Chapter
Electric Charges and Fields

Topic-1: Electric Charges and Coulomb's Law

1 MCQs with One Correct Answer

1. Two beads, each with charge q and mass m, are on a horizontal, frictionless, non-conducting, circular hoop of radius R. One of the beads is glued to the hoop at some point, while the other one performs small oscillations about its equilibrium position along the hoop. The square of the angular frequency of the small oscillations is given by [ε_0 is the permittivity of free space.] **[Adv. 2024]**

   (a) $\frac{q^2}{(4\pi\varepsilon_0 R^3 m)}$ (b) $\frac{q^2}{(32\pi\varepsilon_0 R^3 m)}$

   (c) $\frac{q^2}{(8\pi\varepsilon_0 R^3 m)}$ (d) $\frac{q^2}{(16\pi\varepsilon_0 R^3 m)}$

2. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then **[2007]**
   (a) negative and distributed uniformly over the surface of the sphere
   (b) negative and appears only at the point on the sphere closest to the point charge
   (c) negative and distributed non-uniformly over the entire surface of the sphere
   (d) zero

3. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to : **[1987 - 2 Marks]**

   (a) $-\frac{Q}{2}$ (b) $-\frac{Q}{4}$ (c) $+\frac{Q}{4}$ (d) $+\frac{Q}{2}$

4 Fill in the Blanks

4. Five point charges, each of value $+q$ coul, are placed on five vertices of a regular hexagon of side L metres. The magnitude of the force on the point charge of value $-q$ coulomb placed at the centre of the hexagen is .... newton. **[1992 - 1 Mark]**

5. Two small balls having equal positive charges Q (coulomb) on each are suspended by two insulating strings of equal length L (metre) from a hook fixed to a stand. The whole set up is taken in a satellite into space where there is no gravity (state of weightlessness). The angle between the two strings is .... and the tension in each string is .... newtons. **[1986 - 2 Marks]**

5 True / False

6. A ring of radius R carries a uniformly distributed charge $+Q$. A point charge $-q$ is placed on the axis of the ring at a distance $2R$ from the centre of the ring and released from rest. The particle executes a simple harmonic motion along the axis of the ring. **[1988 - 2 Marks]**

7. Two identical metallic spheres of exactly equal masses are taken. One is given a positive charge Q coulombs and the other an equal negative charge. Their masses after charging are different. **[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

8. Two identical non-conducting solid spheres of same mass and charge are suspended in air from a common point by two non-conducting, massless strings of same length. At equilibrium, the angle between the strings is α. The spheres are now immersed in a dielectric liquid of density 800 kg m^{-3} and dielectric constant 21. If the angle between the strings remains the same after the immersion, then **[Adv. 2020]**
   (a) electric force between the spheres remains unchanged
   (b) electric force between the spheres reduces
   (c) mass density of the spheres is 840 kg m^{-3}
   (d) the tension in the strings holding the spheres remains unchanged

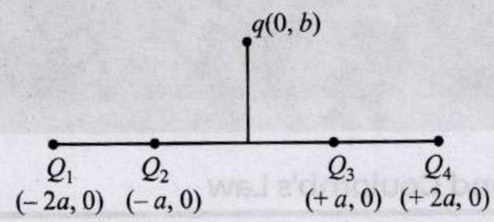
9. Two equal negative charges $-q$ are fixed at points $(0, -a)$ and $(0, a)$ on y – axis. A positive charge Q is released from rest at the point $(2a, 0)$ on the x - axis. The charge Q will **[1984- 2 Marks]**
   (a) execute simple harmonic motion about the origin
   (b) move to the origin remain at rest
   (c) move to infinity
   (d) execute oscillatory but not simple harmonic motion

7 Match the Following

10. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at $x = -2a, -a, +a$ and $+2a$,

respectively. A positive charge q is placed on the positive y axis at a distance $b > 0$. Four options of the signs of these charges are given in List-I. The direction of the forces on the charge q is given in List-II. Match List-I with List-II and select the correct answer using the code given below the lists. **[Adv. 2014]**

	List - I		List - II
P.	Q_1, Q_2, Q_3, Q_4 all positive	**1.**	$+x$



Q.	Q_1, Q_2 positive; Q_3, Q_4 negative	**2.**	$-x$
R.	Q_1, Q_4 positive; Q_2, Q_3 negative	**3.**	$+y$
S.	Q_1, Q_3 positive; Q_2, Q_4 negative	**4.**	$-y$

Codes:

(a) P-3, Q-1, R-4, S-2 (b) P-4, Q-2, R-3, S-1

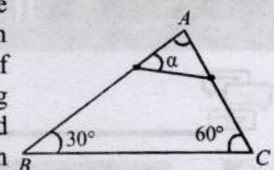
(c) P-3, Q-1, R-2, S-4 (d) P-4, Q-2, R-1, S-3

10 Subjective Problems

11. Three particles, each of mass 1 gm and carrying a charge q, are suspended from a common point by insulated massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side length 3 cm, calculate the charge q on each particle. (Take $g = 10\ m/s^2$). **[1988 - 5 Marks]**

