SOLVED PAPER VITEEE 2014

PART - I (PHYSICS)

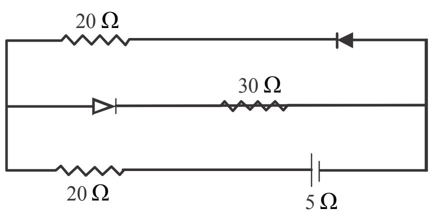
1. The amplification factor of a triode is 50. If the grid potential is decreased by 0.20 V. What increase,in plate potential will keep the plate current unchanged?

(a) 5 V (b) 10 V
(c) 0.2 V (d) 50 V

2. If the nuclear fission,piece of uranium of mass 5.0 g is lost, the energy obtained in kWh is

(a) 1.25×10^7 (b) 2.25×10^7
(c) 3.25×10^7 (d) 0.25×10^7

3. Current in the circuit will be



(a) $\frac{5}{40}$ A (b) $\frac{5}{50}$ A

(c) $\frac{5}{10}$ A (d) $\frac{5}{20}$ A

4. An installation consisting of an electric motor driving a water pump left 75 L of water per second to a height of 4.7 m.If the motor consumes a power of 5 kW, then the efficiency of the installation is

(a) 39% (b) 69%
(c) 93% (d) 96%

5. A potential difference across the terminals of a battery is 50 V when 11 A current is drawn and 60 V,when 1 A current is drawn. The emf and the internal resistance of the battery are

(a) 62 V, 2 Ω (b) 63 V, 1 Ω
(c) 61 V, 1 Ω (d) 64 V, 2 Ω

6. Beyond which frequency, the ionosphere bands any incident electromagnetic radiation but do not reflect it back towards the earth?

(a) 50 MHz (b) 40 MHz
(c) 30 MHz (d) 20 MHz

7. A metallic surface ejects electrons. When exposed to green light of intensity I but no photoelectrons are emitted,when exposed to yellow light of intensity I.It is possible to eject electron from the same surface by

(a) yellow light of same intensity which is more than I
(b) green light of any intensity
(c) red light of any intensity
(d) None of the above

8. An electron moves at right angle to a magnetic field of 5×10^{-2} T with a speed of 6×10^7 m/s. If the specific charge of the electron is 1.7×10^{11} C/kg. The radius of the circular path will be

(a) 2.9 cm (b) 3.9 cm
(c) 2.35 cm (d) 2 cm

9. A solenoid 30 cm long is made by winding 2000 loops of wire on an iron rod whose cross-section is 1.5 cm^2.If the relative permeability of the iron is 6000. What is the self-inductance of the solenoid?

(a) 1.5 H (b) 2.5 H
(c) 3.5 H (d) 0.5 H

10. A coil of resistance 10 Ω and an inductance 5 H is connected to a 100 V battery. The energy stored in the coil is

(a) 325 erg (b) 125 J
(c) 250 erg (d) 250 J

11. A galvanometer has current range of 15 mA and voltage range 750 mV. To convert this galvanometer into an ammeter of range 25 A, the required shunt is

(a) 0.8 Ω (b) 0.93 Ω
(c) 0.03 Ω (d) 2.0 Ω

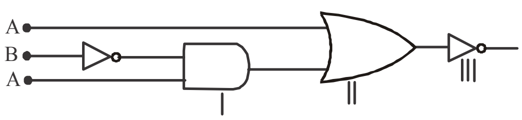
12. The denial cell is balanced on 125 cm length of a potentiometer. Now, the cell is short circuited by a resistance of 2Ω and the balance is obtained at 100 cm. The internal resistance of the denial cell is

(a) $\frac{4}{3}\Omega$ (b) 1.5 Ω
(c) 1.25 Ω (d) 0.5 Ω

13. Four resistance of 10Ω, 60Ω, 100Ω and 200Ω respectively taken in order are used to form a Wheatstone's bridge. A 15V battery is connected to the ends of a 200 Ω resistance, the current through it will be
(a) 7.5×10^{-5} A (b) 7.5×10^{-4} A
(c) 7.5×10^{-3} A (d) 7.5×10^{-2} A

14. A circuit has a self-inductance of 1 H and carries a current of 2A. To prevent sparking, when the circuit is switched off, a capacitor which can withstand 400 V is used. The least capacitance of capacitor connected across the switch must be equal to
(a) 50 μF (b) 25 μF
(c) 100 μF (d) 12.5 μF

15. The output Y of the logic circuit shown in figure is best represented as



(a) $\overline{A}+\overline{B.C}$ (b) $A+\overline{B}.C$
(c) $\overline{A+B.C}$ (d) $\overline{A+\overline{B}.C}$

16. A resistor of 6 kΩ with tolerance 10% and another resistance of 4 kΩ with tolerance 10% are connected in series. The tolerance of the combination is about
(a) 5% (b) 10%
(c) 12% (d) 15%

17. If we add impurity to a metal those atoms also deflect electrons. Therefore,
(a) the electrical and thermal conductivities both increase
(b) the electrical and thermal conductivities both decrease
(c) the electrical conductivity increases but thermal conductivity decreases
(d) the electrical conductivity decrease but thermal conductivity increases

18. A proton and an α -particle, accelerated through the same potential difference, enter a region of uniform magnetic field normally. If the radius of the proton orbit is 10 cm, then radius of α -particle is
(a) 10 cm (b) $10\sqrt{2}\,cm$
(c) 20 cm (d) $5\sqrt{2}\,cm$

19. An ammeter and a voltmeter of resistance R are connected in series to an electric cell of negligible internal resistance. Their reading are A and V respectively. If another resistance R is connected in parallel with the voltmeter, then
(a) both A and V will increase
(b) both A and V will decrease
(c) A will decrease and V will increase
(d) A will increase and V will decrease

20. A neutron is moving with velocity u. It collides head on and elastically with an atom of mass number A. If the initial kinetic energy of the neutron is E, then how much kinetic energy will be retained by the neutron after reflection?

(a) $\left(\frac{A}{A+1}\right)^2 E$ (b) $\frac{A}{(A+1)^2}E$

(c) $\left(\frac{A-1}{A+1}\right)^2 E$ (d) $\frac{(A-1)}{(A+1)^2}E$

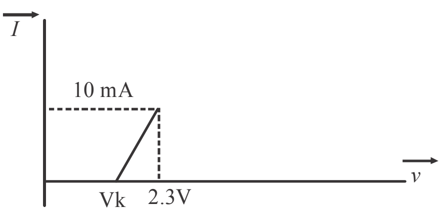
21. If a magnet is suspended at angle 30° to the magnet meridian, the dip of needle makes angle of 45° with the horizontal, the real dip is

(a) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (b) $\tan^{-1}(\sqrt{3})$

(c) $\tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$ (d) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

22. Which has more luminous efficiency ?
(a) A 40 W bulb
(b) A 40W fluorescent tube
(c) Both have same
(d) Cannot say

23. The resistance of a germanium junction diode whose $V-I$ is shown in figure is (V_k = 0.3 V)



(a) $5\,k\Omega$ (b) $0.2\,k\Omega$

(c) $2.3\,k\Omega$ (d) $\left(\frac{10}{2.3}\right)k\Omega$

24. In hydrogen discharge tube, it is observed that through a given cross-section 3.31×10^{15} electrons are moving from right to left and 3.12×10^{5} protons are moving from left to right. The current in the discharge tube and its direction will be

(a) 2 mA towards left
(b) 2 mA, towards right
(c) 1 mA, towards right
(d) 2 mA, towards left

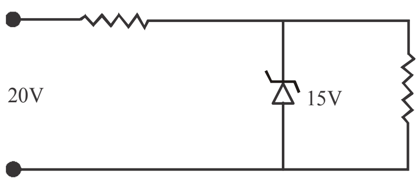
25. In a semiconductor, separation between conduction and valence band is of the order of

(a) 0 eV (b) 1 eV
(c) 10 eV (d) 50 eV

26. If 1000 droplets each of potential 1V and radius r are mixed to form a big drop. Then, the potential of the drop as compared to small droplets, will be

(a) 1000 V (b) 800 V
(c) 100 V (d) 20 V

27. A Zener diode, having breakdown voltage equal to 15 V is used in a voltage regulator circuit shown in figure. The current through the diode is



(a) 10 mA (b) 15 mA
(c) 20 mA (d) 5 mA

28. The activity of a radioactive sample is measured as N_0 counts per minute at $t = 0$ and N_0/C counts per minute at $t = 5$ min. The time, (in minute) at which the activity reduces to half its value, is

(a) $\log_e \frac{2}{5}$ (b) $\frac{5}{\log_e 2}$

(c) $5 \log_{10} 2$ (d) $5 \log_e 2$

29. If the electron in the hydrogen atom jumps from third orbit to second orbit, the wavelength of the emitted radiation in term of Rydberg constant is

(a) $\frac{6}{5R}$ (b) $\frac{36}{5R}$

(c) $\frac{64}{7R}$ (d) None of these

30. Silver has a work function of 4.7 eV. When ultraviolet light of wavelength 100 nm is incident on it a potential of 7.7 V is required to stop the photoelectrons from reaching the collector plate. How much potential will be required to stop photoelectrons, when light of wavelength 200 nm is incident on it?

(a) 15.4 V (b) 2.35 V
(c) 3.85 V (d) 1.5 V

31. If the distance of 100 W lamp is increased from a photocell, the saturation current i in the photocell varies with the distance d as

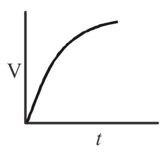
(a) $i \propto d^2$ (b) $i \propto d$

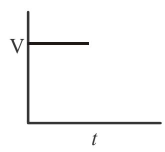
(c) $i \propto \frac{1}{d}$ (d) $i \propto \frac{1}{d^2}$

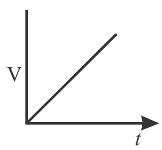
32. Following process is known as $h\nu \longleftrightarrow e^+ + e^-$

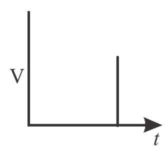
(a) Pair production (b) photoelectric effect
(c) Compton effect (d) Zeeman effect

33. During charging a capacitor, variations of potential V of the capacitor with time t is shown as

(a) 

(b) 

(c) 

(d) 

34. When a resistor of 11 Ω is connected in series with a electric cell. The current following in it is 0.5 A. Instead when a resistor of 5Ω is connected to the same electric cell in series, the current increases by 0.4A. The internal resistance of the cell is

(a) 1.5Ω (b) 2Ω
(c) 2.5Ω (d) 3.5Ω

35. A battery is charged at a potential of 15 V in 8 h when the current flowing is 10A. The battery on discharge supplies a current of 5A for 15 h. The mean terminal voltage during discharge is 14V. The watt-hour efficiency of battery is
(a) 80% (b) 90%
(c) 87.5% (d) 82.5%

36. A circular current carrying coil has a radius R. The distance from the centre of the coil on the axis, where the magnetic induction will be $\frac{1}{8}$th to its value at the centre of the coil is
(a) $\frac{R}{\sqrt{3}}$ (b) $R\sqrt{3}$
(c) $2\sqrt{3}R$ (d) $\frac{2}{\sqrt{3}}R$

37. The incorrect statement regarding the lines of force of the magnetic field B is
(a) magnetic intensity is a measure of lines of force passing through unit area held normal to it
(b) magnetic lines of force forms a close curve
(c) inside a magnet, its magnetic lines of force move from north pole of a magnetic towards its south pole
(d) due to a magnetic lines of force never cut each other

