

CHAPTER

6.5

FREQUENCY-DOMAIN ANALYSIS

Statement for Q.1-2:

An under damped second order system having a transfer function of the form

$$T(s) = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

has a frequency response plot shown in fig. P6.5.1-2.

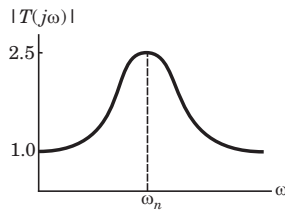


Fig. P6.5.1-2

1. The system gain K is

- (A) 1 (B) 2
(C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

2. The damping factor ξ is approximately

- (A) 0.6 (B) 0.2
(C) 1.8 (D) 2.4

3. For the transfer function

$$G(s)H(s) = \frac{1}{s(s+1)(s+0.5)}$$

the phase cross-over frequency is

- (A) 0.5 rad/sec (B) 0.707 rad/sec
(C) 1.732 rad/sec (D) 2 rad/sec

4. The gain-phase plots of open-loop transfer function of four different system are shown in fig. P6.5.4. The correct sequence of the increasing order of stability of these four system will be

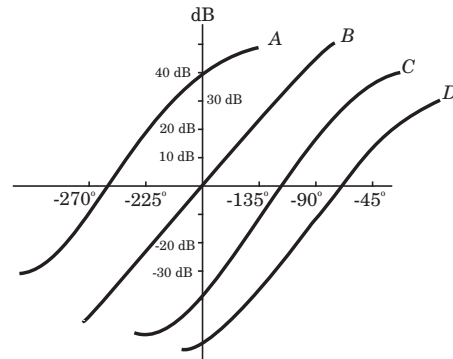


Fig. P6.5.4

- (A) D, C, B, A (B) A, B, C, D
(C) B, C, A, D (D) A, D, B, C

5. The open-loop frequency response of a unity feedback system is shown in following table

ω	$ G(j\omega) $	$\angle G(j\omega)$
2	8.5	-119°
3	6.4	-128°
4	4.8	-142°
5	2.56	-156°
6	1.4	-164°
8	1.00	-172°
10	0.63	-180°

Fig. P6.5.5

The gain margin and phase margin of the system are

- (A) 2 dB, 8° (B) 2 dB, -172°
(C) 4 dB, 8° (D) 4 dB, -172°

Statement for Q.6–7:

Consider the gain-phase plot shown in fig. P6.5.6–7.

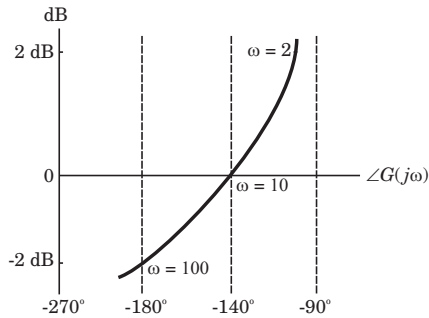


Fig. P6.5.6-7

- 6.** The gain margin and phase margin are
(A) -2 dB, 40° (B) 2 dB, 40°
(C) 2 dB, 140° (D) -2 dB, 140°
- 7.** The gain crossover and phase crossover frequency are respectively
(A) 10 rad/sec, 100 rad/sec
(B) 100 rad/sec, 10 rad/sec
(C) 10 rad/sec, 2 rad/sec
(D) 100 rad/sec, 2 rad/sec
- 8.** The phase margin of a system with the open loop transfer function

$$G(s)H(s) = \frac{(1-s)}{(1+s)(3+s)} \text{ is}$$

- (A) 68.3° (B) 90°
(C) 0 (D) ∞

- 9.** Consider a *u/fb* system having an open-loop transfer function

$$G(s) = \frac{K}{s(0.2s+1)(0.05s+1)}$$

For $K=1$, the gain margin is 28 dB. When gain margin is 20 dB, K will be equal to

- (A) 2 (B) 4
(C) 5 (D) 2.5

- 10.** The gain margin of the *u/fb* system

$$G(s) = \frac{2}{(s+1)(s+2)} \text{ is}$$

- (A) 1.76 dB (B) 3.5 dB
(C) -3.5 dB (D) -1.76 dB

- 11.** The open-loop transfer function of a system is

$$G(s)H(s) = \frac{K}{s(1+2s)(1+3s)}$$

The phase crossover frequency is

- (A) 6 rad/sec (B) 2.46 rad/sec
(C) 0.41 rad/sec (D) 3.23 rad/sec

- 12.** The open-loop transfer function of a *u/fb* system is

$$G(s) = \frac{1+s}{s(1+0.5s)}$$

The corner frequencies are

- (A) 0 and 2 (B) 0 and 1
(C) 0 and -1 (D) 1 and 2

- 13.** In the Bode-plot of a unity feedback control system, the value of magnitude of $G(j\omega)$ at the phase crossover frequency is $\frac{1}{2}$. The gain margin is

- (A) 2 (B) $\frac{1}{2}$
(C) $\frac{1}{3}$ (D) 3

- 14.** In the Bode-plot of a *u/fb* control system, the value of phase of $G(j\omega)$ at the gain crossover frequency is -120°.

