Chapter 9: Optics

EXERCISES, EXERCISE [PAGES 1 - 185]

Exercises | Q 1. (i) | Page 184

Choose the correct option.
As per the recent understanding, light consists of

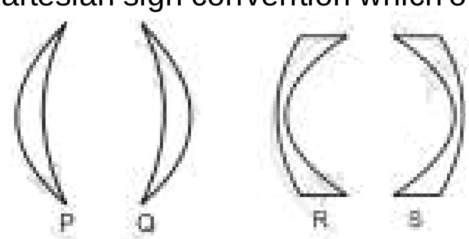
1. rays
2. waves
3. corpuscles
4. **photons obeying the rules of waves**

SOLUTION

Photons obeying the rules of waves

Exercises | Q 1. (ii) | Page 184

Choose the correct option.
Consider the optically denser lenses P, Q, R, and S drawn below. According to the Cartesian sign convention which of these have a positive focal length?



1. Only P
2. **Only P and Q**
3. Only P and R
4. Only Q and S

SOLUTION

Only P and Q

Exercises | Q 1. (iii) | Page 184

Choose the correct option.
Two plane mirrors are inclined at angle of 40° between them. Number of images seen of a tiny object kept between them is

1. Only 8
2. Only 9
3. **8 or 9**

4. 9 or 10

SOLUTION

Two plane mirrors are inclined at angle of 40° between them. Number of images seen of a tiny object kept between them is **8 or 9**

Exercises | Q 1. (iv) | Page 184

Choose the correct option.
A concave mirror of curvature 40 cm, used for shaving purposes produces image of double size as that of the object. Object distance must be

1. 10 cm only
2. 20 cm only
3. 30 cm only
4. **10 cm or 30 cm**

SOLUTION

10 cm or 30 cm

Exercises | Q 1. (v) | Page 184

Choose the correct option.
Which of the following aberrations will NOT occur for spherical mirrors?

1. **Chromatic aberration**
2. Coma
3. Distortion
4. Spherical aberration

SOLUTION

Chromatic aberration

Exercises | Q 1. (vi) | Page 184

Choose the correct option.
There are different fish, monkeys, and water of the habitable planet of the star Proxima b. A fish swimming underwater feels that there is a monkey at 2.5 m on the top of a tree. The same monkey feels that the fish is 1.6 m below the water surface.

Interestingly, height of the tree and the depth at which the fish is swimming are exactly same. Refractive index of that water must be

1. $\frac{6}{5}$
2. $\frac{5}{4}$
3. $\frac{4}{3}$
4. $\frac{7}{5}$

SOLUTION

5/4

Exercises | Q 1. (vii) | Page 184

Choose the correct option.
Consider the following phenomena/ applications: P) Mirage, Q) rainbow, R) Optical fibre and S) glittering of a diamond. Total internal reflection is involved in

1. **Only R and S**
2. Only R
3. Only P, R and S
4. all the four

SOLUTION

Only R and S

Exercises | Q 1. (viii) | Page 184

Choose the correct option.
A student uses spectacles of number -2 for seeing distant objects. Commonly used lenses for her/his spectacles are

1. **bi-concave**
2. double concave
3. concavo-convex
4. convexo-concave

SOLUTION

A student uses spectacles of number -2 for seeing distant objects. Commonly used lenses for her/his spectacles are **bi-concave**

Exercises | Q 1. (ix) | Page 185
Choose the correct option.

A spherical marble, of refractive index 1.5 and curvature 1.5 cm, contains a tiny air bubble at its centre. Where will it appear when seen from outside?

1 cm inside

at the centre

$\frac{5}{3}$ cm inside

2 cm inside

SOLUTION

At the centre

Exercises | Q 1. (x) | Page 185

Choose the correct option.
Select the WRONG statement.

1. **Smaller angle of prism is recommended for greater angular dispersion.**
2. Right-angled isosceles glass prism is commonly used for total internal reflection.
3. Angle of deviation is practically constant for thin prisms.
4. For emergent rays to be possible from the second refracting surface, certain minimum angle of incidence is necessary from the first surface.

SOLUTION

Smaller angle of prism is recommended for greater angular dispersion.

Exercises | Q 1. (xi) | Page 185

Choose the correct option.
Angles of deviation for extreme colours are given for different prisms. Select the one having maximum dispersive power of its material.

1. **7°, 10°**
2. 8°, 11°
3. 12°, 16°
4. 10°, 14°

SOLUTION

7°, 10°

Exercises | Q 1. (xii) | Page 185

Choose the correct option.

Which of the following is not involved in formation of a rainbow?

1. refraction
2. angular dispersion
3. angular deviation
4. **total internal reflection**

SOLUTION

Total internal reflection

Exercises | Q 1. (xiii) | Page 185

Choose the correct option.
Consider the following statements regarding a simple microscope:
(P) It allows us to keep the object within the least distance of distant vision.
(Q) Image appears to be biggest if the object is at the focus.
(R) It is simply a convex lens.

1. Only (P) is correct
2. Only (P) and (Q) are correct
3. Only (Q) and (R) are correct
4. **Only (P) and (R) are correct**

SOLUTION

Only (P) and (R) are correct

Exercises | Q 2. (i) | Page 185

Answer the following question.
As per recent development, what is the nature of light? Wave optics and particle nature of light are used to explain which phenomena of light respectively?

SOLUTION

i. As per recent development, it is now an established fact that light possesses dual nature. Light consists of energy carrier photons. These photons follow the rules of electromagnetic waves.

ii. Wave optics explains the phenomena of light such as interference, diffraction, polarization, Doppler effect, etc.

iii. Particle nature of light can be used to explain phenomena like photoelectric effect, emission of spectral lines, Compton effect, etc.

Exercise | Q 2. (ii) (a) | Page 185

Which phenomena can be satisfactorily explained using ray optics?

SOLUTION

Ray optics or geometrical optics: Ray optics can be used for understanding phenomena like reflection, refraction, double refraction, total internal reflection, etc.

Exercises | Q 2. (ii) (b) | Page 185

Answer the following question.
State the assumptions on which ray optics is based.

SOLUTION

Ray optics is based on the following fundamental laws:

i. Light travels in a straight line in a homogeneous and isotropic medium.

ii. Two or more rays can intersect at a point without affecting their paths beyond that point.

iii. **Laws of reflection:**

a. Reflected ray lies in the plane formed by the incident ray and the normal drawn at the point of incidence; and the two rays are on either side of the normal.

b. Angles of incidence and reflection are equal ($i = r$).

iv. **Laws of refraction:**

a. Refracted ray lies in the plane formed by the incident ray and the normal drawn at the point of incidence; and the two rays are on either side of the normal.

b. Angle of incidence (θ_1) and angle of refraction (θ_2) are related by Snell's law, given by,

$$n_1 \sin \theta_1 = n2 \sin \theta_2$$

where, n_1, n_2 = refractive indices of medium 1 and medium 2 respectively.

Exercises | Q 2. (iii) | Page 1

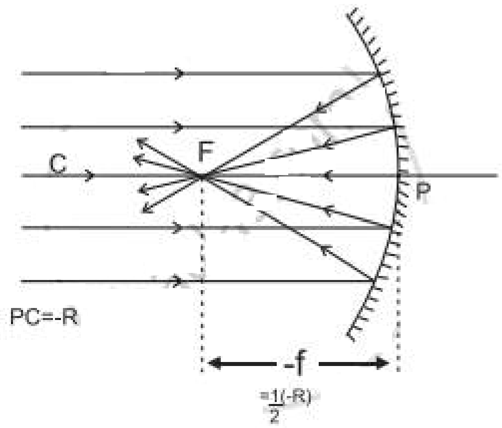
Answer the following question.
What is focal power of a spherical mirror or a lens? What may be the reason for using $P = 1/f$ as its expression?

SOLUTION

i. Converging or diverging the ability of a lens or of a mirror is defined as its focal power.

ii. This implies, the more the power of any spherical mirror or a lens, the more is its ability to converge or diverge the light that passes through it.

iii. In the case of a convex lens or concave mirror, the more the convergence, the shorter is the focal length as shown in the figure.



iv. Similarly, in the case of a concave lens or convex mirror, the more the divergence, the shorter is the focal length.

v. This explains that the focal power of any spherical lens or mirror is inversely proportional to the focal length.

vi. Hence, the expression of focal power is given by the formula, P = 1/f

Exercises | Q 2. (iv) | Page 185

At which positions of the objects do spherical mirrors produce diminished image?

SOLUTION

Amongst the two types of spherical mirrors, convex mirror always produces a diminished image at all positions of the object.

Concave mirror produces diminished image when object is placed:

a. Beyond radius of curvature (i.e., u > 2f)

b. At infinity (i.e., u = ∞)

Exercises | Q 2. (v) | Page 185

At which positions of the objects do spherical mirrors produce a magnified image?

SOLUTION

Amongst the two types of spherical mirrors, the convex mirror always produces a diminished image at all positions of the object.

