

11.EXPONENTS

11.0 Introduction

The population of India according to 2011 census is about 120,00,00,000

The approximate distance between the sun and the earth is 15,00,00,000 km.

The speed of the light in vacuum is about 30,00,00,000 m/sec. Light travels a distance of 30,00,00,000 mts., approximately in 1 sec.,

The population of Andhra Pradesh according to 2011 census is about 8,50,00,000.

These are all very large numbers. Do you find it easy to read, write and understand such large numbers? No, certainly not.

Thus, we need a way in which we can represent such larger numbers in a simpler manner. Exponents help us in doing so. In this chapter you will learn more about exponents and the laws of exponents.

11.1 Exponential Form

Let us consider the following repeated additions:

$$4 + 4 + 4 + 4 + 4$$

$$5 + 5 + 5 + 5 + 5 + 5$$

$$7 + 7 + 7 + 7 + 7 + 7 + 7 + 7$$

We use multiplication to shorten the representation of repeated additions by writing 5×4 , 6×5 and 8×7 respectively.

Now can we express repeated multiplication of a number by itself in a simpler way?

Let us consider the following illustrations.

The population of Bihar as per the 2011 Census is about 10,00,00,000.

Here 10 is multiplied by itself for 8 times i.e. $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$.

So we can write the population of Bihar as 10^8 . Here 10 is called the base and 8 is called the exponent. 10^8 is said to be in exponential form and it is read as 10 raised to the power of 8.

The speed of light in vacuum is 30,00,00,000 m/sec. This is expressed as 3×10^8 m/sec in exponential form. In 3×10^8 , 10^8 is read as '*10 raised to the power of 8*'. 10 is the base and 8 is the exponent.

The approximate distance between the sun and the earth is 15,00,00,000 km. This is expressed as 15×10^7 km in exponential form. In 10^7 , 10 is the base and 7 is the exponent.

The population of Andhra Pradesh according to 2011 census is about 8,50,00,000. This is expressed as 85×10^6 in exponential form. 10^6 is read as '*10 raised to the power of 6*'. Here 10 is the base and 6 is the exponent.

We can also use exponents in writing the expanded form of a given number for example the expanded form of 36584 =
 $(3 \times 10000) + (6 \times 1000) + (5 \times 100) + (8 \times 10) + (4 \times 1)$
 $= (3 \times 10^4) + (6 \times 10^3) + (5 \times 10^2) + (8 \times 10^1) + (4 \times 1)$

Do This



1. Write the following in exponential form. (values are rounded off)

(i) Total surface area of the Earth is 510,000,000 square kilometers.

(ii) Population of Rajasthan is approximately 7,00,00,000

(iii) The approximate age of the Earth is 4550 million years.

(iv) 1000 km in meters

2. Express (i) 48951 (ii) 89325 in expanded form using exponents.

11.1.1 Exponents with other bases

So far we have seen numbers whose base is 10. However, the base can be any number.

For example $81 = 3 \times 3 \times 3 \times 3 = 3^4$

Here 3 is the base and 4 is the exponent.

Similarly, $125 = 5 \times 5 \times 5 = 5^3$

Here 5 is the base and 3 is the exponent.

Example 1: Which is greater 3^4 or 4^3 ?

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$

$$4^3 = 4 \times 4 \times 4 = 64$$

$$81 > 64$$

Therefore, $3^4 > 4^3$

Do This

1. Is 3^2 equal to 2^3 ? Justify.
2. Write the following numbers in exponential form. Also state the
(a) base (b)exponent and (c) how it is read.
(i) 32 (ii) 64 (iii) 256 (iv) 243 (v) 49

Squared and cubed

When any base is raised to the power 2 or 3, it has a special name.

$10^2 = 10 \times 10$ and is read as '**10 raised to the power 2**' or '**10 squared**'.

Similarly, $4^2 = 4 \times 4$ and can be read as '**4 raised to the power of 2**' or '**4 squared**'.

$10 \times 10 \times 10 = 10^3$ is read as '**10 raised to the power 3**' or '**10 cubed**'.

Similarly, $6 \times 6 \times 6 = 6^3$ and can be read as '**6 raised to the power 3**' or '**6 cubed**'.

In general, we can take any positive number 'a' as the base and write.

$a \times a = a^2$ (this is read as 'a raised to the power of 2' or 'a squared')

$a \times a \times a = a^3$ (this is read as 'a raised to the power of 3' or 'a cubed')

$a \times a \times a \times a = a^4$ (this is read as 'a raised to the power of 4')

_____ $= a^5$ (_____)

_____ $= a^6$ (_____) and so on.

Thus, we can say that $a \times a \times a \times a \times a \times a \times \dots$ 'm' times $= a^m$ where 'a' is the base and 'm' is the exponent.

