ગુજરાત રાજ્યના શિક્ષણવિભાગના પત્ર-ક્રમાંક
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PHYSICS

Standard 12

(Semester III)

PLEDGE

India is my country.
All Indians are my brothers and sisters.
I love my country and I am proud of its rich and varied heritage.
I shall always strive to be worthy of it.
I shall respect my parents, teachers and all my elders and treat everyone with courtesy.
I pledge my devotion to my country and its people.
My happiness lies in their well-being and prosperity.

રાજ્ય સરકારની વિનામૂલ્યે યોજના હેઠળનું પુસ્તક

Gujarat State Board of School Textbooks
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PREFACE

The Gujarat State Secondary and Higher Secondary Education Board has prepared new syllabi in accordance with the new national syllabi prepared by N.C.E.R.T. based on N.C.F. 2005 and core-curriculam. These syllabi are sanctioned by the Government of Gujarat.

It is a pleasure for the Gujarat State Board of School Textbooks, to place before the students this textbook of **Physics, Standard 12, (Semester III)** prepared according to the new syllabus.

Before publishing the textbook, its manuscript has been fully reviewed by experts and teachers teaching at this level. Following suggestions given by teachers and experts, we have made necessary changes in the manuscript before publishing the textbook.

The Board has taken special care to ensure that this textbook is interesting, useful and free from errors. However, we welcome any suggestions, from people interested in education, to improve the quality of the textbook.

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FUNDAMENTAL DUTIES

It shall be the duty of every citizen of India

(A) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(B) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(C) to uphold and protect the sovereignty, unity and integrity of India;

(D) to defend the country and render national service when called upon to do so;

(E) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(F) to value and preserve the rich heritage of our composite culture;

(G) to protect and improve the natural environment including forests, lakes, rivers and wild life, and to have compassion for living creatures;

(H) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(I) to safeguard public property and to abjure violence;

(J) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

(K) to provide opportunities for education by the parent or the guardian, to his child or a ward between the age of 6-14 years as the case may be.

* Constitution of India : Section 51 A

I N D E X

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About This Textbook...

We have pleasure in presenting this textbook of physics of Standard 12 to you. This book is on the syllabi based on the courses of National Curriculum Framework (NCF), Core-Curriculum and National Council of Educational Research and Training (NCERT) and has been sanctioned by the State Government keeping in view the National Education Policy.

The State Government has implemented the semester system in science stream. The semester system will reduce the educational load of the students and increase the interest towards study.

In this Textbook of Physics for Standard-12, Seven chapters are included, looking into the depth of the topics, time which will be available for classroom teaching, etc...

The real understanding of the theories of physics is obtained only through solving related problems. Hence, for the new concept, solved problems are given. One of the positive sides of the book is that at the end of each chapter extended summary is given. On the basis of this one can see the whole contents of the chapter at a glance.

Keeping in view the formats of various entrance test conducted on all India basis, we have included MCQs, Short questions, objective questions and problems in this book. At the end of the book, Hints for solving the problems are also included so that students themselves can solve the problems.

This book is published in quite a new look in four-colour printing so that the figures included in the book are much clear. It has been observed, generally, that students do not preserve old text-books, once they go to the higher standard. In the semester system, each semester has its own importance and the look of the book is also very nice so the students would like to preserve this book and it will become a reference book in future.

The previous textbook got excellent support from students, teachers and experts. So a substantial portion from that book is taken in this book either in its original form or with some changes. We are thankful to that team of authors. We are also thankful to the teachers who remained present in the Review workshop and gave their inputs to make this textbook error-free.

Proper care has been taken by authors, subject advisors and reviewers while preparing this book to see that it becomes error-free and concepts are properly developed. We welcome suggestions and comments for the importance of the textbook in future.

Authors/Editors

1

ELECTRIC CHARGE AND ELECTRIC FIELD

1.1 Introduction

Whatever facilities an individual is enjoying in this modern age is due to technological development. From all kinds of energy, electric energy holds an important role for human comfort. Electric energy can be easily stored and can be transferred to another form of energy. There is no exaggeration in calling the electricity is the mother of technology. Electric charges are the foundation stones of electricity.

In this chapter we will study about static charges, their properties and interaction between them. Such a study is called static electricity. Static electricity is used in copier machine, laser printer, television etc. Natural phenomenon such as lightning can be understood through static electricity. Here, we will study about electric fields due to different system of charges and its characteristics.

1.2 Electric Charge

Any matter consists of certain fundamental particles. Fundamental particles are more than 100. Out of them three particles are most important namely electron, proton and neutron. Because of their masses these particles exert gravitational force on each other. For example two electrons 1cm apart exert 5.5×10^{-67} N gravitational force on each other, which is attractive. However, an electron is found to repel another electron at the same distance (1 cm) with a force of 2.3×10^{-24} N. This additional force other than gravitational force is an electric force. **The fundamental intrinsic property due to which such a force acts is called the electric charge.**

Just as masses of two particles are responsible for the gravitational force, charges are responsible for the electric force.

Two protons placed at a distance of 1 cm also repel each other with a force of 2.3×10^{-24} N, which shows that proton has the electric charge. The magnitude of this charge is same as the charge of an electron. Now if a proton and electron are placed 1 cm apart, they exert a force of 2.3×10^{-24} N on each other but this force is attractive.

Thus, we conclude that magnitude of charge on electron and proton is same but they are of opposite type.

Electric charges are of two types : Positive charge and Negative charge. Traditionally, charge of a proton considered positive and that of an electron negative. Though it makes no difference whatsoever to Physics if this sign convention is reversed.

The force acting between two like charges is repulsive and it is attractive between two unlike charges.

All material bodies contain equal number of electrons and equal number of protons in their normal state. So they are electrically neutral. In any substance, electrons are comparatively weakly bound than the force with which the protons are bound inside the nucleus. Hence, whenever there is an exchange of charge between two bodies due to some process (e.g. friction), it is the electrons are transferred from one body to the other. The body that receives the extra electrons, becomes negatively charged. The body that loses the electrons, becomes positively charged because it has more number of protons than electrons. Thus, when a glass rod is rubbed with a silk cloth, some electrons are transferred from the glass rod to the silk cloth. The glass rod becomes positively charged and the cloth becomes negatively charged because it receives extra electrons. To detect these charges a simple device is used, known as **electroscope.**

Electic charge is a fundamental property like mass. It is difficult to define. The SI unit of the quantity of charge is coulomb and abbreviated as C.

One coulomb is the charge flowing through any section of the conductor in one second when the electric current in it is 1 ampere. The charge on a proton is $e = +1.6 \times 10^{-19}$ C. The charge on the electron is $e = -1.6 \times 10^{-19}$ C.

Quantization of Electric Charge

All the experiments carried out so far show that the **magnitude of all charges found in nature are in integral multiple of a fundamental charge.**

$Q = ne$

This fact is known as quantization of charges. The fundamental charge is the charge of an electron or proton. It is denoted by e and it is called the fundamental unit of charge.

Out of all the fundamental particles, the building blocks of all matters, the particles having possesed charge equal to e. For example, charge on proton and positron (positive electron) is $+e$, while charge on electron is $-e$. Thus, charge on any object can be increased or decreased only in step of e. The quantization of charge was first suggested by English scientist Faraday. It was experimentally demonstrated by Millikan in 1912.

No theory, so far, has been able to explain satisfactorily, the quantization of charges.

According to new research, the proton and neutron consists of another fundamental particles called **quarks.**

A proton and neutron consist of three quarks each. These quarks are of two types : the quark possessing $+\frac{2}{3}$e charge is called an up quark (u) and another having $-\frac{1}{3}e$ charge is called a down quark (d). (The composition of proton is indicated as uud and composition of neutron is indicated as udd). Thus, **matter is formed of such quarks and electrons.** The independent existence of quark is not detected so far.

Conservation of Electric Charge

The algebraic sum of electric charges in an electrically isolated system always remains constant irrespective of any process taking place. This statement represents the law of conservation of charge.

In an electrically isolated system, a charge can neither enter from outside nor escape from inside. Any chargeless thing can enter or leave such a system.

In the experiment of glass rod and silk cloth, before rubbing glass rod with silk the net charge on them is zero. After rubbing the glass rod with silk cloth, the glass rod becomes positively charged and same amount of negative charge is received by the silk cloth. Thus, after the process of friction the net charge of system (glass rod + silk cloth) is zero.

Now, to understand the conservation of electric charge we consider another illustration.

As shown in figure 1.1, the initial charge in a box having thin walls is zero. A highly energetic photon enters in the box. A photon is a chargeless particle. As the photon enters through a box it produces an electron-positron pair. After the pair production in the isolated system the net charge is zero because the charges on the electron and positron are equal and opposite type. The initial charge of the system was zero. Thus, in this event also charge is conserved.

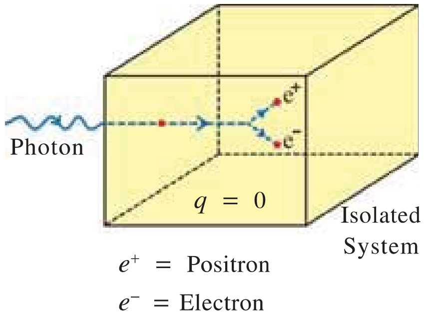


Figure 1.1 Conservation of Electric Charge

In other words in an electrically isolated system, only those processes are possible in which charges of equal magnitude and opposite types are either produced or destroyed.

Charging by Induction

Consider two identical isolated sphere placed on an insulated stand, one carrying net charge +Q (i.e. positively charged) and other having no net charge. If they are brought directly in contact or brought in contact with conducting wire, some of the electrons from the chargeless sphere transferred to positively charged sphere. As a result, the positive charge on the positively charged sphere reduced and chargeless sphere becomes positive, because it loses the electrons. Now, both the spheres will have equal amount of charge$+\frac{Q}{2}$ after the separation because they are identical. Thus we have established $\frac{Q}{2}$ electric charge on the other sphere through contact or that the charging of the second sphere has taken place.

There is another method of charging the object. In that method the charged body does not looses its own charge and without coming in physical contact with other object it will induce opposite charge in that. This phenomenon is called **induction of electric charge.**

Figure 1.2(a) shows an isolated metal sphere. The net charge on the sphere is zero. As shown in figure 1.2(b), a negatively charged plastic rod is brought close to the sphere the free electrons of the sphere move away from the rod because of repulsion and go to the other part of the sphere. Consequently the part of the sphere close to the rod becomes positively charged due to deficiency of electron in that region.

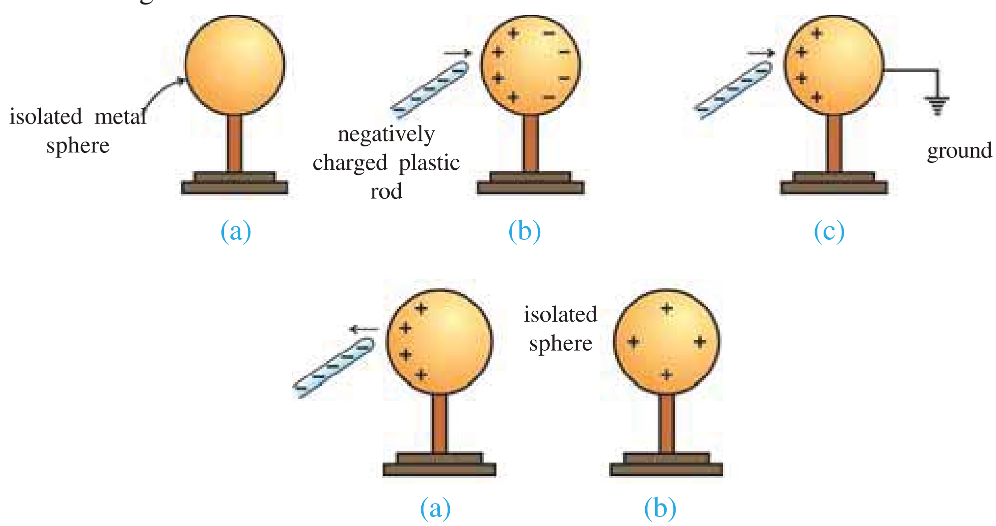


Figure 1.2 Induction of Electric Charge

As shown in figure 1.2(c) when the sphere is connected to the earth through a conducting wire, the some of the electrons of the sphere will flow to the ground. (The earth is a good conductor and it act as a practically infinite source of extra electrons or sink of electrons.)

As shown in figure 1.2(d), even if the connection with the earth is removed, the sphere retains the positive charge. When the plastic rod is moved away from the sphere, the electrons get redistributed on the sphere such that the same positive charge is spread all over the surface of the sphere. (Figure 1.2 (e))

1.3 Coulomb's Law

French scientist Charles Coulomb (1736-1806) measured electrical attraction and repulsion between two electric charges through a number of experiments and deduced the law that governs them, which is known as Coulomb's law. The law is as under :

'The electric force (Coulombian force) between two stationary point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.' This force is along the line joining the two charges.

According to Coulomb's law, the electric force between the two point charges q_1 and q_2 separated by a distance r can be given as,

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\therefore \; F = k\frac{q_1 q_2}{r^2} \quad (1.3.1)$$

Where k is a Coulomb's constant. It's value depends on the unit of q_1, q_2 and r. Experimentally the value of k in vacuum in SI unit is 8.9875×10^9 Nm^2C^{-2}. For practical purposes, $k = 9 \times 10^9$ Nm^2C^{-2}. (In CGS unit value of k is 1).

For the simplification of formula in electrostatic k is expressed as $\frac{1}{4\pi\varepsilon_0}$.

$$k = \frac{1}{4\pi\varepsilon_0}$$

Where, ε_0 is the permittivity of free space. From the above equation,

$$\varepsilon_0 = \frac{1}{4\pi k} = \frac{1}{4\pi \times 8.9875 \times 10^9} \approx 8.854 \times 10^{-12} \; C^2N^{-1}m^{-2}$$

$$\text{Thus, } F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}. \quad (1.3.2)$$

If the charges are in any other insulating medium and not in vacuum, the permittivity of vacuum ε_0 in equation (1.3.2) should be replaced by the permittivity ε of that medium. Hence force in that medium,

$$F_m = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}. \quad (1.3.3)$$

Thus, Coulombian force acting on two point charges is also depend on the medium between the two charges. By taking ratio of equation (1.3.2) and (1.3.3),

$$\frac{F}{F_m} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r = K$$

$$\therefore \; F_m = \frac{F}{K} \quad (1.3.4)$$

Where, ε_r is known as relative permittivity of the medium or dielectric constant (K). A detailed study about this we will learn in Chapter 2. From equation (1.3.4) it is clear that the force between given charges held at a given distance apart in insulating medium is only $\frac{1}{K}$ times (i.e.k-th part) of the force between them in vacuum.

Remember that Coulomb's law holds only for stationary point charges. Generally, this law is also applicable for charged objects whose sizes are much smaller than the distance between them.

Coulomb's law resembles inverse square law of gravitation. The charge q plays the same role in Coulomb's law that the mass m plays in gravitational law. The gravitational forces are always attractive, whereas electrostatic forces can be repulsive or attractive, because electric charges are of two types.

Illustration 1 : The repulsive force between two particles of same mass and charge, separated by a certain distance is equal to the weight of one of them. Find the distance between them.

Mass of particle $= 1.6 \times 10^{-27}$kg

Charge of particle $= 1.6 \times 10^{-19}$C, $k = 9 \times 10^9$ MKS, $g = 10\text{ms}^{-2}$.

Solution : Here,

Repulsive force between two particles = Weight of one of the particles

$$\therefore \; k\frac{q_1 q_2}{r^2} = mg$$

$$\therefore \; r^2 = \frac{kq_1 q_2}{mg} = \frac{9\times10^9 \times (1.6\times10^{-19})^2}{(1.6\times10^{-27})(10)} = 1.44 \times 10^{-2}$$

$\therefore \; r = 0.12$ m.

Illustration 2 : Two spheres of copper, having mass 1g each, are kept 1 m apart. The number of electrons in them are 1% less than the number of protons. Find the electrical force between them. Atomic weight of copper is 63.54 g/mol, atomic number is 29, Avogadro's number $N_A = 6.023 \times 10^{23}$ mol^{-1}. $k = 9 \times 10^9$ SI.

Solution : In a neutral atom of copper the number of electrons and protons are 29 each. Here, the number of electrons are less than that of protons by 1%.

$$\therefore \text{ Net charge on each atom } q' = \left(\begin{matrix}\text{Total Charge} \\ \text{of Protons}\end{matrix}\right) + \left(\begin{matrix}\text{Total Charge} \\ \text{of Electrons}\end{matrix}\right)$$

$$= (+29e) + (-29e) - (-0.29e)$$

$$= +\,0.29e$$

$\therefore$ Net positive charge of 1 g copper,

$$q = \left(\begin{matrix}\text{No. of Atoms} \\ \text{in 1g Copper}\end{matrix}\right) \times 0.29e = \frac{6.023\times10^{23}}{63.54} \times 0.29e$$

$\therefore$ Electric force between two copper spheres,

$$F = k\frac{q\,q}{r^2} = k\frac{q^2}{r^2} = \frac{9\times10^9}{1^2} \times \left(\frac{6.023\times10^{23}\times0.29\times1.6\times10^{-19}}{63.54}\right)^2$$

$$= 1.74 \times 10^{15}\text{N}$$

It can be seen in above example that even a difference of 1% between positive and negative charges in any substance can give rise to a very large force. Most of the matters are electrically neutral so that there is a dominance of weak gravitational force on them.

Illustration 3 : Charge Q is uniformly distributed over a body. How should the body be divided into two parts, so that force acting between the two parts of body is maximum for a given separation between them ?

Solution : Suppose the body is broken into two parts such that the charge on one part of body is q and on the other is $Q - q$. The force existing between the two parts separated by distance r will be,

$$F = k\frac{q(Q-q)}{r^2}$$

The force F to be maximum, the quantity $y = q(Q - q) = Qq - q^2$

should be maximum. For this $\frac{dy}{dq}$ should be zero. $\therefore \frac{dy}{dq} = Q - 2q = 0$

$$\therefore q = \frac{Q}{2}$$

Thus, the body should be divided into two parts such equal charges are present on each part.

Coulomb's Law in Vector from :

Force is a vector quantity, so the Coulomb's law can be represented in vector form as follows :

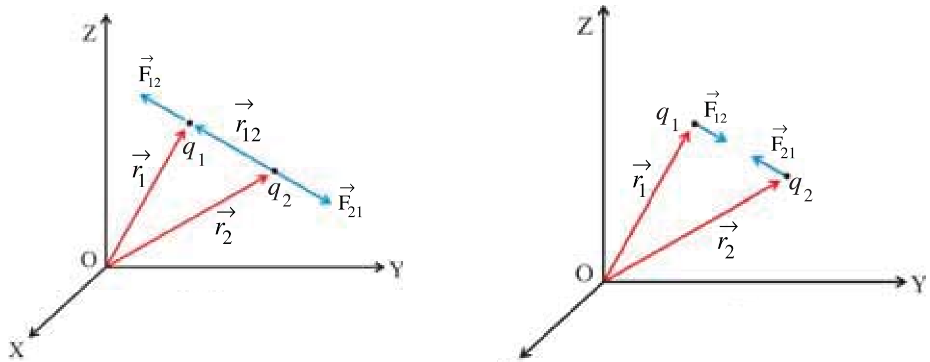


Figure 1.3 Coulomb's Law in Vector Form

As shwon in figure 1.3(a), let $\vec{r}_1$ and $\vec{r}_2$ be the position vectors of the charges q_1 and q_2 respectively in a Cartesian co-ordinate system. Let $\vec{r}_{12}$ be the unit vector pointing from q_2 to q_1, $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$.

According to Coulomb's Law, force acting on charge q_1 due to charge q_2 is,

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} \qquad (1.3.5)$$

Where, $r_{12} = |\vec{r}_1 - \vec{r}_2|$ is the distance between the two charges and $\hat{r}_{12}$ is a unit vector of $\vec{r}_{12}$ in the direction from q_2 to q_1.

$$\hat{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}$$

$$\therefore \vec{F}_{12} = k\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \cdot \frac{(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$$

$$\vec{F}_{12} = k\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) \qquad (1.3.6)$$

Above equation is valid for any sign of the charges whether positive or negative. If q_1 and q_2 are of the same sign (either both positive or both negative)

$\vec{F}_{12}$ is along $\hat{r}_{12}$, which denotes repulsive force. If q_1 and q_2 are of opposite sign, $\vec{F}_{21}$ is along $-\hat{r}_{12}$, which denotes the attraction between the opposite charges. (See Figure 1.3(b)).

The coulombian force on charge q_1 due to charge q_2 can be given by replacing 1 and 2 in equation (1.3.6)

$$\vec{F}_{21} = k\frac{q_1 q_2}{r_{21}^2}\vec{r}_{21} \qquad (1.3.7)$$

$$= k\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1) \qquad (1.3.8)$$

Where, $\vec{r}_{21}$ is a unit vector directed from q_1 to q_2.

Here, $\vec{r}_2 - \vec{r}_1 = -(\vec{r}_1 - \vec{r}_2)$

Thus, from equation (1.3.8),

$$\vec{F}_{21} = -k\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) = -\vec{F}_{12}$$

Thus, Coulomb's Law agrees with the Newton's Third Law.

1.4 Forces between more than two charges : The Superposition Principle

We can use Coulomb's Law to find the force acting between two electric charges. When more than two charges (Suppose they are q_1, q_2, ..., q_n) are present and to calculate the net force acting on any one charge, we have to use superposition principle in addition to Coulomb's Law.

Superposition Principle : When more than one coulombian forces are acting on a charge, the resultant coulombian force acting on it is equal to the vector sum of the individual force.

Thus, the coulombian force acting between two charges is not influenced by the presence of a third charge. Hence, the coulombian force is called a two body force.

Consider a system of charges q_1, q_2, q_3 and q_4 as shown in figure 1.4. Let $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$ and $\vec{r}_4$ are their respective position vectors in a given co–ordinate system. Here, we will find the resultant force $\vec{F}_2$ acting on charge q_2 due to the other charges.

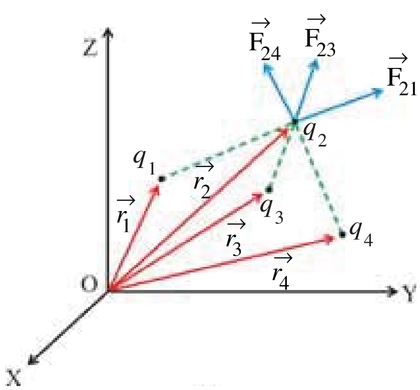


Figure 1.4 Superposition Principle

The force on charge q_2 due to charge q_1 is,

$$\vec{F}_{21} = k\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$$

The force on charge q_2 due to q_3 is,

$$\vec{F}_{23} = k\frac{q_2 q_3}{r_{23}^2}\hat{r}_{23}$$

The force on charge q_2 due to q_4 is,

$$\vec{F}_{24} = k\frac{q_2 q_4}{r_{24}^2}\hat{r}_{24}$$

According to superposition principle,

$$\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24} = k\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + k\frac{q_2 q_3}{r_{23}^2}\hat{r}_{23} + k\frac{q_2 q_4}{r_{24}^2}\hat{r}_{24}$$

$$= kq_2\left[\frac{q_1}{r_{21}^2}\hat{r}_{21} + \frac{q_3}{r_{23}^2}\hat{r}_{23} + \frac{q_4}{r_{24}^2}\hat{r}_{24}\right]$$

$$= kq_2\sum_{\substack{j=1\\ j\neq 2}}^{4}\frac{q_j}{r_{2j}^2}\hat{r}_{2j} \qquad (1.4.1)$$

or

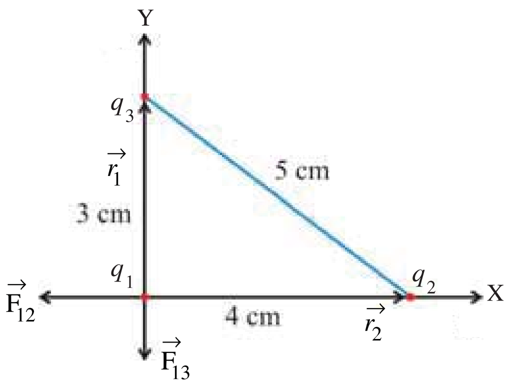
$$\vec{F}_2 = kq_2\sum_{\substack{j=1\\ j\neq 2}}^{4}\frac{q_j}{|\vec{r}_2-\vec{r}_j|^3}(\vec{r}_2-\vec{r}_j) \qquad (1.4.2)$$

In general, the force acting on charge q_i due to system of n electric charges will be,

$$\vec{F}_i = kq_i\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{q_j}{r_{ij}^2}\hat{r}_{ij}$$

$$\vec{F}_i = kq_i\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{q_j}{|\vec{r}_i-\vec{r}_j|^3}(\vec{r}_i-\vec{r}_j) \qquad (1.4.3)$$

Illustration 4 : Three equal charges each having a magnitude of 2.0×10^{-6}C are placed at the three corners of a right angled triangle of sides 3cm, 4cm and 5cm. Find the force on the charge at the right angle corner.



Solution :

The situation is as shown in the figure.

$q_1 = q_2 = q_3 = q = 2 \times 10^{-6}$C

The position vectors of q_1, q_2 and q_3 are respectively $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$.

