

Chapter 2

AC to DC Converters (or) Rectifiers

LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

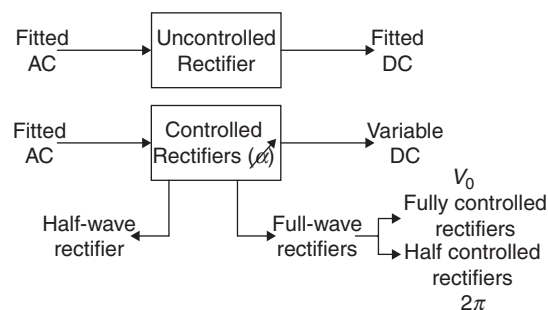
- Single phase half-wave rectifiers: with R load
- Half wave rectifier with R.L. load
- Half wave rectifier with RLC load
- Three phase controlled half-wave converter
- Gating sequence
- Single phase full controlled full wave rectifier
- Single phase dual converter
- Gating sequence
- Effect of source inductance

Earlier, DC power was obtained from motor-generator sets but nowadays phase-controlled AC to DC converters employing thyristors are extensively used for changing constant AC input voltage to controlled DC output voltage.

The members of thyristor family such as SCR and triac are more suitable for power central application, because they can be switched ON and OFF by means of simple gate control circuit.

The variation in load power at constant input voltage which can be obtained by changing the firing angle α is called as phase control.

Many industrial applications make use of controlled DC power.



Examples:

- Steel-rolling mills, paper mills, printing presser and textile mills employing DC drives.
- Magnet power supplies.
- Electromechanical and electro metallurgical process.
- High voltage DC transmission.
- Traction system working on DC.

- SCR power controllers provide a relatively economical means of power central. SCR power controllers cast less and are more efficient than saturable core reactors and variable transformers. Compared to contactors, SCR power controllers offer a much fine degree of central and do not suffer, from the maintenance problems of mechanical devices.
- The DC current is unidirectional, but the DC voltage may have either polarity.
- With one polarity, the flow of power is from the AC source to the DC load is called rectification.
- With a reversal of the DC voltage by the load, the flow of power is from the DC source to the AC supply, this process is called Inversion.
- If the half of the SCR's are replaced by diodes, then it is called half-controlled or semi-converter circuit.

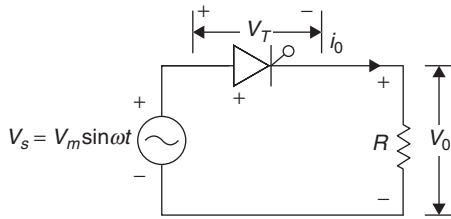
Therefore, the circuit that converts an AC signal into a unidirectional signal is called rectifier. Depending upon the type of input supply, the rectifiers are classified into

1. Single phase and
2. Three phase

Each type can be subdivided into

1. **Semi-converter:** One quadrant converter and polarity of its output voltage and current is the same.
2. **Full converter:** Two quadrant converter and the polarity of its output voltage can either be positive or negative.
3. **Dual converter:** Four quadrant converter and both output voltages and currents can be either positive or negative.

SINGLE-PHASE HALF-WAVE RECTIFIERS: WITH R LOAD



It consists of single thyristor feeding DC power to a resistive load.

So,

$$i_s = i_T = i_0$$

At

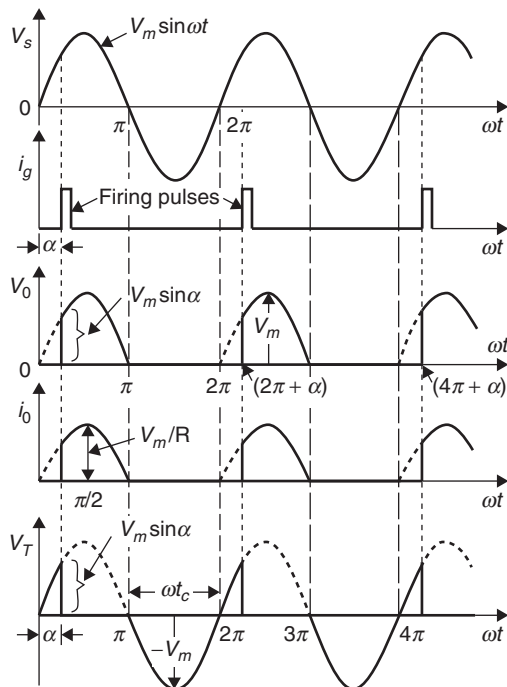
$$\omega t = \alpha$$

$$V_0 = V_m \sin \alpha$$

$$I_0 = \frac{V_m \sin \alpha}{R}$$

Principle of operation of single-phase half-wave rectifier with R load

- During positive half-cycle of the supply voltage, the SCR is forward-biased. And will conduct if a trigger pulse is applied to gate.
- Then the load current flows, output voltage V_0 is same as the input voltage.
- At $t = \pi$, current falls naturally to zero since the SCR is reverse-biased.
- During negative half cycle, SCR blocks the flow of current and no voltage is applied to load.
- ωt_0 to $+2\pi$, again the SCR starts conduct.
- From ωt_0 to 2π it is called firing angle or delay angle and ωt_0 to π is called conduction angle (θ).



Current waveform is same. As V_0 but magnitude is differed depending on the value of R .

Line commutation takes place

Turn-off time $= t_c = \pi/\omega$

Average output voltage

$$V_0 = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t d(\omega t)$$

$$V_0 = \frac{V_m}{2\pi} (1 + \cos \alpha)$$

RMS value of output voltage

$$V_{\text{RMS}} = \sqrt{\left(\frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t d(\omega t) \right)}$$

$$V_{\text{RMS}} = \frac{V_m}{2\sqrt{\pi}} [\pi - \alpha + (\sin 2\alpha)/2]^{1/2}$$

Average output current

$$I_0 = V_0/R$$

RMS output current

$$I_{\text{RMS}} = \frac{V_{\text{RMS}}}{R}$$

Expression for input power factor:

Source Side

$$V_s = V_m \sin \omega t$$

$$V_{\text{sr}} = \frac{V_m}{\sqrt{2}}$$

$$(V_0)_{\text{avg}} = 0 \text{ V}$$

$$(I_s)_{\text{avg}} = \frac{V_0}{R} \frac{V_m}{2\pi R} [1 + \cos \alpha]$$

$$(I_0)_{\text{avg}} = (I_s)_{\text{avg}}$$

$$P_{\text{IN}} = V_{\text{sr}} I_{\text{sr}} \cos \phi_s$$

Control Output

$$V_{\text{or}} = \left\{ \frac{V_m^2}{2\pi} \left[\frac{\pi - \alpha}{2} + \frac{\sin 2\alpha}{4} \right] \right\}^{1/2}$$

$$(V_0)_{\text{avg}} = \frac{V_m}{2\pi} [1 + \cos \alpha]$$

$$P_{\text{OUT}} = V_{\text{or}} I_{\text{or}} \cos \theta$$

$$P_{\text{IN}} = P_{\text{OUT}}$$

$$V_{\text{sr}} I_{\text{sr}} \cos \phi_s = V_{\text{or}} \times I_{\text{or}} \times 1$$

$$\text{Source P.F.} = \cos \phi_s = \frac{V_{\text{or}}}{V_{\text{sr}}}$$

Power Factor (P.F.) of input supply

$$= \frac{\text{Power supplied to load}}{\text{Source volt ampere}}$$

$$\text{P.F.} = \frac{V_{\text{RMS}}^2/R}{V_s I_s} = \frac{V_{\text{RMS}}^2/R}{V_s I_s}$$

$$\left[\because I_s = I_{\text{RMS}} = \frac{V_{\text{RMS}}}{R} \right]$$

$$\text{P.F.} = \frac{V_{\text{RMS}}}{V_s}$$

when $\alpha = 0$

Then $V_0 = \frac{V_m}{\pi} = 0.318 V_m$.

$$I_0 = \frac{0.318 V_m}{R}$$

$$V_{\text{RMS}} = \frac{V_m}{2} = 0.5 V_m I_{\text{RMS}}$$

$$= \frac{V_{\text{RMS}}}{R} = \frac{0.5 V_m}{R}$$

$$P_{\text{DC}} = (0.318 V_m)^2/R$$

$$P_{\text{AC}} = (0.5 V_m)^2/R$$

$$(A) \eta = \frac{P_{\text{DC}}}{P_{\text{AC}}} = \frac{(0.318 V_m)^2/R}{(0.5 V_m)^2/R} = 40.5\%$$

$$(B) \text{ F.F.} = \frac{0.5 V_m}{0.318 V_m} = 157\% = 1.57\%$$

$$(C) \text{ R.F.} = \sqrt{1.57^2 - 1} = 1.21 \text{ or } 121\%$$

$$(D) (V_s)_{\text{RMS}} = \left[\frac{1}{T} \int_0^T V_m \sin \omega t^2 dt \right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

$$(I_s)_{\text{RMS}} = 0.5 V_m / R$$

(E) Peak Inverse Voltage (PIV)

$$\text{PIV} = V_m$$

$$(F) I_{s(\text{peak})} = \frac{V_m}{R} = I_s = 2$$

(G) Input P.F. for a resistive load

$$\text{P.F.} = \frac{P_{\text{AC}}}{VA} = \frac{(0.5)^2}{(0.707) \times 0.5}$$

$$= 0.707$$

$V_s \rightarrow$ Transformer secondary voltage

$I_s \rightarrow$ Transformer secondary current

$$\text{V-A rating} = V_s I_s = 0.707 V_m \times \frac{0.5 V_m}{R}$$

$$\text{TUF} = \frac{P_{\text{AC}}}{(V_s I_s)} = \frac{(0.318)^2}{(0.707) \times 0.5} = 0.286$$

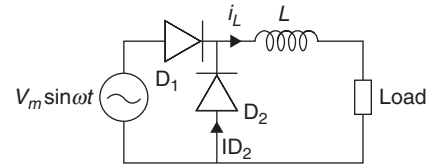
Note:

- The performance of the converter is degraded at higher range of α .
- Varying the delay angle α from 0 to π can vary the average output voltage from V_m/π to 0.

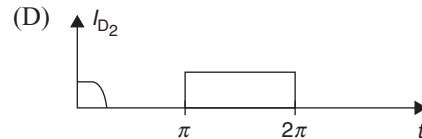
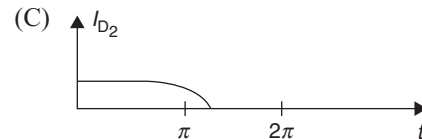
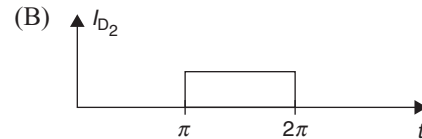
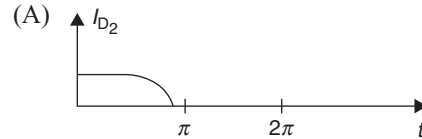
Input transformer can carry DC current, thereby causing a magnetic saturation problem.

Solved Examples

Example 1:



The figure above is that of a single-phase half wave uncontrolled converter fed from the output of a two winding isolating transformer. The current waveform through diode D_2 will be [assume the load current to be constant].



Solution: (B)

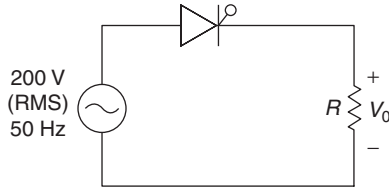
Diode D_2 conducts only at negative half cycles, i.e., from π to 2π .

Choice (A) and (C) gives I_{D_2} during positive cycle hence incorrect.

Choice (D) has a small current I_{D_2} in the positive half cycle hence wrong. The correct option is choice (B).

Example 2: In the phase-controlled half-wave converter shown above, the thyristor is fired in every positive half cycle of the input voltage, at an angle α . The firing angle for the peak value of the instantaneous output voltage of 200 V would be close to

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(A) 90° (B) 135° (C) 45° (D) 78.23°

Solution: (B)

$$\begin{aligned} \text{For } \alpha < 90^\circ \quad V_m &= V_p \\ \text{For } \alpha > 90^\circ \quad V_m \sin \alpha &= V_p \end{aligned}$$

$$V_{\text{RMS}} = 200 \text{ V}$$

$$\Rightarrow V_m = 200\sqrt{2}$$

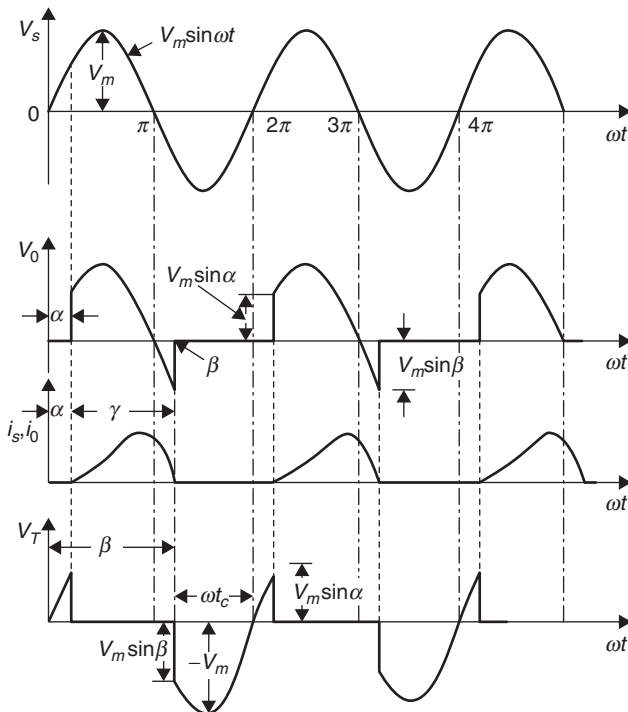
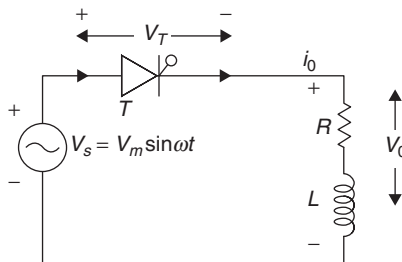
$$200\sqrt{2} \sin \alpha = 200$$

$$\sin \alpha = \frac{1}{\sqrt{2}} \left\{ \text{Since } \alpha > 90^\circ \right\}$$

$$\alpha = 45^\circ$$

$$\alpha = 90 + 45 = 135^\circ$$

Half-wave Rectifier with R.L. Load



Principle of Operation

Circuit turn-off time

$$T_c = \frac{(2\pi - \beta)}{\omega}$$

Average output voltage

$$V_0 = \frac{1}{2\pi} \int_{\alpha}^{\beta} V_m \sin \omega t d(\omega t)$$

$$V_0 = \frac{V_m}{2\pi} (\cos \alpha - \cos \beta)$$

RMS output voltage

$$V_{\text{RMS}} = \sqrt{\left(\frac{1}{2\pi} \int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t d(\omega t) \right)}$$

$$V_{\text{RMS}} = \frac{V_m}{2\sqrt{\pi}} \left[\beta - \alpha + \frac{1}{2} [\sin(2\alpha) - \sin(2\beta)] \right]^{1/2}$$

Average output current

$$I_0 = V_0 / R$$

RMS value of output current,

$$I_{\text{RMS}} = \frac{V_{\text{RMS}}}{R}$$

β is called extinction angle.

