

Ex-10.1

Q1

$$\angle A = 45^\circ, \angle B = 60^\circ \text{ and } \angle C = 75^\circ$$

Using sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\frac{a}{\sin 45} = \frac{b}{\sin 60} = \frac{c}{\sin 75} = k$$

$$\frac{a}{\frac{1}{\sqrt{2}}} = \frac{b}{\frac{\sqrt{3}}{2}} = \frac{c}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = k$$

$$a : b : c = 2 : \sqrt{6} : (\sqrt{3} + 1)$$

Q2

$$\angle C = 105^\circ, \angle B = 45^\circ, a = 2$$

From here we can calculate that

$$\angle A = 30^\circ$$

$$a \sin B = b \sin A$$

$$\Rightarrow 2 \sin 45 = b \sin 30$$

$$\Rightarrow 2 \times \frac{1}{\sqrt{2}} = b \times \frac{1}{2}$$

$$\Rightarrow \sqrt{2} = \frac{b}{2}$$

$$\Rightarrow b = 2\sqrt{2}$$

Q3

$$a = 18, b = 24, c = 30, \angle C = 90^\circ$$

$$\text{let } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{18} = \frac{\sin B}{24} = \frac{\sin 90}{30}$$

$$\frac{\sin A}{18} = \frac{\sin B}{24} = \frac{1}{30}$$

$$\frac{\sin A}{18} = \frac{1}{30} \Rightarrow \sin A = \frac{18}{30} = \frac{3}{5}$$

$$\frac{\sin B}{24} = \frac{1}{30} \Rightarrow \sin B = \frac{24}{30} = \frac{4}{5}$$

$$\therefore \sin A = \frac{3}{5}, \sin B = \frac{4}{5}, \sin C = 1$$

Q4

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$$

Let $a = k \sin A, b = k \sin B$ (Using sine rule)

LHS

$$= \frac{a-b}{a+b}$$

$$= \frac{k \sin A - k \sin B}{k \sin A + k \sin B}$$

$$= \frac{\sin A - \sin B}{\sin A + \sin B}$$

$$= \frac{2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)}{2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)}$$

$$= \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)} = RHS$$

Q5

$$(a-b) \cos \frac{C}{2} = c \sin\left(\frac{A-B}{2}\right)$$

Let $a = k \sin A, b = k \sin B, c = k \sin C$

LHS

$$(a-b) \cos \frac{C}{2}$$

$$= k(\sin A - \sin B) \cos \frac{C}{2}$$

$$= 2k \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) \cos \frac{C}{2}$$

$$= 2k \cos\left(\frac{\pi-C}{2}\right) \sin\left(\frac{A-B}{2}\right) \cos \frac{C}{2}$$

$$= 2k \sin\left(\frac{C}{2}\right) \cos \frac{C}{2} \sin\left(\frac{A-B}{2}\right) \quad \left[\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta\right]$$

$$= k \sin C \sin\left(\frac{A-B}{2}\right)$$

$$= c \sin\left(\frac{A-B}{2}\right) = RHS$$

Q6

$$\frac{c}{a-b} = \frac{\tan\left(\frac{A}{2}\right) + \tan\left(\frac{B}{2}\right)}{\tan\left(\frac{A}{2}\right) - \tan\left(\frac{B}{2}\right)}$$

LHS

$$\begin{aligned} & \frac{c}{a-b} \\ &= \frac{k \sin C}{k \sin A - k \sin B} \\ &= \frac{\sin C}{\sin A - \sin B} \\ &= \frac{2 \sin \frac{C}{2} \cos \frac{C}{2}}{\sin A - \sin B} \\ &= \frac{2 \sin \frac{C}{2} \cos \frac{C}{2}}{2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)} \\ &= \frac{\sin \frac{C}{2} \cos \frac{(\pi - (A+B))}{2}}{\cos\left(\frac{\pi - C}{2}\right) \sin\left(\frac{A-B}{2}\right)} \end{aligned}$$

$$\begin{aligned} &= \frac{\sin \frac{C}{2} \sin \frac{(A+B)}{2}}{\sin \frac{C}{2} \sin\left(\frac{A-B}{2}\right)} \\ &= \frac{\sin \frac{(A+B)}{2}}{\sin\left(\frac{A-B}{2}\right)} \end{aligned}$$

$$= \frac{\sin\left(\frac{A}{2}\right) \cos\left(\frac{B}{2}\right) + \sin\left(\frac{B}{2}\right) \cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right) \cos\left(\frac{B}{2}\right) - \sin\left(\frac{B}{2}\right) \cos\left(\frac{A}{2}\right)}$$

$$= \frac{\tan\left(\frac{A}{2}\right) + \tan\left(\frac{B}{2}\right)}{\tan\left(\frac{A}{2}\right) - \tan\left(\frac{B}{2}\right)} \left[\text{Dividing both Numerator and Denominator by } \cos\left(\frac{A}{2}\right) \cos\left(\frac{B}{2}\right) \right]$$

= RHS

Q7

$$\frac{c}{a+b} = \frac{1 - \tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)}{1 + \tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)}$$

LHS

$$= \frac{c}{a+b}$$

$$= \frac{k \sin C}{k \sin A + k \sin B}$$

$$= \frac{2 \sin \frac{C}{2} \cos \frac{C}{2}}{2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)}$$

$$= \frac{\sin \frac{C}{2} \cos \frac{C}{2}}{\sin\left(\frac{\pi - C}{2}\right) \cos\left(\frac{A-B}{2}\right)}$$

$$= \frac{\sin\left(\frac{\pi - (A+B)}{2}\right) \cos \frac{C}{2}}{\cos\left(\frac{C}{2}\right) \cos\left(\frac{A-B}{2}\right)}$$

$$= \frac{\cos\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A-B}{2}\right)}$$

$$= \frac{\cos \frac{A}{2} \cos \frac{B}{2} - \sin \frac{A}{2} \sin \frac{B}{2}}{\cos \frac{A}{2} \cos \frac{B}{2} + \sin \frac{A}{2} \sin \frac{B}{2}}$$

$$= \frac{1 - \tan \frac{A}{2} \tan \frac{B}{2}}{1 + \tan \frac{A}{2} \tan \frac{B}{2}} \left[\text{Dividing both Numerator and Denominator by } \cos\left(\frac{A}{2}\right) \cos\left(\frac{B}{2}\right) \right]$$