12. A thin fixed ring of radius 1 metre has a positive charge 1×10^{-5} coulomb uniformly distributed over it. A particle of mass 0. 9g and having a negative charge of 1×10^{-6} coulomb is placed on the axis at a distance of 1 m from the centre of the ring. Show that the motion of the negatively charged particle is approxaimately simple harmonic. Calculate the time period of oscillations. **[1982 - 5 marks]**

13. A rigid insulated wire frame, in the form of right triangle ABC is set in a vertical plane. Two beads of equal masses m each carrying charges q_1 and q_2 are connected by a chord of length l and can slide without friction on the wires. Considering the case when the beads are stationary, determine : **[1978]**



(i) the angle α,
(ii) the tension in the chord, and
(iii) the normal reactions on the beads.

If the chord is now cut, what are the values of the charges for which the beads continue to remain stationary?

14. Three charges each of value q, are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the centre of the triangle. **[1978]**

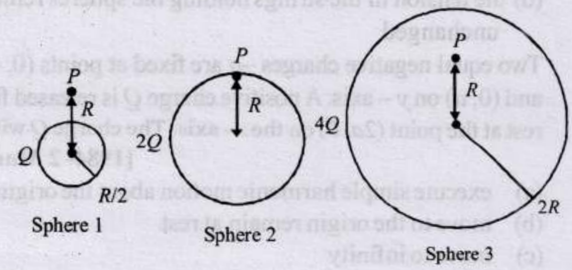
(i) If $Q = -q$, will the charges at the corners move towards centre or fly away from it.

(ii) For what value of Q will the charges remain stationary? In this situation how much work is done in removing the charges to infinity?

Topic-2: Electric Field and Field Lines

1 MCQs with One Correct Answer

1. Charges Q, $2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii $R/2$, R and $2R$ respectively, as shown in figure. If magnitude of the electric fields at point P at a distance R from the centre of sphere 1, 2 and 3 are E_1, E_2 and E_3 respectively, then **[Adv. 2014]**



(a) $E_1 > E_2 > E_3$ (b) $E_3 > E_1 > E_2$

(c) $E_2 > E_1 > E_3$ (d) $E_3 > E_2 > E_1$

2. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5\ Vm^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2 \times 10^{-3}\ ms^{-1}$. Given $g = 9.8\ m\ s^{-2}$, viscosity of the air $= 1.8 \times 10^{-5}\ Ns\ m^{-2}$ and the density of oil $= 900\ kg\ m^{-3}$, the magnitude of q is **[2010]**

(a) $1.6 \times 10^{-19}\ C$ (b) $3.2 \times 10^{-19}\ C$

(c) $4.8 \times 10^{-19}\ C$ (d) $8.0 \times 10^{-19}\ C$

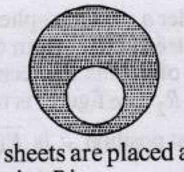
3. Three concentric metallic spherical shells of radii R, $2R$, $3R$, are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is **[2009]**

(a) 1 : 2 : 3 (b) 1 : 3 : 5

(c) 1 : 4 : 9 (d) 1 : 8 : 18

4. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in the figure. The electric field inside the emptied space is **[2007]**

(a) zero everywhere
(b) non-zero and uniform
(c) non-uniform
(d) zero only at its center



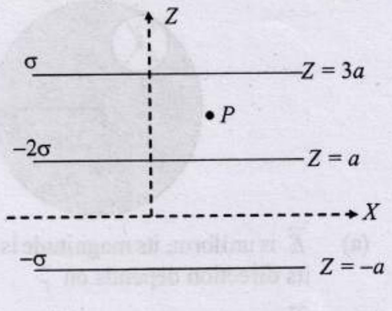
5. Three infinitely long charge sheets are placed as shown in figure. The electric field at point P is **[2005S]**

(a) $\frac{2\sigma}{\varepsilon_0}\hat{k}$

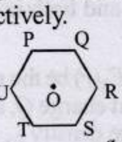
(b) $\frac{4\sigma}{\varepsilon_0}\hat{k}$

(c) $-\frac{2\sigma}{\varepsilon_0}\hat{k}$

(d) $-\frac{4\sigma}{\varepsilon_0}\hat{k}$

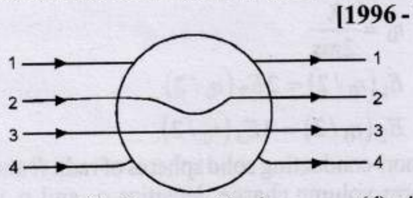


6. Six charges of equal magnitude, 3 positive and 3 negative are to be placed on $PQRSTU$ corners of a regular hexagon, such that field at the centre is double that of what it would have been if only one +ve charge is placed at R. Which of the following arrangement of charge is possible for P, Q, R, S, T and U respectively. **[2004S]**