It depends on the strength of the inductor minimum value of β is π .

α is called conduction angle.

The power from $\alpha \rightarrow \beta$ is less than the power from $\alpha \rightarrow \pi$ because some energy is returned by the inductor to source.

$\Rightarrow$ By using free-wheeling diode, we can avoid this.

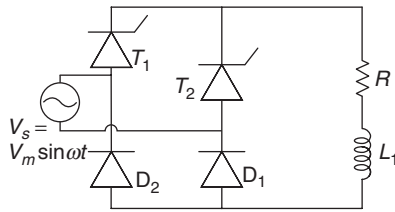
Power factor of supply,

$$\text{P.F.} = \frac{V_{\text{RMS}}}{V_s}$$

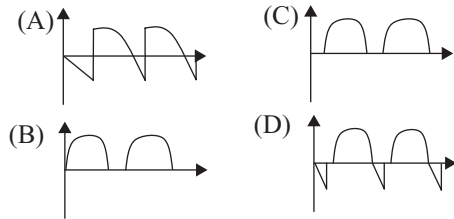
Principle of Operation

- If the SCR is triggered, at a firing angle of ∞ , the load current increases slowly, since the inductance in the load forces the current to lag the voltage.
- The voltage across the load is positive, inductor, stores energy.
- In the negative, region, SCR is reverse biased, energy stored in the inductor is returned and maintains a forward decaying current through the load.
- Current flows until β even though SCR turns off.
- Due to which the average output voltage becomes less than that of purely resistive load.

Example 4:



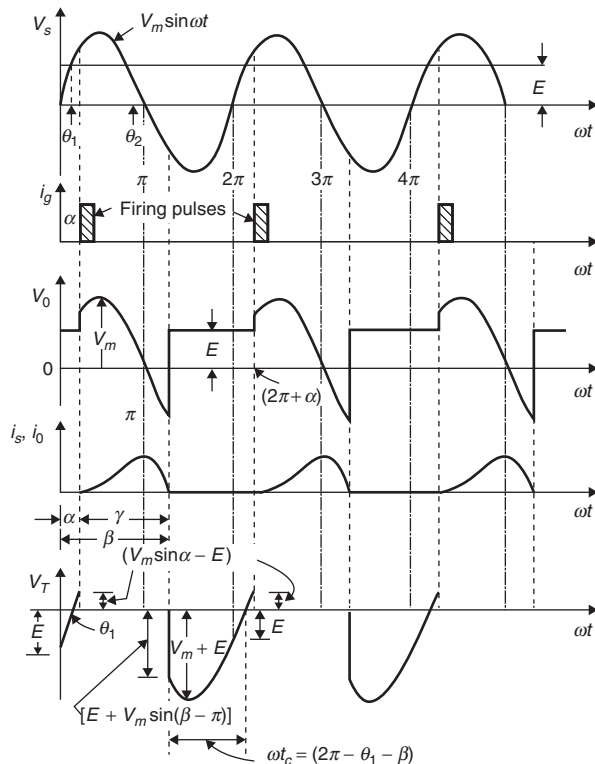
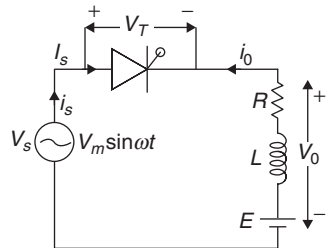
The output DC voltage wave shape of a single-phase-controlled semi-converter shown in the figure, operating at $\alpha = 30^\circ$, will be as shown in the following figure.



Solution: (C)

In a semi-converter, the output voltage will never be negative and only will conduct after firing.

Half-wave Rectifier with RLE Load



- Firing angles ranges from θ_1 to θ_2 .
- β is greater than θ_2 but can be less than π circuit turn-off time.

$$t_c = \frac{2\pi + \theta_1 - \beta}{\omega}$$

Average output voltage

$$V_0 = E + I_0 R$$

Average output current

$$I_0 = \frac{1}{2\pi} \int V_m (\sin \omega t - E) / R d\omega t$$

$$I_0 = \frac{1}{2\pi R} [V_m [\cos \alpha - \cos \beta] - E (\beta - \alpha)]$$

Supply power factor

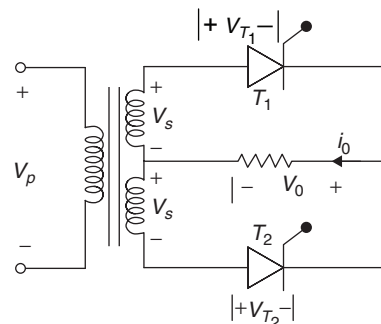
$$P_f = \frac{I_{RMS}^2 R + I_0 E}{V_s I_{RMS}}$$

$$\theta_1 = \sin^{-1}(E/V_m)$$

$$\theta_2 = 180^\circ - \theta_1$$

SINGLE-PHASE FULL WAVE MID-POINT TYPE RECTIFIERS

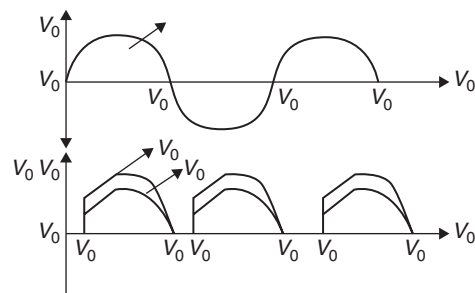
- A full-wave rectifier circuit with a centre tapped transformer is shown below.



- Each half of the transformer with its associated diode acts as a half-wave rectifier and the output of a full-wave rectifier is shown.
- Because there is no DC current flowing through the transformer, there is no DC saturation problem of transformer core.

⇒ It is assumed that the number of turns from primary to each secondary is same.

(R-load) Mid-point Converter:



Average output voltage

$$V_0 = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin(\omega t) d\omega t$$

$$(V_0)_{\text{avg}} = \frac{V_m}{\pi} [1 + \cos \alpha]$$

$$(I_0)_{\text{avg}} = \frac{(V_0)_{\text{avg}}}{R} = \frac{V_m}{\pi R} [1 + \cos \alpha]$$

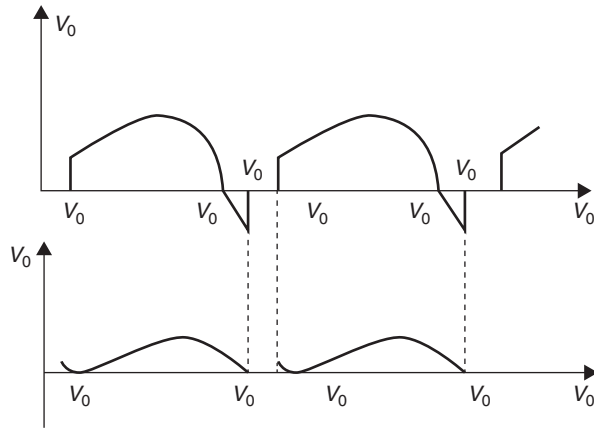
$$V_{\text{RMS}} = \left[\frac{1}{\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t d\omega t \right]^{1/2}$$

$$V_{\text{RMS}} = \left[\frac{V_m^2}{\pi} \left[\frac{\pi - \alpha}{2} + \frac{\sin 2\alpha}{4} \right] \right]^{1/2}$$

RI-load

Circuit turn-off time

$$t_c = \frac{2\pi - \beta}{\omega}$$



Conduction angle = $\beta - \alpha$

Average output voltage,

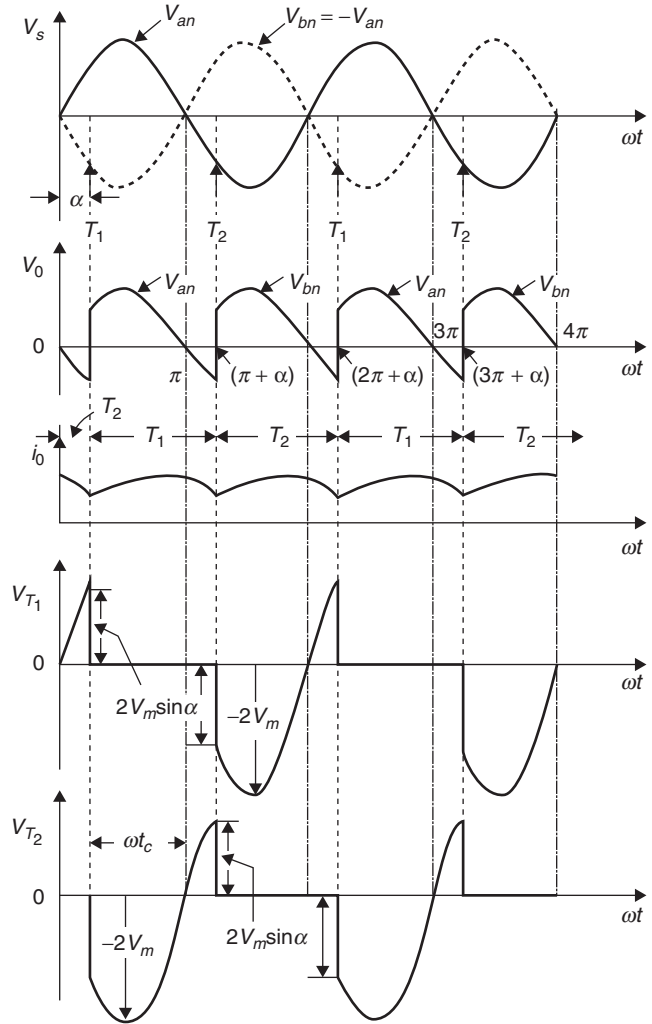
$$V_0 = \frac{1}{\pi} \int_{\alpha}^{\beta} V_m \sin \omega t d\omega t$$

$$V_0 = \frac{V_m}{\pi} [\cos \alpha - \cos \beta]$$

$$V_{\text{or}} = \left[\frac{1}{\pi} \int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t d\omega t \right]^{1/2}$$

$$V_{\text{or}} = \left\{ \frac{V_m^2}{\pi} \left[\frac{\beta - \alpha}{2} + \frac{\sin 2\alpha - \sin 2\beta}{4} \right] \right\}^{1/2}$$

Continuous Conduction Load



Circuit turn-off time

$$T_c = \frac{\pi - \alpha}{\omega}$$

Average output voltage

$$V_0 = \frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} V_m \sin \omega t d(\omega t)$$

$$V_0 = \frac{2V_m}{\pi} \cos \alpha$$

For R-load $V_0 = \frac{V_m}{\pi} [1 + \cos \alpha]$

In this method, thyristor is subjected twice to the supply voltage $2V_m \sin \alpha$, which is not desirable. Also thyristor are under rated.

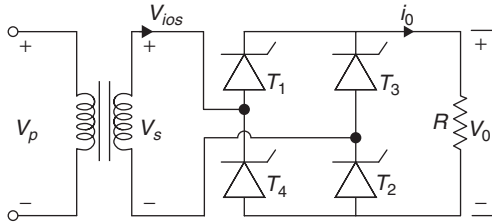
For both mid-point and bridge converters output voltage equations and waveforms are same except their peak inverse voltages.

$$(\text{PIV})_{\text{mid-point}} = 2 (\text{PIV})_{\text{Bridge}}$$

For same rating SCRs used,

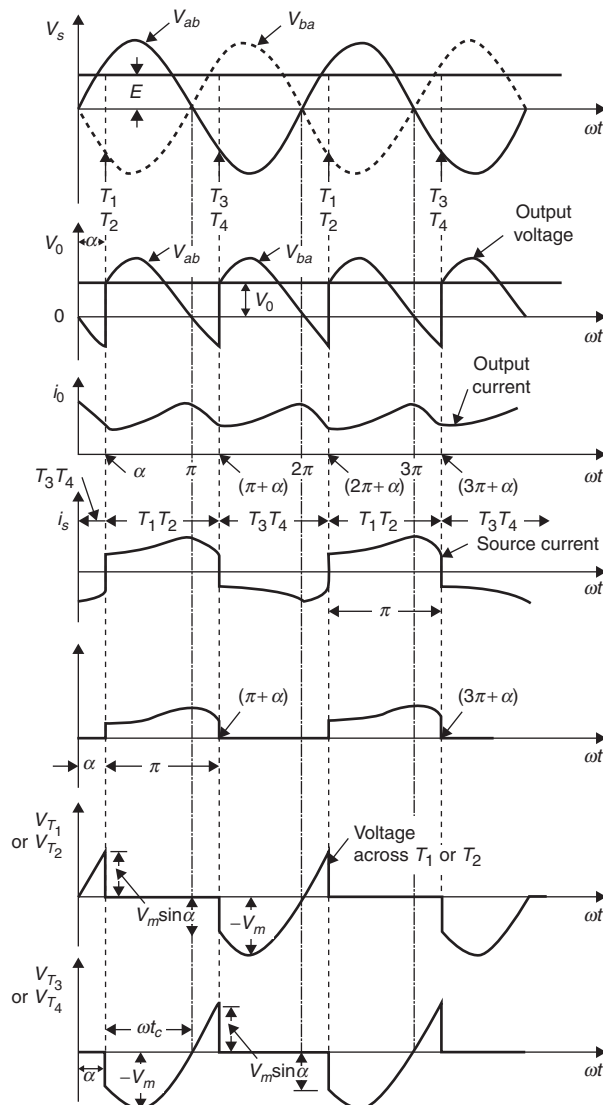
$$\frac{(\text{Power-handling capacity})_{\text{Bridge}}}{2 (\text{Power-handling capacitor})_{\text{mid-point}}}$$

SINGLE-PHASE FULL-WAVE BRIDGE-TYPE RECTIFIER



- During positive half-cycle of input voltage, the power is supplied to the load through diodes D_1 and D_2 .
- During negative half-cycle diodes D_3 and D_4 conduct. The output voltage is shown in figure below. The PIV of a diode is only V_m .