= RHS

Q8

$$\frac{a+b}{c} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}}$$

Let $a = k \sin A, b = k \sin B, c = k \sin C$

LHS

$$\frac{k \sin A + k \sin B}{k \sin C}$$

$$= \frac{\sin A + \sin B}{\sin C}$$

$$= \frac{2 \sin \frac{A+B}{2} \cdot \cos \frac{A-B}{2}}{2 \sin \frac{C}{2} \cdot \cos \frac{C}{2}}$$

$$= \frac{\sin\left(\frac{\pi - C}{2}\right) \cdot \cos \frac{A-B}{2}}{\sin \frac{C}{2} \cdot \cos \frac{C}{2}}$$

$$= \frac{\cos \frac{A-B}{2}}{\sin \frac{C}{2}} = RHS$$

Q9

$$\sin\left(\frac{B-C}{2}\right) = \frac{b-c}{a} \cos\frac{A}{2}$$

Let $a = k \sin A$, $b = k \sin B$, $c = k \sin C$

RHS

$$\begin{aligned} & \frac{b-c}{a} \cos\frac{A}{2} \\ &= \frac{k \sin B - k \sin C}{k \sin A} \cdot \cos\frac{A}{2} \\ &= \frac{\sin B - \sin C}{\sin A} \cdot \cos\frac{A}{2} \\ &= \frac{2 \cos\frac{B+C}{2} \cdot \sin\frac{B-C}{2}}{2 \sin\frac{A}{2} \cdot \cos\frac{A}{2}} \cos\frac{A}{2} \\ &= \frac{\cos\frac{\pi-A}{2} \sin\frac{B-C}{2}}{\sin\frac{A}{2}} \\ &= \frac{\sin\frac{A}{2} \sin\frac{B-C}{2}}{\sin\frac{A}{2}} = \sin\frac{B-C}{2} = \text{RHS} \end{aligned}$$

Q10

$$\text{let } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

LHS,

$$\begin{aligned} & \frac{a^2 - c^2}{b^2} \\ &= \frac{k^2 \sin^2 A - k^2 \sin^2 C}{k^2 \sin^2 B} \\ &= \frac{k^2 (\sin^2 A - \sin^2 C)}{k^2 \sin^2 B} \\ &= \frac{(\sin^2 A - \sin^2 C)}{\sin^2(\pi - (A+C))} \\ &= \frac{\sin(A+C) \sin(A-C)}{\sin^2(A+C)} \\ &= \frac{\sin(A-C)}{\sin(A+C)} = \text{RHS} \end{aligned}$$

Q11

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

RHS,

$$\begin{aligned} & a \sin(B-C) \\ &= a \sin B \cos C - a \sin C \cos B \\ &= a(bk) \left(\frac{a^2 + b^2 - c^2}{2ab} \right) - a(ck) \left(\frac{a^2 + c^2 - b^2}{2ac} \right) \\ &= k \cdot \frac{(a^2 + b^2 - c^2)}{2} - k \frac{(a^2 + c^2 - b^2)}{2} \\ &= 2k \cdot \frac{(b^2 - c^2)}{2} \\ &= b \cdot (kb) - c \cdot (kc) \\ &= b(\sin B) - c(\sin C) \end{aligned}$$

LHS

Q12

$$a^2 \sin(B-C) = (b^2 - c^2) \sin A$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

LHS,

$$\begin{aligned} & a^2 \sin(B-C) \\ &= a^2 \{ \sin B \cos C - \sin C \cos B \} \\ &= a^2 kb \cdot \frac{a^2 + b^2 - c^2}{2ab} - a^2 ck \cdot \frac{a^2 + c^2 - b^2}{2ac} \text{ [Using cos rule and sine rule]} \\ &= a^2 k \cdot \frac{a^2 + b^2 - c^2}{2a} - a^2 k \cdot \frac{a^2 + c^2 - b^2}{2a} \\ &= a^2 k \cdot \left(\frac{a^2 + b^2 - c^2 - a^2 - c^2 + b^2}{2a} \right) \\ &= a^2 k \cdot \left(\frac{2b^2 - 2c^2}{2a} \right) \\ &= ak \cdot (b^2 - c^2) \\ &= \sin A (b^2 - c^2) = RHS \end{aligned}$$

Hence Proved

Q13

$$\frac{\sqrt{\sin A} - \sqrt{\sin B}}{\sqrt{\sin A} + \sqrt{\sin B}} = \frac{a+b-2\sqrt{ab}}{a-b}$$

RHS

$$\frac{a+b-2\sqrt{ab}}{a-b}$$

$$= \frac{(\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{ab}}{(\sqrt{a})^2 - (\sqrt{b})^2}$$

$$= \frac{(\sqrt{a} - \sqrt{b})^2}{(\sqrt{a})^2 - (\sqrt{b})^2}$$

$$= \frac{(\sqrt{a} - \sqrt{b})}{(\sqrt{a} + \sqrt{b})}$$

$$= \frac{(\sqrt{k \sin A} - \sqrt{k \sin B})}{(\sqrt{k \sin A} + \sqrt{k \sin B})}$$

$$= \frac{(\sqrt{\sin A} - \sqrt{\sin B})}{(\sqrt{\sin A} + \sqrt{\sin B})} \text{ [taking } k \text{ common and cancelling them]}$$

LHS

Hence Proved

Q14

LHS,

$$a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B)$$

$$= a \sin B - a \sin C + b \sin C - b \sin A + c \sin A - c \sin B$$

$$= b \sin A - c \sin A + c \sin B - b \sin A + c \sin A - c \sin B [\because b \sin A = a \sin B, b \sin C = c \sin B, c \sin A = a \sin C]$$

$$= 0 = \text{RHS}$$

Hence Proved

Q15

$$\frac{a^2 \sin(B-C)}{\sin A} + \frac{b^2 \sin(C-A)}{\sin B} + \frac{c^2 \sin(A-B)}{\sin C} = 0$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

LHS

$$\begin{aligned} & \frac{a^2 \sin(B-C)}{\sin A} + \frac{b^2 \sin(C-A)}{\sin B} + \frac{c^2 \sin(A-B)}{\sin C} \\ &= ak \sin(B-C) + bk \sin(C-A) + ck \sin(A-B) \\ &= \sin A \sin(B-C) + \sin B \sin(C-A) + \sin C \sin(A-B) \\ &= \sin(\pi - (B+C)) \sin(B-C) + \sin(\pi - (C+A)) \sin(C-A) \\ &\quad + \sin(\pi - (A+B)) \sin(A-B) \\ &= \sin(B+C) \sin(B-C) + \sin(C+A) \sin(C-A) \\ &\quad + \sin(A+B) \sin(A-B) \\ &= \sin^2 B - \sin^2 C + \sin^2 C - \sin^2 A + \sin^2 A - \sin^2 B = 0 = RHS \end{aligned}$$