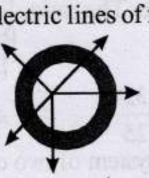
(a) +, +, +, −, −, −
(b) −, +, +, +, −, −
(c) −, +, +, −, +, −
(d) +, −, +, −, +, −

7. A metallic solid sphere is placed in a uniform electric fied. The lines of force follow the path(s) shown in Figure as **[1996 - 2 Marks]**



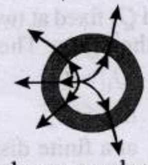
(a) 1 (b) 2 (c) 3 (d) 4

8. A metallic shell has a point charge 'q' kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of forces? **[2003S]**

(a)


(b)


(c)

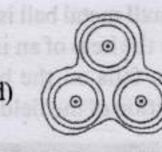

(d)


9. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in **[2001S]**

(a)


(b)

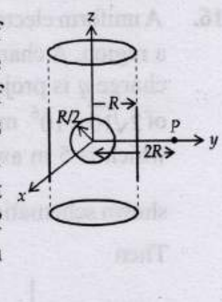

(c)


(d)


2 Integer Value Answer

10. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16K\varepsilon_0}$.

The value of k is **[2012]**

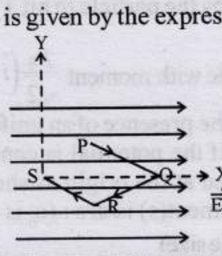


11. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre.

If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at $r = R$, find the value of a. **[2009]**

4 Fill in the Blanks

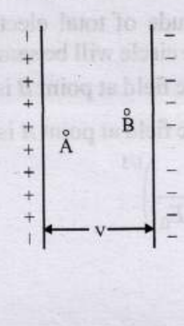
12. A point charge q moves from point P to point S along the path $PQRS$ (fig.) in a uniform electric field E pointing parallel to the positive direction of the X-axis. The cooridnates of the points P, Q, R and S are $(a, b, O), (2a, O, O)$ $(a, -b, \underline{O})$ and (O, O, O) respectively. The work done by the field in the above process is given by the expression .... **[1989 - 2 Marks]**



5 True / False

13. An electric line of forces in the $x - y$ plane is given by the equation $x^2 + y^2 = 1$. A particle with unit positive charge, initially at rest at the point $x = 1, y = 0$ in the $x - y$ plane, will move along the circular line of force. **[1988 - 2 Marks]**

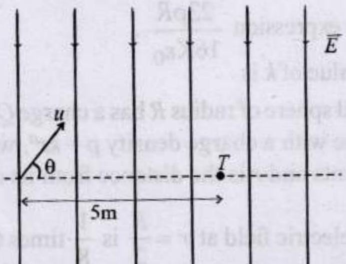
14. Two protons A and B are placed in between the two plates of a parallel plate capacitor charged to a potential difference V as shown in the figure. The forces on the two protons are identical. **[1986 - 3 Marks]**



15. A small metal ball is suspended in a uniform electric field with the help of an insulated thread. If high energy X-ray beam falls on the ball, the ball will be deflected in the direction of the field. **[1983 - 2 Marks]**

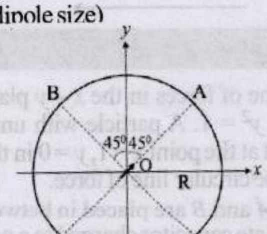
6 MCQs with One or More than One Correct Answer

16. A uniform electric field, $\vec{E} = -400\sqrt{3}\hat{y}$ NC^{-1} is applied in a region. A charged particle of mass m carrying positive charge q is projected in this region with an initial speed of $2\sqrt{10}\times10^6$ ms^{-1}. This particle is aimed to hit a target T, which is 5 m away from its entry point into the field as shown schematically in the figure. Take $\frac{q}{m} = 10^{10}$ Ckg^{-1}. Then **[Adv. 2020]**



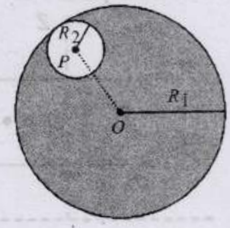
(a) the particle will hit T if projected at an angle 45° from the horizontal
(b) the particle will hit T if projected either at an angle 30° or 60° from the horizontal
(c) time taken by the particle to hit T could be $\sqrt{\frac{5}{6}}$ μs as well as $\sqrt{\frac{5}{2}}$ μs
(d) time taken by the particle to hit T is $\sqrt{\frac{5}{3}}$ μs

17. An electric dipole with moment $\frac{p_0}{\sqrt{2}}(\hat{i}+\hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are : (ε_0 is permittivity of free space. R >>dipole size) **[Adv. 2019]**



(a) The magnitude of total electric field on any two points of the circle will be same
(b) Total electric field at point B is $\vec{E}_B = 0$
(c) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i}+\hat{j})$
(d) $R = \left(\frac{p_0}{4\pi\epsilon_0 E_0}\right)^{1/3}$

18. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\mathrm{r})$, then the correct statement(s) is (are) **[Adv. 2015]**



(a) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$
(b) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$
(c) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$
(d) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$