Concave mirror produces magnified image when the object is placed:

a. between centre of curvature and focus (i.e., $2f > u > f$)

b. between focus and pole of the mirror (i.e., $u < f$)

Exercises | Q 2. (vi) | Page 185

At which positions of the objects do spherical mirrors produce a magnified image?

SOLUTION

Amongst the two types of spherical mirrors, the convex mirror always produces a diminished image at all positions of the object.

Concave mirror produces magnified image when the object is placed:

a. between centre of curvature and focus (i.e., $2f > u > f$)

b. between focus and pole of the mirror (i.e., $u < f$)

Exercises | Q 2. (vii) | Page 185

Answer the following question.
Define absolute refractive index and relative refractive index. Explain in brief with an illustration for each.

SOLUTION

i. Absolute refractive index of a medium is defined as the ratio of the speed of light in a vacuum to that in the given medium.

ii. A stick or pencil kept obliquely in a glass containing water appears broken as its part in water appears to be raised.

iii. As the speed of light is different in two media, the rays of light coming from water undergo refraction at the boundary separating the two media.

iv. Consider the speed of light to be v in water and c in air. (Speed of light in air ≈ speed of light in vacuum)

∴ refractive index of water $= \dfrac{n_w}{n_a} = \dfrac{n_w}{n_{vacuum}} = \dfrac{c}{v}$

v. The relative refractive index of a medium 2 is the refractive index of medium 2 with respect to medium 1 and it is defined as the ratio of the speed of light v_1 in medium 1 to its speed v_2 in medium 2.

∴ The relative refractive index of medium 2,

$^1n_2 = \dfrac{v_1}{v_2}$

vi. Consider a beaker filled with water of absolute refractive index n_1 kept on a transparent glass slab of absolute refractive index n_2.

vii. Thus, the refractive index of water with respect to that of glass will be,

$^gn_w = \dfrac{n_2}{n_1} = \dfrac{\frac{c}{v_2}}{\frac{c}{v_1}} = \dfrac{v_1}{v_2}$.

Exercises | Q 2. (viii) | Page 185

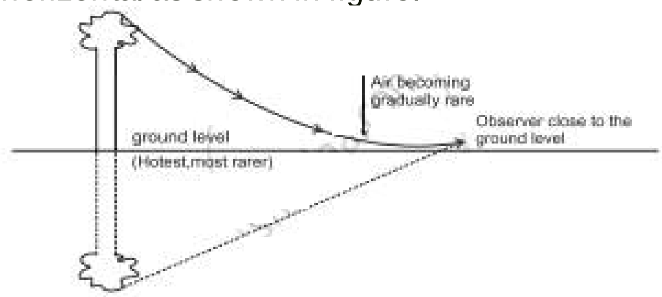
Answer the following question.

Explain 'mirage' as an illustration of refraction.

SOLUTION

i. On a hot clear Sunny day, along a level road, there appears a pond of water ahead of the road. However, if we physically reach the spot, there is nothing but the dry road and water pond again appears some distance ahead. This illusion is called mirage.
ii. Mirage results from the refraction of light through a non-uniform medium.
iii. On a hot day the air in contact with the road is hottest and as we go up, it gets gradually cooler. The refractive index of air thus decreases with height. Hot air tends to be less optically dense than cooler air which results into a non-uniform medium.
iv. Light travels in a straight line through a uniform medium but refracts when traveling through a non-uniform medium.
v. Thus, the ray of light coming from the top of an object get refracted while travelling downwards into less optically dense air and become more and more

horizontal as shown in figure.



vi. As it almost touches the road, it bends (refracts) upward. Then onwards, upward bending continues due to denser air.

vii. As a result, for an observer, it appears to be coming from below thereby giving an illusion of reflection from an (imaginary) water surface.

Exercises | Q 2. (ix) | Page 185

Answer the following question.

Under what conditions are total internal reflection possible? Explain it with a suitable example.

SOLUTION

Conditions for total internal reflection:

i. The light ray must travel from denser medium to rarer medium.

ii. The angle of incidence in the denser medium must be greater than critical angle for the given pair of media.

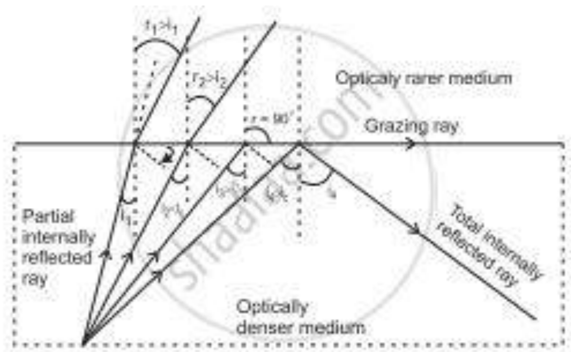
Total internal reflection in optical fibre:

i. Consider an optical fibre made up of the core of refractive index n_1 and cladding of refractive index n_2 such that, $n_1 > n_2$.

ii. When a ray of light is incident from a core (denser medium), the refracted ray is bent away from the normal.

iii. At a particular angle of incidence i_c in the denser medium, the corresponding angle of refraction in the rarer medium is 90°.

iv. For angles of incidence greater than i_c, the angle of refraction becomes larger than 90° and the ray does not enter into the rarer medium at all but is reflected totally into the denser medium as shown in the figure.



Exercises | Q 2. (x) | Page 185

Answer the following question.
Define the critical angle of incidence and obtain an expression for it.

SOLUTION

- The critical angle for a pair of refracting media can be defined as that angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90°.
- Let n be the relative refractive index of a denser medium with respect to the rarer.
- Then, according to Snell's law,

$$n = \frac{n_{denser}}{n_{rarer}} = \frac{\sin r}{\sin i_c} = \frac{\sin 90^\circ}{\sin i_c}$$

$$\therefore \sin(i_c) = \frac{1}{n}$$

Exercises | Q 2. (xi) | Page 185

Answer the following question.
Describe the construction and working of an optical fibre.

SOLUTION

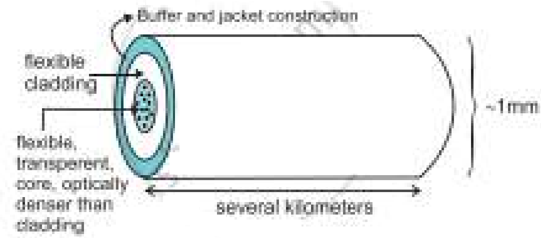
Construction:

i. An optical fibre consists of an extremely thin, transparent, and flexible core surrounded by an optically rarer flexible cover called cladding.

ii. For protection, the whole system is coated by a buffer and a jacket

iii. The entire thickness of the fibre is less than half an mm.

iv. Many such fibres can be packed together in an outer cover.



Construction of Optical fiber

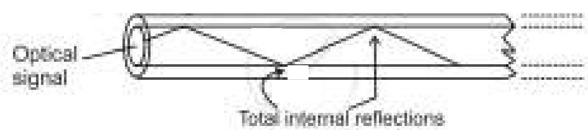
Working:

i. Working of optical fibre is based on the principle of total internal reflection.

ii. An optical signal (a ray of light) entering the core suffers multiple total internal reflections before emerging out after several kilometres.

iii. The optical signal travels with the highest possible speed in the material.

iv. The emerged optical signal has an extremely low loss in signal strength.



Working of Optical fibre

Exercises | Q 2. (xii) | Page 185

Answer the following question.
What are the advantages of optical fibre communication over electronic communication?

SOLUTION

Advantages:

i. **Broad bandwidth (frequency range):** For TV signals, a single optical fibre can carry over 90000 independent signals (channels).

ii. **Immune to EM interference:** Optical fibre being electrically non-conductive, does not pick up nearby EM signals.

iii. **Low attenuation loss:** loss being lower than 0.2 dB/km, a single long cable can be used for several kilometres.

iv. **Electrical insulator:** Optical fibres being electrical insulators, ground loops of metal wires or lightning do not cause any harm.

v. **Theft prevention:** Optical fibres do not use copper or other expensive material which are prone to be robbed.

vi. **Security of information:** Internal damage is most unlikely to occur, keeping the information secure.

Exercises | Q 2. (xiii) | Page 185

Answer the following question.
Why is prism binoculars preferred over traditional binoculars? Describe its working in brief.

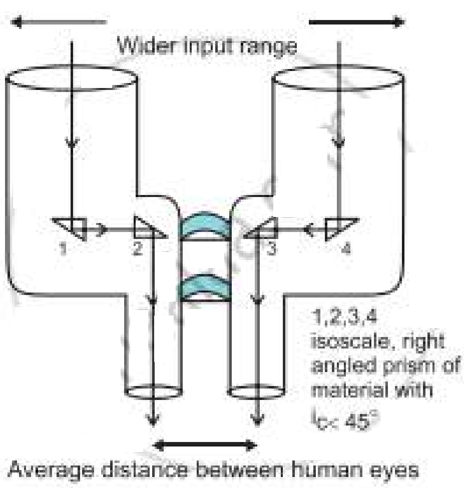
SOLUTION

i. Traditional binoculars use only two cylinders. Distance between the two cylinders can't be greater than that between the two eyes. This creates a limitation of the field of view.

ii. A prism binocular has two right-angled glass prisms that apply the principle of total internal reflection.

iii. The incident light rays are reflected internally twice giving the viewer a wider field of view. For this reason, prism binoculars are preferred over traditional binoculars.

