CBSE Sample Paper -01
SUMMATIVE ASSESSMENT –I
Class – X Mathematics

Time allowed: 3 hours Maximum Marks: 90

General Instructions:

a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

Section A

1. Write the number of solution of the following pair of linear equations: $x+2y-8=0, \quad 2x+4y=16$
2. If two zeros of the polynomial $f(x) = x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then find its third zero.
3. Evaluate: $\tan 5°\tan 25°\tan 30°\tan 65°\tan 85°$
4. Express $\sin 67^o + \text{Cos } 75^o$ in term of trigonometric ratios of angles between 0^o and 45^o.

SECTION – B

5. Find the class marks if classes 15.5 – 18.5 and 50 – 75.
6. Find the values of α and β for which the following system of linear equations has infinite number of solutions. $2x + 3y = 7$, $2\alpha x + (\alpha + \beta)y = 28$
7. The areas of two similar triangles ABC and PQR are 64 cm^2 and 121 cm^2 respectively. If QR = 15.4 cm, find BC.
8. Given that sin(A + B) = sinAcosB + cosAsinB, find the value of sin75°.
9. A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household. Find the mode.

Family size	1-3	3-5	5-7	7-9	9-11
No. of families	7	8	2	4	1

10. The perimeters of two similar triangles are 30 cm and 20 cm. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.

Section C

11. Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.

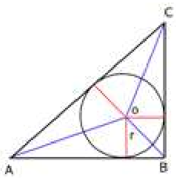
12. Solve: $ax + by = a - b$
$bx - ay = a + b$

13. The lengths of 40 leaves of a plant are measured correctly to the nearest millimeter , and the data obtained is represented in the following table :

Length (in mm)	118-126	127-135	136-144	145-153	154-162	163-171	172-180
Number of Leaves	3	5	9	12	5	4	2

Find the median length of the leaves.

14. ABC is a right-angled triangle right angled at A. A circle is inscribed in it the lengths of two sides containing the right angle are 6 cm and 8 cm. Find the radius of the circle.



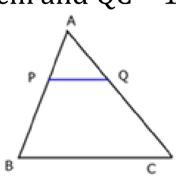
15. Given 15 oct A = 8, find sin A and sec A.

16. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

17. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data.

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

18. P and Q are points on sides AB and AC, respectively of ΔABC. If AP = 3 cm, PB = 6 cm, AQ = 5 cm and QC = 10 cm, show that BC = 3PQ.



19. If α and β are the zeros of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, find the polynomial whose zeros are 2α + 3β and 3α + 2β.

20. Prove $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$.

Section – D

21. Find a cubic polynomial with the sum of the zeros, sum of the products of its zeros taken two at a time, and the product of its zeros as 2, -7, -14 respectively.

22. Draw the graphs of the equations $x - y + 1 = 0$ *and* $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

23. Using Basic Proportionality Theorem, Prove that a line drawn through the mid-point of one sides of a triangle parallel to another side bisects the third side.
24. ABCD is a trapezium in which AB||DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO}=\frac{CO}{DO}$.
25. In ΔPQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the value of sin P, cos P and tan P.
26. If cosecA = 2, find the value of $\frac{1}{\tan A}+\frac{\sin A}{1+\cos A}$.
27. The mean of the following frequency distribution is 62.8. Find the missing frequency x.

Classes	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	8	x	12	7	8

28. If $3\cot A=4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A}=\cos^2 A-\sin^2 A$ or not.
29. Draw a cumulative frequency curve and cumulative frequency polygon for the following frequency distribution by less than method.

Age (in years)	0-9	10-19	20-9	30-39	40-49	50-59	60-69
Number of persons	5	15	20	23	17	11	9

30. A train covered a certain distance at a uniform speed. If the train would have been 6 km/hr faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr, it would have taken 6 hours more than the scheduled time. Find the length of the journey.
31. In order to celebrate Van Mahotsav, the students of a school planned to plant two types of trees in the nearby park. They decided to plant 144 trees of type A and 84 trees of type B. If the two types of plants are to be in the same number of columns, find the maximum number of columns in which they can be planted.
What values do these students possess?

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Class – X
Mathematics

Time allowed: 3 hours | **ANSWERS** | Maximum Marks: 90

SECTION – A

1. Here,

$$\frac{a_1}{a_1}=\frac{1}{2},\frac{b_1}{b_2}=\frac{2}{4}=\frac{1}{2},\frac{c_1}{c_2}=\frac{-8}{-16}=\frac{1}{2}$$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$\therefore$ The given pair of linear equations has infinitely many solutions.