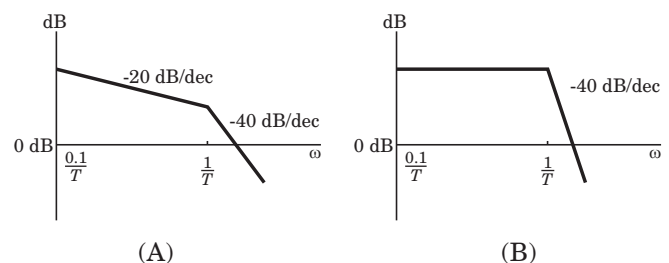
The phase margin of the system is

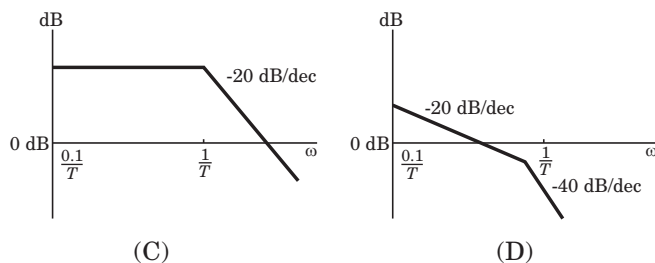
- (A) -120° (B) 60°
(C) -60° (D) 120°

- 15.** The transfer function of a system is given by

$$G(s) = \frac{K}{s(sT+1)} ; K < \frac{1}{T}$$

The Bode plot of this function is





16. The asymptotic approximation of the log-magnitude versus frequency plot of a certain system is shown in fig. P6.5.16. Its transfer function is

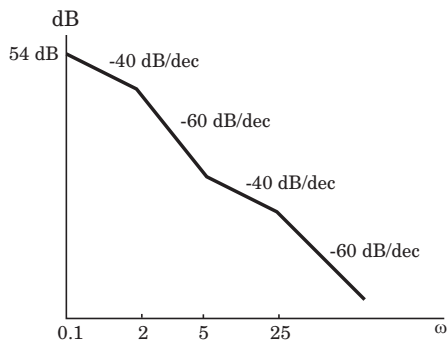


Fig. P6.5.16

- (A) $\frac{50(s+5)}{s^2(s+2)(s+25)}$ (B) $\frac{20(s+5)}{s^2(s+2)(s+25)}$
 (C) $\frac{10s^2(s+5)}{(s+2)(s+25)}$ (D) $\frac{20(s+5)}{s(s+2)(s+25)}$

17. For the Bode plot shown in fig. P6.5.17 the transfer function is

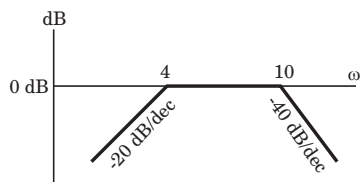


Fig. P6.5.17

- (A) $\frac{100s}{(s+4)(s+10)^2}$ (B) $\frac{100(s+4)}{s(s+10)^2}$
 (C) $\frac{100}{(s+4)(s+10)}$ (D) $\frac{100}{s^2(s+4)(s+10)}$

18. Bode plot of a stable system is shown in the fig. P6.3.18. The open-loop transfer function of the *u/fb* system is

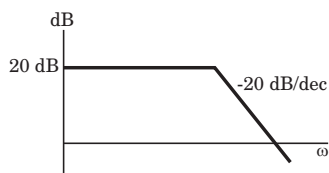


Fig. P6.5.18

- (A) $\frac{100}{s+10}$ (B) $\frac{10}{s+10}$
 (C) $\frac{1}{s+10}$ (D) None of the above

19. Consider the asymptotic Bode plot of a minimum phase linear system given in fig. P6.5.19. The transfer function is

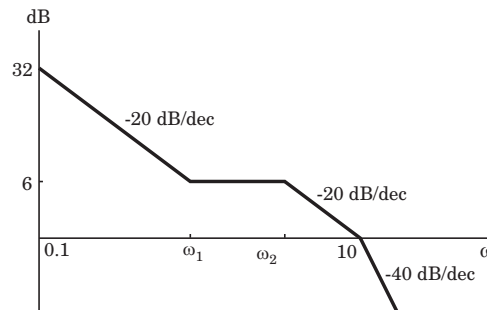


Fig. P6.5.19

- (A) $\frac{8s(s+2)}{(s+5)(s+10)}$ (B) $\frac{4(s+5)}{(s+2)(s+10)}$
 (C) $\frac{4(s+2)}{s(s+5)(s+10)}$ (D) $\frac{8s(s+5)}{(s+2)(s+10)}$

20. The Bode plot shown in fig. P6.5.20 represent

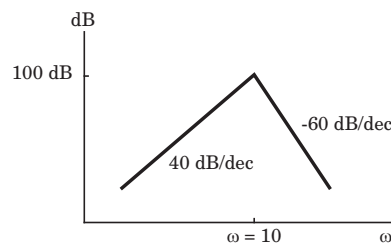


Fig. P6.5.20

- (A) $\frac{100s^2}{(1+0.1s)^3}$ (B) $\frac{1000s^2}{(1+0.1s)^3}$
 (C) $\frac{100s^2}{(1+0.1s)^5}$ (D) $\frac{1000s^2}{(1+0.1s)^5}$

Statement for Q.21-22:

The Bode plot of the transfer function $K/(1+sT)$ is given in the fig. P6.5.21-22.