Working:



Prism binoculars

i. The prism binoculars consist of 4 isosceles, right-angled prisms of a material having a critical angle less than 45°.

ii. The prism binoculars have a wider input range compared to traditional binoculars.

iii. The light rays incident on the prism binoculars, first get total internally reflected by the isosceles, right-angled prisms 1 and 4.

iv. These reflected rays undergo another total internal reflection by prisms 2 and 3 to form the final image.

Exercises | Q 2. (xii) | Page 185

Answer the following question.

A spherical surface separates two transparent media. Derive an expression that relates object and image distances with the radius of curvature for a point object. Clearly state the assumptions, if any.

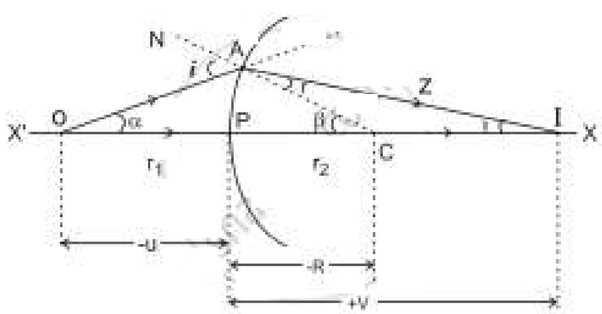
SOLUTION

i. Consider a spherical surface YPY′ of radius curvature R, separating two transparent media of refractive indices n_1 and n_2 respectively with $n_1 < n_2$.

ii. P is the pole and X′PX is the principal axis. A point object O is at a distance u from the pole, in the medium of refractive index n_1.

iii. In order to minimize spherical aberration, we consider two paraxial rays.

iv. The ray OP along the principal axis travels undeviated along with PX. Another ray OA strikes the surface at A.



Refraction at a single refracting surface

v. As $n_1 < n_2$, the ray deviates towards the normal (CAN), travels along with AZ, and a real image of point object O is formed at I.

vi. Let α, β and γ be the angles subtended by incident ray, normal and refracted ray with the principal axis.
$\therefore$ $i = (\alpha + \beta)$ and $r = (\beta - \gamma)$

vii. As the rays are paraxial, all the angles can be considered to be very small.,
i.e., $\sin i \approx i$ and $\sin r \approx r$

Angles α, β, and Y can also be expressed as,

$\alpha = \frac{\text{arcPA}}{\text{OP}} = \frac{\text{arcPA}}{-u}$,

$\beta = \frac{\text{arcPA}}{\text{PC}} = \frac{\text{arcPA}}{R}$

and $\gamma = \frac{\text{arcPA}}{\text{PI}} = \frac{\text{arcPA}}{v}$

viii. According to Snell's law,

$n_1 \sin(i) = n_2 \sin(r)$

For small angles, Snell's law can be written as, $n_1 i = n_2 r$

$\therefore n_1(\alpha + \beta) = n_2(\beta - \gamma)$

$(n_2 - n_1)\beta = n_1\alpha + n_2\gamma$

Substituting values of α, β and Y, we get,

$(n_2 - n_1)\frac{\text{arcPA}}{R} = n_1\left(\frac{\text{arcPA}}{-u}\right) + n_2\left(\frac{\text{arcPA}}{v}\right)$

$\therefore \frac{n_2 - n_1}{R} = \frac{n_2}{v} - \frac{n_1}{u}$

Assumptions:

To derive an expression that relates object and image distances with the radius of curvature for a point object, the two rays considered are assumed to be paraxial thus making the angles subtended by incident ray, normal and refracted ray with the principal axis very small.

Exercises | Q 2. (xiii) | Page 186

Answer the following question.

Derive lens makers' equation. Why is it called so? Under which conditions focal length f and radii of curvature R are numerically equal for a lens?

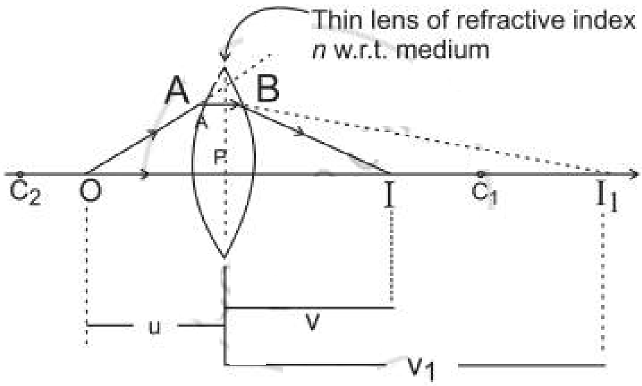
SOLUTION

i. Consider a lens of radii of curvature R_1 and R_2 kept in a medium such that the refractive index of the material of the lens with respect to the medium is denoted as n.

ii. Assuming the lens to be thin, P is the common pole for both the surfaces. O is a point object on the principal axis at a distance u from P.

iii. The refracting surface facing the object is considered as the first refracting surface with radii R_1.

iv. In the absence of the second refracting surface, the paraxial ray OA deviates towards normal and would intersect the axis at I_1. $PI_1 = v_1$ is the image distance for intermediate image I_1.



v. For a curved surface,

$$\frac{(n_2 - n_1)}{R} = \frac{n_2}{v} - \frac{n_1}{u}$$

Thus, in this case,

$\therefore n_2 = n, n_1 = 1, R = R_1, u = u$ and $v = v_1$

$$\therefore \frac{(n-1)}{R_1} = \frac{n}{v_1} = \frac{1}{u} \quad .....(1)$$

vi. Before reaching I_1, the incident rays (AB and OP) strike the second refracting surface. In this case, image I_1 acts as a virtual object for the second surface.

vii. For second refracting surface,

$n_2 = 1, n_1 = n, R = R_2, u = v_1$ and $PI = v$

$$\therefore \frac{(1-n)}{R_2} = \frac{1}{v} - \frac{n}{v_1} - \frac{(n-1)}{R_2} = \frac{1}{v} - \frac{n}{v_1} \quad .....(2)$$

viii. Adding equations (1) and (2),

$$(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{v} - \frac{1}{u}$$

For object at infinity, image is formed at focus, i.e., for $u = \infty$, $v = f$. Substituting this in above equation,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad .....(3)$$

This equation is known as the lens makers' formula.

ix. Since the equation can be used to calculate the radii of curvature for the lens, it is called the lens makers' equation.

x. The numeric value of focal length f and radius of curvature R is the same under the following two conditions:

Case I: For a thin, symmetric, and double convex lens made of glass (n = 1.5), R_1 is positive and R_2 is negative but, $|R_1| = |R_2|$.

In this case,

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = 0.5\left(\frac{2}{R}\right) = \frac{1}{R}$$

$\therefore$ f = R

Case II: Similarly, for a thin, symmetric and double concave lens made of glass (n = 1.5), R_1 is negative and R_2 is positive but, $|R_1| = |R_2|$.

In this case,

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{-R} - \frac{1}{R}\right) = 0.5\left(-\frac{2}{R}\right) = -\frac{1}{R}$$

$\therefore$ f = – R or |f| = |R|

Exercises | Q 3. (i) | Page 186

Answer the following question in detail.

What are the different types of dispersions of light? Why do they occur?

SOLUTION

i. There are two types of dispersions:
   a. Angular dispersion
   b. Lateral dispersion

ii. The refractive index of material depends on the frequency of incident light. Hence, for different colours, the refractive index of the material is different.

iii. For an obliquely incident ray, the angles of refraction are different for each colour and they separate as they travel along different directions resulting in angular dispersion.

iv. When a polychromatic beam of light is obliquely incident upon a plane parallel transparent slab, emergent beam consists of all component colours separated out.

v. In this case, these colours are parallel to each other and are also parallel to their initial direction resulting in lateral dispersion.

Exercises | Q 3. (ii) | Page 186

Answer the following question in detail.

Define angular dispersion for a prism.

SOLUTION

If a polychromatic beam is an incident upon a prism, the emergent beam consists of all the individual colours angularly separated. This phenomenon is known as angular dispersion for a prism.

Exercises | Q 3. (ii) | Page 186

Answer the following question in detail.

Obtain angular dispersion expression for a thin prism. Relate it with the refractive indices of the material of the prism for corresponding colours.