Do This

1. Write the expanded form of the following.
(i) p^7 (ii) l^4 (iii) s^9 (iv) d^6 (v) z^5
2. Write the following in exponential form.
(i) $a \times a \times a \times \dots$ 'l' times
(ii) $5 \times 5 \times 5 \times 5 \dots$ 'n' times
(iii) $q \times q \times q \times q \times q \dots$ 15 times
(iv) $r \times r \times r \times \dots$ 'b' times

11.2 Writing a number in exponential form through prime factorization.

Let us express the following numbers in the exponential form using prime factorization.

- (i) 432 (ii) 450

Solution (i): $432 = 2 \times 216$

$$= 2 \times 2 \times 108$$

$$= 2 \times 2 \times 2 \times 54$$

$$= 2 \times 2 \times 2 \times 2 \times 27$$

$$= 2 \times 2 \times 2 \times 2 \times 3 \times 9$$

$$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$= (2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3)$$

$$= 2^4 \times 3^3$$

Therefore, $432 = 2^4 \times 3^3$

(ii) $450 = 2 \times 225$

$$= 2 \times 3 \times 75$$

$$= 2 \times 3 \times 3 \times 25$$

$$= 2 \times 3 \times 3 \times 5 \times 5$$

$$= 2 \times 3^2 \times 5^2$$

Therefore, $450 = 2 \times 3^2 \times 5^2$

Do This

Write the following in exponential form using prime factorization.

(i) 2500 (ii) 1296 (iii) 8000 (iv) 6300

Exercise - 1

1. Write the base and the exponent in each case. Also, write the term in the expanded form.

(i) 3^4 (ii) $(7x)^2$ (iii) $(5ab)^3$ (iv) $(4y)^5$

2. Write the exponential form of each expression.

(i) $7 \times 7 \times 7 \times 7 \times 7$

(ii) $3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5$

(iii) $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

3. Express the following as the product of exponents through prime factorization.

(i) 288 (ii) 1250 (iii) 2250 (iv) 3600 (v) 2400

4. Identify the greater number in each of the following pairs.

(i) 2^3 or 3^2 (ii) 5^3 or 3^5 (iii) 2^8 or 8^2

5. If $a = 3$, $b = 2$ find the value of (i) $a^b + b^a$ (ii) $a^a + b^b$ (iii) $(a + b)^b$ (iv) $(a - b)^a$

11.3 Laws of exponents

When we multiply terms with exponents we use some rules to find the product easily. These rules have been discussed here.

11.3.1 Multiplying terms with the same base

Example 2 : $2^4 \times 2^3$

Solution : $2^4 \times 2^3 = (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$$\begin{array}{c} \underbrace{\hspace{1.5cm}} \quad \underbrace{\hspace{1.5cm}} \\ 4 \text{ times} \quad \quad 3 \text{ times} \\ = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ \underbrace{\hspace{2.5cm}} \\ 7 \text{ times} \end{array}$$

$$= 2^7 \text{ and this is same as } 2^{4+3} \text{ (as } 4 + 3 = 7\text{)}$$

$$\text{Therefore, } 2^4 \times 2^3 = 2^{4+3}$$

Example 3: $5^2 \times 5^3$

Solution : $5^2 \times 5^3 = (5 \times 5) \times (5 \times 5 \times 5)$

$$\begin{array}{c} \underbrace{\hspace{1.5cm}} \quad \underbrace{\hspace{1.5cm}} \\ 2 \text{ times} \quad \quad 3 \text{ times} \\ = 5 \times 5 \times 5 \times 5 \times 5 \\ \underbrace{\hspace{2.5cm}} \\ 5 \text{ times} \end{array}$$

$$= 5^5 \text{ and this is same as } 5^{2+3} \text{ (as } 2 + 3 = 5\text{)}$$

$$\text{Therefore, } 5^2 \times 5^3 = 5^{2+3}$$

Do This

Find the values of 2^4 , 2^3 and 2^7

verify whether $2^4 \times 2^3 = 2^7$

Find the values of 5^2 , 5^3 and 5^5 and verify whether $5^2 \times 5^3 = 5^5$

Example 4 : $a^4 \times a^5$

Solution: $a^4 \times a^5 = (a \times a \times a \times a) \times (a \times a \times a \times a \times a)$

$$= (a \times a \times a \times a \times a \times a \times a \times a \times a)$$

$$= a^9 \text{ and this is same as } a^{4+5} \text{ (as } 4 + 5 = 9\text{)}$$

$$\text{Therefore, } a^4 \times a^5 = a^{4+5}$$

Based on the above observations we can say that.

