Waves

Question1

The displacement of a travelling wave y = Csin $\frac{2\pi}{\lambda}$ (at −x) where t is time, x is distance and λ is the wavelength, all in S.I. units. Then the frequency of the wave is

[NEET 2024 Re]

Options:

A.

2πλ/a

B.

2πa/λ

C.

λ/a

D.

a/λ

Answer: D

Solution:

$y = c\sin \frac{2\pi}{\lambda}(\text{at} - x)$

$y = c\sin\left(\frac{2\pi}{\lambda}\text{at} - \frac{2\pi}{\lambda}x\right)$

Comparing with $y = A\sin(\omega t - kx)$

$\omega = 2\pi f = \frac{2\pi a}{\lambda}$

$f = \frac{a}{\lambda}$

Question2

Two slits in Young's double slit experiment are 1.5 mm apart and the screen is placed at a distance of 1m from the slits. If the wavelength of light used is 600 × 10^{-9} m then the fringe separation is

[NEET 2024 Re]

Options:

A.

4×10^{-5} m

B.

9×10^{-8} m

C.

4×10^{-7} m

D.

4×10^{-4} m

Answer: D

Solution:

$$\text{Fringe width} = \text{Fringe separation } (\beta) = \frac{\lambda D}{d}$$

$$\Rightarrow \beta = \frac{600 \times 10^{-9} \times 1}{1.5 \times 10^{-3}} = \frac{6 \times 10^{-7}}{1.5 \times 10^{-3}} = 4 \times 10^{-4} \text{m}$$

Question3

Interference pattern can be observed due to superposition of the following waves:
A. y = asin ωt
B. y = asin 2ωt
C. y = asin (ωt − ϕ)
D. y = asin 3ωt
Choose the correct answer from the options given below.

[NEET 2024 Re]

Options:

A.

B and C

B.

B and D

C.

A and C

D.

A and B

Answer: C

Solution:

For interference pattern to be observed the sources must be coherent.

Hence the superposition of waves y = asin (ωt) & y = asin (ωt − ϕ) would result in interference.

Question4

The ratio of frequencies of fundamental harmonic produced by an open pipe to that of closed pipe having the same length is

[NEET 2023]

Options:

A.

2 : 1

B.

1 : 3

C.

3 : 1

D.

1 : 2

Answer: A

Solution:

$$f_0 = f_{\text{open pipe}} = \frac{v}{2l}$$

$$f_c = f_{\text{closed pipe}} = \frac{v}{4l}$$

$$\frac{f_0}{f_c} = \frac{v}{2l} \times \frac{4l}{v}$$

$$f_0 : f_c = 2 : 1$$

Question5

The 4th overtone of a closed organ pipe is same as that of 3rd overtone

of an open pipe. The ratio of the length of the closed pipe to the length of the open pipe is :

[NEET 2023 mpr]

Options:

A.

8:9

B.

9:7

C.

9:8

D.

7:9

Answer: C

Solution:

$$n_{cop} = (2M+1)^{th} \text{ Har.} = (2 \times 4 + 1) \times \frac{V}{4\ell_c} = \frac{9V}{4\ell_c}$$

$$n_{oop} = (M+1)^{th} \text{ Har.} = (3+1)\frac{V}{2\ell_0} = \frac{4V}{2\ell_0}$$

$$\text{now } \frac{9V}{4\ell_c} = \frac{4V}{2\ell_0}$$

$$\frac{\ell_c}{\ell_0} = \frac{18}{16} = \frac{9}{8}$$

Question6

If the initial tension on a stretched string is doubled, then the ratio of the initial and final speeds of a transverse wave along the string is [NEET-2022]

Options:

A. 1 : 1

B. $\sqrt{2}$: 1

C. 1 : $\sqrt{2}$

D. 1 : 2

Answer: C

Solution:

We know, velocity of transverse wave

$v = \sqrt{\frac{T}{\mu}}$

$\therefore\ v_i = \sqrt{\frac{T}{\mu}}$ and $v_f = \sqrt{\frac{2T}{\mu}}$

$\therefore\ \frac{v_i}{v_f} = \frac{1}{\sqrt{2}}$

Question7

An organ pipe filled with a gas at 27˚C resonates at 400 Hz in its fundamental mode. If it is filled with the same gas at 90˚C, the resonance frequency at the same mode will be:
[NEET Re-2022]

Options:

A. 512 Hz

B. 420 Hz

C. 440 Hz

D. 484 Hz

Answer: C

Solution:

$f = \frac{V}{\lambda}$

Velocity of sound $\propto \sqrt{T}$

$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}$

$\frac{400}{f_2} = \sqrt{\frac{273+27}{273+90}} = \sqrt{\frac{300}{363}} = \frac{1}{1.1}$

$\Rightarrow f_2 = 440\,\text{Hz}$

Question8

In a guitar, two strings A and B made of same material are slightly out of tune and produce beats of frequency $6Hz$. When tension in B is slightly decreased, the beat frequency increases to $7Hz$. If the frequency of A is $530Hz$, the original frequency of B will be
[2020]

Options:

A. $524Hz$

B. $536Hz$

C. $537Hz.$

D. $523Hz.$

Answer: A

Solution:

Solution:

(a) Frequency of string, $f = \frac{1}{2l}\sqrt{\frac{T}{m}}$

Frequency $\propto \sqrt{\text{Tension}}$

Difference of f_A and f_B is 6 Hz.

If tension decreases, f_B decreases and becomes f_B'

Now, difference of f_A and $f_B' = 7Hz$ (increases) So, $f_A > f_B$

$f_A - f_B = 6Hz$

$\Rightarrow f_A = 530Hz \Rightarrow f_B = 524Hz$ (original)

Question9

A tuning fork with frequency $800Hz$ produces resonance in a resonance column tube with upper end open and lower end closed by water surface. Successive resonance are observed at length $9.75cm$ $31.25cm$ and $52.75cm$. The speed of sound in air is
(OD NEET 2019)

Options:

A. $500m/s$

B. $156m/s$

C. $344m/s$

D. $172m/s$

Answer: C

Solution:

Frequency (ν) = $800 Hz$
As the pipe is closed at one end, so
$l_3 - l_2 = l_2 - l_1 = \frac{\lambda}{2} = 21.5 cm$
$\therefore \lambda = 43.0 cm$
As $\nu = \frac{v}{\lambda} \Rightarrow v = \nu\lambda$
$\therefore v = \frac{800 \times 43}{100} = 344 ms^{-1}$

Question10

A tuning fork is used to produce resonance in a glass tube. The length of the air column in this tube can be adjusted by a variable piston. At room temperature of $27^{\circ}C$ two successive resonances are produced at $20 cm$ and $73 cm$ of column length. If the frequency of the tuning fork is $320 Hz$, the velocity of sound in air at $27^{\circ}C$ is (NEET 2018)

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Options:

A. $330 ms^{-1}$

B. $339 ms^{-1}$

C. $350 ms^{-1}$

D. $300 ms^{-1}$

Answer: B

Solution:

Solution:
The velocity of sound in air at 27°C is $v = 2(\nu)[L_2 - L_1]$; where $\nu =$ frequency of tuning fork and L_1, L_2 are the successive column length.
$\therefore v = 2 \times 320[73 - 20] \times 10^{-2}$
$= 339.2 ms^{-1} \approx 339 ms^{-1}$.

Question11

The fundamental frequency in an open organ pipe is equal to the third harmonic of a closed organ pipe. If the length of the closed organ pipe is $20 cm$, the length of the open organ pipe is (NEET 2018)

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Options:

A. 13.2cm

B. 8cm

C. 12.5cm

D. 16cm

Answer: A

Solution:

Solution:
For closed organ pipe, third harmonic is $\frac{3v}{4l}$
For open organ pipe, fundamental frequency is $\frac{v}{2l'}$
Given, third harmonic for closed organ pipe = fundamental frequency for open organ pipe.
$\therefore \frac{3v}{4l} = \frac{v}{2'} \Rightarrow l' = \frac{4l}{3 \times 2} = \frac{2l}{3};$
where l and l' are the lengths of closed and open organ pipes respectively.
$\therefore l' = \frac{2 \times 20}{3} = 13.33cm$

Question12

The two nearest harmonics of a tube closed at one end and open at other end are 220 Hz and 260 Hz. What is the fundamental frequency of the system?
(2017 NEET)

Options:

A. 20 Hz

B. 30 Hz

C. 40 Hz

D. 10 Hz

Answer: A

Solution:

Solution:
Nearest harmonics of an organ pipe closed at one end is differ by twice of its fundamental frequency.
$\therefore 260 - 220 = 2\upsilon, \quad \upsilon = 20 \text{ Hz}$

Question13

Two cars moving in opposite directions approach each other with speed

of $22ms^{-1}$ and $16.5ms^{-1}$ respectively. The driver of the first car blows a horn having a frequency 400 Hz. The frequency heard by the driver of the second car is
[velocity of sound is $340ms^{-1}$]
(2017 NEET)

Options:

A. 361 Hz

B. 411 Hz

C. 448 Hz

D. 350 Hz

Answer: C

Solution:

Solution:
The required frequency of sound heard by the driver of second car is given as

$$\upsilon' = \upsilon\left(\frac{v + v_0}{v - v_s}\right)$$

Where v = velocity od sound
v_0 = velocity of obsever, i.e, second car
v_s = velocity of source i.e., first clear

$$\upsilon' = 400\left(\frac{340 + 16.5}{340 - 22}\right) = 400\left(\frac{356.5}{318}\right) \approx 448 \text{ Hz}$$

Question14

A siren emitting a sound of frequency 800 Hz moves away from an observer towards a cliff at a speed of $15ms^{-1}$. Then, the frequency of sound that the observer hears in the echo reflected from the cliff is (Take velocity of sound in air = $330ms^{-1}$) (2016 NEET Phase-I)