38. Two coils have a mutual inductance 0.55 H. The current changes in the first coil according to equation $I = I_0 \sin \omega t$.
where, $I_0 = 10$A and $\omega = 100\,\pi$ rad/s.
The maximum value of emf in the second coil is
(a) 2π (b) 5π
(c) π (d) 4π

39. An *L-C-R* circuit contains $R = 50\,\Omega$, $L = 1$ mH and C = 0.1μF. The impedence of the circuit will be minimum for a frequency of
(a) $\frac{10^5}{2\pi}$ Hz (b) $\frac{10^6}{2\pi}$ Hz
(c) $2\pi \times 10^5$ Hz (d) $2\pi \times 10^6$ Hz

40. An eye can detect 5×10^4 photons per square meter per sec of green light ($\lambda\alpha = 5000$Å) while the ear can detect 10^{-13}W/m^2. The factor by which the eye is more sensitive as a power detector then ear is close to
(a) 5 (b) 10
(c) 10^6 (d) 15

PART - II (CHEMISTRY)

41. The sodium extract of an organic compound on acidification with acetic acid and addition of lead acetate solution gives a black precipitate.
The organic compound contains
(a) nitrogen (b) halogen
(c) sulphur (d) phosphorus

42. The volume strength of 1.5 N H_2O_2 solution is
(a) 16.8 L (b) 8.4 L
(c) 4.2 L (d) 5.2 L

43. $MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$;
E° = 1.51 V
$MnO_2 + 4H^+ + 2e^- \longrightarrow Mn^{2+} + 2H_2O$;
$E° = 1.23$ V $E°_{MnO_4^- \mid MnO_2}$ is
(a) 1.70 V (b) 0.91 V
(c) 1.37 V (d) 0.548 V

44. A metal has bcc structure and the edge length of its unit cell is 3.04Å. The volume of the unit cell in cm^3 will be
(a) 1.6×10^{21} cm^3 (b) 2.81×10^{-23} cm^3
(c) 6.02×10^{-23} cm^3 (d) 6.6×10^{-24} cm^3

45. Among $[Fe(H_2O)_6]^{3+}$, $[Fe(CN)_6]^{3-}$, $[Fe(Cl)_6]^{3-}$ species, the hybridisation state of the Fe atom are, respectively.
(a) $d^2sp^3, d^2sp^3, sp^3d^2$ (b) $sp^3d^2, d^2sp^3, d^2sp^3$
(c) $sp^3d^2, d^2sp^3, sp^3d^2$ (d) None of the above

46. Which of the following hydrogen bonds are strongest in vapour phase?
(a) HF ……….. HF
(b) HF ………... HCl
(c) HCl………… HCl
(d) HF …………… HI

47. The rate constant for forward reaction and backward reaction of hydrolysis of ester are 1.1×10^{-2} and 1.5×10^{-3} per minute respectively. Equilibrium constant for the reaction is
$$CH_3COOC_2H_5 + H_2O \rightleftharpoons CH_3COOH + C_2H_5OH$$
(a) 33.7 (b) 7.33
(c) 5.33 (d) 33.3

48. 19.85 mL of 0.1 N NaOH reacts with 20 mL of HCl solution for complete neutralisation. The molarity of HCl solution is
(a) 9.9 (b) 0.99
(c) 0.099 (d) 0.0099

49. An *f*-shell containing 6 unpaired electrons can exchange

(a) 6 electrons (b) 9 electrons
(c) 12 electrons (d) 15 electrons

50. The standard molar heat of formation of ethane, CO_2 and water (l) are respectively –21.1, –94.1 and –68.3 kcal. The standard molar heat of combustion of ethane will be

(a) –372 kcal (b) 162 kcal
(c) –240 kcal (d) 183.5 kcal

51. The solubility product of Ag_2CrO_4 is 32×10^{-12}. What is the concentration of CrO^-_4 ions in that solution?

(a) 2×10^{-4} M (b) 16×10^{-4} M
(c) 8×10^{-4} M (d) 8×10^{-8} M

52. The equivalent conductivity of a solution containing 2.54g of $CuSO_4$ per L is 91.0 W^{-1} cm^2 eq^{-1}. Its conductivity would be

(a) $2.9 \times 10^{-3}\ \Omega^{-1}\ cm^{-1}$
(b) $1.8 \times 10^{-2}\ \Omega^{-1}\ cm^{-1}$
(c) $2.4 \times 10^{-4}\ \Omega^{-1}\ cm^{-1}$
(d) $3.6 \times 10^{-3}\ \Omega^{-1}\ cm^{-1}$

53. The half-life of two samples are 0.1 and 0.8 s. Their respective concentration are 400 and 50 respectively. The order of the reaction is

(a) 0 (b) 2
(c) 1 (d) 4

54. Which sequence of reactions shows correct chemical relation between sodium and its compounds?

(a) $Na + O_2 \longrightarrow Na_2O \xrightarrow{HCL(aq)} NaCl \xrightarrow{CO_2} Na_2CO_3 \xrightarrow{\Delta} Na$

(b) $Na \xrightarrow{O_2} Na_2O \xrightarrow{H_2O} NaOH \xrightarrow{CO_2} Na_2CO_3 \xrightarrow{\Delta} Na$

(c) $Na + H_2O \longrightarrow NaOH \xrightarrow{HCl} NaCl \xrightarrow{CO_2} Na_2CO_3 \xrightarrow{\Delta} Na$

(d) $Na + H_2O \longrightarrow NaOH \xrightarrow{CO_2} Na_2CO_3 \xrightarrow{HCl} \underset{(molten)}{NaCl} \xrightarrow{Electrolysis} Na$

55. Purest form of iron is

(a) pig iron (b) wrought iron
(c) cast iron (d) steel

56. Which has the smallest size?

(a) Na^+ (b) Mg^{2+}
(c) Al^{3+} (d) P^{5+}

57. In the reaction,

$8Al + 3Fe_3O_4 \longrightarrow 4Al_2O_3 + 9Fe$

the number of electrons transferred from the reductant to the oxidant is

(a) 8 (b) 4
(c) 16 (d) 24

58. The bond angles of NH_3, NH_4^+ and NH_2^- are in the order

(a) $NH_2^- > NH_3 > NH_4^+$
(b) $NH_4^+ > NH_3 > NH_2^-$
(c) $NH_3 > NH_2^- > NH_4^+$
(d) $NH > NH_4^+ > NH_2^-$

59. A gaseous mixture containing He, CH_4 and SO_2 was allowed to effuse through a fine hole then find what molar ratio of gases coming out initially? (Given mixture contains He, CH_4 and SO_2 in 1 : 2 : 3 mole ratio).

(a) $\sqrt{2} : \sqrt{2} : 3$ (b) 2 : 2 : 3
(c) 4 : 4 : 3 (d) 1 : 1 : 3

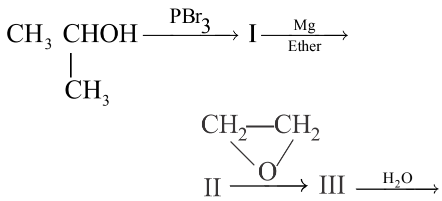
60. According to Bohr's theory, the angular momentum for an electron of 3rd orbit is

(a) $3\hbar$ (b) $1.5\hbar$
(c) $9\hbar$ (d) $2\dfrac{\hbar}{\pi}$

61. 2.76 g of silver carbonate on being strongly heated yields a residue weighing

(a) 3.54 g (b) 3.0 g
(c) 1.36 g (d) 2.16 g

62. The final product (IV) in the sequence of reactions



IV is

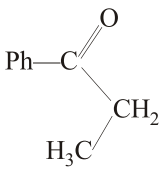
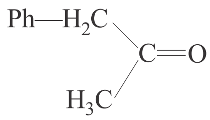
(a) $CH_3-CH(CH_3)OCH_2CH_2OH$

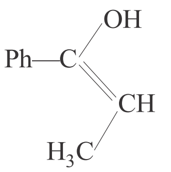
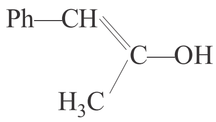
(b) $CH_3-CH(CH_3)CH_2CH_2Br$

(c) $CH_3-CH(CH_3)-CH_2CH_2OH$

(d) $CH_3-CH(CH_3)OCH_2CH_3$

63. $Ph-C \equiv C-CH_3 \xrightarrow{Hg^{2+}/H^+}$ A.

(a)  (b) 

(c)  (d) 

64. Which of the following has an ester linkage?
(a) Nylon-66 (b) Dacron
(c) PVC (d) Bakelite

65. Which of the following pairs give positive Tollen's test?
(a) Glucose,sucrose
(b) Glucose,fructose
(c) Hexanal,acetophenone
(d) Fructose,sucrose

66. Peptisation involves
(a) precipitation of colloidial particles
(b) disintegration of colloidal aggregates
(c) evaporation of dispersion medium
(d) impact of molecules of the dispersion medium on the colloidal particles

67. Which of the following has the maximum number of unpaired d-electrons?
(a) Fe^{2+} (b) Cu^+
(c) Zn (d) Ni^{3+}

68. Iodine is formed when potassium iodide reacts with a solution of
(a) $ZnSO_4$ (b) $CuSO_4$
(c) $(NH_4)_2SO_4$ (d) Na_2SO_4

69. Which of the following does not represent the correct order of the property indicated?
(a) $Sc^{3+} > Cr^{3+} > Fe^{3+} > Mn^{3+}$ — ionic radii
(b) Sc < Ti < Cr < Mn —density
(c) $Mn^{2+} > Ni^{2+} > Co^{2+} < Fe^{2+}$ —ionic radii
(d) FeO < CaO < MnO < CuO —basic nature

70. If the elevation in boiling point of a solution of 10 g of solute (mol. wt. = 100) in 100 g of water is ΔT_b, the ebullioscopic constant of water is
(a) 10 (b) $100\,T_b$
(b) ΔT_b (d) $\frac{\Delta T_b}{10}$

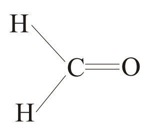
71. Which of the following compounds cannot be prepared singly by the Wurtz reaction?
(a) C_2H_6
(b) $(CH_3)_2CHCH_3$
(c) $CH_3CH_2CH_2CH_3$
(d) All of the above can be prepared

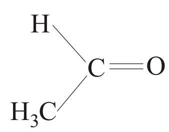
72. Which of the following oxides is strongly basic?
(a) Tl_2O (b) B_2O_3
(c) Al_2O_3 (d) Ga_2O_3

73. In Langmuir's model of adsorption of a gas on a solid surface,
(a) the rate of dissociation of adsorbed molecules from the surface does not depend on the surface covered
(b) the adsorption at a single site on the surface may involve multiple molecules at the same time
(c) the mass of gas striking a given area of surface is proportional to the pressure of the gas
(d) the mass of gas striking a given area of surface is independent of the pressure of the gas

74. How many sigma and pi-bonds are there in the molecule of dicyanoethene (CN—CH=CH—CN)?
(a) 3 sigma and 3 pi (b) 5 sigma and 2 pi
(c) 7 sigma and 5 pi (d) 2 sigma and 3 pi

75. What will be the order of reactivity of the following carbonyl compounds with Grignard's reagent?


I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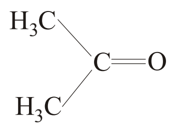
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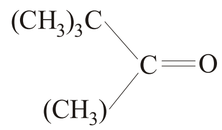
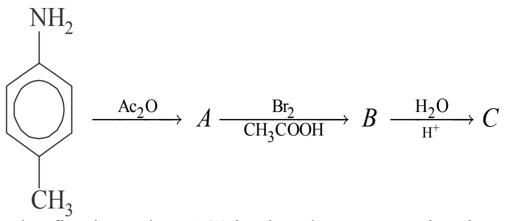
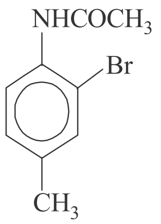
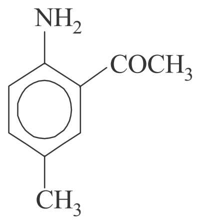
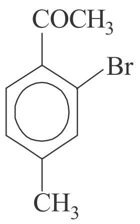
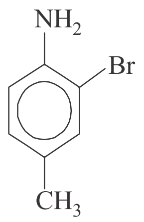

IV

(a) I > II > III > IV (b) IV > III > II > I
(c) II > I > IV > III (d) III > II > I > IV

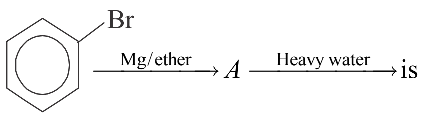
76. 