$\vec{r}_1 = (0,\ 0)$

$\vec{r}_2 = (4,\ 0)\text{cm} = (0.04,\ 0)\text{m}$

$\vec{r}_3 = (0,\ 3)\text{cm} = (0,\ 0.03)\text{m}$.

q_1 is placed at the right angle of right angled triangle. Net force acting on q_1 is,

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}$$

$$= k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + k\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} = kq^2\left[\frac{\hat{r}_{12}}{r_{12}^2} + \frac{\hat{r}_{13}}{r_{13}^2}\right] \quad (\because\ q_1 = q_2 = q) \qquad (1)$$

Now, $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = (0,\ 0) - (0.04,\ 0) = (-0.04,\ 0)\text{m}.$

$\therefore\ r_{12} = \sqrt{(-0.04)^2 + (0)^2} = 0.04\text{m}.$

$$\hat{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_{12}|} = \frac{(-0.04, 0)}{0.04} = (-1,\ 0)\text{m}.$$

$\vec{r}_{13} = \vec{r}_1 - \vec{r}_3 = (0,\ 0) - (0,\ 0.03) = (0,\ -0.03)\text{m}$

$r_{13} = \sqrt{(0)^2 + (-0.03)^2} = 0.03\text{m}$

$$\hat{r}_{13} = \frac{\vec{r}_1 - \vec{r}_3}{|\vec{r}_{13}|} = \frac{(0, -0.03)}{0.03} = (0,\ -1)\text{m}$$

Put all these values in equation (1),

$$\vec{F}_1 = (9 \times 10^9)\ (2 \times 10^{-6})^2 \left[\frac{(-1, 0)}{(0.04)^2} + \frac{(0, -1)}{(0.03)^2}\right]$$

$$= 36 \times 10^{-3}[625\ (-1,\ 0) + 1111.1\ (0,\ -1)]$$

$$= (-22.5,\ -40)\text{N}$$

$\therefore\ |\vec{F}_1| = \sqrt{(-22.5)^2 + (-40)^2} = 45.88\text{N}$

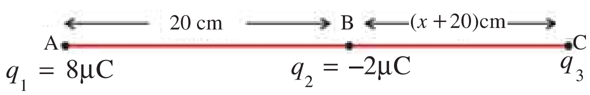
Direction of force,

$$\theta = tan^{-1}\left(\frac{F_y}{F_x}\right) = tan^{-1}\left(\frac{-40}{-22.5}\right) = tan^{-1}(1.777)$$

$$= 60.6^\circ$$

θ is the angle with respect to negative X-axis.

Illustration 5 : Two electric charges having magnitude 8.0μC and −2.0μC are separated by 20cm. Where should a third charge be placed so that the resultant force acting on it is zero ?



Solution : Let the two charges q_1 = 8μC and q_2 = −2μC be placed at points A and B respectively as shown in figure. The resultant force on the third charge q_3 will be zero only if the forces due to two charges are equal in magnitude and opposite in direction. This is possible only if the third charge is placed at a point on the line joining the two charges. Third charge q_3 cannot be placed anywhere between points A and B since q_1 and q_2 have opposite sign. As the magnitude of charge on A is greater than that on B the third charge has to be nearer to B.

Suppose the third charge is placed at point C and BC = x cm.

According to superposition principle, the net force on charge q_3,

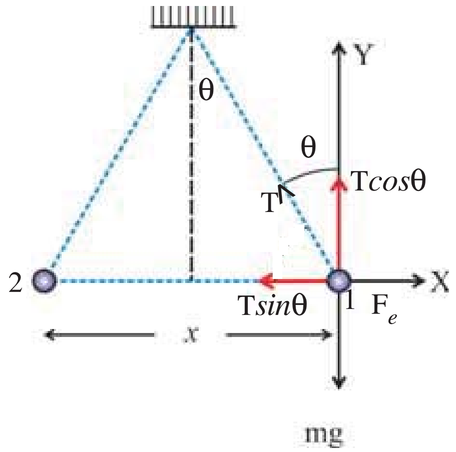
$F_3 = F_{31} + F_{32}$

$$0 = k\frac{q_1 q_3}{(r+x)^2} + k\frac{q_2 q_3}{x^2} = \frac{q_1}{(r+x)^2} + \frac{q_2}{x^2} = \frac{8\times10^{-16}}{(20+x)^2} - \frac{2\times10^{-16}}{x^2}$$

$$\therefore \frac{20+x}{x} = 2 \qquad \therefore x = 20\text{cm}$$

Illustration 6 : Two spheres having same radius and mass are suspended by two strings of equal length from the same point, in such a way that their surfaces touch each other. On depositing 4×10^{-7}C charge on them, they repel each other in such a way that in equilibrium the angle between their strings becomes 60°. If the distance from the point of suspension to the center of the sphere is 20cm, find the mass of each sphere. $k = 9 \times 10^9$ SI and $g = 10\text{ms}^{-2}$.

Solution : If the spheres are identical in all respects then 4×10^{-7}C charge will be distributed equally between them. Hence charge on each sphere is 2×10^{-7}C. The force acting on sphere 1 in equilibrium will be :



(1) Weight mg in the vertically downward direction.

(2) F_e, the repulsive force between the spheres,

(3) The tension T produced in the string.

Under the balanced condition, if we consider the X and Y components in the Cartesian co-ordinate system as shown in the figure,

$F_e = T\sin\theta$

$$\therefore k\frac{q^2}{x^2} = T\sin\theta \qquad (1)$$

$$\text{and } mg = T\cos\theta \qquad (2)$$

$$\frac{kq^2}{x^2 mg} = \tan\theta \Rightarrow m = \frac{kq^2}{x^2 g\tan\theta}$$

From figure, $\sin\theta = \dfrac{\frac{x}{2}}{l} = \dfrac{x}{2l}$

$\therefore x = 2l \sin\theta$

$$\therefore m = \frac{kq^2}{g\, 4l^2 \sin^2\theta \tan\theta}$$

$$\therefore m = \frac{(9\times10^9)(2\times10^{-7})^2}{10\times4(20\times10^{-2})^2\times(\sin30^\circ)^2\times(\tan30^\circ)} = 1.56 \times 10^{-3}\text{kg}$$

1.5 Electric Field

When we place a point charge q_0 in the region around another point charge q in the space, it will exert the electric force on q_0. We may ask the question. If charge q_0 is removed then what is left in the surrounding ? Is there nothing ? If there is nothing in the surrounding, then how does a force act on q_0 ? In order to answer these questions, the concept of electric field is very useful.

A charge produces some effect in the space around it. The region around the charge in which the effect of electric charge is prevailing is called the electric field of the charge. This electric field can interact with another charge q_0 placed in it and exerts the force on it. (It does not exert the force which produce the electric field). Thus, electric field acts as an agency between q and q_0.

Suppose a charge Q is placed at an origin of a co-ordinate system. Now bring a charge q_0 at the given point in the electric field without disturbing the position of charge Q. If the position vector of that point is $\vec{r}$, then electric field at that point can be defined as follows :

$$\vec{E}(\vec{r}) = \frac{\vec{F}(\vec{r})}{q_0} \qquad (1.5.1)$$

Here, $\vec{E}$ is called the electric field or electric field intensity of charge Q at a position vector $\vec{r}$. The quantity $\vec{E}$ is independent of q_0. It is dependent solely on the magnitude of electric charges of the system, their arrangement and the position vector $\vec{r}$ of q_0.

The charge q_0 used to define or to measure intensity of electric field is called a test charge. The charges producing electric field are called the source charges.

In SI system, the unit of electric field is NC^{-1} or Vm^{-1}.

In equation (1.5.1), if $q_0 = 1C$ then $\vec{E} = \vec{F}$ and definition of electric field can be given as follows :

'The force acting on a unit positive charge at a given point in an electric field of a point charge of a system at charges is called the electric field or intensity of electric field $\vec{E}$ at that point.

Electric field is a vector quantity and it is in the direction of force acting on unit positive charge at a given point.

If the system of charges consists of more than one charge, then electric field at a given point can be obtained by using Coulomb's Law and superposition principle.

Consider a system of charges q_1, q_2, ..., q_n with position vectors $\vec{r_1}$, $\vec{r_2}$, ..., $\vec{r_n}$ relative to origin. The electric field is produced in the region surrounding the system due to the system of charges. We want to determine the electric field at a point P(x, y, z) having position vector $\vec{r}$. For this purpose place a very small test charge q_0 at that point and use the superposition principle.

Electric field at point P due to charge q_1 is given by,

$$\vec{E}_1 = \frac{\vec{F}_1}{q_0} = k\frac{q_1}{|\vec{r}-\vec{r}_1|^3}(\vec{r}-\vec{r}_1)$$

Electric field at point P due to charge q_2 is.

$$\vec{E}_2 = \frac{\vec{F}_2}{q_0} = k\frac{q_2}{|\vec{r}-\vec{r}_2|^3}(\vec{r}-\vec{r}_2)$$

Same way, electric field at point P due to charge q_n is

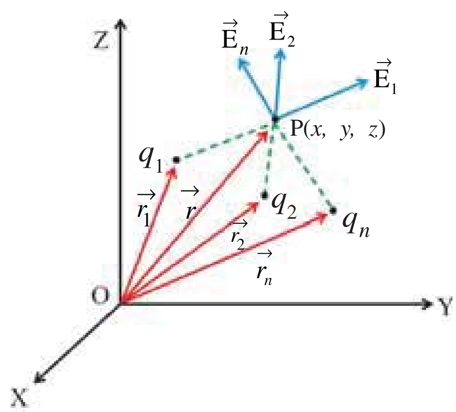


Figure 1.5 Superposition Principle for Electric Field

$$\vec{E}_n = \frac{\vec{F}_n}{q_0} = k\frac{q_n}{|\vec{r}-\vec{r}_n|^3}(\vec{r}-\vec{r}_n)$$

According to superposition principle, net electric field at a point P is.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n$$

$$= k\frac{q_1}{|\vec{r}-\vec{r}_1|^3}(\vec{r}-\vec{r}_1) + k\frac{q_2}{|\vec{r}-\vec{r}_2|^3}(\vec{r}-\vec{r}_2) + \ldots\ldots + k\frac{q_n}{|\vec{r}-\vec{r}_n|^3}(\vec{r}-\vec{r}_n)$$

$$\vec{E} = k\sum_{j=1}^{n}\frac{q_j}{|\vec{r}-\vec{r}_j|^3}(\vec{r}-\vec{r}_j) \qquad (1.5.2)$$

Here, q_1, q_2, ..., q_n are the sources of electric field.

The following points are noteworthy for an electric field :

(1) To determine the electric field there should not be any change in the original system of charges due to the presence of a test charge. So it is necessary that the test charge should be very small. To define electric field more precisely $q_0 \to 0$. But minimum value of q_0 is 1.6×10^{-19}C.

(2) Equation 1.5.2 indicates the force acting on unit positive charge at point $\vec{r}$ (x, y, z). Once $\vec{E}(\vec{r})$ is known, we do not have to worry about the source of electric field. In this sense, the electric field itself is a special representation of the system of charges producing electric field, as far as the effect on other charge is concerned. Once such a representation is done, the force acting on charge q kept at that point in the field can be determined using following equation.

$$\vec{F}(\vec{r}) = q\vec{E}(\vec{r}) \qquad (1.5.3)$$

(3) The direction of force acting on unit positive charge at a given point is the direction of electric field at that point.

(4) Faraday was the first person to introduce the concept of electric field. Electric field is not an imaginary concept but a physical reality.

1.6 Electric Field Due to a Point Charge

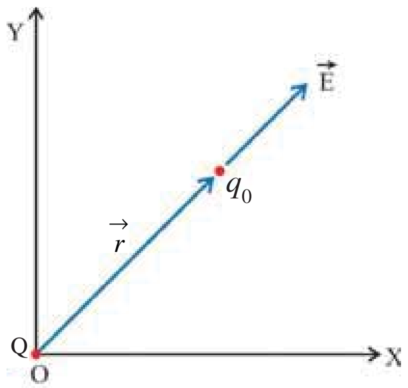


Figure 1.6 Electric Field Due to a Point Charge

As shown in figure 1.6, consider a point charge Q on the origin of a cartesian co-ordinate system.

In order to calculate electric field due to charge Q, consider a test charge q_0 at a distance r from the charge Q. Force acting on charge q due to Q is,

$$\vec{F} = k\frac{Qq_0}{r^2}\hat{r}$$

Therefore, electric field intensity at r due to Q will be,

$$\vec{E} = \frac{\vec{F}}{q_0} = k\frac{Q}{r^2}\hat{r} \qquad (1.6.1)$$

Figure 1.7 shows the electric field due to point charge in two dimensions using field vectors. From figure 1.7 it is clear that for positive charge ($Q > 0$), direction of field vectors are radially outward while those of a negative charge ($Q < 0$) are radially inward. The length of the arrow decreases, indicating the decreasing strength of the electric field, as we go away from the charge.

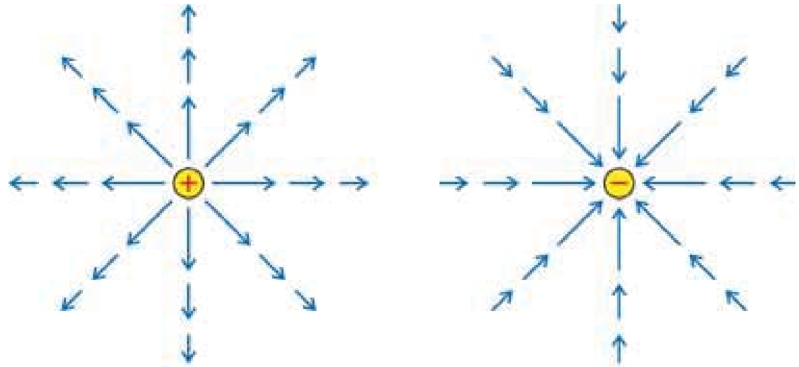
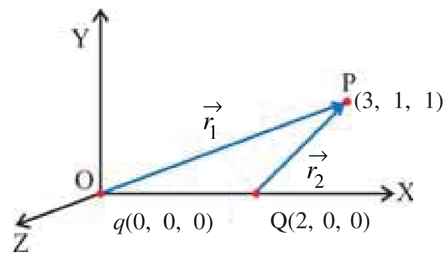


Figure 1.7 Electric Field of a Point Charge

Illustration 7 : A charge $+10^{-9}$C is located at the origin of cartesian co-ordinate system and another charge Q at (2, 0, 0)m. If X-component of electric field at (3, 1, 1)m is zero, calculate the value of Q.

Solution : As shown in the figure, Position vector of $q = 10^{-9}$C is (0, 0, 0) and position vector of Q is (2, 0, 0)m.



The co-ordinates of point P is (3, 1, 1) m.

$$\therefore \vec{r}_1 = (3, 1, 1) - (0, 0, 0) = (3, 1, 1)$$

$$= 3\hat{i} + \hat{j} + \hat{k}$$

$$|\vec{r}_1| = \sqrt{(3)^2 + (1)^2 + (1)^2} = \sqrt{11} \text{ m.}$$

$$\vec{r}_2 = (3, 1, 1) - (2, 0, 0) = (1, 1, 1) \text{ m}$$

$$= \hat{i} + \hat{j} + \hat{k}$$

$$|\vec{r}_2| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3} \text{ m.}$$

Electric field at point P, $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$= k\frac{q}{r_1^3}\vec{r}_1 + k\frac{Q}{r_2^3}\vec{r}_2 = k\left[\frac{10^{-9}(3\hat{i} + \hat{j} + \hat{k})}{(\sqrt{11})^3} + \frac{Q(\hat{i} + \hat{j} + \hat{k})}{(\sqrt{3})^3}\right]$$

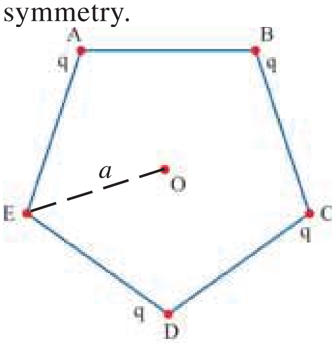
Now, x component of electric field is zero.

$$\therefore E_x = k\left[\frac{10^{-9} \times 3}{(11)^{\frac{3}{2}}} + \frac{Q}{(3)^{\frac{3}{2}}}\right] = 0$$

$$\therefore Q = -\left(\frac{3}{11}\right)^{\frac{3}{2}} \times 3 \times 10^{-9} = -0.43 \times 10^{-9} \text{ C.}$$

Illustration 8 : Four particles, each having a charge q, are placed on the four vertices of a regular pentagon. The distance of each corner from the centre is a. Find the electric field at the centre of the pentagon.

Solution : Let the charges be placed at the vertices A, B, C and D of the pentagon as shown in figure. If we put a charge q at the corner E also, the field at O will be zero by symmetry.



Therefore, $\vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D + \vec{E}_E = 0$

$\therefore \vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D = -\vec{E}_E$

Thus, the field at the centre due to charges at A, B, C and D is equal and opposite to the field due to the charges q at E alone.

The field at the centre due to the charge q at E is.

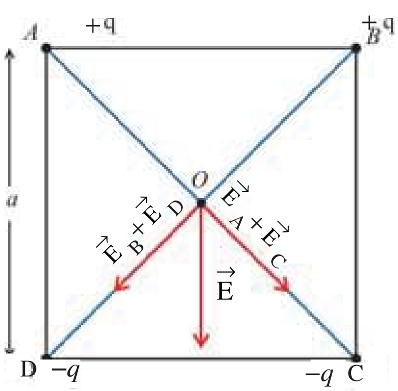
$$\vec{E}_E = k\frac{q}{a^2} \text{ (along EO).}$$

Thus, the field at O due to the charges on A, B, C and D is

$$\vec{E} = k\frac{q}{a^2} \text{ (along OE direction).}$$

Illustration 9 : Four electric charges $+q$, $+q$, $-q$ and $-q$ are respectively placed on the vertices A, B, C and D of a square. The length of the square is a, calculate the intensity of the resultant electric field at the centre.

Solution : All the electric charges are equidistant from the centre O of the square, hence the magnitude of intensity of electric field due to all the charges will be the same at point O. If r is the distance of vertices from the centre, we have,



$$E_A = E_B = E_C = E_D = \frac{kq}{r^2}$$

The directions of these electric field are as shown in figure.

If E' is the resultant field of E_A and E_C, then

$$E' = E_A + E_C = 2\frac{kq}{r^2} \quad (1)$$

In a similar way E'' is the resultant field of E_B and E_D.

$$E'' = E_B + E_D = 2\frac{kq}{r^2} \quad (2)$$

Resultant electric field, $\vec{E} = \vec{E}' + \vec{E}''$

$$\therefore E = \sqrt{(E')^2 + (E'')^2} \quad (\because \text{ Angle between } \vec{E}' \text{ and } \vec{E}'' \text{ is } 90^\circ)$$

$$= \sqrt{\left(2\frac{kq}{r^2}\right)^2 + \left(2\frac{kq}{r^2}\right)^2} \quad \text{(from equation (1) and (2))}$$

$$= \sqrt{\left(\frac{8k^2q^2}{r^4}\right)} = \left(\frac{2\sqrt{2}kq}{r^2}\right) \quad (3)$$

From the figure, $(2r)^2 = a^2 + a^2$

$$\therefore\ 2r = \sqrt{2a^2} \quad \therefore\ r = \frac{a}{\sqrt{2}}$$

Putting the value of r in equation (3),

$$E = \frac{2\sqrt{2}kq}{\left(\frac{a}{\sqrt{2}}\right)^2} = 4\sqrt{2}\frac{kq}{a^2}$$

The direction of E is parallel to AD (or BC).

Illustration 10 : An electron falls through a distance of 1.5 cm in a space, devoid of gravity, having uniform electric field of intensity 2.0×10^4 N C^{-1}. (Figure (a)). The direction of electric field intensity is then reversed keeping its magnitude same, in which a proton falls through the same distance. (Figure (b)). Calculate the time taken by both of them. $m_e = 9.1 \times 10^{-31}$ kg, $m_p = 1.7 \times 10^{-27}$ kg and $e = 1.6 \times 10^{-19}$ C.

Solution : As shown in the Figure (a), the direction of the electric field is vertically upward because of which the electron experiences a force eE in the vertically downward direction.

The acceleration of electron,

$$a_e = \frac{eE}{m_e}$$

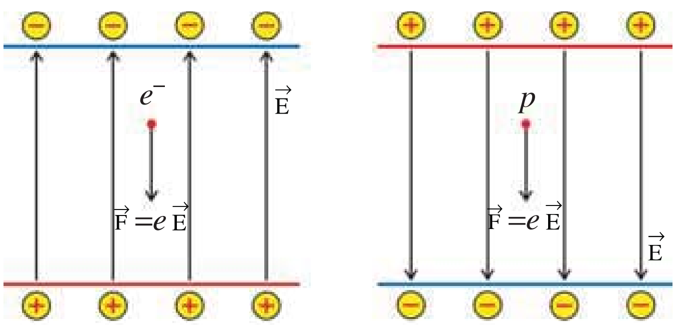


Figure (a) Figure (b)

From the equation of motion $d = v_0 t + \frac{1}{2}at^2$ (considering $v_0 = 0$) the time taken by electron to travel distance h.

$$t_e = \sqrt{\frac{2h}{a_e}} = \sqrt{\frac{2hm_e}{eE}}$$

Substituting the given data, we have $t_e = 2.9 \times 10^{-9}$s = 2.9 ns.

As shown in the figure (b) electric field is now in the vertically downward direction, the proton experiences the electric force eE in the vertically downward direction.

Therefore, acceleration of proton $a_p = \frac{eE}{m_p}$.

Therefore, the time taken by the proton to travel distance h is $t_p = \sqrt{\frac{2hm_p}{eE}}$

Substituting the given data $t_p = 1.3 \times 10^{-7}$s = 0.13μs (microsecond).

Hence, we can see that the time taken by a heavier particle is more than the time taken by a lighter particle having the same magnitude of charge in a uniform electric field.

(On the contrary, as we have studied in Standard 11, the time taken for free fall in gravitational field is independent of the mass.)

1.7 Electric Dipole

A system of two equal and opposite charges, separated by a finite distance is called an electric dipole.

As shown in figure 1.8, the two electric charges of electric dipoles are $+q$ and $-q$ and distance between them is $2a$. Electric dipole moment ($\vec{p}$) of the system can be defined as follows :

$$\vec{p} = q(2\vec{a}) \quad (1.7.1)$$

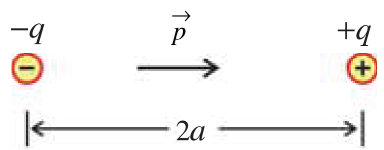


Figure 1.8 Electric Dipole

The SI unit of electric dipole is coulomb-meter (Cm). Electric dipole is a vector quantity and its direction is from the negative charge ($-q$) to positive charge ($+q$).

The net electric charge on an electric dipole is zero but its electric field is not zero, since the position of the two charges is different.

If lim $q \to \infty$ and $2a \to 0$ in $\vec{p} = 2\vec{a}q$, then the electric dipole is called a point dipole.

Electric field of a Dipole :

To find the electric field due to an electric dipole, placed the co-ordinate system such that its Z-axis coincides with the dipole and origin of system coincides with the centre of dipole. The separation between the charges of the dipole $+q$ and $-q$ is $2a$.

Here, we will determine the electric field at the point on the axis as well as point on the equator of a dipole.

Electric field at the point on the axis of a Dipole :

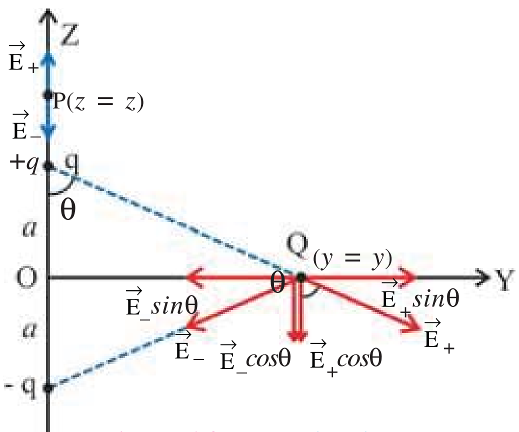


Figure 1.9 Electric Field at the Point on the Axis of a Dipole

As shown in figure 1.9, we want to determine the electric field at a point P on the axis of a dipole. Let the point P be a distance z from the origin. Hence, the distance of point P will be $z - a$ and $z + a$ from charges $+q$ and $-q$ respectively.

Electric field at point P due to charge $+q$ is,

$$\vec{E}_+ = k\frac{q}{(z-a)^2}\hat{p} \quad (1.7.2)$$

Where, $\hat{p}$ is the unit vector along the dipole axis from $-q$ to $+q$.

Now, Electric field at point P due to charge $-q$ is, $\vec{E}_- = -k\frac{q}{(z+a)^2}\hat{p}$ (1.7.3)

According to superposition principle, the net electric field at point P is,

$$\vec{E}(z) = \vec{E}_+ + \vec{E}_- = kq\left[\frac{1}{(z-a)^2} - \frac{1}{(z+a)^2}\right]\hat{p} = kq\frac{4za}{(z^2-a^2)^2}\hat{p}$$

$$\therefore \quad \vec{E}(z) = \frac{2kpz}{(z^2 - a^2)^2}\hat{p} \quad (\because \; 2aq = p) \tag{1.7.4}$$

If $z >> a$, then a^2 can be neglected in comparison with z^2.

$$\vec{E}(z) = \frac{2kp}{z^3}\hat{p} \quad (z >> a) \tag{1.7.5}$$

The direction of this resultant electric field is from O to P.

Electric Field at a Point on the Equator of a Dipole

The perpendicular bisector to the line joining the two electric charges of the dipole is called the equator of a dipole. Here, we want to determine the electric field at a point Q on the equator. Point Q is at a distance y from the centre of a dipole. The magnitude of electric field due to the two charges $+q$ and $-q$ will be same since they are at equal distance from point Q.

Magnitude of Electric field due to $+q$ is,

$$E_+ = k\frac{q}{(y^2 + a^2)} \tag{1.7.6}$$

Magnitude of Electric field due to $-q$ is,

$$E_- = k\frac{q}{(y^2 + a^2)} \tag{1.7.7}$$

The direction of $\vec{E}_+$ and $\vec{E}_-$ at point Q are shown in Figure 1.9.

The components of $\vec{E}_+$ and $\vec{E}_-$ normal to the dipole axis are $E_+ sin\theta$ and $E_- sin\theta$ respectively. These components cancelled each other, since they are of equal magnitude with opposite directions.

The components of $\vec{E}_+$ and $\vec{E}_-$ along the dipole axis are $E_+ cos\theta$ and $E_- cos\theta$ respectively. They will be added up since they are in the same direction.