Continuous Conduction (Highly Inductive)



R-Load

0 to α :

$$T_1, T_2, T_3, T_4 \rightarrow \text{OFF}$$

$$i_0 = 0, i_s = 0, V_T(1, 2, 3, 4) = 0, V_0 = 0.$$

α to π

$$T_1, T_2 \rightarrow \text{ON}, T_3, T_4 \rightarrow \text{OFF}$$

$$V_0 = V_s, i_0 = i_s, V_T(1, 2) = 0, V_T(3, 4) = -V_s$$

$$\text{PIV} = V_m$$

Average output voltage:

$$V_0 = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t \, d\omega t$$

$$V_0 = \frac{V_m}{\pi} (1 + \cos \alpha) V_0$$

$$I_0 = \frac{V_m}{\pi R} (1 + \cos \alpha) I_0$$

RMS Output Voltage

$$V_{or} = \left[\frac{1}{\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t \, d\omega t \right]^{1/2}$$

$$= \left(\frac{V_m^2}{\pi} \left(\frac{\pi - \alpha}{2} + \frac{\sin 2\alpha}{4} \right) \right)^{1/2}$$

Circuit turn-off time,

$$t_c = \frac{\pi - \alpha}{\omega}.$$

RL Load (Discontinuous Condition)

$$(V_0)_{\text{avg}} = \frac{1}{\pi} \int_{\alpha}^{\beta} V_m \sin \omega t \, d\omega t = \frac{V_m}{\pi} (\cos \alpha - \cos \beta).$$

$$V_{or} = \left\{ \frac{V_m^2}{\pi} \left[\frac{\beta - \alpha}{2} + \frac{\sin 2\alpha - \sin 2\beta}{4} \right] \right\}^{1/2}$$

Circuit turn-off time,

$$t_c = \frac{2\pi - \beta}{\omega}$$

RL and RLE Continuous Conduction Load

$$(V_0)_{\text{avg}} = \frac{2V_m}{\pi} \cos \alpha$$

$$V_{or} = V_m / \sqrt{2}$$

Circuit turn-off time, $t_c = \frac{\pi}{\omega}$.

$\Rightarrow$ The thyristors can be subjected to a maximum voltage of V_m only not $2V_m$ as in midpoint type converter.

⇒ SCR are used to full rated voltage.
 ⇒ Power loss and voltage drop is more.
 ⇒ $\alpha > 90^\circ$, output voltage is negative, and can be used for regenerative braking applications.

Performance Parameter

For a full-wave DC rectifier
 Average output voltage is

$$V_{DC} = \frac{2V_m}{\pi} = 0.6366 V_m$$

Average load current is

$$I_{DC} = \frac{V_{DC}}{R} = \frac{0.6366 V_m}{R}$$

$$P_{DC} = (0.6366 V_m)^2 / R$$

RMS value of output voltage and current is

$$V_{RMS} = \left[\frac{2}{T} \int_0^{T/2} (V_m \sin \omega t) d\omega t \right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

$$I_{RMS} = \frac{V_{RMS}}{R} = \frac{0.707 V_m}{R}$$

$$P_{AC} = (0.707 V_m)^2 / R$$

$$(a) \text{ Efficiency } \eta = \frac{P_{DC}}{P_{AC}} = \frac{(0.6366 V_m)^2}{(0.707 V_m)^2} = 81\%$$

$$(b) \text{ Form Factor FF} = \frac{V_{RMS}}{V_{avg}} = \frac{0.707 V_m}{0.6366 V_m}$$

$$\text{FF} = 1.11$$

$$(c) \text{ Ripple Factor RF} = \sqrt{1.11^2 - 1} = 0.482$$

$$\text{RF} = 48.2\%$$

$$(d) V_s = V_m / \sqrt{2} = 0.707 V_m$$

$$I_s = 0.5 V_m / R$$

$$\text{VA rating of transformer} = 2 V_s I_s$$

$$= 2 \times 0.707 V_m \times 0.5 V_m / R$$

$$\text{TUF} = \frac{P_{DC}^2}{VA} = \frac{0.6366^2}{2 \times 0.707 \times 0.5} = 0.5732$$

$$\text{TUF} = 57.32 \%$$

$$(e) \text{ PIV} = 2 V_m$$

$$(f) (I_s)_{\text{peak}} = V_m / R \text{ and } I_s = \frac{0.707 V_m}{R}$$

$$\text{C.F. of input current is } \frac{(I_s)_{\text{peak}}}{I_s}$$

$$\text{C.F.} = \frac{1}{0.707} = \sqrt{2}$$

(g) The input P.F. for a resistive load

$$\text{P.F.} = \frac{P_{AC}}{VA} = \frac{0.707^2}{2 \times 0.707 \times 0.5} = 0.707$$

Note:

$$\bullet \frac{1}{\text{TUF}} = \frac{1}{0.5732} = 1.75 \text{ signifies that the input trans-}$$

former, if present, must be 1.75 times larger than when it is used to deliver power from a pure AC sinusoidal voltage.

- It has R.F. = 48.2% and a rectification of 81%.
- The performance of a full-wave rectifier is significantly improved compared with that of a half-wave rectifier.

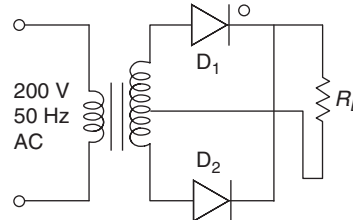
Example 5: If a ripple free current is assumed to be flowing through a highly inductive load, which is supplied from a single-phase diode bridge rectifier. The input side current wave form will be

- (A) square (B) sinusoidal
 (C) triangular (D) constant

Solution: (A)

Ripple free output current will have constant magnitude and thereby the supply current will take a square-wave form.

Example 6: The figure shown below is a full-wave rectifiers fed from a centre tapped transformer.



The diodes D_1 and D_2 would have a peak reverse voltage of

- (A) 30 V (B) 60 V
 (C) $30\sqrt{2}$ V (D) $60\sqrt{2}$ V

Solution: (B)

Peak reverse voltage across the diode = $2V_m$

V_m = Peak value of each secondary voltage and the secondary of centre tapped transformer

$$V_s = 30(\text{RMS})$$

$$V_{s(\text{max})} = 30\sqrt{2} \text{ V} = V_m$$

$$\text{Peak reverse voltage} = 2 V_m = 2 \times 30 \times \sqrt{2}$$

$$= 60\sqrt{2} \text{ V}$$

Example 7: A single-phase fully controlled thyristor bridge converter is fed from a 230 V, 50 Hz supply is operating at a firing angle of 30° and constant DC output current of 10 A. The displacement factor at input is [assume overlap angle of 10°]

- (A) 0.7 (B) 0.6 (C) 0.8 (D) 0.85

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Solution: (A)

$$\text{Displacement factor D.F.} = \frac{V_0 I_0}{V_s I_s}$$

$$V_0 = \frac{2V_m \cos \alpha}{\pi} - \frac{\omega L_s I_0}{\pi}$$

$$I_0 = \frac{V_m}{\omega L_s} [\cos \alpha - \cos(\alpha + \mu)]$$

$$\omega = 2\pi f = 2\pi \times 50 = 314.16$$

$$L_s = \frac{V_m}{\omega I_0} [\cos \alpha - \cos(\alpha + \mu)]$$

$$= \frac{230 \times \sqrt{2}}{314.16 \times 10} [\cos 30 - \cos(30 + 10)]$$

$$L_s = 0.0103$$

$$V_0 = \frac{2 \times 230 \times \sqrt{2} \cos 30}{\pi} - \frac{314.16 \times 0.0103 \times 10}{\pi}$$

$$= \frac{1}{\pi} [531.024] = 169 \text{ V}$$

$$\text{D.F.} = \frac{169 \times 10}{230 \times 10} = 0.73$$

Performance Analysis 1- ϕ Full Converter

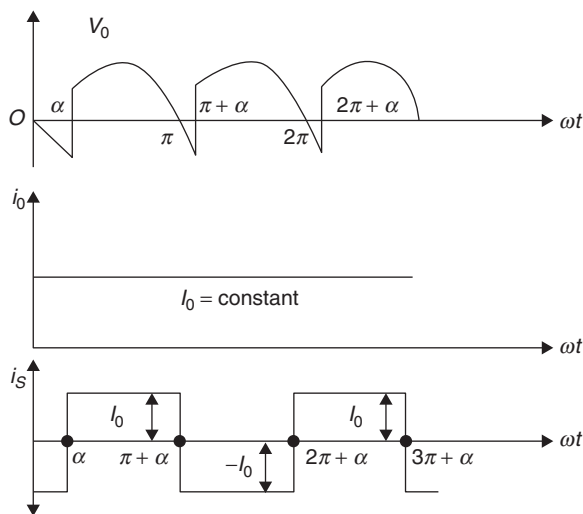
Single-phase fully controlled rectifier in continuous conduction mode draws a constant load current, I_0 .

$$(V_0)_{\text{avg}} = \frac{2V_m}{\pi} \cos \alpha$$

$$V_{\text{or}} = V_{\text{sr}} = \frac{V_m}{\sqrt{2}}$$

$$I_{\text{sr}} = I_0 = I_{\text{or}}$$

$$I_{s(\text{avg})} = 0$$



Source current is not a sinusoidal wave.

Applying Fourier series analysis to it to carry out the harmonic distortion analysis,

$$i_s = \sum_{n=1,3,5}^{\infty} \frac{4I_0}{n\pi} \sin(n\omega t + \phi_n)$$

where $\phi_n = -n\alpha$

$$\begin{aligned} \text{RMS value of } n^{\text{th}} \text{ harmonic current, } I_{\text{sn(RMS)}} &= \frac{4I_0}{n\pi \times \sqrt{2}} \\ &= \frac{2\sqrt{2}I_0}{n\pi} \end{aligned}$$

$$(i) \text{ Distortion Factor} = \frac{\text{Fundamental RMS}}{\text{Overall RMS}} = \frac{(I_{s1})_{\text{RMS}}}{I_{\text{sr}}}$$

$$(ii) \frac{\frac{2\sqrt{2}I_0}{\pi}}{I_0} = \frac{2\sqrt{2}}{\pi}$$

$$(iii) \text{ Fundamental Displacement Factor} = \cos \phi_1 = \cos(-\alpha) = \cos \alpha$$

$$(iv) \text{ Source Power Factor} = (\text{Distortion Factor}) (\text{Fundamental Displacement Factor})$$

$$\frac{2\sqrt{2}}{\pi} \cos \alpha$$

$$(iv) \text{ Total Harmonic Distortion (THD)}$$

$$= \frac{(\text{Useless harmonics})_{\text{RMS}}}{(\text{Fundamental})_{\text{RMS}}}$$

$$\text{THD} = \sqrt{\left(\frac{I_{\text{sr}}}{I_{s1}}\right)^2 - 1}$$

$$\text{THD} = \sqrt{\frac{\pi^2}{8} - 1} = 48.34\%$$

$$(v) \text{ From Factor}$$

$$= \frac{\text{RMS output voltage}}{\text{Average output voltage}} = \frac{V_m / \sqrt{2}}{\frac{2V_m}{\pi} \cos \alpha}$$

$$= \frac{1}{\frac{2\sqrt{2}}{\pi} \cos \alpha} = \frac{1}{\text{Source Power Factor}}$$

$$(vi) \text{ Ripple Factor (voltage ripple factor)}$$

$$= \frac{\text{Ripple output voltage}}{\text{Average output voltage}}$$

$$= \sqrt{\frac{V_{\text{or}}^2 - V_0^2}{V_0^2}}$$

$$= \sqrt{(\text{Form factor})^2 - 1}$$

$$= \sqrt{\frac{\pi^2}{8 \cos^2 \alpha} - 1}$$

$$(vii) \text{ Source Active Power} = V_{sr} I_{sr} \cos \phi_s$$

$$= \frac{V_m}{\sqrt{2}} \times I_0 \times \frac{2\sqrt{2}}{\pi} \cos \alpha$$

$$= \frac{2V_m}{\pi} \cos \alpha \cdot I_0 = (V_0)_{avg} (I_0)_{avg}$$

$$P_{IN} = P_{OUT}$$

$$(viii) \text{ Source Reactive Power} = V_{sr} I_{sr} \sin \phi_s$$

$$= \frac{V_m}{\sqrt{2}} \times I_0 \times \frac{2\sqrt{2}}{\pi} \sin \alpha = \frac{2V_m}{\pi} \cos \alpha \cdot I_0 \cdot \tan \alpha$$

$$= V_0 I_0 \tan \alpha$$

SINGLE-PHASE SEMI-CONVERTER

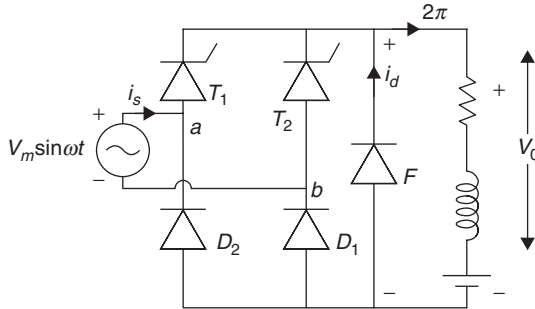


Figure 1 Symmetrical semi converter

For R-Loads

$$\text{Symmetrical: Circuit turn-off time, } t_c = \frac{\pi - \alpha}{\omega}$$

$$\text{Asymmetrical: Circuit turn-off time, } t_c = \frac{\pi}{\omega}$$

$$(V_0)_{avg} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t \, d\omega t = \frac{V_m}{\pi} [1 + \cos \alpha]$$

$$\text{Output RMS voltage, } V_{RMS}$$

$$= \left[\frac{1}{\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t \, d\omega t \right]^{1/2}$$

$$V_{RMS} = \frac{V_m}{\pi} \left[\frac{\pi - \alpha}{2} + \frac{\sin 2\alpha}{4} \right]^{1/2}$$

For Continuous Conduction Loads (RL, RLE)