Options:

A. 838 Hz

B. 885 Hz

C. 765 Hz

D. 800 Hz

Answer: A

Solution:

Here, frequency of sound emitted by siren,
$\upsilon_0 = 800\mathrm{H\,z}$
Speed of source, $v_s = 15\mathrm{ms}^{-1}$
Speed of sound in air,$v = 330\mathrm{ms}^{-1}$
Apparent frequency of sound at the cliff = frequency heard by observer $= \upsilon$
Using Doppler's effect of sound
$\upsilon = \left(\frac{v}{v - v_s}\right)\upsilon_0 = \frac{330}{300 - 15} \times 800 \approx 838\mathrm{Hz}$

Question15

An air column, closed at one end and open at the other, resonates with a tuning fork when the smallest length of the column is 50 cm. The next larger length of the column resonating with the same tuning fork is (2016 NEET Phase-I)

Options:

A. 150 cm

B. 200 cm

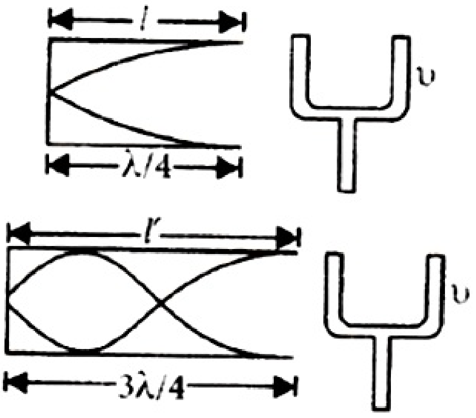
C. 66.7 cm

D. 100 cm

Answer: A

Solution:

Solution:



From figure
First harmonic is obtained at $l = \frac{\lambda}{4} = 50\mathrm{cm}$
Third harmonic is obtained for resonance,
$l' = \frac{3\lambda}{4} = 3 \times 50 = 150$ cm

Question16

A uniform rope of length L and mass m_1 hangs vertically from a rigid support. A block of mass m_2 is attached to the free end of the rope. A

transverse pulse of wavelength λ_1 is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope is λ_2.The ratio $\frac{\lambda_2}{\lambda_1}$ is
(2016 NEET Phase-I)

Options:

A. $\sqrt{\frac{m_2}{m_1}}$

B. $\sqrt{\frac{m_1 + m_2}{m_1}}$

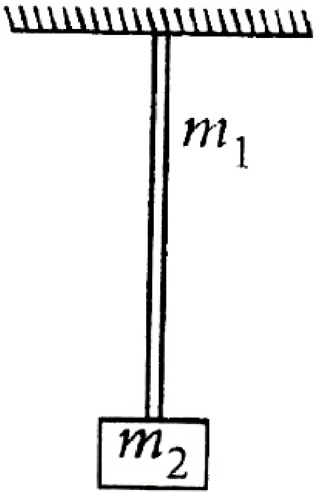
C. $\sqrt{\frac{m_1}{m_2}}$

D. $\sqrt{\frac{m_1 + m_2}{m_2}}$

Answer: D

Solution:

Solution:



Wavelength of pulse at the lower end $(\lambda_1) \propto$ velocity $(v_1) = \sqrt{\frac{T_1}{\mu}}$

Similarly, $\lambda_2 \propto v_2 = \sqrt{\frac{T_2}{\mu}}$

$\therefore \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{(m_1 + m_2)g}{m_2 g}}$

$= \sqrt{\frac{m_1 + m_2}{m_2}}$

Question17

The second overtone of an open organ pipe has the same frequency as

the first overtone of a closed pipe L metre long. The length of the open pipe will be
(2016 NEET Phase-II)

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Options:

A. L

B. 2L

C. $\frac{L}{2}$

D. 4L

Answer: B

Solution:

Second overtone of an open organ pipe
=Third harmonic $= 3 \times \upsilon'_0 = 3 \times \frac{v}{2L'}$
First overtone of a closed organ pipe
= Third harmonic $= 3 \times \upsilon_0 = 3 \times \frac{v}{4L}$
According to question,
$3\upsilon'_0 = 3\upsilon_0 \Rightarrow 3 \times \frac{v}{2L'} = 3 \times \frac{v}{4L} \Rightarrow L' = 2L$

Question18

Three sound waves of equal amplitudes have frequencies $(n-1), n(n+1)$. They superimpose to give beats. The number of beats produced per second will be
(2016 NEET Phase-II)

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Options:

A. 1

B. 4

C. 3

D. 2

Answer: D

Solution:

Solution:
Beats are defined as the periodic repetition of fluctuating intensities of sound waves. This occurs when two sound waves

of similar frequencies interfere with one another. It is characterised by waves whose amplitude varies at a regular rate.
The beats oscillate to and fro between amplitude zero and maximum amplitude.
The positive amplitude is called crest and the negative amplitude is called trough. A loud sound is heard when the waves interfere constructively. This happens when two crests or two troughs interfere. Similarly, no sound is heard when the waves interfere destructively. This happens when one crest and one trough interferes.
The beat frequency or beats per second is the difference between frequencies of two notes which interfere to produce beats.
Here since frequencies $(n-1)$, n, $(n+1)$ superimpose to form beats, then the difference in frequency $(n-1)$, n is 1. i.e. $n-(n-1)=n-n+1=1$
Similarly the difference in frequency n, $(n+1)$ is 1.i.e. $n+1-n=1$
Then the total difference between $(n-1)$, n, $(n+1)$ is 2.i.e $[n-(n-1)]+[n+1-n]=1+1=2$.
Hence the answer is 2

Question19

A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. The lowest resonant frequency for this string is (2015)

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Options:

A. 10.5 Hz

B. 105 Hz

C. 155 Hz

D. 205 Hz

Answer: B

Solution:

For a string fixed at both ends, the resonant frequencies are
$\upsilon_n = \frac{nv}{2L}$ where $n = 1, 2, 3, ...$
The difference between two consecutive resonant frequencies is
$\Delta\upsilon_n = \upsilon_{n+1} - \upsilon_n = \frac{(n+1)v}{2L} - \frac{nv}{2L} = \frac{v}{2L}$
which is also the lowest resonant frequency (n = 1).
Thus the lowest resonant frequency for the given string
$420 Hz - 315 Hz = 105 Hz$

Question20

4.0 g of a gas occupies 22.4 litres at NTP. The specific heat capacity of the gas at constant volume is $5.0 J K^{-1} mol^{-1}$. If the speed of sound in this gas at NTP is $952 ms^{-1}$, then the heat capacity at constant pressure is (Take gas constant $R = 8.3 J K^{-1} mol^{-1}$)

(2015)

Options:

A. $7.0 \mathrm{J\,K^{-1} mol^{-1}}$

B. $8.5 \mathrm{J\,K^{-1} mol^{-1}}$

C. $8.0 \mathrm{J\,K^{-1} mol^{-1}}$

D. $7.5 \mathrm{J\,K^{-1} mol^{-1}}$

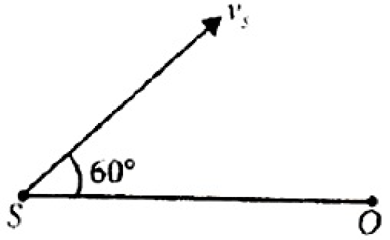
Answer: C

Solution:

Solution:
Since 4.0 g of a gas occupies 22.4 litres at NTP, so the molecular mass of the gas is
$M = 4.0 \mathrm{g mol^{-1}}$
As the speed of the sound in the gas is
$v = \sqrt{\frac{\gamma RT}{M}}$
Where γ is the ratio of two specific heats, R is the universal gas constant and T is the temperature of the gas.
$\therefore \gamma = \frac{M v^2}{RT}$
Here,
$M = 4.0 \mathrm{gml^{-1}} = 4.0 \times 10^{-3} \mathrm{kg mol^{-1}}$,
$v = 952 \mathrm{ms^{-1}}$, $R = 8.3 \mathrm{J\,K^{-1} mol^{-1}}$
and $T = 273 \mathrm{K}$ (at NTP)
$\therefore \gamma = \frac{(4.0 \times 10^{-3} \mathrm{kg mol^{-1}})(952 \mathrm{ms^{-1}})^2}{(8.3 \mathrm{J\,K^{-1} mol^{-1}})(273 \mathrm{K})} = 1.6$
By definition, $\gamma = \frac{C_p}{C_v}$ or $C_p = C_V$
But $\gamma = 1.6$ and $C_v = 5.0 \mathrm{J\,K^{-1} mol^{-1}}$
$\therefore C_p = (1.6)\left(5.0 \mathrm{J\,K^{1} mol^{-1}}\right) = 8.0 \mathrm{J\,K^{-1} mol^{-1}}$

Question21

A source of sound S emitting waves of frequency 100 Hz and an observer O are located at some distance from each other. The source is moving with a speed of 19.4 m s'1 at an angle of 60° with the source observer line as shown in the figure. The observer is at rest.
The apparent frequency observed by the observer(velocity of sound in air $330 \mathrm{ms^{-1}}$), is
(2015)



Options:

A. 106 Hz

B. 97 Hz

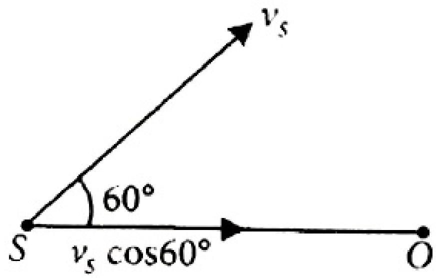
C. 100 Hz

D. 103 Hz

Answer: D

Solution:

Solution:



Here, Frequency of source, $\upsilon_0 = 100$ Hz
Velocity of source, $v_s = 19.4 ms^{-1}$
Velocity of sound in air, $v = 330 ms^{-1}$
As the velocity of source along the source observer line is $v_s \cos 60°$ and the observer is at rest, so the apparent frequency observed by the observer is

$$\upsilon = \upsilon_0 \left(\frac{v}{v - v_s \cos 60°} \right)$$

$$= (100 Hz) \left(\frac{330 ms^{-1}}{330 ms^{-1} - (19.4 ms^{-1}) \left(\frac{1}{2}\right)} \right)$$

$$= (100 Hz) \left(\frac{330 ms^{-1}}{330 ms^{-1} - 9.7 ms^{-1}} \right)$$

$$= (100 Hz) \left(\frac{330 ms^{-1}}{320.3 ms^{-1}} \right) = 103 Hz$$

Question22

The fundamental frequency of a closed organ pipe of length 20 cm is equal to the second overtone of an organ pipe open at both the ends. The length of organ pipe open at both the ends is
(2015, Cancelled)

Options:

A. 120 cm

B. 140 cm

C. 80 cm

D. 100 cm

Answer: A

Solution:

For closed organ pipe, fundamental frequency is given by

$\upsilon_c = \frac{v}{4l}$

For open organ pipe, fundamental frequency is given by

$\upsilon_o = \frac{v}{2l'}$

2nd overtone of open organ pipe $\upsilon' = 3\upsilon_o$; $\upsilon' = \frac{3v}{2l'}$

According to question, $\upsilon_c = \upsilon'$

$\frac{v}{4l} = \frac{3v}{2l'}$

$l' = 6l$

Here, $l = 20cm$, $l' = ?$

$\therefore l' = 6 \times 20 = 120cm$

Question23

The number of possible natural oscillations of air column in a pipe closed at one end of length 85 cm whose frequencies lie below 1250 Hz are
(Velocity of sound $= 340ms^{-1}$)
(2014)

Options:

A. 4

B. 5

C. 7

D. 6

Answer: D

Solution:

Solution:

Fundamental frequency of the closed organ pipe is

$\upsilon = \frac{v}{4L}$

Here $v = 340ms^{-1}$, $L = 85cm = 0.85m$

$\therefore \upsilon = \frac{340ms^{-1}}{4 \times 0.85m} = 100$ Hz

The natural frequencies of the closed organ pipe will be

$\upsilon_n = (2n-1)\upsilon = \upsilon, 3\upsilon, 5\upsilon, 7\upsilon, 9\upsilon, 11\upsilon, 13\upsilon, \ldots\ldots\ldots\ldots$

$= 100Hz, 300Hz, 500Hz, 700Hz, 900Hz, 1100Hz, 1300Hz, \ldots.$

and so on.

Thus, the natural frequencies lies below the 1250 Hz is 6.

Question24

A speeding motorcyclist sees traffic jam ahead him. He slows down to $36 kmhour^{-1}$ He finds that traffic has eased and a car moving ahead of him at $18 kmhour^{-1}$ is honking at a frequency of 1392 Hz. If the speed of sound is $343 ms^{-1}$, the frequency of the honk as heard by him will be (2014)

Options:

A. 1332 Hz

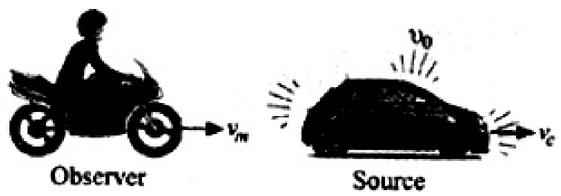
B. 1372 Hz

C. 1412 Hz

D. 1454 Hz

Answer: C

Solution:



Here, speed of motorcyclist, $v_m = 36 kmhour^{-1} = 10 ms^{-1}$
Speed of car, $v_c = 18 kmhour^{-1} = 5 ms^{-1}$
Frequency of source, $\upsilon_0 = 1392 Hz$
Speed of sound, $v = 34 ms^{-1}$
The frequency of the honk heard by the motorcyclist is

$$\upsilon' = \upsilon_0 \left(\frac{v + v_m}{v + v_c} \right) = 1392 \left(\frac{343 + 10}{343 + 5} \right) = 1412 Hz$$

Question25

If we study the vibration of a pipe open at both ends, then the following statement is not true.
(2013 NEET)

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Options:

A. All harmonics of the fundamental frequency will be generated

B. Pressure change will be maximum at both ends.

C. Open end will be antinode.

D. Odd harmonics of the fundamental frequency will be generated.

Answer: B

Solution:

Pressure change will be minimum at both ends.

Question26

A wave travelling in the +ve x-direction having displacement along y-direction as 1 m, wavelength 2πm and frequency of $\frac{1}{\pi}$Hz is represented by
(2013 NEET)

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Options:

A. y = sin(10πx − 20πt)

B. y = sin(2πx + 2πt)

C. y = sin(x − 2t)

D. y = sin(2πx − 2πt)

Answer: C

Question27

A source of unknown frequency gives 4 beats/s when sounded with a source of known frequency 250 Hz. The second harmonic of the source of unknown frequency gives five beats per second, when sounded with a source of frequency 513 Hz. The unknown frequency is
(2013 NEET)

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Options:

A. 240 Hz

B. 260 Hz

C. 254 Hz

D. 246 Hz

Answer: C

Solution:

Let the unknown frequency be f.
Beat frequency $b = |f - f'|$
For $f' = 250 Hz$ $b = 4$
$\therefore |f - 250| = 4 \ldots\ldots (1)$
For $f' = 513 b = 5$
$\therefore |2f - 513| = 5 \ldots\ldots (2)$
Case 1 : $2f > 513 Hz$ and $f > 250 Hz$
We get $f - 250 = 4 \Longrightarrow f = 254 Hz$
and $2f - 513 = 5 \Longrightarrow f = 259 Hz$
which is not possible.
Case 2: $2f < 513 Hz$ and $f > 250 Hz$
We get $f - 250 = 4 \Longrightarrow f = 254 Hz$
And $513 - 2f = 5 \Longrightarrow f = 254 Hz$
Case 3 : $2f < 513 Hz$ but $f < 250 Hz$
We get $250 - f = 4 \Longrightarrow f = 246 Hz$
And $513 - 2f = 5 \Longrightarrow f = 254 Hz$
Which is not possible.
So the unkown frequency $f = 254 Hz$

Question28

The length of the wire between two ends of a sonometer is 100cm. What should be the positions of two bridges below the wire so that the three segments of the wire have their fundamental frequencies in the ratio 1: 3: 5
(KN NEET 2013)

Options:

A. $\frac{1500}{23}$cm, $\frac{500}{23}$cm

B. $\frac{1500}{23}$cm, $\frac{300}{23}$cm

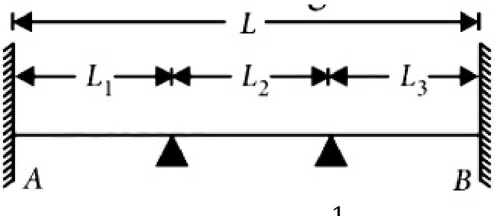
C. $\frac{300}{23}$cm, $\frac{1500}{23}$cm

D. $\frac{1500}{23}$cm, $\frac{2000}{23}$cm

Answer: D

Solution:

Solution:
Let $L(=100 cm)$ be the length of the wire and L_1, L_2 and L_3 are the lengths of the segments as shown in the figure.



Fundamental frequency, $v \propto \frac{1}{L}$
As the fundamental frequencies are in the ratio of 1: 3: 5 ,
$\therefore L_1 : L_2 : L_3 = \frac{1}{1} : \frac{1}{3} : \frac{1}{5} = 15 : 5 : 3$
Let x be the common factor. Then
$15x + 5x + 3x = L = 100$
$23x = 100$ or $x = \frac{100}{23}$

$\therefore L_1 = 15 \times \frac{100}{23} = \frac{1500}{23}$cm

$L_2 = 5 \times \frac{100}{23} = \frac{500}{23}$cm; $L_3 = 3 \times \frac{100}{23} = \frac{300}{23}$cm

∴ The bridges should be placed from A at $\frac{1500}{23}$cm and $\frac{2000}{23}$cm respectively.