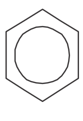
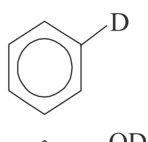
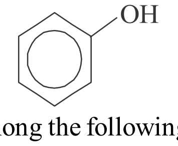
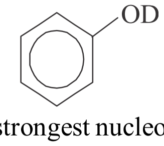
The final product 'C' in the above reacrtion is

(a) 
(b) 
(c) 
(d) 

77. Which of the following isomerism is shown by ethyl acetoacetate?
(a) Geometrical isomerism
(b) Keto-enol tautomerism
(c) Enantiomerism
(d) Diastereoisomerism

78. The final product obtained in the reaction,



(a) 
(b) 
(c) 
(d) 

79. Among the following the strongest nucleophile is
(a) C_2H_5SH (b) CH_3COO^-
(c) CH_3NH_2 (d) $NCCH_2$

80. Which set has different class of compounds?
(a) Tranquillizers-Equanil, heroin,valium
(b) Antiseptics-Bithional,dettol,boric acid
(c) Analgesics-Naproxen,morphine,asprin
(d) Bactericidal-penicillin,aminoglycosides, ofloxacin

PART - III (MATHEMATICS)

81. The solution of $\frac{dy}{dx} = \frac{x^2+y^2+1}{2xy}$, satisfying $y(1)=0$ is given by
(a) hyperbola (b) circle
(c) ellipse (d) parabola

82. If $x.\frac{dy}{dx} + y = x.\frac{f(xy)}{f'(xy)}$, then $f(xy)$ is equal to
(a) $k.e^{\frac{x^2}{2}}$ (b) $k.e^{y^{2/2}}$
(c) $k.e^{x^2}$ (d) $k.e^{\frac{xy}{2}}$

83. The differential equation of the rectangular hyperbola hyperbola, where axes are the asymptotes of the hyperbola, is
(a) $y\frac{dy}{dx} = x$ (b) $x\frac{dy}{dx} = -y$
(b) $x\frac{dy}{dx} = y$ (d) $xdy + ydx = c$

84. The length of longer diagonal of the parallelogram constructed on $5a+2b$ and $a-3b$, if it is given that $|a| = 2\sqrt{2}$, $|b| = 3$ and the angle between a and b is $\frac{\pi}{4}$, is
(a) 15 (b) $\sqrt{113}$
(c) $\sqrt{593}$ (d) $\sqrt{369}$

85. If $r = \alpha\, b \times c + \beta\, c \times a + \gamma\, a \times b$ and $[a\ b\ c] = 2$, then $\alpha + \beta + \gamma$ is equal to
(a) $r.[b \times c + c \times a + a \times b]$
(b) $\frac{1}{2} r.(a+b+c)$
(c) $2r.(a+b+c)$
(d) 4

86. If a, b, c are three non-coplanar vectors and p, q, r are reciprocal vectors, then $(la + mb + nc).(lp + mq + nr)$ is equal to
(a) $l+m+n$ (b) $l^3+m^3+n^3$
(c) $l^2+m^2+n^2$ (d) None of these

87. If the integers m and n are chosen at random from 1 to 100, then the probability that a number of the form $7^n + 7^m$ is divisible by 5, equals to
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{8}$ (d) $\frac{1}{3}$

88. Let X denote the sum of the numbers obtained when two fair dice are rolled. The variance and standard deviation of X are

(a) $\frac{31}{6}$ and $\sqrt{\frac{31}{6}}$ (b) $\frac{35}{6}$ and $\sqrt{\frac{35}{6}}$

(c) $\frac{17}{6}$ and $\sqrt{\frac{17}{6}}$ (d) $\frac{31}{6}$ and $\sqrt{\frac{35}{6}}$

89. A four digit number is formed by the digits 1, 2, 3, 4 with no repetition. The probability that the number is odd, is

(a) zero (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) None of these

90. If the vertices of a triangle are A(0, 4, 1), B(2, 3, –1) and C(4, 5, 0), then the orthocentre of ΔABC, is

(a) (4, 5, 0) (b) (2, 3, –1)

(c) (–2, 3, –1) (d) (2, 0, 2)

91. The equation of normal to the curve $y = (1+x)^y + \sin^{-1}(\sin^2 x)$ at $x = 0$ is

(a) $x+y=1$ (b) $x-y=1$

(c) $x+y=-1$ (d) $x-y=-1$

92. The value of c from the Lagrange's mean value theorem for which $f(x) = \sqrt{25-x^2}$ in [1,5], is

(a) 5 (b) 1

(c) $\sqrt{15}$ (d) None of these

93. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$, then A . (adj A) is equal to

(a) A (b) |A|

(c) |A| (d) None of these

94. If there is an error of k% in measuring the edge of a cube, then the percent error in estimating its volume is

(a) k (b) 3k

(c) $\frac{k}{3}$ (d) None of these

95. If the system of equations $x + ky - z = 0$, $3x - ky - z = 0$ and $x - 3y + z = 0$, has non-zero solution, then k is equal to

(a) –1 (b) 0

(c) 1 (d) 2

96. If the points (1, 2, 3) and (2, –1, 0) lie on the opposite sides of the plane $2x + 3y - 2z = k$, then

(a) $k < 1$ (b) $k > 2$

(c) $k < 1$ or $k > 2$ (d) $1 < k < 2$

97. If $\Delta(x) = \begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x-\cos x \\ \sin x & \sin x & 1 \end{vmatrix}$, then $\int_0^{\pi/4} \Delta(x)\,dx$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 0 (d) $-\frac{1}{4}$

98. Let $f'(x)$, be differentiable $\forall x$. If $f(1) = -2$ and $f'(x) \geq 2 \ \forall \ x \in [1, 6]$, then

(a) $f(6) < 8$ (b) $f(6) \geq 8$

(c) $f(6) \geq 5$ (d) $f(6) \leq 5$

99. If $\Delta_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(\mathrm{m}^2) & \sin^2(\mathrm{m}) & \sin^2(\mathrm{m}+1) \end{vmatrix}$, then the value of $\sum_{r=0}^{m} \Delta_r$, *is*

(a) 1 (b) 0

(c) 2 (d) None of these

100. Two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = z$ intersect at a point, if k is equal to

(a) $\frac{2}{9}$ (b) $\frac{1}{2}$ (c) $\frac{9}{2}$ (d) $\frac{1}{6}$

101. The minimum value of $\frac{x}{\log x}$ is

(a) e (b) $\frac{1}{e}$

(c) e^2 (d) e^3

102. The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point (1,1) and the coordinate axes lies in the first quadrant. If its area is 2, then the value of b is

(a) –1 (b) 3

(c) –3 (d) 1

103. The statement (p $\Rightarrow$ q) $\Leftrightarrow$ $(\sim p \wedge q)$ is a

(a) tautology

(b) contradiction

(c) Neither (a) nor (b)

(d) None of these

104. If $x + iy = \frac{3}{2+\cos\theta + i\sin\theta}$, then $x^2 + y^2$ is equal to

(a) $3x-4$ (b) $4x-3$

(c) $4x+3$ (d) None of these

105. The negation of $(\sim p \wedge q) \vee (p \wedge \sim q)$ is
(a) $(p \vee \sim q) \vee (\sim p \vee q)$
(b) $(p \vee \sim q) \wedge (\sim p \vee q)$
(c) $(p \wedge \sim q) \wedge (\sim p \vee q)$
(d) $(p \wedge \sim q) \wedge (p \vee \sim q)$

106. The normals at three points P, Q and R of the parabola $y^2 = 4ax$ meet at (h, k). The centroid of the $\Delta\, PQR$ lies on
(a) $x = 0$ (b) $y = 0$
(c) $x = -a$ (d) $y = a$

107. The minimum area of the triangle formed by any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the coordinate axes is
(a) $a^2 + b^2$ (b) $\frac{(a+b)^2}{2}$
(c) ab (d) $\frac{(a-b)^2}{2}$

108. If the line $lx + my - n = 0$ will be a normal to the hyperbola, then $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{k}$, where k is equal to
(a) n (b) n^2
(c) n^3 (d) None of these

109. If $\cos\alpha + i\sin\alpha$, $b = \cos\beta + i\sin\beta$, $c = \cos\gamma + i\sin\gamma$ and $\frac{b}{c} + \frac{c}{a} + \frac{a}{b} = 1$, then $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)$ is equal to
(a) $\frac{3}{2}$ (b) $-\frac{3}{2}$
(c) 0 (d) 1

110. If $|z + 4| \le 3$, then the greatest and the least value of $|z + 1|$ are
(a) $-1, 6$ (b) $6, 0$
(c) $6, 3$ (d) None of these

111. The angle between lines joining the origin to the point of intersection of the line $\sqrt{3}\,x + y = 2$ and the curve $y^2 - x^2 = 4$ is
(a) $\tan^{-1}\frac{2}{\sqrt{3}}$ (b) $\frac{\pi}{6}$
(c) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (d) $\frac{\pi}{2}$

112. If the area of the triangle on the complex plane formed by the points z, $z + iz$ and iz is 200, then the value of $3\,|z|$ must be equal to
(a) 20 (b) 40
(c) 60 (d) 80

113. Equation of the chord of the hyperbola $25x^2 - 16y^2 = 400$ which is bisected at the point $(6, 2)$ is
(a) $6x - 7y = 418$ (b) $75x - 16y = 418$
(c) $25x - 4y = 400$ (d) None of these

114. If a plane meets the coordinate axes at A, B and C such that the centroid of the triangle is $(1, 2, 4)$, then the equation of the plane is
(a) $x + 2y + 4z = 12$ (b) $4x + 2y + z = 12$
(c) $x + 2y + 4z = 3$ (d) $4x + 2y + z = 3$

115. The volume of the tetrahedron included between the plane $3x + 4y - 5z - 60 = 0$ and the coordinate planes is
(a) 60 (b) 600
(c) 720 (d) 400

116. $\int_0^{2x} (\sin x + |\sin x|)\, dx$ is equal to
(a) 0 (b) 4
(c) 8 (d) 1

117. The value of $\int_0^{\sqrt{2}} [x^2]\, dx$, where [.] is the greatest integer function, is
(a) $2 - \sqrt{2}$ (b) $2 + \sqrt{2}$
(c) $\sqrt{2} - 1$ (d) $\sqrt{2} - 2$

118. If $l(m, n) = \int_0^1 t^m\, (1+t)^n\, dt$, then the expression for $l(m, n)$ in terms of $l(m+1, n+1)$ is
(a) $\frac{2^n}{m+1} - \frac{n}{m+1}.l(m+1, n-1)$
(b) $\frac{n}{m+1}.l(m+1, n-1)$
(c) $\frac{2n}{m+1} + \frac{n}{m+1}l.\,(m+1, n-1)$
(d) $\frac{m}{n+1}.l\,(m+1, n-1)$