On the assumption of constant load current, I_0

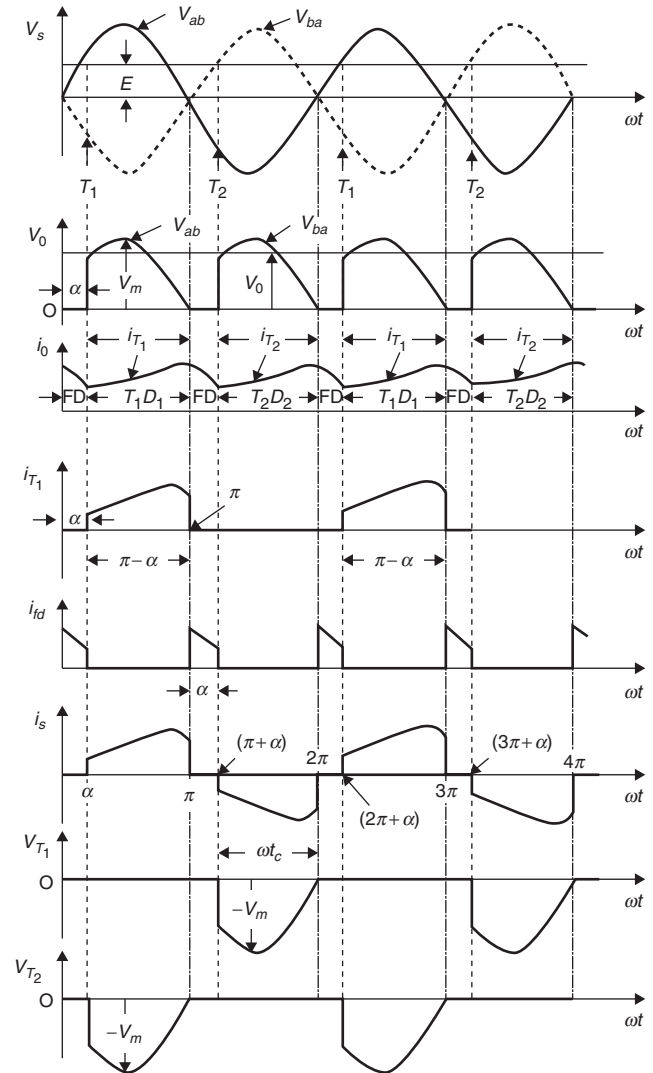
$$(I_{T1})_{avg} = I_0 \left(\frac{\pi - \alpha}{2\pi} \right)$$

$$(I_{T1})_{RMS} = I_0 \left(\frac{\pi - \alpha}{2\pi} \right)^{1/2}$$

$$V_0 = \frac{V_m}{\pi} [1 + \cos \alpha]$$

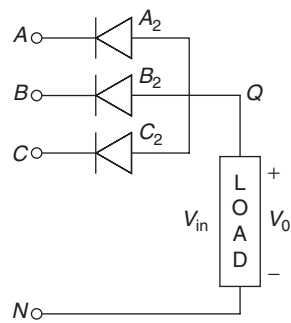
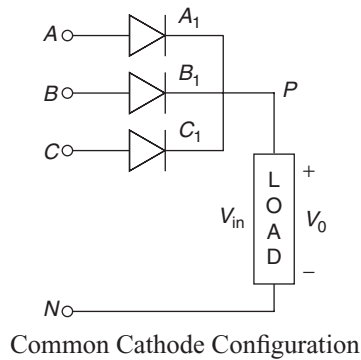
$$V_{or} = \left\{ \frac{V_m^2}{\pi} \left[\frac{\pi - \alpha}{2} + \frac{\sin 2\alpha}{4} \right] \right\}^{1/2}$$

$$I_{sr} = I_0 \left[\frac{\pi - \alpha}{\pi} \right]^{1/2}$$



No negative voltage is obtained in this case. Since only two thyristors are used cost is less.

THREE-PHASE UNCONTROLLED HALF-WAVE RECTIFIER



- In common cathode configuration, the diode is subjected to maximum positive voltage will be in conduction. Load is subjected only to phase voltages.
- The diode which is subjected to maximum negative voltage will be in conduction in common anode configuration.

Average output voltage

$$V_0 = \frac{1}{2\pi/3} \int_{\pi/6}^{5\pi/6} V_m \sin \omega t d(\omega t)$$

$$V_0 = \frac{3\sqrt{3}}{2\pi} V_{mp}$$

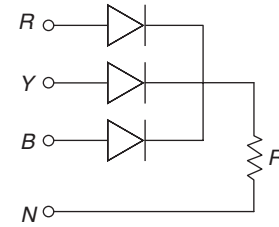
$$V_0 = \frac{3\sqrt{6}}{2\pi} V_{ph}$$

- If diodes are replaced with thyristors,

$$V_0 = \frac{3\sqrt{6}}{2\pi} V_{ph} \cos \alpha$$

- This type of converter is rarely used in industry because it introduces DC component in the supply current.

Example 8: The ripple frequency of a 3-phase half-wave rectifier, which is connected to a 380 V (line to line) 200 Hz symmetrical 3-Ø, 4wire supply, as shown below, would be



(A) 200 Hz (B) 800 Hz (C) 400 Hz (D) 600 Hz

Solution: (D)

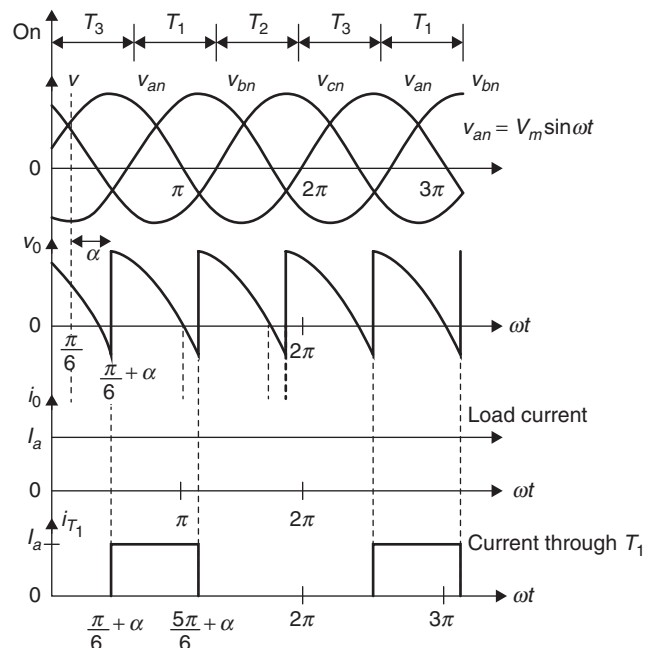
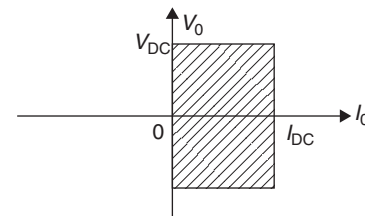
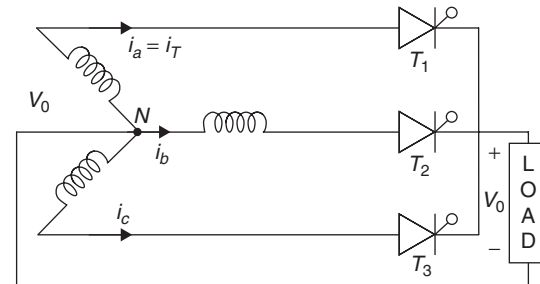
$$V_1 = 380 \text{ V}$$

$$f = 200 \text{ Hz}$$

In a 3-phase rectifier one cycle of supply voltage, the output voltage has three pulses. The ripple frequency = $3f$

$$= 3 \times 200 = 600 \text{ Hz}$$

THREE-PHASE CONTROLLED HALF-WAVE CONVERTER



$$V_{an} = V_m \sin \omega t$$

$$V_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3} \right)$$

$$V_{cn} = V_m \sin \left(\omega t - \frac{4\pi}{3} \right)$$

- When T_1 is fired at $\omega t = \pi/6 + \alpha$, the phase voltage V_{an} appears across the load until thyristor T_2 is fired at $\omega t = 5\pi/6 + \alpha$.
- When T_2 is fired, T_1 is reverse-biased, because $V_{ab} = (V_{an} - V_{bn})$ is negative and T_1 is turned off. The phase voltage appears across the load until T_3 is fired at $\omega t = \frac{3\pi}{2} + \alpha$.
- When T_3 is fired, T_2 is turned off and V_{cn} appears across the load until T_1 is fired again at the beginning of next cycle

$$V_{DC} = \frac{3}{2\pi} \int_{\left(\frac{\pi}{6} + \alpha\right)}^{\left(\frac{5\pi}{6} + \alpha\right)} V_m \sin \omega t \, d\omega t$$

$$V_{DC} = \frac{3\sqrt{3}}{2\pi} V_m \cos \alpha$$

Maximum average output voltage that occurs at $\alpha = 0$

$$V_{DM} = \frac{3\sqrt{3}}{2\pi} V_m$$

Normalized average output voltage

$$V_n = \frac{V_{DC}}{V_{DM}} = \cos \alpha$$

RMS value of output voltage

$$V_{RMS} = \left[\frac{3}{2\pi} \left(\int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} V_m^2 \sin^2 \omega t \, (d\omega t) \right) \right]^{\frac{1}{2}}$$

$$V_{RMS} = \sqrt{3} V_m \left(\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha \right)^{\frac{1}{2}}$$

For R-load continuous conduction $\left(\alpha < \frac{\pi}{6} \right)$:

$$V_0 = \frac{3\sqrt{3}}{2\pi} V_m \cos \alpha$$

For R-load discontinuous conduction $\left(\alpha \geq \frac{\pi}{6} \right)$

$$V_0 = \frac{3V_m}{2\pi} \left[1 + \cos \left(\alpha + \frac{\pi}{6} \right) \right]$$

Gating Sequence

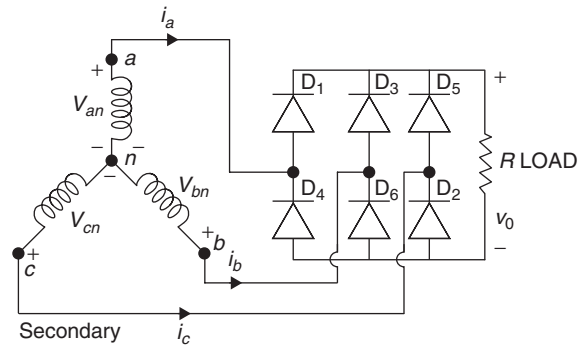
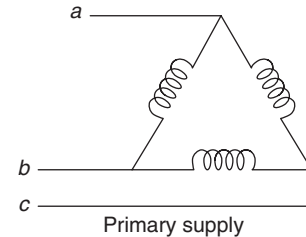
1. Generate a pulse signal at positive zero crossing of the phase voltage V_{an} . Delay the pulse by desired angle $(\alpha + \pi/6)$ and apply it to gate and cathode terminals of T_1 through a gate isolating circuit.
2. Generate two more pulses of delay angles $\alpha + 5\pi/6$ and $\alpha + 9\pi/6$ for gating T_2 and T_3 through gate isolating circuits.

Note:

- The frequency of the output ripple is three times the supply frequency.
- For $\alpha > \pi/6$ with a resistive load, the current is discontinuous.
- This converter is not normally used in practical applications.

THREE-PHASE UNCONTROLLED FULL-WAVE RECTIFIER

- It is commonly used in high power applications. It can operate with or without transformer and gives a six-pulse ripples on the output voltage. The diodes are numbered in the order of conduction sequences and each one conducts for 120° .
- The conduction sequence for diode is D_1-D_2 , D_3-D_2 , D_3-D_4 , D_5-D_6 and D_1-D_6 .
- The pair of diodes, connected between that pair of supply lines having the highest amount of instantaneous line to line voltage will conduct.



If V_{an} , V_{bn} , V_{cn} are the instantaneous values of phase voltages, then they can be expressed in terms of peak voltage as

$$V_{an} = V_m \sin \omega t$$

$$V_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3} \right)$$

$$V_{cn} = V_m \sin\left(\omega t - \frac{4\pi}{3}\right)$$

Since line-to-line voltages lead the phase voltages by 30° , the instantaneous value of line-to-line voltages can be described by

$$V_{ab} = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

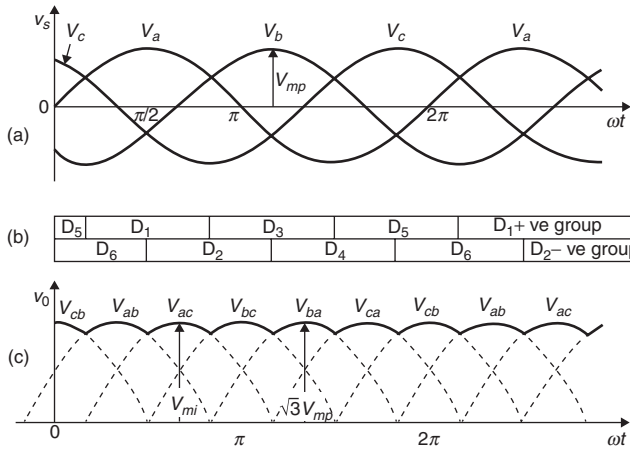
$$V_{bc} = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$V_{ca} = \sqrt{3}V_m \sin\left(\omega t - \frac{7\pi}{6}\right)$$

$D_1 D_3 D_5 \rightarrow$ Common Cathode configuration

$D_4 D_6 D_2 \rightarrow$ Common Anode configuration

The load is connected between two common points.



- The diode which is subjected to maximum positive voltage and the diode which is subjected to maximum negative voltage will be in conduction simultaneously.