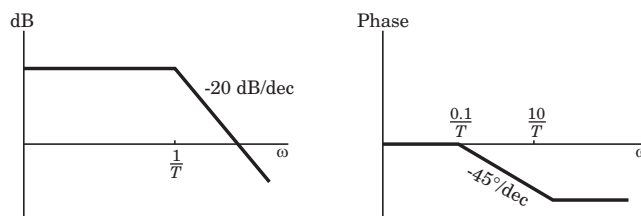


Fig. P6.5.21-22

Statement for Q.29–30:

Consider the Bode plot of a u/fb system shown in fig. P6.5.29–30.

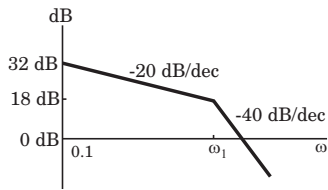


Fig. P6.5.29-30

29. The steady state error corresponding to a ramp input is

- (A) 0.25 (B) 0.2
(C) 0 (D) ∞

30. The damping ratio is

- (A) 0.063 (B) 0.179
(C) 0.483 (D) 0.639

31. The Nyquist plot of a open-loop transfer function $G(j\omega)H(j\omega)$ of a system encloses the $(-1, j0)$ point. The gain margin of the system is

- (A) less than zero (B) greater than zero
(C) zero (D) infinity

32. Consider a u/fb system

$$G(s) = \frac{K}{s(1 + sT_1)(1 + sT_2)(1 + sT_3)}$$

The angle of asymptote, which the Nyquist plot approaches as $\omega \rightarrow 0$, is

- (A) -90° (B) 90°
(C) 180° (D) -45°

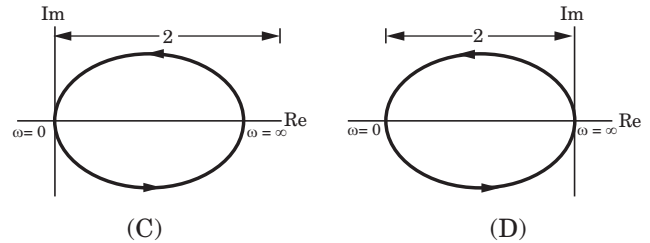
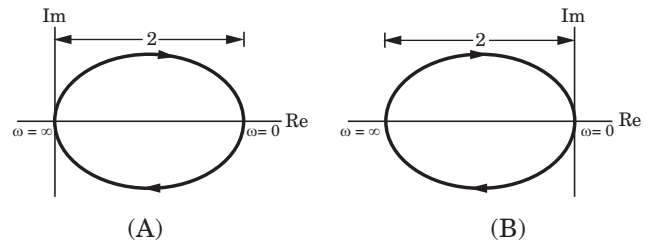
33. If the gain margin of a certain feedback system is given as 20 dB, the Nyquist plot will cross the negative real axis at the point

- (A) $s = -0.05$ (B) $s = -0.2$
(C) $s = -0.1$ (D) $s = -0.01$

34. The transfer function of an open-loop system is

$$G(s)H(s) = \frac{s + 2}{(s + 1)(s - 1)}$$

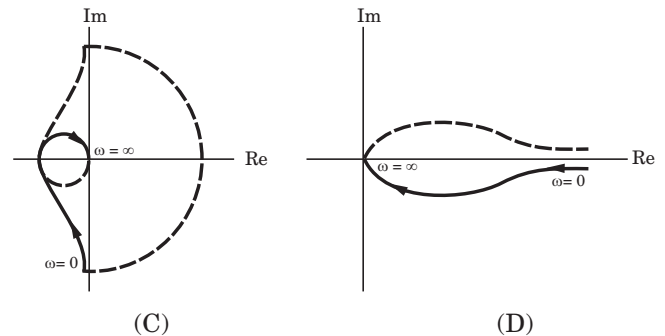
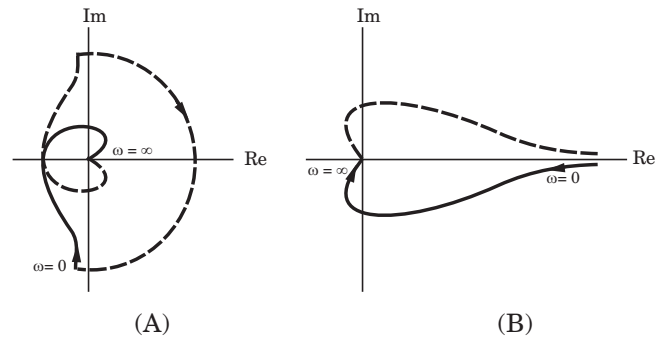
The Nyquist plot will be of the form



