} =$ **[West Bengal JEE 1991; Roorkee 1976]**

(a) $x\frac{dy}{dx} - y$ (b) $\left(x\frac{dy}{dx} - y\right)^2$ (c) $y\frac{dy}{dx} - x$ (d) $\left(y\frac{dy}{dx} - x\right)^2$

254. $\frac{d^2}{dx^2}(2\cos x\cos 3x) =$ **[Rajasthan PET 2003]**

(a) $2^2(\cos 2x + 2^2\cos 4x)$ (b) $2^2(\cos 2x - 2^2\cos 4x)$ (c) $2^2(-\cos 2x + 2^2\cos 4x)$ (d) $-2^2(\cos 2x + 2^2\cos 4x)$

255. If $x = t^2$, $y = t^3$, then $\frac{d^2y}{dx^2} =$

(a) $3/2$ (b) $3/(4t)$ (c) $3/(2t)$ (d) $3t/2$

256. If $y = ae^{mx} + be^{-mx}$, then $\frac{d^2y}{dx^2} - m^2y =$ **[MP PET 1987]**

(a) $m^2(ae^{mx} - be^{-mx})$ (b) 1 (c) 0 (d) None of these

257. If $y = x^2e^{mx}$, where m is a constant, then $\frac{d^3y}{dx^3} =$ **[MP PET 1987]**

(a) $me^{mx}(m^2x^2 + 6mx + 6)$ (b) $2m^3xe^{mx}$ (c) $me^{mx}(m^2x^2 + 2mx + 2)$ (d) None of these

258. If f be a polynomial, then the second derivative of $f(e^x)$ is

(a) $f'(e^x)$ (b) $f''(e^x)e^x + f'(e^x)$ (c) $f''(e^x)e^{2x} + f''(e^x)$ (d) $f''(e^x)e^{2x} + f'(e^x)e^x$

259. If $y = ae^x + be^{-x} + c$ where a, b, c are parameters then $y''' =$

(a) y (b) y' (c) 0 (d) y''

260. If $y = a\cos(\log x) + b\sin(\log x)$ where a, b are parameters then $x^2y'' + xy' =$ **[EAMCET 2002]**

(a) y (b) $-y$ (c) $2y$ (d) $-2y$

261. If $y = x^3\log\log_e(1+x)$ then $y''(0)$ equals **[AMU 1999]**

(a) 0 (b) -1 (c) $6\log_e 2$ (d) 6

262. $\frac{d^2x}{dy^2}$ is equal to **[AMU 2001]**

(a) $\frac{1}{(dy/dx)^2}$ (b) $\frac{(d^2y/dx^2)}{(dy/dx)^2}$ (c) $\frac{d^2y}{dx^2}$ (d) $\frac{(-d^2y/dx^2)}{(dy/dx)^2}$

263. If $x = e^t\sin t$, $y = e^t\cos t$, t is a parameter, then $\frac{d^2y}{dx^2}$ at (1, 1) is equal to **[AMU 2001]**

(a) $-1/2$ (b) $-1/4$ (c) 0 (d) $1/2$

264. $\frac{d^n}{dx^n}(\sin 2x) =$

(a) $\sin\left(\frac{n\pi}{2}+x\right)$ (b) $2^n \sin\left(\frac{n\pi}{2}+2x\right)$ (c) $2^n \sin\left(\frac{\pi}{2}+2x\right)$ (d) None of these

265. $\frac{d^n}{dx^n}(\log x)=$ **[Rajasthan PET 2002]**

(a) $\frac{(n-1)!}{x^n}$ (b) $\frac{n!}{x^n}$ (c) $\frac{(n-2)!}{x^n}$ (d) $(-1)^{n-1}\frac{(n-1)!}{x^n}$

266. $\frac{d^n}{dx^n}(e^{2x}+e^{-2x})=$

(a) $e^{2x}+(-1)^n e^{-2x}$ (b) $2^n(e^{2x}-e^{-2x})$ (c) $2^n[e^{2x}+(-1)^n e^{-2x}]$ (d) None of these