Performance Parameters

Average value of output voltage

$$V_{DC} = \frac{2}{2\pi/6} \int_0^{\pi/6} \sqrt{3}V_m \cos \omega t d(\omega t) = \frac{3\sqrt{3}V_m}{\pi}$$

$$V_{DC} = 1.654V_m$$

$$I_{DC} = 1.654V_m/R$$

$$P_{DC} = \frac{(1.654V_m)^2}{R}$$

RMS value of output voltage

$$V_{RMS} = \left[\frac{2}{2\pi/6} \int_0^{\pi/6} 3V_m^2 \cos^2 \omega t d(\omega t) \right]^{\frac{1}{2}}$$

$$= \left(\frac{3}{2} + \frac{9\sqrt{3}}{4\pi} \right)^{\frac{1}{2}} V_m$$

$$V_{RMS} = 1.6554 V_m$$

$$I_{RMS} = 1.6554 V_m/R$$

$$P_{AC} = (1.6554 V_m)^2/R$$

$$I_{RMS} = \left[\frac{4}{2\pi} \int_0^{\pi/6} I_m^2 \cos^2 \omega t d(\omega t) \right]^{\frac{1}{2}}$$

$$= I_m \left[\frac{1}{\pi} \left(\frac{\pi}{6} + \frac{1}{2} \sin \frac{2\pi}{6} \right) \right]^{\frac{1}{2}}$$

$$I_{RMS} = 0.5515 I_m. \text{ If load is purely resistive.}$$

where $I_m = \sqrt{3}V_m/R$

RMS value of secondary current

$$I_s = \left[\frac{8}{2\pi} \int_0^{\pi/6} I_m^2 \cos^2 \omega t d(\omega t) \right]^{\frac{1}{2}}$$

$$I_m \left[\frac{2}{\pi} \left(\frac{\pi}{6} + \frac{1}{2} \sin \frac{2\pi}{6} \right) \right]^{\frac{1}{2}}$$

$$I_s = 0.7804 I_m$$

$$(a) \eta = \frac{P_{DC}}{P_{AC}} = \frac{(1.654V_m)^2}{(1.6554V_m)^2}$$

$$\eta = 99.83\%$$

$$(b) \text{F.F.} = \frac{1.6554}{1.654} = 1.0008$$

$$\text{F.F.} = 100.08\%$$

$$(c) \text{R.F.} = \sqrt{1.0008^2 - 1} = 0.04$$

$$\text{R.F.} = 4\%$$

$$(d) V_s = 0.707 V_m$$

$$I_s = 0.7804 I_m = 0.7804 \times \frac{\sqrt{3} V_m}{R}$$

$$\text{T.U.F.} = \frac{1.654^2}{3 \times \sqrt{3} \times 0.707 \times 0.7804} = 0.9542$$

(e) Peak line to neutral voltage

$$V_m = \frac{280.7}{1.654} = 169.7$$

$$\text{PIV} = \sqrt{3} V_m = \sqrt{3} \times 169.7$$

$$\text{PIV} = 293.9 \text{ V}$$

(f) Average current through each diode is

$$I_d = \left[\frac{4}{2\pi} \int_0^{\pi/6} I_m \cos \omega t d(\omega t) \right]$$

$$= I_m \times \frac{2}{\pi} \sin \frac{\pi}{6}$$

$$I_d = 0.3183 I_m$$

Note:

- A three-phase bridge rectifier has considerably improved performances compared with those of single-phase rectifiers
- If diode is replaced by thyristor

$$V_0 = \frac{3V_{ml} \cos \alpha}{\pi}$$

For R-load continuous conduction $\left(\alpha < \frac{\pi}{3}\right)$

$$V_0 = \frac{3V_{ml}}{\pi} \cos \alpha$$

For R-load discontinuous conduction $\left(\alpha \geq \frac{\pi}{3}\right)$

$$V_0 = \frac{3V_{ml}}{\pi} \left[1 + \cos \left(\alpha + \frac{\pi}{3} \right) \right]$$

Example 9: A three-phase 230 V (L–L) AC source is supplying power to a purely resistive load through an AC to DC diode bridge rectifier. The average DC voltage across the load will be

- (A) 300 V (B) 294.20 V (C) 310.61 V (D) 330.5 V

Solution: (C)

Average DC voltage across the load $= \frac{3V_{ml}}{\pi}$

$V_{ml} \rightarrow$ Peak value of L–L voltage

$$V_{ml} = \sqrt{2} \times 230$$

$$\begin{aligned} \text{Average DC voltage} &= \frac{2 \times \sqrt{2} \times 230}{\pi} \\ &= 310.609 \end{aligned}$$

Example 10: With an increase of overlap angle in a three-phase-controlled bridge rectifier, the output DC voltage would

- (A) Not change (B) Increase
(C) Depend on load (D) Decrease

Solution: (D)

Example 11: The ratio of peak to peak voltage ripple to peak output DC voltage at the output of a fully controlled natural commutated 3-phase bridge rectifier operating at a firing angle of $\alpha = 30^\circ$ is

- (A) 0.866 (B) 0.5 (C) 0.73 (D) 0.134

Solution: (B)

$$V_{p-pr} = V_m - V_m \cos \left(\frac{\pi}{6} + \alpha \right)$$

$$\frac{V_{p-pr}}{V_m} = 1 - \cos \left(\frac{\pi}{6} + \alpha \right)$$

$$= 1 - \cos(30 + 30) = 0.5$$

Example 12: A three-phase diode bridge rectifier is connected to a 200 V (RMS) 50 Hz, 3- ϕ supply. The peak instantaneous output voltage across a purely resistive load connected across the output of a rectifier would be

- (A) $200\sqrt{2}$ V (B) 200 V

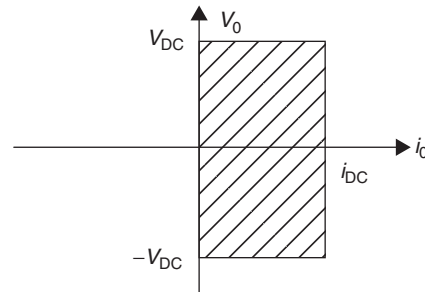
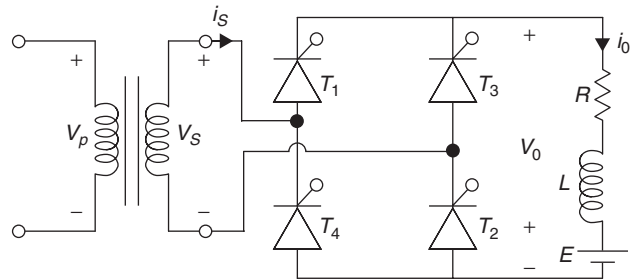
- (C) $200 \times \sqrt{\frac{2}{3}}$ V (D) $\frac{200}{\sqrt{3}}$ V

Solution: (A)

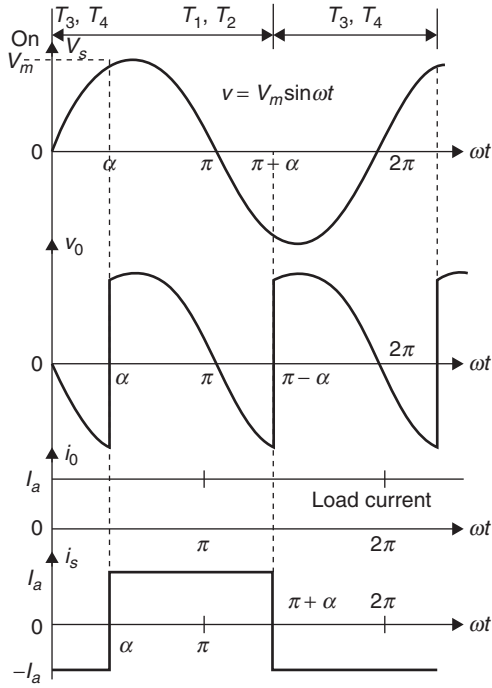
Peak instantaneous value of output voltage (resistive load)

$$\begin{aligned} &= \sqrt{2} V_{ml} \\ &= 200\sqrt{2} \end{aligned}$$

SINGLE-PHASE FULL-CONTROLLED FULL-WAVE RECTIFIER



- During positive half-cycle, thyristors T_1 and T_2 are forward biased and when these two thyristors are fired simultaneously at $\omega t = \alpha$, the load is connected to the input supply through T_1 and T_2 . Due to inductive load, T_1 and T_2 continue to conduct beyond $\omega t = \pi$ even though the input voltage is already negative.
- During negative half cycle of the input voltage, T_3 and T_4 are forward biased and firing of T_3 and T_4 applies the supply voltage across thyristors T_1 and T_2 as reverse blocking voltage. T_1 and T_2 are turned off due to line or natural commutation and load current is transferred from T_1 and T_2 to T_3 and T_4 .
- The diagram below shows the waveforms for input–output voltages and currents.



- During the period α to π the input voltage V_s and input current are positive and power flows from supply to load. The converter is said to be operated in rectification mode.
- During the period from π to $\pi + \alpha$, the input voltage V_s is negative and input current is positive and reverse power flows from the load to the supply. The converter is said to be in inversion mode.
- Depending on the value of α , the average output voltage could be either positive or negative and it provides two-quadrant operation.

Average output voltage, V_{DC}

$$= \frac{2}{\pi} \int_{\alpha}^{\pi+\alpha} V_m \sin \omega t d(\omega t)$$

$$V_{DC} = \frac{2V_m}{\pi} \cos \alpha$$

V_{DC} can be varied from $2V_m/\pi$ to $-2V_m/\pi$ by varying α from 0 to π .

Maximum average output voltage

$$V_{dm} = 2V_m/\pi$$

Normalized average output voltage

$$V_n = \frac{V_{DC}}{V_{DM}} = \cos \alpha$$

$$V_{RMS} = \left[\frac{2}{2\pi\alpha} \int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2 \omega t d(\omega t) \right]^{\frac{\pi}{2}}$$

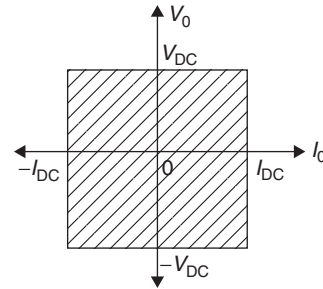
$$V_{RMS} = \frac{V_m}{\sqrt{2}} = V_s$$

Note:

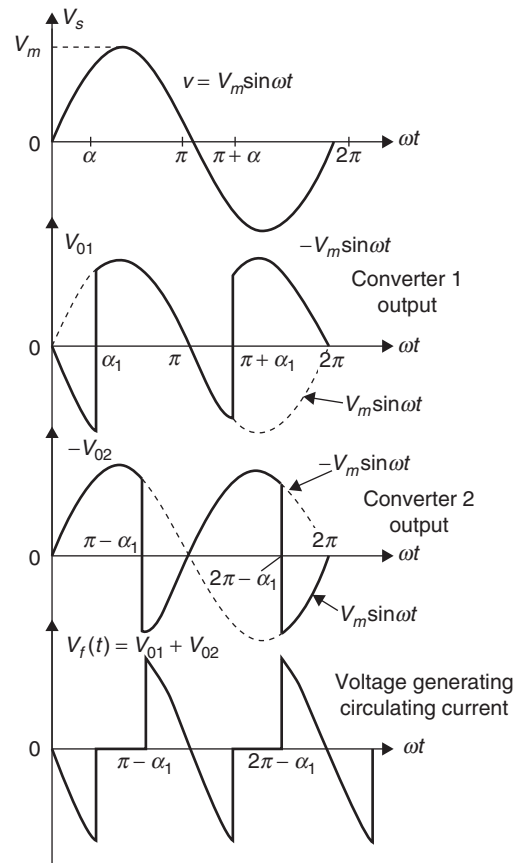
- Varying the delay angle α from 0 to π can vary the average output voltage from $2V_m/\pi$ to $-2V_m/\pi$ provided the load is highly inductive and its current is continuous.

- For a purely resistive load, the delay angle α can be varied from 0 to $\pi/2$ producing an output voltage ranging from $2V_m/\pi$ to 0.
- The full converter can operate in two quadrants for a highly inductive load and in one quadrant only for a purely resistive load.

SINGLE-PHASE DUAL CONVERTER



- If two full converters are connected back to back as shown in the above figure, both the output voltage and load current flow can be reversed.
- The system provides a four-quadrant operation and is called a dual converter.
- If α_1 and α_2 are the delay angles of converters 1 and 2, respectively, the corresponding average output voltage are V_{DC1} and V_{DC2} . The delay angles are controlled such that one converter operates as inverter. But both the converters produce same average output voltage.



- The above diagram shows the output waveforms of two converters where two average output voltages are the same.

The average output voltages are

$$V_{DC1} = \frac{2V_m}{\pi} \cos \alpha_1$$

$$V_{DC2} = \frac{2V_m}{\pi} \cos \alpha_2$$

Since one converter is rectifying and the other is inverting,

$$V_{DC1} = -V_{DC2} \text{ (or) } \cos \alpha_2 = -\cos \alpha_1 = \cos(\pi - \alpha_1)$$

$$\alpha_2 = \pi - \alpha_1$$

- Since the instantaneous output voltages of the two converters are out of phase, there can be instantaneous voltage difference and this can result in circulating current between two converters. This circulating current cannot flow through the load and is normally limited by a circulating current reactor L_r .
- Circulating current can be expressed as

$$i_r = \frac{1}{\omega L_r} \int_{\pi - \alpha_1}^{\omega t} V_r d(\omega t) = \frac{1}{\omega L_r} \int_{\pi - \alpha_1}^{\omega t} (V_{01} + V_{02}) d\omega t$$

$$= \frac{V_m}{\omega L_r} \int_{2\pi - \alpha_1}^{\omega t} \sin \omega t d(\omega t) - \int_{2\pi - \alpha_1}^{\omega t} -\sin \omega t d(\omega t)$$

$$= \frac{2V_m}{\omega L_r} (\cos \alpha_1 - \cos \alpha_2)$$

$$i_r > 0 \text{ for } 0 \leq \alpha_1 \leq \frac{\pi}{2}$$

$$i_r < 0 \text{ for } \frac{\pi}{2} \leq \alpha_1 \leq \pi$$

- For $\alpha_1 = 0$, converter 1 operates.
- For $\alpha_1 = \pi$, converter 2 operates.
- For $0 \leq \alpha_1 \leq \pi/2$, converter 1 supplies a positive load current $+i_0$ and thus circulating current can only be positive.
- For $\pi/2 < \alpha_1 < \pi$, converter 2 supplies a negative load current $-i_0$ and thus only a negative circulating current can flow.
- At $\alpha_1 = \pi/2$, converter 1 supplies positive circulating current during first half cycle and converter 2 supplies negative circulating current during the second half cycle.
- The instantaneous circulating current depends on the delay angle. For $\alpha_1 = 0$ its magnitude is maximum when $\omega t = n\pi$ ($n = 1, 3, 5, \dots$) and minimum when $\omega t = n\pi$ ($n = 0, 2, 4, \dots$).