$$a^m \times a^n = (a \times a \times a \dots \dots \dots 'm' \text{ times}) \times (a \times a \times a \times \dots \dots \dots 'n' \text{ times}) = a^{m+n}$$

For any non-zero integer 'a', and integers 'm' and 'n'

$$a^m \times a^n = a^{m+n}$$

Do This

1. Simplify the following using the formula $a^m \times a^n = a^{m+n}$
(i) $3^{11} \times 3^9$ (ii) $p^5 \times p^8$
2. Find the appropriate number in place of the symbol ‘?’ in the following.

Let 'k' be any non zero integer

(i) $k^3 \times k^4 = k^?$ (ii) $k^{15} \times k^? = k^{31}$

11.3.2 Exponent of exponent

Example 5 : Consider $(3^2)^3$

Solution : Here ' 3^2 ' is the base and '3' is the exponent

$$\begin{aligned}(3^2)^3 &= 3^2 \times 3^2 \times 3^2 \\ &= 3^{2+2+2} \text{ (multiplying terms with the same base)} \\ &= 3^6 \text{ and this is the same as } 3^{2 \times 3} \text{ (as } 2 \times 3 = 6\text{)} \\ \text{Therefore, } (3^2)^3 &= 3^{2 \times 3}\end{aligned}$$

Do This

Compute 3^6 , cube of 3^2 and verify whether $(3^2)^3 = 3^6$?

Example 6 : Let us consider $(4^5)^3$

Solution : $(4^5)^3 = 4^5 \times 4^5 \times 4^5$
 $= 4^{5+5+5}$ (multiplying terms with the same base)
 $= 4^{15}$ and this is same as $4^{5 \times 3}$ (as $5 \times 3 = 15$)
 Therefore, $(4^5)^3 = 4^{5 \times 3}$

Example 7: $(a^m)^4$

Solution : $(a^m)^4 = a^m \times a^m \times a^m \times a^m$
 $= a^{m+m+m+m}$ (multiplying terms with the same base)
 $= a^{4m}$ and this is same as $a^{m \times 4}$ (as $4 \times m = 4m$)
 Therefore, $(a^m)^4 = a^{m \times 4}$

Based on all the above we can say that $(a^m)^n = a^m \times a^m \times a^m \dots n \text{ times} = a^{m+m+m+\dots n \text{ times}} = a^{mn}$

For any non-zero integer ‘a’ and integers ‘m’ and ‘n’

$$(a^m)^n = a^{mn}$$

11.3.3 Exponent of a product

Example 8 : Consider $3^5 \times 4^5$

Solution : Here 3^5 and 4^5 have the same exponent 5 but different bases.

$$\begin{aligned} 3^5 \times 4^5 &= (3 \times 3 \times 3 \times 3 \times 3) \times (4 \times 4 \times 4 \times 4 \times 4) \\ &= (3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4) \\ &= (3 \times 4)^5 \end{aligned}$$

Therefore, $3^5 \times 4^5 = (3 \times 4)^5$

Example 9: Consider $4^4 \times 5^4$

Solution : Here 4^4 and 5^4 have the same exponent 4 but have different bases.

$$\begin{aligned} 4^4 \times 5^4 &= (4 \times 4 \times 4 \times 4) \times (5 \times 5 \times 5 \times 5) \\ &= (4 \times 4 \times 4 \times 4 \times 5 \times 5 \times 5 \times 5) \\ &= (4 \times 5) \times (4 \times 5) \times (4 \times 5) \times (4 \times 5) \\ &= (4 \times 5)^4 \end{aligned}$$

Therefore, $4^4 \times 5^4 = (4 \times 5)^4$

Example 10 : Consider $p^7 \times q^7$

Solution : Here p^7 and q^7 have the same exponent 7 but different bases.