267. If $y=\sin x \sin 3x$, then $y_n=$

(a) $\frac{1}{2}\left[\cos\left(2x+n\frac{\pi}{2}\right)-\cos\left(4x+n\frac{\pi}{2}\right)\right]$ (b) $\frac{1}{2}\left[2^n\cos\left(2x+n\frac{\pi}{2}\right)-4^n\cos\left(4x+n\frac{\pi}{2}\right)\right]$

(c) $\frac{1}{2}\left[4^n\cos\left(4x+n\frac{\pi}{2}\right)-2^n\cos\left(2x+n\frac{\pi}{2}\right)\right]$ (d) None of these

268. The n^{th} derivative of $\frac{x}{1-x}$ is

(a) $\frac{(-1)^n n!}{(1-x)^{n+1}}$ (b) $\frac{n!}{(1-x)^{n+1}}$ (c) $\frac{(-1)^n}{(1-x)^{n+1}}$ (d) $\frac{1}{(1-x)^{n+1}}$

269. If $y=\sin^2 x$, then value of y_n is

(a) $2^n\cos\left(2x+\frac{n\pi}{2}\right)$ (b) $-2^n\cos\left(2x+\frac{n\pi}{2}\right)$ (c) $-2^{n-1}\cos\left(2x+\frac{n\pi}{2}\right)$ (d) None of these

270. If $y = \sin 2x \cos 2x$, then value of y_n is

(a) $2^{2n-1}\sin\left(4x+\frac{n\pi}{2}\right)$ (b) $2^{2n}\sin\left(4x+\frac{n\pi}{2}\right)$ (c) $2^{2n-1}\cos\left(4x+\frac{n\pi}{2}\right)$ (d) None of these

271. If $y=e^{6-5x}$, then the value of y_n is

(a) $5^n e^{6-5x}$ (b) $(-5)^n e^{6-5x}$ (c) $5^{n-1}e^{6-5x}$ (d) $(-5)^{n-1}e^{6-5x}$

272. If $y=8^x$, then the value of y_n is

(a) $\frac{8^x}{\log_e 8}$ (b) $\frac{8^x}{(\log_e 8)^n}$ (c) $8^x\log_e 8$ (d) $8^x(\log_e 8)^n$

273. $D^n[f(ax+b)]$ is equal to

(a) $n!f_n(ax+b)$ (b) $a^n f_n(ax+b)$ (c) $(n-1)!a^n f_n(ax+b)$ (d) 0

274. If $y=x^{n-1}\log x$, then which of the following statement is true

(a) $xy_n=n!$ (b) $xy_n=(n-1)!$ (c) $xy_n=(n-2)!$ (d) $x^2y_n=n!$

Advance Level

275. If $x = f_1(t)$ and $y = f_2(t)$, then $\frac{d^2y}{dx^2} =$

(a) $\frac{f_1' f_2'' - f_2' f_1''}{(f_1')^2}$ (b) $\frac{f_1' f_2'' - f_2' f_1''}{(f_1')^3}$ (c) $\frac{f_1''(t)}{f_2''(t)}$ (d) $\frac{-f_1''(t)}{f_2''(t)}$

276. If $y^2 = p(x)$ is a polynomial of degree three, then $2\frac{d}{dx}\left\{y^3 . \frac{d^2y}{dx^2}\right\} =$ **[IIT 1988; Rajasthan PET 2000]**

(a) $p'''(x) + p'(x)$ (b) $p''(x).p'''(x)$ (c) $p(x).p'''(x)$ (d) Constant

277. If $x = a\cos\theta$, $y = b\sin\theta$, then $\frac{d^3y}{dx^3}$ is equal to

(a) $-\frac{3b}{a^3}\operatorname{cosec}^4\theta \cot^4\theta$ (b) $\frac{3b}{a^3}\operatorname{cosec}^4\theta \cot\theta$ (c) $-\frac{3b}{a^3}\operatorname{cosec}^4\theta\cot\theta$ (d) None of these