35. Consider a u/fb system whose open-loop transfer function is

$$G(s) = \frac{K}{s(s^2 + 2s + 2)}$$

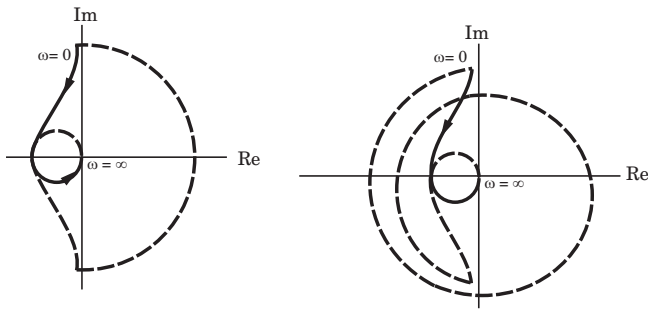
The Nyquist plot for this system is



36. The open loop transfer function of a system is

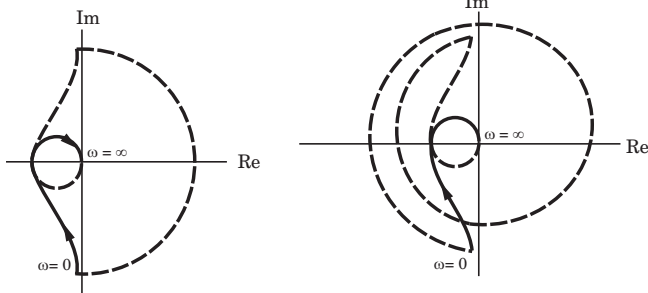
$$G(s)H(s) = \frac{K(1 + s)^2}{s^3}$$

The Nyquist plot for this system is



(A)

(B)



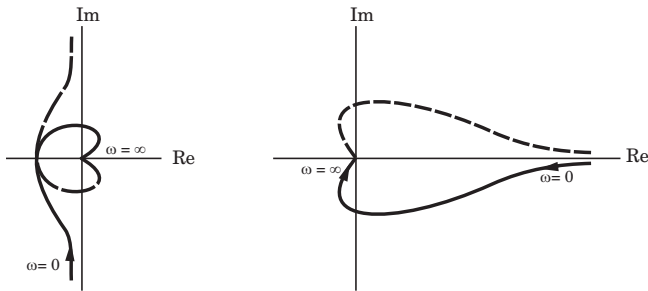
(C)

(D)

37. For the certain unity feedback system

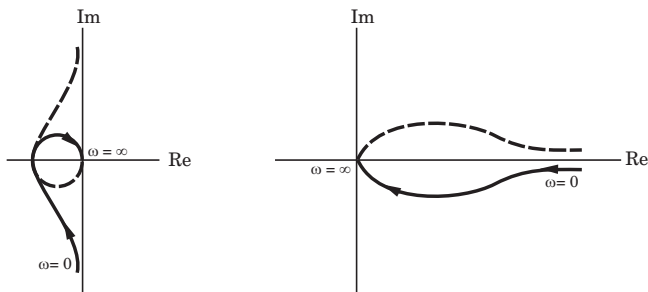
$$G(s) = \frac{K}{s(s+1)(2s+1)(3s+1)}$$

The Nyquist plot is



(A)

(B)



(C)

(D)

38. The Nyquist plot of a system is shown in fig. P6.5.38. The open-loop transfer function is

$$G(s)H(s) = \frac{4s+1}{s^2(s+1)(2s+1)}$$

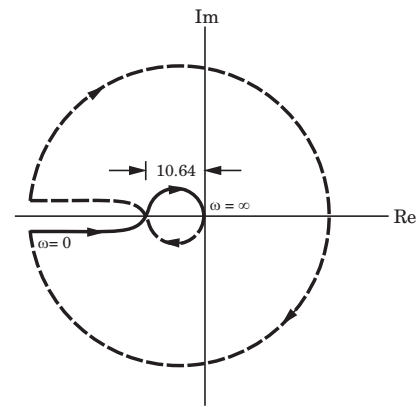


Fig. P6.5.38

The no. of poles of closed loop system in RHP are

(A) 0

(B) 1

(C) 2

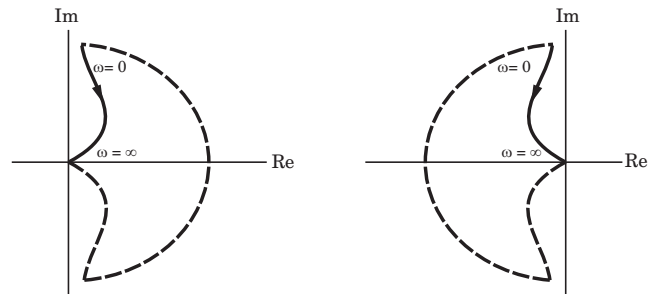
(D) 4

Statement for Q.39–40:

The open-loop transfer function of a feedback control system is

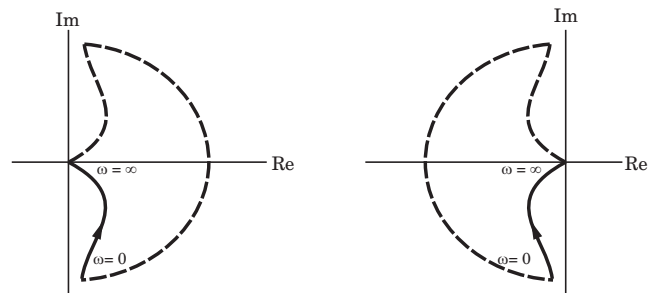
$$G(s)H(s) = \frac{-1}{2s(1-20s)}$$

39. The Nyquist plot for this system is



(A)

(B)



(C)

(D)

40. Regarding the system consider the statements

1. Open-loop system is stable

2. Closed-loop system is unstable

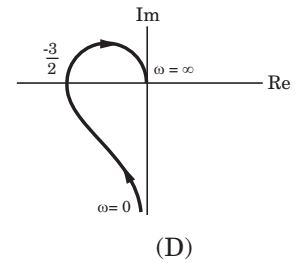
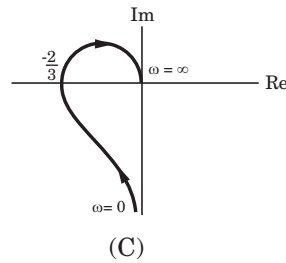
3. One closed-loop poles is lying on the RHP

The correct statements are

- (A) 1 and 2 (B) 1 and 3
(C) only 2 (D) All

41. The Nyquist plot shown in the fig. P6.5.41 is for

- (A) type-0 system (B) type-1 system
(C) type-2 system (D) type-3 system

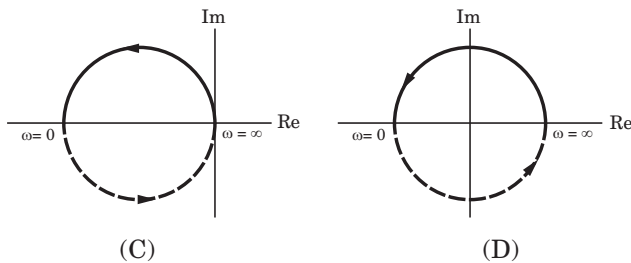
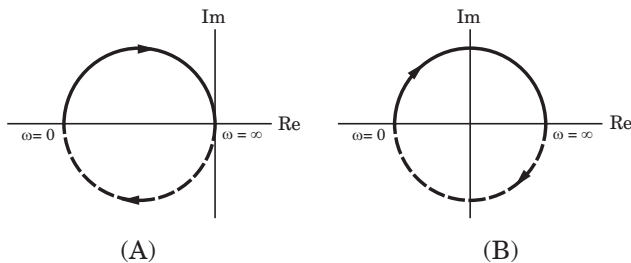


Statement for Q.42–43:

The open-loop transfer function of a feedback system is

$$G(s)H(s) = \frac{K(1+s)}{(1-s)}$$

42. The Nyquist plot of this system is



43. The system is stable for K

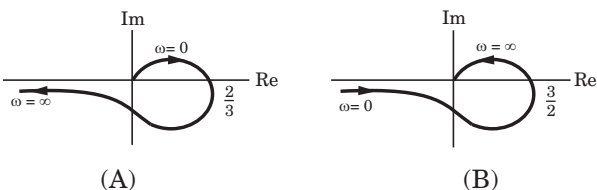
- (A) $K > 1$ (B) $K < 1$
(C) any value of K (D) unstable

Statement for Q.44–46:

A unity feedback system has open-loop transfer function

$$G(s) = \frac{1}{s(2s+1)(s+1)}$$

44. The Nyquist plot for the system is



45. The phase crossover and gain crossover frequencies are

- (A) 1.414 rad/sec, 0.57 rad/sec
(B) 1.414 rad/sec, 1.38 rad/sec
(C) 0.707 rad/sec, 0.57 rad/sec
(D) 0.707 rad/sec, 1.38 rad/sec

46. The gain margin and phase margin are

- (A) -3.52 dB, -168.5° (B) -3.52 dB, 11.6°
(C) 3.52 dB, -168.5° (D) 3.52 dB, 11.6°

SOLUTIONS

$$1. (A) T(s) = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$T(j\omega) = \frac{K\omega_n^2}{-\omega^2 + 2j\xi\omega_n\omega + \omega_n^2}$$

$$|T(j\omega)|^2 = \frac{K^2\omega_n^4}{(\omega_n^2 - \omega^2)^2 + 4\xi^2\omega_n^2\omega^2}$$