Q16

$$a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2(\cos^2 A - \cos^2 B) = 0$$

LHS

$$\begin{aligned} &= a^2(1 - \sin^2 B - 1 + \sin^2 C) + b^2(1 - \sin^2 C - 1 + \sin^2 A) \\ &\quad + c^2(1 - \sin^2 A - 1 + \sin^2 B) \\ &= a^2(\sin^2 C - \sin^2 B) + b^2(\sin^2 A - \sin^2 C) + c^2(\sin^2 B - \sin^2 A) \\ &= a^2(k^2 c^2 - k^2 b^2) + b^2(k^2 a^2 - k^2 c^2) + c^2(k^2 b^2 - k^2 a^2) \\ &= k^2(a^2 c^2 - a^2 b^2 + b^2 a^2 - b^2 c^2 + b^2 c^2 - a^2 c^2) \\ &= k^2 \times 0 = 0 = RHS \end{aligned}$$

Q17

$$\text{Let } a = k \sin A, b = k \sin B, c = k \sin C$$

LHS

$$\begin{aligned} & b \cos B + c \cos C \\ &= k \sin B \cos B + k \sin C \cos C \\ &= \frac{k}{2} (2 \sin B \cos B + 2 \sin C \cos C) \\ &= \frac{k}{2} (\sin 2B + \sin 2C) \\ &= \frac{k}{2} 2 \sin(B+C) \cos(B-C) \\ &= k \sin(\pi - A) \cos(B-C) \\ &= k \sin A \cos(B-C) \\ &= a \cos(B-C) = RHS \end{aligned}$$

Q18

$$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \frac{1}{a^2} - \frac{1}{b^2}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

LHS

$$= \frac{1 - 2\sin^2 A}{a^2} - \frac{1 - 2\sin^2 B}{b^2}$$

$$= \frac{1}{a^2} - \frac{1}{b^2} - 2\left(\frac{\sin^2 A}{a^2} - \frac{\sin^2 B}{b^2}\right)$$

$$= \frac{1}{a^2} - \frac{1}{b^2} - 2(k^2 - k^2) \text{ [Using sine rule]}$$

$$= \frac{1}{a^2} - \frac{1}{b^2} = RHS$$

hence Proved

Q19

$$\frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c+a} + \frac{\cos^2 A - \cos^2 B}{a+b} = 0$$

LHS

$$\frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c+a} + \frac{\cos^2 A - \cos^2 B}{a+b}$$

$$= \frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c+a} + \frac{\cos^2 A - \cos^2 B}{a+b}$$

$$= \frac{1 - \sin^2 B - 1 + \sin^2 C}{b+c} + \frac{1 - \sin^2 C - 1 + \sin^2 A}{c+a} + \frac{1 - \sin^2 A - 1 + \sin^2 B}{a+b}$$

$$= \frac{\sin^2 C - \sin^2 B}{b+c} + \frac{\sin^2 A - \sin^2 C}{c+a} + \frac{\sin^2 B - \sin^2 A}{a+b}$$

$$= \frac{k^2 c^2 - k^2 b^2}{b+c} + \frac{k^2 a^2 - k^2 c^2}{c+a} + \frac{k^2 b^2 - k^2 a^2}{a+b}$$

$$= k^2 \left(\frac{c^2 - b^2}{b+c} + \frac{a^2 - c^2}{c+a} + \frac{b^2 - a^2}{a+b} \right)$$

$$= k^2 (c - b + a - c + b - a) \text{ [Using } b^2 - a^2 = (b-a)(b+a)]$$

$$= 0 = RHS$$

Hence Proved

Q20

We know $a \sin B = b \sin A, c \sin B = b \sin C, a \sin C = c \sin B$

$$a \sin \frac{A}{2} \sin \left(\frac{B-C}{2} \right) + b \sin \frac{B}{2} \sin \left(\frac{C-A}{2} \right) + c \sin \frac{C}{2} \sin \left(\frac{A-B}{2} \right) = 0$$

LHS

$$= a \sin \left(\frac{\pi - (B+C)}{2} \right) \sin \left(\frac{B-C}{2} \right) + b \sin \left(\frac{\pi - (C+A)}{2} \right) \sin \left(\frac{C-A}{2} \right)$$

$$+ c \sin \left(\frac{\pi - (A+B)}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$= a \cos \left(\frac{B+C}{2} \right) \sin \left(\frac{B-C}{2} \right) + b \cos \left(\frac{C+A}{2} \right) \sin \left(\frac{C-A}{2} \right)$$

$$+ c \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$= a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B)$$

$$= a \sin B - a \sin C + b \sin C - b \sin A + c \sin A - c \sin B$$

$$= b \sin A - a \sin C + b \sin C - b \sin A + a \sin C - b \sin C$$

$$= 0 = \text{RHS}$$

Q21

$$\frac{b \sec B + c \sec C}{\tan B + \tan C} = \frac{c \sec C + a \sec A}{\tan C + \tan A} = \frac{a \sec A + b \sec B}{\tan A + \tan B}$$

$$\frac{b \sec B + c \sec C}{\tan B + \tan C}$$

$$= \frac{k \sin B \sec B + k \sin C \sec C}{\tan B + \tan C}$$

$$= \frac{k \sin B \frac{1}{\cos B} + k \sin C \frac{1}{\cos C}}{\tan B + \tan C}$$

$$= \frac{k \tan B + k \tan C}{\tan B + \tan C} = \frac{k(\tan B + \tan C)}{\tan B + \tan C} = k$$