278. $\frac{d^{20}y}{dx^{20}}(2\cos x\cos 3x) =$ **[EAMCET 1994]**

(a) $2^{20}(\cos 2x - 2^{20}\cos 4x)$ (b) $2^{20}(\cos 2x + 2^{20}\cos 4x)$ (c) $2^{20}(\sin 2x + 2^{20}\sin 4x)$ (d) $2^{20}(\sin 2x - 2^{20}\sin 4x)$

279. If $u = x^2 + y^2$ and $x = s + 3t$, $y = 2s - t$, then $\frac{d^2u}{ds^2} =$ **[Orissa JEE 2002]**

(a) 12 (b) 32 (c) 36 (d) 10

280. If $y = \sin x + e^x$, then $\frac{d^2x}{dy^2} =$ **[KCET 1999; UPSEAT 2001; Haryana CEE 2002]**

(a) $(-\sin x + e^x)^{-1}$ (b) $\frac{\sin x - e^x}{(\cos x + e^x)^2}$ (c) $\frac{\sin x - e^x}{(\cos x + e^x)^3}$ (d) $\frac{\sin x + e^x}{(\cos x + e^x)^3}$

281. If $x = at^2$, $y = 2at$, then $\frac{d^2y}{dx^2} =$ **[Karnataka CET 1993]**

(a) $-\frac{1}{t^2}$ (b) $\frac{1}{2at^3}$ (c) $-\frac{1}{t^3}$ (d) $-\frac{1}{2at^3}$

282. If $I_n = \frac{d^n}{dx^n}(x^n \log x)$, then $I_n - nI_{n-1} =$ **[EAMCET 2003]**

(a) n (b) $n - 1$ (c) $n\,!$ (d) $(n - 1)!$

283. If $y = (\sin^{-1}x)^2 + k\sin^{-1}x$ then which is true

(a) $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$ (b) $(1+x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$ (c) $(1-x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$ (d) $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$

284. If $y = e^{\tan^{-1}x}$ then which is true

(a) $(1+x^2)y_2 + (2x-1)y_1 = 0$ (b) $(1+x^2)y_2 + (2x+1)y_1 = 0$ (c) $(1+x^2)y_2 - (2x-1)y_1 = 0$ (d) $(1+x^2)y_2 - (2x+1)y_1 = 0$

285. The function $u = e^x\sin x$, $v = e^x\cos x$ satisfy the equation

(a) $v\frac{du}{dv} = u\frac{dv}{dx} + u^2 + v^2$ (b) $\frac{d^2u}{dx^2} = 2v$ (c) $\frac{d^2v}{dx^2} = -2u$ (d) None of these

286. If $x^2+y^2=a^2$ and $k=\frac{1}{a}$, then k is equal to

(a) $\frac{y''}{\sqrt{1+y'}}$ (b) $\frac{|y''|}{\sqrt{(1+y'^2)^3}}$ (c) $\frac{2y''}{\sqrt{1+y'^2}}$ (d) $\frac{y''}{2\sqrt{(1+y'^2)^3}}$

287. If $(a+bx)e^{y/x}=x$, then the value of $x^3\frac{d^2y}{dx^2}$ is

(a) $\left(y\frac{dy}{dx}-x\right)^2$ (b) $\left(x\frac{dy}{dx}-y\right)^2$ (c) $x\frac{dy}{dx}-y$ (d) None of these

288. If $y=[\log(x+\sqrt{x^2+1})]^2$ then which is correct

(a) $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=2$ (b) $(1+x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$ (c) $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$ (d) $(1+x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0$

289. If $y=\frac{1}{x^2-a^2}$, then $\frac{d^2y}{dx^2}$ equals

(a) $\frac{3x^2+a^2}{(x^2-a^2)^3}$ (b) $\frac{3x^2+a^2}{(x^2-a^2)^4}$ (c) $\frac{2(3x^2+a^2)}{(x^2-a^2)^3}$ (d) $\frac{2(3x^2+a^2)}{(x^2-a^2)^4}$