Question29

Two sources P and Q produce notes of frequency 660 Hz each. A listener moves from P to Q with a speed of $1ms^{-1}$. If the speed of sound is 330m / s, then the number of beats heard by the listener per second will be
(KN NEET 2013)

Options:

A. 4

B. 8

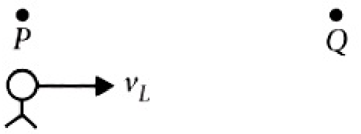
C. 2

D. zero

Answer: A

Solution:

Solution:



The situation as shown in the figure.
Here,
Speed of listener,$v_L = 1ms^{-1}$
Speed of sound,$v = 330ms^{-1}$
Frequency of each source, $\upsilon = 660Hz$
Apparent frequency due to P,

$$\upsilon' = \frac{\upsilon(v - v_L)}{v}$$

Apparent frequency due to Q,

$$\upsilon'' = \frac{\upsilon(v + v_L)}{v}$$

Number of beats heard by the listener per second is

$$\upsilon'' - \upsilon' = \frac{\upsilon(v + v_L)}{v} - \upsilon\frac{(v - v_L)}{v}$$

$$= \frac{2\upsilon v_L}{v} = \frac{2 \times 660 \times 1}{330} = 4$$

Question30

When a string is divided into three segments of length l_1, l_2 and l_3 the fundamental frequencies of these three segments are υ_1, υ_2 and υ_3

respectively. The original fundamental frequency (υ) of string is (2012)

Options:

A. $\sqrt{\upsilon} = \sqrt{\upsilon_1} + \sqrt{\upsilon_2} + \sqrt{\upsilon_3}$

B. $\upsilon = \upsilon_1 + \upsilon_2 + \upsilon_3$

C. $\frac{1}{\upsilon} = \frac{1}{\upsilon_1} + \frac{1}{\upsilon_2} + \frac{1}{\upsilon_3}$

D. $\frac{1}{\sqrt{\upsilon}} = \frac{1}{\sqrt{\upsilon_1}} + \frac{1}{\sqrt{\upsilon_2}} + \frac{1}{\sqrt{\upsilon_3}}$

Answer: C

Solution:

Solution:
Let l be the length of the string. Fundamental frequency is given by
$\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$
or $\upsilon \propto \frac{1}{l}$ ($\because$ T and μ are constants)
or $\upsilon = \frac{k}{l}$ where k is a constant
Here $l_1 = \frac{k}{\upsilon_1}, l_2 = \frac{k}{\upsilon_2}, l_3 = \frac{k}{\upsilon_3}$ and $l = \frac{k}{\upsilon}$
But $l = l_1 + l_2 + l_3$ $\quad \therefore \frac{1}{\upsilon} = \frac{1}{\upsilon_1} + \frac{1}{\upsilon_2} + \frac{1}{\upsilon_3}$

Question31

Two sources of sound placed close to each other, are emitting progressive waves given by $y_1 = 4\sin 600\pi t$ and $y_2 = 5\sin 608\pi t$ An observer located near these two sources of sound will hear (2012)

Options:

A. 4 beats per second with intensity ratio 25 : 16 between waxing and waning.

B. 8 beats per second with intensity ratio 25 : 16 between waxing and waning.

C. 8 beats per second with intensity ratio 81 :1 between waxing and waning

D. 4 beats per second with intensity ratio 81 :1 between waxing and waning.

Answer: D

Solution:

Given $y_1 = 4\sin 600\pi t$, $y_2 = 5\sin 608\pi t$
$\therefore \omega_1 = 600\pi$ or $2\pi\upsilon_1 = 600\pi$ or $\upsilon_1 = 300$
$A_1 = 4$
and $\omega_2 = 680\pi$ or $2\pi\upsilon_2 = 608\pi$ or $\upsilon_2 = 304$
$A_2 = 5$
Number of beats heard per second
$= \upsilon_2 - \upsilon_1 = 304 - 300 = 4$
$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{(4+5)^2}{(4-5)^2} = \frac{81}{1}$$

Question32

The equation of a simple harmonic wave is given by $y = 3\sin\frac{\pi}{2}(50t - x)$, where x and y are in metres and t is in seconds. The ratio of maximum particle velocity to the wave velocity is (2012 Mains)

Options:

A. 2π

B. $\frac{3}{2}\pi$

C. 3π

D. $\frac{2}{3}\pi$

Answer: B

Solution:

Solution:
The given wave equation is
$y = 3\sin\frac{\pi}{2}(50t - x)$
$y = 3\sin\left(25\pi t - \frac{\pi}{2}x\right)$ ....(i)
The standard wave equation is
$y = A\sin(\omega t - kx)$ ......(ii)
Comparing(i) and (ii), we get
$\omega = 25\pi, k = \frac{\pi}{2}$
Wave velocity, $v = \frac{\omega}{k} = \frac{25\pi}{(\pi / 2)} = 50ms^{-1}$
Particle velocity, $v_p = \frac{dy}{dt} = \frac{d}{dt}\left(3\sin\left(25\pi t - \frac{\pi}{2}\right)\right)$
$= 75\pi\cos\left(25\pi t - \frac{\pi}{2}\right)$
Maximum particle velocity, $(v_p)_{max} = 75\pi m\ s^{-1}$
$\therefore \frac{(v_p)_{max}}{v} = \frac{75\pi}{50} = \frac{3}{2}\pi$

Question33

A train moving at a speed of 220ms^{-1} towards a stationary object, emits a sound of frequency 1000 Hz. Some of the sound reaching the object gets reflected back to the train as echo. The frequency of the echo as detected by the driver of the train is (Speed of sound in air is 330ms^{-1}) (2012 Mains)

Options:

A. 3500 Hz

B. 4000 Hz

C. 5000 Hz

D. 3000 Hz

Answer: C

Solution:

Solution:
Here,
Speed of the train,v_T = 220ms^{-1}
Speed of sound in air,v = 330ms^{-1}
The frequency of the echo detected by the driver of the train is

$$\upsilon' = \upsilon\left(\frac{v + v_T}{v - v_T}\right) = 1000\left(\frac{330 + 220}{330} - 220\right)$$

$$= 1000 \times \frac{550}{110} = 5000\text{Hz}$$

Question34

Two waves are represented by the equations
$y_1 = a\sin(\omega t + kx + 0.57)$m and
$y_2 = a\cos(\omega t + kx)$m, where x is in meter and 7 in sec. The phase difference between them is
(2011)

Options:

A. 1.0 radian

B. 1.25 radian

C. 1.57 radian

D. 0.57 radian

Answer: A

Solution:

$y_1 = a\sin(\omega t + kx + 0.57)$ ∴ phase $\phi_1 = \omega t + kx + 0.57$

$y_2 = a\cos(\omega t + kx) = a\sin\left(wt + kx + \frac{\pi}{2}\right)$

∴ phase $\phi_2 = \omega t + kx + \frac{\pi}{2}$

Phase difference $\Delta\phi = \phi_2 - \phi_1$

$= \left(\omega t + kx + \frac{\pi}{2}\right) - (\omega t + kx + 0.57) = \frac{\pi}{2} - 0.57$

$= (1.57 - 0.57)$ radian $= 1$ radian

Question35

Sound waves travel at 350 m/s through a warm air and at 3500 m/s through brass. The wavelength of a 700 Hz acoustic wave as it enters brass from warm air (2011)

Options:

A. decrease by a factor 10

B. increase by a factor 20

C. increase by a factor 10

D. decrease by a factor 20

Answer: C

Solution:

Solution:

Here, $v_{air} = 350\frac{m}{s}$, $v_{brass} = 3500\frac{m}{s}$

When a sound wave travels from one medium to another medium its frequency remains the same,∴ Frequency,$\upsilon = \frac{v}{\lambda}$

Since υ remains the same in both the medium

$\Rightarrow \frac{v_{air}}{\lambda_{air}} = \frac{v_{brass}}{\lambda_{brass}}$

$\lambda_{brass} = \lambda_{air} \times \frac{v_{brass}}{v_{air}} = \lambda_{air} \times \frac{3500}{350} = 10\lambda_{air}$

Question36

Two identical piano wires, kept under the same tension T have a fundamental frequency of 600 Hz. The fractional increase in the tension of one of the wires which will lead to occurrence of 6 beats/s when both the wires oscillate together would be

(2011 Mains)

Options:

A. 0.01

B. 0.02

C. 0.03

D. 0.04

Answer: B

Solution:

Solution:

As $\upsilon = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ $\quad \therefore \frac{\Delta\upsilon}{\upsilon} = \frac{1}{2}\frac{\Delta T}{T}$

$\frac{\Delta T}{T} = 2\frac{\Delta\upsilon}{\upsilon} = 2 \times \frac{6}{600} = 0.02$

Question37

A transverse wave is represented by y = Asin(ωt − kx).For what value of the wavelength is the wave velocity equal to the maximum particle velocity?
(2010)

Options:

A. $\frac{\pi A}{2}$

B. πA

C. $2\pi A$

D. A

Answer: C

Solution:

Solution:

The given wave equation is $y = A\sin(\omega t - kx)$

Wave velocity, $v = \frac{\omega}{k}$........(i)

Particle velocity, $v_p = \frac{dy}{dt} = A\omega\cos(\omega t - kx)$

Maximum particle velocity, $(v_p)_{max} = A\omega$......(ii)

According to the given question

$v = (v_p)_{max}$

$\frac{\omega}{k} = A\omega$ (Using (i) and (ii))

$\frac{1}{k} = A$ or $\frac{\lambda}{2\pi} = A$ $\left(\because k = \frac{2\pi}{\lambda}\right)$

$\lambda = 2\pi A$

Question38

A tuning fork of frequency 512 Hz makes 4 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per sec when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was (2010)

Options:

A. 510 Hz

B. 514 Hz

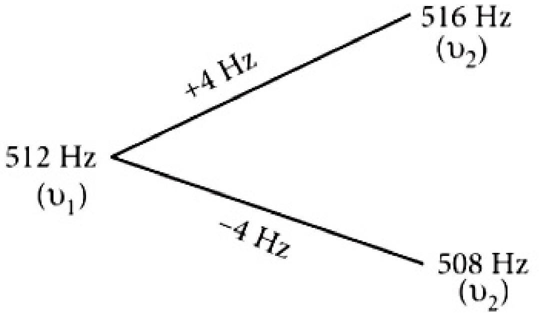
C. 516 Hz

D. 508 Hz

Answer: D

Solution:

Solution:
Let the frequencies of tuning fork and piano string be υ_1 and υ_2 respectively.
$\therefore \upsilon_2 = \upsilon_1 \pm 4 = 512\mathrm{Hz} \pm 4 = 516\mathrm{Hz}$ or $508\mathrm{Hz}$



Increase in the tension of a piano string increases its frequency.
If $\upsilon_2 = 516\mathrm{Hz}$, further increase in υ_2, resulted in an increase in the beat frequency. But this is not given in the question.
If $\upsilon_2 = 508\mathrm{Hz}$, further increase in υ_2 resulted in decrease in the beat frequency. This is given in the question. When the beat frequency decreases to 2 beats per second. Therefore, the frequency of the piano string before increasing the tension was $508\mathrm{Hz}$.