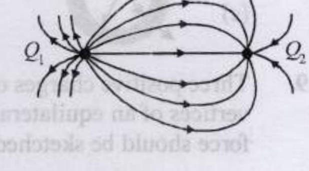
19. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric field at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then **[Adv. 2014]**

(a) $Q = 4\sigma\pi r_0^2$
(b) $r_0 = \frac{\lambda}{2\pi\sigma}$
(c) $E_1(r_0/2) = 2E_2(r_0/2)$
(d) $E_2(r_0/2) = 4E_3(r_0/2)$

20. Two non-conducting solid spheres of radii R and 2R, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be **[Adv. 2013]**

(a) –4 (b) $-\frac{32}{25}$ (c) $\frac{32}{25}$ (d) 4

21. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that **[2010]**



(a) $|Q_1| > |Q_2|$
(b) $|Q_1| < |Q_2|$
(c) at a finite distance to the left of Q_1 the electric field is zero
(d) at a finite distance to the right of Q_2 the electric field is zero

22. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre **[1998S - 2 Marks]**

(a) increases as r increases, for $r < R$.
(b) decreases as r increases, for $0 < r < \infty$.
(c) decreases as r increases, for $R < r < \infty$.
(d) is discontinuous at $r = R$.

7 Match the Following

23. The electric field E is measured at a point P (0, 0, d) generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II. **[Adv. 2018]**

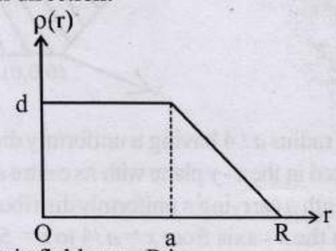
LIST–I	LIST–II
P. E is independent of d	**1.** A point charge Q at the origin
Q. $E \propto 1/d$	**2.** A small dipole with point charges Q at (0, 0, l) and – Q at (0, 0, –l). Take $2l << d$
R. $E \propto 1/d^2$	**3.** An infinite line charge coincident with the x-axis, with uniform linear charge density λ
S. $E \propto 1/d^3$	**4.** Two infinite wires carrying uniform linear charge density parallel to the x - axis. The one along (y = 0, z = l) has a charge density +λ and the one along (y = 0, z = –l) has a charge density –λ Take 2l << d
	5. Infinite plane charge coincident with the xy-plane with uniform surface charge density

(a) $P \to 5; Q \to 3, 4; R \to 1; S \to 2$
(b) $P \to 5; Q \to 3; R \to 1, 4; S \to 2$
(c) $P \to 5; Q \to 3; R \to 1, 2; S \to 4$
(d) $P \to 4; Q \to 2, 3; R \to 1; S \to 5$

8 Comprehension/Passage Based Questions

Passage

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density ρ (r) [charge per unit volume] is dependent only on the radial distance r from the centre of the nucleus as shown in figure The electric field is only along the radial direction. **[2008]**



24. The electric field at r = R is
(a) independent of a
(b) directly proportional to a
(c) directly proportional to a^2
(d) inversely proportional to a

25. For a = 0, the value of d (maximum value of ρ as shown in the figure) is –

(a) $\frac{3Ze}{4\pi R^3}$ (b) $\frac{3Ze}{\pi R^3}$ (c) $\frac{4Ze}{3\pi R^3}$ (d) $\frac{Ze}{3\pi R^3}$

26. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies.
(a) a = 0 (b) a = R/2 (c) a = R (d) a = 2R/3

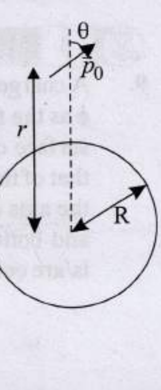
10 Subjective Problems

27. Two uniformly charged large plane sheets S_1 and S_2 having charge densities σ_1 and σ_2 ($\sigma_1 > \sigma_2$) are placed at a distance d parallel to each other. A charge q_0 is moved along a line of length $a(a < d)$ at an angle 45° with the normal to S_1. Calculate the work done by the electric field **[2004]**

Topic-3: Electric Dipole, Electric Flux and Gauss's Law

1 MCQs with One Correct Answer

1. A small electric dipole $\vec{p}_0$, having a moment of inertia I about its center, is kept at a distance r from the center of a spherical shell of radius R. The surface charge density σ is uniformly distributed on the spherical shell. The dipole is initially oriented at a small angle θ as shown in the figure. While staying at a distance r, the dipole is free to rotate about its center. **[Adv. 2024]**



If released from rest, then which of the following statement(s) is(are) correct?

[ϵ_0 is the permittivity of free space.]