$$\text{Similarly, } \frac{c \sec C + a \sec A}{\tan C + \tan A} = k$$

$$\text{Similarly, } \frac{a \sec A + b \sec B}{\tan A + \tan B} = k$$

Q22

$$a \cos A + b \cos B + c \cos C = 2b \sin A \sin C = 2c \sin A \sin B$$

LHS

$$\begin{aligned} & a \cos A + b \cos B + c \cos C \\ &= k \sin A \cos A + k \sin B \cos B + k \sin C \cos C \\ &= \frac{k}{2} (\sin 2A + \sin 2B + \sin 2C) \\ &= \frac{k}{2} (2 \sin(A+B) \cdot \cos(A-B) + 2 \sin C \cdot \cos C) \\ &= \frac{2k}{2} (\sin(\pi - C) \cdot \cos(A-B) + \sin C \cdot \cos C) \\ &= k (\sin C \cdot \cos(A-B) + \sin C \cdot \cos C) \\ &= k \sin C (\cos(A-B) + \cos C) \\ &= k \sin C \cdot 2 \cos\left(\frac{A-B+C}{2}\right) \cdot \cos\left(\frac{A-B-C}{2}\right) \\ &= k \sin C \cdot 2 \cos\left(\frac{\pi - 2B}{2}\right) \cdot \cos\left(\frac{A - \pi + A}{2}\right) \\ &= k \sin C \cdot 2 \sin B \cdot \cos\left(\frac{2A - \pi}{2}\right) \\ &= k \sin C \cdot 2 \sin B \cdot \cos\left(\frac{\pi - 2A}{2}\right) \\ &= k \sin C \cdot 2 \sin B \cdot \sin A \\ &= 2 \sin B \sin C (k \sin A) = 2a \sin B \sin C \\ &= RHS \end{aligned}$$

$$\text{Similarly, } a \cos A + b \cos B + c \cos C = 2c \sin A \sin B$$

Q23

$$\begin{aligned} & a(\cos B \cos C + \cos A) = b(\cos A \cos C + \cos B) = c(\cos A \cos B + \cos C) \\ & a(\cos B \cos C - \cos(\pi - (B + C))) \\ &= a(\cos B \cos C - \cos(B + C)) \\ &= a(\cos B \cos C - \cos B \cdot \cos C + \sin B \sin C) \\ &= a \sin B \sin C \\ &= k \sin A \sin B \sin C \end{aligned}$$

$$\text{Similarly, } b(\cos A \cos C + \cos B) = k \sin A \sin B \sin C$$

$$\text{Similarly, } c(\cos A \cos B + \cos C) = k \sin A \sin B \sin C$$

Q24

Let $a = k \sin A$

$$a(\cos C - \cos B) = 2(b - c) \cos^2 \frac{A}{2}$$

LHS

$$= a(\cos C - \cos B)$$

$$= a 2 \sin \frac{C+B}{2} \cdot \sin \frac{B-C}{2}$$

$$= 2k \sin A \sin \frac{\pi - A}{2} \cdot \sin \frac{B-C}{2}$$

$$= 2k 2 \sin \frac{A}{2} \cdot \cos \frac{A}{2} \cdot \cos \frac{A}{2} \cdot \sin \frac{B-C}{2}$$

$$= 2k \cos^2 \frac{A}{2} \left(2 \sin \frac{B-C}{2} \cdot \sin \frac{A}{2} \right)$$

$$= 2k \cos^2 \frac{A}{2} \left(2 \sin \frac{B-C}{2} \cdot \sin \frac{\pi - (B+C)}{2} \right)$$

$$= 2k \cos^2 \frac{A}{2} \left(2 \sin \frac{B-C}{2} \cdot \cos \frac{B+C}{2} \right)$$

$$= 2k \cos^2 \frac{A}{2} (\sin B - \sin C)$$

$$= 2 \cos^2 \frac{A}{2} (k \sin B - k \sin C)$$

$$= 2 \cos^2 \frac{A}{2} (b - c) = RHS$$

Q25

$$b \cos \theta = c \cos(A - \theta) + a \cos(C + \theta)$$

Let $a \sin C = c \sin A$ [Using sine rule]

RHS

$$= c \cos(A - \theta) + a \cos(C + \theta)$$

$$= c \cos A \cos \theta + c \sin A \sin \theta + a \cos C \cos \theta - a \sin C \sin \theta$$

$$= k \sin C \cos A \cos \theta + k \sin C \sin A \sin \theta + k \sin A \cos C \cos \theta$$

$$- k \sin A \sin C \sin \theta$$

$$= k \sin C \cos A \cos \theta + k \sin A \cos C \cos \theta$$

$$= k \cos \theta (\sin C \cos A + \sin A \cos C)$$

$$= k \cos \theta \sin(C + A)$$

$$= k \cos \theta \sin(\pi - B)$$

$$= k \cos \theta \sin B$$

$$= k \sin B \cdot \cos \theta = b \cos \theta = LHS$$

Q26

Let $\sin A = ak, \sin B = bk, \sin C = ck$

$$\sin^2 A + \sin^2 B = \sin^2 C$$

$$\Rightarrow k^2 a^2 + k^2 b^2 = k^2 c^2 \text{ [Using sine rule]}$$

$$\Rightarrow a^2 + b^2 = c^2$$

Since the triangle satisfies the Pythagoras theorem, therefore it is right angled.

Q27

a^2, b^2, c^2 are in A.P.

$$\Rightarrow -2a^2, -2b^2, -2c^2 \text{ are in A.P.}$$

$$\Rightarrow (a^2 + b^2 + c^2) - 2a^2, (a^2 + b^2 + c^2) - 2b^2, (a^2 + b^2 + c^2) - 2c^2 \text{ are in A.P.}$$

$$\Rightarrow (b^2 + c^2 - a^2), (c^2 + a^2 - b^2), (b^2 + a^2 - c^2) \text{ are in A.P.}$$

$$\Rightarrow \frac{(b^2 + c^2 - a^2)}{2abc}, \frac{(c^2 + a^2 - b^2)}{2abc}, \frac{(b^2 + a^2 - c^2)}{2abc} \text{ are in A.P.}$$

$$\Rightarrow \frac{1}{a} \frac{(b^2 + c^2 - a^2)}{2bc}, \frac{1}{b} \frac{(c^2 + a^2 - b^2)}{2ac}, \frac{1}{c} \frac{(b^2 + a^2 - c^2)}{2ab} \text{ are in A.P.}$$

