Number System and Operations

NOTES

Integers

Integers are the set of all positive and negative numbers including zero.

Addition of Integers

- The sum of two integers is an integer or integers are closed under addition.
- The sum of two integers is commutative.
- The addition of integers is associative.
- 0 is the additive identity for integers.

Subtraction of Integers

Integers are closed under subtraction but they are neither commutative nor associative. Thus, if a and b are two integers then a - b is also an integer.

Multiplication of Integers

- The product of two integers is an integer or integers are closed under multiplication.
- The product of two integers is commutative.
- Multiplication of integers is associative.
- 1 is the multiplicative identity for integers.
- For any integer a, $a \times 0 = 0 \times a = 0$

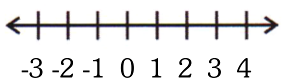
While multiplying a positive integer and a negative integer, we multiply them as whole numbers and put a minus sign before the product.

Division of integers

- Division of integers is not communicative.
- For any integer a, a $\div$ 0 is not defined but 0 $\div$ a = 0 for a $\neq 0$.
- Any integer divided by 1 gives the same integer.
- Division of integers is not associative.

Comparing Integers

We can compare two different integers by looking at their positions on the number line. For any two different integer on the number line, the integer on the right is greater than the integer on left.



➢ **Example:**

Simplify: $\mathbf{14-\left[3+15\ \{15\times 3-2\left(13-25\right)\}\right]}$

(a) 1024 (b) -1024

(c) 1038 (d) -1038

(e) None of these

Ans. (b)

Explanation: $14-\left[3+15\{15\times 3-2\left(13-25\right)\}\right]$

$=14-\left[3+15\{15\times 3-2\left(-12\right)\}\right]=14-\left[3+15\{15\times 3+24\}\right]$

$=14-\left[3+15\times 69\right]=14-1038=-\ 1024$

➢ **Example:**

Find the value of $\mathbf{244-[13+25\{15\div 3-(13-\overline{24-12})\}]}$.

(a) -144 (b) -156

(c) 144 (d) 131

(e) None of these

Ans. (d)

Explanation: $244-[13+25\ \{15\div 3-(13-\overline{24-12})\}]$

$=244-\left[13+25\{15\div 3-\left(13-12\right)\}\right]=244-\left[13+25\{15\div 3-1\}\right]$

$=244-\left[13+25\times 4\right]=244-113=131$

Absolute Value of an Integer

Absolute value of any integer a is defined as follows

$$|\,a\,|=\begin{cases} a,\ if\ a\geq 0 \\ -a, if\ a<0 \end{cases}$$

➢ **Example:**

Evaluate: $\mathbf{|\,44-[1+5\{12\div 4-2(1-\overline{4-3})\}]\,|}$

(a) 17 (b) -14

(c) 28 (d) 12

(e) None of these

Ans. (c)

Explanation: $|\,44-[1+5\{12\div 4-2(1-\overline{4-3})\}]\,|=\left|\ 44-\left[1+5\ \{12\div 4-2\ \left(1-1\right)\}\right]\ \right|$

$=|\,44-\left[1+5\{12\div 4-0\}\right]\ |=|\,44-\left[1+5\times 3\right]\ |=|\ 44-\ 16\ |=|28|=28$

Fraction

Fractional number is defined as a part of whole.

Properties of Fractions

- Multiplication by the same number with numerator and denominator of the fraction results the same fraction.
- For every two fractions with equal denominators, the larger fraction is the fraction with the larger numerator.
- For every two fractions with equal numerators, the larger fraction is the fraction with the smaller denominator.

Addition and Subtraction of the Fractions

Add the numerators of the given fractions after making their denominators equal by taking their LCM. Subtract the numerators of the given fractions after making their denominators equal by taking their LCM.

Multiplication and Division of the Fractions

Multiplication of fractions is similar to the multiplication of arithmetic numbers. If this fact is kept in mind the student will have little difficulty in mastering multiplication in algebra. For instance: we recall that to multiply a fraction by a whole number, it is simply the multiplication of the numerator by the whole numbers.

For multiplying a fraction by another fraction follow these steps:

Step 1: Multiply the numerators of the given fractions.

Step 2: Multiply the denominators of the given fractions.

Step 3: Simplify the fraction if needed.

To divide the given fractions, first find the reciprocal of divisor and then multiply the given fraction by that reciprocal.

Comparison of Fractions

For comparing two fractions $\frac{a}{b} and \frac{c}{d}$ multiply numerator of the first fraction with the denominator of the second and vice versa. Compare the product $a \times d$ and $b \times c$.

if $a \times d > b \times c$ *then* $\frac{a}{b} > \frac{c}{d}$

if $a \times d > b \times c$ *then* $\frac{a}{b} < \frac{c}{d}$

- **Example:**

A pillar has three colours, of which $\frac{7}{8}$ m of it yellow, $\frac{12}{24}$ m of it is green and $3\frac{1}{2}$ m of it is white. Find the length of the pillar.