Question39

Each of the two strings of length 51.6 cm and 49.1 cm are tensioned separately by 20 N force. Mass per unit length of both the strings is same and equal to 1 g/m. When both the strings vibrate simultaneously the number of beats is (2009)

Options:

A. 7

B. 8

C. 3

D. 5

Answer: A

Solution:

Solution:
$l_1 = 0.516\text{m}, l_2 = 0.491\text{m}, T = 20\text{N}$
Mass per unit length, $\mu = 0.001\text{kg/m}$
Frequency, $\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$
$\upsilon_1 = \frac{1}{2 \times 0.516}\sqrt{\frac{20}{0.001}}, \upsilon_2 = \frac{1}{2 \times 0.491}\sqrt{\frac{20}{0.001}}$

$\therefore$ Number of beats $= \upsilon_1 - \upsilon_2 = 7$

Question40

The driver of a car travelling with speed 30 m/sec towards a hill sounds a horn of frequency 600 Hz. If the velocity of sound in air is 330 m/s, the frequency of reflected sound as heard by driver is (2009)

Options:

A. 555.5 Hz

B. 720 Hz

C. 500 Hz

D. 550 Hz

Answer: B

Solution:

Solution:
Car is the source and the hill is observer.
Frequency heard at the hill, υ_1
$\therefore \upsilon_1 = \frac{\upsilon \times v}{v - V} = \frac{600 \times 330}{330 - 30}$
Now for reflection, the hill is the source and the driver the observer.

$\therefore \upsilon_2 = \upsilon_1 \times \frac{(330 + 30)}{330}$

$\Rightarrow \upsilon_2 = \frac{600 \times 330}{330} \times \frac{360}{330} \Rightarrow \upsilon_2 = 720\text{Hz}$

Question41

A wave in a string has an amplitude of 2 cm. The wave travels in the +ve direction of x axis with a speed of 128 m/sec and it is noted that 5 complete waves fit in 4 m length of the string. The equation describing the wave is
(2009)

Options:

A. y = (0.02) m sin (15.7x - 2010t)

B. y = (0.02) m sin (15.7x +2010t)

C. y = (0.02) m sin (7.85x - 1005t)

D. y = (0.02) m sin (7.85x + 1005t)

Answer: C

Solution:

Solution:
Amplitude = 2 cm = 0.02 m, v = 128 m/s

$\lambda = \frac{4}{5} = 0.8\text{m}; \upsilon = \frac{128}{0.8} = 160\text{Hz}$

$\omega = 2\pi\upsilon = 2\pi \times 160 = 1005; k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.8} = 7.85$

$\therefore y = 0.02\sin(7.85x - 1005t)$

Question42

A point performs simple harmonic oscillation of period T and the equation of motion is given by x = asin $\left(\omega t + \frac{\pi}{6}\right)$.After the elapse of what fraction of the time period thevelocity of the point will be equal to half of its maximum velocity?
(2008)

Options:

A. $\frac{T}{3}$

B. $\frac{T}{12}$

C. $\frac{T}{8}$

D. $\frac{T}{6}$

Answer: B

Solution:

Solution:

$x = a \sin\left(\omega t + \frac{\pi}{6}\right)$

$\frac{dx}{dt} = a\omega \cos\left(\omega t + \frac{\pi}{6}\right)$

Max. velocity $= a\omega$

$\therefore a\omega 2 = a\omega \cos\left(\omega t + \frac{\pi}{6}\right)$

$\therefore \cos\left(\omega t + \frac{\pi}{6}\right) = \frac{1}{2}$

$\Rightarrow 60°$ or $\frac{2\pi}{6}$ radian $= \frac{2\pi}{T} . t + \frac{\pi}{6}$

$\Rightarrow \frac{2\pi}{T} . t = \frac{2\pi}{6} - \frac{\pi}{6} = +\frac{\pi}{6}$

$\therefore = +\frac{\pi}{6} \times \frac{T}{2\pi} = \left| +\frac{T}{12} \right|$

Question43

Two periodic waves of intensities I_1 and I_2 pass through a region at the same time in the same direction. The sum of the maximum and minimum intensities is
(2008)

Options:

A. $(\sqrt{I_1} - \sqrt{I_2})^2$

B. $2(I_1 + I_2)$

C. $I_1 + I_2$

D. $(\sqrt{I_1} + \sqrt{I_2})^2$

Answer: B

Solution:

Solution:

Other factors such as ω and v remaining the same, $I = A^2 \times$ constant K, or $A = \sqrt{\frac{I}{K}}$

On superposition

$A_{max} = A_1 + A_2$ and $A_{min} = A_1 - A_2$
$\therefore A_{max}^2 = A_1^2 + A_2^2 + 2A_1A_2$
$\Rightarrow \frac{I_{max}}{K} = \frac{I_1}{K} + \frac{I_2}{K} + \frac{2\sqrt{I_1 I_2}}{K}$
$A_{min}^2 = A_1^2 + A_2^2 - 2A_1A_2$
$\Rightarrow \frac{I_{min}}{K} = \frac{I_1}{K} + \frac{I_2}{K} - \frac{2\sqrt{I_1 I_2}}{K}$
$\therefore I_{max} + I_{min} = 2I_1 + 2I_2 = 2(I_1 + I_2)$

Question44

The wave described by $y = 0.25 \sin(10\pi x - 2\pi t)$, where x and y are in meters and 7 in seconds, is a wave travelling along the (2008)

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Options:

A. +ve x direction with frequency 1 Hz and wavelength $\lambda = 0.2m$

B. -ve x direction with amplitude 0.25 m and wavelength $\lambda = 0.2m$

C. -ve x direction with frequency 1 Hz.

D. +ve x direction with frequency 7t Hz and wavelength $\lambda = 0.2m$

Answer: A

Solution:

Solution:
$y = 0.25\sin(10\pi x - 2\pi t)$
$y_{max} = 0.25$
$k = \frac{2\pi}{\lambda} = 10\pi \Rightarrow \lambda = 0.2m$
$\omega = 2\pi v = 2\pi \Rightarrow v = 1Hz$
The sign is negative inside the bracket. Therefore this wave travels in the positive x-direction.

Question45

Two vibrating tuning forks produce waves given by $y_1 = 4\sin 500\pi t$ and $y_2 = 2\sin 506\pi t$Number of beats produced per minute is (2006)

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Options:

A. 360

B. 180

C. 60

D. 3

Answer: B

Solution:

$Y_1 = 4 \sin 500 \pi t$, $Y_2 = 2 \sin 506 \pi t$
$\omega_1 = 500\pi = 2\pi v_1 \Rightarrow v_1 = 250 Hz$
$\omega_2 = 506\pi = 2\pi v_2 \Rightarrow v_2 = 253 Hz$
$v = v_2 - v_1 = 253 - 250 = 3$ beats /s
Number of beats per minute $= 3 \times 60 = 180$.

Question46

Two sound waves with wavelengths 5.0m and 5.5m respectively, each propagate in a gas with velocity 330m / s. We expect the following number of beats per second.
(2006)

Options:

A. 6

B. 12

C. 0

D. 1 .

Answer: A

Solution:

Solution:
Frequency $= \frac{\text{velocity}}{\text{wavelength}}$
$\therefore v_1 = \frac{v}{\lambda_1} = \frac{330}{5} = 66 Hz$
and $v_2 = \frac{v}{\lambda_2} = \frac{330}{5.5} = 60 Hz$
Number of beats per second $= v_1 - v_2$
$= 66 - 60 = 6$

Question47

The time of reverberation of a room A is one second. What will be the time (in seconds) of reverberation of a room, having all the dimensions double of those of room A ?
(2006)

Options:

A. 1

B. 2

C. 4

D. $\frac{1}{2}$.

Answer: B

Solution:

Solution:

Reverberation time, $T = \frac{0.61V}{aS}$

where V is the volume of room in cubic metres, a is the average absorption coefficient of the room, S is the total surface area of room in square metres.

or, $T \propto \frac{V}{S}$ or, $\frac{T_1}{T_2} = \left(\frac{V_1}{V_2}\right)\left(\frac{S_2}{S_1}\right) = \left(\frac{V}{8V}\right)\left(\frac{4S}{S}\right) = \frac{1}{2}$

or, $T_2 = 2T_1 = 2 \times 1 = 2$ sec. ($\because T_1 = 1$ sec)

Question48

A transverse wave propagating along x -axis is represented by $y(x, t) = 8.0 \sin\left(0.5\pi x - 4\pi t - \frac{\pi}{4}\right)$ where x is in metres and t is in seconds. The speed of the wave is (2006)

Options:

A. $8 m / s$

B. $4\pi m / s$

C. $0.5\pi m / s$

D. $\frac{\pi}{4} m / s$

Answer: A

Solution:

Solution:

$y(x, t) = 8.0 \sin\left(0.5\pi x - 4\pi t - \frac{\pi}{4}\right)$

Compare with a standard wave equation,

$y = a\sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T} + \phi\right)$

we get $\frac{2\pi}{\lambda} = 0.5\pi$ or, $\lambda = \frac{2\pi}{0.5\pi} = 4m$

$\frac{2\pi}{T} = 4\pi$ or, $T = \frac{2\pi}{4\pi} = \frac{1}{2}$ sec.

$v = \frac{1}{T} = 2Hz$

Wave velocity, $v = \lambda v = 4 \times 2 = 8m/sec$.