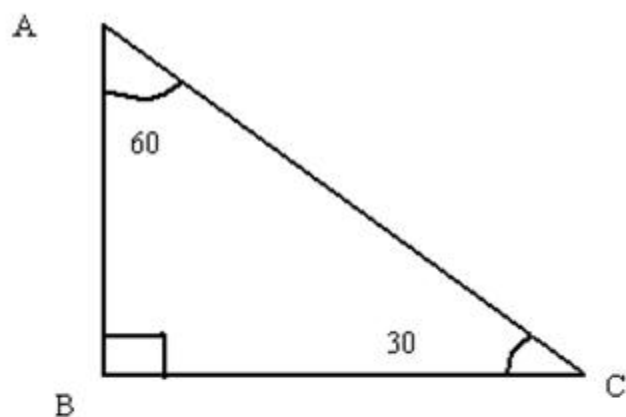
$$\Rightarrow \frac{1}{a} \cos A, \frac{1}{b} \cos B, \frac{1}{c} \cos C \text{ are in A.P.}$$

$$\Rightarrow \frac{k}{a} \cos A, \frac{k}{b} \cos B, \frac{k}{c} \cos C \text{ are in A.P.}$$

$$\Rightarrow \frac{\cos A}{\sin A}, \frac{\cos B}{\sin B}, \frac{\cos C}{\sin C} \text{ are in A.P.}$$

$$\Rightarrow \cot A, \cot B, \cot C \text{ are in A.P.}$$

Q28



$BC=15\text{m}, AB=h$

From the diagram we can calculate, $\angle A = 60^\circ$

Using sine rule,

$$\frac{\sin A}{15} = \frac{\sin C}{h}$$

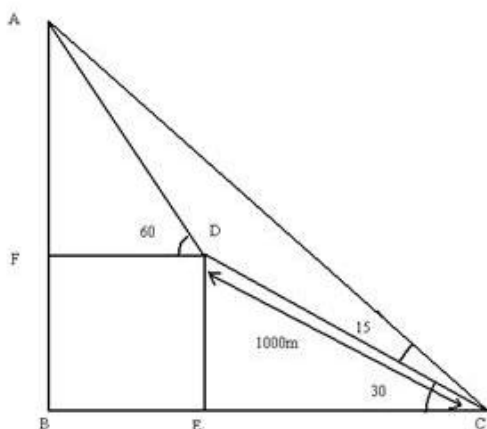
$$\Rightarrow \frac{\sin 60}{15} = \frac{\sin 30}{h}$$

$$\Rightarrow \frac{\sqrt{3}}{2 \times 15} = \frac{1}{2 \times h}$$

$$\Rightarrow \frac{\sqrt{3}}{15} = \frac{1}{h}$$

$$\Rightarrow h = \frac{15}{\sqrt{3}} \Rightarrow h = 5\sqrt{3}$$

Q29



$$DE = 1000 \sin 30 = 1000 \times \frac{1}{2} = 500m = FB$$

$$EC = 1000 \cos 30 = 1000 \times \frac{\sqrt{3}}{2} = 500\sqrt{3}m$$

Let $AF = x$ m

$$DF = \frac{x}{\sqrt{3}} m = BE$$

We know,

From $\triangle ABC$,

$$\tan 45 = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{AF + FB}{BE + EC}$$

$$\Rightarrow 1 = \frac{x + 500}{\frac{x}{\sqrt{3}} + 500\sqrt{3}}$$

$$\Rightarrow \frac{x}{\sqrt{3}} + 500\sqrt{3} = x + 500$$

$$\Rightarrow x + 1500 = x\sqrt{3} + 500\sqrt{3}$$

$$\Rightarrow 1500 - 500\sqrt{3} = x\sqrt{3} - x$$

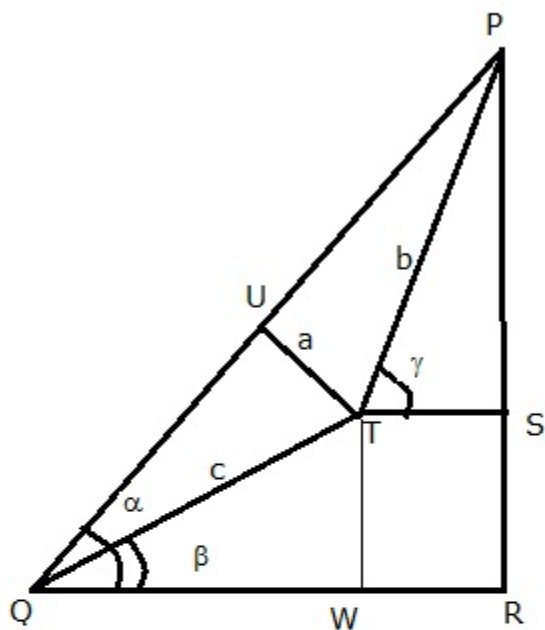
$$\Rightarrow 500\sqrt{3}(\sqrt{3} - 1) = x(\sqrt{3} - 1)$$

$$\therefore x = 500\sqrt{3}m$$

The height of the triangle is $AB = AF + FB = 500(\sqrt{3} + 1)m$

Q30

Consider the following figure.



The person is observing the peak P from the point Q .

The distance he travelled is $QT = c$ metres and the angle of inclination of QT is β .

He is observing the peak from the point and the angle of inclination is γ .

Now consider the triangle ΔQUT .

$$\angle TQU = \beta - \alpha$$

$$\text{Thus, } \sin(\alpha - \beta) = \frac{a}{c}$$

$$\Rightarrow a = c \times \sin(\alpha - \beta) \dots (1)$$

Now consider the triangle ΔPQR .

We know that $\angle QPR = 90^\circ - \alpha$

In triangle ΔPTS , $\angle TPS = 90^\circ - \gamma$

Thus, $\angle TPU = \angle QPR - \angle TPS$

$$\Rightarrow \angle TPU = (90^\circ - \alpha) - (90^\circ - \gamma)$$

$$\Rightarrow \angle TPU = \gamma - \alpha$$

Now consider the ΔTPU ,

$$\text{Thus, } \sin(\gamma - \alpha) = \frac{a}{b}$$

$$\Rightarrow b = \frac{a}{\sin(\gamma - \alpha)}$$

Substituting the value of a from equation (1), we have,

$$b = \frac{c \times \sin(\alpha - \beta)}{\sin(\gamma - \alpha)} \dots (2)$$

We need to find the total height of the peak PR .