$$\begin{aligned}
 p^7 \times q^7 &= (\mathbf{p} \times \mathbf{p} \times \mathbf{p} \times \mathbf{p} \times \mathbf{p} \times \mathbf{p} \times \mathbf{p}) \times (\mathbf{q} \times \mathbf{q} \times \mathbf{q} \times \mathbf{q} \times \mathbf{q} \times \mathbf{q} \times \mathbf{q}) \\
 &= (\mathbf{p} \times \mathbf{p} \times \mathbf{p} \times \mathbf{p} \times \mathbf{p} \times \mathbf{p} \times \mathbf{p} \times \mathbf{q} \times \mathbf{q} \times \mathbf{q} \times \mathbf{q} \times \mathbf{q} \times \mathbf{q} \times \mathbf{q}) \\
 &= (\mathbf{p} \times \mathbf{q}) \times (\mathbf{p} \times \mathbf{q}) \times (\mathbf{p} \times \mathbf{q}) \times (\mathbf{p} \times \mathbf{q}) \times (\mathbf{p} \times \mathbf{q}) \times (\mathbf{p} \times \mathbf{q}) \times (\mathbf{p} \times \mathbf{q}) \\
 &= (\mathbf{p} \times \mathbf{q})^7
 \end{aligned}$$

Therefore, $p^7 \times q^7 = (p \times q)^7$

Based on all the above we can conclude that $a^m \times b^m = (a \times b)^m = (ab)^m$

For any two non-zero integers 'a', 'b' and any positive integer 'm'

$$a^m \times b^m = (ab)^m$$

Do This

1. Simplify the following using the law $a^m \times b^m = (a \times b)^m$

(i) $(2 \times 3)^4$ (ii) $x^p \times y^p$ (iii) $a^8 \times b^8$ (iv) $(5 \times 4)^{11}$

11.3.4 Division of exponents

Before discussing division of exponents we will now discuss about negative exponents.

11.3.4(a) Negative exponents

Observe the following pattern.

$2^5 = 32$	$3^5 = 243$
$2^4 = 16$	$3^4 = 81$
$2^3 = 8$	$3^3 = 27$
$2^2 = 4$	$3^2 = 9$
$2^1 = 2$	$3^1 = 3$
$2^0 = 1$	$3^0 = 1$
$2^{-1} = \dots\dots$	$3^{-1} = \dots\dots$
(Hint: half of 1)	(Hint: one-third of 1)
$2^{-2} = \dots\dots$	$3^{-2} = \dots\dots$

What part of 32 is 16?

What is the difference between 2^5 and 2^4 ?

You will find that each time the exponent decreases by 1, the value becomes half of the previous.

From the above patterns we can say.

$$2^{-1} = \frac{1}{2} \text{ and } 2^{-2} = \frac{1}{4} \text{ and}$$

$$3^{-1} = \frac{1}{3} \text{ and } 3^{-2} = \frac{1}{9}$$

Furthermore, we can see that $2^{-2} = \frac{1}{4} = \frac{1}{2^2}$

similarly, $3^{-1} = \frac{1}{3}$ and $3^{-2} = \frac{1}{9} = \frac{1}{3^2}$

For any non-zero integer 'a' and any integer 'n'

$$a^{-n} = \frac{1}{a^n}$$

Do This

1. Write the following, by using $a^{-n} = \frac{1}{a^n}$, with positive exponents.

(i) x^{-7} (ii) a^{-5} (iii) 7^{-5} (iv) 9^{-6}

11.3.4(b) Zero exponents

In the earlier discussion we have seen that

$$2^0 = 1, 3^0 = 1$$

Similarly we can say $4^0 = 1$ $5^0 = 1$ and so on

Thus for non zero integer 'a'

$$a^0 = 1$$

11.3.4(c) Division of exponents having the same base

Example 11 : Consider $\frac{7^7}{7^3}$

Solution : $\frac{7^7}{7^3} = \frac{7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7} = 7 \times 7 \times 7 \times 7$

$$= 7^4 \text{ which is same as } 7^{7-3} \text{ (as } 7 - 3 = 4 \text{)}$$

Therefore, $\frac{7^7}{7^3} = 7^{7-3}$

Example 12: Consider $\frac{3^8}{3^3}$

Solution : $\frac{3^8}{3^3} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3} = 3 \times 3 \times 3 \times 3$
 $= 3^5$ which is same as 3^{8-3} (as $8 - 3 = 5$)

Therefore, $\frac{3^8}{3^3} = 3^{8-3}$

Example 13: Consider $\frac{5^8}{5^5}$

Solution : $\frac{5^8}{5^5} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5} = \frac{1}{5 \times 5 \times 5} = \frac{1}{5^3}$
 $\frac{1}{5^3}$ which is same as $\frac{1}{5^{8-5}}$ (as $8 - 5 = 3$)

Therefore, $\frac{5^8}{5^5} = \frac{1}{5^{8-5}}$

Example 14: Consider $\frac{a^2}{a^7}$

Solution : $\frac{a^2}{a^7} = \frac{a \times a}{a \times a \times a \times a \times a \times a \times a} = \frac{1}{a \times a \times a \times a \times a}$
 $= \frac{1}{a^5}$ which is the same as $\frac{1}{a^{7-2}}$ (as $7 - 2 = 5$)

Therefore, $\frac{a^2}{a^7} = \frac{1}{a^{7-2}}$

Based on all the above examples we can say that-

$\frac{a^m}{a^n} = a^{m-n}$ if $m > n$ and $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ if $m < n$
Non-zero integer 'a' and integers 'm', 'n'
 $\frac{a^m}{a^n} = a^{m-n}$ if $m > n$ and $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ if $n > m$

What happens when $m = n$? Give your answer.