SOLUTION

i. For any two-component colours, angular dispersion is given by,
$\delta_{21} = \delta_2 - \delta_1$

ii. For white light, we consider two extreme colours viz., red and violet.
$\therefore \delta_{VR} = \delta_V - \delta_R$

iii. For thin prism,
$\delta = A(n - 1)$
$\therefore \delta_{21} = \delta_2 - \delta_1$
$= A(n_2 - 1) - A(n_1 - 1) = A(n_2 - n_1)$

where n_1 and n_2 are refractive indices for the two colours.

iv. For white light,
$\delta_{VR} = \delta_V - \delta_R$
$= A(n_V - 1) - A(n_R - 1) = A(n_V - n_R)$.

Exercises | Q 3. (iii) | Page 186

Answer the following question in detail.

Explain and define the dispersive power of transparent material. Obtain its expressions in terms of angles of deviation and refractive indices.

SOLUTION

i. The ability of an optical material to disperse constituent colours is its dispersive power.

ii. It is measured for any two colours as the ratio of angular dispersion to the mean deviation for those two colours. Thus, for the extreme colours of white light, dispersive power is given by,

$$\omega = \frac{\delta_V - \delta_R}{\left(\frac{\delta_V + \delta_R}{2}\right)} \approx \frac{\delta_V - \delta_R}{\delta_Y} = \frac{A(n_V - n_R)}{A(n_Y - 1)} = \frac{n_V - n_R}{n_Y - 1}$$

Exercises | Q 3. (iv) (i) | Page 186

Answer the following question in detail.
State the conditions under which a rainbow can be seen.

SOLUTION

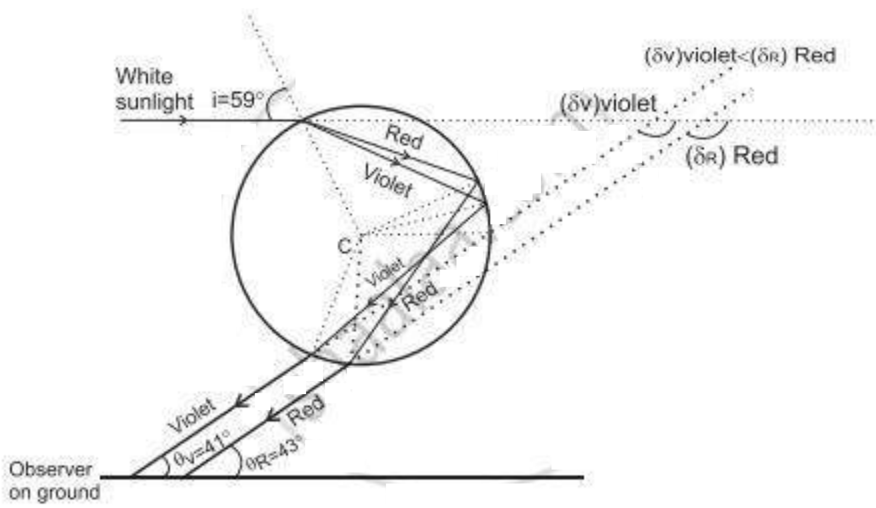
A rainbow can be observed when there is a light shower with a relatively large raindrop occurring during morning or evening time with enough sunlight around.

Exercises | Q 3. (iv) (ii) | Page 186

Answer the following question in detail.
Explain the formation of a primary rainbow. For which angular range with the horizontal is it visible?

SOLUTION

1. A ray AB incident from Sun (white light) strikes the upper portion of a water drop at an incident angle i.
2. On entering into the water, it deviates and disperses into constituent colours. The figure shows the extreme colours (violet and red).



Formation of primary rainbow

3. Refracted rays BV and BR strike the opposite inner surface of water drop and suffer internal reflection.

4. These reflected rays finally emerge from V′ and R′ and can be seen by an observer on the ground.

5. For the observer, they appear to be coming from the opposite side of the Sun.

6. Minimum deviation rays of red and violet colour are inclined to the ground level at $\theta_R = 42.8° \approx 43°$ and $\theta_V = 40.8 \approx 41°$ respectively. As a result, in the rainbow, the red is above and violet is below.

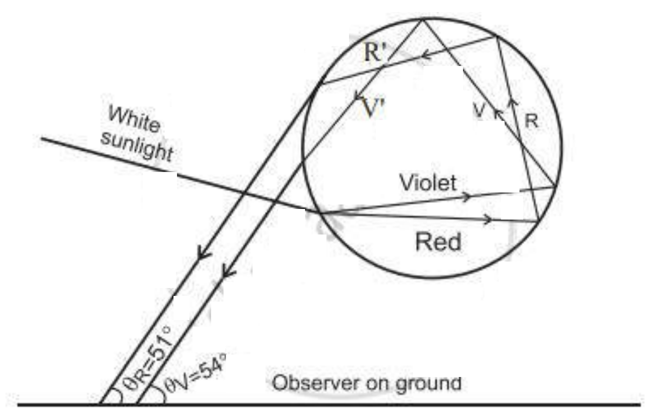
Exercises | Q 3. (iv) (iii) | Page 186

Answer the following question in detail.
Explain the formation of a secondary rainbow. For which angular range with the horizontal is it visible?

SOLUTION

1. A ray AB incident from Sun (white light) strikes the lower portion of a water drop at an incident angle i.

2. On entering into the water, it deviates and disperses into constituent colours. The figure shows the extreme colours (violet and red).



Formation of secondary rainbow

3. Refracted rays BV and BR finally emerge the drop from V' and R' after suffering two internal reflections and can be seen by an observer on the ground.

4. Minimum deviation rays of red and violet colour are inclined to the ground level at $\theta_R \approx 51°$ and $\theta_V \approx 53°$ respectively. As a result, in the rainbow, the violet is above and red is below.

Exercises | Q 3. (iv) (iv) | Page 186

Answer the following question in detail.

Is it possible to see primary and secondary rainbow simultaneously? Under what conditions?

SOLUTION

Yes, it is possible to see primary and secondary rainbows simultaneously. This can occur when the centres of both the rainbows coincide.

Exercises | Q 3. (v) (i) | Page 186

Answer the following question in detail.

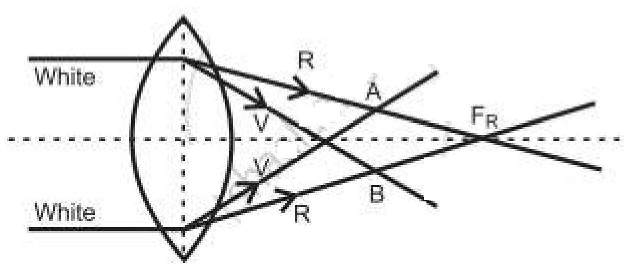
Explain chromatic aberration for spherical lenses. State a method to minimize or eliminate it.

SOLUTION

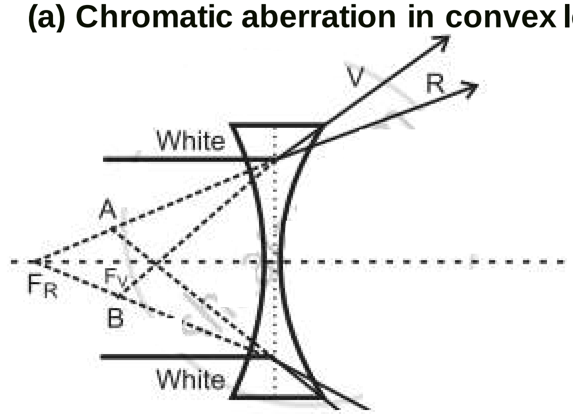
1. Lenses are prepared by using a transparent material medium having a different refractive index for different colours. Hence angular dispersion is present.

2. If the lens is thick, this will result in notably different foci corresponding to each colour for a polychromatic beam, like a white light. This defect is called chromatic aberration.

3. As violet light has maximum deviation, it is focussed closest to the pole.



(a) Chromatic aberration in convex lens



(b) Chromatic aberration in concave lens

Reducing/eliminating chromatic aberration:

i. Eliminating chromatic aberrations for all colours is impossible. Hence, it is minimised by eliminating aberrations for extreme colours.

ii. This is achieved by using either a convex and a concave lens in contact or two thin convex lenses with proper separation. Such a combination is called achromatic combination.

Exercises | Q 3. (v) (ii) | Page 186

Answer the following question in detail.
What is achromatism? Derive a condition to achieve achromatism for a lens combination. State the conditions for it to be converging.

SOLUTION

1. To eliminate chromatic aberrations for extreme colours from a lens, either a convex and a concave lens in contact or two thin convex lenses with proper separation are used.

2. This combination is called an achromatic combination. The process of using this combination is termed as achromatism of a lens.

3. Let ω_1 and ω_2 be the dispersive powers of materials of the two-component lenses used in contact for an achromatic combination.