Here, $PR = PS + SR \dots (3)$

From the triangle PST ,

$$\sin \gamma = \frac{PS}{PT} = \frac{PS}{b}$$

$$\Rightarrow PS = b \sin \gamma \dots (4)$$

From the triangle QTW ,

$$\sin \beta = \frac{TW}{QT} = \frac{TW}{c}$$

$$\Rightarrow TW = SR = c \sin \beta \dots (5)$$

Substituting the values of PS and SR from equations (4) and (5)

in equation (3), we have

$$PR = PS + SR$$

$$\Rightarrow PR = b \sin \gamma + c \sin \beta$$

$$\Rightarrow PR = \frac{c \times \sin(\alpha - \beta)}{\sin(\gamma - \alpha)} \sin \gamma + c \sin \beta \quad [\text{from equation (2)}]$$

$$\Rightarrow PR = \frac{c \times \sin(\alpha - \beta) \times \sin \gamma + c \sin \beta \times \sin(\gamma - \alpha)}{\sin(\gamma - \alpha)}$$

$$\Rightarrow PR = c \left[\frac{\sin \alpha \times \cos \beta \times \sin \gamma - \cos \alpha \times \sin \beta \times \sin \gamma + \sin \beta \times \sin \gamma \times \cos \alpha - \sin \beta \times \sin \alpha \times \cos \gamma}{\sin(\gamma - \alpha)} \right]$$

$$\Rightarrow PR = c \left[\frac{\sin \alpha \times \cos \beta \times \sin \gamma - \sin \beta \times \sin \alpha \times \cos \gamma}{\sin(\gamma - \alpha)} \right]$$

$$\Rightarrow PR = \frac{c \sin \alpha \times (\cos \beta \times \sin \gamma - \sin \beta \times \cos \gamma)}{\sin(\gamma - \alpha)}$$

$$\Rightarrow PR = \frac{c \sin \alpha \times \sin(\gamma - \beta)}{\sin(\gamma - \alpha)}$$

Q31

If the sides a, b, c of a ΔABC are in H.P.

$\therefore \frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in AP

$$\therefore \frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\Rightarrow \frac{a-b}{ba} = \frac{b-c}{ca}$$

$$\Rightarrow \frac{\sin A - \sin B}{\sin B \sin A} = \frac{\sin B - \sin C}{\sin C \sin B} \dots \dots \dots [\text{Using sine rule}]$$

$$\Rightarrow \frac{2 \sin \frac{A-B}{2} \cos \frac{A+B}{2}}{\sin A} = \frac{2 \sin \frac{B-C}{2} \cos \frac{B+C}{2}}{\sin C}$$

But $A + B + C = \pi$

$$A + B = \pi - C$$

$$\cos \frac{A+B}{2} = \cos \left(\frac{\pi}{2} - \frac{C}{2} \right) = \sin \frac{C}{2}$$

$$\sin^2 \frac{C}{2} \cos \frac{C}{2} \sin \frac{A-B}{2} = \sin \frac{B-C}{2} \cos \frac{A}{2} \sin^2 \frac{A}{2}$$

$$\sin^2 \frac{C}{2} \sin \frac{A+B}{2} \sin \frac{A-B}{2} = \sin \frac{B-C}{2} \cos \frac{B+C}{2} \sin^2 \frac{A}{2}$$

$$\sin^2 \frac{C}{2} \left[\sin^2 \frac{A}{2} - \sin^2 \frac{B}{2} \right] = \sin^2 \frac{A}{2} \left[\sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} \right]$$

$$\sin^2 \frac{C}{2} \sin^2 \frac{A}{2} - \sin^2 \frac{C}{2} \sin^2 \frac{B}{2} = \sin^2 \frac{A}{2} \sin^2 \frac{B}{2} - \sin^2 \frac{A}{2} \sin^2 \frac{C}{2}$$

$$\frac{1}{\sin^2 \frac{B}{2}} - \frac{1}{\sin^2 \frac{A}{2}} = \frac{1}{\sin^2 \frac{C}{2}} - \frac{1}{\sin^2 \frac{B}{2}}$$

Hence $\frac{1}{\sin^2 \frac{A}{2}}, \frac{1}{\sin^2 \frac{B}{2}}, \frac{1}{\sin^2 \frac{C}{2}}$ are in AP.

$\therefore \sin^2 \frac{A}{2}, \sin^2 \frac{B}{2}, \sin^2 \frac{C}{2}$ are in HP.

Ex-10.2

Q1

The area of a triangle ABC is given by

$$\begin{aligned}\Delta &= \frac{1}{2}ab \sin C \\ &= \frac{1}{2} \times 5 \times 6 \sin 60^\circ \\ &= \frac{15\sqrt{3}}{2} \text{ sq. unit}\end{aligned}$$

Q2

The area of a triangle ABC is given by

$$\begin{aligned}\Delta &= \frac{1}{2}ab \sin C \\ \cos C &= \frac{a^2 + b^2 - c^2}{2ab} \\ &= \frac{2 + 3 - 5}{2\sqrt{6}} \\ &= 0 \\ \sin C &= \sqrt{1 - \cos^2 C} \\ &= 1 \\ \text{Therefore,} \\ \Delta &= \frac{1}{2}ab \sin C \\ &= \frac{1}{2}\sqrt{6}\end{aligned}$$

Q3

We have, $a = 4$, $b = 6$ and $c = 8$

$$\begin{aligned}\cos A &= \frac{b^2 + c^2 - a^2}{2bc} = \frac{7}{8} \\ \cos B &= \frac{a^2 + c^2 - b^2}{2ac} = \frac{11}{16} \\ \cos C &= \frac{a^2 + b^2 - c^2}{2ab} = -\frac{1}{4} \\ 8\cos A + 16\cos B + 4\cos C &= 8 \times \frac{7}{8} + 16 \times \frac{11}{16} + 4 \times \left(-\frac{1}{4}\right) \\ &= 17\end{aligned}$$

Q4

In any $\triangle ABC$, we have

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

we have,

$$a = 18, b = 24, c = 30$$

Therefore,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{1152}{1440} = \frac{4}{5}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{648}{1080} = \frac{3}{5}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{0}{864} = 0$$