Example 15 : Consider $\frac{4^3}{4^3}$

Solution : $\frac{4^3}{4^3} = \frac{4 \times 4 \times 4}{4 \times 4 \times 4} = \frac{1}{1} = 1$ (1)

Also we know that $\frac{a^m}{a^n} = a^{m-n}$
 $\therefore \frac{4^3}{4^3} = 4^{3-3} = 4^0 = 1$ from (1)

Similarly find $\frac{7^4}{7^4} = ?$

What do you observe from above?

Also consider $\frac{a^4}{a^4} = \frac{a \times a \times a \times a}{a \times a \times a \times a} = 1$

But from $\frac{a^m}{a^n} = a^{m-n}$

We have $\frac{a^4}{a^4} = a^{4-4} = a^0 = 1$

For any non zero number 'a' we have $a^0 = 1$.

Observe here m, n ($m = n$)

$\frac{a^m}{a^n} = 1$

Thus if $m = n$

Do This

1. Simplify and write in the form of a^{m-n} or $\frac{1}{a^{n-m}}$

(i) $\frac{13^8}{13^5}$ (ii) $\frac{3^4}{3^{14}}$

2. Fill the appropriate number in the box.

Ex : $\frac{8^8}{8^3} = 8^{[8-3]} = 8^{[5]}$

$$(i) \frac{12^{12}}{12^7} = 12^{\square} = 12^{\square} \quad (ii) \frac{a^{18}}{a^{\square}} = a^{\square} = a^{10}$$

11.3.4(d) Dividing terms with the same exponents

Example 16: Expand and simplify $\left(\frac{7}{4}\right)^5$

Solution :

$$\begin{aligned} \left(\frac{7}{4}\right)^5 &= \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4} \\ &= \frac{7 \times 7 \times 7 \times 7 \times 7}{4 \times 4 \times 4 \times 4 \times 4} \\ &= \frac{7^5}{4^5} \text{ (by the definition of exponent)} \end{aligned}$$

Therefore, $\left(\frac{7}{4}\right)^5 = \frac{7^5}{4^5}$

Example 17: Expand and simplify $\left(\frac{p}{q}\right)^6$

Solution :

$$\begin{aligned} \left(\frac{p}{q}\right)^6 &= \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right) \\ &= \frac{p \times p \times p \times p \times p \times p}{q \times q \times q \times q \times q \times q} \\ &= \frac{p^6}{q^6} \text{ (By the definition of exponent)} \end{aligned}$$

Therefore, $\left(\frac{p}{q}\right)^6 = \frac{p^6}{q^6}$

Based on the above observations we can say that.

$$\left(\frac{a}{b}\right)^m = \frac{a \times a \times a \times \dots \times a \text{ 'm' times}}{b \times b \times b \times \dots \times b \text{ 'm' times}} = \frac{a^m}{b^m}$$

For any non-zero integers a, b and integer 'm' $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Do This

1. Complete the following

(i) $\left(\frac{5}{7}\right)^3 = \frac{5^3}{\square}$ (ii) $\left(\frac{3}{2}\right)^{\square} = \frac{3^5}{2^5}$

(iii) $\left(\frac{8}{3}\right)^4 = \frac{\square}{\square}$ (iv) $\left(\frac{x}{y}\right)^{11} = \frac{\square}{y^{11}}$

11.3.5 Terms with negative base

Example 18 : Evaluate $(1)^4$, $(1)^5$, $(1)^7$, $(-1)^2$, $(-1)^3$, $(-1)^4$, $(-1)^5$

Solution : $(1)^4 = 1 \times 1 \times 1 \times 1 = 1$

$(1)^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1$

$(1)^7 = 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$

$(-1)^2 = (-1) \times (-1) = 1$

$(-1)^3 = (-1) \times (-1) \times (-1) = -1$

$(-1)^4 = (-1) \times (-1) \times (-1) \times (-1) = 1$

$(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$

From the above illustrations we observe that:

(i) 1 raised to any power is 1.