2. Let α = $\sqrt{3}$ and β = $-\sqrt{3}$ be the given zeros and γ be the third zero. Then,

$$\alpha+\beta+\gamma=-\left(\frac{-4}{1}\right) \qquad \left[\text{Using } \alpha+\beta+\gamma=\frac{\text{Coeff. of } x^2}{\text{Coeff. of } x^3}\right]$$

$$\Rightarrow \quad \sqrt{3}-\sqrt{3}+\gamma=4$$

$$\Rightarrow \quad \gamma=4$$

Hence, third zero is 4.

3. We have,

$$\tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ$$

$$=\left(\tan 5^\circ \tan 85^\circ\right)\left(\tan 25^\circ \tan 65^\circ\right)\tan 30^\circ$$

$$=\left(\tan 5^\circ \cot 5^\circ\right)\left(\tan 25^\circ \cot 25^\circ\right)\tan 30^\circ \qquad \left[\begin{array}{l}\because \tan 85^\circ=\tan\left(90^\circ-5^\circ\right)=\cot 5^\circ \\ \tan 65^\circ=\tan\left(90^\circ-25^\circ\right)=\cot 25^\circ\end{array}\right]$$

$$=1\times 1\times\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}$$

4. $\text{Sin}\,67^o+\text{Cos}\,75^o$

$$=\text{Sin}(90^o-23^o)+\text{Cos}(90^o-15^o)=\text{Cos}\,23^o+\text{Sin}\,15^o$$

Section B

5. Class marks $=\frac{\text{Upper limit + lower limit}}{2}$

$\therefore$ Class Marks of 15.5 - 18.5 $=\frac{18.5+15.5}{2}=\frac{34}{2}=17$

Class Marks of 50 – 70 $= \frac{75+50}{2} = \frac{125}{2} = 62.5.$

6. The given system of equations will have infinite number of solutions, if

$$\frac{2}{2\alpha} = \frac{3}{\alpha+\beta} = \frac{7}{28}$$

$$\Rightarrow \frac{1}{\alpha} = \frac{3}{\alpha+\beta} = \frac{1}{4}$$

$$\Rightarrow \frac{1}{\alpha} = \frac{1}{4} \text{ and } \frac{3}{\alpha+\beta} = \frac{1}{4}$$

$\Rightarrow$ $\alpha = 4$ and $\alpha + \beta = 12$

$\Rightarrow$ $\alpha = 4$ and $\beta = 12 - 4 = 8$

7. Given that $\Delta ABC \sim \Delta PQR$

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \quad \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\Rightarrow \quad \frac{8\times 15.4}{11} = 11.2 \text{ cm}$$

$$\Rightarrow \quad BC = \frac{8}{11} = \frac{BC}{15.4}$$

8. Putting A = 45° and B = 30° in sin(A + B) = sinAcosB + cosAsinB, we get

sin(45° + 30°) = sin45°cos30° + cos45°sin30°

$$\Rightarrow \sin 75^\circ = \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\times\frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

9. Since the maximum frequency = 8 and it corresponds to the class 3-5

Modal class = 3-5

Here, $l = 3, h = 2, f_1 = 8, f_0 = 7, f_2 = 2$

We know that mode Mo is given by

$$Mo = l + h\frac{f_1 - f_0}{2f_1 - f_0 - f_2}$$

$$= 3 + 2\frac{(8-7)}{2(8)-7-2}$$

$$= 3 + 2\frac{(1)}{7} = 3 + \frac{2}{7}$$

$$= 3 + 0.2857 = 3.286 \text{ nearly}$$

10. Let ΔABC and ΔDEF be two similar triangles of perimeters P_1 and P_2 respectively. Also, let AB = 12 cm, P_1 = 30 cm and P_2 = 20 cm. Then,

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{P_1}{P_2}$$

$$\Rightarrow \frac{AB}{DE} = \frac{P_1}{P_2}$$

$$\Rightarrow \frac{12}{DE} = \frac{30}{20} \qquad \Rightarrow DE = \frac{12 \times 20}{30} = 8 \text{ cm}$$

Thus, the corresponding side of the second triangle is 8 cm.