The net electric field at point Q is opposite to $\hat{p}$ we have,

$$\vec{E}(y) = -(E_+ + E_-)cos\theta\,\hat{p}$$

$$= -\left(\frac{kq}{(y^2 + a^2)} + \frac{kq}{(y^2 + a^2)}\right)\left(\frac{a}{(y^2 + a^2)^{\frac{1}{2}}}\right)\hat{p} = -k\frac{(2aq)}{(y^2 + a^2)^{\frac{3}{2}}}\hat{p}$$

$$\therefore \quad \vec{E}(y) = -\frac{kp}{(y^2 + a^2)^{\frac{3}{2}}}\hat{p} \tag{1.7.8}$$

If $y >> a$ then, $\vec{E}(y) = -\frac{kp}{y^3}\hat{p} \qquad (y >> a)$ (1.7.9)

From the equations (1.7.5) and (1.7.9) it is clear that electric field of dipole at large distance falls of not as $\frac{1}{r^2}$ but as $\frac{1}{r^3}$.

Behaviour of an Electric Dipole in a Uniform Electric Field :

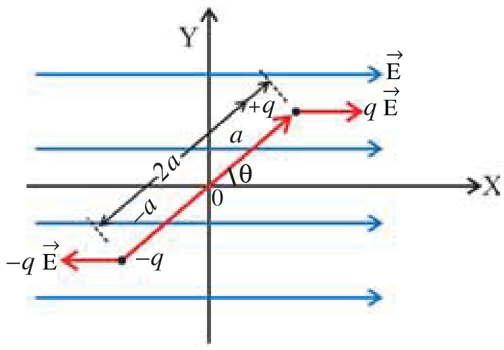


Figure 1.10 Electric Dipole in Uniform Electric Field

As shown in figure 1.10, an electric dipole of $|\vec{p}| = q|2\vec{a}|$ is kept in a uniform electric field $\vec{E}$. The origin O of co-ordinate system coincides with the centre of a dipole and electric field $\vec{E}$ is in positive X-axis. Suppose, at any instant, the angle between $\vec{p}$ and $\vec{E}$ is θ.

The forces $q\vec{E}$ and $-q\vec{E}$ are acting on $+q$ and $-q$ charges respectively. These forces are equal and in opposite direction. The resultant force being zero, keeps the dipole in translational equilibrium.

But, the two forces have different lines of action, hence the dipole will experience a torque. A torque acting on $+q$ with respect to point O, due to force $q\vec{E}$ is,

$$\vec{\tau}_1 = (\vec{a} \times q\vec{E}) \quad (1.7.10)$$

In a similar way, the torque acting on $-q$ with respect to point O due to force $-q\vec{E}$ is,

$$\vec{\tau}_2 = (-\vec{a}) \times (-q\vec{E}) = (\vec{a} \times q\vec{E}) \quad (1.7.11)$$

Here, $\vec{a}$ and $-\vec{a}$ are the position vectors of $+q$ and $-q$ respectively.

From the equation (1.7.10) and (1.7.11) the resultant torque acting on a dipole,

$$\vec{\tau} = (\vec{a} \times q\vec{E}) + (\vec{a} \times q\vec{E}) = 2\vec{a} \times q\vec{E} = 2\vec{a}q \times \vec{E}$$

$$\therefore \vec{\tau} = \vec{p} \times \vec{E} \quad (1.7.12)$$

Magnitude of torque, $|\vec{\tau}| = pE\sin\theta$ (1.7.13)

The direction of torque is perpendicular to the paper, going inside of it.

The torque rotates the dipole in such a way that the angle θ reduces (In this case dipole rotates in clockwise direction), when the dipole align itself along the direction of the electric field ($\theta = 0$), the torque becomes zero. This is the normal position of dipole in electric field. If the dipole is to be rotated by an angle θ from this position, work is required to be done against the torque. This work is equal to the change in the potential energy of the dipole.

Behaviour of electric dipole in non-uniform electric field :

If the electric field is non-uniform the intensity of electric field will be different at different points as a result the electric force acting on the positive charge and negative charge of the dipoles will also be different. In this situation both the net force and torque are acting on the dipole. As a result dipole experiences a linear displacement in addition to rotation. This rotation of dipole continues only till the dipole aligns in the direction of the electric field. But linear motion of the dipole will continue.

Our common experience is that when a dry comb is rubbed against dry hair, it attracts the small pieces of papers.

Here, the comb acquires negative charge through friction. But the paper is not charged, then why does paper attract by comb ?

The non-uniform electric field is produced by the charge on the comb. Electric dipole is induced along the direction of non-uniform electric field in the small pieces of papers. When charged comb is brought near to the small pieces of paper, this non-uniform electric field exerts a net force on the small pieces of paper and paper move in the direction of comb.

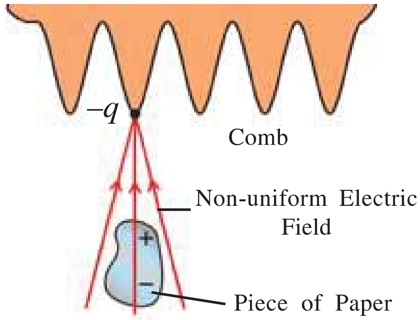


Figure 1.11 Electric Field of Comb

Illustration 11 : Calculate the magnitude of the torque on an electric dipole having dipole moment of 4×10^{-9} Cm placed in a uniform electric field of intensity of 5×10^{4} NC^{-1} making an angle 30^{o} with the field.

Solution : $p = 4 \times 10^{-9}$ Cm, E = 5×10^{4} NC^{-1}, $\theta = 30^{o}$, τ = ?

$\tau = pEsin\theta = (4 \times 10^{-9})(5 \times 10^{4})sin30^{o} = 10^{-4}$ Nm.

1.8 Continuous Distribution of Charges

We can determine the net electric force acting on a point charge due to the discrete charges in the space using Coulomb's law and superposition principle. But, in practical situation we need to work with the continuous charge distribution. For example, a continuous charge distribution on a surface. In this situation, it is difficult to describe the effect of these charges using superposition principle. Therefore, we use the concept of charge density to describe the system of continuous charge distribution. It is not necessary that charge density will be uniform in the system.

The continuous charge distribution of electric charge can be of three types :

(1) Linear charge distribution, (2) Surface charge distribution and (3) Volume charge distribution.

(1) Linear Charge Distribution : Consider a continuous charge distribution over a line as shown in figure 1.12. We want to determine the force acting on a charge q situated at point P due to this charge distribution.

Let the amount of charge per unit length of line be λ. It is called the linear charge density.

$$\lambda = \frac{\text{Total Charge on a Line}}{\text{Length of a Line}} = \frac{Q}{l}, \text{ Unit of } \lambda \text{ is Cm}^{-1}$$

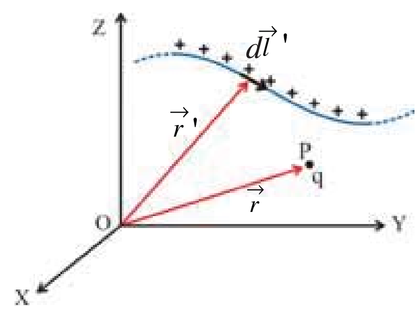


Figure 1.12 Linear Charge Distribution

If the charge distribution is not uniform, then λ will be different at different points on the line. In that case linear charge density is represented as $\lambda(\vec{r}\,')$ at a point on a line having position vector $\vec{r}\,'$.

Imagine the line to be divided into a large number of small segments of length $\vec{dl}'$. Such a line element $\vec{dl}'$ having position vector $\vec{r}\,'$ with respect to O is shown in figure 1.12. Hence, the charge in line element $\vec{dl}'$ will be,

$$dq = \lambda(\vec{r}\,')\,|\vec{dl}'| \tag{1.8.1}$$

The force acting on charge q having position vector $\vec{r}$ will be,

$$d\vec{F} = k\frac{(q)(dq)}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,') \quad (1.8.2)$$

In order to calculate total force acting on charge q we have to add the forces like $d\vec{F}$ due to all the line elements of entire linear charge distribution. If the line elements of the charges are distributed continuously then the sum results into integration.

Total force,

$$\vec{F} = \int_l d\vec{F} = \int_l \frac{k(q)(dq)}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')$$

$$\therefore \vec{F} = kq\int_l \frac{\lambda(\vec{r}\,')|d\vec{l}\,'|}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,') \text{ (from equation 1.8.2)} \quad (1.8.3)$$

If the charge q situated at point P is very small ($q \to 0$), then the intensity of electric field at that point due to linear charge distribution will be,

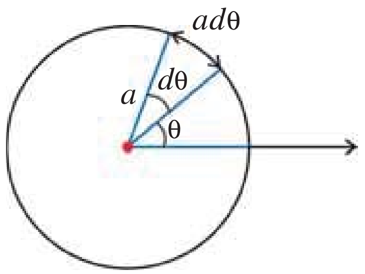
$$\vec{E} = \frac{\vec{F}}{q} = k\int_l \frac{\lambda(\vec{r}\,')|d\vec{l}\,'|}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,') \quad (1.8.4)$$

Illustration 12 : A circle, as shown in the figure, having radius 'a' has line charge distribution over its circumference having linear charge density $\lambda = \lambda_o cos^2\theta$. Calculate the total electric charge residing on the circumference of the circle. [**Hint :** $\int_0^{2\pi} cos^2\theta\, d\theta = \pi$]

Solution : The length of an infinitesimally small line element shown in the figure is $ad\theta$, then the charge on the line element is

$dq = \lambda ad\theta = \lambda_0 cos^2\theta\ ad\theta$

In order to calculate the total charge Q residing on the surface, we have to integrate dq over the entire surface.



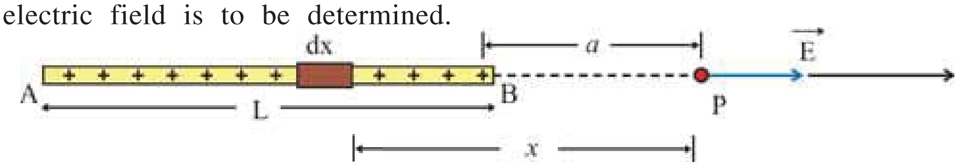
$$\therefore Q = \oint dq$$

Here the symbol $\oint$ indicates the integration over the entire closed path (circumference of the circle)

$$\therefore Q = \int_0^{2\pi} \lambda_0 cos^2\theta\, ad\theta = a\lambda_0 \int_0^{2\pi} cos^2\theta d\theta = \pi a\lambda_0$$

Illustration 13 : A conducting wire of length L carries a total charge q which is uniformly distributed on it. Find the electric field at a point located on the axis of the wire at a distance 'a' from the nearer end. (Neglect the thickness of a wire).

Solution : Consider a small element dx of the rod located at a distance x from point P where the electric field is to be determined.



The charge in this element will be, $dq = \frac{q}{L} dx$

Hence, magnitude of electric field at point P will be,

$$dE = k\frac{dq}{x^2} = k\frac{q}{L}\frac{dx}{x^2}$$

Now, electric field at P due to entire wire,

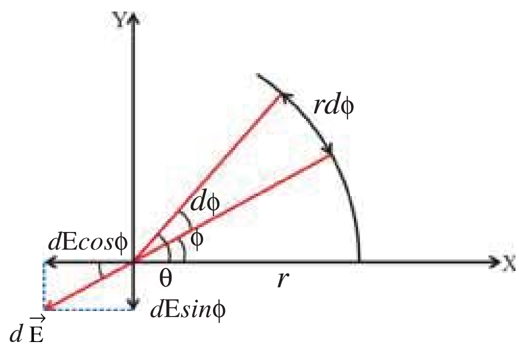
$$E = \int_a^{L+a} dE = \frac{kq}{L}\int_a^{L+a}\frac{dx}{x^2} = \frac{kq}{L}\left[-\frac{1}{x}\right]_a^{L+a}$$

$$= \frac{kq}{L}\left[-\frac{1}{L+a}+\frac{1}{a}\right] = k\frac{q}{a(L+a)} = \frac{1}{4\pi\varepsilon_0}\frac{q}{a(L+a)}$$

Note : If L << a, then $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$ which is same as electric field due to a point charge. If the charge q is positive, the direction of the field will be along the AP.

Illustration 14 : An arc of raidus r subtends an angle θ at the centre with the X-axis in a cartesian co-ordinate system. A charge is distributed over the arc such that the linear charge density is λ. Calculate the electric field at the origin.

Solution : The electric charge distributed on the portion of arc making an angle $d\phi$ is $d_q = \lambda r d\phi$.



The electric field at the origin due to this charge

$$dE = \frac{k\lambda r d\phi}{r^2}$$

The electric field vector $d\vec{E}$ is shown in the figure. Taking two components of $d\vec{E}$:

$$d\vec{E}_x = -\frac{k\cdot\lambda r d\phi}{r^2}\cos\phi\ \hat{i} \text{ and}$$

$$d\vec{E}_y = -\frac{k\cdot\lambda r d\phi}{r^2}\sin\phi\ \hat{j}$$

Now, $$\vec{E}_x = \int_0^{\theta} dE_x = -\frac{k\lambda}{r}\int_0^{\theta}\cos\phi\, d\phi\, \hat{i} = -\frac{k\lambda}{r}[\sin\phi]_0^{\theta}\ \hat{i}$$

$$\therefore \vec{E}_x = -\frac{k\lambda}{r}\sin\theta\,\hat{i} \qquad (1)$$

Now, $$\vec{E}_y = -\frac{k\lambda}{r}\int_0^{\theta}\sin\phi\, d\phi\,\hat{j} = -\frac{k\lambda}{r}[-\cos\phi]_0^{\theta}\ \hat{j}$$

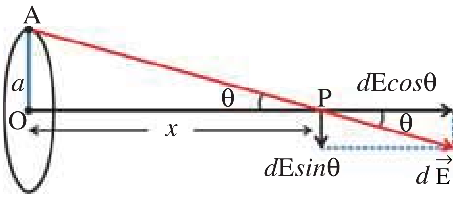
$$\therefore \vec{E}_y = -\frac{k\lambda}{r}[\cos\theta - 1]\,\hat{j} \qquad (2)$$

$$\vec{E} = \vec{E}_x + \vec{E}_y = -\frac{k\lambda}{r}\sin\theta\,\hat{i} + \frac{k\lambda}{r}(\cos\theta - 1)\,\hat{j}$$ (From equations (1) and (2))

$$\therefore \vec{E} = \frac{k\lambda}{r}[(-\sin\theta)\,\hat{i} + (\cos\theta - 1)\,\hat{j}]$$

Illustration 15 : A charge Q is uniformly distributed on the circumference of a circular ring of radius a. Find the intensity of electric field at a point at a distance x from the centre on the axis of the ring.

Solution : Given situation is depicted in the figure. Consider an infinitesimal element at point A on the circumference of the ring. Let charge on this element be dq. The magnitude of the intensity of electric field $d\vec{E}$, at a point P situated at a distance x from the centre on its axis is,



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{AP^2} = k\frac{dq}{(a^2+x^2)} \quad (1)$$

Its direction is from A to P. Now consider two components of $d\vec{E}$, (i) $dE\sin\theta$, perpendicular to the axis of the ring and (ii) $dE\cos\theta$, parallel to the axis.

Here it is clear that in the vector sum of intensities due to all such elements taken all over the circumference, the $dE\sin\theta$ components of the diametrically opposite elements will meet each other as they are mutually opposite. Hence only $dE\cos\theta$ components only should be considered for integration.

∴ The total intensity of electric field at point P.

$$E = \int dE\cos\theta = \int k\frac{dq}{(a^2+x^2)}\frac{OP}{AP} \qquad (\because \cos\theta = \frac{OP}{AP})$$

$$E = k\int\frac{dq}{(a^2+x^2)}\frac{x}{(a^2+x^2)^{\frac{1}{2}}} \qquad \text{(from equation 1)}$$

$$\therefore\ E = k\frac{x}{(a^2+x^2)^{\frac{3}{2}}}\int_{surface} dq = \frac{kxQ}{(a^2+x^2)^{\frac{3}{2}}} = \frac{1}{4\pi\varepsilon_0}\frac{xQ}{(a^2+x^2)^{\frac{3}{2}}}$$

(2) Surface Charge Distribution :

As shown in figure 1.13 suppose the charge is distributed continuously over a surface. We want to determine the force on the charge q placed at point P having position vector $\vec{r}$ due to these charge distribution.

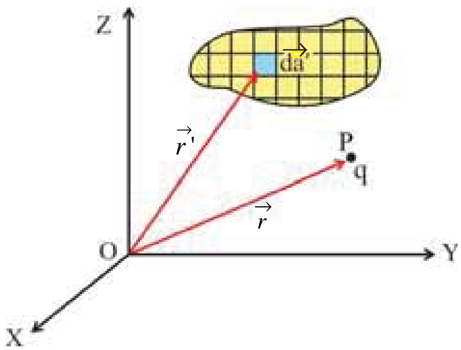


Figure 1.13
Surface Charge Distribution

Here, charge is distributed continuously over a surface having surface charge density $\sigma(\vec{r}\,')$.

Surface charge density is the charge per unit area.

$$\sigma = \frac{\text{Total Charge on the Surface}}{\text{Surface Area}} = \frac{Q}{A},\ \text{Unit of } \sigma \text{ is } Cm^{-2}.$$

Imagine the entire surface to be divided into large number of small surface element of $\vec{da}\,'$. The charge in area element $\vec{da}\,'$ will be,

$$dq = \sigma(\vec{r}\,')\,|\vec{da}\,'| \quad (1.8.5)$$

Force acting on charge q due to this charge (dq) will be,

$$d\vec{F} = k\frac{(q)(dq)}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,') \quad (1.8.6)$$

The total force on q due to charge on the surface can be determined by taking surface integration of above equation, From equation 1.8.5 and 1.8.6,

$$\vec{F} = \int_s d\vec{F} = kq\int_s \frac{\sigma(\vec{r}\,')|d\vec{a}\,'|}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,') \qquad (1.8.7)$$

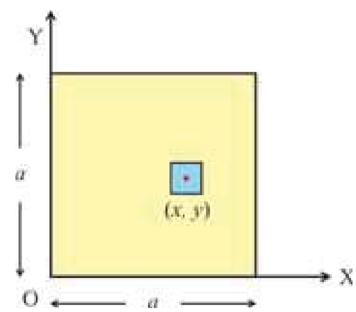
If the charge at point P is very small, then electric field at that point,

$$\vec{E} = \frac{\vec{F}}{q} = k\int_s \frac{\sigma(\vec{r}\,')|d\vec{a}\,'|}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')$$

Illustration 16 : As shown in the figure, a square having length a has electric charge distribution of surface charge density $\sigma = \sigma_0 xy$. Calculate the total electric charge on the square. The Cartesian co-ordinate system is shown in the figure.

Solution : As shown in the figure, consider an element of area $dxdy$ at a point (x, y). The charge on the area element is,

$dq = \sigma_0 xy \; dx \; dy$



∴ Therefore, the total electric charge on the surface,

$$Q = \sigma_0 \int_0^a x\,dx \cdot \int_0^a y\,dy = \sigma_0\left[\frac{x^2}{2}\right]_0^a \left[\frac{y^2}{2}\right]_0^a = \sigma_0\left(\frac{a^2}{2}\right)\left(\frac{a^2}{2}\right)$$

$$\therefore\ Q = \frac{\sigma_0 a^4}{4}$$

(3) Volume Charge Distribution :

As shown in figure 1.14, suppose electric charge is distributed continuously over some volume and volume charge density is $\rho(\vec{r}\,')$.

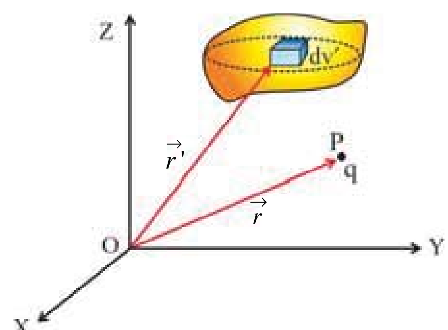


Figure 1.14 Volume Charge Distribution

Volume charge density is charge per unit volume.

$$\rho = \frac{\text{Total Charge}}{\text{Total Volume}} = \frac{Q}{V}, \text{ Unit of } \rho \text{ is } Cm^{-3}.$$

Imagine the entire volume divided into small volume elements dV'. The charge in this volume element will be,

$dq = \rho(\vec{r}\,')dV'$

Force acting on the charge q at point P having position vector $\vec{r}$ due to charge dq will be $d\vec{F} = k\dfrac{k(q)(dq)}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')$

As explained earlier, the total force acting on charge q can be determined by taking volume integration of above equation.

$$\vec{F} = \int_V d\vec{F} = kq\int_V \frac{\rho(\vec{r}\,')dV'}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')$$

If charge q is very small, then electric field at point P will be,

$$\vec{E} = \frac{\vec{F}}{q} = k\int_V \frac{\rho(\vec{r}\,')dV'}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')$$

1.9 Electric Field Lines

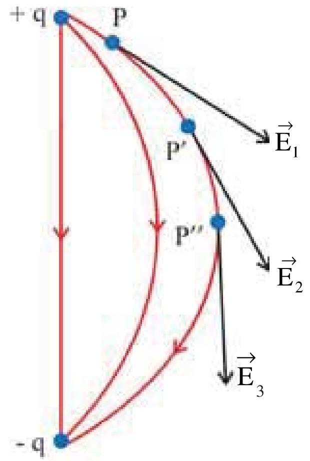


Figure 1.15 To Draw Electric Field Line of an Electric Dipole

Electric field lines are the pictorial representation of the electric field produced by the electric charge. Scientist Michael Faraday introduced the concept of electric field lines and obtained important results of electric field. (Faraday called these electric field of lines as lines of electric force.)

An electric field line is a curve drawn in an electric field in such a way that the tangent to the curve at any point is in the direction of net electric field at that point.

In fact, an electric line of field is the path along which a positive charge would move if free to do so.

Now we consider an example of electric dipole to understand the method of drawing electric field lines.

We can use the equation of electric field to determine the intensity of electric field at any point. As shown in figure 1.15 draw a vector of electric field ($\vec{E}_1$) at a point P, according to magnitude and direction at electric field at that point. Now consider another point P' close to P and draw a vector of electric field $\vec{E}_2$ at that point according to its magnitude and direction. Similarly draw a vector $\vec{E}_3$ at point P'', very close to P'. Same way other vectors of electric field can be drawn.

P, P', P'' all these points are so close to each other that a continuous curve passes through the tails of these vectors can be drawn. This curve represents the **electric field line.** Thus the curve on which the tangents drawn at different points like P, P', P'' ... and so on, represent the direction of the electric field at the respective points, is called the **electric field line.**

Characteristics of Electric Field Lines :

(1) Electric field lines start from positive charges and end at negative charges.

(2) The tangent drawn at any points on the electric field lines shows the direction of electric field at that point.

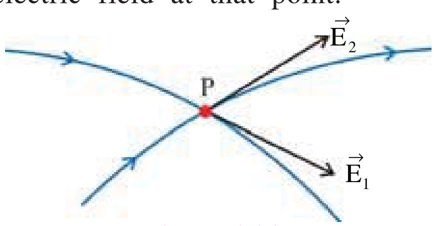


Figure 1.16

(3) Two field lines never cross each other.

If two lines intersect at a point, two tangents can be drawn at that point indicating two directions of electric field at that point which is not possible.

(4) Electric field lines of stationary electric charge distribution do not form closed loops.

(5) The separation of neighbouring field lines in a region at electric field indicates the strength of electric field in that region.

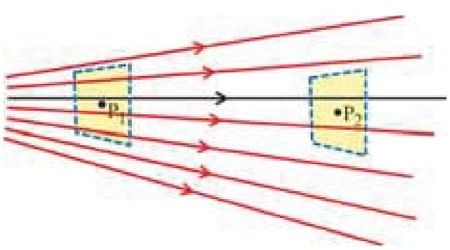


Figure 1.17 Intensity of Electric Field

In practice, the number of field lines are so restricted that the number of field lines passing through unit cross sectional area about a point, kept perpendicular to electric field lines is proportional to the intensity of electric field at that point. If the field lines are close to each other, the electric field in that region is relatively strong, if the field lines are far apart, the field is weak in that region.

From the figure 1.17, it is clear that at point P_1 electric field is relatively strong than electric field at point P_2.

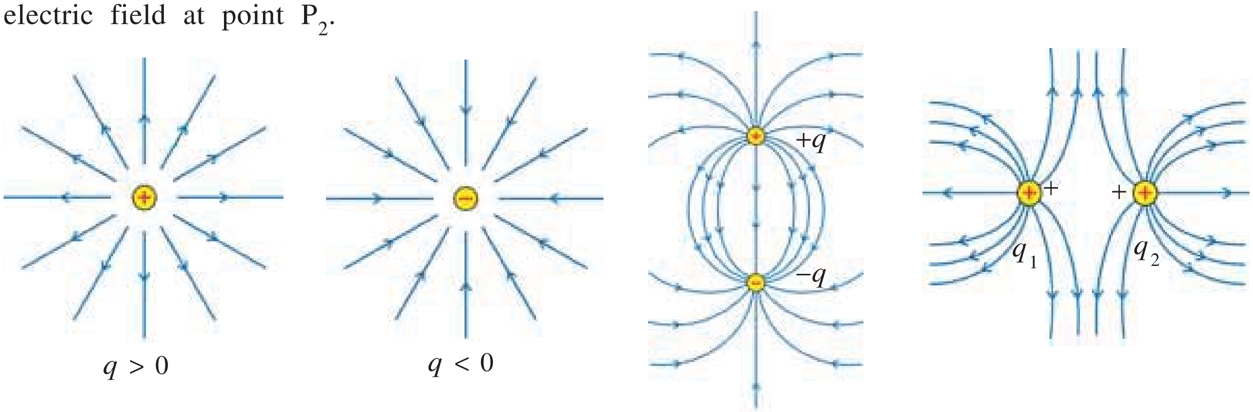


Figure 1.18 Electric Field Lines of Some Systems of Charges

(6) Field lines of uniform electric field are mutually parallel and equidistant.

Note : The electric field lines are geometrical representation of electric field and are not real. But electric field is a reality.