(ii) (-1) raised to odd power is (-1) and (-1) raised to even power is $(+1)$.

Thus $(-a)^m = -a^m$ If 'm' is odd

$(-a)^m = a^m$ If 'm' is even

Now, let us look at some more examples.

$(-3)^4 = (-3) (-3) (-3) (-3) = 81$

$(-a)^4 = (-a) (-a) (-a) (-a) = a^4$

$$(-a)^{-3} = \frac{1}{(-a)^3} = \frac{1}{(-a)} \times \frac{1}{(-a)} \times \frac{1}{(-a)} = \frac{1}{(-a)^3} = \frac{-1}{a^3}$$

Example 19 : Express $\frac{-27}{125}$ in exponential form

Solution : $-27 = (-3)(-3)(-3) = (-3)^3$

$$125 = 5 \times 5 \times 5 = (5)^3$$

Therefore, $\frac{-27}{125} = \frac{(-3)^3}{(5)^3}$ as $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

Thus, $\frac{-27}{125} = \left(\frac{-3}{5}\right)^3$

Do This

1. Write in expanded form.

(i) $(a)^{-5}$ (ii) $(-a)^4$ (iii) $(-7)^{-5}$ (iv) $(-a)^m$

2. Write in exponential form

(i) $(-3) \times (-3) \times (-3)$ (ii) $(-b) \times (-b) \times (-b) \times (-b)$

(iii) $\frac{1}{(-2)} \times \frac{1}{(-2)} \times \frac{1}{(-2)} \dots$ 'm' times

Exercise 2

1. Simplify the following using laws of exponents.

(i) $2^{10} \times 2^4$ (ii) $(3^2) \times (3^2)^4$ (iii) $\frac{5^7}{5^2}$

(iv) $9^2 \times 9^{18} \times 9^{10}$ (v) $\left(\frac{3}{5}\right)^4 \times \left(\frac{3}{5}\right)^3 \times \left(\frac{3}{5}\right)^8$ (vi) $(-3)^3 \times (-3)^{10} \times (-3)^7$

(vii) $(3^2)^2$ (viii) $2^4 \times 3^4$ (ix) $2^{4a} \times 2^{5a}$

(x) $(10^2)^3$ (xi) $\left[\left(\frac{-5}{6}\right)^2\right]^3$ (xii) $2^{3a+7} \times 2^{7a+3}$

(xiii) $\left(\frac{2}{3}\right)^5$ (xiv) $(-3)^3 \times (-5)^3$ (xv) $\frac{(-4)^6}{(-4)^3}$

(xvi) $\frac{9^7}{9^{15}}$ (xvii) $\frac{(-6)^5}{(-6)^9}$ (xiii) $(-7)^7 \times (-7)^8$

(xix) $(-6^4)^4$ (xx) $a^x \times a^y \times a^z$

2. By what number should 3^{-4} be multiplied so that the product is 729?

3. If $5^6 \times 5^{2x} = 5^{10}$, then find x.

4. Evaluate $2^0 + 3^0$

5. Simplify $\left(\frac{x^a}{x^b}\right)^a \times \left(\frac{x^b}{x^a}\right)^a \times \left(\frac{x^a}{x^a}\right)^b$

6. State true or false and justify your answer.

(i) $100 \times 10^{11} = 10^{13}$ (ii) $3^2 \times 4^3 = 12^5$ (iii) $5^0 = (100000)^0$

(iv) $4^3 = 8^2$ (v) $2^3 > 3^2$ (vi) $(-2)^4 > (-3)^4$

(vii) $(-2)^5 > (-3)^5$

Classroom Project

Collect the annual income particulars of any ten families in your locality and round it to the nearest thousands / lakhs and express the income of each family in the exponential form.

11.3.6 Expressing large numbers in standard form

The mass of the Earth is about 5976×10^{21} kg.

The width of the Milky Way Galaxy from one edge to the other edge is about 946×10^{15} km.

These numbers are still not very easy to comprehend. Thus, they are often expressed in standard form. In standard form the:

Mass of the Earth is about 5.976×10^{24} kg

Width of the Milky Way Galaxy from one edge to the other edge is about 9.46×10^{17} km.

Thus, in standard form (Scientific notation) a number is expressed as the product of largest integer exponent of 10 and a decimal number between 1 and 10.

Exercise 3

Express the number appearing in the following statements in standard form.

- (i) The distance between the Earth and the Moon is approximately 384,000,000m.
- (ii) The universe is estimated to be about 12,000,000,000 years old.
- (iii) The distance of the sun from the center of the Milky Way Galaxy is estimated to be 300,000,000,000,000,000 m.
- (iv) The earth has approximately 1,353,000,000 cubic km of sea water.