Question49

Which one of the following statements is true? (2006)

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Options:

A. both light and sound waves can travel in vacuum

B. both light and sound waves in air are transverse

C. the sound waves in air are longitudinal while the light waves are transverse

D. both light and sound waves in air are longitudinal.

Answer: C

Solution:

Solution:
Light waves are electromagnetic waves. Light waves are transverse in nature and do not require a medium to travel, hence they can travel in vacuum. Sound waves are longitudinal waves and require a medium to travel. They do not travel in vacuum.

Question50

A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at distances of 2m and 3m respectively from the source. The ratio of the intensities of the waves at P andQ is (2005)

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Options:

A. 3: 2

B. 2: 3

C. 9: 4

D. 4: 9 .

Answer: C

Solution:

$d_1 = 2m, d_2 = 3m$
Intensity $\propto \frac{1}{(\text{ distance})^2}$
$I_1 \propto \frac{1}{2^2}$ and $I_2 \propto \frac{1}{3^2}$
$\therefore \frac{I_1}{I_2} = \frac{9}{4}$

Question51

The phase difference between two waves, represented by
$$y_1 = 10^{-6} \sin\left[100t + \left(\frac{x}{50}\right) + 0.5\right] m$$
$$y_2 = 10^{-6} \cos\left[100t + \left(\frac{x}{50}\right)\right] m$$
where x is expressed in metres and t is expressed in seconds, is approximately.
(2004)

Options:

A. 1.07 radians

B. 2.07 radians

C. 0.5 radians

D. 1.5 radians

Answer: A

Solution:

Solution:
$y_1 = 10^{-6} \sin\left[100t + \left(\frac{x}{50}\right) + 0.5\right]$
$y_2 = 10^{-6} \cos\left[100t + \left(\frac{x}{50}\right)\right]$ $\left[\text{using} \cos x = \sin\left(x + \frac{\pi}{2}\right)\right]$
$= 10^{-6} \sin\left[100t + \left(\frac{x}{50}\right) + \frac{\pi}{2}\right]$
$= 10^{-6} \sin\left[100t + \left(\frac{x}{50}\right) + 1.57\right]$
The phase difference $= 1.57 - 0.5 = 1.07$ radians [or using $\sin x = \cos\left(\frac{\pi}{2} - x\right)$, we get the same result.

Question52

A car is moving towards a high cliff. The driver sounds a horn of

frequency f . The reflected sound heard by the driver has frequency 2f . If v is the velocity of sound, then the velocity of the car, in the same velocity units, will be
(2004)

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Options:

A. $\frac{v}{\sqrt{2}}$

B. $\frac{v}{3}$

C. $\frac{v}{4}$

D. $\frac{v}{2}$

Answer: B

Solution:

Solution:
1^{st} the car is the source and at the cliff, one observes f'.
$\therefore f' = \frac{v}{v - v_s} f$
2^{nd} cliff is now source. It emits frequency f' and the observer is now the driver who observes f''.
$\therefore f'' = \left[\frac{v + v_o}{v}\right] f'$ or $2f = \left[\frac{v + v_o}{v - v_s}\right] f$
$\Rightarrow 2v - 2v_o = v + v_o$ [as $v_s = v_o$]
$\Rightarrow v_o = \frac{v}{3}$

Question53

An observer moves towards a stationary source of sound with a speed $\frac{1}{5}$ th of the speed of sound. The wavelength and frequency of the source emitted are lambda and f respectively. The apparent frequency and wavelength recorded by the observer are respectively
(2003)

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Options:

A. $1.2f$, 1.2λ

B. $1.2f$, λ

C. f , 1.2λ

D. $0.8f$, 0.8λ

Answer: B

Solution:

Solution:
Apparent frequency,

$$f' = \frac{v+v_o}{v}f = \frac{v+\left(\frac{1}{5}\right)v}{v}f = 1.2f$$

Wavelength does not change by motion of observer.

Question54

A whistle revolves in a circle with angular speed $\omega = 20 \, rad/s$ using a string of length $50cm$. If the frequency of sound from the whistle is $385Hz$, then what is the minimum frequency heard by an observer which is far away from the centre (velocity of sound $= 340m/s$) (2002)

Options:

A. $385Hz$

B. $374Hz$

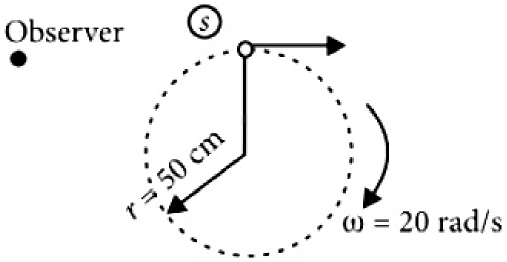
C. $394Hz$

D. $333Hz$.

Answer: B

Solution:

Solution:
The whistle is revolving in a circle of radius $50cm$. So the source (whistle) is moving and the observer is fixed.



The minimum frequency will be heard by the observer when the linear velocity of the whistle (source) will be in a direction as shown in the figure, i.e. when the source is receding.

The apparent frequency heard by the observer is then given by $v' = v\left(\frac{v}{V+v}\right)$

where V and v are the velocities of sound and source respectively and v is the actual frequency.
Now, $v = r\omega = 0.5 \times 20 = 10m/s$
$V = 340m/s, v = 385Hz$

$$\therefore v' = 385 \times \frac{340}{340+10} = 374Hz$$

Question55

If a wave travelling in positive x-direction with $A = 0.2m/s$, velocity $= 360m/s$ and $\lambda = 60m$, then correct expression for the wave is:

(2002)

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Options:

A. $y = 0.2 \sin\left[2\pi\left(6t + \frac{x}{60}\right)\right]$

B. $y = 0.2 \sin\left[\pi\left(6t + \frac{x}{60}\right)\right]$

C. $y = 0.2 \sin\left[2\pi\left(6t - \frac{x}{60}\right)\right]$

D. $y = 0.2 \sin\left[\pi\left(6t - \frac{x}{60}\right)\right]$

Answer: C

Solution:

Solution:
General equation for a plane progressive wave travelling along positive x-direction is given b
$y = A \sin(wt - kx)$
$= A \sin\left[2\pi\left(v - \frac{x}{\lambda}\right)\right] \ldots$ (i)
Here, it is given
$v = 360 m / s, \lambda = 60 m$
We know that, frequency $v = \frac{v}{\lambda} = \frac{360}{60} = 6 Hz.$
Substituting $A = 0.2 m / s, v = 6 Hz$
$\lambda = 60 m$ in Eq. (i), we get
$y = 0.2 \sin\left[2\pi\left(6t - \frac{x}{60}\right)\right]$

Question56

The equation of a wave is represented by $y = 10^{-4} \sin\left(100t - \frac{x}{10}\right)$ m, then the velocityof wave will be
(2001)

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Options:

A. $100 m / s$

B. $4 m / s$

C. $1000 m / s$

D. $10 m / s$

Answer: C

Solution:

Solution:

Comparing the given equation with general equation, $y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$, we get

$T = \frac{2\pi}{100}$ and $\lambda = 20\pi$

$\therefore v = v\lambda = \frac{100}{2\pi} \times 20\pi = 1000 m / s$

Question57

Two waves having equation
$x_1 = a\sin(\omega t - kx + \phi_1)$
$x_2 = a\sin(\omega t - kx + \phi_2)$
If in the resultant wave the frequency and amplitude remain equal to amplitude of superimposing waves, the phase difference between them is
(2001)

Options:

A. $\frac{\pi}{6}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: B

Solution:

Solution:
Resultant amplitude $= 2a(1 + \cos\phi) = a$
$\therefore (1 + \cos\phi) = \frac{1}{2}; \cos\phi = -\frac{1}{2}; \phi = \frac{2\pi}{3}$

Question58

If the tension and diameter of a sonometer wire of fundamental frequency n is doubled and density is halved then its fundamental frequency will become
(2001)

Options:

A. $\frac{n}{4}$

B. $\sqrt{2}n$

C. n

D. $\frac{n}{\sqrt{2}}$

Answer: C

Solution:

Solution:

$n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}}$

$\rho_1' = \frac{\rho}{2}$; $T' = 2T$ and $D' = 2D$ or $r' = 2r$

$n' = \frac{1}{2l}\sqrt{\frac{2T}{\pi (2r)^2 \frac{\rho}{2}}}$

After solving, $n' = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} = n$.

Question59

The equations of two waves acting in perpendicular directions are given as
$x = a\cos(\omega t + \delta)$ and $y = a\cos(\omega t + \alpha)$
where $\delta = \alpha + \frac{\pi}{2}$, the resultant wave represents
(2000)

Options:

A. a parabola

B. a circle

C. an ellipse

D. a straight line

Answer: B

Solution:

Given : $x = a\cos(\omega t + \delta)$
and $y = a\cos(\omega t + \alpha)$ ....(i)

where, $\delta = \alpha + \frac{\pi}{2}$

$\therefore x = a\cos\left(\omega t + \alpha + \frac{\pi}{2}\right)$

$= -a\sin(\omega t + \alpha)$ ......(ii)

Given the two waves are acting in perpendicular direction with the same frequency and phase difference $\frac{\pi}{2}$.

From equations (i) and (ii),
$x^2 + y^2 = a^2$
which represents the equation of a circle.