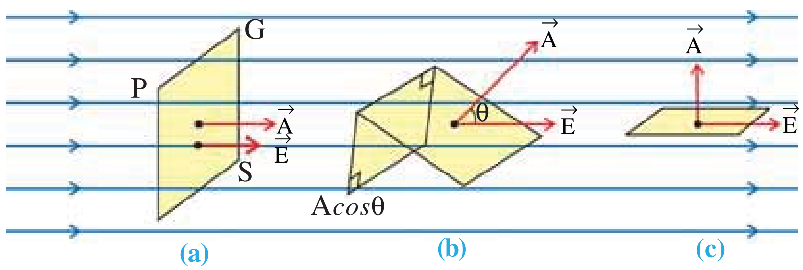
Figure 1.18 shows the electric field lines of some systems of charges.

Here, the field lines are drawn in a plane but are actually they are in three dimensional space.

1.10 Electric Flux

Coulomb's law is the fundamental law in electrostatics, we can apply Coulomb's law to find electric field at any point. Another equivalent of Coulomb's law is Gauss's law. Gauss's law is useful to determine the electric field of the system of charges having symmetry. Before we discuss Gauss's law we discuss the concept of electric flux.

The concepts of electric flux relates the electric field with its source. Flux is simply a mathematical concept which can be interpreted physically. Flux is a characteristics of all types of the vector fields.



$\theta = 0,\ \phi = EA$ $\quad \phi = EA\cos\theta$ $\quad \theta = 90^o,\ \phi = 0$

Figure 1.19 Electric Flux for Uniform Electric Field

Electric flux is quantity proportional to the number of electric field lines passing through surface. (Here we use the word proportional because the number of lines we choose to draw arbitrary.) Consider a surface of area $\vec{A}$ placed in a uniform electric field $\vec{E}$. Surface $\vec{A}$ is perpendicular to $\vec{E}$ as shown in figure 1.19. Area $\vec{A}$ is vector quantity and its direction is along the outward drawn normal to the area. Here, area vector $\vec{A}$ and $\vec{E}$ both are in the same direction.

Electric field can also be defined in terms of the electric field lines. Electric field at any point is the number of electric field lines are passing through a surface of unit area placed perpendicular to the electric field at that point. Therefore, the number of lines passing through surface of area A will be EA. This is an electric flux ϕ associated with the given surface.

Thus, electric flux is the number of lines passing through the surface. It is represented as ϕ.

$$\therefore \ \phi = EA \qquad (1.10.1)$$

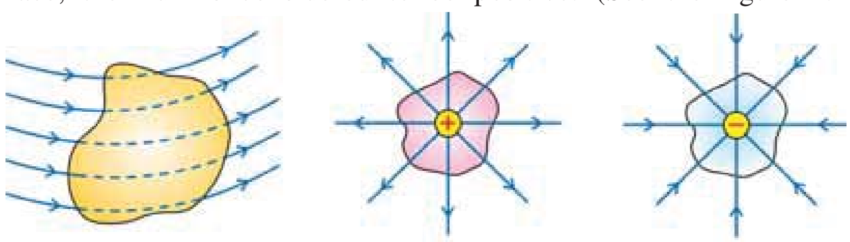
If the surface under the consideration is not perpendicular to the field, the number of lines passing through it must be less. As shown in figure 1.19(b), if the surface of area $\vec{A}$ is making an angle θ with the direction of electric field $\vec{E}$, then to determine the electric flux linked with the surface, we have to consider the $A\cos\theta$ component of the $\vec{A}$ parallel (or anti-parallel) to the electric field. Hence, Electric flux linked with the surface is,

$$\phi = EA\cos\theta \qquad (1.10.2)$$

In the vector form,

$$\phi = \vec{E} \cdot \vec{A} \qquad (1.10.3)$$

Electric flux is a scalar quantity. Its SI unit is $Nm^2 C^{-1}$ or V m. From equation 1.10.2, it is clear that flux can be positive, negative or zero. If the surface is parallel to the electric field then, $\vec{A} \perp \vec{E}$. Hence, the flux linked with surface will be $\phi = EA\cos 90^o = 0$. For $\theta < 90^o$, flux is positive and for $\theta > 90^o$, it is negative. If the field lines are entering in the close surface, then flux linked with this surface is considered to be negative and if the field lines are leaving the surface, the flux is considered to be positive. (See the figure 1.20).



$\phi = 0$ $\qquad$ $\phi > 0$ (Positive Flux) $\qquad$ $\phi < 0$ (Negative Flux)

Figure 1.20 **Electric flux**

Now, we will discuss the general definition of electric flux.

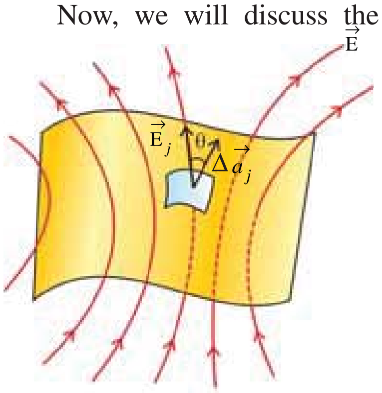


Figure 1.21 **Electric Flux Linked with the Surface Placed in Non-uniform Electric Field**

As shown in figure 1.21, consider an arbitrary surface in the electric field. Divide this imaginary surface into small surface elements. If the element is infinitesimally small and surface is not highly irregular, each surface element can be considered as a plane. In such a small element we can consider an electric field to be uniform. Each of these small surface element can be represented by an area vector. The magnitude of this vector should be equal to the area of surface element and direction is along the normal to the surface. If the surface is closed, i.e. surface enclosed the volume, then such vectors are drawn in outward direction of closed surface.

Suppose the vector $\Delta\vec{a}_j$ is an area vector of j^{th} element and the electric field at this element is $\vec{E}_j$. As the area of this element is very small the electric field does not change appreciably at all the points over the element. Hence, the electric flux associated with the j^{th} surface element will be.

$$\phi_j = \vec{E}_j \cdot \Delta\vec{\phi}_j \qquad (1.10.4)$$

Total flux ϕ linked with the surface can be determined by adding the flux associated with all such elements.

$$\phi = \sum_j \vec{E}_j \cdot \Delta\vec{\phi}_j \qquad (1.10.5)$$

Taking $\lim |\Delta \vec{a}_j| \to 0$, i.e. considering each element as small as possible, the summation taken in equation 1.10.5 can be written as integration.

$$\phi = \lim_{|\Delta \vec{a}_j| \to 0} \sum_j \vec{E}_j \cdot \Delta\vec{\phi}_j$$

$$\phi = \int_{\text{surface}} \vec{E} \cdot d\vec{a} \qquad (1.10.6)$$

Equation 1.10.6 is known as the surface integration of $\vec{E}$ over surface a.

Thus, the general definition of electric flux can be given as follows :

'The flux lined with any surface is the surface integration of the electric field over the given surface.'

1.11 Gauss's Law

The integration of electric field over a closed surface which enclosed the charges, leads us towards the Gauss's law. Gauss's law is one of the fundamental laws of nature. To understand this law, consider the following example :

Consider a point charge $+q$ located at the centre O of the sphere of raidus r. See figure 1.22. Now, we will determine the total flux linked with the surface of a sphere.

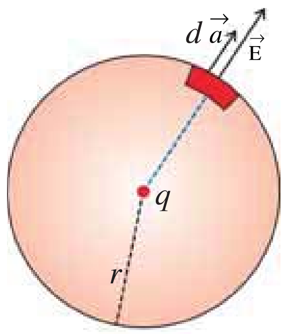


Figure 1.22 Flux Associated with the Sphere

According to definition of flux, total flux linked with the surface,

$$\phi = \int_s \vec{E} \cdot d\vec{a} = \int_s E\, da\, cos\theta \qquad (1.11.1)$$

All the points on the surface are at equidistant from the centre, hence magnitude of $\vec{E}$ will be same at every points at the surface. The electric field due to a point charge is radially outward.

Hence, area vector $\vec{da}$ of each surface element will be along the direction of $\vec{E}$ ($\theta = 0$). From equation 1.11.1

$$\phi = \int_s E\, da \quad (\because cos 0 = 1)$$

$$= E \int da = \frac{q}{4\pi\varepsilon_0 r^2} \times 4\pi r^2 \quad \text{(Area of the surface of sphere is } 4\pi r^2\text{)}$$

$$\phi = \frac{q}{\varepsilon_0} \qquad (1.11.2)$$

Here, the flux is independent of the radius of the sphere; hence it is true for any closed surface. Equation (1.11.2) is the general result of Gauss's law. Gauss's law statement is as follows, we will accept it without proof.

Guass's Law : The total electric flux associated with any closed surface is equal to the ratio of the net electric charge enclosed by the surface to ε_0.

Flux associated with any closed surface, $\phi = \int_s \vec{E} \cdot \vec{da} = \frac{\Sigma q}{\varepsilon_0}$ (1.11.3)

The law implies that the total electric flux through a closed surface is zero if no charge is enclosed by the surface.

Let us note some important points regarding this law :

(1) Gauss's Law is true for any closed surface, no matter what its shape or size.

(2) The term q on the right side of equation (1.11.3), includes the sum of all charges enclosed by the surface. The charges may be located anywhere inside the surface.

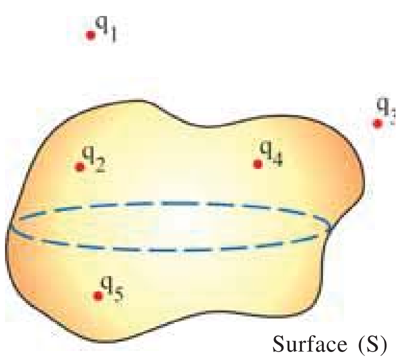


Figure 1.23

(3) The electric field appearing on the left hand side of equation 1.11.3 is the electric field produced due to a system of charges, whether enclosed by the surface or outside it.

As for example q_1, q_2, q_3, q_4 and q_5 as shown in figure 1.23. To calculate the electric flux passing through surface S, $\vec{E}$ is determined by taking the vector addition of the electric fields at the surface due to all the charges, which is used in the left side of the equation (1.11.3). But on the right side of the equation (1.11.3) we should consider the charges q_2, q_4 and q_5 to calculate net charge Σq.

Flux linked with the surface S,

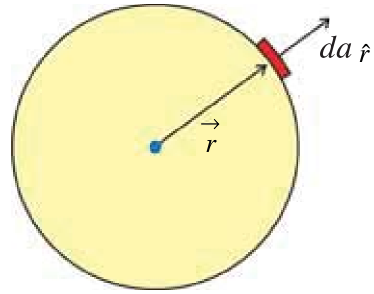
$$\phi = \frac{q_2 + q_4 + q_5}{\varepsilon_0}$$

(4) The surface that we choose for the application of Gauss's Law is called Gaussian surface.

(5) Gauss's Law is useful towards a much easier calculation of electric field when system has some symmetry.

Illustration 17 : An electric field prevailing in a region depends only on x and y co-ordinates according to an equation, $\vec{E} = b\frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ where b is a constant. Find the flux passing through a sphere of radius r whose centre is on the origin of the co-ordinate system.

Solution : As shown in the figure, $\hat{r}$ is the unit vector in the direction of $\vec{r}$.



$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{r}$ Now, $\vec{E} = b\frac{x\hat{i}+y\hat{j}}{x^2+y^2}$

$$\therefore \vec{E}\cdot d\vec{a} = b\left(\frac{x\hat{i}+y\hat{j}}{x^2+y^2}\right)\cdot\frac{x\hat{i}+y\hat{j}+z\hat{k}}{r}da = \frac{bda}{r}\frac{x^2+y^2}{x^2+y^2} = \frac{b}{r}da$$

$$\therefore \int\vec{E}\cdot d\vec{a} = \frac{b}{r}\int da = \frac{b}{r}.4\pi r^2 = 4\pi br$$

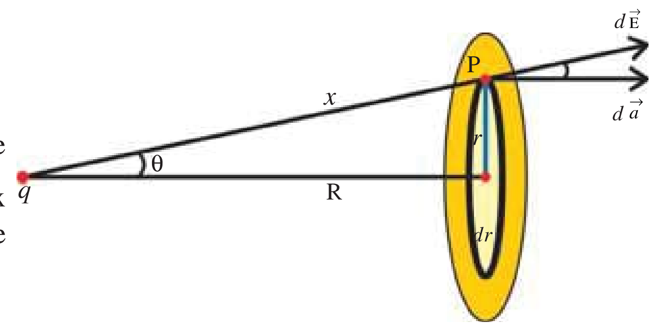
$\therefore \phi = 4\pi br$

Illustration 18 : Calculate the total electric flux linked with a circular disc of radius a, situated at a distance R from a point charge q.

[**Hint :** $\int\frac{rdr}{(R^2+r^2)^{\frac{3}{2}}} = -\frac{1}{\sqrt{(R^2+r^2)}}$]

Solution : Consider a thin circular ring of radius r and width dr as shown in figure. The electric field intensity at some point P on the ring is given by,

$|d\vec{E}| = \frac{kq}{x^2}$



The area of the ring is $|d\vec{a}| = 2\pi r dr$.

$d\vec{a}$ is perpendicular to the plane of the ring and makes an angle θ with $d\vec{E}$. The flux passing through the small area element of the disc is given by,

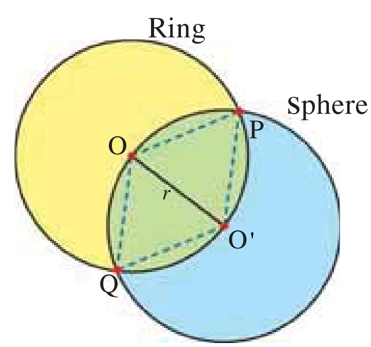
$d\phi = |d\vec{E}|\ |d\vec{a}|cos\theta$

$$= \frac{kq}{x^2}\times 2\pi rdr\times\frac{R}{x} = 2\pi kqR\times\frac{rdr}{x^3} = 2\pi kqR\times\frac{rdr}{(R^2+r^2)^{\frac{3}{2}}} \quad (\because x^2 = R^2 + r^2)$$

$$\therefore \text{Total flux } \phi = 2\pi kqR\int_0^a\frac{rdr}{(R^2+r^2)^{\frac{3}{2}}} = 2\pi kqR\left[-\frac{1}{\sqrt{(R^2+r^2)}}\right]_0^a = 2\pi kqR\left[\frac{1}{R}-\frac{1}{\sqrt{(R^2+a^2)}}\right]$$

Illustration 19 : Q amount of electric charge is uniformly distributed on a ring of radius r. A sphere of radius r is drawn in such a way that the centre of the sphere lies on the surface of the ring. Calculate the electric flux associated with the surface of the sphere.

Solution : It is evident from the geometry of the sphere that OP = OO' and O'P = O'O. Hence, ΔOPO' is an equilateral triangle.



$\therefore$ ∠POO' = 60^o or ∠POQ = 120^o

Hence, the chord PO'Q of the ring will subtend an angle 120^o at its centre. Hence, it is evident that the length of the chord will be equal to one third of the circumference of the ring. The total charge residing on this chord (enclosed by sphere) PO'Q will be equal to $\frac{Q}{3}$.

From Gauss's Law, the total flux passing through surface of the sphere is equal to $\frac{Q}{3\varepsilon_0}$.

1.12 Applications of Gauss's Law

The electric field of any symmetric charge distribution can be easily determined by using Gauss's Law. Let us consider some examples.

(1) Electric Field Due to an Infinitely Long Straight Uniformly Charged Wire :

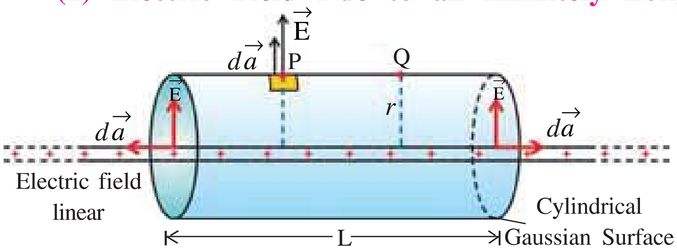


Figure 1.24 Infinitely Long Wire having Linear Charge Distribution

An infinitely long and linear charge distribution having uniform linear charge density λ is shown in figure 1.24. We want to find the intensity of electric field at point P situated at a perpendicular distance r from the linear charge distribution.

Since the wire is of infinite length, the electric field at all points line P, Q, ... situated at the same perpendicular distance from the wire will be same.

Now imagine a cylindrical Gaussian surface of radius r and length L, whose axis coincides with the line of linear charge distribution. At all the points at such a cylindrical surface the electric field is same and directed radially outward. The area of this curved surface of cylinder is $2\pi r L$ and cross sectional area is πr^2. The charge enclosed by the cylinder of length L is $q = \lambda L$.

As shown in figure, electric flux associated with cylindrical surface of radius r and length L is,

$$\phi_1 = \int \vec{E} \cdot d\vec{a} = \int E\, da \cos 0 = E \int da$$

$$\therefore \phi_1 = E(2\pi r L) \qquad (1.12.1)$$

Now, the flux associated with the two end sides of cylinder perpendicular to axis is,

$$\phi_2 = \int \vec{E} \cdot d\vec{a} = \int E\, da \cos 90^\circ = 0$$

$$\therefore \text{Total flux, } \phi = \phi_1 + \phi_2 = (2\pi r L)E$$

According to Gauss's Law, $\phi = (2\pi r L)E = \dfrac{q}{\varepsilon_0} \Rightarrow (2\pi r L)E = \dfrac{\lambda L}{\varepsilon_0}$

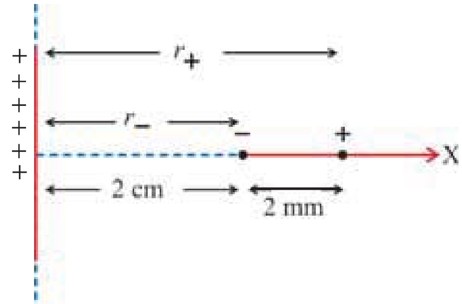
$$\therefore E = \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{r} \qquad (1.12.3)$$

Electric field is in the direction of raidus, hence taking $\hat{r}$ as the unit vector in the direction of radius.

$$\therefore \vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{r} \hat{r} \qquad (1.12.4)$$

Illustration 20 : An electric dipole is prepared by taking two electric charges of 2×10^{-8}C separated by distance 2 mm. This dipole is kept near a line charge distribution having density 4×10^{-4}C/m in such a way that the negative electric charge of the dipole is at a distance 2 cm from the wire as shown in the figure. Calculate the force acting on the dipole. Take $k = 9 \times 10^9$ Nm^2C^{-2}.

Solution : The electric field intensity at some point r from continuous line charge distribution having density λ is given by the following formula.



From $E = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r} = \frac{2k\lambda}{r}$, $\vec{F}_- = \frac{-2k\lambda q}{r_-}\hat{i}$ and $\vec{F}_+ = \frac{2k\lambda q}{r_+}\hat{i}$

$\therefore$ Resultant force, $\vec{F} = \vec{F}_+ + \vec{F}_- = 2k\lambda q\left[\frac{1}{r_+} - \frac{1}{r_-}\right]\hat{i}$

$$= 2 \times 9 \times 10^9 \times 4 \times 10^{-4} \times 2 \times 10^{-8}\left[\frac{1}{2.2\times10^{-2}} - \frac{1}{2.0\times10^{-2}}\right]\hat{i}$$

$$= -0.65\,\hat{i}\ \text{N}$$

(2) Electric Field Due to a Uniformly Charged Infinite Plane Sheet :

As shown in Figure 1.25, we want to find the electric field at point P situated at a perpendicular distance r from the infinite plane sheet of uniform surface charge density σ.

(The figure shows only a small part at the infinite plane sheet.)

It can be inferred from the symmetry that on either side and equidistant from the plane, points like P and P' the magnitude of electric field will be same. But the direction of electric field at these two points are perpendicular to the plane and mutually opposite. (If the charge on the plane is positive / negative, the direction of the field will be away / towards the plane).

As shown in the figure 1.25 consider a close cylindrical Gaussian surface having cross-sectional area A and equal length on either side of the cylinder. The charge enclosed by the close cylinder is $q = \sigma A$ since the surface charge density of plane is σ.

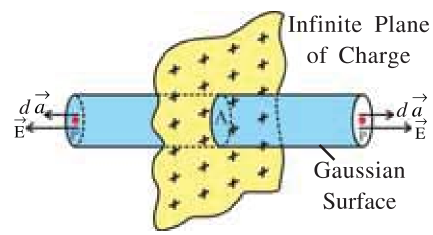


Figure 1.25 Electric Field due to Infinite Plane Sheet of Uniform Surface Charge Density

The flux linked with the curved surface of cylinder is

$$\phi_1 = \int \vec{E}\cdot d\vec{a} = \int E\, da \cos 90^\circ = 0 \qquad (1.12.5)$$

because for curved surface, $\vec{E}$ and $\vec{da}$ both are perpendicular to each other.

The flux linked with the surface of area A at the end of cylinder at point P is,

$$\phi_p = \int \vec{E}\cdot d\vec{a} = \int E\, da \cos 0 = \int E\, da = EA \qquad (1.12.6)$$

Same way, flux linked with surface area A at point P' is, $\phi_{p'} = EA$ (1.12.7)

Thus, total flux, $\phi = \phi_1 + \phi_p + \phi_{p'} = 0 + EA + EA = 2EA$

According to Gauss's Law, $\phi = 2EA = \frac{q}{\varepsilon_0}$

$\Rightarrow 2EA = \frac{\sigma A}{\varepsilon_0}$ $(\because\ q = \sigma A)$

$$\therefore E = \frac{\sigma}{2\varepsilon_0} \qquad (1.12.8)$$

This equation shows that electric field at a point is independent of the distance of the point from the plane.

Electric field in a vector form is represented as,

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n} \qquad (1.12.9)$$

Where, $\hat{n}$ is a unit vector normal to the plane and going away from it. If the charge on a plane is negative, then $\vec{E}$ is towards the plane and perpendicular to it.

Equation (1.12.8) is used to calculate the electric field intensity and direction of the electric field between two planes having surface charge density σ_1 and σ_2.

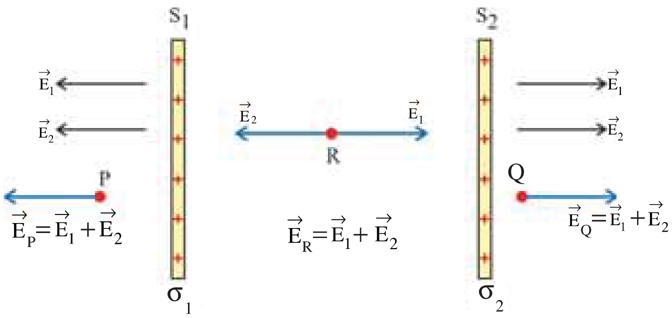


Figure 1.26

Two parallel planes S_1 and S_2 having surface charge density σ_1 and σ_2 respectively are shown in figure 1.26. $\vec{E}_1$ and $\vec{E}_2$ are the electric field produced due to the charge on the S_1 and S_2 respectively.

From the figure, electric field at point P,

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} = \frac{\sigma_1+\sigma_2}{2\varepsilon_0} \quad (\text{in } S_2S_1 \text{ direction})$$

Electric field at point Q,

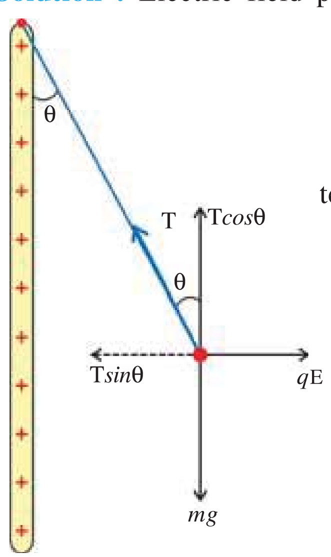
$$\vec{E}_Q = \vec{E}_1 + \vec{E}_2 = \frac{\sigma_1+\sigma_2}{2\varepsilon_0} \quad (\text{in } S_1S_2 \text{ direction})$$

Electric field at point R, (for $\sigma_1 > \sigma_2$)

$$\vec{E}_R = \vec{E}_1 + \vec{E}_2 = \frac{\sigma_1-\sigma_2}{2\varepsilon_0} \quad (\text{in } S_1S_2 \text{ direction}) \qquad (1.12.10)$$

Illustration 21 : A particle of mass *m* and charge *q* is attached to one end of a thread. The other end of the thread is attached to a large, vertical positively charged plate, having surface charge density σ. Find the angle the thread makes with the plate vertical in equilibrium.

Solution : Electric field produced due to positively charged plane is,



$$E = \frac{\sigma}{2\varepsilon_0}$$

The forces acting on the charge and components of the tension (T) produced in string are shown in the figure.

In the equilibrium,

$$T\cos\theta = mg \text{ and } T\sin\theta = qE$$

$$\therefore \tan\theta = \frac{qE}{mg} = \frac{q\sigma}{2mg\varepsilon_0}$$

$$\therefore \theta = \tan^{-1}\left(\frac{q\sigma}{2mg\varepsilon_0}\right)$$

(3) Electric Field Due to a Uniformly Charged Thin Spherical Shell :

Let σ be the surface charge density on a spherical shell having radius R, as shown in figure 1.27. Therefore, total charge on the shell,

$q = \sigma A = \sigma(4\pi R^2)$ (1.12.11)

The electric field produced from such a spherical shell is radial. We want to determine the electric field at points inside and outside the spherical shell.

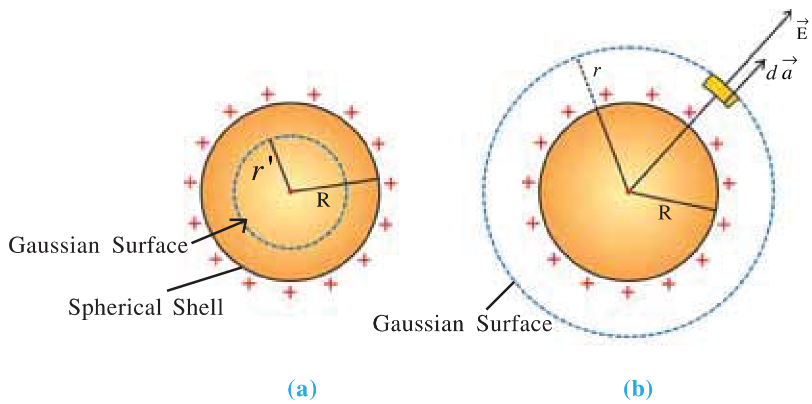


Figure 1.27 Electric Field of a Spherical Shell

(1) For a Point Lying Inside a Shell : Consider a spherical Gaussian surface of radius $r'(r' < R)$, concentric with the shell (See figure 1.27)

Since the charge enclosed by such a surface is zero then according to Gauss's Law,

$$\int \vec{E} \cdot d\vec{a} = \frac{q}{\varepsilon_0} = 0 \qquad (\because q = 0)$$

$$\therefore \vec{E} = 0 \qquad (1.12.12)$$

Thus, electric field inside the charged spherical shell is zero.