Looking Back

- Very large numbers are easier to read, write and understand when expressed in exponential form.
- $10,000 = 10^4$ (10 raised to the power of 4); $243 = 3^5$ (3 raised to the power of 5); $64 = 2^6$ (2 raised to the power of 6). In these examples 10, 3, 2 are the respective bases and 4, 5, 6 are the respective exponents.

• Laws of Exponents: For any non-zero integers 'a' and 'b' and integers 'm' and 'n'

(i) $a^m \times a^n = a^{m+n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $a^m \times b^m = (ab)^m$

(iv) $a^{-n} = \frac{1}{a^n}$ (v) $\frac{a^m}{a^n} = a^{m-n}$ if $m > n$

(vi) $\frac{a^m}{b^n} = \frac{1}{a^{n-m}}$ if $n > m$ (vii) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

(viii) $a^0 = 1$ (where $a \neq 0$)



2 432

2 216

2 108

2 54

3 27

3 9

3 3

1

2 450

3 225

3 75

5 25

5 5

1





Free distribution by A.P. Government



(by the definition of exponent)

Therefore, $\left(\frac{7}{4}\right)^5 = \frac{7^5}{4^5}$

Example 17: Expand and simplify $\left(\frac{p}{q}\right)^6$

Solution : $\left(\frac{p}{q}\right)^6 = \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right)$
 $= \frac{p \times p \times p \times p \times p \times p}{q \times q \times q \times q \times q \times q}$
 $= \frac{p^6}{q^6}$ (By the definition of exponent)

Therefore, $\left(\frac{p}{q}\right)^6 = \frac{p^6}{q^6}$

Based on the above observations we can say that.

$$\left(\frac{a}{b}\right)^m = \frac{a \times a \times a \times \dots \times a \text{ 'm' times}}{b \times b \times b \times \dots \times b \text{ 'm' times}} = \frac{a^m}{b^m}$$

For any non-zero integers a, b and integer 'm', $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Do This

1. Complete the following

(i) $\left(\frac{5}{7}\right)^3 = \frac{5^3}{\square}$ (ii) $\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5}$

(iii) $\left(\frac{8}{3}\right)^4 = \frac{\square}{\square}$ (iv) $\left(\frac{x}{y}\right)^{11} = \frac{\square}{y^{11}}$

11.3.5 Terms with negative base

Example 18 : Evaluate $(1)^4$, $(1)^5$, $(1)^7$, $(-1)^2$, $(-1)^3$, $(-1)^4$, $(-1)^5$

Solution : $(1)^4 = 1 \times 1 \times 1 \times 1 = 1$

$(1)^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1$

$(1)^7 = 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$

$(-1)^2 = (-1) \times (-1) = 1$

$(-1)^3 = (-1) \times (-1) \times (-1) = -1$

$(-1)^4 = (-1) \times (-1) \times (-1) \times (-1) = 1$

$(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$

From the above illustrations we observe that:

(i) 1 raised to any power is 1.

(ii) (-1) raised to odd power is (-1) and (-1) raised to even power is $(+1)$.

Thus $(-a)^m = -a^m$ If 'm' is odd

$(-a)^m = a^m$ If 'm' is even

Now, let us look at some more examples.

$(-3)^4 = (-3) (-3) (-3) (-3) = 81$

$(-a)^4 = (-a) (-a) (-a) (-a) = a^4$

$(-a)^{-3} = \frac{1}{(-a)^3} = \frac{1}{(-a)} \times \frac{1}{(-a)} \times \frac{1}{(-a)} = \frac{1}{(-a)^3} = \frac{-1}{a^3}$

Example 19 : Express $\frac{-27}{125}$ in exponential form

Solution : $-27 = (-3) (-3) (-3) = (-3)^3$

$125 = 5 \times 5 \times 5 = (5)^3$

Therefore, $\frac{-27}{125} = \frac{(-3)^3}{(5)^3}$ as $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

Thus, $\frac{-27}{125} = \left(\frac{-3}{5}\right)^3$

Do This

1. Write in expanded form.

(i) $(a)^{-5}$ (ii) $(-a)^4$ (iii) $(-7)^{-5}$ (iv) $(-a)^m$

2. Write in exponential form

(i) $(-3) \times (-3) \times (-3)$ (ii) $(-b) \times (-b) \times (-b) \times (-b)$

(iii) $\frac{1}{(-2)} \times \frac{1}{(-2)} \times \frac{1}{(-2)} \dots$ 'm' times