Gating Sequence

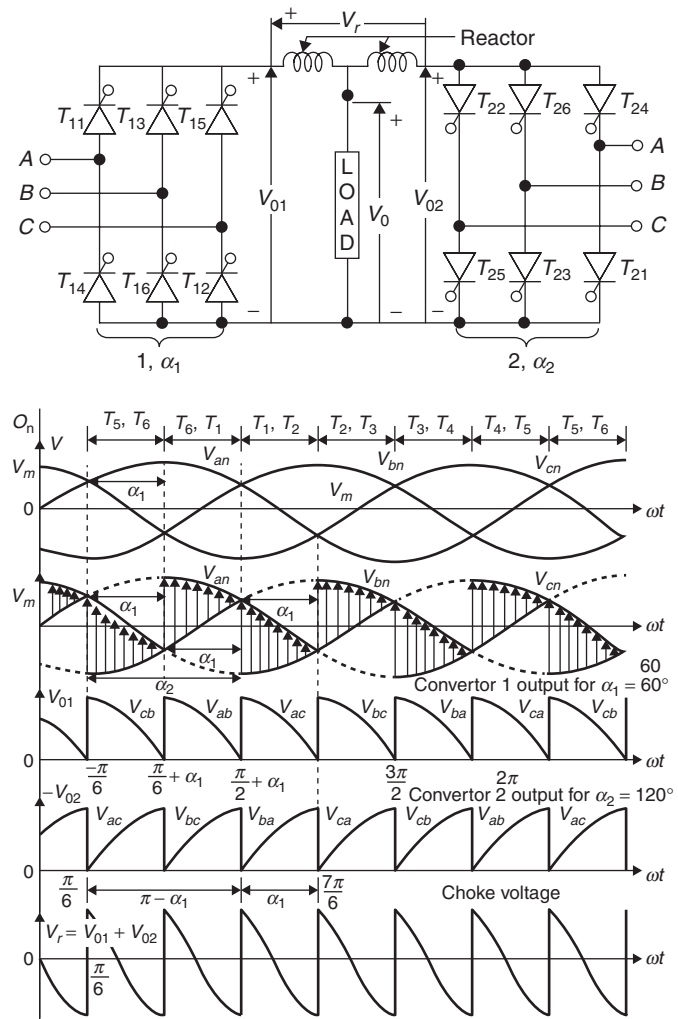
- Triggering the positive converter with a delay angle of $\alpha_1 = \alpha$.
- Triggering the negative converter with a delay angle of $\alpha_2 = \pi - \alpha$ through gate-isolating circuit.

Note:

- Dual converter consists of two, full converters: one converter producing output voltage and another converter producing negative output voltage.
- Varying the delay angle α from 0 to π can vary the average output voltage from $2V_m/\pi$ to $-2V_m/\pi$ provided the load is highly inductive and its current is continuous.
- For a highly inductive load, the dual converter can operate in four quadrants. The current can flow in and out of the load. A DC inductor is needed to reduce the circulating current.

THREE-PHASE DUAL CONVERTER

- They are used in applications up to 2000 kW level, variable speed drives where a four-quadrant operation is generally required.



- The above diagram shows the three-phase dual converter where two, three-phase converters are connected back to back.
- The principle of operation is similar to that of a three-phase full converter.

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- During interval $(\pi/6 + \alpha_1) \leq \omega t \leq (\pi/2 + \alpha_1)$, the line to line voltage V_{ab} appears across the output of converter 1 and V_{bc} appears across converter 2.

If L-N voltages are defined as $V_{an} = V_m \sin \omega t$,
 $V_{bn} = V_m \sin(\omega t - 2\pi/3)$, $V_{cn} = V_m \sin(\omega t + 2\pi/3)$

Corresponding L-L voltages are

$$V_{ab} = V_{an} - V_{bn} = \sqrt{3} V_m \sin \left(\omega t + \frac{\pi}{6} \right)$$

$$V_{bc} = \sqrt{3} V_m \sin \left(\omega t - \frac{\pi}{2} \right),$$

$$V_{ca} = \sqrt{3} V_m \sin \left(\omega t + \frac{5\pi}{6} \right)$$

If V_{01} and V_{02} are output voltages of converters 1 and 2, respectively, the instantaneous voltage across the inductor during interval $(\pi/6 + \alpha_1) \leq \omega t \leq (\pi/2 + \alpha_1)$ is

$$V_r = V_{01} + V_{02} = V_{ab} - V_{bc}$$

$$V_r = \sqrt{3} V_m \cos(\omega t - \pi/6)$$

Circulating current

$$i_r(t) = \frac{1}{\omega L_r} \int_{\pi/6 + \alpha_1}^{\omega t} 3V_m \cos \left(\omega t - \frac{\pi}{6} \right) d\omega t$$

$$i_r(t) = \frac{3V_m}{\omega L_r} \left[\sin \left(\omega t - \frac{\pi}{6} \right) - \sin \alpha_1 \right]$$

- The circulating current depends on delay angle α_1 and on inductance L_r . This current becomes maximum when $\omega t = 2\pi/3$ and $\alpha_1 = 0$. Even without any external load the converters would be continuously running due to circulating current as a result of ripple voltage across the inductor.
- This allows smooth reversal of load current during the change over from one quadrant to another and provides fast dynamic responses especially for electrical motor drives.

Gating Sequence

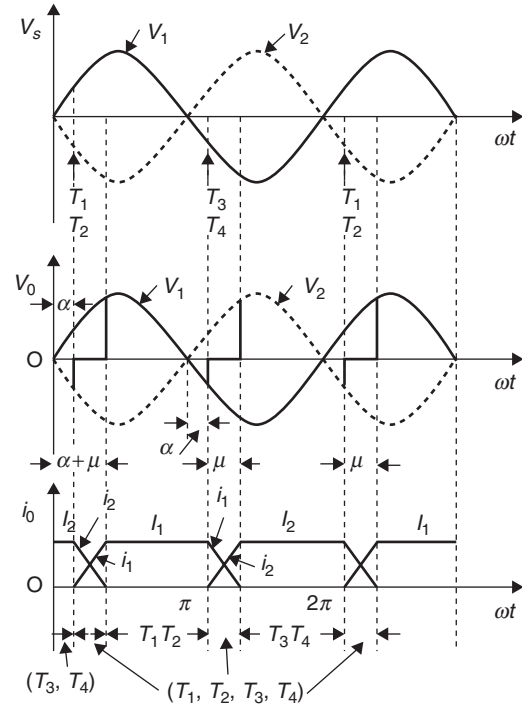
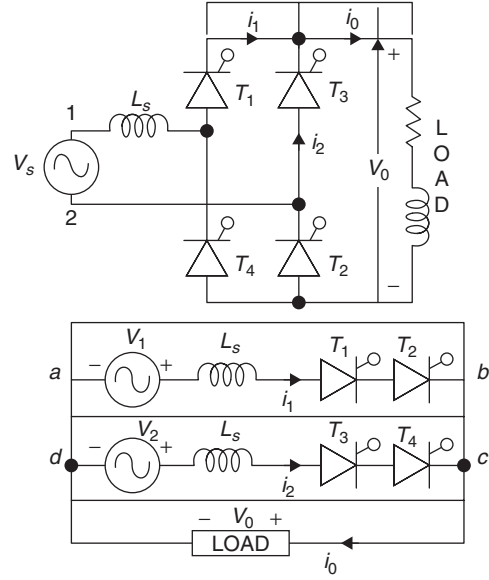
- Trigger the positive converter with a delay angle of $\alpha_1 = \alpha$.
- Trigger the negative converter with a delay angle of $\alpha_2 = \pi - \alpha$ through gate-isolating circuits.

Note:

- Three-phase dual converters are used for high-power applications up to 2000 kW.
- For a highly inductive load, the dual converter can operate in 4 quadrants. The current can flow in and out of the load.
- A DC inductor is needed to reduce the circulating current.

Effect of Source Inductance

In a full-wave rectifier, when conduction shifts from $T_1 T_2$ to $T_3 T_4$ all the four thyristors are in conduction which short circuits the source. T_0 limit the short. Circuit current, L_s is included in the circuit.



By KVL $V_1 - L_s \frac{di_1}{dt} = V_2 - L_s \frac{di_2}{dt}$

$$V_1 - V_2 = L_s \left[\frac{di_1}{dt} - \frac{di_2}{dt} \right]$$

$$\frac{di_1}{dt} - \frac{di_2}{dt} = \frac{V_1 - V_2}{L_s}$$

$$= \frac{2V_m \sin \omega t}{L_s}$$

$$\frac{di_1}{dt} - \frac{di_2}{dt} = \frac{2V_m \sin \omega t}{L_s}$$

During the overlapping angle $I_0 = i_1 + i_2$

$$\frac{di_1}{dt} + \frac{di_2}{dt} = \frac{di_0}{dt} = 0$$

Adding above two equations,

$$\frac{di_1}{dt} = \frac{V_m}{L} \sin \omega t$$

$$\int_0^{I_0} di_1 = \int_{\alpha}^{\alpha+\mu} \frac{V_m}{L} \sin(\omega t) d(\omega t)$$

$$I_0 = \frac{V_m}{\omega L} [\cos \alpha - \cos(\alpha + \mu)]$$

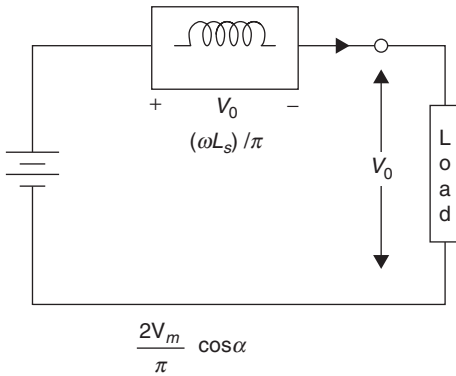
$$V_0 = \frac{1}{\pi} \int_{(\alpha+\mu)}^{\pi+\alpha} V_m \sin \omega t d\omega t$$

$$V_0 = \frac{V_m}{\pi} (\cos(\alpha + \mu) + \cos \alpha)$$

$$\text{But } \cos(\alpha + \mu) = \frac{-L_s}{V_m} I_0 + \cos \alpha$$

$\therefore$

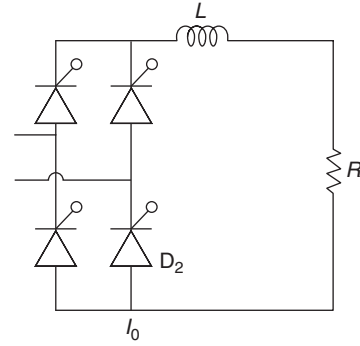
$$V_0 = \frac{2V_m}{\pi} \cos \alpha - \frac{\omega L_s}{\pi} I_0$$



The inductance of source results in a lesser value of voltage. For a 3- ϕ converter,

$$V_0 = \frac{3\sqrt{6}}{\pi} V_{ph} \cos \alpha - \frac{3\omega L_s I_0}{\pi}$$

Example 13: The figure shows a fully controlled thyristor converter fed from a single-phase source.



The DC output voltage of the converter is 400 V when $\alpha = 0^\circ$. At what value of α will the DC output voltage of the converter be 200 V (assuming continuous conduction)?

- (A) 30° (B) 45° (C) 60° (D) 90°

Solution: (C)

Given a full-wave rectifier feeding inductive load and assuming continuous conduction

$$V_0 = \frac{2V_m}{\pi} \cos \alpha$$

$$V_0 = 400, \alpha = 0^\circ$$

$$\frac{2V_m}{\pi} = 400 \text{ For } V_0 = 200 \text{ } \alpha = ?$$

$$\frac{2V_m}{\pi} \cos \alpha = 200$$

$$\cos \alpha = \frac{200}{400} \text{ and } \alpha = 60^\circ$$

Example 14: The displacement power factor of a single fully controlled rectifier feeding constant DC current into the load and operating at a firing angle of 18° would be

- (A) 0.95 (B) 0.86 (C) 0.77 (D) 0.5

Solution: (A)

Displacement factor of rectifier

$$= \cos \alpha = \cos 18^\circ = 0.95$$

Example 15: A thyristarized three-phase fully controlled converter feed a load that drawn constant DC current. The converter must be fed an input line current which has

- (A) An equivalent average value to that of the DC load current.
(B) An equivalent peak value to that of the DC load current.
(C) An equivalent RMS value to that of the load current.
(D) None of the above.

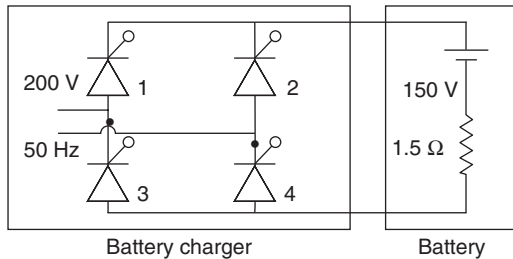
Solution: (B)

EXERCISES

Practice Problems I

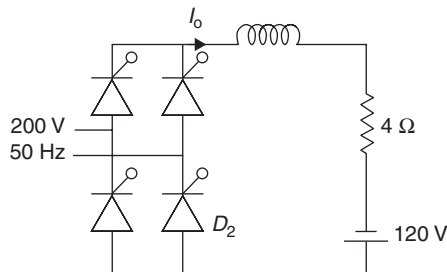
Directions for questions 1 to 15: Select the correct alternative from the given choices.

1.



In the figure shown above if the SCRs are triggered by a constant DC signal. The average charging current if the SCR gets open circuited would be

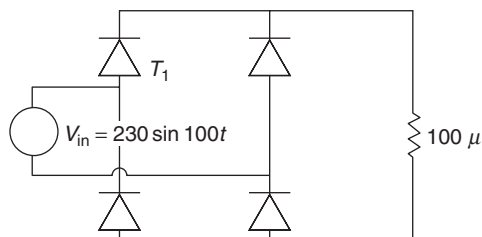
- (A) 11.9 A (B) 15.9 A
(C) 12.65 A (D) 18.73 A
2. A constant and ripple free load current is supplied by a single-phase full bridge converter. The input power factor for a triggering angle of 60° would be
- (A) 0.65 (B) 0.45
(C) 0.75 (D) 0.55
3. A single excited DC motor employs a single-phase fully controlled bridge converter for electrical braking. The equivalent circuit is as shown below



The triggering angle of the bridge for a load current of 5 A would be [Assume the load inductance is sufficient to ensure continuous and ripple free load current]

- (A) 56° (B) 129°
(C) 124° (D) 51°

4.