Q5

$$b(c \cos A - a \cos C) = c^2 - a^2$$

RHS

$$= c^2 - a^2$$

$$= k^2 \sin^2 C - k^2 \sin^2 A$$

$$= k^2 (\sin^2 C - \sin^2 A)$$

$$= k^2 \sin(C + A) \sin(C - A)$$

$$= k^2 \sin(\pi - B) \sin(C - A)$$

$$= k^2 \sin B \sin(C - A)$$

$$= k \sin B \cdot k \sin(C - A)$$

$$= bk \sin(C - A)$$

$$= bk (\sin C \cos A - \sin A \cos C)$$

$$= b(k \sin C \cos A - k \sin A \cos C)$$

$$= b(c \cos A - a \cos C) = LHS$$

Q6

$$\begin{aligned} & c(a \cos B - b \cos A) \\ &= ac \cos B - bc \cos A \\ &= ac \cdot \frac{a^2 + c^2 - b^2}{2ac} - bc \cdot \frac{b^2 + c^2 - a^2}{2bc} \\ &= \frac{a^2 + c^2 - b^2}{2} - \frac{b^2 + c^2 - a^2}{2} \\ &= \frac{a^2 + c^2 - b^2 - b^2 - c^2 + a^2}{2} \\ &= \frac{2a^2 - 2b^2}{2} = (a^2 - b^2) = RHS \end{aligned}$$

Q7

$$\begin{aligned} & 2(bc \cos A + ca \cos B + ab \cos C) = a^2 + b^2 + c^2 \\ & LHS \\ &= 2bc \cos A + 2ca \cos B + 2ab \cos C \\ &= 2bc \frac{b^2 + c^2 - a^2}{2bc} + 2ca \frac{a^2 + c^2 - b^2}{2ca} + 2ab \frac{a^2 + b^2 - c^2}{2ab} \\ &= b^2 + c^2 - a^2 + a^2 + c^2 - b^2 + a^2 + b^2 - c^2 \\ &= a^2 + b^2 + c^2 = RHS \end{aligned}$$

Q8

For any $\triangle ABC$, we have

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

therefore,

$$\begin{aligned} (c^2 + b^2 - a^2) \tan A &= (c^2 + b^2 - a^2) \frac{\sin A}{\cos A} \\ &= (c^2 + b^2 - a^2) \frac{ka}{\frac{b^2 + c^2 - a^2}{2bc}} \\ &= 2kabc \end{aligned}$$

Also,

$$\begin{aligned} (a^2 + c^2 - b^2) \tan B &= (a^2 + c^2 - b^2) \frac{\sin B}{\cos B} \\ &= (a^2 + c^2 - b^2) \frac{kb}{\frac{a^2 + c^2 - b^2}{2ac}} \\ &= 2kabc \end{aligned}$$

Now,

$$\begin{aligned} (a^2 + b^2 - c^2) \tan C &= (a^2 + b^2 - c^2) \frac{\sin C}{\cos C} \\ &= (a^2 + b^2 - c^2) \frac{kc}{\frac{a^2 + b^2 - c^2}{2ab}} \\ &= 2kabc \end{aligned}$$

Hence proved.

Q9

$$\frac{c - b \cos A}{b - c \cos A} = \frac{\cos B}{\cos C}$$

LHS

$$\begin{aligned} &= \frac{c - b \cos A}{b - c \cos A} \\ &= \frac{k \sin C - k \sin B \cos A}{k \sin B - k \sin C \cos A} \\ &= \frac{\sin(\pi - (A + B)) - \sin B \cos A}{\sin(\pi - (A + C)) - \sin C \cos A} \\ &= \frac{\sin(A + B) - \sin B \cos A}{\sin(A + C) - \sin C \cos A} \\ &= \frac{\sin A \cos B + \cos A \sin B - \sin B \cos A}{\sin A \cos C + \cos A \sin C - \sin C \cos A} \\ &= \frac{\sin A \cos B}{\sin A \cos C} \\ &= \frac{\cos B}{\cos C} = RHS \end{aligned}$$

Q10

In any $\triangle ABC$, we have

$$a = b \cos C + c \cos B$$

$$b = c \cos A + a \cos C$$

$$c = a \cos B + b \cos A$$

Therefore,

$$\begin{aligned} L.H.S &= a(\cos B + \cos C - 1) + b(\cos C + \cos A - 1) + c(\cos A + \cos B - 1) \\ &= a \cos B + a \cos C - a + b \cos C + b \cos A - b + c \cos A + c \cos B - c \\ &= c - b \cos A + a \cos C - a + a - c \cos B + b \cos A - b + b - a \cos C + c \cos B - c \\ &= 0 \\ &= R.H.S \end{aligned}$$

Hence proved.

Q11

By sine rule we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$k \sin A = a, k \sin B = b, k \sin C = c$$

$$a \cos A + b \cos B + c \cos C = k \sin A \cos A + k \sin B \cos B + k \sin C \cos C$$

$$= \left(\frac{1}{2}\right)k[2 \sin A \cos A + 2 \sin B \cos B + 2 \sin C \cos C]$$

$$= \left(\frac{1}{2}\right)k[\sin 2A + \sin 2B + \sin 2C]$$

$$= k[\sin(A+B) \cos(A-B) + \sin C \cos C]$$

$$= k[\sin(\pi - C) \cos(A-B) + \sin C \cos(\pi - (A+B))]$$

$$= k[\sin C \cos(A-B) - \sin C \cos(A+B)]$$

$$= k[\sin C(\cos(A-B) - \cos(A+B))]$$

$$= k \sin C[2 \sin A \sin B]$$

$$= 2 \sin C(k \sin A) \sin B$$

$$= 2a \sin B \sin C$$

Q12

We know that by cosine rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow 2bc \cos A = b^2 + c^2 - a^2$$

$$\Rightarrow a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Rightarrow a^2 = b^2 + c^2 - 2bc \left(2 \cos^2 \frac{A}{2} - 1 \right)$$

$$\Rightarrow a^2 = b^2 + c^2 + 2bc - 4bc \cos^2 \frac{A}{2}$$

$$\Rightarrow a^2 = (b+c)^2 - 4bc \cos^2 \frac{A}{2}$$

Q13

$$4 \left(bc \cos^2 \frac{A}{2} + ca \cos^2 \frac{B}{2} + ab \cos^2 \frac{C}{2} \right) = (a+b+c)^2$$