290. If $y^{1/m}+y^{-1/m}=2x$, then $(x^2-1)y_2+xy_1$ is equal to

(a) m^2y (b) $-m^2y$ (c) $\pm m^2y$ (d) $\pm my$

291. If $y=(\sin^{-1}x)^2+(\cos^{-1}x)^2$, then $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}$ is equal to

(a) 4 (b) 3 (c) 1 (d) 0

292. If $y=e^{\sqrt{x}}+e^{-\sqrt{x}}$, then $xy_2+\frac{1}{2}y_1-\frac{1}{4}y$ is equal to

(a) 0 (b) 1 (c) – 1 (d) 2

293. If $y=\sin 2x$ then $\frac{d^6y}{dx^6}$ at $x=\frac{\pi}{2}$ is equal to

(a) – 64 (b) 0 (c) 64 (d) None of these

294. $\frac{d^n}{dx^n}\cos^2 x=$

(a) $2^{n-1}\cos\left(2x+\frac{\pi}{2}\right)$ (b) $2^{n-1}\cos\left(2x-\frac{\pi}{2}\right)$ (c) $2^{n-1}\cos\left(2x+\frac{n\pi}{2}\right)$ (d) $2^{n-1}\cos\left(2x-\frac{n\pi}{2}\right)$

295. If $y=\cos^4 x$, then y_n is equal to

(a) $2^{2n-3}\cos\left(4x+\frac{n\pi}{2}\right)+2^{n-1}\cos\left(2x+\frac{n\pi}{2}\right)$ (b) $2^{2n-3}\cos\left(2x+\frac{n\pi}{2}\right)+2^{n-1}\cos\left(4x+\frac{n\pi}{2}\right)$

(c) $\cos\left(4x+\frac{n\pi}{2}\right)+\cos\left(2x+\frac{n\pi}{2}\right)$ (d) None of these

296. If $y=\sin^2 x\sin 2x$ then y_n is equal to

(a) $2^{n-1}\sin\left(2x+\frac{n\pi}{2}\right)+4^{n-1}\sin\left(4x+\frac{n\pi}{2}\right)$ (b) $2^{n-1}\sin\left(2x+\frac{n\pi}{2}\right)+4^{n-1}\sin\left(4x+\frac{n\pi}{2}\right)$

(c) $2\sin\left(2x+\frac{n\pi}{2}\right)+\sin\left(4x+\frac{n\pi}{2}\right)$

(d) None of these

n^{th} Derivative Using Partial Fractions

Basic Level

297. n^{th} derivative of $\frac{1}{3x^2-5x+2}$ is

(a) $(-1)^n n!\left\{\frac{1}{(x-1)^{n+1}}+\frac{3^{n+1}}{(3x-2)^{n+1}}\right\}$

(b) $n!\left\{\frac{1}{(x-1)^{n+1}}-\frac{3^{n+1}}{(3x-2)^{n+1}}\right\}$

(c) $(-1)^n n!\left\{\frac{1}{(x-1)^{n+1}}-\frac{3^{n+1}}{(3x-2)^{n+1}}\right\}$

(d) None of these

298. n^{th} derivative of $\frac{1}{x^2+5x+6}$ is

(a) $(-1)^n n!\left[\frac{1}{(x+2)^{n+1}}+\frac{1}{(x+3)^{n+1}}\right]$ (b) $(-1)^n n!\left[\frac{1}{(x+3)^{n+1}}-\frac{1}{(x+2)^{n+1}}\right]$ (c) $(-1)^n n!\left[\frac{1}{(x+2)^{n+1}}-\frac{1}{(x+3)^{n+1}}\right]$ (d) None of these