(a) The dipole will undergo small oscillations at any finite value of r.
(b) The dipole will undergo small oscillations at any finite value of $r > R$.
(c) The dipole will undergo small oscillations with an angular frequency of $\sqrt{\frac{2\sigma p_0}{\epsilon_0 I}}$ at $r = 2R$

(d) The dipole will undergo small oscillations with an angular frequency of $\sqrt{\dfrac{\sigma p_0}{100\epsilon_0 I}}$ at $r = 10R$

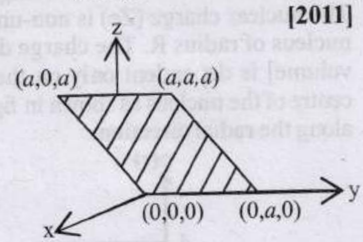
2. Consider an electric field $\vec{E} = E_0\hat{x}$ where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is **[2011]**

(a) $2E_0a_2$

(b) $\sqrt{2}E_0a^2$

(c) $E_0 a^2$

(d) $\dfrac{E_0a^2}{\sqrt{2}}$



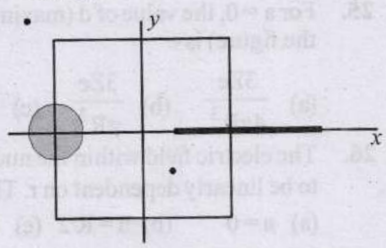
3. A disc of radius $a/4$ having a uniformly distributed charge 6C is placed in the x-y plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge 8C is placed on the x - axis from $x = a/4$ to $x = 5a/4$. Two point charges -7 C and 3 C are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2, y = \pm a/2, z = \pm a/2$. The electric flux through this cubical surface is **[2009]**

(a) $\dfrac{-2C}{\varepsilon_0}$

(b) $\dfrac{2C}{\varepsilon_0}$

(c) $\dfrac{10C}{\varepsilon_0}$

(d) $\dfrac{12C}{\varepsilon_0}$



4. A Gaussian surface in the figure is shown by dotted line. The electric field on the surface will be **[2004S]**

(a) due to q_1 and q_2 only

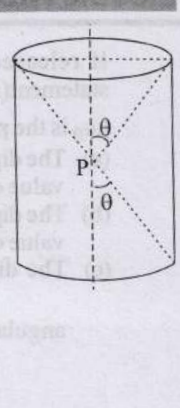
(b) due to q_2 only

(c) zero

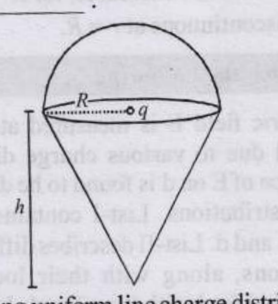
(d) due to all

2 Integer Value Answer

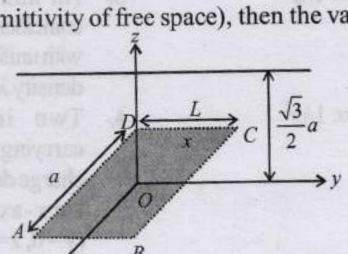
5. A charge is kept at the central point P of a cylindrical region. The two edges subtend a half-angle θ at P, as shown in the figure. When θ = 30°, then the electric flux through the curved surface of the cylinder is Φ. If θ = 60°, then the electric flux through the curved surface becomes $\dfrac{\Phi}{\sqrt{n}}$, where the value of n is______. **[Adv. 2024]**



6. A charge q is surrounded by a closed surface consisting of an inverted cone of height h and base radius R, and a hemisphere of radius R as shown in the figure. The electric flux through the conical surface is $\dfrac{nq}{6\epsilon_0}$ (in SI units). The value of n is ______. **[Adv. 2022]**



7. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the $y-z$ plane at $z = \dfrac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the $x - y$ plane with its centre at the origin is $\dfrac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is **[Adv. 2015]**



3 Numeric / New Stem Based Questions

8. A circular disc of radius R carries surface charge density $\sigma(r) = \sigma_0\left(1 - \dfrac{r}{R}\right)$, where σ_0 is a constant and r is the distance from the center of the disc. Electric flux through a large spherical surface that encloses the charged disc completely is ϕ_0. Electric flux through another spherical surface of radius $\dfrac{R}{4}$ and concentric with the disc is ϕ. Then the ratio $\dfrac{\phi_0}{\phi}$ is______. **[Adv. 2020]**

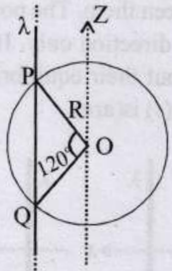
6 MCQs with One or More than One Correct Answer

9. A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct ? **[Adv. 2019]**

[ε_0 is the permittivity of free space]

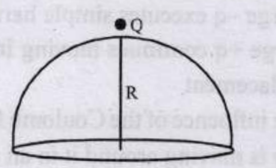
(a) If $h > 2R$ and $r = 3R/5$ then $\phi = Q/5\varepsilon_0$
(b) If $h > 2R$ and $r > R$ then $\phi = Q/\varepsilon_0$
(c) If $h < 8R/5$ and $r = 3R/5$ then $\phi = 0$
(d) If $h > 2R$ and $r > 4R/5$ then $\phi = Q/5\varepsilon_0$

10. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ϵ_0. Which of the following statements is (are) true? **[Adv. 2018]**