4. Let V, R and Y denote the focal lengths for violet, red and yellow colours respectively.

5. For lens 1, let

   $K_1 = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)_1$ and $K_2 = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)_2$

6. For the combination to be achromatic, the resultant focal length of the combination must be the same for both the colours,

   $\therefore f_V = f_R$

   $$\therefore \frac{1}{f_V} = \frac{1}{f_R}$$

   For two thin lenses in contact, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

   $$\therefore \frac{1}{(f_V)_1} + \frac{1}{(f_V)_2} = \frac{1}{(f_R)_1} + \frac{1}{(f_R)_2}$$

7. From Lens maker's formula,

   $[(n_V)_1 - 1]K_1 + [(n_V)_2 - 1]K_2 = [(n_R)_1 - 1]K_1 + [(n_R)_2 - 1]K_2$

   $$\therefore \frac{K_1}{K_2} = \frac{(n_V)_2 - (n_R)_2}{(n_V)_1 - (n_R)_1} \quad ....(1)$$

8. Similarly, for mean colour (yellow),

$$\frac{1}{f_Y} = \frac{1}{(f_Y)_1} + \frac{1}{(f_Y)_2} \quad ...(2)$$

$$\frac{K_1}{K_2} = \frac{(n_Y)_2 - 1}{(n_Y)_1 - 1} \times \frac{(f_Y)_2}{(f_Y)_1} \quad ....(3)$$

9. From equations (1) and (3)

$$\frac{(f_Y)_2}{(f_Y)_1} = -\frac{(n_V)_2 - (n_R)_2}{(n_V)_1 - (n_R)_1} \times \frac{(n_Y)_2 - 1}{(n_Y)_1 - 1} \quad ...(4)$$

Now, dispersive power $\omega_1 = \frac{(n_V)_2 - (n_R)_2}{(n_Y)_2 - 1}$ and

$\omega_2 = \frac{(n_V)_2 - (n_R)_2}{(n_Y)_2 - 1}$

10. Substituting values of ω_1 and ω_2 in equation (4), we get,

$$\frac{(f_Y)_2}{(f_Y)_1} = \frac{\omega_2}{\omega_1}$$

This is the condition for achromatism of a combination of lenses.

Condition for converging:

For this combination to be converging, f_Y must be positive.

Using equation (3), for f_Y to be positive,

$(f_Y)_1 < (f_Y)_2 \Rightarrow \omega_1 < \omega_2$

Exercises | Q 3. (vi) | Page 186

Answer the following question in detail.

Describe spherical aberration for spherical lenses. What are the different ways to minimize or eliminate it?

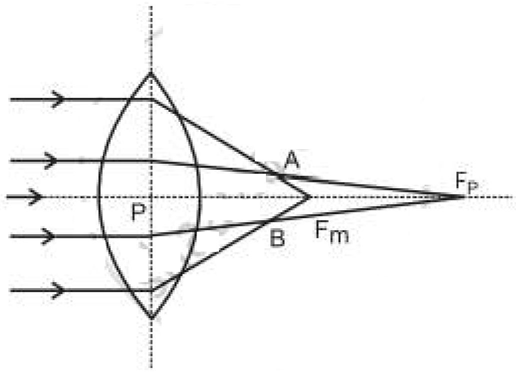
SOLUTION

i. All the formulae used for image formation by lenses are based on some assumptions. However, in reality, these assumptions are not always true.

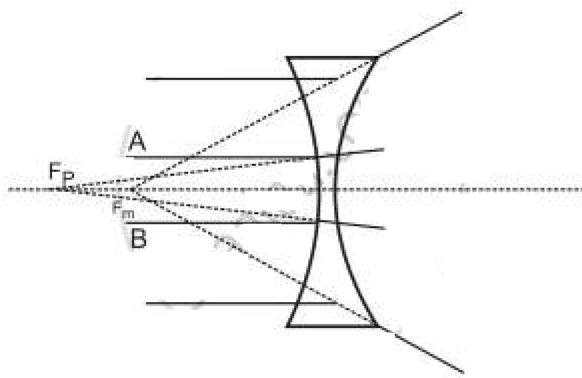
ii. A single point focus in case of lenses is possible only for small aperture spherical lenses and for paraxial rays.

iii. The rays coming from a distant object farther from the principal axis no longer remain parallel to the axis. Thus, the focus gradually shifts towards the pole.

iv. This defect arises due to the spherical shape of the refracting surface, hence known as spherical aberration. It results in a blurred image with unclear boundaries.



Spherical aberration in convex lens



Spherical aberration in concave lens

v. As shown in the figure, the rays near the edge of the lens converge at focal point F_M. Whereas, the rays near the principal axis converge at point F_P. The distance between F_M and F_P is measured as the longitudinal spherical aberration.

vi. In absence of this aberration, a single point image can be obtained on a screen. In the presence of spherical aberration, the image is always a circle.

vii. At a particular location of the screen (across AB in the figure), the diameter of this circle is minimum. This is called the circle of least confusion. The radius of this circle is transverse spherical aberration.

Methods to eliminate/reduce spherical aberration in lenses:

1. The cheapest method to reduce spherical aberration is to use a planoconvex or planoconcave lens with a curved side facing the incident rays.

2. Certain ratio of radii of curvature for a given refractive index almost eliminates the spherical aberration. For n = 1.5, the ratio is $\frac{R_1}{R_2} = \frac{1}{6}$ and n = 2, $\frac{R_1}{R_2} = \frac{1}{5}$.
3. The use of two thin converging lenses separated by a distance equal to the difference between their focal lengths with the lens of larger focal length facing the incident rays considerably reduces spherical aberration
4. Spherical aberration of a convex lens is positive (for real image), while that of a concave lens is negative. Thus, a suitable combination of them can completely eliminate spherical aberration.

Exercises | Q 3. (vii) | Page 186

Answer the following question in detail.
Define and describe the magnifying power of an optical instrument.

SOLUTION

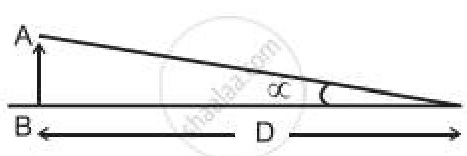
Angular magnification or magnifying power of an optical instrument is defined as the ratio of the visual angle made by the image formed by that optical instrument (β) to the visual angle subtended by the object when kept at the least distance of distinct vision (α).

Exercises | Q 3. (viii) | Page 186

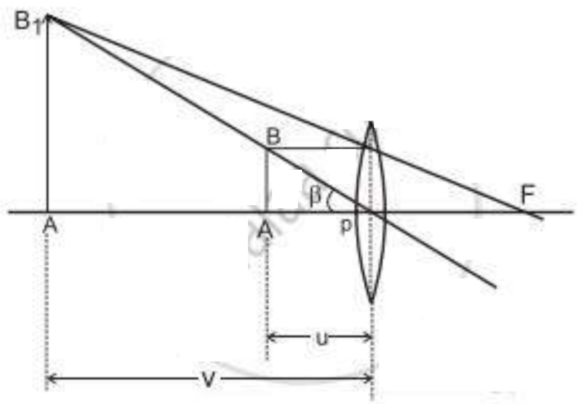
Answer the following question in detail.
Derive an expression for the magnifying power of a simple microscope. Obtain its minimum and maximum values in terms of its focal length.

SOLUTION

1. Figure (a) shows the visual angle α made by an object when kept at the least distance of distinct vision (D = 25 cm). Without an optical instrument, this is the greatest possible visual angle as we cannot get the object closer than this.



(a) Visual Angle α



(b) Visual Angle β

2. Figure (b) shows a convex lens forming an erect, virtual, and magnified image of the same object when placed within the focus.

3. The visual angle β of the object and the image, in this case, is the same. However, this time the viewer is looking at the image which is not closer than D. Hence the same object is now at a distance smaller than D.