SECTION – C

11. It is given that on dividing 398 by the required number, there is a remainder of 7. This means that 398 – 7 = 391 is exactly divisible by the required number. In other words, required number is a factor of 391.

Similarly, required positive integer is a factor of 436 – 11 = 425 and 542 – 15 = 527.

Clearly, required number is the HCF of 391, 425 and 527.

Using the factor tree, the prime factorisations of 391, 425 and 527 are as follows:

391 = 17 × 23

425 = 5^2 × 17

527 = 17 × 31

$\therefore$ HCF of 391, 425 and 527 is 17.

Thus, the required number is 17.

12. The given system of equations may be written as

$$ax + by = (a - b) = 0$$

$$bx - ay = (a + b) = 0$$

By cross-multiplication, we have

$$\frac{x}{\begin{matrix} b & -(a-b) \\ -a & -(a+b) \end{matrix}} = \frac{-y}{\begin{matrix} a & -(a-b) \\ b & -(a+b) \end{matrix}} = \frac{1}{\begin{matrix} a & b \\ b & -a \end{matrix}}$$

$$\Rightarrow \frac{x}{b \times -(a+b) - (-a) \times -(a-b)} = \frac{-y}{a \times -(a+b) - b \times -(a-b)} = \frac{1}{-a^2 - b^2}$$

$$\Rightarrow \frac{x}{-b(a+b) - a(a-b)} = \frac{-y}{-a(a+b) + b(a-b)} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \frac{x}{-b^2 - a^2} = \frac{-y}{-a^2 - b^2} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \frac{x}{-(a^2+b^2)} = \frac{y}{(a^2+b^2)} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow x = -\frac{(a^2+b^2)}{-(a^2+b^2)} = 1 \quad and \quad y = \frac{(a^2+b^2)}{-(a^2+a^2)} = -1$$

Hence, the solution of the given system of equations is $x = 1, y = -1$.

13. Here, the classes are not in inclusive form. So, we first convert them in inclusive form by subtracting $\frac{h}{2}$ from the lower limit and adding $\frac{h}{2}$ to the upper limit of each class, where h is the difference between the lower limit of a class and the upper limit of preceding class.

Now, we have

Class interval	**Number of leaves (f)**	**Cumulative frequency (cf)**
117.5-126.5	2	3
126.5-135.5	5	8
135.5-144.5	9	17
144.5-153.5	12	29
153.5-162.5	5	34
162.5-171.5	4	38
171.5-180.5	2	40
Total	$\sum f_i = 40$	

We have, $n = 40 \Rightarrow \frac{n}{2} = 20$

And, the cumulative frequency just greater than $\frac{n}{2}$ is 29 and corresponding class is 144.5-153.5. So median class is 144.5-153.5.

Here, we have $\frac{n}{2} = 20, l = 144.5, h = 9, f = 12, cf = 17$

$$\therefore Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h = 144.5 + \left(\frac{20-17}{12} \right) \times 9$$

$$= 144.5 + \frac{3}{12} \times 9 = 144.5 + \frac{9}{4}$$
$$= 144.5 + 2.25 = 146.75 mm.$$

Hence, the median length of the leaves is 146.75 mm.

14. Using Pythagoras theorem in ΔBAC, we have

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = 6^2 + 8^2 = 100$

$\Rightarrow$ BC = 10 cm

Now,

Area of ΔABC = Area of ΔOAB + Area of ΔOBC + Area of ΔOCA

$\Rightarrow \frac{1}{2}$ AB × AC = $\frac{1}{2}$ AB × r + $\frac{1}{2}$ BC × r + $\frac{1}{2}$ CA × r

$\Rightarrow \frac{1}{2}$ 6 × 8 = $\frac{1}{2}$ × 6 × r + $\frac{1}{2}$ × 10 × r + $\frac{1}{2}$ × 8 × r

$\Rightarrow 48 = 24r$

$\Rightarrow r = 2$ cm

15. Let us first draw a right ΔABC, in which $\angle B = 90^o$.

Now, we have, 15 cot A = 8

$\therefore \cot A = \frac{8}{15} = \frac{AB}{BC} = \frac{Base}{Perpendicular}$

Let AB = 8k and BC = 15k

Then, $AC = \sqrt{(AB)^2 + (BC)^2}$ (By Pythagoras Theorem)

$= \sqrt{(8k)^2 + (15k)^2} = \sqrt{64k^2 + 225k^2} = \sqrt{289k^2} = 17k$

$\therefore \sin A = \frac{Perpendicular}{Hypotenuse} \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

and, $\sin A = \frac{Hypotenuse}{Base} \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$.