Advance Level

299. n^{th} derivative of $\frac{2x+3}{5x+7}$

(a) $\frac{(-1)^n n!5^{n-1}}{(5x+7)^{n+1}}$

(b) $\frac{(-1)^n n!5^{n-1}}{(5x+7)^{n-1}}$

(c) $\frac{(-1)^n n!5^{n+1}}{(5x+7)^{n+1}}$

(d) $\frac{(-1)^n n!5^{n+1}}{(5x+7)^{n-1}}$

300. n^{th} derivative of $\frac{1}{x^2-a^2}$ is

(a) $\frac{(-1)^n n!}{2a}[(x-a)^{n+1}-(x+a)^{n-1}]$

(b) $\frac{(-1)^n n!}{2a}[(x+a)^{n+1}-(x-a)^{n+1}]$

(c) $\frac{(-1)^n n!}{2a}\left[\frac{1}{(x-a)^{n+1}}-\frac{1}{(x+a)^{n+1}}\right]$

(d) $\frac{(-1)^n n!}{2a}[(x-a)^{n+1}+(x+a)^{n+1}]$

Differentiation of Determinants

Basic Level

301. If $f(x)=\begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$, then $f'(x)$ is

(a) x^2

(b) $6x$

(c) $6x^2$

(d) 1

302. If $f(x)=\begin{vmatrix} \sec\theta & \tan^2\theta & 1 \\ \theta\sec x & \tan x & x \\ 1 & \tan x-\tan\theta & 0 \end{vmatrix}$, then $f'(\theta)$ is

(a) 0 (b) – 1 (c) Independent of θ (d) None of these

303. Let f, g, h and k be differentiable in (a, b) and F is defined as $F(x) = \begin{vmatrix} f(x) & g(x) \\ h(x) & k(x) \end{vmatrix}$ for all $x \in (a,b)$ then F' is given by

(a) $\begin{vmatrix} f & g \\ h & k \end{vmatrix} + \begin{vmatrix} f & g \\ h' & k \end{vmatrix}$ (b) $\begin{vmatrix} f' & g' \\ h & k \end{vmatrix} + \begin{vmatrix} f & g \\ h' & k' \end{vmatrix}$ (c) $\begin{vmatrix} f & g' \\ h & k' \end{vmatrix} + \begin{vmatrix} f' & g \\ h & k' \end{vmatrix}$ (d) $\begin{vmatrix} f & g \\ h' & k' \end{vmatrix} + \begin{vmatrix} f' & g \\ h & k \end{vmatrix}$

Advance Level

304. $f(x) = \begin{vmatrix} x^3 & x^2 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$, here p is a constant, then $\frac{d^3 f(x)}{dx^3}$ is

(a) Proportional to x^2 (b) Proportional to x (c) Proportional to x^3 (d) A constant

305. If $y = \sin px$ and y_n is the n^{th} derivative of y, then $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$ is equal to **[AMU 2002]**

(a) 1 (b) 0 (c) – 1 (d) None of these

Differentiation of Integral Functions

Basic Level

306. Let $f(t) = \log(t)$, then $\frac{d}{dx}\left(\int_{x^2}^{x^3} f(t)dt\right)$

(a) Has a value 0 when x = 0

(b) Has a value 0 when x = 1 and $x = \frac{4}{9}$

(c) Has a value $9e^2 - 4$ when $x = e$

(d) Has a differential coefficient 27e – 8 for $x = e$

307. If $f(x) = \int_0^x t \sin t \, dt$, then $f'(x) =$

(a) $x \sin x$ (b) $x \cos x$ (c) $\sin x + \cos x$ (d) $x^2/2$

Advance Level

308. If $F(x) = \frac{1}{x^2}\int_4^x (4t^2 - 2F'(t))\,dt$, then $F'(4)$ equals

(a) $32/9$ (b) $64/3$ (c) $64/9$ (d) None of these

Leibneitz's theorem

Basic Level

309. If $y = x \sin x$, then at $x = 0$ the value of y_{15} equal to

(a) 0 (b) – 15 (c) 15! (d) –(15)!