(2) For a Point Lying Outside the Shell :

To determine electric field outside the shell, consider a spherical Gaussian surface of radius $r(r > R)$. (See figure 1.27 (b))

According to Gauss's Law, flux linked with this surface,

$$\int \vec{E} \cdot d\vec{a} = \frac{q}{\varepsilon_0}$$

$$\int E\, da \cos 0 = \frac{q}{\varepsilon_0} \qquad (\because \vec{E} \text{ and } d\vec{a} \text{ are in the same direction.})$$

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad (1.12.3)$$

For an electric field on the surface of a shell, put $r = R$.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} \qquad (1.12.4)$$

Equations 1.12.3 and 1.12.4 shows that for an electric field outside the sphere the entire charge on a shell can be treated as concentrated at its centre.

Putting, $q = (4\pi R^2)\sigma$ in equation 1.12.4

$$E = \frac{1}{4\pi\varepsilon_0}\frac{(4\pi R^2)\sigma}{r^2}$$

$$\therefore\ E = \frac{\sigma}{\varepsilon_0}\cdot\frac{R^2}{r^2} \qquad (1.12.5)$$

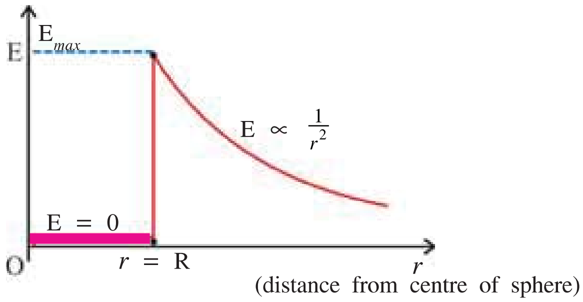


Figure 1.28 Electric Field of a Spherical Shell

Figure 1.28 shows the variation of electric field intensity with distance from the centre O to the region outside the uniformly charged spherical shell. Inside the shell $\vec{E} = 0$. The magnitude of E is maximum on the surface (r = R). However, outside the shell electric field decreases) according to $\frac{1}{r^2}$.

(4) Electric Field Intensity Due to Uniformly Charged Sphere :

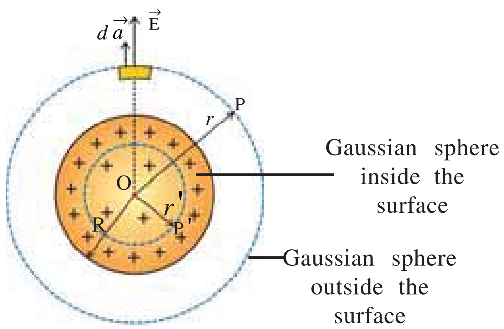


Figure 1.29 Electric Field due to Uniformly Charged Sphere

Let ρ be the volume charge density of a charged sphere having radius R as shwon in figure 1.29. The charge inside the sphere is

$$q = (\tfrac{4}{3}\pi R^3)\rho. \qquad (1.12.6)$$

The electric field due to such a sphere is radial. We want to determine the electric field at points inside and outside for such a charge sphere.

(1) For Point Lying Inside the Sphere : Imagine spherical Gaussian surface of radius r' (where $r' < R$) concentric with sphere to determine the electric field at a distance r' (point P') from the centre of sphere. The charge enclosed by such a sphere is.

$$q' = (\tfrac{4}{3}\pi r'^3)\rho \qquad (1.12.7)$$

$$= \frac{4}{3}\pi r'^3 \times \frac{q}{\frac{4}{3}\pi R^3} \qquad \text{(From equation 1.12.6)}$$

$$\therefore\ q' = q\frac{r'^3}{R^3} \qquad (1.12.8)$$

The flux linked with the Gaussian surface.

$$\int \vec{E}\cdot d\vec{a} = \frac{q'}{\varepsilon_0}$$

$$E(4\pi r'^2) = \frac{q\,r'^3}{\varepsilon_0 R^3} \qquad \text{(From equation 1.12.8)}$$

$$\therefore \; E = \frac{q}{4\pi\varepsilon_0}\frac{r'}{R^3} \qquad (\text{for } r' \le R) \qquad (1.12.9)$$

i.e. Inside the sphere, $E \propto r'$

By putting the value of q from equation 1.12.6, we can represent electric field in terms of charge density.

$$E = \frac{\rho r'}{3\varepsilon_0} \qquad (\text{for } r' \le R) \qquad (1.12.10)$$

(2) For Point Lying Outside the Sphere : Now, consider a Gaussian surface of radius r (where $r > R$). The centres of two spheres coincide with each other. The charge enclosed by this surface is q. According to Gaussian's Law.

$$\int \vec{E} \cdot d\vec{a} = \frac{q}{\varepsilon_0}$$

$$\int E\, da = \frac{q}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\therefore \; E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad (\text{for } r \ge R) \qquad (1.12.11)$$

This shows that a point outside the sphere the entire charge of the sphere can be considered as concentrated at its centre. Thus, for a point outside the sphere, $E \propto \frac{1}{r^2}$.

In above equation put $q = (\frac{4}{3}\pi R^3)\rho$,

We can have electric field in terms of ρ.

$$E = \frac{R^3\rho}{3r^2\varepsilon_0} \qquad (1.12.12)$$

Figure 1.30 shows the variation of electric field intensity with distance r from the centre O to the region outside the charged sphere. Note that electric field on a surface of sphere is maximum ($E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$)

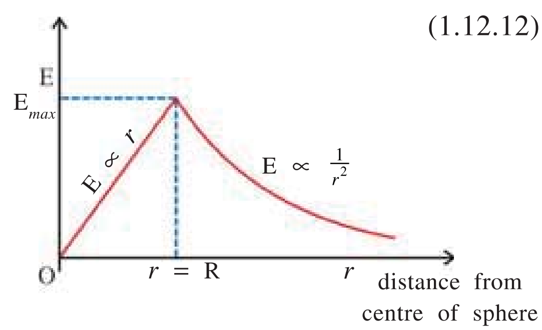


Figure 1.30 Electric Field of a Charged Sphere

SUMMARY

1. **Electric Charge :** Just as masses of two particles are responsible for the gravitational force, charges are responsible for the electric force. Electric charge is an intrinsic property of a particle.

   Charges are of two types : (1) Positive charge (2) Negative charge.

   The force acting between two like charges is repulsive and it is attractive between two unlike charges.
   The SI unit of charge is coulomb (C).

2. **Quantization of Electric Charge :** The magnitude of all charges found in nature are in integral multiple of a fundamental charge. $Q = ne$ where, e is the fundamental unit of charge.
3. **Conservation of Electric Charge :** Irrespective of any process taking place, the algebraic sum of electric charges in an electrically isolated system always remains constant.
4. **Coulomb's Law :** The electric force between two stationary point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.

$$F = k\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

If $q_1 q_2 > 0$, then there is a repulsion between the two charges and for $q_2 q_2 < 0$ there is an attractive force between the charges.

5. **Electric Field Intensity :** The force acting on a unit positive charges at a given point in an electric field of a system of charges is called the electric field or the intensity of electric field ($\vec{E}$) at that point.

$$\vec{E} = \frac{\vec{F}}{q}$$

The SI unit of $\vec{E}$ is NC^{-1} or Vm^{-1}.

If $\vec{r_1}$, $\vec{r_2}$, ... $\vec{r_n}$ are the position vectors of the charge q_1, q_2, ... q_n respectively then neot electric field at a point of position vector $\vec{r}$ is,

$$\vec{E} = k\sum_{j=1}^{n}\frac{q_j}{|\vec{r}-\vec{r_j}|^3}(\vec{r}-\vec{r_j})$$

6. **Electric Dipole :** A system of two equal and opposite charges, separated by a finite distance is called an electric dipole.

Electric dipole moment $\vec{p} = (2\vec{a})q$

The direction of $\vec{p}$ is from the negative electric charge to the positive electric charge.

7. Electric field of a dipole on the axis of the dipole at point $z = z$,

$$\vec{E}(z) = \frac{2kp}{z^3}\hat{p} \qquad (\text{for } z >> a)$$

Electric field of a dipole on the equator of the dipole at point y = y.

$$\vec{E}(y) = -\frac{kp}{y^3}\hat{p} \qquad (\text{for } y >> a)$$

8. The torque acting on the dipole place in the electric field at an angle θ,

$$\vec{\tau} = \vec{p} \times \vec{E}, \; |\vec{\tau}| = pE\sin\theta$$

9. **Electric Flux :** Electric flux associated with surface of area $\vec{A}$, placed in the uniform electric field.

$\phi = \vec{E} \cdot \vec{A} = EA\cos\theta$

where θ is the angle between $\vec{E}$ and $\vec{A}$.

Its unit is Nm^2C^{-1} or Vm.

10. **Gauss's Law :** The total electric flux associated with the closed surface,

$$\phi = \int_s \vec{E} \cdot d\vec{a} = \frac{\Sigma q}{\varepsilon_0}$$

where Σq is the net charge enclosed by the surface.

11. Electric field due to an infinitely long straight charged wire,

$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{r} \hat{r}$, where, r is the perpendicular distance from the wire.

12. Electric field due to uniformly charged infinite plane, $E = \frac{\sigma}{2\varepsilon_0}$

13. Electric field due to uniformly charged thin spherical shell,

(1) Electric field inside the shell $\vec{E} = 0$.

(2) Electric field at a distance r from the centre outside the shell,

$E = k\frac{q}{r^2} = \frac{\sigma}{\varepsilon_0} \frac{R^2}{r^2}$, where R = radius of spherical shell.

14. Electric field due to a uniformly charged sphere of radius R,

(1) Electric field inside the region of the sphere :

$$E = \frac{Q}{4\pi\varepsilon_0} \frac{r}{R^3} = \frac{\rho r}{3\varepsilon_0}$$

(2) Electric field outside the sphere,

$$E(r) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2} = \frac{R^3\rho}{3r^2\varepsilon_0}$$

where, Q is the total charge inside the sphere.

EXERCISE

For the following statements choose the correct option from the given options :

1. The force acting between two point charges kept at a certain distance is ϕ. Now magnitudes of charges are doubled and distance between them is halved, the force acting between them is ....

(A) ϕ (B) 4ϕ (C) 8ϕ (D) 16ϕ

2. An electric dipole is placed in a uniform field. The resultant force acting on it ....

(A) always be zero (B) depends on its relative position

(C) never be zero (D) depends on its dipole moment.

3. An electric dipole is placed in an electric field of a point charge, then ....

(A) the resultant force acting on the dipole is always zero

(B) the resultant force acting on the dipole may be zero

(C) torque acting on it may be zero

(D) torque acting on it is always zero.

4. When an electron and a proton are both placed in an electric field ....

(A) the electric forces acting on them are equal in magnitude as well as direction.

(B) only the magnitudes of forces are same

(C) accelerations produced in them are same

(D) magnitudes of accelerations produced in them are same.

5. The electric force acting between two point charges kept at a certain distance in vacuum is α. If the same two charges are kept at the same distance in a medium of dielectric constant K. The electric force acting between them is ....

(A) α (B) $K\alpha$ (C) $K^2\alpha$ (D) α/K

6. The distance between two point charges $4q$ and $-q$ is r. A third charge Q is placed at their midpoint. The resultant force acting on $-q$ is zero then Q = ...

(A) $-q$ (B) q (C) $-4q$ (D) $4q$

7. The linear charge density on the circumference of a circle of radius 'a' varies as $\lambda = \lambda_0 cos\theta$. The total charge on it is ....

(A) zero (B) infinite (C) $\pi a\lambda_0$ (D) $2\pi a$

8. Two identical metal spheres A and B carry same charge q. When the two spheres are at distance r from each other, the force acting between them is F. Another identical sphere C is first brought in contact with A, then it is touched to sphere B and then separated from it. Now the force acting between A and B at the same distance is ....

(A) F (B) 2F (C) $\frac{3F}{8}$ (D) $\frac{F}{4}$

9. Two point charges of q and $4q$ are kept 30 cm apart. At a distance ...., on the straight line joining them, the intensity of electric field is zero.

(A) 20 cm from $4q$ (B) 7.5 cm from q

(C) 15 cm from $4q$ (D) 5 cm from q

10. The dimensions of permittivity $[\varepsilon_0]$ are .... .Take Q as the dimension of charge.

(A) $M^1L^{-2}T^{-2}Q^{-2}$ (B) $M^{-1}L^2T^{-3}Q^{-1}$ (C) $M^{-1}L^{-3}T^2Q^2$ (D) $M^{-1}L^3T^{-2}Q^{-2}$

11. The electric dipole moment of an HCL atom is 3.4×10^{-30} Cm. The charges on both atoms are unlike and of same magnitude. Magnitude of this charge is .... The distance between the charges is 1 $\overset{o}{A}$

(A) 1.7×10^{-20} C (B) 3.4×10^{-20} C (C) 6.8×10^{-20} C (D) 3.4×10^{-10} C

12. There exists an electric field of 100 N/C along Z–direction. The flux passing through a square of 10 cm sides placed on XY plane inside the electric field is ....

(A) 1.0 Nm^2/C (B) 2.0 Vm (C) 10 Vm (D) 4.0 Nm^2/C

13. The radius of a conducting spherical shell is 10 mm and a 100 μC charge is spread on it. The force acting on a 10 μC charged placed at its centre is .... $k = 9 \times 10^9$ MKS.

(A) 10^3N (B) 10^2N (C) zero (D) 10^5N

14. When a 10 μC charge is enclosed by a closed surface, the flux passing through the surface is ϕ. Now another −10 μC charge is placed inside the closed surface, then the flux passing through the surface is ....

(A) 2ϕ (B) ϕ (C) 4ϕ (D) zero

15. An electric dipole is placed at the centre of a sphere. The flux passing through the surface of the sphere is ....

(A) Infinity (B) zero (C) cannot be found (D) $\frac{2q}{\varepsilon_0}$

16. Two spheres carrying charge q are hanging from a same point of suspension with the help of threads of length 1 m, in a space free from gravity. The distance between them will be ....

(A) 0 (B) 0.5

(C) 2 m (D) cannot be determined.

17. One point electric charge Q is placed at P. A closed surface is placed near the point P. The electrical total flux passing through a surface of the sphere will be

(A) $Q\ \varepsilon_0$ (B) $\frac{\varepsilon_0}{Q}$ (C) $\frac{Q}{\varepsilon_0}$ (D) zero

18. Charge Q each is placed on $(n - 1)$ corners of a polygon of sides n. The distance of each corner from the centre of the polygon is r. The electric field at its centre is ....

(A) $k\frac{Q}{r^2}$ (B) $(n - 1)\ k\frac{Q}{r^2}$ (C) $\frac{n}{n-1}k\frac{Q}{r^2}$ (D) $\frac{n-1}{n}k\frac{Q}{r^2}$

19. When two spheres having 2Q and −Q charge are placed at a certain distance, the force acting between them is F. Now they are connected by a conducting wire and again separated from each other. How much force will act between them if the separation now is the same as before ?

(A) F (B) $\frac{F}{2}$ (C) $\frac{F}{4}$ (D) $\frac{F}{8}$

20. The number of electric field of lines emerged out from 1 C charge is ....

($\varepsilon_0 = 8.85 \times 10^{-12}$ MKS)

(A) 9×10^9 (B) 8.85×10^2 (C) 1.13×10^{11} (D) infinite

21. When 10^{19} electrons are removed from a neutral metal plate through some process, the charge on it becomes ....

(A) −1.6 C (B) + 1.6 C (C) 10^9 C (D) 10^{-19} C

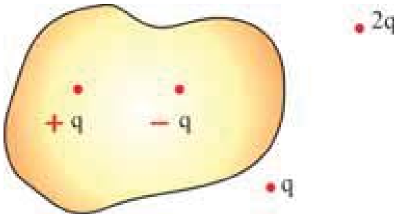
22. A charge Q is placed at the centre of a cube. The electric flux emerging from any one surface of the cube is ....

(A) $\frac{Q}{\varepsilon_0}$ (B) $\frac{Q}{2\varepsilon_0}$ (C) $\frac{Q}{4\varepsilon_0}$ (D) $\frac{Q}{6\varepsilon_0}$

23. The liquid drop of mass m has a charge q. What should be the magnitude of electric field E to balance this drop ?

(A) $\frac{mg}{q}$ (B) $\frac{E}{m}$ (C) mgq (D) $\frac{mq}{g}$

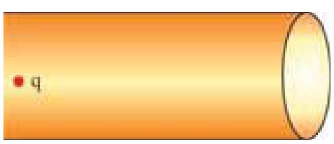
24. As shown in figure the electric flux associated with close surface is ....



(A) $\frac{3q}{\varepsilon_0}$ (B) $\frac{2q}{\varepsilon_0}$

(C) $\frac{q}{\varepsilon_0}$ (D) zero

25. As shown in the figure, q charge is placed at the open end of the cylinder with one end open. The total flux emerging from the surface of cylinder is ....



(A) $\frac{q}{\varepsilon_0}$ (B) $\frac{2q}{\varepsilon_0}$

(C) $\frac{q}{2\varepsilon_0}$ (D) zero

ANSWERS

1. (D)	2. (A)	3. (C)	4. (B)	5. (D)	6. (A)
7. (A)	8. (C)	9. (A)	10. (C)	11. (B)	12. (A)
13. (C)	14. (D)	15. (B)	16. (C)	17. (D)	18. (A)
19. (D)	20. (C)	21. (B)	22. (D)	23. (A)	24. (D)
25. (C)					

Answer the following questions in brief :

1. How many number of protons of the charge is equivalent to a 1 μC ?
2. Two identical metal spheres of equal radius are taken. One of the spheres has charge of 1000 electrons and another has charge of 600 protons. When the two spheres are brought in contact with copper wire and removed, what will be the charges on each sphere ?
3. If $q_1q_2 > 0$, which type of the force acting between two charges ?
4. What is a test charge ? What should be its magnitude ?
5. Define the electric dipole moment and give its SI unit.
6. What will be the torque acting on the dipole if it is placed parallel to the electric field.
7. Explain the behaviour of electric dipole placed in the non-uniform electric field.
8. Give the statement of Gauss's Law.
9. Why does the two electric field lines not intersecting each other ?
10. Draw the electric field lines of electric dipole.
11. A charge enclosed by the spherical Gaussian surface is 8.85×10^{-8}C. What is the electric flux linked with this surface ? If the radius of sphere is doubled, what is the electric flux ?

12. Write the conservation law of electric charge.
13. An electric dipole is placed at the centre of the cube. What is the total electric flux linked with the surfaces of the cube ?

Answer the following questions :

1. Write the Coulomb's Law and represent the forces between the two charges in vector form.
2. Explain the linear charge density, surface charge density and volume charge density. Also give their units.
3. What is electric field ? Explain, giving the characteristics of the electric field.
4. Obtain the expression of the electric field at a point on the axis of the electric dipole.
5. Obtain the expression for the torque acting on the electric dipole place in the uniform electric field.
6. Write the characteristics of the lines of the electric field.
7. Write and explain the Gauss's Law.
8. Obtain the expression of the electric field due to an infinitely long linear charged wire along the perpendicular distance from the wire.
9. Derive the expression of the electric field produced due to uniformly charged infinite plane.
10. Using Gauss's Law, find the intensity of the electric field inside and outside the charged sphere having uniform volume charge density.

Solve the following examples :

1. A metal sphere is suspended through a nylon thread. When another charged sphere (identical to A) is brought near to A and kept at a distance d, a force of repulsion F acts between them. Now A is brought in contact with an identical uncharged spheres C and B also brought in contact with an identical uncharged sphere D and then they are separated from each other. What will be the force between the spheres A and B when they are at a distance $\frac{d}{2}$? [**Ans. :** F]
2. Two identically charged spheres are suspended by strings of equal length. When they are immersed in kerosene, the angle between their strings remains the same as it was in the air. Find the density of the spheres. The dieletric constant of kerosene is 2 and its density is 800 kg m^{-3}. [**Ans. :** 1600 kg m^{-3}]
3. Three point charges 0.5 μC, −0.25 μC and 0.1 μC are placed at the vertices A, B and C of an equilateral triangle ABC. The length of the side of triangle is 5.0 cm. Calculate resultant force acting on the charge placed at point C. $k = 9 \times 10^9$ SI.

   [**Ans. :** $\vec{F}_3 = 0.045\ (3, \sqrt{3})$N]
4. Three identical charges q are placed on the vertices of an equilateral triangle. Find the resultant force acting on the charge $2q$ kept at its centroid. (The distance of the centroid from vertices is 1 m). [**Ans. :** Zero]
5. An electric dipole of momentum $\vec{p}$ is placed in a uniform electric field. The dipole is rotated through a very small angle θ from equilibrium and is released. Prove that it executes simple harmonic motion with frequency $f = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}$. Where, I = moment of inertia of the dipole.

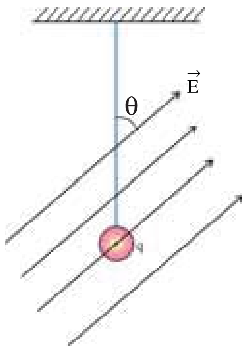
6. The surface charge density of a very large surface is $-3.0 \times 10^{-6} Cm^{-2}$. From what distance should an electron of 150 eV energy be projected towards the plane so that its velocity becomes zero on reaching the plane ? Charge of an electron = 1.6×10^{-19}C, 1 eV = 1.6×10^{-19}J, $\varepsilon_0 = 9 \times 10^{-12}$ SI. [**Ans. :** 9×10^{-4}m]

7. Two small, identical spheres, one positively charged and another negatively charged are placed 0.5m apart attract each other with a force of 0.108N. If they are brought in contact for some time and again separated by 0.5m, they repelled each other with force of 0.036N. What were the initial charges on the spheres ?

[**Ans. :** $q_1 = \pm 3.0 \times 10^{-6}$C, $q_2 = \mp 1.0 \times 10^{-6}$C]

8. Two charged particles of mass m and 2m have charges $+2q$ and $+q$ respectively. They are kept in a uniform electric field far away from each other and then allowed to move for some time t. Find the ratio of their kinetic energy. [**Ans. :** 8 : 1]

9.

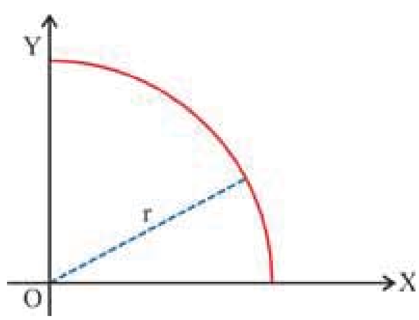


A simple pendulum is suspended in a uniform electric field $\vec{E}$ as shown in the figure. What will be its period if its length is l ? Charge on the bob of pendulum is q and mass is m.

[**Ans. :** $T = 2\pi \sqrt{\dfrac{l}{\left(g^2 + \dfrac{q^2E^2}{m^2} - \dfrac{2gqE}{m}\cos\theta\right)^{\frac{1}{2}}}}$]

10. A charge of 4×10^{-8}C is uniformly distributed over the surface of sphere of radius 1cm. Another hollow sphere of radius 5cm is concentric with the smaller sphere. Find the intensity of the electric field at a distance 2cm from the centre. $k = 9 \times 10^{-9}$ SI. [**Ans. :** 9×10^5 NC^{-1}]

11.



An arc of radius r, lying in the first quadrant is shown in the figure. The linear charge density on the arc is λ. Calculate the magnitude and direction of electric field intensity at the point of origin.

[**Ans. :** $E = \dfrac{\sqrt{2}k\lambda}{r}$, making an angle 45° with X-axis in the third quadrant]

12. A particle of mass 5×10^{-9} kg is held at some distance from very large uniformly charged plane. The surface charge density on the plane is $4 \times 10^{-6} C/m^2$. What should be the charge on the particle so that the particle remains stationary even after releasing it ? $\varepsilon_0 = 8.85 \times 10^{-12} C^{-2}N^{-1}m^{-2}$, $g = 9.8$ ms^{-2}

[**Ans. :** $q = 2.17 \times 10^{-13}$C)

13. In the hydrogen atom, an electron revolves around a proton in a circular orbit of radius 0.53 Å. Calculate the radial acceleration and the angular velocity of the electron. $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$C.

[**Ans. :** $a_r = 9.01 \times 10^{22}$ m/s^2, ω = 3 9 × ^{16} rad/s)

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2

ELECTROSTATIC POTENTIAL AND CAPACITANCE

2.1 Introduction

In Chapter 1, we learned about the types of electric charge, the forces acting between the charges, the electric fields produced by a point charge and by different charge distributions and Gauss' theorem. The **force** acting on a given **charge** q can be found by knowing the electric field. Now, if the electric charge is able to move due to this force, it will start moving and in such a motion work will be done. So, now in this chapter we shall study in detail, the physical quantities like electrostatic energy, electrostatic potential that give information about the work done on the charge. Moreover electric potential and electric field, both the quantities can be obtained from each other. We will also know the relation between them.

A simple device which stores the electric charge and electrical energy is a **capacitor**. We shall also study about the capacitance of a capacitor, the series and parallel combinations of capacitors, the electrical energy stored in it, etc. The capacitors are used in different electrical and electronic circuits e.g. electric motor, flashgun of a camera, pulsed lasers, radio, TV etc. At the end of the chapter we shall see about a device–with the help of which we can get a very large potential difference–Van de Graaff generator.