16. $\sin\theta + \cos\theta = \sqrt{3}$

$\Rightarrow \quad (\sin\theta + \cos\theta)^2 = 3$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow \quad 2\sin\theta\cos\theta = 2 \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow \quad \sin\theta.\cos\theta = 1 = \sin^2\theta + \cos^2\theta$

$\Rightarrow \quad 1 = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\Rightarrow \quad 1 = \tan\theta + \cot\theta$

17. The modal class is 40-50, since it has the maximum frequency.

$l = 40, f_1 = 20, f_0 = 12, f_2 = 11, h = 10$

Mode = $l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 40 + \left(\dfrac{20-12}{2\times 20 - 12 - 11}\right)\times 10$

$= 40 + \left(\dfrac{8}{17}\right)\times 10$

= 40 + 4.71

= 44.71 cars

18. S**olution:**

We have,

AB = AP + PB = 3 + 6 = 9 cm

And, AC = AQ + QC = 5 + 10 = 15 cm

$\therefore \quad \dfrac{AP}{AB} = \dfrac{3}{9} = \dfrac{1}{3} \text{ and } \dfrac{AQ}{AC} = \dfrac{5}{15} = \dfrac{1}{3}$

$\Rightarrow \quad \dfrac{AP}{AB} = \dfrac{AQ}{QC}$

Thus, in triangles APQ and ABC, we have

$\dfrac{AP}{AB} = \dfrac{AQ}{QC} \text{ and } \angle A = \angle A$

Therefore, by SAS criterion of similarity, we have

$\Delta APQ \sim \Delta ABC$

$\Rightarrow \quad \dfrac{AP}{AB} = \dfrac{PQ}{BC} = \dfrac{AQ}{AC}$

$\Rightarrow \quad \dfrac{PQ}{BC} = \dfrac{AQ}{AC}$

$\Rightarrow \quad \frac{PQ}{BC} = \frac{5}{15} = \frac{1}{3}$

$\Rightarrow \quad BC = 3PQ$

19. **Solution:**

Since α and β are the zeros of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$

$\therefore \quad \alpha + \beta = -\left(-\frac{5}{2}\right) = \frac{5}{2}$ and $\alpha\beta = \frac{7}{2}$

Let S and P denote respectively the sum and product of zeros of the required polynomial.

Then, $S = (2\alpha + 3\beta) + (3\alpha + 2\beta) = 5(\alpha + \beta) = 5 \times \frac{5}{2} = \frac{25}{2}$

And, $P = (2\alpha + 3\beta)(3\alpha + 2\beta)$

$= 6(\alpha^2 + \beta^2) + 13\alpha\beta$

$= 6\alpha^2 + 6\beta^2 + 12\alpha\beta + \alpha\beta$

$= 6(\alpha + \beta)^2 + \alpha\beta$

$= 6 \times \left(\frac{5}{2}\right)^2 + \frac{7}{2} = 6 \times \frac{25}{4} + \frac{7}{2} = \frac{75}{2} + \frac{7}{2} = 41$

Hence, the required polynomial is given by $g(x) \quad = k(x^2 - Sx + P)$

$= \left(x^2 - \frac{25}{2}x + 41\right)$, where k is any non-zero real number.

20. We have,

LHS = $\sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}}$

$= \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta} \times \frac{1 - \sin\theta}{1 - \sin\theta}}$

$= \sqrt{\frac{(1 - \sin\theta)^2}{1 - \sin^2\theta}}$

$= \sqrt{\frac{(1 - \sin\theta)^2}{\cos^2\theta}}$

$= \sqrt{\left(\frac{1 - \sin\theta}{\cos\theta}\right)^2}$

$= \frac{1 - \sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \sec\theta - \tan\theta$ = RHS