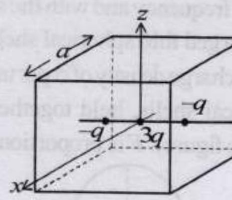
(a) The electric flux through the shell is $\sqrt{3}R\lambda/\epsilon_0$
(b) The z-component of the electric field is zero at all the points on the surface of the shell
(c) The electric flux through the shell is $\sqrt{2}R\lambda/\epsilon_0$
(d) The electric field is normal to the surface of the shell at all points

11. A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct? **[Adv. 2017]**



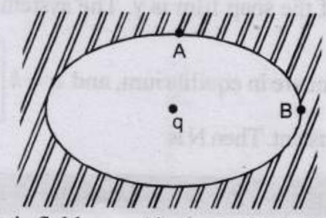
(a) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$
(b) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$
(c) The component of the electric field normal to the flat surface is constant over the surface
(d) The circumference of the flat surface is an equipotential

12. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $(0, -a/4, 0)$, $+3q$ at $(0, 0, 0)$ and $-q$ at $(0, +a/4, 0)$. Choose the correct options(s) **[2012]**



(a) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$
(b) The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$.
(c) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$
(d) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$.

13. An ellipsoidal cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then **[1999S - 3 Marks]**



(a) electric field near A in the cavity = electric field near B in the cavity
(b) charge density at A = charge density at B
(c) potential at A = potential at B
(d) total electric field flux through the surface of the cavity is q/ε_0

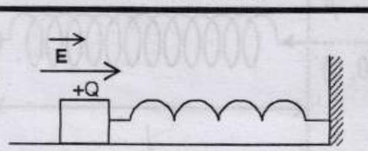
14. The magnitude of electric field $\vec{E}$ in the annular region of a charged cylindrical capacitor. **[1996 - 2 Marks]**

(a) is same throughout
(b) is higher near the outer cylinder than near the inner cylinder
(c) varies as $1/r$, where r is the distance from axis
(d) varies as $1/r^2$ where r is the distance from the axis.

Topic-4: Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

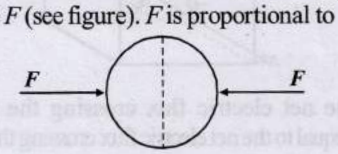
1. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge +Q on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be **[2011]**



(a) of the same frequency and with shifted mean position.

(b) of the same frequency and with the same mean position
(c) of changed frequency and with shifted mean position.
(d) of changed frequency and with the same mean position.

2. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to **[2010]**



(a) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (b) $\frac{1}{\varepsilon_0}\sigma^2 R$

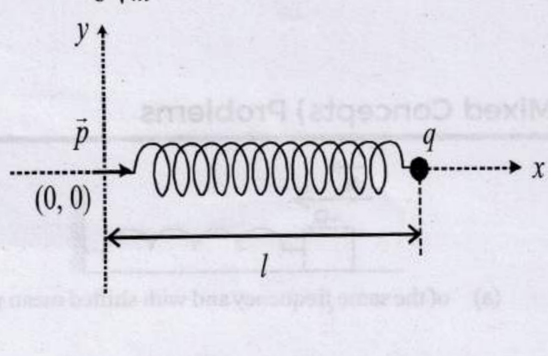
(c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (d) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

2 Integer Value Answer

3. Four point charges, each of +q, are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where 'k' is a constant. Then N is **[2011]**

3 Numeric / New Stem Based Questions

4. One end of a spring of negligible unstretched length and spring constant k is fixed at the origin (0, 0). A point particle of mass m carrying a positive charge q is attached at its other end. The entire system is kept on a smooth horizontal surface. When a point dipole $\vec{p}$ pointing towards the charge q is fixed at the origin, the spring gets stretched to a length l and attains a new equilibrium position (see figure below). If the point mass is now displaced slightly by $\Delta l << l$ from its equilibrium position and released, it is found to oscillate at frequency $\frac{1}{\delta}\sqrt{\frac{k}{m}}$. The value of δ is ______.



5 True / False

5. The work done in carrying a point charge from one point to another in an electrostatic field depends on the path along which the point charge is carried. **[1981- 2 Marks]**

6 MCQs with One or More than One Correct Answer

6. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and –q are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is(are) **[Adv. 2015]**

(a) Both charges execute simple harmonic motion
(b) Both charges will continue moving in the direction of their displacement
(c) Charge +q executes simple harmonic motion while charge –q continues moving in the direction of its displacement
(d) Charge –q executes simple harmonic motion while charge +q continues moving in the direction of its displacement

7. Under the influence of the Coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s). **[2009]**
(a) The angular momentum of the charge $-q$ is constant
(b) The linear momentum of the charge $-q$ is constant
(c) The angular velocity of the charge $-q$ is constant
(d) The linear speed of the charge $-q$ is constant