2.2 Work done during the Motion of an Electric Charge in the Electric Field

We had seen in Chapter-1 that when an electric charge q is placed at a point in an electric field $\vec{E}$, a force $\vec{F} = q\vec{E}$, acts on it. Now, if this charge is able to move, it starts moving. To discuss the work done during such a motion, initially we will consider a unit positive charge.

As shown in the figure 2.1, we want to take a unit positive charge ($q = +1$ C charge) from point A to point B, in the electric field produced by a point charge (Q), and also want to find the **work done by the electric field** during this motion. Many different paths can be thought of to go from A to B. In the figure 2.1 ACB and ADB paths are shown as illustrations.

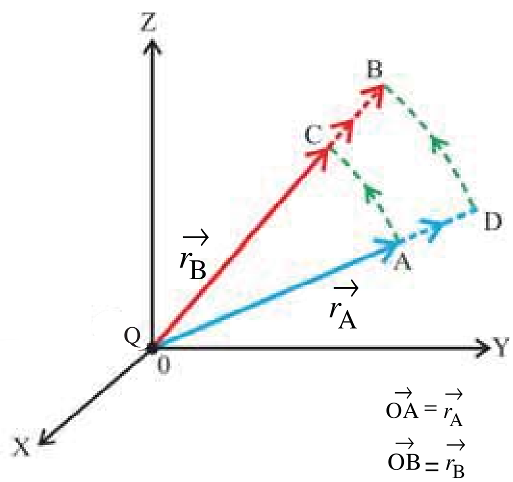


Figure 2.1 Work during the Motion of a Charge

According to the definition, the force on the unit positive charge at a given point, is the electric field $\vec{E}$ at that point. According to the formula $E = \frac{kQ(1)}{r^2}$ this force varies continuously with distance. Hence the work done by the electric field on unit positive charge in a small displacement is given by $dW = \vec{E} \cdot \vec{dr}$ and the work done during

A to B by $W_{AB} = \int_A^B \vec{E} \cdot \vec{dr}$ (2.2.1)

Here, $\int_A^B \vec{E} \cdot \vec{dr}$ is called the **line integral of electric field** between the points A and B.

ACB Path : (1) First, we go from A to C on the circular arc AC having radius OA and then we go from C to B in $\vec{OC}$ direction. The electric field produced by Q, is normal to the arc AC at every point on it (the angle between $\vec{E}$ and $\vec{dr}$ = 90°). Hence $W_{AC} = \int_A^C \vec{E} \cdot \vec{dr} = 0$.

The work done by the electric field on the path CB, is

$$W_{CB} = \int_C^B \vec{E} \cdot \vec{dr} \qquad (2.2.2)$$

$$= \int_C^B \frac{kQ}{r^2} \hat{r}_B \cdot dr\, \hat{r}_B = kQ \int_C^B \frac{1}{r^2} dr = kQ \left[-\frac{1}{r}\right]_{r_C}^{r_B}$$

$$W_{CB} = kQ \left[\frac{1}{r_C} - \frac{1}{r_B}\right] \qquad (2.2.3)$$

Thus, on the path ACB, the work done by the electric field

$$W_{ACB} = W_{AC} + W_{CB} = kQ \left[\frac{1}{r_C} - \frac{1}{r_B}\right] \qquad (2.2.4)$$

Here, since $r_C < r_B$, it is self-evident that this work is positive.

(2) Path ADB : From A to D, just like the above, the work done by the electric field is obtained as $W_{AD} = kQ \left[\frac{1}{r_A} - \frac{1}{r_D}\right]$. Moreover, since the electric field is normal to the arc DB, the work done in this motion = 0.

Hence the work done by the electric field on ADB path is

$$W_{ADB} = W_{AD} + W_{DB} = kQ\left[\frac{1}{r_A} - \frac{1}{r_D}\right] \qquad (2.2.5)$$

Here $|\vec{r_D}| = |\vec{r_B}|$ and $|\vec{r_A}| = |\vec{r_C}|$. Hence from equations 2.2.4 and 2.2.5,

$$W_{ACB} = W_{ADB} = W_{AB} = kQ \left[\frac{1}{r_A} - \frac{1}{r_B}\right] \qquad (2.2.6)$$

Thus, in an electric field, the work done by the **electric field** in moving a unit positive charge from one point to the other, **depends only on the positions of those two points and does not depend on the path joining them.**

Now, if we move the unit positive charge from point B to A, **on any** path, the work done by the electric field, will be given by (according to equation 2.2.6)

$$W_{BA} = kQ\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \quad (2.2.7)$$

If a unit positive charge is taken from point A to B **on any path** and then is brought back to A on any path, a closed loop is formed (e.g. ACBDA or ADBCA) and on this closed loop the total work done by the electric field ($\oint \vec{E} \cdot \vec{dr}$); will be $W_{AB} + W_{BA} = 0$ (using equations 2.2.6 and 2.2.7). You are aware of the fact that a field with this property is known as a **conservative field.** Thus electric field is also a conservative field. [In Standard 11 you had also seen that the gravitational field is also a conservative field.]

Although we have considered the work done on unit positive charge, all these aspects are also applicable to the work done on any charge q, but for that, the right hand side of the above equations for the work, should be multiplied by q. e.g., Work for A to B will be $W_{AB} = \int_A^B q\vec{E} \cdot \vec{dr}$. Moreover, you will be able to understand that instead of the work done **by the electric field,** if we want to find the **work required to be done by the external force against the electric field** (for the motion without acceleration), then the **negative sign** will have to be put on the right hand side of the above equation (2.2.1) for the work. Hence for unit positive charge, such a work will be given by $W'_{AB} = -\int_A^B \vec{E} \cdot \vec{dr}$, which is the same in magnitude as work given by equation 2.2.1 but has the opposite sign to it. For charge q such a work will be given by $W''_{AB} = -\int_A^B q\vec{E} \cdot \vec{dr}$.

From this discussion we should remember that $\int_A^B \vec{E} \cdot \vec{dr}$, that is the line integral of electric field between A to B – is the work done by the electric field in moving a unit positive charge from A to B and it does not depend on the path. Moreover, $\oint \vec{E} \cdot \vec{dr} = 0$. $\vec{E} \cdot \vec{dr}$ is also sometimes written as $\vec{E} \cdot \vec{dl}$ where $\vec{dl}$ is also a small displacement vector

2.3 Electrostatic Potential

We know that the work done by the electric field in moving a unit positive (+1 C) charge from one point to the other, in the electric field, depends only on the positions of those two points and not on the path joining them.

If we take a reference point A, and take the unit positive charge from point A to B; A to C; A to D; ..... etc in the electric field, then the work done by the electric field is obtained as $W_{AB} = \int_A^B \vec{E} \cdot \vec{dr}$, $W_{AC} = \int_A^C \vec{E} \cdot \vec{dr}$, $W_{AD} = \int_A^D \vec{E} \cdot \vec{dr}$,.... respectively. But the reference point A is already fixed, hence the above mentioned work depends on the position of the other points (B, C, D, ...) only. Conventionally the reference point is taken as a point at infinite distance from the source of electric field. Hence to bring a unit positive charge from that point to a point P in the field, the work done by the electric field is given by the formula $W_{\infty P} = \int_\infty^P \vec{E} \cdot \vec{dr}$

and it becomes the function only of the position of point P. But, if we want to find the work required to be done **against** the electric field; in order that the motion becomes **"motion without acceleration,"**

the formula $W'_{\infty P} = -\int_{\infty}^{P} \vec{E} \cdot \vec{dr}$ has to be used.

An important characteristic of an electric field is called **electrostatic potential** and with reference to the work done on unit positive charge, it is defined as under :

"Work required to be done against the electric field in bringing a unit positive charge from infinite distance to the given point in the electric field is called the electrostatic potential (V) at that point."

Here the meaning of "against the electric field" is actually **"against the force by the electric field".** We will call the electrostatic potential as electric potential in short.

According to the above definition, the electric potential at a point P is given by the formula :

$$V_P = -\int_{\infty}^{P} \vec{E} \cdot \vec{dr} \tag{2.3.1}$$

In other words this formula represents the definition of electric potential.

From this formula the potential difference betwen points Q and P is given by

$$V_Q - V_P = \left(-\int_{\infty}^{Q} \vec{E} \cdot \vec{dr}\right) - \left(-\int_{\infty}^{P} \vec{E} \cdot \vec{dr}\right) \tag{2.3.2}$$

$$= \int_{Q}^{\infty} \vec{E} \cdot \vec{dr} + \int_{\infty}^{P} \vec{E} \cdot \vec{dr} = \int_{Q}^{P} \vec{E} \cdot \vec{dr} \tag{2.3.3}$$

$$= -\int_{P}^{Q} \vec{E} \cdot \vec{dr} \tag{2.3.4}$$

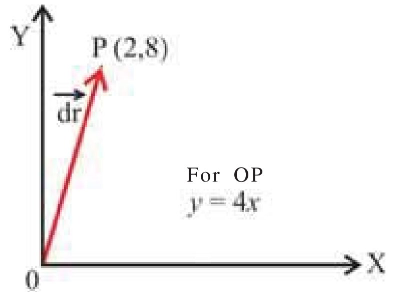
This potential difference shows the **work required to be done to take a unit positive charge from P to Q, against the electric field** and in that sense it also shows the potential of Q with respect to P. Very often the potential difference is in short written as p.d. also. The unit of electric potential (and hence that of the potential difference also) is joule / coulomb which is called volt (symbol V) in memory of the scientist Volta. i.e., volt $= \frac{\text{joule}}{\text{coulomb}}$ or $V = \frac{J}{C}$. It's dimensional formula is $M^1L^2T^{-3}A^{-1}$.

Electric potential is a scalar quantity. Moreover, we have obtained electric potential from the vector quantity-electric field $\vec{E}$ (See equation 2.3.1). In future we will also obtain electric field from the electric potential. In the calculations involving electric field $\vec{E}$, its three components E_x, E_y, E_z have to be considered and the calculations become longer, while in the calculations involving the electric potential, **only one scalar** appears and hence the calculations become shorter and easier. Hence the concept of electric potential is widely used. Absolute value of electric potential has no importance, only the difference in potential is important.

[For Information Only : Galvani (1737–1798) produced electricity by placing two different metallic electrodes in the tissue of frog. He called it Animal Electricity. Volta explained that the above process had nothing to do with the characteristics of the frog, but one can generate electricity by placing two dissimilar metallic electrodes on any wet body. He was the one who designed the electro chemical cell, which we studied earlier as voltaic cell.

The importance of electric potential in electricity is similar to the importance of temperature in thermodynamics and the height of fluid in hydrostatics. The electricity flows (i.e. the electric current flows) from an electrically charged material having higher electric potential to an electrically charged material having lower electric potential. Quite similar to water, which flows from a higher level to a lower level or like the flow of heat which flows from a region having higher temperature to a region having lower temperature. Thus, the direction of the flow of electric current between two materials depends on their electric potentials.]

Illustration 1 : Suppose an electric field due to a stationary charge distribution is given by $\vec{E} = ky\hat{i} + kx\hat{j}$, where k is a constant. (a) Find the line integral of electric field on the linear path joining the origin O with point P(2, 8), in the Figure. (b) Obtain the formula for the electric potential at any point on the line OP, with respect to (0, 0)



Solution : (a) The displacement vector $\vec{dr}$ on the line OP is $\vec{dr} = dx\hat{i} + dy\hat{j}$

$$\therefore \vec{E} \cdot \vec{dr} = (ky\hat{i} + kx\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$= kydx + kxdy = k(ydx + xdy)$$

Moreover, on the entire OP line $y = 4x$ ($\because$ the slope of a straight line is constant)

$\therefore \ dy = 4dx$

$\therefore$ The line integral of electric field from O to P, is

$$\int_O^P \vec{E} \cdot \vec{dl} = k\int_O^P (ydx + xdy) = k\int_{(0,0)}^{(2,8)} [4xdx + x(4dx)] = k\int_0^2 8x\,dx \qquad \text{(A)}$$

$$= 8k\left[\frac{x^2}{2}\right]_0^2 = 16k$$

(b) In order to obtain the potential at any point Q(x, y) on the line OP with respect to (0, 0) we can use $V(Q) = -\int_O^Q \vec{E} \cdot \vec{dl}$

$$\therefore \ V(Q) = -\int_{(0)}^{(x)} 8kx\,dx \ldots \text{(from equation A)}$$

$$= -\,8k\left[\frac{x^2}{2}\right]_0^x = -4kx^2$$

Illustration 2 : The electric field at distance r perpendicularly from the length of an infinitely long wire is $E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}$, where λ is the linear charge density of the wire. Find the potential at a point having distance b from the wire with respect to a point having distance a from the wire ($a > b$). [**Hint :** $\int \frac{1}{r} dr = ln\ r$].

Solution : $V_b - V_a = -\int_a^b \vec{E} \cdot \vec{dr}$

$$= -\int_a^b \frac{\lambda}{2\pi \in_0 r} dr \quad (\because \vec{E} \parallel \vec{dr})$$

$$= -\frac{\lambda}{2\pi\varepsilon_0} \int_a^b \frac{1}{r} dr = -\frac{\lambda}{2\pi\varepsilon_0} [ln\ r]_a^b = -\frac{\lambda}{2\pi\varepsilon_0 r} [ln\, b - ln\, a]$$

$$= \frac{\lambda}{2\pi\varepsilon_0} [ln\, a - ln\, b]$$

$$= \frac{\lambda}{2\pi\varepsilon_0} ln \left(\frac{a}{b}\right)$$

For reference point a, taking $V_a = 0$

$$\therefore V_b = \frac{\lambda}{2\pi\varepsilon_0} ln \left(\frac{a}{b}\right)$$

Illustration 3 : An electric field is represented by $\vec{E} = Ax\hat{i}$, where $A = 10 \frac{V}{m^2}$. Find the potential of the origin with respect to the point (10, 20)m.

Solution : $\vec{E} = Ax\hat{i} = 10x\hat{i}$

$$V(0, 0) - V(10, 20) = -\int_{(10, 20)}^{(0, 0)} \vec{E} \cdot \vec{dr}$$

$$= -\int_{(10, 20)}^{(0, 0)} (10x\hat{i}) \cdot (dx\hat{i} + dy\hat{j}) = -\int_{10}^{0} 10x\, dx$$

$$= -10\left[\frac{x^2}{2}\right]_{10}^{0} = [0 - (-500)] = 500 \text{ volt}$$

Since V(10, 20) is to be taken as zero,

V(0, 0) = 500 volt.

2.4 Electrostatic Potential Energy

In the previous article (2.2), we had discussed the work done by the electric field on a unit positive charge and then also on the charge q, during the motion in the electric field. Moreover we had also talked about the work required to be done by the external force against the electric field, in which the **motion of charge is without acceleration only.** Hence its velocity remains constant and its kinetic energy does not change. But the work done by this external force is stored in the form of potential energy of that charge. From this, the electric potential energy is defined as under :

"The work required to be done against the electric field in bringing a given charge (q), from infinite distance to the given point in the electric field is called the electric potential energy of that charge at that point." Here "motion without acceleration" is implied when we mentioned" "work required to be done."

From the definitions of electric potential energy and the electric potential, we can write the electric potential energy of charge q at point P, as

$$U_P = -\int_{\infty}^{P} q\vec{E}\cdot\vec{dr} = -q\int_{\infty}^{P}\vec{E}\cdot\vec{dr} \qquad (2.4.1)$$

$$= qV_P \qquad (2.4.2)$$

Moreover, we can also call the electric potential at point P as the electric potential energy of unit positive charge (q = +1 C) at that point. That is,

$$\begin{Bmatrix}\text{electric potential}\\ \text{at a given point}\end{Bmatrix} = \begin{Bmatrix}\text{electric potential energy of unit}\\ \text{positive charge at that point}\end{Bmatrix}$$

For more clarity in this discussion, we note a few important points as under :

(1) When we bring charge q (or a unit positive charge) from infinite distance to the given point or when we move it from one point to the other in the field, the **positions of the sources (charges) producing the field are not changed.** (We will imagine these sources as being clamped on their positions by some invisible force !!)

(2) The absolute value of the electric potential energy is not at all important, only the difference in its value is important. Here, in moving a charge q, from point P to Q, **without acceleration,** the **work required to be done by the external force,** shows the difference in the electric potential energies ($U_Q - U_P$) of this charge q, at those two points.

$$\therefore\ U_Q - U_P = -q\int_{P}^{Q}\vec{E}\cdot\vec{dr} \qquad (2.4.3)$$

(3) Here, electric potential energy is **of the entire system** of the sources producing the field and the charge that is moved, for **some one configuration,** and when the configuration changes the electric potential energy of the system also changes. e.g., when the distance between them is r, it is one configuration and if distance r changes, the configuration is also said to be changed and hence the electric potential energy of the system is also said to be changed. But as the conditions of the sources producing the field are not changed, the entire change in the electric potential energy is **experienced by this charge *q* only which we have moved.** Hence we are able to write $U_Q - U_P$ as the difference in potential energy **of this charge *q* only.** Because of this reason we have mentioned "potential energy of charge q" for equation 2.4.1 and "potential energy of unit positive charge" in the discussion that followed it.

2.5 Electric Potential due to a Point Charge

We want to find the electric potential V(P), due to a point charge q, at some point P, at a distance r from it.

For this we will put the origin of co-ordinate axes 0, at the position of that charge. See figure 2.2. Here $\vec{OP} = \vec{r}$. According to the definition of electric potential we can use the equation.

$$V(P) = -\int_{\infty}^{P}\vec{E}\cdot\vec{dr} \qquad (2.5.1)$$

Moreover, we can also write this equation in another form as

$$V(P) = \int_{P}^{\infty}\vec{E}\cdot\vec{dr} \qquad (2.5.2)$$

because, $\int_{\infty}^{P}\vec{E}\cdot\vec{dr} = -\int_{P}^{\infty}\vec{E}\cdot\vec{dr}$.

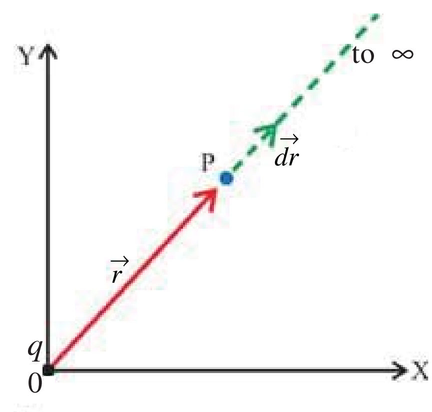


Figure 2.2 Potential due to Point Charge

At this point P, $\vec{E} = \dfrac{kq}{r^2}\hat{r}$ (2.5.3)

∴ From equation 2.5.2

$$V(P) = \int_P^{\infty} \frac{kq}{r^2} \hat{r} \cdot dr\hat{r} = \int_r^{\infty} \frac{kq}{r^2} dr$$

$$= kq \int_r^{\infty} \frac{1}{r^2} dr = kq \left[-\frac{1}{r}\right]_r^{\infty}$$

$$V(P) = \frac{kq}{r} \quad (2.5.6)$$

or $$V(P) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \quad (2.5.7)$$

This equation is true for any charge, positive or negative. The potential due to a positive charge is positive and that due to a negative charge is negative (as q is to be put with negative sign in the above equation.)

It is self evident from equation 2.5.6 that as the distance r increases, the electric potential decreases as $\frac{1}{r}$. In case of potential also superposition principle is applicable. To find the electric potential due to many point charges we should find the potential due to every charge according to equation 2.5.7 and they should be added algebraically.

Illustration 4 : A point P is 20 m away from a 2 μC point charge and 40 m away from a 4 μC point charge. Find the electric potential at P.

(1) Find the work required to be done to bring 0.2 C charge from infinite distance to the point P.

(2) Find the work required to be done to bring −0.4 C charge from infinite distance to the point P. [$k = 9 \times 10^9$ N m^2 C^{-2}]

Solution : $$V_P = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = k\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$$

$$= 9 \times 10^9 \left[\frac{2 \times 10^{-6}}{20} + \frac{4 \times 10^{-6}}{40}\right] = 1800 \text{ volt}$$

(1) $W_1 = V_P\, q_1' = (1800)(0.2) = 360$ J.

(2) $W_2 = V_P\, q_2' = (1800)(-0.4) = -720$ J

2.6 Electric Potential due to an Electric Dipole

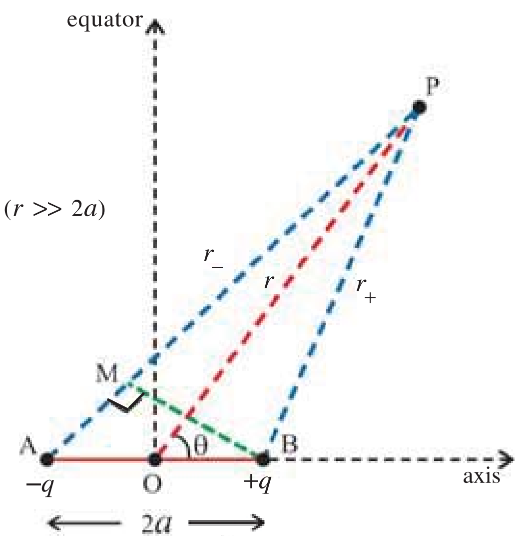


Figure 2.3 Potential due to an Electric Dipole

We have seen in Chapter-1 that two equal and opposite charges (+ q and − q) separated by a finite distance (= $2a$) constitute an electric dipole.

Such a dipole is shown in the figure 2.3, with the origin of co-ordinate system O at its mid-point. The magnitude of the dipole moment of the dipole is $p = q(2a)$ and its direction is from negative to the positive charge that is, in AB direction.

We want to find the electric potential at point P far away from the mid-point O of dipole and in the direction making an angle θ with the axis of the dipole. Let OP = r, AP = r_-, and BP = r_+. At P, the electric potential is equal to the sum of the potentials produced by each of the charges.

$$\therefore \quad V(P) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{r_-} \quad (2.6.1)$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_+} - \frac{1}{r_-}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r_- - r_+}{r_- r_+}\right] \quad (2.6.2)$$

Since P is a far distant point, $r >> 2a$ and hence we can take AP || OP || BP. In this condition the figure 2.3 shows that

$$\left\{\begin{array}{l}\text{for numerator of equation (2.6.2), } r_- - r_+ = AM = 2a\ cos\theta \\ \text{and for denominator, } r_- \approx r_+ \approx r\end{array}\right\} \quad (2.6.3)$$

We have considered a very far distant point as compared to the length ($2a$) of the dipole. The molecular dipoles are very small and such an approximation is very well applicable to them. From equations (2.6.2) and (2.6.3), we get

$$V(r) = \frac{q}{4\pi\varepsilon_0}\left(\frac{2a\ cos\theta}{r^2}\right) \quad (2.6.4)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p\ cos\theta}{r^2} \quad (2.6.5)$$

Writing the unit vector in the direction $\overrightarrow{OP}$ as $\hat{r}$, we can wirte $\vec{p}\cdot\hat{r} = p\ cos\theta$.

$$\therefore \quad V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2} \text{ (for } r >> 2a) \quad (2.6.6)$$

Note : The dipole obtained in the limits $q \to \infty$ and $a \to 0$, is called the point dipole. For such a point dipole the above equation is more accurate, while for the physical dipole - found in practice - this equation gives an approximate value of the electric potential. Let us note a few points evident from equation (2.6.4), as under :

(1) Potential on the Axis : For a point on the axis of the dipole

$$\theta = 0 \text{ or } \pi. \ \therefore \ V = \pm\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

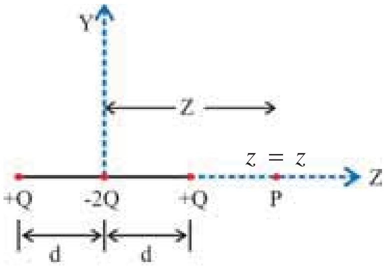
From the given point, if the nearer charge is $+q$, then we get V as positive and if it is $-q$, then we get V as negative.

(2) Potential on the Equator : For a point on the equator $\theta = \frac{\pi}{2} \ \therefore \ V = 0$

(3) The potential at any point depends on the angle between its position vector $\vec{r}$ and $\vec{p}$.

(4) The potential due to a dipole decreases as $\frac{1}{r^2}$ with distance (while the potential due to a point charge decreases as $\frac{1}{r}$ with distance). We have seen in Chapter 1 that the electric field due to a dipole decreases as $\frac{1}{r^3}$)

Ilustration 5 : When two dipoles are lined up in opposite direction, the arrangement is known as a quadruple (as shown in the Figure). (1) Calculate the electric potential at a point $z = z$ along the axis of the quadruple and (2) If $z >> d$, then show that,



$$V(z) = \frac{Q}{4\pi\varepsilon_0}\frac{2d^2}{z^3}$$

Note : $2|Q|d^2$ is called the quadruple moment.

Solution : (1) Let z be the Z co-ordinate of point P.

The electric potential at point P, due to +Q charge (which is at the left hand side of the origin) is,

$$V_1 = \frac{kQ}{z+d} \qquad (1)$$

The electric potential at point P due to the +Q charge which is at the right hand side of the origin is,

$$V_2 = \frac{kQ}{z-d} \qquad (2)$$

The electric potential at point P, due to −2Q charge present at the origin is,

$$V_3 = -\frac{k(2Q)}{z} \qquad (3)$$

∴ The total potential at point P,

$$V(z) = V_1 + V_2 + V_3$$

$$= kQ\left[\frac{1}{z+d}+\frac{1}{z-d}-\frac{2}{z}\right] = kQ\left[\frac{2z}{z^2-d^2}-\frac{2}{z}\right] = kQ\left[\frac{2d^2}{z(z^2-d^2)}\right]$$

(2) If $z >> d$, we can neglect d^2 in comparison with z^2 in the denominator of right hand side of the above equation.

$$\therefore\ V(z) = \frac{kQ(2d^2)}{z^3} = \frac{Q}{4\pi\varepsilon_0}\frac{2d^2}{z^3}$$

Illustration 6 : Charge Q is distributed uniformly over a non-conducting sphere of radius R. Find the electric potential at distance r from the centre of the sphere (r < R). The electric field at a distance r from the centre of the sphere is given as $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\,r\hat{r}$. Also find the potential at the centre of the sphere.