Exercise 2

1. Simplify the following using laws of exponents.

(i) $2^{10} \times 2^4$ (ii) $(3^2) \times (3^2)^4$ (iii) $\frac{5^7}{5^2}$

(iv) $9^2 \times 9^{18} \times 9^{10}$ (v) $\left(\frac{3}{5}\right)^4 \times \left(\frac{3}{5}\right)^3 \times \left(\frac{3}{5}\right)^8$ (vi) $(-3)^3 \times (-3)^{10} \times (-3)^7$

(vii) $(3^2)^2$ (viii) $2^4 \times 3^4$ (ix) $2^{4a} \times 2^{5a}$

(x) $(10^2)^3$ (xi) $\left[\left(\frac{-5}{6}\right)^2\right]^3$ (xii) $2^{3a+7} \times 2^{7a+3}$

(xiii) $\left(\frac{2}{3}\right)^5$ (xiv) $(-3)^3 \times (-5)^3$ (xv) $\frac{(-4)^6}{(-4)^3}$

(xvi) $\frac{9^7}{9^{15}}$ (xvii) $\frac{(-6)^5}{(-6)^9}$ (xiii) $(-7)^7 \times (-7)^8$

(xix) $(-6^4)^4$ (xx) $a^x \times a^y \times a^z$

2. By what number should 3^{-4} be multiplied so that the product is 729?

3. If $5^6 \times 5^{2x} = 5^{10}$, then find x .

4. Evaluate $2^0 + 3^0$

5. Simplify $\left(\frac{x^a}{x^b}\right)^a \times \left(\frac{x^b}{x^a}\right)^a \times \left(\frac{x^a}{x^a}\right)^b$

6. State true or false and justify your answer.

(i) $100 \times 10^{11} = 10^{13}$ (ii) $3^2 \times 4^3 = 12^5$ (iii) $5^0 = (100000)^0$

(iv) $4^3 = 8^2$ (v) $2^3 > 3^2$ (vi) $(-2)^4 > (-3)^4$

(vii) $(-2)^5 > (-3)^5$

Classroom Project

Collect the annual income particulars of any ten families in your locality and round it to the nearest thousands / lakhs and express the income of each family in the exponential form.

11.3.6 Expressing large numbers in standard form

The mass of the Earth is about 5976×10^{21} kg.

The width of the Milky Way Galaxy from one edge to the other edge is about 946×10^{15} km.

These numbers are still not very easy to comprehend. Thus, they are often expressed in standard form. In standard form the:

Mass of the Earth is about 5.976×10^{24} kg

Width of the Milky Way Galaxy from one edge to the other edge is about 9.46×10^{17} km.

Thus, in standard form (Scientific notation) a number is expressed as the product of largest integer exponent of 10 and a decimal number between 1 and 10.

Exercise 3

Express the number appearing in the following statements in standard form.

(i) The distance between the Earth and the Moon is approximately 384,000,000m.

(ii) The universe is estimated to be about 12,000,000,000 years old.

(iii) The distance of the sun from the center of the Milky Way Galaxy is estimated to be 300,000,000,000,000,000 m.

(iv) The earth has approximately 1,353,000,000 cubic km of sea water.

Looking Back

- Very large numbers are easier to read, write and understand when expressed in exponential form.
- $10,000 = 10^4$ (10 raised to the power of 4); $243 = 3^5$ (3 raised to the power of 5); $64 = 2^6$ (2 raised to the power of 6). In these examples 10, 3, 2 are the respective bases and 4, 5, 6 are the respective exponents.
- Laws of Exponents: For any non-zero integers 'a' and 'b' and integers 'm' and 'n'

$$(i) a^m \times a^n = a^{m+n} \quad (ii) (a^m)^n = a^{mn} \quad (iii) a^m \times b^m = (ab)^m$$

$$(iv) a^{-n} = \frac{1}{a^n} \quad (v) \frac{a^m}{a^n} = a^{m-n} \quad \text{if } m > n$$

$$(vi) \frac{a^m}{b^n} = \frac{1}{a^{n-m}} \quad \text{if } n > m \quad (vii) \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

$$(viii) a^0 = 1 \quad (\text{where } a \neq 0)$$

11



$$2 \ 432$$

$$2 \ 216$$

$$2 \ 108$$

$$2 \ 54$$

$$3 \ 27$$

$$3 \ 9$$

$$3 \ 3$$

$$1$$

$$2 \ 450$$

$$3 \ 225$$

$$3 \ 75$$

$$5 \ 25$$

$$5 \ 5$$

$$1$$





Free distribution by A.P. Government



1. [Untitled-1](#)