8. A positively charged thin metal ring of radius R is fixed in the xy plane with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of P is **[1998S - 2 Marks]**
(a) periodic, for all values of z_0 satisfying $0 < z_0 < \infty$
(b) simple harmonic, for all values of z_0 satisfying $0 < z_0 \le R$
(c) approximately simple harmonic, provided $z_0 << R$
(d) such that P crosses O and continues to move along the negative z axis towards $z = -\infty$

Answer Key

Topic-1 : Electric Charges and Coulomb's Law

1. (b) **2.** (d) **3.** (b) **4.** $\left(\frac{1}{a\pi\varepsilon_0}\frac{q^2}{L^2}\right)$ **5.** $(180°, \frac{kQ^2}{4L^2})$ **6.** (False) **7.** (True) **8.** (a, c) **9.** (d)
10. (a)

Topic-2 : Electric Field and Field Lines

1. (c) **2.** (d) **3.** (b) **4.** (b) **5.** (c) **6.** (c) **7.** (d) **8.** (c) **9.** (c) **10.** (6)
11. (2) **12.** (– qEA) **13.** False **14.** True **15.** True **16.** (b, c) **17.** (b, d) **18.** (d) **19.** (c) **20.** (b, d)
21. (a, d) **22.** (a, c) **23.** (b) **24.** (a) **25.** (b) **26.** (c)

Topic-3 : Electric Dipole, Electric Flux and Gauss's Law

1. (b,d) **2.** (c) **3.** (a) **4.** (d) **5.** (3) **6.** (3) **7.** (6) **8.** (6.40) **9.** (a,b,c) **10.** (a, b)
11. (a, d) **12.** (a, c, d) **13.** (c, d) **14.** (c)

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. (a) **2.** (a) **3.** (3) **4.** (3.14) **5.** False **6.** (c) **7.** (a) **8.** (a, c)

Hints & Solutions

Topic-1: Electric Charges and Coulomb's Law

1. **(b)** Here $r = 2R\cos\phi$ also $\theta = 2\phi \Rightarrow \theta = \frac{\phi}{2}$

And $\theta = \frac{x}{R}$

If θ is the small angular displacement of free charge, then

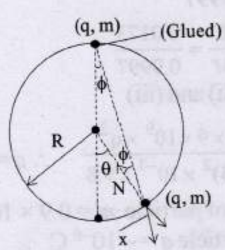
$$F(\phi) = \frac{Kq^2}{r^2}$$

So, restoring force towards mean position is $F_{(R)} = \frac{Kq^2}{r^2}\sin\phi$

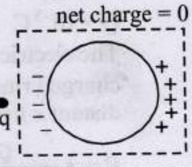
$$a_R = \frac{F_{(R)}}{m} = \frac{-Kq^2}{mr^2}\cdot\sin\phi = \frac{-Kq^2\cdot\sin\phi}{m\,4R^2\cos^2\phi}$$

$$\Rightarrow a_R = -\frac{Kq^2}{4mR^2\cos^2\left(\frac{\phi}{2}\right)}\cdot\sin\left(\frac{\theta}{2}\right) \approx \frac{-Kq^2}{4mR^2}\frac{1}{2}\cdot\frac{x}{R} = \omega^2\cdot x$$

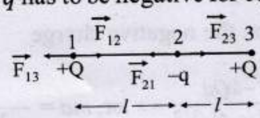
$$\therefore \omega^2 = \frac{q^2}{32\pi\varepsilon_0 mR^3}$$



2. **(d)** When a positive point charge is placed outside a conducting sphere, a rearrangement of charge takes place on the surface as shown in the figure. But net charge on the sphere is zero as no charge has left or entered the sphere.



3. **(b)** Charge q has to be negative for equilibrium.

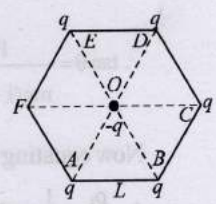


Considering equilibrium of charge

$$F_{13} = F_{12}$$

$$\frac{KQ\times Q}{(2\ell)^2} = \frac{KQ(-q)}{\ell^2} \quad \therefore q = -\frac{Q}{4}$$

4. $\left(\frac{1}{4\pi\in_0}\frac{q^2}{L^2}\right)$

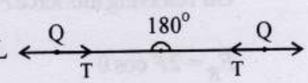


Force on (–q) at O due to charge at D will get cancelled out by force on (–q) due to charge on A. Similarly force on –q due to charge at E will get cancelled out due to charge on B.

So the net force will be because of charge q on C only at 'O' $F = \frac{1}{4\pi\in_0}\frac{q^2}{L^2}$ newton and attractive in nature.