119. The area in the first quadrant between $x^2 + y^2 = \pi^2$ and $y = \sin x$ is
(a) $\frac{\pi^3 - 8}{4}$ (b) $\frac{\pi^3}{4}$
(c) $\frac{\pi^3 - 16}{4}$ (b) $\frac{\pi^3 - 8}{2}$

120. The area bounded by $y = xe^{|x|}$ and lines $|x| = 1$, $y = 0$ is
(a) 4 sq units (b) 6 sq units
(c) 1 sq unit (d) 2 sq units

SOLUTIONS

PART - I (PHYSICS)

1. (b) Amplification factor of a triode, $\mu = \frac{\Delta V_p}{\Delta V_g}$

$\Rightarrow \Delta V_p = -\mu \times \Delta V_s = -50\,(-20) = 10V$

2. (a) As we know, energy E = Δmc^2
$= 0.5 \times 10^{-3} \times (3\times10^8)^2$
$= 4.5 \times 10^{13}$J

$= \frac{4.5\times10^{13}}{3.6\times10^{6}} = 1.25\times10^{7}$ kWh

3. (b) Here, diode in lower branch is forward and in upper branch is reversed biased

$\therefore$ $i = \frac{5}{20+30} = \frac{5}{50}$A

4. (b) Power consumed by motor = 5 kW
$= 5 \times 10^{3}$ W = 5000W

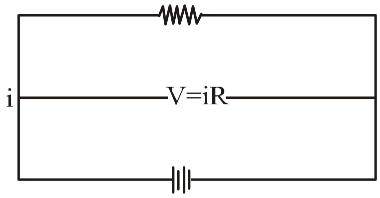
Power used in lifting water = $\frac{mgh}{t}$

$= 7.5 \times 9.8 \times 4.7 = 3454.5$ kW

Efficiency $= \frac{\text{Power used}}{\text{Power consumed}} \times 100\%$

$= \frac{3454.5}{5000} \times 100 = 69\%$

5. (c) For a closed circuit cell supplies a constant current in the circuit.



For cell E = V + Ir
For V = 50 V
E = 50 + 11r
Similarly, for V = 60 V
E = 60 + r
From eqs. (i) and (ii), we get
E = 61V

6. (b) The ionosphere can reflect electromagnetic waves of frequency less than 40 MHz but not of frequency more than 40 MHz.

7. (a) $\lambda_{red} > \lambda_{violet}$
As wavelength λ less than that of yellow colour and hence can initiate photoelectric effect irrespective of intensity.

8. (c) Radius of circular path

$r = \frac{mV}{B} = \frac{V}{\left(\frac{e}{m}\right)B}$

$r = \frac{6\times10^{7}}{1.7\times10^{11}\times15\times10^{-2}}$

$= 2.35 \times 10^{-2}$ m = 2.35 m

9. (a) As we know, Self-inductance of the solenoid

$L = \frac{\mu_r.\mu_0 N^2 A}{I}$

$= \frac{600\times4\pi\times10^{-7}\times(2000)^2\times(1.5)\times10^{-4}}{0.3}$

= 1.5 H

10. (d) Current $I = \frac{V}{R} = \frac{100}{10} = 10A$

Energy, $E = \frac{1}{2}LI^2$

$= \frac{1}{2} \times 5 \times (10)^2 = 250J$

11. (c) Given : $V = 750\times10^{-3}$ V;
$I_g = 15\times10^{-3}$A and I = 25A

Using, $a = \frac{y}{I_g}$

$= \frac{750\times10^{-3}}{15\times10^{-3}} = 50\,\Omega$

$I_g = \frac{S}{S+a} \times I$

$15 \times 10^{-3} = \left(\frac{S}{S+50}\right) \times 25$

$\therefore$ S = 0.03 Ω

12. (d) Here, $r = R\left(\frac{I_1}{I_2} - 1\right) = 2\left(\frac{125}{100} - 1\right)$

$$= 2\left(\frac{5}{4} - 1\right) = 2 \times \frac{1}{4} = 0.05\,\Omega$$

13. (d) Resistances 10Ω, 60Ω and 100Ω are in series and they together are in parallel to 200Ω resistance. When a potential difference of 15 V is applied across 200Ω then current through it

$$I = \frac{15}{200} = 7.5 \times 10^{-2} A$$

14. (b) Energy stored in capacitor = energy stored in inductance

i.e., $\frac{1}{2}CV^2 = \frac{1}{2}LI^2$

$$\Rightarrow C = \frac{LI^2}{V^2} = \frac{1 \times (2)^2}{(400)^2} = 25\ \mu F$$

15. (d) Boolean expression for Logic gate-1

$B.C = Y'$

Boolean expression for Logic gate-II

$A + (\vec{B}.\vec{C}) = Y''$

Boolean expression for Logic gate-III

$A + \vec{B}.\vec{C} = Y$

16. (b) In series combination equivalent resistance,

$R = R_1 + R_2$

$= 6 + 4 = 10\,k\Omega$

Error in combination,

$\Delta R = \Delta R_1 + \Delta R_2$

$= \frac{10}{100} \times 6 + \frac{10}{100} \times 4$

$= 0.6 + 0.4 = 1$

$$\frac{\Delta R}{R} = \frac{1}{10} = 10\,\%$$

17. (b) If the number of electrons increase, their number of collision, increasing the thermal and electrical resistance.

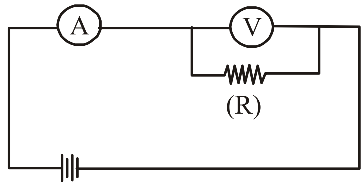
So, electrical and thermal conductivities both decrease.

18. (b) Radius of path $r_{time} = \frac{1}{\beta}\sqrt{\frac{2mv}{q}}$

$$\therefore \quad \frac{r_\alpha}{r_p} = \sqrt{\frac{m_\alpha}{m_p}}\sqrt{\frac{q_p}{q_\alpha}}$$

or, $\frac{r_\alpha}{10} = \sqrt{\frac{4}{2}} \Rightarrow r_\alpha = 10\sqrt{2}\text{cm}$.

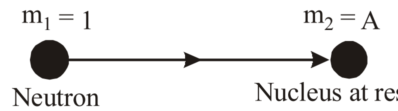
19. (d) The effective resistance will decrease when resistance R is connected in parallel with the voltmeter.



According to Ohm's law, $V = IR$ or, $R = \frac{V}{I}$

Here, as R decreases, so V decrease and I should increase.

20. (c) Fraction retained by nucleus



$$\left(\frac{\Delta k}{k}\right)_{retained} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)^2 = \left(\frac{A-1}{A+1}\right)^2$$

After collision kinetic energy retained by neutron $\left(\frac{A-1}{A+1}\right)^2 E$

21. (d) Here, $\tan\delta' = \frac{\tan\delta}{\cos\theta} = \frac{\tan 45°}{\cos 30°}$

$$\tan\delta = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$$

or $\delta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

22. (b) Luminous efficiency for the same power supply, 40 W fluorescent tube gives more light. Hence, 40 W fluorescent tube has greater

23. (b) Resistance, $R = \frac{\Delta V}{\Delta I} = \frac{2.3 - 0.3}{10 \times 10^{-3}}$

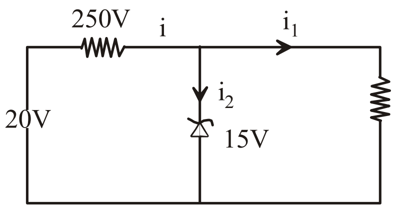
$R = \frac{2}{10} \times 10^3 = 0.2 \times 10^3\,\Omega = 0.2\,k\Omega$

24. (c) Here, number of electrons $n_e = 3.13 \times 10^{15}$ and number of protons $n_p = 3.12 \times 10^{15}$
Current $I = n_e q_e + n_p q_p$
$= 3.13 \times 10^{15} \times 1.6 \times 10^{-17} + 3.12 \times 10^{15} \times 1.6 \times 10^{-19}$
$= 1 \times 10^{-3} = 1$ mA
Now, due to excess charge on electrons, the direction of the current will be towards right.

25. (b) In conductor separation between conduction and valence bands is zero and in insulator, it is greater than 1eV.
Hence, in semiconductor the separation between conduction and valence band is 1 eV.

26. (c) Potential of big drop $= q \times n^{2/3} = 100$ V

27. (d)



For $R = 1k\,\Omega$

$i_1 = \frac{15}{1}$ mA $= 15$ mA

$R = 250\,\Omega$

$i_{250} = \frac{20 - 15}{250} = \frac{5}{250} = 20$ mA

$i_{Zener} = 20 - 15 = 5$mA

28. (d) After n half-lives

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/7}$

$N = \frac{N_0}{e} = \frac{cN_0}{cN_0} = \left(\frac{1}{2}\right)^{5/7}$

$\Rightarrow \frac{1}{e} = \left(\frac{1}{2}\right)^{5/7}$

Taking log on both sides, we get

$\log 1 - \log e = \frac{5}{7} \log \frac{1}{2}$

$-1 = \frac{5}{7}(-\log 2)$

$T = 5 \log_e 2$
Now, let t' be the time after which activity reduces to half

$\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{1'/5\log_e 2}$

$t' = 5\ o\log_e 2$

29. (b) As we know $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right)$

$\frac{1}{\lambda} = R\left(\frac{9-4}{36}\right) = \frac{5R}{36}$

$\therefore \quad \lambda = \frac{36}{5R}$

30. (d) From Einstein's Photoelectric equation,

$ev_0 + \phi = \frac{hc}{\lambda}$

and $ev_0 + \phi_0 = \frac{hc}{\lambda'}$

$\frac{ev'_0 + \phi}{eV_0 + \phi} = \frac{\lambda}{\lambda'} = \frac{100}{200} = \frac{1}{2}$

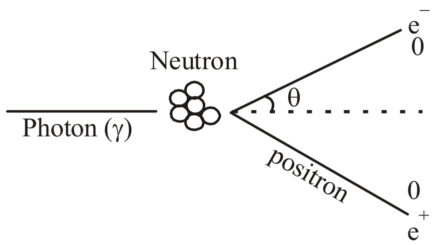
$2ev'_0 + 2\phi = ev_0 + \phi$

$ev'_0 = \frac{ev_0 - \phi}{2} = \frac{7.7 - 4.7}{2} = 1.5V$

31. (d) As we know, Photoelectric current depends on the intensity of incident radiation i.e., $i \propto I$

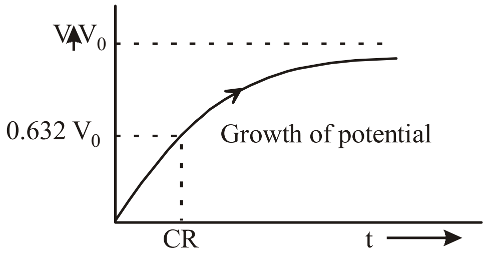
But, intensity of radiation $I \propto \frac{1}{d^2}$ so, $i \propto \frac{1}{d^2}$

32. (a) The creation of an elementary particle and its antiparticle usually from a photon (or another neutral boson) is called Pair production. This is allowed, provided there is enough energy available to create the pair.