SECTION – D

21. Let the cubic polynomial be $p(x) = ax^3 + bx^2 + cx + d$. Then,

Sum of zeros $= \frac{-b}{a} = 2$

Sum of the products of zeros taken two at a time $= \frac{c}{a} = -7$

And product of the zero $= \frac{-d}{a} = -14$

$\frac{b}{a}$=-2, $\frac{c}{a}$=-7, $-\frac{d}{a}$=-14 or $\frac{d}{a}$=14

Therefore, p(x) = $ax^3 + bx^2 + cx + d$

p(x) = $a\left[x^3 + \frac{b}{a}x^2 + \frac{c}{a} + \frac{d}{a}\right]$

p(x) = $a\left[a^3 + (+2)x^2 + (-7)x + 14\right]$

p(x) = $a\left[x^3 - 2x^2 - 7x + 14\right]$

for real value of a = 1

p(x) = $x^3 - 2x^2 - 7x + 14$

22. We have,

$x - y + 1 = 0$ *and* $3x + 2y - 12 = 0$

Thus,

$x - y = 1 \Rightarrow x = y - 1 \quad ...(i)$

$3x + 2y = 12 \Rightarrow x = \frac{12 - 2y}{3} \quad ...(ii)$

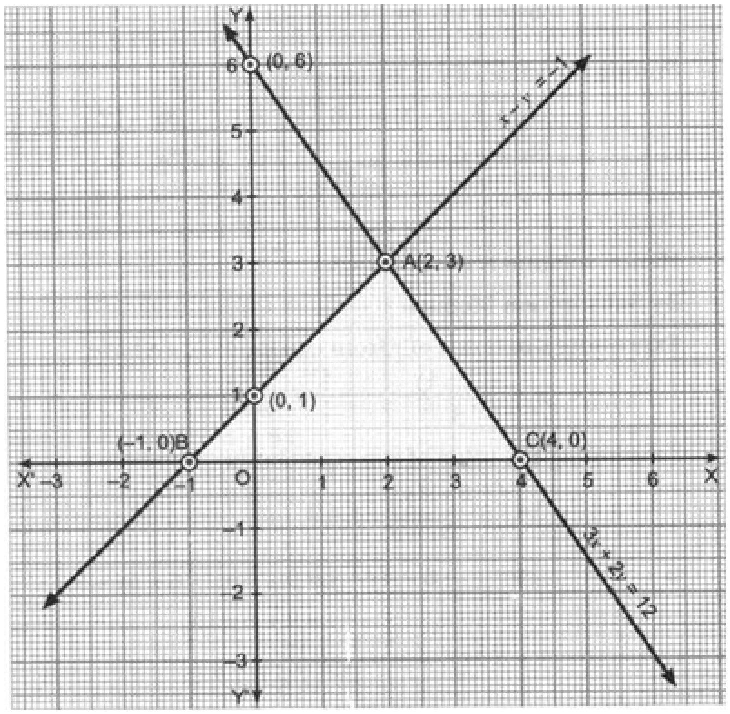
From equation (i), we have

x	-1	0	2
y	0	1	3

From equation (ii), we have

x	0	4	2
y	6	0	3

Plotting this, we have

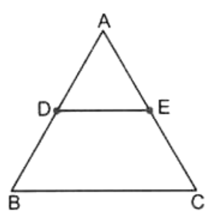


ABC is the required (shaded) region and point of intersection is (2,3).

$\therefore$ The vertices of the triangle are (-1,0),(4,0),(2,3).

23. Given: A ΔABC in which D is the mid-point of AB and DE is drawn parallel to BC, which AC at E.

To Prove: AE = EC



Proof: In ΔABC, DE||BC

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{AD}{DB}=\frac{AE}{EC} \qquad \text{(i)}$$

Now, since D is the mid-point of AB

$\Rightarrow \quad AD = BD \qquad$ (ii)

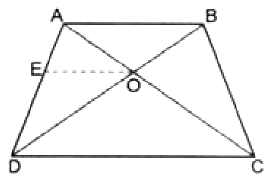
From (i) and (ii), we have

$$\frac{BD}{BD} = \frac{AE}{EC} \Rightarrow 1 = \frac{AE}{EC}$$

$$\Rightarrow \quad AE = EC$$

Hence, E is the mid-point of AC.

24. Given: ABCD is a trapezium, in which AB||DC and its diagonals intersect each other at the point o.



To prove: $\frac{AO}{BO} = \frac{CO}{DO}$

Construction: Through O, draw OE||B i.e., OE||DC.