LHS,

$$4 \left(bc \cos^2 \frac{A}{2} + ca \cos^2 \frac{B}{2} + ab \cos^2 \frac{C}{2} \right)$$

$$= 2 \left(bc \cdot 2 \cos^2 \frac{A}{2} + ca \cdot 2 \cos^2 \frac{B}{2} + ab \cdot 2 \cos^2 \frac{C}{2} \right)$$

$$= 2 (bc(1 - \cos A) + ca(1 - \cos B) + ab(1 - \cos C))$$

$$= 2bc - 2bc \cos A + 2ca - 2ca \cos B + 2ab - 2ab \cos C$$

$$= 2bc - 2bc \frac{b^2 + c^2 - a^2}{2bc} + 2ca - 2ca \frac{a^2 + c^2 - b^2}{2ca} + 2ab$$

$$- 2ab \frac{b^2 + a^2 - c^2}{2ab} [\cos \text{ rule}]$$

$$= 2bc - b^2 - c^2 + a^2 + 2ca - a^2 - c^2 + b^2 + 2ab - b^2 - a^2 + c^2$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$= (a+b+c)^2 = RHS$$

Q14

$$\begin{aligned}
& \sin^3 A \cos(B-C) + \sin^3 B \cos(C-A) + \sin^3 C \cos(A-B) \\
&= \sin^2 A \sin A \cos(B-C) + \sin^2 B \sin B \cos(C-A) + \sin^2 C \sin C \cos(A-B) \\
&= \sin^2 A \sin(\pi - (B+C)) \cos(B-C) + \sin^2 B \sin(\pi - (A+C)) \cos(C-A) \\
&\quad + \sin^2 C \sin(\pi - (A+B)) \cos(A-B) \\
&= \sin^2 A \sin(B+C) \cos(B-C) + \sin^2 B \sin(C+A) \cos(C-A) \\
&\quad + \sin^2 C \sin(A+B) \cos(A-B) \\
&= \sin^2 A (\sin 2B + \sin 2C) + \sin^2 B (\sin 2C + \sin 2A) + \sin^2 C (\sin 2A + \sin 2B) \\
&= \sin^2 A (2 \sin B \cos B + 2 \sin C \cos C) + \sin^2 B (2 \sin C \cos C + 2 \sin A \cos A) \\
&\quad + \sin^2 C (2 \sin A \cos A + 2 \sin B \cos B) \\
&= \sin^2 A (2 \sin B \cos B + 2 \sin C \cos C) + \sin^2 B (2 \sin C \cos C + 2 \sin A \cos A) \\
&\quad + \sin^2 C (2 \sin A \cos A + 2 \sin B \cos B) \\
&= \sin^2 A 2 \sin B \cos B + \sin^2 A 2 \sin C \cos C + \sin^2 B 2 \sin C \cos C \\
&\quad + \sin^2 B 2 \sin A \cos A + \sin^2 C 2 \sin A \cos A + \sin^2 C 2 \sin B \cos B \\
&= k^2 a^2 2kb \cos B + k^2 a^2 2kc \cos C + k^2 b^2 2ka \cos C \\
&\quad + k^2 b^2 2ka \cos A + k^2 c^2 2ka \cos A + k^2 c^2 2kb \cos B \\
&= k^3 ab(a \cos B + b \cos A) + k^3 ac(a \cos C + c \cos A) + k^3 bc(c \cos B + b \cos C) \\
&= k^3 abc + k^3 acb + k^3 bca \\
&= k^3 3abc \\
&= 3(k \sin A k \sin B k \sin C) \\
&= 3abc = RHS
\end{aligned}$$

Q15

$$\text{Let } \frac{b+c}{12} = \frac{c+a}{13} = \frac{a+b}{15} = \lambda (\text{say})$$

$$b+c=12\lambda, c+a=13\lambda, a+b=15\lambda$$

$$(b+c+c+a+a+b)=12\lambda+13\lambda+15\lambda$$

$$2(a+b+c)=40\lambda$$

$$a+b+c=20\lambda$$

$$b+c=12\lambda \text{ and } a+b+c=20\lambda \Rightarrow a=8\lambda$$

$$c+a=13\lambda \text{ and } a+b+c=20\lambda \Rightarrow b=7\lambda$$

$$a+b=15\lambda \text{ and } a+b+c=20\lambda \Rightarrow c=5\lambda$$

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{49\lambda^2+25\lambda^2-64\lambda^2}{70\lambda^2} = \frac{1}{7}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{64\lambda^2+25\lambda^2-49\lambda^2}{80\lambda^2} = \frac{1}{2}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{64\lambda^2+49\lambda^2-25\lambda^2}{112\lambda^2} = \frac{11}{14}$$

$$\cos A : \cos B : \cos C = \frac{1}{7} : \frac{1}{2} : \frac{11}{14} = 2 : 7 : 11$$

Q16

We have, $\angle B = 60^\circ$

$$\cos B = \frac{1}{2} \Rightarrow \frac{a^2+c^2-b^2}{2ac} = \frac{1}{2}$$

$$\Rightarrow a^2+c^2-b^2=ac$$

$$\Rightarrow a^2+c^2-ac=b^2 \quad \dots\dots(i)$$

$$(a+b+c)(a-b+c)=3ca$$

$$a^2-ab+ac+ab-b^2+bc+ac-bc+c^2=3ac$$

$$a^2+c^2-b^2+2ac-3ac=0$$

$$a^2+c^2-ac=b^2$$

which is given.

Q17

Consider the given equation:

$$\cos^2 A + \cos^2 B + \cos^2 C = 1$$

$$\Rightarrow 1 - \sin^2 A + 1 - \sin^2 B + 1 - \sin^2 C = 1$$

$$\Rightarrow 3 - \sin^2 A + 1 - \sin^2 B + 1 - \sin^2 C = 1$$

Q18

Let $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$. Then, $\sin A = ka$, $\sin B = kb$, $\sin C = kc$

$$\text{Now, } \cos C = \frac{\sin A}{2 \sin B}$$

$$2 \sin B \cos C = \sin A$$

$$2 \left(\frac{a^2 + b^2 - c^2}{2ab} \right) kb = ka$$

$$a^2 + b^2 - c^2 = a^2$$

$$b^2 = c^2$$

$$b = c$$

$\triangle ABC$ is isosceles.

Q19

Let P and Q be the position of two ships at the end of 3 hours.

Then,

$$OP = 3 \times 24 = 72 \text{ km and } OQ = 3 \times 32 = 96 \text{ km}$$

Using cosine formula in $\triangle OPQ$, we get

$$PQ^2 = OP^2 + OQ^2 - 2OP \times OQ \cos 90^\circ$$

$$PQ^2 = 72^2 + 96^2 - 2 \times 72 \times 96 \cos 90^\circ$$

$$PQ^2 = 14400$$

$$PQ = 120 \text{ km}$$