(a) $4\frac{7}{8}$ m

(b) $5\frac{3}{4}$ m

(c) $4\frac{7}{24}$ m

(d) $8\frac{2}{9}$ m

(e) None of these

Ans. (a)

Explanation: LCM of 8, 24 and 2 is 24. Therefore,

$\frac{7}{8}=\frac{7}{8}\times\frac{3}{3}=\frac{21}{24}$ and $\frac{7}{2}=\frac{7}{2}\times\frac{12}{12}=\frac{84}{24}$

Now, $\frac{21}{24}+\frac{12}{24}+\frac{84}{24} = \frac{21+12+84}{24}=\frac{117}{24}=\frac{39}{8}=4\frac{7}{8}$

Decimals

Decimal number is an another way to write a fraction. For example, ₹ 4.25, ₹ 0.25. The number after the decimal or right side from the decimal is part of one rupee. ₹ 0.25 can also be written as $\frac{25}{100}$ of a rupee. The base of decimal number system is 10. Base 10 or decimal number system can always change the place value of the given number by one position, either multiplying or dividing by 10.

Note: A decimal number 2.234444 .... can be written as $2.23\overline{4}$.

➢ **Example:**

Find the value of $\left(\frac{2.05\times2.05+2.05\times1.34+1.34\times1.34}{2.05\times2.05\times2.05-1.34\times1.34\times1.34}\right)$ correct upto 3 decimal places.

(a) 1.404

(b) 1.308

(c) 1.409

(d) 1.508

(e) None of these

Ans. (c)

Explanation: let a = 2.05 and b = 1.34,

Then $\left(\frac{2.05\times2.05+2.05\times1.34+1.34\times1.34}{2.05\times2.05\times2.05-1.34\times1.34\times1.34}\right)=\frac{a^2+ab+b^2}{a^3-b^3}$

$=\frac{1}{a-b}$, Put the values of a and b and get the result.

➢ **Example:**

The ratio of copper and zinc in an alloy is 8 : 7. If the weight of the copper in the alloy is 1.12 kg, the weight of zinc in it is __________

(a) 9.8 kg

(b) 0.98 kg

(c) 98 kg

(d) 1.26 kg

(e) None of these

Ans. (b)

Explanation: Weight of zinc $= \frac{1.12}{8} \times 7 = 0.98$ kg

- **Example:**

Express one second into hours.

(a) 0.0025 hours (b) 1.0256 hours

(c) 0.00028 hours (d) 1.000126 hours

(e) None of these

Ans. (c)

Explanation: One second $= \frac{1}{60 \times 60}$ *hours* = .00028 hours

Rational Numbers

A number which is in the form of $\frac{a}{b}$, where a and b are integers and $b \neq 0$ is called a rational number.

Rational numbers are of two types:

- Positive rational numbers
- Negative rational numbers

In the standard form we always write the denominator as positive.

Properties of Rational Numbers

- A rational number remains unaltered if we multiply numerator and denominator by the same non - zero numbers.
- A rational number remains same if we divide numerator and denominator by the same non - zero numbers.
- Every whole number is a rational number but the rational number may or may not be a whole number.
- Every integers is a rational number but the rational number may or may not be an integer.

Equivalent Rational Numbers

If we multiply or divide the numerator and denominator of a rational number by the same non-zero integers then we get equivalent rational number.

Comparing Rational Numbers

For two rational numbers which are in the standard form $\frac{a}{b}$ and $\frac{c}{d}$. We find the product ad and bc

- If ad > bc then $\frac{a}{b} > \frac{c}{d}$

- If ad < bc then $\frac{a}{b} < \frac{c}{d}$

- **Example:**

Which one of the following rational numbers is in standard form?

$\mathbf{\frac{2}{-5}, \frac{3}{-4}, \frac{-5}{7}, \frac{-7}{-8}}$

(a) $\frac{2}{-5}$

(b) $\frac{3}{-4}$

(c) $\frac{-5}{7}$

(d) $\frac{-7}{-8}$

(e) None of these

Ans. (c)

Explanation: standard form of $\frac{2}{-5}$ is $\frac{-2}{5}$ and standard form of $\frac{3}{-4}$ is $\frac{-3}{4}$

Also, standard form of $\frac{-5}{7}$ is $\frac{-5}{7}$ and standard form of $\frac{-7}{-8} = \frac{7}{8}$

- **Example:**

Which one of the two fractions is greater $\mathbf{\frac{-7}{8}}$ or $\mathbf{\frac{-6}{7}}$?

(a) $\frac{-7}{-8}$

(b) $\frac{-6}{7}$

(c) both are equal

(d) cannot compare

(e) None of these

Ans. (b)

Explanation: $\frac{-7}{8} = \frac{-7}{8} \times \frac{7}{7} = \frac{-49}{56}$ and $\frac{-6}{7} = \frac{-6}{7} \times \frac{8}{8} = \frac{-48}{56}$

Since $-48 > -49$, therefore $\frac{-6}{7} > \frac{-7}{8}$