Proof: In, ΔADC , we have OE||DC (Construction)

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{AE}{ED} = \frac{AO}{CO} \qquad \text{(i)}$$

Now, in ΔABD , we have OE||AB (Construction)

$\therefore$ Basic Proportionality Theorem, we have

$$\frac{ED}{AE} = \frac{DO}{BO}$$

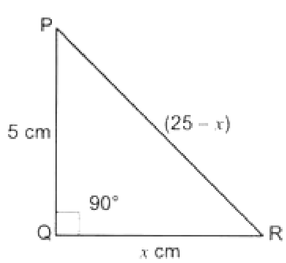
$$\Rightarrow \frac{AE}{ED} = \frac{BO}{DO} \qquad \text{(ii)}$$

From (i) and (ii), we have

$$\frac{AO}{CO} = \frac{BO}{DO} \Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$

25. We have a right-angled ΔPQR in which $\angle Q = 90^o$.

Let QR = x cm



Therefore, PR = (25 – x) cm

By Pythagoras Theorem, we have

$$PR^2 = PQ^2 + QR^2$$

$$(25-x)^2 = 5^2 + x^2$$

$$\Rightarrow (25-x)^2 - x^2 = 5^2$$

$$\Rightarrow (25-x-x)(25-x+x) = 25$$

$$\Rightarrow (25-2x)25 = 25 \qquad \Rightarrow 25-2x = 1$$

$$\Rightarrow 25-1 = 2x \qquad \Rightarrow 24 = 2x$$

$$\therefore \qquad x = 12cm.$$

Hence, QR = 12 cm.

$$PR = (25-x)cm = 25-12 = 13cm$$

$$PQ = 5cm$$

$$\therefore \sin P = \frac{QR}{PR} = \frac{12}{13};\ \cos P = \frac{Pq}{PR} = \frac{5}{13};\ \tan P = \frac{QR}{PQ} = \frac{12}{5}$$

26. We have,

cosecA = $\frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{2}{1}$

So, we draw a right triangle, right angled at B such that

Perpendicular = BC = 1, Hypotenuse = AC = 2

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow$ $2^2 = AB^2 + 1^2$

$\Rightarrow$ $AB^2 = 4 - 1$

$\Rightarrow$ $AB^2 = 3$

$\Rightarrow$ AB = $\sqrt{3}$

Now,

tanA = $\frac{BC}{AB} = \frac{1}{\sqrt{3}}$, sinA = $\frac{BC}{AC} = \frac{1}{2}$ and cosA = $\frac{AB}{AC} = \frac{\sqrt{3}}{2}$

$$\therefore \quad \frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{1}{\frac{1}{\sqrt{3}}} + \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$$

$= \frac{\sqrt{3}}{1} + \frac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}}$

$= \frac{\sqrt{3}}{1} + \frac{1}{2+\sqrt{3}}$

$= \sqrt{3} + \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \sqrt{3} + \frac{2-\sqrt{3}}{2^2 - \left(\sqrt{3}\right)^2}$

$= \sqrt{3} + \frac{2-\sqrt{3}}{4-3}$

$= \sqrt{3} + \left(2-\sqrt{3}\right) = 2$

27. We have,

Class interval	Frequency (f_i)	**Class mark**	$f_i x_i$
0-20	5	10	50
20-40	8	30	240
40-60	X	50	50x
60-80	12	70	840
80-100	7	90	630
100-120	8	110	880
Total	$\sum f_i = 40 + x$		$\sum f_i x_i = 2640 + 50x$

Here, $\sum f_i x_i = 2640 + 50x, \sum f_i = 40 + x, \bar{X} = 62.8$

$\therefore Mean\,(\bar{X}) = \frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow 62.8 = \frac{2640 + 50x}{40 + x}$

$\Rightarrow 2512 + 62.8x = 2640 + 50x$

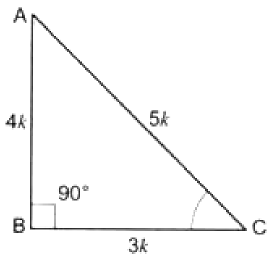
$\Rightarrow 62.8x - 50x = 2640 - 2512$

$\Rightarrow 12.8x = 128$

$\therefore\ x = \frac{128}{12.8} = 10$

Hence, the missing frequency is 10.