Solution : The electric potential on the surface of such a sphere is,

$$V(R) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

As a result, we can use the equation $V(r) - V(R) = -\int_R^r \vec{E}\cdot\vec{dr}$

$$\therefore\ V(r) - V(R) = -\int_R^r \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\,rdr\ \hat{r}\cdot\hat{r} \qquad (\because \vec{dr} = dr\,\hat{r})$$

$$= -\frac{Q}{4\pi\varepsilon_0 R^3}\int_R^r rdr = -\frac{Q}{4\pi\varepsilon_0 R^3}\left[\frac{r^2}{2}\right]_R^r$$

$$= -\frac{Q}{4\pi\varepsilon_0 R^3}\left[\frac{r^2}{2}-\frac{R^2}{2}\right]$$

$$\therefore\ V(r) = V(R) + \frac{Q}{4\pi\varepsilon_0 R^3}\left[\frac{R^2}{2} - \frac{r^2}{2}\right]$$

$$\therefore\ V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} + \frac{Q}{8\pi\varepsilon_0 R^3}\ (R^2 - r^2)$$

$$\therefore\ V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R}\left(3 - \frac{r^2}{R^2}\right),\ r < R$$

At the centre of the sphere $r = 0$, $\therefore$ V (centre) $= \frac{1}{4\pi\varepsilon_0}\left(\frac{3Q}{2R}\right)$.

2.7 Electric Potential due to a System of Charges

In a system of charges, point charges could have been distributed descretely (separated from each other) while in some system they could have been distributed continuously with each other. In some system of charges the distribution of charges could be a mixture of any type of these two distributions.

(a) Descrete Distribution of Charges :

In figure 2.4, point charges q_1, q_2, q_3 ...... q_n are shown as distributed descretely. The position vectors of these charges with respect to the origin of co-ordinate system are $\vec{r}_1$, $\vec{r}_2$,.... $\vec{r}_n$ respectively. We want to find the electric potential due to this system, at point P with position vector $\vec{r}$. For this we will find the electric potential due to every point charge and then will make summation.

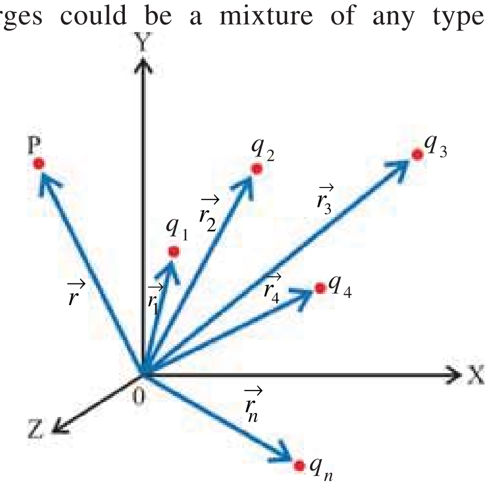


Figure 2.4 Potential Due to Descrete Charges

That is, $V = V_1 + V_2 + ... + V_n$ (2.7.1)

$$\therefore\ V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1p}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2p}} + ... + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_{np}} \qquad (2.7.2)$$

Where r_{1p} = distance of P from $q_1 = |\vec{r} - \vec{r}_1|$.

Similarly r_{2p}, ..., r_{np} are the corresponding distances.

$$\therefore\ V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{|\vec{r} - \vec{r}_1|} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{|\vec{r} - \vec{r}_2|} + ... + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{|\vec{r} - \vec{r}_n|} \qquad (2.7.3)$$

$$\therefore\ V = \sum_{i=1}^{n}\frac{kq_i}{|\vec{r} - \vec{r}_i|} \qquad (2.7.4)$$

(b) Electric Potential due to a Continuous Distribution of Charges :

Suppose in a certain region electric charge is distributed continuously. Imagine this region to be divided in a large number of volume-elements, each one with extremely small volume. If the volume of such an element having position vector $\vec{r}\,'$ is $d\tau'$ and at this position the volume-density of charge is $\rho(\vec{r}\,')$, then the charge in this element is $\rho(\vec{r}\,')\ d\tau'$, and it can be treated as a point charge. The electric potential due to this small, volume element at point P having the position vector $\vec{r}$, is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\rho(\vec{r}\,')d\tau'}{|\vec{r}-\vec{r}\,'|}. \qquad (2.7.5)$$

By integrating this equation over the entire volume of this distribution, we get the total potential at point P, which can be written as under :

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{\text{volume}}\frac{\rho(\vec{r}\,')d\tau'}{|\vec{r}-\vec{r}\,'|} \qquad (2.7.6)$$

If the charge distribution is uniform, $\rho(\vec{r}\,')$ can be taken as canstant (= ρ).

(c) A Spherical Shell with Uniform Charge Distribution :

In Chapter 1, we had seen that the electric field **at a point outside** and **at a point on the surface** of spherical shell with uniform charge distribution is equal to the electric field obtained by considering the entire charge of the shell as cencentrated.at the centre of the shell. We have obtained the electric potential from the electric field ($V = -\int_{\infty}^{r}\vec{E}\cdot\vec{dr}$). For electric potential also the entire charge can be considered as concentrated at the centre of the shell. Hence the potential at a point outside and at a point on the surface of the shell having charge q and radius R, is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad (\text{for } r \geq R) \qquad (2.7.7)$$

where r = distance of the given point from centre of shell.

Moreover, we also know that the electric field inside the shell is zero. Hence during the **motion** of **unit positive charge inside the shell** no work is required to be done. Hence the potentials at **all points** inside the shell are equal having the value equal to the potential on the surface of that shell. i.e. $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$ (for $r \leq R$) (2.7.8)

(Note that here, only that work is accounted for which is done during the motion of unit positive charge from ∞ to the surface of the shell.)

2.8 Equipotential Surfaces

An equipotential surface is that surface **on which the electric potentials at all points are equal.**

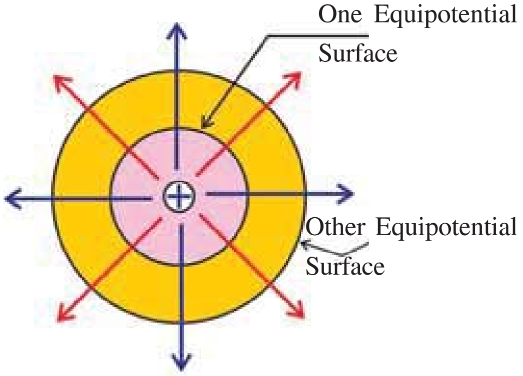


Figure 2.5 Equipotential Surfaces

The electric potential due to a point charge is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$. Hence if r is constant, V also becomes constant. From this we can say that for a single point charge q, the equipotential surfaces are the surfaces of the spheres drawn by taking this charge as the centre. (See figure 2.5). The potentials on two such different surfaces are different but for all the points on the **same surface** the potentials **are equal.** The electric field produced by a point charge is along the radial directions drawn from it. [For $+q$ they are in radial directions going away from it and for $-q$ coming towards it.]. These radial lines are normal to those equipotential surfaces at every point. Hence at a given point the direction of electric field is normal to an equipotential surface passing through that point. We shall now prove that this is true not only for a point charge but in general for any charge configuration.

Suppose a unit positive charge is given a small displacement $d\vec{l}$ **on the** equipotential surface **(along this surface),** from a given point. In this process the work required to be done against the electric field (by the external force) is $dW = -\vec{E}\cdot d\vec{l}$ = potential difference between those two points.

But the potential difference on the equipotential surface = 0.

$\therefore \vec{E}\cdot d\vec{l} = 0 \Rightarrow E\, dl\, cos\theta = 0$, where θ = angle between $\vec{E}$ and $d\vec{l}$.

But $E \neq 0$ and $dl \neq 0$ $\therefore cos\theta = 0$ $\therefore \theta = \frac{\pi}{2}$ $\therefore \vec{E} \perp d\vec{l}$.

But $d\vec{l}$ is along this surface. Hence the electric field $\vec{E}$ is normal to the equipotential surface at that point.

Like the field lines, the equipotential surface is also a useful concept to represent an electric field. For a uniform electric field prevailing in X-direction, the field lines are parallel to X-axis and equispaced, while the equipotential surfaces are normal to X-axis (i.e. parallel to YZ plane.) See figure 2.6.

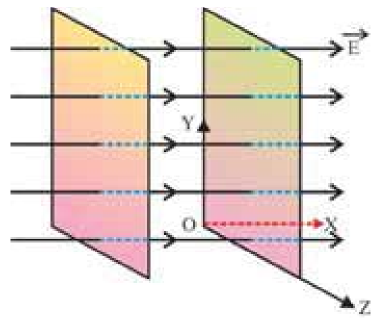
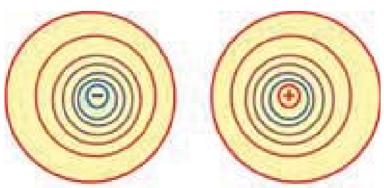
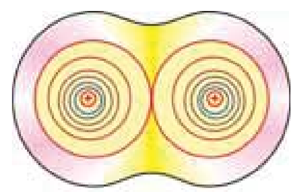


Figure 2.6 Equipotential Surface for a Uniform Electric Field



(a) Equipotential Surfaces of a Dipole (Only For Information)



(b) Equipotential Surfaces of a System of Two Positive and Equal Charge (Only for Information)

Figure 2.7

The equipotential surfaces of an electric dipole are shown in figure 2.7(a).

The equipotential surfaces of a system of two positive charges of equal magnitude are shown in figure 2.7(b).

2.9 Relation between the Electric Field and the Electric Potential

In article 2.3, we have obtained the electric potential $V = (-\int_{\infty}^{P} \vec{E}\cdot\vec{dr})$ from the electric field. Now, if we know about the electric potential in a certain region, we can get the electric field from it as well.

We have seen in article 2.3, that from the line integral of electric field between points P and Q, we can get the potential difference between those two points. (Equation 2.3.4) as

$$V_Q - V_P = \Delta V = -\int_P^Q \vec{E} \cdot \vec{dr} \qquad (2.9.1)$$

Now, if these points P and Q are very close to each other, then for such a small displacement $d\vec{l}$, integration is not required and only one term $\vec{E} \cdot d\vec{l}$ can be kept.

$$\therefore \; dV = -\vec{E} \cdot d\vec{l} \qquad (2.9.2)$$

If $d\vec{l}$ is in the direction of $\vec{E}$, $\vec{E} \cdot d\vec{l} = E \, dl \, cos 0^{o} = E \, dl$

$$\therefore \; dV = -E \, dl$$

$$\therefore \; E = \frac{-dV}{dl} \qquad (2.9.3)$$

This equation gives the magnitude of electric field in the direction of displacement $d\vec{l}$. Here $\frac{dV}{dl}$ = potential difference per unit distance. It is called the **potential gradient.** Its unit is $\frac{V}{m}$. From equation (2.9.3) the unit of electric field is also written as $\frac{V}{m}$, which is equivalent to $\frac{N}{C}$.

If we had taken the displacement $d\vec{l}$ **not** in the direction of electric field, but in some other direction, then $\frac{-dV}{dl}$ would give us the **component of electric field in the direction of that displacement.** e.g. If the electric field is in X-direction only and the displacement is in any direction (in three dimensions), then

$$\vec{E} = E_x \hat{i} \text{ and } d\vec{l} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\therefore \; dV = -(E_x \hat{i}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= -E_x \, dx \qquad (2.9.4)$$

$$\therefore \; E_x = \frac{-dV}{dx} \qquad (2.9.5)$$

Similarly, if the electric field was only in Y and only in Z direction respectively, we would get,

$$E_y = \frac{-dV}{dy} \qquad (2.9.6)$$

$$E_z = \frac{-dV}{dz} \qquad (2.9.7)$$

Now, if the electric field also has all the three ($x-$, $y-$, $z-$) components then from equations (2.9.5) (2.9.6) and (2.9.7) we can write as under.

$$E_x = \frac{-\partial V}{\partial x}, \; E_y = \frac{-\partial V}{\partial y}, \; E_z = \frac{-\partial V}{\partial z} \qquad (2.9.8)$$

$$\text{and } \vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k} \right) \qquad (2.9.9)$$

Here $\frac{\partial V}{\partial x}$, $\frac{\partial V}{\partial y}$, $\frac{\partial V}{\partial z}$ show the partial differentiation of $V(x, y, z)$ with respect to x, y, z respectively. Moreover, the partial differentiation of $V(x, y, z)$ with respect to x means the differentiation of V with respect to **only x** (i.e. $\frac{\partial V}{\partial x}$) by taking y and z in the formula of V, as constants.

In equation (2.9.1), the values of $\vec{E}$ at all points between P and Q come in the calculation, while equations (2.9.3) and (2.9.8) give relation between the potential difference near a given point and the electric field at that point.

The direction of electric field is that in which the rate of decrease of electric potential with distance $\left(\frac{-dV}{dr}\right)$ is maximum and this direction is always normal to the equipotential surface.

This entire discussion is based on the property that electric field is a conservative field.

2.10 Potential Energy of a System of Point Charges

As shown in the figure 2.8, in a system of charges three point charges q_1, q_2 and q_3 are lying stationary at points A, B and C respectively. Their position vectors from the origin of a co-ordinate system are $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ respectively. We want to find the potential energy of this system.

$\vec{OA} = \vec{r_1}$, $\vec{OB} = \vec{r_2}$, $\vec{OC} = \vec{r_3}$

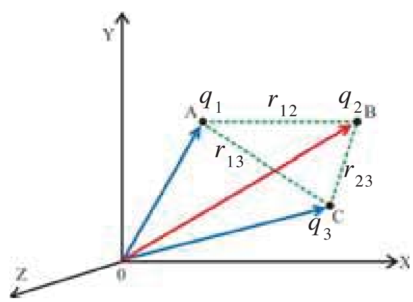


Figure 2.8 System of Point Charges

In the beginning we shall imagine that these charges are lying at infinite distances from the origin and also from each other. In this condition the electric force between them is zero, and their potential energy is also zero.

Moreover, the electric fields at A, B and C are also zero. From such a condition the work required to be done by the external forces (against the electric fields) to arrange them in the above mentioned configuration is stored in the form of potential energy of this system.

First, we bring the charge q_1 from infinite distance to point A. In this process since no electric field is present, the work done by the external force against the electric field is W_1 = zero. (You know that here the field produced by this charge itself is not to be considered.)

Now the charge set on q_1, produces an electric field and electric potential around it. The potential due to this charge q_1 at point B separated by distances r_{12} from it is (from equation 2.5.7) is

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}} \quad (2.10.1)$$

Where $r_{12} = |\vec{r_2} - \vec{r_1}|$

Hence the work required to be done by the external force to bring charge q_2 from infinite distance to point B, is $W_2 = q_2 V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$ (2.10.2) (from equation 2.4.2).

(If we want to consider a system of these **two charges only,** then the total work W_1 + $W_2 = \frac{1}{4\pi\varepsilon_0}\frac{q\,q_2}{r_{12}}$ is the electric potential energy of this system.)

Now q_1 and q_2 both will produce electric fields and electric potentials around them. The electric potential produced due to them at point C is $V_C = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{23}}$ (2.10.3)

Therefore, the work required to be done to bring charge q_3 from infinite distance to point C is

$$W_3 = (V_C)q_3$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2q_3}{r_{23}} \qquad (2.10.4)$$

Hence the total work to be done to set these three charges in the above arrangement (= $W_1 + W_2 + W_3$) is the electric potential energy U of this system.

$$\therefore\ U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2q_3}{r_{23}} \qquad (2.10.5)$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right] \qquad (2.10.6)$$

$$= k\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right] \qquad (2.10.7)$$

From this, in general, the potential energy of a system of n−charges can be wirtten as

$$U = \sum_{\substack{i=1 \\ i<j}}^{n} \frac{kq_iq_j}{q_{ij}} \qquad (2.10.8)$$

As the electric field is conservative; it does not matter, which charge comes earlier or later. In that case the electric potential energy does not change (and given by equation 2.10.8 only)

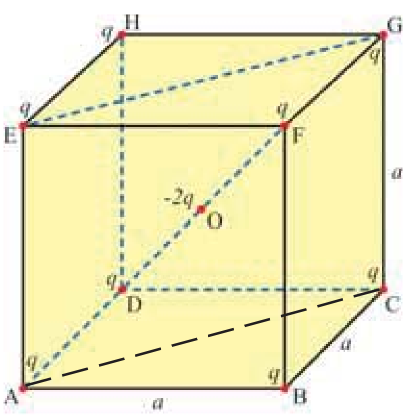


Illustration 7 : Calculate the potential energy of the system of charges, shown in the Figure.

Solution : The total potential energy of the system of charges is equal to the sum of the potential energy of all the pairs of charges.

(1) There are 12 pairs of charges like the AB pair. The distance between the electric charges in such pairs is equal to a.

The potential energy of all such pairs is

$$U_1 = \frac{kq^2}{a} \times 12 \qquad (1)$$

(2) There are 12 pairs of charges like the AC pair. The distance between charges in such a pair is $a\sqrt{2}$. ($\because$ AC = $\sqrt{AB^2 + AC^2}$ = $\sqrt{a^2 + a^2} = a\sqrt{2}$). Their potential energy is,

$$U_2 = \frac{kq^2}{a\sqrt{2}} \times 12 \qquad (2)$$

(3) There are 4 pairs of charges like the AG pair. The distance between charges in these pairs is equal to $a\sqrt{3}$. ($\because$ AG = $\sqrt{AC^2+CG^2}$ = $\sqrt{2a^2+a^2} = a\sqrt{3}$)

Their potential energy is $U_3 = \frac{kq^2}{a\sqrt{3}} \times 4$

(4) There are eight pairs of electric charges similar to AO in which distance between charges is $\frac{a\sqrt{3}}{2}$. (AO = $\frac{AG}{2}$ = $\frac{a\sqrt{3}}{2}$)

Their potential energy is $U_4 = -\frac{kq\,2q}{\left(a\frac{\sqrt{3}}{2}\right)} \times 8$ (4)

$\therefore$ total potential energy $U = U_1 + U_2 + U_3 + U_4$

$$\therefore\ U = \frac{12kq^2}{a} + \frac{12kq^2}{a\sqrt{2}} + \frac{4\,kq^2}{a\sqrt{3}} - \frac{32kq^2}{a\sqrt{3}}$$

$$= \frac{kq^2}{a}\left[12+\frac{12}{\sqrt{2}}+\frac{4}{\sqrt{3}}-\frac{32}{\sqrt{3}}\right] = \frac{kq^2}{a}\left[12(1+\frac{1}{\sqrt{2}})-\frac{28}{\sqrt{3}}\right]$$

2.11 The Potential Energy of an Electric Dipole in an External Electric Field

As shown in figure 2.9, an electric dipole AB is placed in a uniform electric field $\vec{E}$ in X–direction such that the axis of the dipole makes an angle θ with the field $\vec{E}$. Its dipole moment is $q(2a)$ in AB direction. The electric potential energy of this dipole means the algebraic sum of the electric potential energies of both of its charges ($+q$ and $-q$). We arbitrarily take the potential at the position of $-q$ charge as zero. Hence its potential energy becomes zero. Now we will find the potential enegy of $+q$ charge with respect to it and it will become the potential energy of the entire dipole.

AB = $2a$, AC = AB$cos\theta$

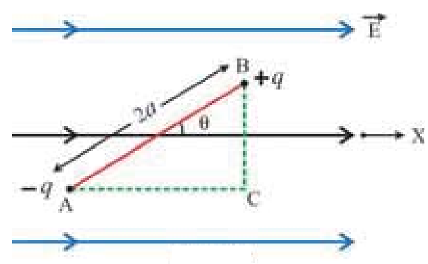


Figure 2.9 Potential Energy of Dipole

As the electric field is only in X-direction,

$$E = \frac{-\Delta V}{\Delta x} = \frac{-(V_B - V_A)}{AC}$$

$$= \frac{-V_B}{2a\,cos\theta} \quad (\because\ V_A = 0) \qquad (2.11.1)$$

$$\therefore\ V_B = -E\ (2a\ cos\theta) \qquad (2.11.2)$$

$\therefore$ Potential energy of $+q$ at B, is

$$U = qV_B = q[-E\ 2a\ cos\theta] \qquad (2.11.3)$$

$$= -E(q\ 2a\ cos\theta)$$

$$= -E\ p\ cos\theta\ [\because\ q(2a) = p]$$

$$= -\vec{E}\cdot\vec{p} \qquad (2.11.4)$$

$\therefore$ The potential energy of the entire dipole $U = -\vec{E} \cdot \vec{p} = -\vec{P} \cdot \vec{p}$ (2.11.5)

We note a few points :

(i) If the axis of the dipole is normal to the electric field, then $\theta = \frac{\pi}{2}$ and

$$U = Ep\ cos\frac{\pi}{2} = 0$$

(ii) If the axis of the dipole is parallel to the field. ($\vec{AB} \parallel \vec{E}$)

Then $\theta = 0$ $\therefore$ $U = -pE$. This is the **minimum value** of potential energy. Hence the dipole tries to arrange its axis parallel to the electric field, so that $\vec{p}$ becomes parallel to $\vec{E}$. In this condition dipole remains in stable equilibrium. (A system always tries to remain in such a state that its potential energy becomes minimum.) (For $\theta = \pi$, the dipole is in an unstable equilibrium.)

2.12 Electrostatics of Conductors

It is interesting to know the effects produced when metallic conductors are placed in the electric field or when electric charges are placed on such conductors.

(a) Effect of External Electric Field on Conductors :

In a metallic conductor there are positive ions situated at the lattice points and the free electrons are moving randomly between these ions. They are free to move within the metal but not free to come out of the metal. When such a conductor is placed in an external electric field $\vec{E}\,'$, the free electrons move under the effect of the force in the direction opposite to the field and get deposited on the surface of one end of conductor. And an equal amount of **positive charge** can be considered as deposited on the other end. Thus electric charges are **induced.** These induced charges produce an electric field $\vec{E}\,''$ inside the conductor, in the direction **opposite** to the external electric field $\vec{E}\,'$. When these two electric fields become equal in magnitude, the resultant (net) electric field ($\vec{E}$) inside the conductor becomes zero. (See figure 2.10). Now the motion of charges in the conductor stops, and the charges become steady (stationary) on the end-surfaces.

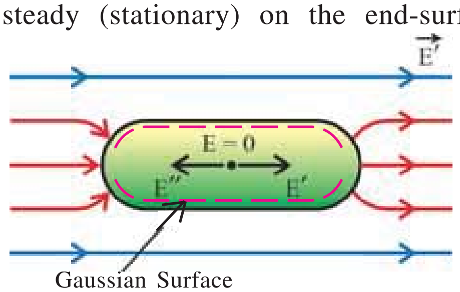


Figure 2.10 Conductor in Electric Field

Now let us consider a Gaussian Surface shown by dotted line, inside the conductor and close to the surface, as shown in figure 2.10. Every point on this surface is a point inside the conductor; the electric field $\vec{E}$ on this entire surface is zero. Hence the electric charge enclosed by it is also zero. ($\because \int \vec{E} \cdot \vec{dr} = \frac{q}{\varepsilon_0}$).

Thus in the case of a metallic conductor, placed in an external electric field,

(1) A steady electric charge distribution is induced on the surface of the conductor.

(2) The net electric field inside the conductor is zero.

(3) The net electric charge inside the conductor is zero.

(4) On the outer surface of the conductor, the electric field at every point is locally normal (perpendicular) to the surface. If the electric field were not normal (perpendicular) a component of electric field parallel to the surface would exist and due to it the charge would move on the surface. But now the motion is stoppd and the charges have become steady. Thus the component of electric field parallel to the surface would be zero, and hence the electric field would be normal to the surface.

(5) Since $\vec{E} = 0$ at every point inside the conductor, the electric potential everywhere inside the conductor is constant and equal to the value of potential on the surface.

(6) If there is a cavity inside the conductor then even when the conductor is placed in an external electric field ($\vec{E}'$), the net electric field inside the coductor is zero and also inside the cavity it is zero. Consider a Gaussian Surface around the cavity as shown in the figure 2.11. Since every point on this surface is a point inside the conductor, the electric field on this entire surface is zero.

Hence the total charge on the surface of the cavity is zero, ($\int \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$). And there is no charge inside the cavity. Hence the electric field everywhere inside the cavity is zero.)

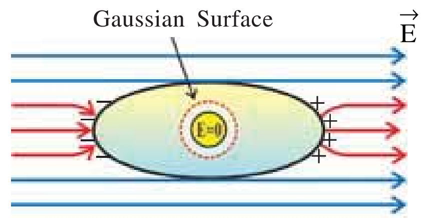


Figure 2.11
Cavity in a Conductor

This fact is called electrostatic shielding. If we are sitting in a car and suppose lightning strickes, we should close the doors of the car. (we suppose the car is fully made of metal !) By doing so, we happen to be in the cavity of car and we are protected due to electrostatic shielding.

(b) Effects Produced by Putting Charge on the Conductor :

In the above discussion we considered the effects produced when a metallic conductor is placed in an external electric field. Now we note the effects produced when a charge is placed on a metallic body, in the absence of an external electric field.

(1) Whether a metallic conductor is put in an external electric field or not and whether a charge is put or not, on it, in all such (but stable) conditions the **electric field everywhere inside** the conductor is always **zero.** This is a very important and a general fact. (This can be taken as a property to define a conductor).

(2) The charge placed on a coductor is always **distributed only on the outer surface** of the conductor. We can understand this by the fact that the electric field inside a coductor is zero. Consider a Gaussian Surface shown by the dots inside the surface and very close to it, (figure 2.12). Every point on it is inside the surface and not on the surface of conductor Hence the electric field at every point on this surface is zero. Hence according to Gauss's theorem the charge enclosed by that surface is also zero.

(3) In a stable condition these charges are steady on the surface. This shows that the electric field is locally normal to the surface. (See figure 2.12).