The power dissipated in the load resistor of 100 connected across a single-phase diode bridge rectifier as shown below approximately is

- (A) $\frac{3680}{\pi}$ (B) $\frac{230}{\pi}$
(C) 920 W (D) 265 W
5. A 230 V, 50 Hz supply is feeding a 3-phase fully controlled bridge converter with a free-wheeling diode and operating at a firing angle of 30° . Due to high load inductance, the load current is assumed to be constant at 8 A. If 'K' is the ratio of the input power factor to input displacement factor, the value of K is
- (A) 0.5 (B) 0.55
(C) 0.65 (D) 0.45
6. A DC supply of 230 V, 50 Hz is fed from a synchronous generator set of 200 V through a 3- ϕ fully controlled bridge converter. The load current is maintained constant at 10 A by the large inductance connected to the DC circuit. If the internal of the generator is 0.4 then each thyristor will be reverse biased for a period of
- (A) 129° (B) 55°
(C) 125° (D) 60°
7. A constant DC load current of 120 A is fed by a three-phase fully controlled converter. Each thyristor of the converter would carry an RMS current of
- (A) 40 A (B) 80 A
(C) 97.98 A (D) 69.28 A
8. The circulating current in a dual converter
- (A) Flows only if there is no inter connecting inductor.
(B) Improves response time and allows smooth reversal of load current.
(C) Reduces response time and does not allow smooth reversal of load current.
(D) Increases response time and allows smooth reversal of load current.
9. If θ_1 and θ_2 are triggering angles of an AC to DC circulating current, dual converter, the converter is operated with the following relationship between θ_1 and θ_2
- (A) $\theta_2 + \theta_1 = 90^\circ$
(B) $\theta_2 - \theta_1 = 180^\circ$
(C) $\theta_2 + \theta_1 = 360^\circ$
(D) None of the above
10. Which among the following statements are true regarding zero current switching (ZCS) and zero voltage switching (ZVS) type resonant converters?
- (1) ZCS can eliminate switching losses at turn off and reduce switching losses at turn ON.
(2) ZVS eliminates the capacitive turn on loss.

- (3) ZCS operates with a constant OFF time control.
 (4) ZVS operates with a constant ON time control.
 (A) 3 and 4 only (B) 1 and 2 only
 (C) 2 and 3 only (D) All 1, 2, 3 and 4
11. A 380 V, 50 Hz, 3- ϕ , balanced power supply is feeding a six-pulse thyristor rectifier bridge. The lowest harmonic component in the AC source line current [assuming a constant DC output current] is
 (A) 50 Hz (B) 100 Hz
 (C) 150 Hz (D) 250 Hz
12. A full-wave bridge converter (single-phase half-controlled) feeds an inductive load. A common DC bus is connected to two SCR's in the converter. The converter should be equipped with a freewheel diode because
 (A) A high AC current will flow through one SCR if a gate pulse to the other SCR is missed.
 (B) Free-wheeling action is not provided by the converter for high triggering angles.
 (C) Shorting of the AC triggering pulses due to free-wheeling action of the converter shall be compensated.
 (D) Inherent free-wheeling is not provided by the converter.
13. A half-wave uncontrolled AC-DC converter is employed as a battery charger to charge a 12 V battery. The converter is connected to a single-phase 120 V, 50 Hz supply mains through a 4:1 stepdown transformer. If the battery is connected to the converter through a current-limiting resistance of 5.7 Ω , the charging current is
 (A) 1.4 A (B) 0.5 A
 (C) 3.4 A (D) 2.22 A
14. A two-pulse midpoint converter feeds a constant ripple free current to a load and the current is I_d at all firing angles. The parameters are such that it has an overlap angle of 45° at 0° triggering angle. The overlap angle, at $\alpha = 45^\circ$ would be
 (A) 65.35° (B) 0°
 (C) 20.35° (D) 12.97°
15. Which among the following are true regarding the effects of free-wheeling diode in a single pulse converter feeding RL load
 (A) Improved power factor due to reduced reactive power requirements
 (B) Reduced ripple content in the load current
 (C) Continuous conduction at smaller value of α
 (D) All of the above

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. In a single-phase half-wave circuit with RL load, and a free-wheeling diode across the load, extinction angle β is more than π . For a firing angle α , SCR and free-wheeling diode would conduct, for
 (A) $\beta - \alpha, \pi - \alpha$ (B) $\beta - \alpha, \beta$
 (C) $\pi - \alpha, \beta - \pi$ (D) $\beta - \pi, \pi - \alpha$
2. A single-phase half-wave controlled rectifier has $300 \sin(314 t)$ V as the input voltage and R as the load. For a firing angle of 60° for the SCR, the average output voltage is
 (A) $\frac{900}{4\pi}$ V (B) $\frac{600}{\pi}$ V
 (C) $\frac{200}{\pi}$ V (D) $\frac{300}{\pi}$ V
3. In a single-phase one-pulse circuit with RL load, and a freewheeling diode across the load, extinction angle is less than π . For a firing angle α , the SCR and free-wheeling diode would conduct for
 (A) $\pi - \alpha, \alpha$ (B) $\beta - \alpha, \alpha$
 (C) $\beta - \alpha, \pi - \alpha$ (D) $\beta - \alpha, 0^\circ$
4. A single-phase two pulse bridge converter has an average output voltage and power output of 400 V and 6 kW, respectively. The SCRs used in the two-pulse bridge converter are now re-employed to form a single-phase two-pulse mid-point converter. This new controlled converter would give an average output voltage and power output of
 (A) 400 V, 6 kW (B) 200 V, 6 kW
 (C) 200 V, 1.5 kW (D) 200 V, 3 kW
5. In a single-phase full converter, for discontinuous load current and extinction angle $\beta > \pi$, each SCR conducts for
 (A) $\pi + \alpha$ (B) π
 (C) $\beta - \alpha$ (D) $\beta + \alpha$
6. Commutation overlap in the phase-controlled AC to DC converter is due to
 (A) Load inductance
 (B) Harmonic content of load current
 (C) Switching operation in the converter
 (D) Source inductance
7. A single-phase fully controlled line commutated AC to DC converter operates as an inverter, when
 (A) $0 \leq \alpha \leq 90^\circ$
 (B) $90^\circ \leq \alpha \leq 180^\circ$
 (C) It supplies to a back emf load
 (D) $90^\circ \leq \alpha \leq 180^\circ$ and there is a suitable DC source in the load circuit
8. A single-phase full-bridge converter feeds an inductive load. $R_L = 15.53 \Omega$ and it has a large inductance providing constant and ripple free DC current. Input to

converter is from a 230 V, 50 Hz single-phase source. If $\alpha = 60^\circ$, the average value of output current is

- (A) 10 A
- (B) 8.165 A
- (C) 5.774 A
- (D) 3.33 A

9. The overlap angle of a phase-controlled converter would increase on increasing the

- (1) Supply voltage
- (2) Supply frequency
- (3) Load current
- (4) Source inductance of these statements

- (A) 1, 2 and 3 are correct
- (B) 2, 3 and 4 are correct
- (C) 1, 2 and 4 are correct
- (D) 1, 3 and 4 are correct

10. A converter which can operate both in 3-pulse and 6-pulse modes is a

- (A) 1-phase full-converter
- (B) 3-phase half-wave converter
- (C) 3-phase semi-converter
- (D) 3-phase full-converter

11. The quality of output AC voltage of a cyclo converter is improved with a/an

- (A) Increase in output voltage at reduced frequency
- (B) Increase in output voltage at increased frequency
- (C) Decrease in output voltage at reduced frequency
- (D) Decrease in output voltage at increased frequency

12. A separately-excited DC motor is required to be controlled from a 3-phase source for two quadrant operation. The most preferred converter would be

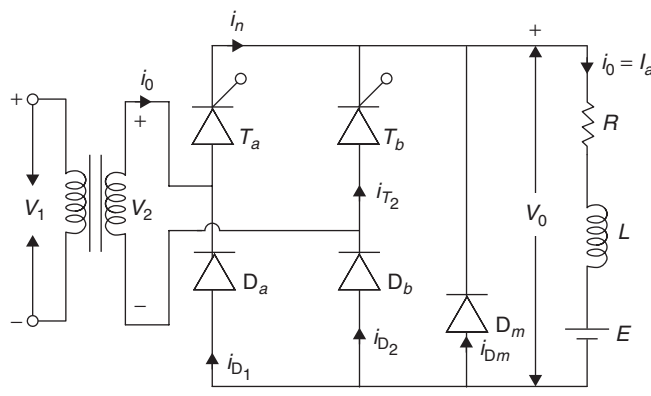
- (A) Half-controlled converter
- (B) Fully controlled converter
- (C) Dual converter
- (D) Half-wave converter

13. The ratio of form factor of a full-wave rectifier to that of a half-wave rectifier would be

- (A) $\frac{1}{\sqrt{2}}$
- (B) $\sqrt{2}$
- (C) $\sqrt{3}$
- (D) $\frac{1}{\sqrt{3}}$

Common Data Questions 14 and 15

If the semi-converter given below is operated from a 230 V, 50 Hz supply. The load current with an average value of I_a is continuous. If the thyristors are operated at a delay angle of 30° and transformers had a turns ratio of 1.



14. The harmonic factor of input current is

- (A) 0.32
- (B) 0.23
- (C) 0.427
- (D) 0.11

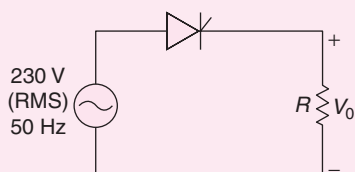
15. Input power factor would be

- (A) 0.86
- (B) 0.92
- (C) 0.77
- (D) 0.65

PREVIOUS YEARS' QUESTIONS

1. Consider a phase-controlled converter shown in figure shown below. The thyristor is fired at an angle α in every positive half cycle of the input voltage. If the peak value of the instantaneous output voltage equals 230 V, the firing angle α is close to

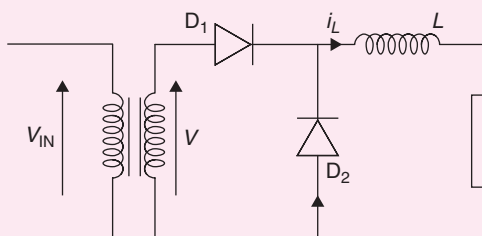
[2005]

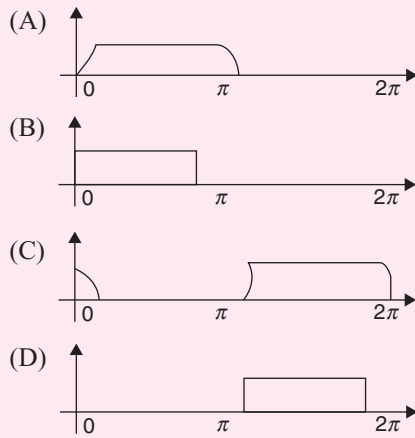


- (A) 45°
- (B) 135°
- (C) 90°
- (D) 83.6°

2. A single-phase half-wave uncontrolled converter circuit is shown in figure. A 2-winding transformer is used at the input for isolation. Assuming the load current to be constant and $v = V_m \sin \omega t$, the current waveform through diode D_2 will be

[2006]





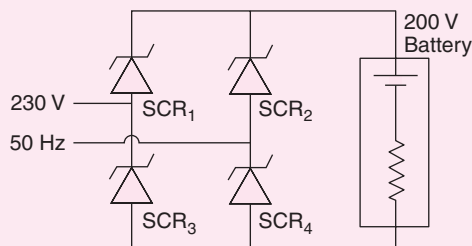
3. A 3-phase fully controlled bridge converter with free-wheeling diode is fed from 400 V, 50 Hz AC source and is operating at a firing angle of 60° . The load current is assumed constant at 10 A due to high load inductance. The input displacement factor (IDF) and the input power factor (PF) of the converter will be [2006]

(A) IDF = 0.867 IPF = 0.828
 (B) IDF = 0.867 IPF = 0.552
 (C) IDF = 0.5 IPF = 0.478
 (D) IDF = 0.5 IPF = 0.318

4. A solar cell of 350 V is feeding power to an AC supply of 440 V, 50 Hz through a 3-phase fully controlled bridge converter. A large inductance is connected in the DC circuit to maintain the DC current at 20 A. If the solar cell resistance is 0.5Ω , then each thyristor will be reverse biased for a period of [2006]

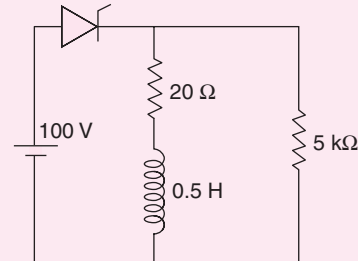
(A) 125° (B) 120° (C) 60° (D) 55°

5. A single-phase bridge converter is used to charge a battery of 200 V having an internal resistance of 0.2Ω as shown in figure. The SCRs are triggered by a constant DC signal. If SCR 2 gets open circuited, then what will be the average charging current? [2006]



(A) 23.8 A (B) 15 A
 (C) 11.9 A (D) 3.54 A

6. An SCR having a turn ON time of $5 \mu\text{sec}$, latching current of 50 mA and holding current of 40 mA is triggered by a short duration pulse and is used in the circuit shown in figure. The minimum pulse width required to turn the SCR ON will be [2006]



(A) $251 \mu\text{sec}$ (B) $150 \mu\text{sec}$
 (C) $100 \mu\text{sec}$ (D) $5 \mu\text{sec}$

7. A single-phase fully controlled thyristor bridge AC–DC converter is operating at a firing angle of 25° and an overlap angle 10° with constant DC output current of 20 A. The fundamental power factor (displacement factor) at input AC mains is [2007]

(A) 0.78 (B) 0.827
 (C) 0.866 (D) 0.9

8. A three-phase, fully controlled thyristor bridge converter is used as line commutated inverter to feed 50 kW power at 420 V DC to a three-phase, 415 V (line), 50 Hz AC mains. Consider DC link current to be constant. The RMS current of the thyristor is [2007]

(A) 119.05 (B) 79.37 A
 (C) 68.73 A (D) 39.68 A

9. A single-phase full-wave half-controlled bridge converter feeds an inductive load. The two SCRs in the converter are connected to a common DC bus. The converter has to have a freewheeling diode [2007]

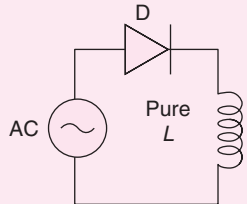
(A) because the converter inherently does not provide for free-wheeling
 (B) because the converter does not provide for free-wheeling for high values of triggering angles
 (C) or else the free-wheeling action of the converter will cause shorting of the AC supply
 (D) or else if a gate pulse to one of the SCRs is missed, it will subsequently cause a high load current in the other SCR

10. A single-phase, 230 V, 50 Hz AC mains fed step down transformer (4:1) is supplying power to a half-wave uncontrolled AC–DC converter used for charging a battery (12 V DC) with the series current

limiting resistor being 19.04Ω . The charging current is [2007]