Question60

A string is cut into three parts, having fundamental frequencies n_1, n_2, n_3 respectively. Then original fundamental frequency n related by the expression as (2000)

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Options:

A. $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

B. $n = n_1 \times n_2 \times n_3$

C. $n = n_1 + n_2 + n_3$

D. $n = \frac{n_1 + n_2 + n_3}{3}$

Answer: A

Solution:

As $n \propto \left(\frac{1}{l}\right)$ and $l = l_1 + l_2 + l_3$

$\therefore \frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

Question61

Two stationary sources each emitting waves of wavelength λ, an observer moves from one source to another with velocity u. Then number of beats heard by him (2000)

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Options:

A. $\frac{2u}{\lambda}$

B. $\frac{u}{\lambda}$

C. $\sqrt{u\lambda}$

D. $\frac{u}{2\lambda}$

Answer: A

Solution:

Solution:

$f' = \frac{v-u}{v}f$; $f'' = \frac{v+u}{v}f$

Number of beats $= f'' - f' = \frac{2u}{\lambda}$

Question62

Two waves of lengths 50cm and 51cm produced 12 beats per sec. The velocity of sound is (1999)

Options:

A. 340m / s

B. 331m / s

C. 306m / s

D. 360m / s

Answer: C

Solution:

Solution:

Number of beats produced per second $= v_1 - v_2 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2}$

$12 = v\left[\frac{1}{50} - \frac{1}{51}\right]$ or $12 = \frac{v \times 1}{50 \times 51}$

or, $v = 12 \times 50 \times 51 cm / s = 306 m / s.$

Question63

A transverse wave is represented by the equation $y = y_0 \sin \frac{2\pi}{\lambda}(vt - x)$ For what value of λ, is the maximum particle velocity equal to two times the wave velocity? (1998)

Options:

A. $\lambda = \frac{\pi y_0}{2}$

B. $\lambda = \frac{\pi y_0}{3}$

C. $\lambda = 2\pi y_0$

D. $\lambda = \pi y_0$

Answer: D

Solution:

Solution:

The given equation of wave is $y = y_0 \sin \frac{2\pi}{\lambda}(vt - x)$

Particle velocity $= \frac{dy}{dt} = y_0 \cos \frac{2\pi}{\lambda}(vt - x) \frac{2\pi v}{\lambda}$

$\left(\frac{dy}{dt}\right)_{max} = y_0 \frac{2\pi}{\lambda} v$

$\therefore y_0 \frac{2\pi}{\lambda} v = 2v$ or $\lambda = \pi y_0$

Question64

A vehicle, with a horn of frequency n is moving with a velocity of 30m / s in a direction perpendicular to the straight line joining the observer and the vehicle. The observer perceives the sound to have a frequency $n + n_1$. Then (if the sound velocity in air is 300m / s)
(1998)

Options:

A. $n_1 = 0.1n$

B. $n_1 = 0$

C. $n_1 = 10n$

D. $n_1 = -0.1n$

Answer: B

Solution:

$n' = n + n_1 = \frac{nv}{v - v_s \cos\theta}$

$= \frac{nv}{v} [\because \cos 90° = 0]$

$n' = n \therefore n_1 = 0$

Question65

A standing wave having 3 nodes and 2 antinodes is formed between two atoms having a distance 1.21Å between them. The wavelength of the standing wave is
(1998)

Options:

A. 6.05Å

B. 2.42Å

C. 1.21Å

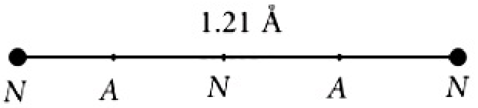
D. 3.63 Å

Answer: C

Solution:

Solution:

Distance between a node and adjoining antinode $= \frac{\lambda}{4}$



From figure, distance between two atoms $= 4 \times \frac{\lambda}{4} = 1.21$Å or, $\lambda = 1.21$Å

Question66

In a sinusoidal wave, the time required for a particular point to move from maximum displacement to zero displacement is 0.170s . The frequency of wave is
(1998)

Options:

A. 0.73Hz

B. 0.36Hz

C. 1.47Hz

D. 2.94 Hz

Answer: C

Solution:

Displacement, $Y_{max} = a$, $Y_{min} = 0$

Time taken $= \frac{T}{4}$

$\therefore \frac{T}{4} = 0.170 \;\; \therefore T = 0.68$

The frequency of wave $= \frac{1}{T} = 1.47 Hz$

Question67

Standing waves are produced in 10m long stretched string. If the string vibrates in 5 segments and wave velocity is 20m / s, the frequency is (1997)

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Options:

A. $5 Hz$

B. $10 Hz$

C. $2 Hz$

D. $4 Hz$

Answer: A

Solution:

Solution:
The frequency of standing wave,
$v = \frac{n}{2l} v = \frac{5 \times 20}{2 \times 10} = 5 Hz.$

Question68

A cylindrical tube, open at both ends has fundamental frequency f in air. The tube is dipped vertically in water, so that half of it is in water. The fundamental frequency of air column is now (1997)

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Options:

A. $\frac{f}{2}$

B. $\frac{3f}{4}$

C. $2f$

D. f

Answer: D

Solution:

Solution:

For the cylindrical tube open at both ends, $f = \frac{v}{2l}$.

When half of the tube is in water, it behaves asa closed pipe of length $\frac{l}{2}$.

$\therefore f' = \frac{v}{4\left(\frac{l}{2}\right)} = \frac{v}{2l} \therefore f' = f$

Question69

The equation of a sound wave is $y = 0.0015 \sin(62.4x + 316t)$ The wavelength of this wave is (1996)

Options:

A. 0.3 unit

B. 0.2 unit

C. 0.1 unit

D. cannot be calculated.

Answer: C

Solution:

Solution:
Sound wave equation is
$y = 0.0015 \sin(62.4x + 316t)$
Comparing it with the general equation of motion
$y = A \sin 2\pi\left[\frac{x}{\lambda} + \frac{t}{T}\right]$, we get $\frac{2\pi}{\lambda} = 62.4$

or $\lambda = \frac{2\pi}{62.4} = 0.1$ unit.

Question70

Two sound waves having a phase difference of 60° have path difference of (1996)

Options:

A. $\frac{\lambda}{6}$

B. $\frac{\lambda}{3}$

C. 2λ

D. $\frac{\lambda}{2}$

Answer: A

Solution:

Solution:

Phase difference $\theta = 60° = \frac{\pi}{3}$ radPhase difference $(\theta) = \frac{\pi}{3} = \frac{2\pi}{\lambda} \times$ Path difference

Therefore Path difference $= \frac{\pi}{3} \times \frac{\lambda}{2\pi} = \frac{\lambda}{6}$.

Question71

The length of a sonometer wire AB is 110cm. Where should the two bridges be placed from A to divide the wire in 3 segments whose fundamental frequencies are in the ratio of 1 : 2 : 3? (1995)

Options:

A. 60cm and 90cm

B. 30cm and 60cm

C. 30cm and 90cm

D. 40cm and 80cm.

Answer: A

Solution:

Solution:

Length of sonometer wire $(l) = 110$cm and ratio of frequencies $= 1 : 2 : 3$

Frequency $(v) \propto \frac{1}{l}$ or $l \propto \frac{1}{v}$

Therefore AC : CD : DB $= \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2$

Therefore AC $= 6 \times \frac{110}{11} = 60$cm and CD $= 3 \times \frac{110}{11} = 30$cm

Thus AD $= 60 + 30 = 90$cm

Question72

A hospital uses an ultrasonic scanner to locate tumours in a tissue. The operating frequency of the scanner is 4.2MHz. The speed of sound in a tissue is 1.7km / s. The wavelength of sound in the tissue is close to (1995)

Options:

A. 4×10^{-3}m

B. 8×10^{-3}m

C. 4×10^{-4}m

D. 8×10^{-4}m.

Answer: C

Solution:

Frequency (υ) = $4.2\mathrm{MHz}$
= $4.2 \times 10^{6}\mathrm{Hz}$ and
speed of sound (v) = $1.7\mathrm{km/s} = 1.7 \times 10^{3}\mathrm{m/s}$
Wavelength of sound in tissue (λ) = $\frac{v}{\upsilon}$
$= \frac{1.7 \times 10^{3}}{4.2 \times 10^{6}} = 4 \times 10^{-4}\mathrm{m}$

Question73

**A source of sound gives 5 beats per second, when sounded with another source of frequency 100 second $^{-1}$. The second harmonic of the source, together with a source of frequency 205sec^{-1} gives 5 beats per second. What is the frequency of the source?
(1995)**

Options:

A. 105 second $^{-1}$

B. 205 second $^{-1}$

C. 95 second $^{-1}$

D. 100 second $^{-1}$

Answer: A

Solution:

5 beats/sec $= 100$ and frequency of second source with 5 beats $/\sec = 205$. The frequency of the first source $= 100 \pm 5 = 105$ or $95\mathrm{Hz}$.
Therefore frequency of second harmonic source $= 210\mathrm{Hz}$ or $190\mathrm{Hz}$.
As the second harmonic gives 5 beats/second with the sound of frequency $205\mathrm{Hz}$, therefore frequency of second harmonic source should be $210\mathrm{Hz}$. The frequency of source $= 105\mathrm{Hz}$

Question74

A star, which is emitting radiation at a wavelength of 5000Å, is approaching the earth with a velocity of $1.5 \times 10^4 \mathrm{m/s}$. The change in wavelength of the radiation as received on the earth is (1995)

Options:

A. 25Å

B. 100Å

C. zero

D. 2.5Å

Answer: A

Solution:

Solution:
Wavelength $(\lambda) = 5000\text{Å}$ and velocity $(v) = 1.5 \times 10^4 \mathrm{m/s}$
Wavelength of the approaching star,
$(\lambda') = \ = \lambda \frac{c - v}{c}$
or $\frac{\lambda'}{\lambda} = 1 - \frac{v}{c}$ or, $\frac{v}{c} = 1 - \frac{\lambda'}{\lambda} = \frac{\lambda - \lambda'}{\lambda} = \frac{\Delta\lambda}{\lambda}$
Therefore $\Delta\lambda = \lambda \times \frac{v}{c} = 5000\text{Å} \times \frac{1.5 \times 10^6}{3 \times 10^8} = 25\text{Å}$
(where $\Delta\lambda$ is the change in the wavelength)

Question75

Which one of the following represents a wave? (1994)

Options:

A. $y = A\sin(\omega t - kx)$

B. $y = A\cos(at - bx + c)$

C. $y = A\sin kx$

D. $y = A \sin \omega t$.

Answer: B

Solution:

Solution:
(a) represents a harmonic progressive wave in the standard form whereas(b) also represents a harmonic progressive wave, both travelling in the positive x - direction. In (b), a is the angular velocity, ω and b is k; c is the initial phase.(d) represents only S.H.M.