4. Angular magnification or magnifying power, in this case, is given by

$$M = \frac{\text{Visual angle of the image}}{\text{Visual angle of the object at D}} = \frac{\beta}{\alpha}$$

For small angles

$$M = \frac{\beta}{\alpha} \approx \frac{\tan(\beta)}{\tan(\alpha)} = \frac{BA/PA}{BA/D} = \frac{D}{u}$$

5. For maximum magnifying power, the image should be at D. For a thin lens, considering thin lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

In case of simple microscope,

f = +f, v = – D, u = – u and M = M_{max}

$$\therefore \frac{1}{f} = \frac{1}{-D} - \frac{1}{-u}$$

$$\therefore \frac{D}{f} = \frac{D}{-D} + \frac{D}{u}$$

As, $M = \frac{D}{u}$,

$$M_{max} = 1 + \frac{D}{f}$$

6. For minimum magnifying power, v = ∞ and u = f (numerically)

$$\therefore M_{min} = \frac{D}{u} = \frac{D}{f}$$

Exercises | Q 3. (ix) | Page 186

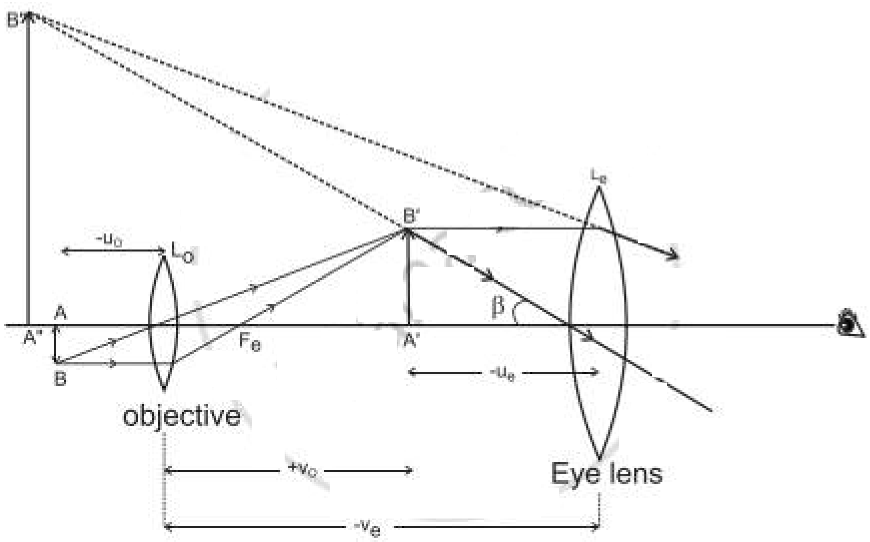
Answer the following question in detail.

Derive the expressions for the magnifying power and the length of a compound microscope using two convex lenses.

SOLUTION

1. The final image formed in compound microscope (A″ B″) as shown in the figure, makes a visual angle β at the eye.

2. The visual angle made by the object from distance D is α.



Compound Microscope

3. From figure,

$\tan \beta = \frac{\text{A"B"}}{v_e} = \frac{\text{A'B'}}{u_e}$

and $\tan \alpha = \frac{\text{AB}}{\text{D}}$

4. Angular magnification or magnifying power of a compound microscope is given by,

$M = \frac{\beta}{\alpha} \approx \frac{\tan(\beta)}{\tan(\alpha)}$

$= \frac{\text{A'B'}}{u_e} \times \frac{\text{D}}{\text{AB}} = \frac{\text{A'B'}}{\text{AB}} \times \frac{\text{D}}{u_e}$

Where, $\left(\frac{\text{A'B'}}{\text{AB}}\right) = \frac{v_o}{u_o}$ = m_o is the linear (lateral) magnification of objective and $\left(\frac{\text{D}}{u_e}\right)$ = M_e is the angular magnification or magnifying power of the eye lens.

$\therefore M = m_o \times M_e$

5. The length of the compound microscope is the distance between the two lenses, $L = v_o + u_e$.

Exercises | Q 3. (x) | Page 186

Answer the following question in detail.
What is a terrestrial telescope and an astronomical telescope?

SOLUTION

i. Telescopes used to see the objects on the Earth, like mountains, trees, players playing a match in a stadium, etc. are called terrestrial telescopes.

ii. In such a case, the final image must be erect. The eye lens used for this purpose must be concave and such a telescope is popularly called a binocular.

iii. Most of the binoculars use three convex lenses with proper separation. The image formed by the second lens is inverted with respect to the object. The third lens again inverts this image and makes the final image erect with respect to the object.

iv. An astronomical telescope is the telescope used to see objects like planets, stars, galaxies, etc. In this case, there is no necessity for an erect image. Such telescopes use a convex lens as an eye lens.

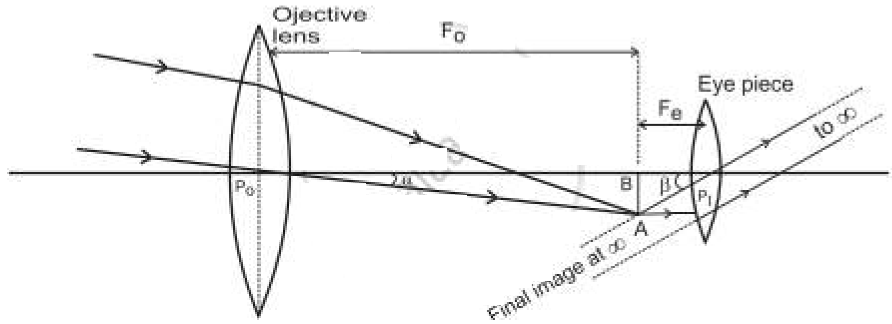
Exercises | Q 3. (xi) | Page 186

Answer the following question in detail.
Obtain the expressions for magnifying power and the length of an astronomical telescope under normal adjustments.

SOLUTION

1. For telescopes, α is the visual angle of the object from its own position, which is practically at infinity.

2. The visual angle of the final image is β and its position can be adjusted to be at D. However, under normal adjustments, the final image is also at infinity making a greater visual angle than that of the object.



Telescope

3. The parallax at the cross wires can be avoided by using the telescopes in normal adjustments.

4. The objective of focal length f_o focusses the parallel incident beam at a distance from the objective giving an inverted image AB.

5. For normal adjustment, the intermediate image AB forms at the focus of the eye lens. Rays refracted beyond the eye lens form a parallel beam inclined at an angle β with the principal axis.

6. Angular magnification or magnifying power for telescope is given by,

$$M = \frac{\beta}{\alpha} \approx \frac{\tan(\beta)}{\tan(\alpha)} = \frac{BA/P_eB}{BA/P_oB} = \frac{f_o}{f_e}$$

7. Length of the telescope for normal adjustment is, $L = f_o + f_e$.

Exercises | Q 3. (xii) (i) | Page 186

Answer the following question in detail.

What is the limitation in increasing the magnifying powers of a simple microscope?

SOLUTION

In the case of a simple microscope,

$$M = \frac{D}{u} = 1 + \frac{D}{f}$$

Thus, the limitation in increasing the magnifying power is determined by the value of focal length and the closeness with which the lens can be held near the eye.

Exercises | Q 3. (xii) (ii) | Page 186

Answer the following question in detail.

What is the limitation in increasing the magnifying powers of a compound microscope?

SOLUTION

In the case of a compound microscope,

$$M = m_o \times M_e = \frac{v_o}{u_o} \times \frac{D}{u_e}$$

Thus, in order to increase m_o, we need to decrease u_o. Thereby, the object comes closer and closer to the focus of the objective. This increases v_o and hence the length of the microscope. Therefore, m_o can be increased only within the limitation of the length of the microscope.

Exercises | Q 3. (xii) (iii) | Page 186

Answer the following question in detail.
What is the limitation in increasing the magnifying powers of the astronomical telescope?

SOLUTION

In the case of telescopes,

$$M = \frac{f_o}{f_e}$$

Where f_o = focal length of the objective

f_e =focal length of the eye-piece

The length of the telescope for normal adjustment is, $L = f_o + f_e$.

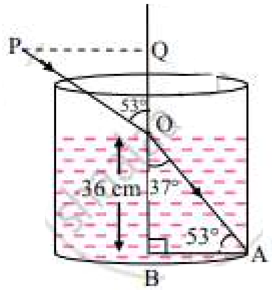
Thus, the magnifying power of the telescope can be increased only within the limitations of the length of the telescope.

Exercises | Q 4. (i) | Page 186

Solve Numerical example.

A monochromatic ray of light strike the water (n = 4/3) surface in a cylindrical vessel at angle of incidence 53°. Depth of water is 36 cm. After striking the water surface, how long will the light take to reach the bottom of the vessel? [Angles of the most popular Pythagorean triangle of sides in the ratio 3:4:5 are nearly 37°, 53°, and 90°]

SOLUTION



From figure,
ray PO = incident ray

ray OA = refracted ray
QOB = Normal to the water surface.

Given that, ∠POQ = angle of incidence (θ_1) = 53°

Seg OB = 36 cm and $n_{water} = \frac{4}{3}$

From Snell's law,

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\therefore n_{water} = \frac{\sin\theta_1}{\sin\theta_2}$

Or $\sin\theta_2 = \frac{\sin\theta_1}{n_{water}} = \frac{\sin(53^\circ) \times 3}{4}$

$\therefore \theta_2 \approx 37°$

ΔOBA forms a Pythagorean triangle with angles 53°, 37°, and 90°.

Thus, sides of ΔOBA will be in ratio 3: 4: 5

Such that OA forms the hypotenuse. From the figure, we can infer that,

Seg OB = 4x = 36 cm

$\therefore$ x = 9

$\therefore$ seg OA = 5x = 45 cm and

seg AB = 3x = 27 cm.

This means the light has to travel 45 cm to reach the bottom of the vessel.

The speed of the light in water is given by,

$v = \frac{c}{n}$

$\therefore v = \frac{3 \times 10^8}{4/3} = \frac{9}{4} \times 10^8 \text{m/s}$

$\therefore$ Time taken by light to reach the bottom of vessel is,

$t = \frac{s}{v} = \frac{45 \times 10^{-2}}{\frac{9}{4} \times 10^8} = 20 \times 10^{-10} = 2 \text{ ns or } 0.002 \text{ μs}$

The light will take **2 ns or 0.002 μs** to reach the bottom of the vessel

Exercises | Q 4. (ii) | Page 186

Solve Numerical example.