33. (a) For charging the capacitor, $q = q_0 \left(1 - e^{\frac{-t}{CR}}\right)$

And, Potential difference $V = V_0\,(1 - e^{-t/CR})$



34. (c) Here, $i = \frac{E}{R + r}$

$0.5 = \frac{E}{11 + r} \Rightarrow E = 5.5 + 0.5\,r$

$0.9 = \frac{E}{5 + r}$ or, $E = 4.5 + 0.9r$

On solving we get $r = 2.5\Omega$

35. (c) Power of battery, when charged is given by
$P_1 = V_1 I_1$
Electrical energy dissipated id given by
$E_1 = P_1 t_1$
$E_1 = V_1 I_1\, t_1 = 15 \times 10 \times 8 = 1200$ Wh
Similarly, the electrical energy dissipated during the discharge of battery is given by
$E_2 = V_2\, I_2 t_2$
$= 14 \times 5 \times 15 = 1050$ Wh
Hence, watt-hour efficiency of the battery

$\Rightarrow \eta = \frac{E_2}{E_1} \times 100\;\; 0.875 \times 100 = 87.5\,\%$

36. (b) Here the ratio, $\frac{B_{Centre}}{B_{axis}} = \left(1 + \frac{x^2}{R^2}\right)^{3/2}$

Also, $B_{axis} = \frac{1}{8} B_{centre}$

$\frac{8}{1} = \left(1 + \frac{x^2}{R^2}\right)^{3/2} \quad \Rightarrow \quad 4 = 1 + \frac{x^2}{R^2}$

$3 = \frac{x^2}{R^2} \Rightarrow X^2 = 3R^2$

or, $x = \sqrt{3}\, R$

37. (c) Inside a magnet, magnetic lines of force move from south pole to north pole.

38. (b) E.M.F. $e = M \frac{di}{dt} = 0.005 \times \frac{d}{dt}\,(i_0 \sin \omega\, t)$
$= 0.005 \times i_0 \cos \omega\, t$
$e_{max} = 0.005 \times 10 \times 100\;\pi = 5\,\pi$

39. (a) Impedance of L-C-R circuit will be minimum for a resonant frequency so,

$v_0 = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\pi\sqrt{1\times10^{-3}\times0.1\times10^{-6}}} = \frac{10^5}{2\pi}$ Hz

40. (a) Energy $= \frac{12375}{5000} = 2.475$ eV $= 4 \times 10^{-19}$ J
Minimum intensity to which the eye can respond.
l_{eye} = (photon flux) × energy of a photon
$l_{eye} = (5 \times 10^4) \times 4 \times 10^{-19}$
$= 2 \times 10^{-14}$ W/m^2
Now, lesser the intensity required by a detector for detection more sensitive it will

be $= \frac{l_{ear}}{l_{eye}} = \frac{10^{-13}}{2\times10^{-14}} = 5$

PART - II (CHEMISTRY)

41. (c) The organic compounds containing sulphur when react with sodium metal give Na_2S. The Na_2S when react with lead acetate forms black ppt. of PbS.

$Na_2S + (CH_3COO)_2Pb \longrightarrow$

$PbS + 2CH_3COONa$
Black
ppt

42. (b) Volume strength = 5.6 × normality
= 5.6 × 1.5
= 8.4 L

43. (a) On subtracting eqn. (ii) from (i) we get

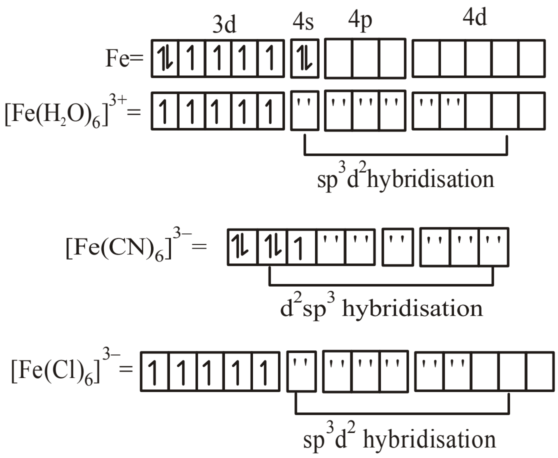
$MnO_4^- + 4H^+ + 3e^- \longrightarrow MnO_2 + 2H_2O$

$\therefore\;\; -E_3 = \frac{-1.51\times5 + 2\times1.23}{3}$

$\therefore\;\; E_3 = 1.70$ V

44. (b) Volume of unit cell (V) = a^3
$\therefore\;\; = (3.04 \times 10^{-8}$ cm$)^3$
$= \;\; 2.81 \times 10^{-23}$ cm^3

45. (c)



46. (a) A compound having maximum electronegative element will form strong hydrogen bond. F is the most negative element among halogens hence form strongest hydrogen bond.

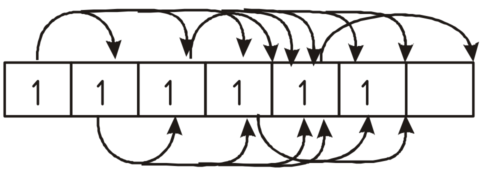
47. (b) Given $k_f = 1.1 \times 10^{-2}$, $k_b = 1.5 \times 10^{-3}$

$$k_c = \frac{k_f}{k_b} = \frac{1.1\times10^{-2}}{1.5\times10^{-3}} = 7.33$$

48. (c) Molarity of base $= \frac{\text{normality}}{\text{Acidity}} = \frac{0.1}{1} = 0.1$

$M_1V_1 = M_2V_2$
$0.1 \times 19.85 = M_2 \times 20$
$M_2 = 0.09925 \approx 0.099$

49. (d)



$5+4+3+2+1=15$

$\therefore$ Equation of normal of x = 0 and y = 1 is
$y-1=-1\,(x-0)$
$\Rightarrow$ $y-1=-x \Rightarrow x+y=1$

50. (a) Given,

(a) $2C + 3H_2 \longrightarrow C_2H_6;\ \Delta H = -21.1$ kcal

(b) $C + O_2 \longrightarrow CO_2;\ \Delta H = -94.1$ kcal

(c) $H_2 + \frac{1}{2}O_2 \longrightarrow H_2O;\ \Delta H = -68.3$ kcal

Now, eqs. $2 \times (b) + 3 \times (c) - (a)$

$$C_2H_6 + \frac{3}{2}O_2 \longrightarrow 2CO_2 + 3H_2O$$

Heat of combustion of Ethane $\Delta x = 2(-94.1)$
$+ 3(-68.3) - (-21.1)$
$= (-188.2) + (-204.9) - 21.1$
$= -372$ kcal

51. (a) $Ag_2CrO_4 \longrightarrow 2Ag^+ + CrO_4^{2-}$

S 2S S

$K_{SP} = (2s)^2 s = 4s^2.s = 4s^3$

$$S = \left(\frac{K_{sp}}{4}\right)^{1/3} \Rightarrow \left(\frac{32\times10^{-12}}{4}\right)^{1/3}$$

$= 2\times10^{-4}$ M

52. (a) We know that,

$\kappa = \Lambda_{eq}.C$

Given, $\Lambda eq = 91.0\ \Omega^{-1}\ cm^2\ eq^{-1}$

$$\kappa = \left(91\Omega^{-1}cm^2eq^{-1}\right)$$

$$\left(\frac{2.54}{159/2\times1000}eq.cm^{-3}\right)$$

$= 2.9 \times 10^{-3}\ \Omega^{-1}cm^{-1}$

53. (b) It is known,

$$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left[\frac{a_2}{a_1}\right]^{(n-1)}$$

Here, n = order of the reaction
Given, $(t_{1/2})_1 = 0.1$ s, $a_1 = 400$
$(t_{1/2})_2 = 0.8$ s, $a_2 = 50$
On putting the values,

$$\frac{0.1}{0.8} = \left[\frac{50}{400}\right]^{(n-1)}$$

Taking log on both sides

$$\log\frac{0.1}{0.8} = (n-1)\log\frac{50}{400}$$

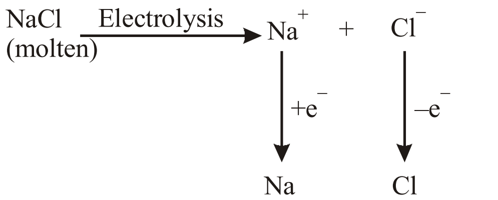
$$\log\frac{1}{8} = (n-1)\log\frac{1}{8}$$

$n - 1 = 1 \Rightarrow n = 2$

54. (d) $2Na + 2H_2O \longrightarrow 2NaOH + H_2\uparrow$

$2NaOH + CO_2 \longrightarrow Na_2CO_3 + H_2O$

$Na_2CO_3 + 2HCl \longrightarrow 2NaCl + H_2O + CO_2$



55. (b) The purest form of iron is wrought or malleable.

56. (d) P^{5+} having maximum nuclear charge per electron. Therefore, its size is smallest.

57. (d) $8Al \longrightarrow 8Al^{3+} + 24e^-$

$9Fe^{8/3+} + 24e^- \longrightarrow 9Fe$

Total 24 electrons are transferred.

58. (b) On increases the number of lone pairs of electrons, bond angle decreases. Therefore, order of bond angle is

$NH_4^+ > NH_3 > NH_2^-$

(no/p) (one/p) (two/p)

59. (c) $\dfrac{n'_{He}}{n'_{CH_4}} = \dfrac{1}{2}\sqrt{\dfrac{16}{4}} = \dfrac{1}{1}$

$\dfrac{n'_{He}}{n'_{SO_2}} = \dfrac{1}{3}\sqrt{\dfrac{64}{4}} = \dfrac{4}{3}$

So, molar ratio will be,

$n'_{He} : n'_{CH_4} : n'_{SO_2} = 4:4:3.$

60. (a) Angular momentum, mvr

$= \dfrac{nh}{2\pi} = \dfrac{3 \times h}{2\pi} = \dfrac{1.5h}{\pi}$

$= 3\hbar \left[\because \hbar = \dfrac{h}{2\pi}\right]$

61. (d) Silver carbonate on being strongly heated decomposes as

$Ag_2CO_3 \xrightarrow{\Delta} 2Ag + CO_2 + \frac{1}{2}O_2$

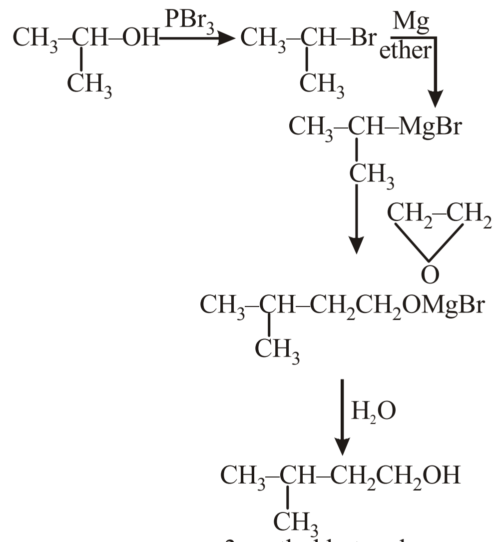
276g 216g

As 276g of Ag_2CO_3 gives = 216g of Ag

Hence, 2.76g of Ag_2CO_3 will give

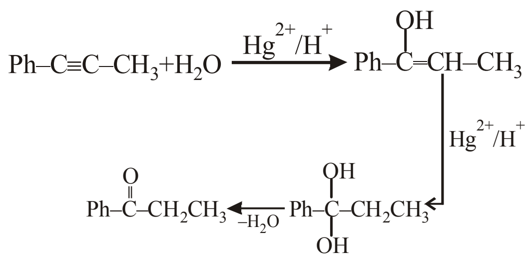
$= \dfrac{2.76 \times 216}{276} = 2.16g$

62. (c)

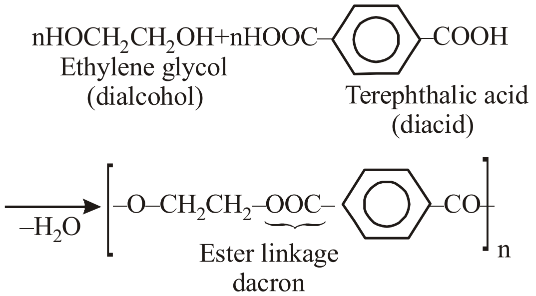


3-methyl butanol

63. (a)



64. (b) Condensation of diacid with dialcohol leads to ester linkage,



65. (b) Aldehydes and α-hydroxy ketones give positive Tollen's test. Glucose being a polyhydroxy aldehyde and fructose an α-hydroxy ketone give positive Tollen's test.