Question76

A wave of frequency $100Hz$ travels along a string towards its fixed end. When this wave travels back, after reflection, a node is formed at a distance of 10cm from the fixed end. The speed of the wave (incident and reflected) is (1994)

Options:

A. $20m / s$

B. $40m / s$

C. $5m / s$

D. $10m / s$

Answer: A

Solution:

Solution:
Frequency (υ) = $100Hz$ and distance from fixed end $= 10cm = 0.1m$. When a stationary wave is produced, the fixed end behaves as a node.
Thus wavelength (λ) = $2 \times 0.1 = 0.2m$.
Therefore velocity $v = v\lambda = 100 \times 0.2 = 20m / s$

Question77

A stationary wave is represented by $y = A \sin(100t) \cos(0.01x)$, where y and Aare in millimetres, t is in seconds and x is in metres. The velocity of the wave is (1994)

Options:

A. $10^4 m / s$

B. not derivable

C. $1 m / s$

D. $10^2 m / s$

Answer: A

Solution:

Solution:
$y = A \sin(100t) \cos(0.01x)$
Comparing it with standard equation
$y = A \sin\left(\frac{2\pi}{T}t\right) \cos\left(\frac{2\pi}{\lambda}x\right)$,
we get $T = \frac{\pi}{50}$ and $\lambda = 200\pi$
Therefore velocity, $(v) = \frac{\lambda}{T} = \frac{200\pi}{\pi / 50} = 200 \times 50$
$= 10000 = 10^4 m / s$

Question78

A source of frequencyv gives 5 beats/second when sounded with a source of frequency $200 Hz$. The second harmonic of frequency 2υ of source gives 10 beats/second when sounded with a source of frequency $420 Hz$. The value of υ is (1994)

Options:

A. $205 Hz$

B. $195 Hz$

C. $200 Hz$

D. $210 Hz$.

Answer: A

Solution:

Solution:
First case: Frequency $= \upsilon$;
No. of beats/sec. $= 5$ and
frequency (sounded with) $= 200 Hz$
. Second case: Frequency $= 2\upsilon$;
No. of beats/sec $= 10$ and frequency (sounded with) $= 420 Hz$.
In the first case, frequency $(\upsilon) = 200 \pm 5 = 205$ or $195 Hz$. And in the second case, frequency $(2\upsilon) = 420 \pm 10$ or $\upsilon = 210 \pm 5 = 205$ or 215. So common value of υ in both the cases is $205 Hz$..

Question79

Wave has simple harmonic motion whose period is 4 seconds while another wave which also possesses simple harmonic motion has its period 3 second. If both are combined, then the resultant wave will have the period equal to
(1993)

Options:

A. 4s

B. 5s

C. 12s

D. 3s

Answer: C

Solution:

Solution:
Beats are produced. Frequency of beats will be $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$
Hence time period $= 12$s.

Question80

A stretched string resonates with tuning fork frequency $512Hz$ when length of the string is $0.5m$. The length of the string required to vibrate resonantly with a tuning fork of frequency $256Hz$ would be (1993)

Options:

A. $0.25m$

B. $0.5m$

C. $1m$

D. $2m$

Answer: C

Solution:

Solution:
$f = \frac{1}{2l}\left[\frac{T}{\mu}\right]^{\frac{1}{2}}$ when f is halved, the length is doubled.
∴ Length is $1m$

Question81

The temperature at which the speed of sound becomes double as was at 27°C is
(1993)

Options:

A. 273°C

B. 0°C

C. 927°C

D. 1027°C

Answer: C

Solution:

Velocity of sound, $v \propto \sqrt{T}$

$\therefore \frac{v}{2v} = \frac{\sqrt{273 + 27}}{\sqrt{T}}$ or $T = 1200K = 927°C$

Question82

For production of beats the two sources must have
(1992)

Options:

A. different frequencies and same amplitude

B. different frequencies

C. different frequencies, same amplitude and same phase

D. different frequencies and same phase

Answer: B

Solution:

Solution:
For production of beats different frequencies are essential. The different amplitudes effect the minimum and maximum amplitude of the beats. If frequencies are different, phases will be different.

Question83

The frequency of sinusodial wave $y = 0.40\cos[2000t + 0.80]$ would be (1992)

Options:

A. $1000\pi Hz$

B. $2000Hz$

C. $20Hz$

D. $\frac{1000}{\pi}Hz$

Answer: D

Solution:

Compare with the equation,
$y = a\cos(2\pi\upsilon t + \phi)$
This give $2\pi v = 2000$
$\upsilon = \frac{1000}{\pi}Hz$

Question84

With the propagation of a longitudinal wave through a material medium, the quantities transmitted in the propagation direction are (1992)

Options:

A. energy, momentum and mass

B. energy

C. energy and mass

D. energy and linear momentum

Answer: B

Solution:

Solution:
With the propagation of a longitudinal wave, energy alone is propagated.

Question85

Two trains move towards each other with the same speed. The speed of sound is 340m / s. If the height of the tone of the whistle of one of them heard on the other changes to $\frac{9}{8}$ times, then the speed of each train should be
(1991)

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Options:

A. 20m / s

B. 2m / s

C. 200m / s

D. 2000m / s

Answer: A

Solution:

Solution:
Here $\upsilon' = \frac{9}{8}\upsilon$
Source and observer are moving in opposite direction, apparent frequency
$\upsilon' = \upsilon \times \frac{(v+u)}{(v-u)}$
$\frac{9}{8}\upsilon = \upsilon \times \frac{340+u}{340-u}$
$\Rightarrow 9 \times 340 - 9u = 8 \times 340 + 8u$
$\Rightarrow 17u = 340 \times 1 \Rightarrow u = \frac{340}{17} = 20m / s$

Question86

A closed organ pipe (closed at one end) is excited to support the third overtone. It is found that air in the pipe has
(1991)

Options:

A. three nodes and three antinodes

B. three nodes and four antinodes

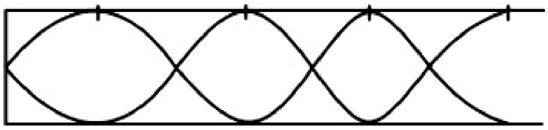
C. four nodes and three antinodes

D. four nodes and four antinodes

Answer: D

Solution:

Third overtone has a frequency $4n$, 4^{th} harmonic = three full loops + one half loop, which would make four nodes and four antinodes.



Question87

Velocity of sound waves in air is 330m / s For a particular sound wave in air, a path difference of 40cm is equivalent to phase difference of 1.6π. The frequency of this wave is
(1990)

Options:

A. $165Hz$

B. $150Hz$

C. $660Hz$

D. $330Hz$

Answer: C

Solution:

Solution:

From $\Delta x = \frac{\lambda}{2\pi}\Delta\phi$

$\lambda = 2\pi\frac{\Delta x}{\Delta\phi} = \frac{2\pi(0.4)}{1.6\pi} = 0.5m$

$\upsilon = \frac{v}{\lambda} = \frac{330}{0.5} = 660Hz$

Question88

A 5.5 metre length of string has a mass of 0.035kg. If the tension in the string in 77N , the speed of a wave on the string is
(1989)

Options:

A. $110ms^{-1}$

B. $165ms^{-1}$

C. $77ms^{-1}$

D. $102 ms^{-1}$

Answer: A

Solution:

Solution:
$\mu = \frac{0.035}{5.5} kg / m, T = 77N$
where μ is mass per unit length.
$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{77 \times 5.5}{0.035}} = 110 m / s$

Question89

If the amplitude of sound is doubled and the frequency reduced to one fourth, the intensity of sound at the same point will be (1989)

Options:

A. increasing by a factor of 2

B. decreasing by a factor of 2

C. decreasing by a factor of 4

D. unchanged

Answer: C

Solution:

Solution:
Intensity $\propto (\text{amplitude})^2$ and also intensity $\propto (\text{frequency})^2$.
Therefore original $I \propto A^2 \omega^2$
$I' \propto 4A^2 \frac{\omega^2}{16}$ i.e., $\frac{1}{4} I$

Question90

The velocity of sound in any gas depends upon (1988)

Options:

A. wavelength of sound only

B. density and elasticity of gas

C. intensity of sound waves only

D. amplitude and frequency of sound

Answer: B

Solution:

Solution:
Velocity of sound in any gas depends upon density and elasticity of gas.
$v = \sqrt{\frac{\gamma P}{\rho}}$ or $\sqrt{\frac{B}{\rho}}$

Question91

Equation of progressive wave is given by $y = 4\sin\left[\pi\left(\frac{t}{5} - \frac{x}{9}\right) + \frac{\pi}{6}\right]$ where y, x are in cm and t is in seconds. Then which of the following is correct? (1988)

Options:

A. $v = 5cm$

B. $\lambda = 18cm$

C. $a = 0.04cm$

D. $f = 50Hz$

Answer: B

Solution:

The standard equation of a progressivewave is
$y = a\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \phi\right]$
The given equation can be written as
$y = 4\sin\left[2\pi\left(\frac{t}{10} - \frac{x}{18}\right) + \frac{\pi}{6}\right]$
$\therefore a = 4cm, T = 10s$
$\lambda = 18cm \text{ and } \phi = \frac{\pi}{6}$