Advance Level

310. If $y = xe^x$ then the value of y_n is

(a) $(n+1)e^x$ (b) $(x+1)e^x$ (c) $(x+n)e^x$ (d) $(x-n)e^x$

Miscellaneous Problems

Basic Level

311. Given that $d/dx\, f(x) = f'(x)$. The relationship $f'(a+b) = f'(a) + f'(b)$ is valid if $f(x)$ is equal to

(a) x (b) x^2 (c) x^3 (d) x^4

312. $f(x)$ and $g(x)$ are two differentiable function on [0, 2] such that $f''(x) - g''(x) = 0, f'(1) = 2, g'(1) = 4, f(2) = 3, g(2) = 9$, then $f(x) - g(x)$ at $x = 3/2$ is

(a) 0 (b) 2 (c) 10 (d) – 5

313. If $y = \dfrac{ax^2}{(x-a)(x-b)(x-c)} + \dfrac{bx}{(x-b)(x-c)} + \dfrac{c}{x-c} + 1$, then $\dfrac{y'}{y} =$ **[IIT 1998]**

(a) $\left(\dfrac{a}{a-x} + \dfrac{b}{b-x} + \dfrac{c}{c-x}\right)$ (b) $\left(\dfrac{a}{a+x} + \dfrac{b}{b+x} + \dfrac{c}{c+x}\right)$ (c) $\dfrac{1}{x}\left(\dfrac{a}{a-x} + \dfrac{b}{b-x} + \dfrac{c}{c-x}\right)$ (d) $\dfrac{1}{y}\left(\dfrac{a}{a-x} + \dfrac{b}{b-x} + \dfrac{c}{c-x}\right)$

314. If $y = \dfrac{1}{1 + x^{a-b} + x^{a-c}} + \dfrac{1}{1 + x^{b-a} + x^{b-c}} + \dfrac{1}{1 + x^{c-a} + x^{c-b}}$, then $\dfrac{dy}{dx}$ equals

(a) $ax^{-1} + bx^{-1} + cx^{-1}$ (b) 0 (c) 1 (d) $a + b + c$

315. Let $f(x)$ be a polynomial function of the second degree. If $f(1) = f(-1)$ and a_1, a_2, a_3 are in A.P. then $f'(a_1), f'(a_2), f'(a_3)$ are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

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Answer Sheet

Assignment (Basic & Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d	d	c	b	d	c	b	a	b	b	a	b	d	a	a	c	a	d	c	b
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
d	d	c	b	b	d	d	a	c	c	d	c	d	a	b	c	d	c	d	d
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
d	b	a	b	a	d	c	b	b	a	c	b	b	b	d	b	c	c	c	a
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
a	a	c	c	a	a	a	a	c	c	a	a	a	b	a	c	c	a	c	c
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
a	d	c	b	b	b	a	c	c	a	d	a	c	c	b	a	a	c	a	a
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
d	d	c	c	a	b	c	a	b	c	d	b	b	d	a	c	b	c	a	d
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
c	c	a	b	d	a	a	a	a	a	a	c	c	c	a	a	b	a	b	b
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
b	c	b	b	a	b	c	a	c	b	c	c	c	c	c	c	c	d	a	c
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
a	a	b	c	b	a	c	c	a	a	a	a	b	a	b	b	a	b	c	a
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
a	d	a	a	b	b	a	a	a	d	b	b	a	a	a	c	c	a	a	a
201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220
d	c	b	b	d	a	b	c	a	a	c	c	b	d	b	a	b	c	a	b
221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240
d	c	d	d	d	b	a	d	a	c	a	c	b	a	c	a	c	b	b	d
241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260
c	b	b	c	a	c	c	d	b	d	b	b	b	d	b	c	a	d	b	b
261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280
a	d	a	b	d	c	b	b	a	a	b	d	b	b	b	c	c	b	d	c
281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300
b	d	a	a	b	b	c	a	c	a	a	a	a	c	c	b	c	c	a	c
301	302	303	304	305	306	307	308	309	310	311	312	313	314	315					
c	c	b	d	b	b	a	a	d	c	b	d	c	b	a					