Estimate the number of images produced if a tiny object is kept in between two plane mirrors inclined at 35°, 36°, 40° and 45°.

SOLUTION

1. For $\theta_1 = 35°$

   $n_1 = \frac{360}{\theta_1} = \frac{360}{35} = 10.28$

   As n_1 is non-integer, N_1 = integral part of $n_1 = 10$

2. For $\theta_2 = 36°$

   $n_2 = \frac{360}{36} = 10$

   As n_2 is even integer, $N_2 = (n_2 - 1) = 9$

3. or $\theta_3 = 40°$

   $n_3 = \frac{360}{40} = 9$

   As n_3 is odd integer,

   Number of images seen $(N_3) = n_3 - 1 = 8$

   (if the object is placed at the angle bisector)

   or Number of images seen $(N_3) = n_3 = 9$

   (if the object is placed off the angle bisector)

4. For θ4 = 45°

$$n_4 = \frac{360}{45} = 8$$

As n_4 is even integer,

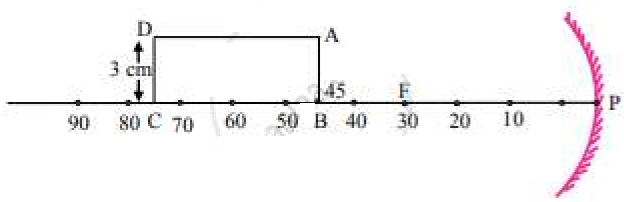
$N_4 = n_4 - 1 = 7$

The number of images seen when mirrors are inclined at 35°, 36°, 40°, 45° are **10, 9, 8** or **9, 7** respectively.

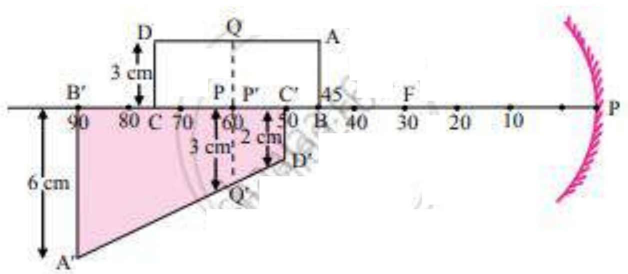
Exercises | Q 4. (iii) | Page 186

Solve Numerical example.

A rectangular sheet of length 30 cm and breadth 3 cm is kept on the principal axis of a concave mirror of focal length 30 cm. Draw the image formed by the mirror on the same diagram, as far as possible on scale.



SOLUTION



From the given condition,

1. For edge AB,

   $u_{AB} = -45$ cm, f = − 30 cm

   ∴ Using mirror formula,

$$\frac{1}{f} = \frac{1}{v_{AB}} = \frac{1}{u_{AB}}$$

$$\therefore \frac{1}{v_{AB}} = \frac{1}{f} - \frac{1}{u_{AB}} = \frac{1}{-30} - \frac{1}{-45} = \frac{1}{-90}$$

$\therefore v_{AB} = -90$ cm

$\therefore$ Magnification for edge AB,

$$m_{AB} = -\frac{v_{AB}}{u_{AB}} = -\frac{(-90)}{(-45)} = -2$$

Negative sign indicates that image is inverted.

$$\therefore m_{AB} = \frac{I_{AB}}{O_{AB}} = -2$$

$\therefore I_{AB} = -2 \times 3 = -6$ cm

2. For edge CD,

$u_{CD} = -75$ cm .....($\because$ length of the sheet is 30 cm)

$$\therefore \frac{1}{v_{CD}} = \frac{1}{f} - \frac{1}{u_{CD}} = \frac{1}{-30} - \frac{1}{-75} = -\frac{1}{50}$$

$\therefore v_{CD} = -50$ cm

Also, $m_{CD} = -\frac{v_{CD}}{u_{CD}} = -\frac{(-50)}{(-75)} = -\frac{2}{3}$

$\therefore \frac{I_{CD}}{O_{CD}} = -\frac{2}{3}$

$\therefore$ ICD $= -\frac{2}{3} \times 3 = -2$ cm

3. For the part of sheet at centre of curvature, PQ, ($u_{PQ} = -60$ cm) the image formed will be of same size ($I_{PQ} = -3$ cm) and on the same point i.e., ($v_{PQ} = -60$ cm). The resulting image is shown in the figure.

Exercises | Q 4. (iv) | Page 186

Solve Numerical example.

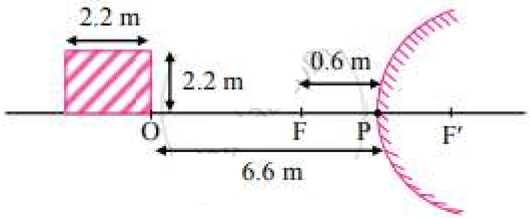
A car uses a convex mirror of curvature 1.2 m as its rear-view mirror. A minibus of cross-section 2.2 m × 2.2 m is 6.6 m away from the mirror. Estimate the image size.

SOLUTION

For a convex mirror,

$$f = \frac{R}{2} = \frac{1.2}{2} = +0.6\text{m}$$

Given that, a minibus, approximately the shape of square is at distance of 6.6 m from the mirror.
i.e., u = – 6.6 m



From mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{0.6} - \frac{1}{(-6.6)}$$

$$= \frac{7.2}{3.96} = \frac{20}{11}$$

$\therefore v = 0.55$ m

As, lateral magnification,

$$m = \frac{h_2}{h_1} = -\frac{v}{u}$$

$$\therefore \frac{h_2}{(2.2)} = -\frac{(0.55)}{(-6.6)}$$

$$\therefore h_2 = \frac{0.55 \times 2.2}{6.6}$$

$$= \frac{0.55}{3}$$

$\therefore h_2 = 0.183$ m

i.e., $h_2 \approx 0.2$ m

Hence, the size of the image of minibus is nearly a square of edge **0.2 m**.

Exercises | Q 4. (v) | Page 187

Solve Numerical example.

A glass slab of thickness 2.5 cm having refractive index 5/3 is kept on an ink spot. A transparent beaker of very thin bottom, containing water of refractive index 4/3 up to 8 cm, is kept on the glass block. Calculate apparent depth of the ink spot when seen from the outside air.

SOLUTION

When observed from the outside air, the light coming from the ink spot gets refracted twice; once through the glass and once through the water.

∴ When observed from water,

$$\frac{n_g}{n_w} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\therefore \frac{5/3}{4/3} = \frac{2.5}{\text{Apparent depth}}$$

$$\therefore \text{Apparent depth} = \frac{2.5}{5/4}$$

$$\therefore \text{Apparent depth} = 2 \text{ cm}$$

∴ Apparent depth = 2 cm

Now when observed from outside air, the total real depth of ink spot can be taken as (8 + 2) cm = 10 cm.

$$\therefore \frac{n_w}{n_{air}} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\therefore \text{Apparent depth} = \frac{10}{4/3}$$

$$= \frac{10 \times 3}{4} = 7.5 \text{ cm}$$

Apparent depth of the ink spot when seen from the outside air is **7.5 cm**.

Exercises | Q 4. (vi) | Page 187

Solve Numerical example.

A convex lens held some distance above a 6 cm long pencil produces its image of SOME size. On shifting the lens by a distance equal to its focal length, it again produces the image of the SAME size as earlier. Determine the image size.

SOLUTION

For a convex lens, it is given that the image size remains unchanged after shifting the lens through a distance equal to its focal length. From given conditions, it can be inferred that the object distance should be u = - f/2.

Also, $h_1 = 6$ cm, $v_1 = v_2$

From the formula for thin lenses,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

i.e., $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\therefore \frac{1}{v} = \frac{1}{f} + \left(-\frac{2}{f}\right)$$

$\therefore v = -f$

Now, magnification of the lens is,

$$m = \frac{h_2}{h_1} = \frac{v}{u} = \frac{-f}{-\frac{f}{2}} = 2$$

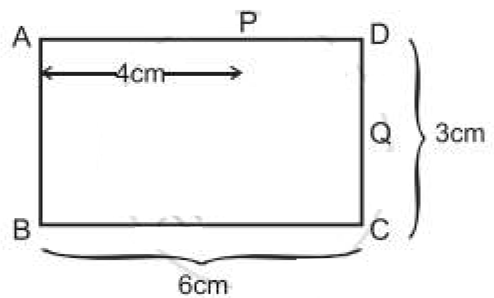
$\therefore h_2 = h_1 \times 2 = 6 \times 2 = 12$ cm

The size of the image formed will be **12 cm**.