From the fig. P6.5.1-2, $|T(j0)| = 1$

$$|T(j0)|^2 = \frac{K^2\omega_n^4}{\omega_n^4} = K^2 = 1 \Rightarrow K = 1$$

2. (B) The peak value of $T(j\omega)$ occurs when the denominator of function $|T(j\omega)|^2$ is minimum i.e. when

$$\omega_n^2 - \omega^2 = 0 \Rightarrow \omega = \omega_n$$

$$|T(j\omega_n)|^2 = \frac{K^2\omega_n^4}{4\xi^2\omega_n^4} = \frac{K^2}{4\xi^2} \Rightarrow |T(j\omega_n)| = \frac{K}{2\xi} = 2.5$$

$$\xi = \frac{K}{5} = 0.2$$

$$3. (B) G(j\omega)H(j\omega) = \frac{1}{j\omega(j\omega + 1)(0.5 + j\omega)}$$

$$\phi = -90^\circ - \tan^{-1} 2\omega - \tan^{-1} \omega$$

At phase cross over point $\phi = -180^\circ$

$$-\tan^{-1} 2\omega - \tan^{-1} \omega - 90^\circ = -180^\circ$$

$$\tan^{-1} 2\omega + \tan^{-1} \omega = 90^\circ$$

$$\frac{2\omega + \omega}{1 - (2\omega)(\omega)} = \tan 90^\circ = \infty$$

$$1 - (2\omega)\omega = 0 \Rightarrow \omega = \frac{1}{\sqrt{2}} = 0.707 \text{ rad/sec}$$

4. (B) For a stable system gain at 180° phase must be negative in dB. More magnitude more stability.

5. (C) At 180° gain is 0.63. Hence gain margin is

$$= 20 \log \frac{1}{0.63} = 4 \text{ dB}$$

At unity gain phase is -172° ,

$$\text{Phase margin} = 180^\circ - 172^\circ = 8^\circ$$

6. (A) At $\angle G(j\omega) = 180^\circ$ gain is -2 dB . Hence gain margin is 2 dB . At 0 dB gain phase is -140° . Hence phase margin is $180^\circ - 140^\circ = 40^\circ$.

7. (A) At $\omega = 100 \text{ rad/sec}$ phase is 180° . Phase cross-over frequency $\omega_\pi = 100 \text{ rad/sec}$.

At $\omega = 10 \text{ rad/sec}$ gain is 0 dB . Gain cross over frequency $\omega = 10 \text{ rad/sec}$.

8. (D) $|GH(j\omega)| \neq 1$, for any value of ω . Thus phase margin is ∞ .

9. (D) For 28 dB gain Nyquist plot intersect the real axis at a ,

$$20 \log \frac{1}{a} = 28 \Rightarrow a = 0.04$$

For 20 dB gain Nyquist plot should intersect at b ,

$$20 \log \frac{1}{b} = 20 \Rightarrow b = 0.1$$

This is achieved if the system gain is increased by factor $\frac{0.1}{0.04} = 2.5$. Thus $K = 2.5$.

$$10. (B) \text{ Here } K = 2, T_1 = 1, T_2 = \frac{1}{2}$$

$$\text{Gain Margin} = \left[\frac{KT_1T_2}{T_1 + T_2} \right]^{-1} = \left[\frac{(2)(0.5)}{1 + 0.5} \right]^{-1} = 1.5 = 3.5 \text{ dB}$$

11. (C) For phase crossover frequency

$$\angle GH(j\omega) = -180^\circ$$

$$GH(j\omega) = \frac{K}{j\omega(1 + 2j\omega)(1 + 3j\omega)}$$

$$-90^\circ - \tan^{-1} 2\omega_\pi - \tan^{-1} 3\omega_\pi = -180^\circ$$

$$\tan^{-1} 2\omega_\pi + \tan^{-1} 3\omega_\pi = 90^\circ$$

$$\frac{2\omega_\pi + 3\omega_\pi}{1 - (2\omega_\pi)(3\omega_\pi)} = \tan 90^\circ$$

$$1 - 6\omega_\pi^2 = 0 \Rightarrow \omega_\pi = 0.41 \text{ rad/s}$$

$$12. (D) G(s) = \frac{s + 1}{s(1 + 0.5s)} = \frac{s + 1}{s\left(\frac{s}{2} + 1\right)}$$

The Bode plot of this function has break at $\omega = 1$ and $\omega = 2$. These are the corner frequencies.

$$13. (A) \text{ G.M.} = \frac{1}{|GH(j\omega_\pi)|} = \frac{1}{\frac{1}{2}} = 2$$

$$14. (B) \text{ P.M.} = 180^\circ + \angle GH(j\omega_\pi) = 180^\circ - 120^\circ = 60^\circ$$

15. (D) Due to pole at origin initial plot has a slope of -20 dB/decade . At $s = j\omega = \frac{1}{T}$. Slope increases to -40

dB/decade . At $\omega = \frac{1}{T}$,

$$|G(j\omega)| \approx KT < 1, \text{ Gain in dB} < 0.$$

27. (C) Initially slope is -20 dB/decade. Hence there is a pole at origin and system type is 1. For type-1 system position error coefficient is ∞ .