66. (b) Peptisation is the process in which freshly prepared precipitate disintegrates into colloidal solution.

67. (a) $Fe^{2+}(24) = [Ar]\ 3d^6\ 4s^0$ It has 4 unpaired electrons
$Cu^{+}(28) = [Ar]\ 3d^{10} 4s^0$ It has 0 unpaired electron
$Zn(30) = [Ar]\ 3d^{10}\ 4s^2$ It has 0 unpaired electron
$Ni^{3+}(25) = [Ar]\ 3d^7\ 4s^0$ It has 3 unpaired electrons

68. (b) $CuSO_4 + 2KI \longrightarrow CuI_2 + K_2SO_4$

$$2CuI_2 \longrightarrow \underset{\text{Cuprous iodide}}{2CuI} + I_2$$

69. (a) The correct order of ionic radii is $Cr^{3+} > Mn^{3+} > Fe^{3+} > Sc^{3+}$.

70. (c) We know that, $\Delta T_b = \dfrac{1000 \times K_b \times w}{W \times M}$

$$M = \frac{1000 \times K_b \times w}{W \times \Delta T_b}$$

$$\Delta T_b = \frac{1000 \times K_b \times 10}{100 \times 100}$$

$$\Delta T_b = K_b$$

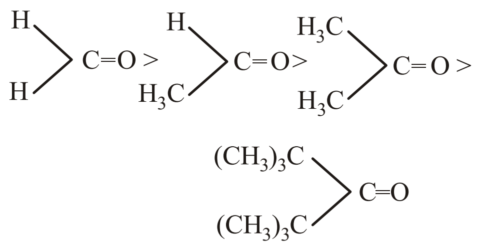
71. (b) If two different alkyl halides (R_1–X and R_2–X) are used, a mixture of three alkanes is obtained which is difficult to separate.

72. (b) On moving down the group, the nature of the oxides of group 13 elements changes from weakly acidic to amphoteric and amphoteric to basic. Tl_2O in aqueous solution gives TlOH which soluble and a strong base.

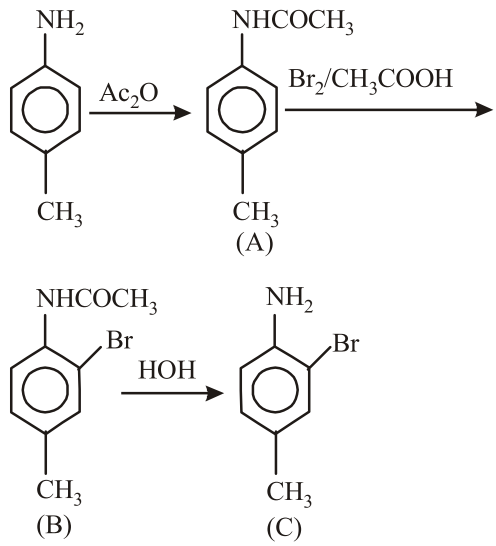
73. (c) According to Langmuir's adsorption isotherm, the mass of gas striking a given area of surface is proportional to the pressure of the gas as $\dfrac{x}{m} = \dfrac{k'p}{1+kp}$

74. (c)
$$\overset{2\pi\ \sigma\ \ \sigma\ \ \sigma\ \ \sigma}{\underset{1\sigma\ \ \sigma|^{\pi}|\sigma\ \ 2\pi}{N{\equiv}C{-}C{=}C{-}C{\equiv}N}}$$
$$H\ \ H$$
7 sigma and 5 pi

75. (a) As the number and the size of the alkyl groups increases, reactivity decreases. Hence, the correct order of reactivity is

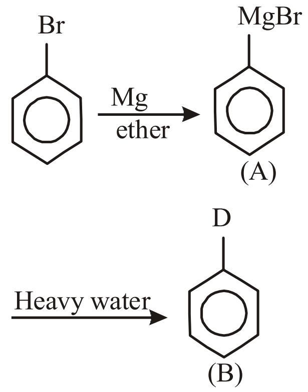


76. (d)



77. (b)
$$\underset{\text{(Keto)}}{CH_3-\overset{\overset{O}{\|}}{C}-CH_2-\overset{\overset{O}{\|}}{C}-OC_2H_5} \rightleftharpoons$$
$$\underset{\text{(enol)}}{CH_3-\overset{\overset{OH}{|}}{C}=CH-\overset{\overset{O}{\|}}{C}-OC_2H_5}$$

78. (b)



79. (a) Nucleophiles are the species which have tendency to donate a pair of electrons. They can be neutral or negatively charged. The nucleophilic power depends on the tendency of species to donate the electrons. Due to the presence of +I effect, it increases. Hence, higher the +I effect, higher the nucleophilic power.

80. (a) Heroin is a narcotic analgesic and not used as tranquilizer.

PART - III (MATHEMATICS)

81. (a) $\frac{dy}{dx} = \frac{x^2+y^2+1}{2xy}$

$\Rightarrow 2xy\,dy = \left(x^2+1\right)dx + y^2dx$

$\Rightarrow \frac{xd\left(y^2\right)-y^2dx}{x^2} = \left(\frac{x^2+1}{x^2}\right).dx$

$\Rightarrow \int d\left(\frac{y^2}{x}\right) = \int\left(1+\frac{1}{x^2}\right)dx$

$\Rightarrow \frac{y^2}{x} = x - \frac{1}{x} + C$

$\Rightarrow y^2 = \left(x^2 - 1 + Cx\right)$

When $x = 1, y = 0$

$\Rightarrow 0 = 1-1+C$

$\Rightarrow C = 0$

$\therefore$ The solution is $x^2-y^2 = 1$ i.e., hyperbola.

82. (a) $x.\frac{dy}{dx} + y = x.\frac{f(xy)}{f'(xy)}$

i.e., $\frac{d}{dx}(xy) = x\frac{f(x,y)}{f'(x,y)}$

$\Rightarrow \frac{f'(xy)}{f(xy)}d(xy) = x\,dx$

$\Rightarrow \int\frac{f'(xy)}{f(xy)}d(xy) = \int x dx$

$\Rightarrow \log\left[f(xy)\right] = \frac{x^2}{2} + C$

$\Rightarrow f(xy) = e^{\left(x^2/2+C\right)}$

$= e^{\frac{x^2}{2}}.e^C = k.e^{\frac{x^2}{2}}$

83. (b) The differential equation of the rectangular hyperbola $xy = c^2$ is

$y + x\frac{dy}{dx} = 0$

$\Rightarrow x\frac{dy}{dx} = -y$

84. (c) **Given :** $|\mathbf{a}| = 2\sqrt{2}, |\mathbf{b}| = 3$

One diagonal is $5\mathbf{a}+2\mathbf{b}+\mathbf{a}-3\mathbf{b} = 6\mathbf{a}-\mathbf{b}$

Length of one diagonal

$= |6\mathbf{a}-\mathbf{b}|$

$= \sqrt{36\mathbf{a}^2+\mathbf{b}^2-2\times6|\mathbf{a}|.|\mathbf{b}|.\cos45°}$

$= \sqrt{36\times8+9-12\times2\sqrt{2}\times3\times\frac{1}{\sqrt{2}}}$

$= \sqrt{288+9-12\times6} = \sqrt{225} = 15$

other diagonal is $(5\mathbf{a}+2\mathbf{b})-(\mathbf{a}-3\mathbf{b})$

$= 4\mathbf{a}+5\mathbf{b}$

Its length is

$= \sqrt{(4\mathbf{a})^2+(5\mathbf{b}^2)+2\times|4\mathbf{a}||5\mathbf{b}|\cos45°}$

$= \sqrt{16\times8+25\times9+40\times6} = \sqrt{593}$

85. (b) $\mathbf{r}.\mathbf{a} = \alpha\left(\mathbf{a}.\mathbf{b}\times\mathbf{c}\right)+\beta\left(\mathbf{a}.\mathbf{c}\times\mathbf{a}\right)+\gamma\left(\mathbf{a}.\mathbf{a}\times\mathbf{b}\right)$

$= \alpha\left[\mathbf{abc}\right]+0+0$

Similarly, $\mathbf{r}.\mathbf{b} = \beta\left[\mathbf{abc}\right]$ and $\mathbf{r}.\mathbf{c} = \gamma\ [\mathbf{abc}]$

$\therefore \frac{1}{2}\mathbf{r}.\left(\mathbf{a}+\mathbf{b}+\mathbf{c}\right) = \frac{1}{2}\left(\mathbf{r}.\mathbf{a}+\mathbf{r}.\mathbf{b}+\mathbf{r}.\mathbf{c}\right)$

$= \frac{1}{2}\left(\alpha+\beta+\gamma\right)\left[\mathbf{abc}\right]$

$= \frac{1}{2}\left(\alpha+\beta+\gamma\right)\times2 = \alpha+\beta+\gamma$

86. (c) p, q and r are reciprocal vectors of **a**, **b** and **c** respectively.

So, $\boldsymbol{p}.\boldsymbol{r} = 1, \boldsymbol{p}.\boldsymbol{b} = 0 = \boldsymbol{p}.\boldsymbol{c}$

$\boldsymbol{q}.\boldsymbol{a} = 0, \boldsymbol{q}.\boldsymbol{b} = 1, \boldsymbol{q}.\boldsymbol{c} = 0$

$\boldsymbol{r}.\boldsymbol{a} = 0, \boldsymbol{r}.\boldsymbol{b} = 0, \boldsymbol{r}.\boldsymbol{c} = 1$

$\therefore (\boldsymbol{la}+\boldsymbol{mb}+\boldsymbol{nc}).(\boldsymbol{lp}+\boldsymbol{mq}+\boldsymbol{nr})$

$= \boldsymbol{l}^2+\boldsymbol{m}^2+\boldsymbol{n}^2$

87. (a) Let $I = 7^n + 7^m$, then we observe that 7^i, 7^2, 7^3 and 7^4 ends in 7, 9, 3 and 1, respectively. Thus, 7^i ends in 7, 9, 3 or 1 according as i is of the form 4k + 1, 4k+2, 4k–1, respectively.