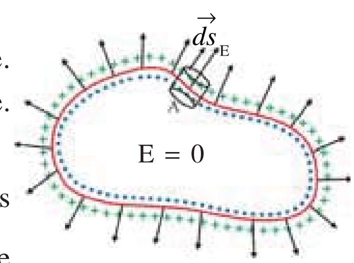


Figure 2.12

(4) The electric field at any point on the charged conductor is $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$, where $\hat{n}$ = unit vector coming out from the surface normally. To prove this, we consider a Gaussian surface of a pill-box (a cylinder) of extremely small length and extremely small cross-section ds. A fraction of it is inside the surface and the remaining part is outside the surface. The total charge enclosed by this pill-box is $q = \sigma ds$; where σ = surface density of charge on the conductor. At every point on the surface of the conductor $\vec{E}$ is perpendicular to the local surface element. Hence it is parallel to surface vector ($\vec{E} \parallel \vec{ds}$).

But inside the surface $\vec{E} = 0$. Hence the flux coming out from the cross-section of pill-box inside the surface = 0. For its side the area vector (surface vector) is normal to $\vec{E}$. Hence flux through it is zero. The flux coming out from the cross-section of pill-box outside the surface is $\vec{E} \cdot \vec{ds} = Eds$.

$\therefore$ Total flux = E ds

According to Gauss's theorem, $E\,ds = \frac{q}{\varepsilon_0} = \frac{\sigma\,ds}{\varepsilon_0}$ (2.12.1)

$\therefore\ E = \frac{\sigma}{\varepsilon_0}$ (2.12.2)

In the vector from $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$ (2.12.3)

If σ is positive, $\vec{E}$ is in the direction of normal coming out from the surface. If σ is negative $\vec{E}$ is in the direction of normal entering into the surface.

(5) If some charge is placed inside a cavity in the conductor, then the charges are so induced on the surface of the cavity and on the outer surface of conductor that the electric field in the region which is inside the conductor but outside the cavity becomes zero. The electric field inside the cavity is non-zero and the electric field outside the conductor due to that charge is also non-zero.

[**Note (For information only) :** In the above discussion we have considered the coductors to be insulated.

The sharp ends of the conductor have a large electric charge density. The electric field near such a region is very strong. This strong electric field can strip the electrons from the surface of the metal. This event is known as Corona discharge. In general, this event is called dielectric breakdown.

The electrons escaping the surface of a metal perform an accelerated motion, colliding with the air particles coming in their way. The excited atoms of the energetic particles emit electromagnetic waves and a greenish glow is observed. Apart from the above process, the ionization of the air molecules also takes palce, during collision

Sailors long ago saw these glows at the pointed tops of their masts and spars and dubbed the phenomenon St. Elmo's fire.]

2.13 Capacitors and Capacitance

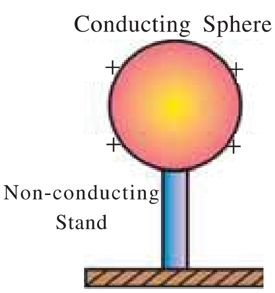
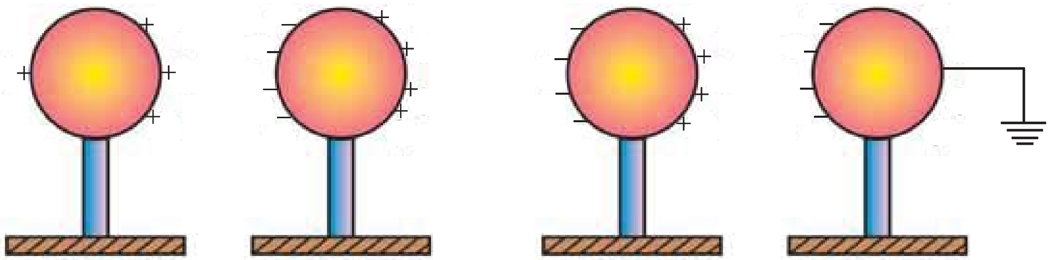


Figure 2.13

Consider an insulated conducting sphere as shown in the figure 2.13. Suppose we go on gradually adding positive charge on this sphere. As the charge on the sphere is gradually increased, the potential (V) on the surface of the sphere and the electric field around the sphere also go on gradually increasing. In this process at some one stage the electric field becomes sufficiently strong to ionize the air particles around the sphere. Hence the charge on the sphere is conducted through air and insulating property of air gets distroyed (i.e. it is not sustained.). This effect is called dielectric breckdown.

Thus the charge on the sphere is leaked and now the sphere is not able to store any additional charge. During this entire process the ratio of the charge (Q) on the sphere and the potential (V) on the sphere remains constant. This ratio is called the capacitance of the sphere. [$C = \frac{Q}{V}$]



(a) (b) (c)

Figure 2.14

The maximum electric field upto which an insulating (non-conducting) medium can maintain its insulating property is called the **dielectric strength** of that medium (or the minimum electric field which starts ionization in a given non-conducting-medium is called its **dielectric strength.**). For air the dielectric strength is nearly 3000 $\frac{V}{mm}$.

Now, if we want to increase the capacity of the above mentioned sphere to store charge (capacitance C), then place another, insulated conducting sphere near the first one. So, electric charge is induced in this second sphere. See figure 2.14(b). If the second sphere is connected to Earth, as in figure 2.14(c) electrons from Earth will flow to it and neutralize the positive charge in it. Now due to negative charge on the second sphere the potential on the surface of the first sphere and the electric field near it are decreased. Now the capacity to store charge on the first sphere increases, as compared to earlier. In this condition also the ratio of the electric charge Q and the p.d. (V) between two spheres at every stage is found to be constant. This ratio is called the capacitance C of this system of two spheres. The value of this capacitance depends on the dimensions of the spheres, their relative arrangement and the medium between them.

"A device formed by two conductors insulated from each other is called a capacitor." These conductors are called the plates of the capacitor. The conductor with positive charge is called the positive plate and the one with negative charge is called the negative plate. The charge on the positive plate is called the charge on the capacitor and the potential difference between the two conductors is called the potential difference (V) between the two plates of the capacitor. Here the capacitance of the capacitor is $C = \frac{Q}{V}$.

The SI unit of capacitance is coulomb / volt and in memory of the great sceintist Michael Faraday it is known as Farad. Its symbol is F. Farad is a large unit for practical purposes and hence smaller units microfarad ($1\ \mu F = 10^{-6} F$) nanofarad ($1\ nF = 10^{-9} F$) and picofarad ($1\ pF = 10^{-12} F$) are used in practice.

A capacitor having a definite capacitance is shown by the symbol ⊣⊢ and the one having a variable capacitance is shown by the symbol ⊣↗⊢.

Moreover, a single conducting sphere of radius R and having charge Q can also be considered as a capacitor, because it also has 'some' capacity to store charge. For such a capacitor other conductor (with $-Q$ charge) is considered to be at infinite distance (separation). Taking the potential at infinite distance from the sphere as zero, the potential on the surface of this sphere is $V = \frac{kQ}{R}$. Hence the potential difference between this sphere and the other one imagined at infinite distance is also $V = \frac{kQ}{R}$.

∴ The capacitance of this sphere is $C = \frac{Q}{V} = \frac{QR}{kQ} = \frac{R}{k} = 4\pi\varepsilon_0 R$ ($\because K = \frac{1}{4\pi\varepsilon_0}$). Earth can also be considered as a capacitor. You may calculate its capacitance.

2.14 Parallel Plate Capacitor

In such a capacitor, two conducting parallel plates of equal area (A) are insulated from each other and kept at a separation of (d). (See figure 2.15)

Considering vacuum (or air) as the non-conducting medium between them, we shall obtain the formula for its capacitance.

Suppose, the electric charge on this capacitor is Q. Therefore, the value of the surface density of charge on its plates is $\sigma = \frac{Q}{A}$. The value of d is kept very small as compared to the dimension of each plate. Due to this, the non-uniformity of the electric field near the ends of the plates can be neglected and in the entire region between the plates the electric field $\vec{E}$ can be taken as constant.

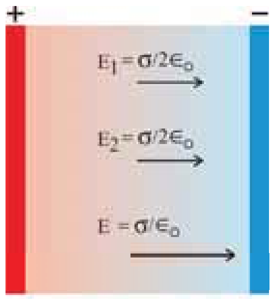


Figure 2.15 Parallel Plate Capacitor

The uniform electric field in the region between two plates due to the positive plate is $E_1 = \frac{\sigma}{2\varepsilon_0}$ in the direction from positive to negative plate. (2.14.1)

Similarly the uniform electric field in the same region due to the negative plate, is $E_2 = \frac{\sigma}{2\varepsilon_0}$ (2.14.2)

(Also in the direction from positive to negative plate.)

Since these two fields are in the same direction, the resultant uniform electric field is

$$E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} \quad (2.14.3)$$

It is in the direction from positive to negative plate.

$$\therefore E = \frac{Q}{\varepsilon_0 A} \quad (2.14.4)$$

In the regions on the other sides of the plates, E_1 and E_2 being equal but in opposite direction, the resultant electric field becomes zero.

If the potential difference between these two plates is V, then $V = Ed$ (2.14.5)

∴ From equations (2.14.4) and (2.14.5),

$$V = \frac{Q}{\varepsilon_0 A} d \quad (2.14.6)$$

∴ From the formula $C = \frac{Q}{V}$, we get the capacitance of parallel plate capacitor as

$$C = \frac{\varepsilon_0 A}{d} \quad (2.14.7)$$

From equation (2.4.7), it is clear that if the distance between two plates each of 1 m × 1 m is 1 mm, its capacitance is $C = \frac{(8.85\times10^{-12})(1)}{10^{-3}} = 8.85 \times 10^{-9}$F.

If we want 1F capacitance, then the area of each plate kept at a separation of 1 mm should be $A = \frac{Cd}{\varepsilon_0} = \frac{(1\times10^{-3})}{8.85\times10^{-12}} = 1.13 \times 10^8$ m^2. Thus each of the length and the breadth of each plate should be nearly 1×10^4 m = 10 km.

2.15 Combinations of Capacitors

The system, formed by the combination of capacitors having capacitances C_1, C_2, ..., C_n has some equivalent (effective) capacitance C. We shall discuss two types of combinations.

(a) Series Combination of Capacitors

The arrangement formed by joining the capacitors having capacitances C_1, C_2, C_3, ..., C_n by conducting wires as shown in figure 2.16 is called the series combination of capacitors.

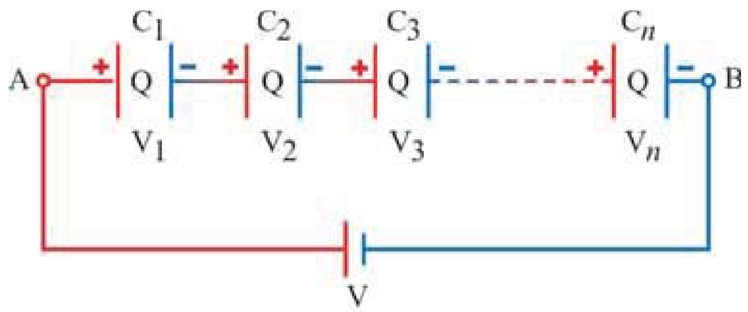


Figure 2.16 Series Combination of Capacitors

In such a condition the charge on every capacitor has the same value Q. As (−Q) charge is deposited by the battery on one plate, it induces (+Q) charge on the other plate. For this (−Q) charge from the second plate will be deposited on the near plate of the next capacitor. This induces +Q charge on the other plate. This continues further. Thus all capacitors have equal charge. but the potential difference between the two plates of different capacitors is different. From the figrue it is clear that

$$V = V_1 + V_2 + V_3 + ... + V_n \tag{2.15.1}$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + ... + \frac{Q}{C_n} \tag{2.15.2}$$

$\left(\because C_1 = \frac{Q}{V_1}, ... \text{ etc.}\right)$

$$\therefore \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n} \tag{2.15.3}$$

If the effective capacitance of this combination is C,

$$\frac{V}{Q} = \frac{1}{C} \tag{2.15.4}$$

$$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n} \tag{2.15.5}$$

Thus the value of effective capacitance is even smaller than the smallest value of capacitance in the combination.

[Note that here the formula obtained for series combination is similar to the formula for effective (equivalent) resistance obtained for the parallel combination of the resistances.]

(b) Parallel Combination of Capacitors

The arrangement formed by joining the capacitors having capacitances C_1, C_2, C_3 by conducting wires as shown in figure 2.17 is called the parallel combination of capacitors.

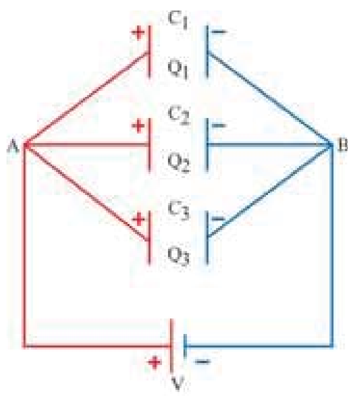


Figure 2.17 Parallel Combination of Capacitors

In such a combination the potential difference (V) between the plates of every capacitor is the same and is equal to the potential difference between their common points A and B. But the charge Q on every capacitor is different.

Here,
$$\left.\begin{aligned} Q_1 &= C_1 V \\ Q_2 &= C_2 V \\ Q_3 &= C_3 V \end{aligned}\right\} \quad (2.15.6)$$

And the total electric charge

$$Q = Q_1 + Q_2 + Q_3$$
$$= C_1V + C_2V + C_3V$$
$$= (C_1 + C_2 + C_3)V \quad (2.15.7)$$

If the effective capacitance of this parallel combination is C, then

$$C = \frac{Q}{V} = C_1 + C_2 + C_3 \quad (2.15.8)$$

If such n-capacitors are joined in parallel connection, the effective capacitance is

$$C = C_1 + C_2 + C_3 + \text{......} + C_n \quad (2.15.9)$$

Here, as the values of capacitances are added the value of effective capacitance is even greater than the largest value of capacitance in the connection.

[Note that the formula obtained here for parallel combination is similar to the formula for effective (equivalent) resistance obtained for the series combination of resistances.]

Illustration 8 : Prove that the force acting on one plate due to the other in a parallel plate capacitor is $F = \frac{1}{2}\frac{CV^2}{d}$.

Solution : The electric field due to one plate is $E_1 = \frac{\sigma}{2\varepsilon_0}$ (1)

A second plate having charge σA is present in the above electric field.

∴ The force acting on the second plate is

$F = (\sigma A)E_1$

Substituting the value of E_1 from (1), we have,

$$F = \frac{\sigma^2 A}{2\varepsilon_0}$$

But $\sigma = \frac{Q}{A}$

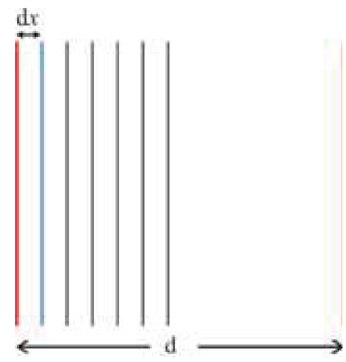
$$\therefore F = \frac{\frac{Q^2}{A^2}.A}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A} = \frac{Q^2/d}{2\varepsilon_0 A/d} = \frac{Q^2}{2dC} \quad \left(\because \frac{\varepsilon_0 A}{d} = C\right)$$

$$\therefore F = \frac{1}{2}\frac{CV^2}{d} \quad (\because Q = CV)$$

Illustration 9 : Figure shows an infinite number of conducting plates of infinitesimal thickness such that consecutive plates are sparated by a small distance dx spread over a distance d to form a capacitor. Calculate the value of the capacitance of such an arrangement.

Solution : The capacitance of each of the capacitors in the above arrangement, $dC = \frac{\varepsilon_0 A}{dx}$

All these capacitors are in series combination with each other.
Therefore the total capacitance C is obtained from



$$\frac{1}{C} = \frac{1}{dC} + \frac{1}{dC} + \ldots$$

$$= \frac{dx}{\varepsilon_0 A} + \frac{dx}{\varepsilon_0 A} + \ldots$$

$$= \frac{1}{\varepsilon_0 A}(dx + dx + \ldots\ldots + dx)$$

$$\therefore \frac{1}{C} = \frac{d}{\varepsilon_0 A}$$

$$\therefore C = \frac{\varepsilon_0 A}{d}$$

This is equivalent to the capacitance of the capacitor formed by the first and the last plate of the above arrangement.

2.16 Energy Stored in a Charged Capacitor

In order to establish a charge on the capacitor, work has to be done on the charge. This work is stored in the form of the potential energy of the charge. Such a potential energy is called the energy of capacitor.

Suppose the charge on a parallel plate capacitor is Q. In this condition each plate of the capacitor is said to be lying in the electric field of the other plate.

The magnitude of the uniform electric field produced by one plate of capacitor is

$$= \frac{\sigma}{2\varepsilon_0} \ldots. \quad (2.16.1)$$

where $\sigma = \frac{Q}{A}$ and A = area of each plate.

Hence by taking arbitrarily the potential on this plate as zero, that of the other plate at distance d from it will be $= \left(\frac{\sigma}{2\varepsilon_0}\right)d$ (2.16.2)

From this, the potential energy of the first plate is zero and that of the second plate will be = (potential) (charge Q on it)

$$= \left(\frac{\sigma d}{2\varepsilon_0}\right)Q \quad (2.16.3)$$

$\therefore$ Energy stored in the capacitor

$$U_E = \frac{\sigma d Q}{2\varepsilon_0} = \left(\frac{Q}{A}\right)\frac{dQ}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A/d} \quad (2.16.4)$$

$$= \frac{Q^2}{2C} \quad (2.16.5)$$

where, $C = \frac{\varepsilon_0 A}{d}$ = capacitance of capacitor.

Moreover, $C = \frac{Q}{V}$. From equation (2.16.5) and this formula we can write

$$U_E = \frac{VQ}{2} \quad (2.16.6)$$

and $U_E = \frac{1}{2}CV^2$ (2.16.7)

We have derived these equations (2.16.5), (2.16.6) and (2.16.7) for the parallel plate capacitor, but in general they are true for all types of capacitor.

To show energy stored in the capacitor in the form of energy density :

The energy stored in the capacitor is $U_E = \frac{1}{2}CV^2$. This energy is stored in the region between the two plates, that is, in the volume Ad, where A = area of each plate and d = separation between them. Hence, if we write the energy stored **per unit volume** in the region between the plates – that is energy density – as ρ_E, then

$$\rho_E = \frac{U_E}{\text{Volume}} = \frac{\frac{1}{2}CV^2}{Ad} \tag{2.16.8}$$

$$= \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)\frac{V^2}{Ad} \tag{2.16.9}$$

$$= \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)\left(\frac{V}{d}\right) \tag{2.16.10}$$

$$= \frac{1}{2}\varepsilon_0 E^2 \qquad \left(\because \ \frac{V}{d} = E\right) \tag{2.16.11}$$

Where $\frac{V}{d}$ = E = electric field between the two plates. Thus the energy stored in the capacitor can be considered as the energy stored in the electric field between its plates.

We have obtained this equation for a parallel plate capacitor but it is a result in general and can be used for the electric field of any arbitrary charge distribution.

Illustration 10 : A capacitor of 4 μF value is charged to 50 V. The above capacitor is then connected in parallel to a 2 μF capacitor. Calculate the total energy of the above system. The second capacitor is not charged prior to its connection with the 4 μF capacitor.

Solution : The energy stored in the capacitor of 4 μF will be

$$W_1 = \frac{1}{2}C_1V^2$$

$$= \frac{1}{2} \times 4 \times (50)^2 = 2 \times 2500 = 5000 \ \mu J$$

The two capacitors are connected in parallel. Let q_1 and q_2 be the electrical charges on capacitors C_1 and C_2 respectively after connection. If V' is their common potential difference across the capacitors. $\left(V' = \frac{q_1}{C_1} = \frac{q_2}{C_2}\right)$

$$\frac{q_1}{q_2} = \frac{C_1}{C_2}$$

$$\therefore \ \frac{q_1+q_2}{q_2} = \frac{C_1+C_2}{C_2} \tag{1}$$

By the law of conservation of charge.

$$q_1 + q_2 = Q \tag{2}$$

Where Q is the initial charge

Now, $Q = C_1V = (4)(50)$

$= 200 \ \mu C$

Putting equation (2) in equation (1) and substituting the value of Q, we have,

$$\frac{200}{q_2} = \frac{(4+2)}{2}$$

$$\therefore\ q_2 = \frac{200\times 2}{6} = \frac{200}{3}\ \mu C$$

From Equation (2)

$$q_1 = 200 - \frac{200}{3}$$

$$= \frac{400}{3}\ \mu C$$

Calculation of energy : The energy of the first capacitor

$$\frac{q_1^2}{2C_1} = \left(\frac{400}{3}\right)^2 \times \frac{1}{2\times 4} = 2222\ \mu J$$

The energy of the second capacitor

$$\frac{q_2^2}{2C_2} = \left(\frac{200}{3}\right)^2 \times \frac{1}{2\times 2} = 1111\ \mu J$$

The total energy of the system, after combination = 2222 + 1111 = 3333 = μJ

Thus the energy decreased by 5000 − 3333 = 1667 μJ. This energy is dissipated in the from of heat.

2.17 Dielectric Substances and their Polarisation

Non-conducting materials are called **dielectric**. Faraday found that when a dielectric is introduced between the plates of a capacitor, the capacitance of the capacitor is increased. In order to understand how does this happen, we should know about the effects produced when a dielectric is placed in an electric field. Dielectric materials are of two types (1) polar and (2) non-polar.

A dielectric is called a polar dielectric if its molecules possess a permanent dipole moment (e.g. HC*l*, H_2O, ... etc.) If the molecules of the dielectric do not possess a permanent dipole moment, then that dielectric is called a non-polar dielectric (e.g. H_2, O_2, CO_2, ... etc.)

(a) Non-polar Molecule : In a non-polar molecule, the centre of the positive charge and the centre of the negative charge coincide with each other. Hence they do not possess a permanent dipole moment. Now, when it is placed in a uniform electric field ($\vec{E}_0$), these centres are displaced in mutually opposite directions. Hence they now, possess a dipole moment $p = qd$, where d = the distance between centres of positive and negative charges after being displaced, q = the value of positive or negative charge (See figure 2.18).

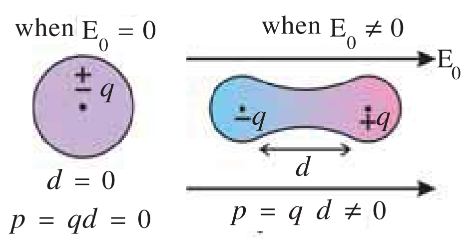


Figure 2.18 Polarisation of a Non-polar Molecule

Thus an electric dipole is induced in it. In other words due to an external electric field a dielectric made of such molecules is said to be polarised. If the external electric field ($\vec{E}_0$) is not very strong, it is found that this dipole moment of molecule is proportional to $\vec{E}_0$.

$$\therefore\ \vec{p} = \alpha \vec{E}_0 \qquad (2.17.1)$$

where α is called the polarisability of the molecule.

From units of $\vec{p}$ and $\vec{E}_0$, the unit of α is $C^2\ m\ N^{-1}$

(b) Polar Molecule : A polar molecule possesses a permanent dipole moment $\vec{p}$, but such dipole moments of different molecules of the substance are randomly oriented in all possible directions and hence the resultant dipole moment of the substance becomes zero.

Now, on applying an external electric field a torque acts on every molecular dipole. Therefore, it rotates and tries to become parallel to the electric field. Thus a resultant dipole moment is produced. In this way the dielectric made up of such molecules is said to be polarised. Moreover, due to thermal oscillations the dipole moment also gets deviated from being parallel to electric field. If the temperature is T, the dipoles will be arranged in such an equilibrium condition that the average thermal energy per molecule ($\frac{3}{2}k_B T$) balances the potential energy of dipole ($U = -\vec{p} \cdot \vec{E}_0$) in the electric field. At 0 K temperature since the thermal energy is zero, the dipoles become parallel to the electric field. We shall only discuss such an ideal situation.

(c) When there is air (or vacuum) between the charged plates of a capacitor, the electric field between the plates is $E_0 = \frac{\sigma_f}{\varepsilon_0}$. (2.17.2)

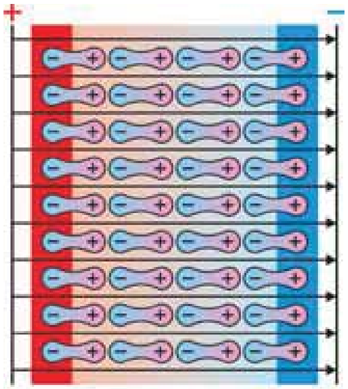


Figure 2.19 Polarisation in Dielectric

where σ_f = value of surface charge density on each plate.

The charge on these plates is called the free charge, because its value can be adjusted at our will (by joining proper battery).

Here, the area of each plate is = A. Now on placing a slab of dielectric material (polar or non-polar) in the region between the plates, the polarisation produced by the electric field $\vec{E}_0$ is shown in the figure 2.19. We want to find the electric field inside the dielectric.

It is clear from the Figure that the opposite charges in the successive dipoles inside the slab cancel the effect of each other, as they are very close to each other and a net (resultant) charge resides only on the faces of the slab, close to the plates. These charges are called **induced charges** or the **bound charges** or the **polarisation charges.** The charge induced on the surface of the slab close to the positive plate is $-\sigma_b A$ and that on the surface close to the negative plate is $+\sigma_b A$, where $-\sigma_b$ and $+\sigma_b$ are, the surface densities of the **bound charges** on **the** respective **surfaces.** This induced charges form a dipole. Its dipole moment is $P_{total} = (\sigma_b A)d$ (2.17.3)

where, d = thickness of the slab = distance between two plates. (if sides of slab touch the plates)

Here, Ad = volume of slab = V (2.17.4)

The dipole moment produced per unit volume is called the **intensity of polarisation** or in short **polarisation (P).**

$$\therefore\ P = \frac{P_{total}}{volume} = \frac{(\sigma_b A)d}{Ad} = \sigma_b \qquad (2.17.5)$$

Thus the magnitude of polarisation(P) in a dielectric is equal to the **surface density** of **bound charges (σ_b),** induced on its surface. The electric field produced by these induced charges is in the direction opposite to the external electric field $\vec{E}_0$. Hence, now the resultant electric field E inside the dielectric can be considered as produced due to ($\sigma_f - \sigma_b$).