$$20 \log K = 6 \Rightarrow K = 2,$$

28. (B) The system is type -0 ,

$$20 \log K_p = 40, \quad K_p = 100, \quad e_{step}(\infty) = \frac{1}{1 + K_p} = \frac{1}{101}.$$

29. (A) The Bode plot is as shown in fig. S6.5.29

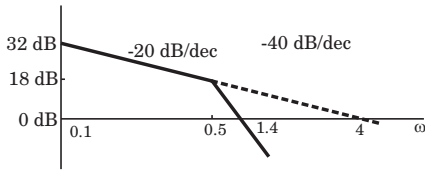


Fig.S6.5.29

$$K_v = 4, \quad e_{ramp}(\infty) = \frac{1}{K_v} = \frac{1}{4} = 0.25$$

$$30. (B) \text{ From fig. S6.5.29 } \xi = \frac{\omega_2}{2\omega_3} = \frac{0.5}{2(1.4)} = 0.179$$

31. (A) If Nyquist plot encloses the point $(-1, j0)$, the system is unstable and gain margin is negative.

$$32. (A) \quad GH(j\omega) = \frac{K}{j\omega(1 + j\omega T_1)(1 + j\omega T_2)(1 + j\omega T_2)}$$

$$\lim_{\omega \rightarrow 0} GH(j\omega) = \lim_{\omega \rightarrow 0} \frac{K}{j\omega} = \lim_{\omega \rightarrow 0} \frac{K}{\omega} \angle -90^\circ$$

Hence, the asymptote of the Nyquist plot tends to an angle of -90° as $\omega \rightarrow 0$.

$$33. (C) \quad 20 \log \frac{1}{|GH(j\omega)|} = 20$$

$$\frac{1}{|GH(j\omega)|} = 10 \Rightarrow |GH(j\omega)| = 0.1$$

Since system is stable, it will cross at $s = -0.1$.

$$34. (B) \quad GH(s) = \frac{s+2}{(s^2-1)}$$

$$GH(j\omega) = \frac{j\omega+2}{(-1-\omega^2)}$$

$$\text{At } \omega=0, \quad GH(j\omega) = 2 \angle -180^\circ$$

$$\text{At } \omega=\infty, \quad GH(j\omega) = 0 \angle -270^\circ$$

Hence (B) is correct option.

$$35. (C) \quad GH(j\omega) = \frac{K}{j\omega(-\omega^2 + 2j\omega + 2)}$$

$$\angle GH(j\omega) = -\tan^{-1} \frac{2\omega}{2-\omega^2} - 90^\circ$$

$$|GH(j\omega)| = \frac{K}{\omega \sqrt{(2-\omega^2)^2 + 4\omega^2}}$$

$$\text{At } \omega=0, \quad GH(j\omega) = \infty \angle -90^\circ,$$

$$\text{At } \omega=\infty \quad GH(j\omega) = 0 \angle -270^\circ,$$

$$\text{At } \omega=1, \quad GH(j\omega) = \frac{K}{\sqrt{5}} \angle -153.43^\circ,$$

$$\text{At } \omega=2, \quad GH(j\omega) = \frac{K}{2\sqrt{18}} \angle -206.6^\circ,$$

Due to s there will be a infinite semicircle. Hence (C) is correct option.

$$36. (B) \quad GH(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3}$$

$$|GH(j\omega)| = \frac{K(1+\omega^2)}{\omega^3}$$

$$\angle GH(j\omega) = -270^\circ + 2 \tan^{-1} \omega$$

$$\text{For } \omega=0, \quad GH(j\omega) = \infty \angle -270^\circ$$

$$\text{For } \omega=1, \quad \angle GH(j\omega) = -180^\circ$$

$$\text{For } \omega=\infty, \quad GH(j\omega) = 0 \angle -90^\circ$$

As ω increases from 0 to ∞ , phase goes -270° to -90° .

Due to s^3 term there will be 3 infinite semicircle.

$$37. (A) \quad |GH(j\omega)| = \frac{K}{\sqrt{1+\omega^2} \sqrt{1+4\omega^2} \sqrt{1+9\omega^2}},$$

$$\angle GH(j\omega) = -90^\circ - \tan^{-1} \omega - \tan^{-1} 2\omega - \tan^{-1} 3\omega,$$

$$\text{For } \omega=0, \quad GH(j\omega) = \infty \angle -90^\circ,$$

$$\text{For } \omega=\infty, \quad GH(j\omega) = 0 \angle -360^\circ,$$

Hence (A) is correct option.

38. (C) The open-loop poles in RHP are $P=0$. Nyquist path enclosed 2 times the point $(-1+j0)$. Taking clockwise encirclements as negative $N=-2$.