If S is the sample space, then $n(S) = (100)^2$

$7^m + 7^n$ is divisible by 5, if

(i) m is of the form 4k + 1 and n is of the form 4k – 1 or

(ii) m is of the form 4k + 2 and n is of the form 4k or

(iii) m is of the form 4k–1 and n is of the form 4k+1 or

(iv) m is of the form 4k and n is of the form 4k + 1 or

So, number of favourable ordered pairs (m, n) = 4 × 25 × 25

$\therefore$ Required probability $= \dfrac{4\times25\times25}{(100)^2} = \dfrac{1}{4}$

88. (b) Let **x** denote the sum of the numbers obtained when two fair dice are rolled. So, X may have values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

$P(X=2) = P(1,1) = \dfrac{1}{36}$

$P(X=3) = P\{(1,2),(2,1)\} = \dfrac{2}{36}$

$P(X=4) = \dfrac{3}{36}, P(X=5) = \dfrac{4}{36}:$

$P(X=6) = \dfrac{5}{36}; P(X=7) = \dfrac{6}{36};$

$P(X=8) = \dfrac{5}{36};$

$P(X=9) = \dfrac{4}{36}; P(X=10) = \dfrac{3}{36}; P(X=11)$

$= \dfrac{2}{36};$

$P(X=12) = \dfrac{1}{36}$

$\therefore$ Probability distribution table is given below

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Mean $\overline{X} = \sum XP(X)$

$$= \frac{\begin{bmatrix} 2\times1+3\times2+4\times3+5\times4+6\times5+7\times6+ \\ 8\times5+9\times4+10\times3+11\times2+12\times1 \end{bmatrix}}{36}$$

$$= \frac{252}{36} = 7$$

Variance $= \sum X^2P(X) - \overline{X}^2$

$$= \frac{\begin{bmatrix} 2^2\times1+3^2\times2+4^2\times3+5^2\times4 \\ +6^2\times5+7^2\times6+8^2\times5+9^2\times \\ 4+10^2\times3+11^2\times2+12^2\times1 \end{bmatrix}}{36} - 7^2$$

$$= \frac{1974}{36} - 49$$

$$= \frac{1974-1764}{36}$$

$$= \frac{210}{36} = \frac{35}{6}$$

$\therefore$ Variance $= \dfrac{35}{6}$

And, SD $= \sqrt{\dfrac{35}{6}}$

89. (d) Given digits are 1, 2, 3, 4.

Possibilities for units place digit (either 1 or 3) = 2

Possibilities for ten's digit = 3

Possibilities for hundred's place digit = 2

Possibilities for thousand place's digit = 1

$\therefore$ Number of favourable outcomes = 2 × 3 × 2 × 1 = 12

Number of numbers formed by 1, 2, 3, 4 (without repetitions) = 4!

$\therefore$ Required probability $= \dfrac{12}{4\times3\times2} = \dfrac{1}{2}$

90. (b) Vertices of Δ ABC are A(0, 4, 1), B(2, 3, –1) and C(4, 5, 0).

$AB = \sqrt{(2-0)^2+(3-4)^2+(-1-1)^2}$

$= \sqrt{4+1+4} = 3$

$BC = \sqrt{(4-2)^2+(5-3)^2+(0+1)^2}$

$= \sqrt{4+4+1} = 3$

and $CA = \sqrt{(4-0)^2+(5-4)^2+(0-1)^2}$

$= \sqrt{16+1+1} = 3\sqrt{2}$

$AB^2 + BC^2 = AC^2$

$\therefore$ ΔABC is a right angled triangle.

We know that, the orthocentre of a right angled triangle is the vertex containing the right angle.

$\therefore$ Orthocentre is point B (2, 3, –1).

91. (a) Given curve is

$y = (1 + x)^y + \sin^{-1}(\sin^2 x)$

On differentiating w.r.t x, we get

$$\frac{dy}{dx} = (1+x)^y\left[\frac{y}{1+x} + \log(1+x)\frac{dy}{dx}\right] + \frac{2\sin x\cos x}{\sqrt{1-\sin^4 x}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{at(0,1)} = 1 \quad [\because \text{at } x = 0, y = 1]$$

Slope of normal at $(x = 0) = -1$

$\therefore$ Equation of normal at $x = 0$ and $y = 1$ is

$y - 1 = -1\,(x - 0)$

$\Rightarrow y - 1 = -x \Rightarrow x + y = 1$

92. (c) It is clear that f (x) has a definite and unique value for each $x \in [1, 5]$.

Thus, for every point in the interval [1, 5], the value of f (x) exists.

So, f(x) is continuous in the interval [1, 5].

Also, $f'(x) = \dfrac{-x}{\sqrt{25-x^2}}$, which clearly exists for all x in an open interval (1, 5).

So, f′(x) is differentiable in (1, 5).

So, there must be a value $c \in [1, 5]$ such that

$$f'(c) = \frac{f(5)-f(1)}{5-1} = \frac{0-\sqrt{24}}{4}$$

$$= \frac{0-2\sqrt{6}}{4} = \frac{-\sqrt{6}}{2}$$

But $f'(c) = \dfrac{-c}{\sqrt{25-c^2}}$

$$\Rightarrow \frac{-c}{\sqrt{25-c^2}} = -\frac{\sqrt{6}}{2}$$

$\Rightarrow 4c^2 = 6\,(25 - c^2)$

$\Rightarrow 4c^2 = 150 - 6c^2 \Rightarrow 10c^2 = 150$

$\Rightarrow c^2 = 15 \Rightarrow c = \pm\sqrt{15}$

$\therefore c = \sqrt{15} \in [1, 5]$

93. (c) $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$

$|A| = 21 - 20 = 1$

$$\therefore A(\text{adj}A) = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1.\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = |A|\,.\,I$$

94. (b) Volume V of a cube of side x is given by $V = x^3$

$$\Rightarrow \frac{dv}{dx} = 3x^2$$

Let the change in x be $\Delta x = K\%$ of

$$x = \frac{kx}{100}$$

Now, the change in volume,

$$\Delta V = \left(\frac{dV}{dx}\right)\Delta x = 3x^3(\Delta x)$$

$$= 3x^2\left(\frac{kx}{100}\right) = \frac{3x^3.k}{100}$$

$\therefore$ Approximate change in volume

$$= \frac{3kx^3}{100} = \frac{3k}{100}.x^3$$

$= 3K\%$ of original volume

95. (c) The system has non- zero solution, if

$$\begin{vmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(-k-3) - k(3+1) - 1(-9+k) = 0$

$\Rightarrow -6k + 6 = 0$

$\Rightarrow k = 1$

96. (d) The points (1, 2, 3) and (2, –1, 0) lie on the opposite sides of the plane $2x + 3y - 2z - k = 0$

So, $(2 + 6 - 6 - k)(4 - 3 - k) < 0$

$\Rightarrow (k - 1)(k - 2) < 0$ .... (i)

$\therefore 1 < k < 2$

97. (d) $$\Delta(x) = \begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x-\cos x \\ \sin x & \sin x & 1 \end{vmatrix}$$

Applying $C_3 \to C_3 + C_2 - C_1$

$$\Delta(x) = \begin{vmatrix} 1 & \cos x & 0 \\ 1+\sin x & \cos x & 0 \\ \sin x & \sin x & 1 \end{vmatrix}$$

$= \cos x - \cos x\,(1 + \sin x)$

$[\because$ expanding along $C_3] = -\cos x . \sin x$

$= -\frac{1}{2}\sin 2x$

$$\therefore \int_0^{\pi/4} \Delta(x)dx = -\frac{1}{2}\int_0^{\pi/4} \sin 2x\,dx$$

$$= -\frac{1}{2}\left[-\frac{\cos 2x}{2}\right]_0^{\pi/4}$$

$$= +\frac{1}{2\times 2}\left[\cos\frac{\pi}{2} - \cos 0°\right]$$

$$= \frac{1}{4}(0-1) = -\frac{1}{4}$$

98. (b) $f'(x)$ is differentiable $\forall\ x \in [1, 6]$

By Lagrange's mean value theorem,

$$f'(x) = \frac{f(6) - f(1)}{6-1}$$

$f'(x) \geq 2 \forall x \in [1, 6]$ (given)

$\Rightarrow \frac{f(6)+2}{5} \geq 2$ $\quad [\because f(1) = -2]$

$\Rightarrow f(6) \geq 10 - 2 \Rightarrow f(6) \geq 8$

99. (b) $\Delta_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$

$$\therefore \sum_{r=0}^{m} \Delta_r = \begin{vmatrix} \sum_{r=0}^{m}(2r-1) & \sum_{r=0}^{m}{}^mC_r & \sum_{r=0}^{m}1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$$

$$= \begin{vmatrix} m^2-1 & 2^m & m+1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$$

$= 0$ $(\because$ two rows are identical)

100. (c) $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = r\,(\text{say})$

$\Rightarrow x = 2r + 1, y = 3r - 1, z = 4r + 1$

Since, the two lines intersect.

So, putting above values in second line, we get

$$\frac{2r+1-3}{1} = \frac{3r-1-k}{2} = \frac{4r+1}{1}$$

$2r - 2 = 4r + 1$

$\Rightarrow r = -3/2$

Also $3r - 1 - k = 8r + 2$

$\Rightarrow k = -5r - 3 = \frac{15}{2} - 3 = \frac{9}{2}$

101. (a) Let $f(x) = \frac{x}{\log x}$

$\Rightarrow f'(x) = \frac{\log x - 1}{(\log x)^2}$

For maxima and minima, put $f'(x) = 0$

$\log x - 1 = 0$

$\Rightarrow x = e$

Now, $f''(x)$

$$= \frac{(\log x)^2 . \frac{1}{x} - (\log x - 1) . \frac{2\log x}{x}}{(\log x)^4}$$

$$\Rightarrow f''(e) = \frac{\frac{1}{e} - 0}{1} = \frac{1}{e} > 0$$

$\therefore f(x)$ is minimum at $x = e$.

Hence, minimum value of $f(x)$ at $x = e$ is

$$f(e) = \frac{e}{\log e} = e$$

102. (c) Given curve is $y = f(x) = x^2 + bx - b$

$\Rightarrow f'(x) = 2x + b$

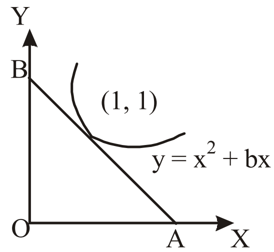
The equation of tangent at point (1, 1) is

$$y - 1 = \left(\frac{dy}{dx}\right)_{(1,1)}(x-1)$$

$\Rightarrow \; y - 1 = (b + 2)(x - 1)$

$\Rightarrow \; (2 + b)x - y = 1 + b$

$$\Rightarrow \frac{x}{\left(\frac{1+b}{2+b}\right)} - \frac{y}{(1+b)} = 1$$



So, $OA = \frac{1+b}{2+b}$

and $OB = -(1 + b)$

Now, area of $\Delta AOB = \frac{1}{2} \times$

$$\frac{(1+b)\left[-(1+b)\right]}{(2+b)} = 2$$

$\Rightarrow \; 4(2 + b) + (1 + b)^2 = 0$

$\Rightarrow \; b^2 + 6b + 9 = 0$

$\Rightarrow \; (b + 3)^2 \Rightarrow \; b = -3$

103. (c)

p	q	p	$p \Rightarrow q$	$\sim p \wedge q$	$(p \Rightarrow q) \Leftrightarrow (\sim p \wedge q)$
T	T	F	T	F	F
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	F	F

Hence, given statement is neither tautology nor contradiction.