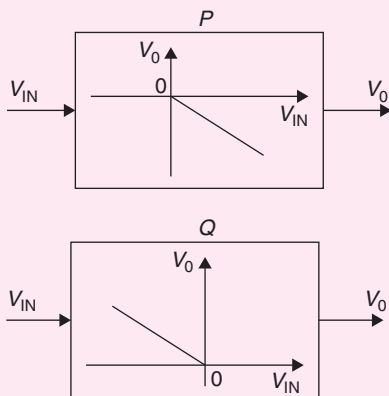
- (A) 2.43 A (B) 1.65 A
(C) 1.22 A (D) 1.0 A

11. In the circuit of adjacent figure the diode connects the AC source to a pure inductance L . The diode conducts for [2007]

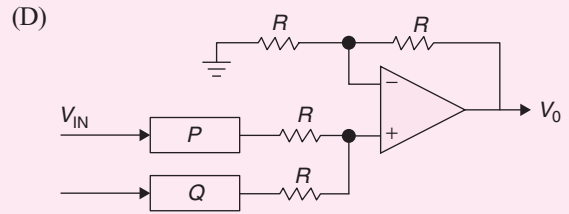
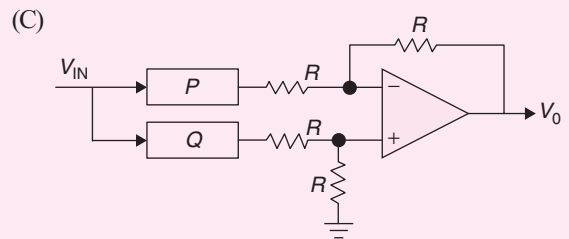
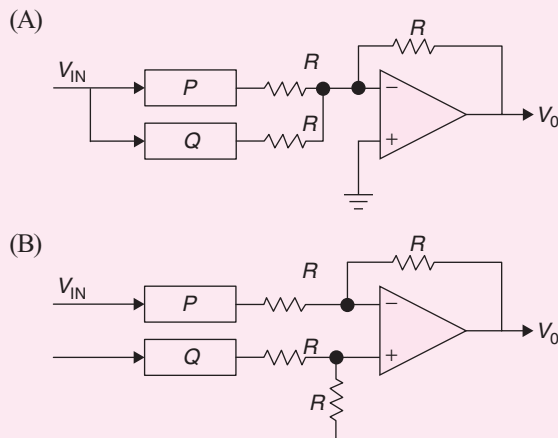


- (A) 90° (B) 180° (C) 270° (D) 360°

12. The block diagrams of two types of half-wave rectifiers are shown in the figure. The transfer characteristics of the rectifiers are also shown within the block. [2008]

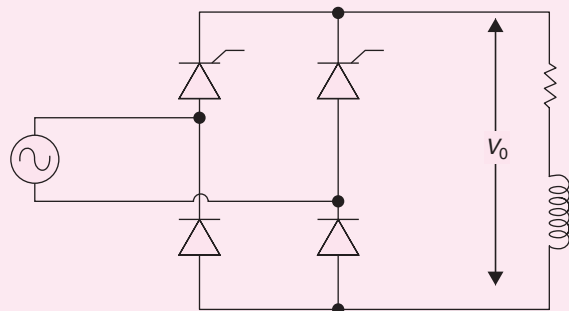


It is desired to make full-wave rectifier using above two half-wave rectifiers. The resultant circuit will be

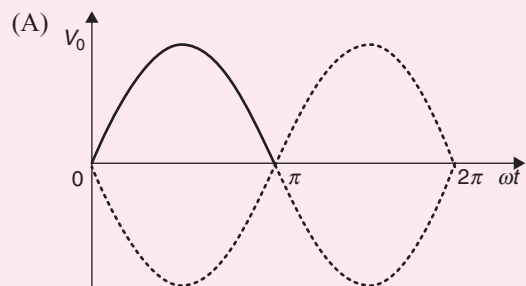


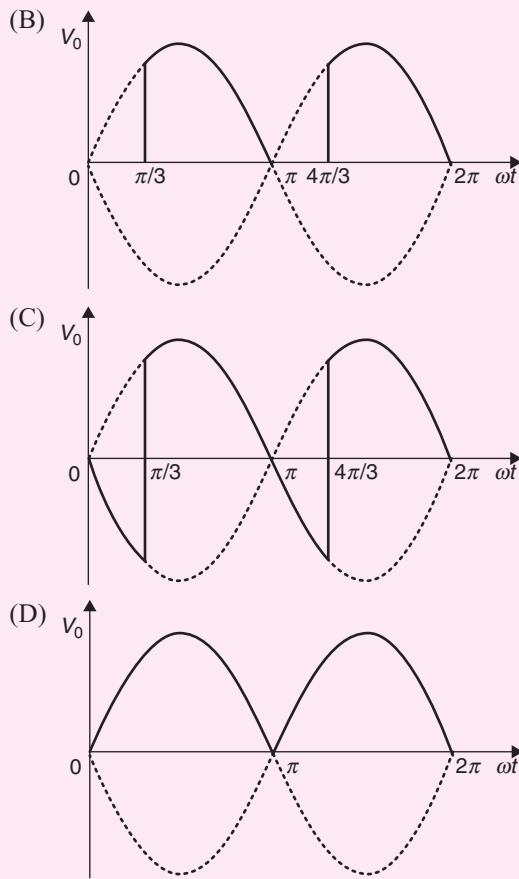
13. A single-phase fully controlled bridge converter supplies a load drawing constant and ripple free load current. If the triggering angle is 30° , the input power factor will be [2008]
(A) 0.65 (B) 0.78 (C) 0.85 (D) 0.866

14. A single-phase half-controlled converter shown in the figure is feeding power to highly inductive load. The converter is operating at a firing angle of 60° . [2008]



If the firing pulses are suddenly removed, the steady-state voltage (V_0) waveform of the converter will become





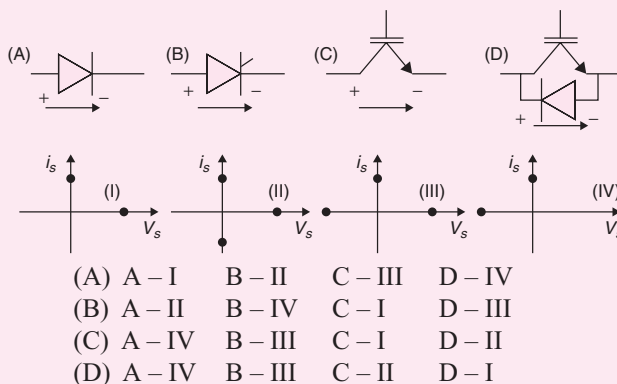
15. A three-phase fully controlled bridge converter is feeding a load drawing a constant and ripple free load current of 10 A at a firing angle of 30° . The approximate total harmonic distortion (% THD) and the RMS value of fundamental component of the input current will, respectively, be

[2008]

- (A) 31% and 6.8 A (B) 31% and 7.8 A
(C) 66% and 6.8 A (D) 66% and 7.8 A

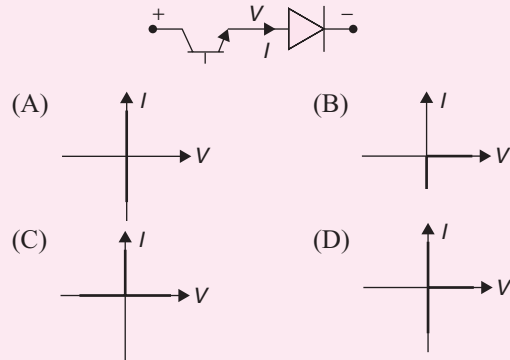
16. Match the switch arrangements on the top row to the steady-state $V-I$ characteristics on the lower row. The steady-state operating points are shown by large black dots.

[2009]



17. Figure shows a composite switch consisting of a power transistor (BJT) in series with a diode. Assuming that the transistor switch and the diode are ideal, the $I-V$ characteristic of the composite switch is

[2010]



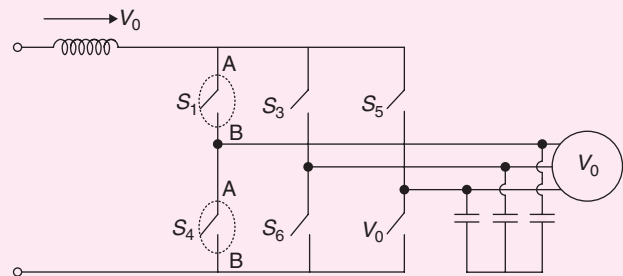
18. Circuit turn-off time of an SCR is defined as the time

[2011]

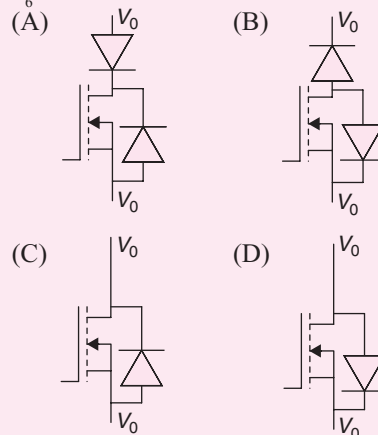
- (A) taken by the SCR to turn off
(B) required for the SCR current to become zero
(C) for which the SCR is reverse biased by the commutation circuit.
(D) for which the SCR is reverse biased to reduce its current below the holding current

19. A three-phase current source inverter used for the speed control of an induction motor is realized using MOSFET switches as shown below. Switches S_1 to S_6 are identical switches.

[2011]

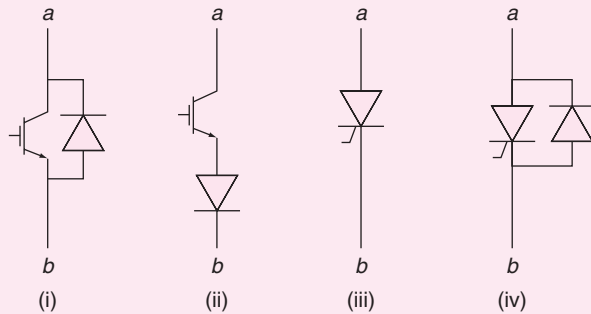


The proper configuration for realizing switches S_1 to S_6 is



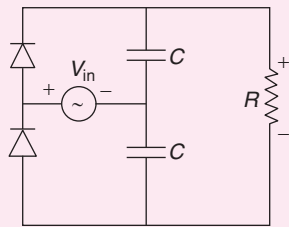
20. The typical ratio of latching current to holding current in a 20 A thyristor is [2012]
 (A) 5.0 (B) 2.0 (C) 1.0 (D) 0.5

21. Figure shows four electronic switches (i), (ii), (iii) and (iv). Which of the switches can block voltages of either polarity (applied between terminals 'a' and 'b') when the active device is in OFF state? [2014]



- (A) (i), (ii) and (iii) (B) (ii), (iii) and (iv)
 (C) (ii) and (iii) (D) (i) and (iv)

22. In the following circuit, the input voltage V_{in} is $100 \sin(100\pi t)$. For $100\pi RC = 50$, the average voltage across R (in Volts) under steady-state is nearest to [2015]



- (A) 100 (B) 31.8
 (C) 200 (D) 63.6

23. In the given rectifier, the delay angle of the thyristor T_1 measured from the positive going zero crossing of V_s is 30° . If the input voltage V_s is $100 \sin(100\pi t)$ V, the average voltage across R (in Volt) under steady-state is _____. [2015]

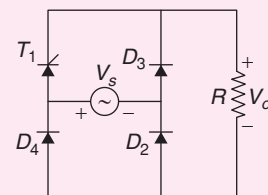
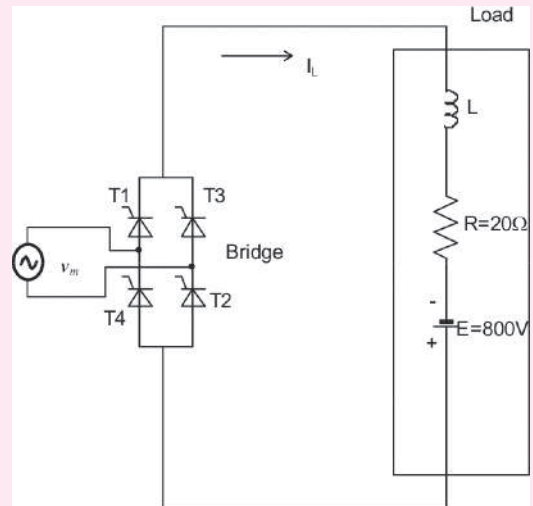
24. A single-phase full-bridge voltage source inverter (VSI) is fed from a 300 V battery. A pulse of 120°

duration is used to trigger the appropriate devices in each half-cycle. The rms value of the fundamental component of the output voltage, in volts, is [2016]

- (A) 234 (B) 245
 (C) 300 (D) 331

25. A three-phase diode bridge rectifier is feeding a constant DC current of 100A to a highly inductive load. If three-phase, 415V, 50Hz AC source is supplying to this bridge rectifier then the rms value of the current in each diode, in ampere, is _____. [2016]

26. A full-bridge converter supplying an RLE load is shown in figure. The firing angle of the bridge converter is 120° . The supply voltage $v_m(t) = 200\pi \sin(100\pi t)$ V, $R = 20\Omega$, $E = 800$ V. The inductor L is large enough to make the output current I_L a smooth dc current. Switches are lossless. The real power feedback to the source, in kW, is _____. [2016]



ANSWER KEYS**EXERCISES****Practice Problems 1**

- | | | | | | | | | | |
|-------|-------|-------|-------|-------|------|------|------|------|-------|
| 1. D | 2. B | 3. C | 4. D | 5. B | 6. A | 7. D | 8. B | 9. D | 10. B |
| 11. C | 12. C | 13. A | 14. C | 15. D | | | | | |

Practice Problems 2

- | | | | | | | | | | |
|-------|-------|-------|-------|-------|------|------|------|------|-------|
| 1. C | 2. A | 3. D | 4. D | 5. C | 6. A | 7. B | 8. D | 9. B | 10. C |
| 11. C | 12. B | 13. A | 14. A | 15. B | | | | | |

Previous Years' Questions

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|-------|-------|--------------|-------|-------|------------|-------|-------|-------|-------|
| 1. B | 2. D | 3. C | 4. D | 5. C | 6. B | 7. A | 8. C | 9. C | 10. D |
| 11. D | 12. B | 13. B | 14. B | 15. B | 16. C | 17. C | 18. C | 19. A | 20. B |
| 21. C | 22. C | 23. 61 to 62 | | 24. A | 25. 57.735 | 26. 6 | | | |