$N = P - Z$, $-2 = 0 - Z$, $Z = 2$ which implies that two poles of closed-loop system are on RHP.

$$39. (B) \quad G(s)H(s) = \frac{-1}{2s(1-20s)},$$

$$|GH(j\omega)| = \frac{1}{2\omega \sqrt{1+400\omega^2}}$$

$$\angle GH(j\omega) = 180^\circ - 90^\circ - \tan^{-1} \frac{-20\omega}{1},$$

$$\text{At } \omega=0 \quad GH(j\omega) = \infty \angle 90^\circ$$

$$\text{At } \omega=\infty \quad GH(j\omega) = 0 \angle 180^\circ$$

$$\text{At } \omega=0.1 \quad GH(j\omega) = 2.24 \angle 153.43^\circ$$

$$\text{At } \omega=0.01 \quad GH(j\omega) = 49 \angle 91.15^\circ$$

40. (C) One open-loop pole is lying on the RHP. Thus open-loop system is unstable and $P = 1$. There is one clockwise encirclement. Hence $N = -1$.

$$Z = P - N = 1 - (-1) = 2,$$

Hence there are 2 closed-loop poles on the RHP and system is unstable.

41. (B) There is one infinite semicircle. Which represent single pole at origin. So system type is 1.

$$\mathbf{42. (D)} \quad |GH(j\omega)| = \frac{K\sqrt{1+\omega^2}}{\sqrt{1+\omega^2}} = K$$

$$\angle GH(j\omega) = \tan^{-1} \omega - \tan^{-1} \frac{-\omega}{1}$$

$$\text{At } \omega = 0 \quad GH(j\omega) = K \angle 0^\circ,$$

$$\text{At } \omega = 1 \quad GH(j\omega) = K \angle 90^\circ,$$

$$\text{At } \omega = 2 \quad GH(j\omega) = K \angle 127^\circ,$$

$$\text{At } \omega = \infty \quad GH(j\omega) = K \angle 180^\circ,$$

43. (A) RHP poles of open-loop system $P = 1$, $Z = P - N$.

For closed loop system to be stable,

$$Z = 0, \quad 0 = 1 - N \Rightarrow N = 1$$

There must be one anticlockwise rotation of point $(-1 + j0)$. It is possible when $K > 1$.

$$\mathbf{44. (C)} \quad G(s) = \frac{1}{s(2s+1)(s+1)}, \quad H(s) = 1$$

$$GH(s) = \frac{1}{s(2s+1)(s+1)}$$

$$GH(j\omega) = \frac{1}{j\omega(2j\omega+1)(j\omega+1)}$$

$$\lim_{\omega \rightarrow 0} GH(j\omega) = \lim_{\omega \rightarrow 0} \frac{1}{j\omega} = \infty \angle -90^\circ$$

$$\lim_{\omega \rightarrow \infty} GH(j\omega) = \lim_{\omega \rightarrow \infty} \omega \frac{1}{2(j\omega)^3} = 0 \angle -270^\circ$$

The intersection with the real axis can be calculated as

$$\text{Im}\{GH(j\omega)\} = 0, \quad \text{The condition gives } \omega(2\omega^2 - 1) = 0$$

$$\text{i.e. } \omega = 0, \quad \frac{1}{\sqrt{2}}, \quad GH\left(j \frac{1}{\sqrt{2}}\right) = \frac{-2}{3}$$

With the above information the plot in option (C) is correct.

45. (C) The Nyquist plot crosses the negative real axis at $\omega = \frac{1}{\sqrt{2}}$ rad/sec. Hence phase crossover frequency is

$$\omega_\pi = \frac{1}{\sqrt{2}} = 0.707 \text{ rad/sec.}$$

The frequency at which magnitude unity is

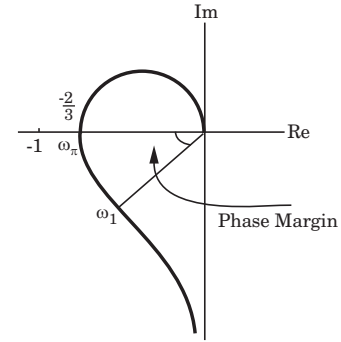


Fig.S6.5.44

$$\omega_1^2 (1 + \omega_1^2) (1 + 4\omega_1^2) = 1$$

$$\omega^2 = 0.326, \quad \omega_1 = 0.57 \text{ rad/sec}$$

$$\mathbf{46. (D)} \quad \text{G.M.} = 20 \log \frac{1}{|GH(j\omega_\pi)|}, \quad |GH(j\omega_\pi)| = \frac{2}{3}$$

$$\text{Gain Margin} = 20 \log \frac{3}{2} = 3.52 \text{ dB.}$$

$$\angle GH(j\omega) = -90^\circ - \tan^{-1} \omega - \tan^{-1} 2\omega,$$

$$\text{At unit gain } \omega_1 = 0.57 \text{ rad/sec,}$$

Phase at this frequency is

$$\angle GH(j\omega_1) = -90^\circ - \tan^{-1} 0.57 - \tan^{-1} 2(0.57) = -168.42^\circ$$

$$\text{Phase margin} = -168.42^\circ + 180^\circ = 11.6^\circ$$

Note that system is stable. So gain margin and phase margin are positive value. Hence only possible option is (D).