104. (b) $x + iy$

$$= \frac{3}{2 + \cos\theta + i\sin\theta} = \frac{3(2 + \cos\theta - i\sin\theta)}{(2+\cos\theta)^2 + \sin^2\theta}$$

$$= \frac{6 + 3\cos\theta - 3i\sin\theta}{4 + \cos^2\theta + 4\cos\theta + \sin^2\theta}$$

$$= \frac{6 + 3\cos\theta - 3i\sin\theta}{5 + 4\cos\theta}$$

$$= \left(\frac{6 + 3\cos\theta}{5 + 4\cos\theta}\right) + \left(\frac{-3\sin\theta}{5 + 4\cos\theta}\right)$$

On equating real and imaginary parts, we get

$$x = \frac{3(2+\cos\theta)}{5 + 4\cos\theta}$$

And $y = \frac{-3\sin\theta}{5 + 4\cos\theta}$

$$\therefore x^2 + y^2 = \frac{9[4 + \cos^2\theta + 4\cos\theta + \sin^2\theta]}{(5 + 4\cos\theta)^2}$$

$$= \frac{9}{5 + 4\cos\theta} = 4\left(\frac{6 + 3\cos\theta}{5 + 4\cos^2\theta}\right) - 3$$

$= \; 4x - 3$

105. (b) Let $S : (\sim p \wedge q) \vee (p \wedge \sim q)$

$\Rightarrow \; \sim S : \sim [(\sim p \wedge q) \vee (p \wedge \sim q)]$

$\Rightarrow \; \sim S : \sim (\sim p \wedge q) \wedge \sim (p \wedge \sim q)$

$\Rightarrow \; \sim S : (p \vee \sim q) \wedge (\sim p \vee q)$

106. (b) The sum of ordinates of feet of normals drawn from a point to the parabola, $y^2 = 4ax$ is always zero.

Now, as normals at three points P, Q and R of parabola $y^2 = 4ax$ meet at (h, k).

$\Rightarrow$ The normals from (h, k) to $y^2 = 4ax$ meet the parabola at P, Q and R.

$\Rightarrow$ y-coordinate y_1, y_2, y_3 of these points and R will be zero.

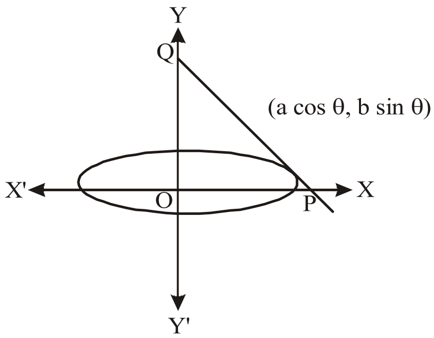
$\Rightarrow$ y-coordinate of the centroid of ΔPQR

i. e., $\frac{y_1 + y_2 + y_3}{3} = \frac{0}{3} = 0$

$\therefore$ centroid lies on $y = 0$

107. (c) Equation of tangent at $(a\cos\theta, b\sin\theta)$ to the ellipse is

$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$



Coordinates of P and Q are

$\left(\frac{a}{\cos\theta},0\right)$ and $\left(0,\frac{b}{\sin\theta}\right)$, respectively.

Area of $\Delta OPQ=\frac{1}{2}\left|\frac{a}{\cos\theta}\times\frac{b}{\sin\theta}\right|=\frac{ab}{|\sin 2\theta|}$

$\therefore$ Minimum area = ab

108. (b) The equation of any normal

to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $ax\cos\phi+by\cot\phi$ $=a^2+b^2$

$\Rightarrow$ $ax\cos\phi+by\cot\phi-(a^2+b^2)=0$ .....(i)

The straight line $lx+my-n=0$ will be normal to the hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then eq. (i) and $lx+my-n=$ 0 represent the same line,

$\therefore \frac{a\cos\phi}{l}=\frac{b\cot\phi}{m}=\frac{a^2+b^2}{n}$

$\Rightarrow \sec\phi=\frac{na}{l\left(a^2+b^2\right)}$

and $\tan\phi=\frac{nb}{m\left(a^2+b^2\right)}$

$$\frac{n^2a^2}{l^2\left(a^2+b^2\right)^2}-\frac{n^2b^2}{m^2\left(a^2+b^2\right)^2}=1$$

$[\because \sec^2\phi-\tan^2\phi=1]$

$\Rightarrow \frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{\left(a^2+b^2\right)^2}{n^2}$

But given equation of normal is

$\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{\left(a^2+b^2\right)^2}{k}$

$\therefore k=n^2$

109. (d) Given: a = $\cos\alpha+i\sin\alpha$

b = $\cos\beta+i\sin\beta$

and c = $\cos\gamma+i\sin\gamma$

Now, $\frac{b}{c}=\frac{\cos\beta+i\sin\beta}{\cos\gamma+i\sin\gamma}\times\frac{\cos\gamma-i\sin\gamma}{\cos\gamma-i\sin\gamma}$

$\cos\beta.\cos\gamma+\sin\beta.\sin\gamma+i$ $[\sin\beta.\cos\gamma-\sin\gamma.\cos\beta]$

$\Rightarrow \frac{b}{c}=\cos(\beta-\gamma)+i\sin(\beta-\gamma)$ .....(i)

Similarly, $\frac{c}{a}=\cos(\gamma-\alpha)+i\sin(\gamma-\alpha)$ ....(ii)

and $\frac{a}{b}=\cos(\alpha-\beta)+i\sin(\alpha-\beta)$ ....(iii)

On adding Eqs. (i), (ii) and (iii), we get

$\cos(\beta-\alpha)+\cos(\gamma-\alpha)+\cos(\alpha+\beta)+i$ $\left[\sin(\beta-\gamma)+\sin(\gamma-\alpha)+\sin(\alpha-\beta)\right]=1$

$\left[\because \frac{b}{c}+\frac{c}{a}+\frac{a}{b}=1\right]$

On equating real parts, we get

$\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)=1$

110. (b) $|z+4|\le 3 \Rightarrow -3\le z+4\le 3$

$\Rightarrow -6\le z+1\le 0$

$\Rightarrow 0\le -(z+1)\le 6$

$\Rightarrow 0\le |z+1|\le 6$

Hence, the greatest and least values are 6 and 0.

111. (c) On homogenising $y^2-x^2=4$ with the help of the line $\sqrt{3}x+y=2$, we get

$$y^2 - x^2 = 4\frac{\left(\sqrt{3}x + y\right)^2}{4}$$

$$\Rightarrow y^2 - x^2 = 3x^2 + y^2 + 2\sqrt{3}xy$$

$$\Rightarrow 4x^2 + 2\sqrt{3}xy = 0$$

On comparing with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 4$, $h = \sqrt{3}$ and $b = 0$

We know that,

$$\tan\theta = 2\frac{\sqrt{h^2 - ab}}{a+b} = \frac{2.\sqrt{3} - 0}{4+0}$$

$\therefore$ The angle between the lines is

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

112. (c) Let $z = x + iy$, then
$z + iz = x + iy + i(x + iy) = (x - y) + i(x + y)$
and $iz = i(x + iy) = -y + ix$,
Then, the area of the triangle formed by these lines is

$$\Delta = \frac{1}{2}\left\|\begin{matrix} x & y & 1 \\ (x-y) & (x+y) & 1 \\ -y & x & 1 \end{matrix}\right\|$$

Applying $R_2 \to R_2 - (R_1 + R_3)$,

$$\Delta = \frac{1}{2}\left\|\begin{matrix} x & y & 1 \\ 0 & 0 & -1 \\ -y & x & 1 \end{matrix}\right\| = \frac{1}{2}\left(x^2 + y^2\right)$$

$$\Rightarrow \frac{1}{2}|z|^2 = 200 \qquad \text{(given)}$$

$$|z|^2 = 400 \Rightarrow |z| = 20$$

$$\therefore 3|z| = 3 \times 20 = 60$$

113. (b) Given hyperbola is $25x^2 - 16y^2 = 400$
If (6, 2) is the midpoint of the chord, then equation of chord is $T = S_1$

$\Rightarrow$ $25(6x) - 16(2y) = 25(36) - 16(4)$

$\Rightarrow$ $75x - 16y = 450 - 32$

$\Rightarrow$ $75x - 16y = 418$

114. (b) Let the equation of the plane is

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$$

Then, $A(\alpha, 0, 0)$, $B(0, \beta, 0)$ and $C(0, 0, \gamma)$ are the points on the coordinate axes,
The centroid of the triangle is (1, 2, 4).

$$\therefore \frac{\alpha}{3} = 1 \Rightarrow \alpha = 3$$

$$\frac{\beta}{3} = 2 \Rightarrow \beta = 6$$

and $\frac{\gamma}{3} = 4 \Rightarrow \gamma = 12$

$\therefore$ The equation of the plane is

$$\frac{x}{3} + \frac{y}{6} + \frac{z}{12} = 1$$

$$\Rightarrow 4x + 2y + z = 12$$

115. (b) The given equation of the plane is $3x + 4y - 5z - 60 = 0$. It can be written in the form $\frac{x}{20} + \frac{y}{15} + \frac{z}{-12} = 1$

which meets the coordinate axes at the points A(20, 0, 0), B(0, 15, 0) and C(0, 0, –12).
The coordinates of the origin are O (0, 0, 0).
Therefore, volume of the tetrahedron OABC is =

$$= \frac{1}{6}\left\|\begin{matrix} 20 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & -12 \end{matrix}\right\| = \frac{1}{6}\left|20 \times 15 \times (-12)\right| = 600$$

116. (b) $\int_0^{2\pi} \left(\sin x + |\sin x|\right) dx$

$$= \int_0^{\pi} \left(\sin x + \sin x\right) dx + \int_{\pi}^{2\pi} \left(\sin x - \sin x\right) dx$$

$$= 2\int_0^{\pi} \sin x\, dx + 0 = -2\int_0^{\pi} \cos x\, dx$$

$$= -2(\cos\pi - \cos 0) = -2(-1 - 1) = 4$$

117. (c) $\int_0^{\sqrt{2}} \left[x^2\right] dx = \int_0^1 \left[x^2\right] dx + \int_1^{\sqrt{2}} \left[x^2\right] dx$

$$= \int_0^1 0\, dx + \int_1^{\sqrt{2}} 1\, dx$$

$= \quad [x]_1^{\sqrt{2}} = \sqrt{2} - 1$

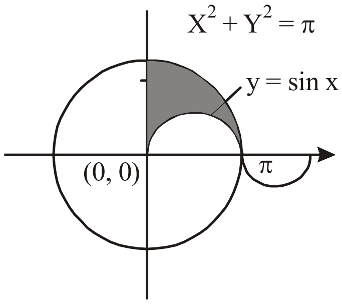
118. (a) $l(m, n) = l = \int_0^1 t^m (1+t)^n \, dt$

$$\Rightarrow l(m,n) = \left[(1+t)^n . \frac{t^{m+1}}{m+1}\right]_0^1$$

$$- \frac{n}{m+1}\int_0^1 (1+t)^{n-1} . t^{m+1} .dt$$

$$= \frac{2^n}{m+1} - \frac{n}{m+1}.l(m+1, n-1)$$

119. (a) $x^2 + y^2 = \pi^2$ is a circle of radius π and centre at origin.



Required area

$= \quad$ Area of circle (1st quadrant) $- \int_0^\pi \sin x \, dx$

$$= \frac{\pi.\pi^2}{4} - [-\cos x]_0^\pi = \frac{\pi^3}{4} + (\cos\pi - \cos 0)$$

$$= \frac{\pi^3}{4} + (-1-1) = \frac{\pi^3}{4} - 2 = \frac{\pi^3 - 8}{4}$$

120. (d) $|x| = 1$

$\therefore \quad x = \pm 1$

$$\therefore \quad y = xe^{|x|} = \begin{cases} x.e^{-x}, \ -1 < x < 0 \\ x.e^x, 0 < x < 1 \end{cases}$$

$$\therefore \text{Required area} = \left|\int_{-1}^0 xe^{-x} dx + \int_0^1 xe^x dx\right|$$

$$= \left|\left[-x.e^{-x} - e^{-x}\right]_{-1}^0 + \left\{x.e^x - e^x\right\}_0^1\right|$$

$$= \left|\{(0-1) - (1.e - e)\}\right| + \left|\{(e-e) - (0-1)\}\right|$$

$= 1 + 1 = 2$ sq units