28. Let us consider a right triangle ABC in which $\angle B = 90^o$.



Now, $\cot A = \frac{Base}{Perpendicular} = \frac{AB}{BC} = \frac{4}{3}$

Let AB = 4k and BC = 3k

$\therefore$ By Pythagoras Theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (4k)^2 + (3k)^2 = 16k^2 + 9k^2$

$AC^2 = 25k^2 \quad \therefore AC = 5k$

Therefore, $\tan A = \frac{Perpendicular}{Base} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

And, $\sin A = \frac{Perpendicular}{Hypotenuse} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

$\cos A = \frac{Base}{Hypotenuse} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

Now, L.H.S $= \frac{1-\tan^2 A}{1+\tan^2 A}$

$$= \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{16-9}{16+9} = \frac{7}{25}$$

R.H.S $= \cos^2 A - \sin^2 A$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

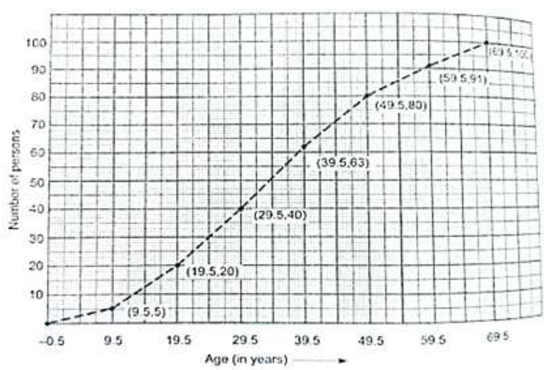
Hence, $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A.$

29. **Solution:**

The given frequency distribution is not continuous. So, we first make it continuous and prepare the cumulative frequency distribution as under.

Age (in years)	Frequency	Age less than	Cumulative frequency
–0.5-9.5	5	9.5	5
9.5-19.5	15	19.5	20
19.5-29.5	20	29.5	40
29.5-39.5	23	39.5	63
39.5-49.5	17	49.5	80
49.5-59.5	11	59.5	91
59.5-69.5	9	69.5	100

Now, we plot points (9.5, 5), (19.5, 20), (29.5, 40), (39.5, 63), (49.5, 80), (59.5, 91) and (69.5, 100) and join them by a free hand smooth curve to obtain the required ogive as shown in the figure.



The cumulative frequency polygon is obtained by joining these points by line segments as shown below.

30. **Solution:**

Let the actual speed of the train be x km/hr and the actual time taken by y hours. Then,

Distance covered = (xy) km ...(i) [∵ Distance = Speed × Time]

If the speed is increased by 6 km/hr, then time of journey is reduced by 4 hours, i.e., when speed is $(x + 6)$ km/hr, time of journey is $(y - 4)$ hours.

∴ Distance covered = $(x + 6)(y - 4)$

⇒ $xy = (x + 6)(y - 4)$ [using (i)]

⇒ $-4x + 6y - 24 = 0$

⇒ $-2x + 3y - 12 = 0$ ...(ii)

When the speed is reduced by 6 km/hr, then the time of journey is increased by 6 hours, i.e., when speed is $(x - 6)$ km/hr, time of journey is $(y + 6)$ hours.

$\therefore$ Distance covered = $(x - 6)(y + 6)$

$\Rightarrow$ $xy = (x - 6)(y + 6)$ [using (i)]

$\Rightarrow$ $6x - 6y - 36 = 0$

$\Rightarrow$ $x - y - 6 = 0$ ...(iii)

Thus, we obtain the following system of equations:

$-2x + 3y - 12 = 0$

$x - y - 6 = 0$

By using cross-multiplication, we have,

$$\frac{x}{3\times-6-(-1)\times12}=\frac{-y}{-2\times-6-1\times-12}=\frac{1}{-2\times-1-1\times3}$$

$$\Rightarrow \frac{x}{-30}=\frac{-y}{24}=\frac{1}{-1}$$

$\Rightarrow$ $x = 30$ and $y = 24$.

Putting the values of x and y in equation (i), we get

Distance = 30 × 24 = 720 km

Hence, the length of the journey is 720 km.

31. Maximum number of columns in which the two types of plants can be planted = HCF of 144 and 84

Since 144 > 84

So, by division lemma,

$144 = 84\times1+60$

Again, applying division lemma (since remainder ^ 0), we get

$84 = 60\times1+24$

Continuing the same way,

$60 = 24\times2+12$

$24 = 12\times2+0$

Y Remainder at this stage = 0

Therefore, HCF (144, 84) = 12

Environmental protection, sincerity, social work, cooperation.