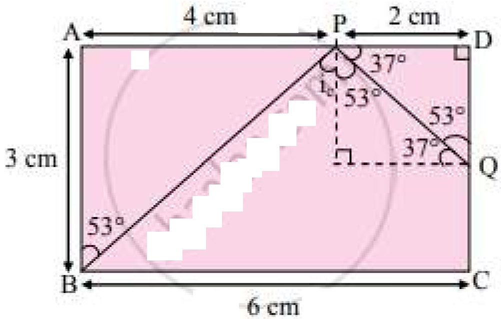
Exercises | Q 4. (vii) | Page 187

Solve Numerical example.

Figure below shows the section ABCD of a transparent slab. There is a tiny green LED light source at the bottom left corner B. A certain ray of light from B suffers total internal reflection at nearest point P on the surface AD and strikes the surface CD at point Q. Determine refractive index of the material of the slab and distance DQ. At Q, the ray PQ will suffer partial or total internal reflection? [Angles of the most popular Pythagorean triangle of sides in the ratio 3:4:5 are nearly 37°, 53°, and 90°]



SOLUTION



As the light ray undergoes total internal reflection at P, the ray BP may be incident at a critical angle. For a Pythagorean triangle with sides in ratio 3: 4: 5 the angle opposite to side 3 units is 37° and that opposite to 4 units is 53°.

Thus, from the figure, we can say, in ΔBAP,

$\angle ABP = 53°$

$\therefore \angle BPN - i_c = 53°$

$\therefore n_{glass} = \frac{1}{\sin i_c} = \frac{1}{\sin(53^\circ)} \approx \frac{1}{08} = \frac{5}{4}$

∴ Refractive index (n) of the slab is $\frac{5}{4}$.

From symmetry, ΔPDQ is also a Pythagorean triangle with sides in ratio QD: PD: PQ = 3: 4: 5.

∴ PD = 2 cm ⇒ QD = 1.5 cm.

As the critical angle is ic = 53° and the angle of incidence at Q, ∠PQN = 37° is less than the critical angle, there will be a partial internal reflection at Q.

The refractive index of a material is $\frac{5}{4}$. The ray PQ will suffer **partial internal reflection** at Q.

Exercises | Q 4. (viii) | Page 187

Solve Numerical example.

A point object is kept 10 cm away from one of the surfaces of a thick double convex lens of refractive index 1.5 and radii of curvature 10 cm and 8 cm. Central thickness of the lens is 2 cm. Determine location of the final image considering paraxial rays only.

SOLUTION

Given that, R_1 = 10 cm, R_2 = – 8 cm, u = – 10 cm and n = 1.5

From lens maker's equation,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \frac{1}{f} = (1.5-1)\left(\frac{1}{10} - \frac{1}{-8}\right)$$

$$= 0.5 \times \frac{9}{40} = \frac{9}{80}$$

$$\therefore f = \frac{80}{9} \text{ cm}$$

Now,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\therefore \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{9}{80} + \frac{1}{-10} = \frac{1}{80}$$

$$\therefore v = 80 \text{ cm}$$

The final location of the image will be at **80 cm** from the lens.

Exercises | Q 4. (ix) | Page 187

Solve Numerical example.
A monochromatic ray of light is incident at 37° on an equilateral prism of refractive index 3/2. Determine angle of emergence and angle of deviation. If angle of prism is adjustable, what should its value be for emergent rays to be just possible for the same angle of incidence?

SOLUTION

By Snell's law, in case of prism,

$$n = \frac{\sin(i)}{\sin(r_1)}$$

$$\therefore \frac{3}{2} = \frac{\sin(37^{\circ})}{\sin(r_1)}$$

$$\therefore r1 = \sin^{-1}\left(\frac{0.6018}{3/2}\right)$$

= sin−1 (0.4012)

= 23°39′

For equilateral prism, A = 60°

Also, A= $r_1 + r_2$

$\therefore r_2 = A - r_1 = 60° - 23°39' = 36°21'$

Applying snell's law on the second surface of prism,

$$\frac{1}{n} = \frac{\sin(r_2)}{\sin(e)}$$

$$\therefore \frac{2}{3} = \frac{\sin(36^{\circ}21')}{\sin(e)}$$

$$\therefore e = \sin^{-1}\left(\frac{0.5927}{2/3}\right)$$

= sin−1 (0.889)

= 62°44′

≈ 63

For any prism,

i + e = A + δ

∴ δ = (i + e) − A

= (37 + 63) − 60

= 40°

For an emergent ray to just emerge, the angle r_2' acts as a critical angle.

$\therefore r_2' = \sin^{-1}\left(\frac{1}{n}\right)$

$= \sin^{-1}\left(\frac{2}{3}\right)$

= 41′48°

As, $A = r_1 + r_2$ and i to be kept the same.

$\Rightarrow A' = r_1 + r_2' = 23°39' + 41°48'$

= 65°27’

i. Angle of emergence and angle of deviation in first case are **63°** and **40°** respectively.

ii. For emergent ray to just emerge with same incident angle the angle of prism should have the value of **65°27’**.

Q 4. (x) | Page 187

Solve Numerical example.
From the given data set, determine angular dispersion by the prism and dispersive power of its material for extreme colours. $n_R = 1.62$ $n_V = 1.66$, $\delta_R = 3.1°$

SOLUTION

Given: $n_R = 1.62$, $n_V = 1.66$, $\delta_R = 3.1°$

To find: i. Angular dispersion (δ_{VR})

ii. Dispersive power (ω_{VR})

Formula: i. $\delta = A\,(n - 1)$

ii. $\delta_{VR} = \delta_V - \delta_R$

iii. $\omega = \dfrac{\delta_V - \delta_R}{\left(\frac{\delta_V + \delta_R}{2}\right)}$

Calculation: From formula (i),

$\delta_R = A(n_R - 1)$

$$\therefore A = \frac{\delta_R}{(n_R - 1)} = \frac{3.1}{(1.62 - 1)} = \frac{3.1}{0.62}$$

$\therefore \delta_V = A(n_V - 1) = 5 \times (1.66 - 1) = 3.3°$

From formula (ii),

$\delta_{VR} = 3.3 - 3.1 = 0.2°$

From formula (iii),

$$\omega_{VR} = \frac{3.3 - 3.1}{\left(\frac{3.3+3.1}{2}\right)} = \frac{0.2}{6.4} \times 2 = \frac{0.2}{3.2} = \frac{1}{16}$$

$= 0.0625$

i. Angular dispersion by prism is **0.2°**

ii. Dispersive power of material of prism is **0.0625.**

Exercises | Q 4. (xi) | Page 187

Solve Numerical example.

Refractive index of a flint glass varies from 1.60 to 1.66 for visible range. Radii of curvature of a thin convex lens are 10 cm and 15 cm. Calculate the chromatic aberration between extreme colours.

SOLUTION

Given the refractive indices for extreme colours. As, $n_R < n_V$

$n_R = 1.60$ and $n_V = 1.66$

For convex lens,

$R_1 = 10$ cm and $R_2 = -15$ cm

$$\therefore \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{1}{10} - \frac{1}{-15}\right) = \frac{1}{6}$$

For red colour,

$$\frac{1}{f_R} = (n_R - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.60 - 1)\frac{1}{6}$$

$= 0.1$

$\therefore f_R = 10$ cm

Similarly, for violet colour,

$$\frac{1}{f_V} = (n_V - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.66 - 1)\frac{1}{6}$$

$= 0.11$

$\therefore f_V = 11$ cm

$\therefore$ Longitudinal chromatic aberration

$= f_V - f_R = 11 - 10 = 1$ cm

The chromatic aberration between extreme colours will be **1 cm**.

Exercises | Q 4. (xii) | Page 187

Solve Numerical example.

A person uses spectacles of 'number' 2.00 for reading. Determine the range of magnifying power (angular magnification) possible. It is a concavoconvex lens (n = 10.5) having a curvature of one of its surfaces to be 10 cm. Estimate that of the other.

SOLUTION

For a single concavo-convex lens, the magnifying power will be the same as that for a simple microscope As the number represents the power of the lens,

$$P = \frac{1}{f} = 2 \Rightarrow f = 0.5 \text{ m.}$$

$\therefore$ Range of magnifying power of a lens will be,

$$M_{min} = \frac{D}{f} = \frac{0.25}{0.5} = 0.5$$

$$\text{and } M_{max} = 1 + \frac{D}{f} = 1 + 0.5 = 1.5$$

Given that, n = 1.5, $|R_1|$ = 10 cm

f = 0.5 m = 50 cm

From lens maker's equation,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{50} = (1.5-1)\left(\frac{1}{10} - \frac{1}{R_2}\right)$$

$$\therefore 0.04 = \left(\frac{1}{10} - \frac{1}{R_2}\right)$$

$$\therefore \frac{1}{R_2} = \frac{1}{10} - 0.04 = \frac{3}{50}$$

$$\therefore R_2 = \frac{50}{3} \text{ cm}$$

50/3 cm.