5. **$\left(180°, \frac{KQ^2}{4L^2}\right)$** Here only electrostatic force of repulsion is acting which will take the two balls as far as possible. The angle between the two strings will be 180°. The tension in each string will be equal to the electrostatic force of repulsion

Here, $Q_1 = Q_2 = Q$ and $R = 2L$



$$T = \frac{1}{4\pi\varepsilon_0}\times\frac{Q\times Q}{(2L)^2} = \frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{4L^2}$$

6. **(False)** Electric field along axis of ring.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(R^2+x^2\right)^{3/2}} = \frac{1Q\times 2R}{4\pi\varepsilon_0\left[R^2+(2R)^2\right]^{3/2}} = \frac{2QR}{4\pi\varepsilon_0 5\sqrt{5}R^3}$$

$\therefore$ Force $= -qE$

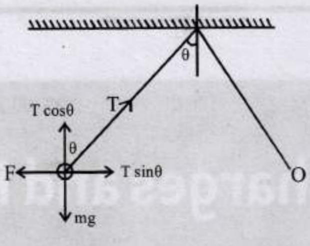
$\frac{-2QR}{4\pi\varepsilon_0 5\sqrt{5}R^2}$ *i.e.*, The force is not proportional to displacement.

Hence charge particle will not execute simple harmonic motion.

7. **(True)** The metallic sphere which gets negatively charged gains electrons and hence its mass increases.

The metallic sphere which gets positively charged loses electrons and hence its mass decreases.

8. **(a, c)** The net electric force on any sphere is lesser but by Coulomb law the force due to one sphere to another remain the same.



In equilibrium
T cosθ = mg
and T sin θ = F

$\therefore \tan\theta = \dfrac{F}{mg}$ ...(i)

As force between two charge bodies according to coulomb's law does not depend upon the medium, hence force between them remain same.
Hence after immersed in dielectric medium
As given no change in angle θ.

$T'\sin\theta = \dfrac{F}{\varepsilon_r}$ and $T'\cos\theta = mg\left(1-\dfrac{\rho_l}{\rho_s}\right)$

$\therefore \tan\theta = \dfrac{F}{mg(1-\frac{\rho_l}{\rho_s})}\dfrac{1}{\varepsilon_r}$ ...(ii)

Now equating eqn. (i) & (ii)

$1-\dfrac{\rho_l}{\rho_s} = \dfrac{1}{\varepsilon_r} \Rightarrow 1-\dfrac{800}{\rho_s} = \dfrac{1}{21}$

$\therefore \rho_s = 840 \text{ kg/m}^3$
Hence option (c) is correct

And tension, $T' = \dfrac{T}{\varepsilon_r} = \dfrac{T}{21}$

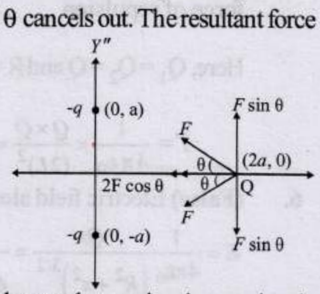
Hence option (d) is incorrect.

9. **(d)** Let us consider the positive charge Q at a distance x from the origin. Let force between Q and $-q$ be F. On resolving this force $F \sin\theta$ cancels out. The resultant force

$F_R = 2F\cos\theta$

$= 2\times\dfrac{kQq}{(x^2+a^2)}\times\dfrac{x}{\sqrt{x^2+a^2}}$

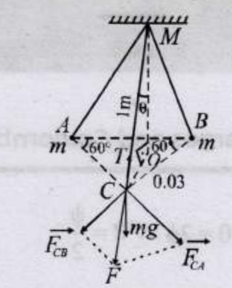
$= \dfrac{2kQqx}{(x^2+a^2)^{3/2}}$

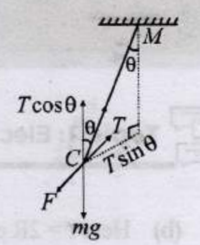


Since F_R is not proportional to x, the motion is not simple harmonic. The charge Q will accelerate till the origin and gain velocity. At the origin the net force is zero but due to momentum it will cross the origin and move towards left. As it comes on negative x-axis, the force is again towards the origin.
Hence charge Q will execute oscillatory but not simple harmonic motion.

10. **(a)** If Q_1, Q_2, Q_3 and Q_4 are all positive, then the force will be along + y-direction as components of forces along x-axis cancell out each other.
If Q_1, Q_2 are positive and Q_3, Q_4 are negative the force will act along + x-direction as components of forees along y-axis cancell out each other.
If Q_1, Q_4 are positive and Q_2, Q_3 are negative then attractive force will dominate repulsive force and the force will be along − y direction.
If Q_1, Q_3 positive and Q_2, Q_4 negative components of forces along y-axis can cel out each other. So net force on charge q along x-axis.

11. Each particle will be in equilibrium under the act of three force tension of string T, weighting, resultant electrostatic force F of the two other charges. Force F and mg are perpendicular.





Resolving T in the direction of mg and F and applying the condition of equilibrium,

$T\cos\theta = mg;$ $T\sin\theta = F$

$\therefore \tan\theta = \dfrac{F}{mg}$ ...(i)

$F = \sqrt{F_{CA}^2+F_{CB}^2+2F_{CA}F_{CB}\cos\alpha}$

$\therefore F = \sqrt{F_{CA}^2+F_{CA}^2+2F_{CA}^2\times\frac{1}{2}}$

$F = \sqrt{3}F_{CA} = \sqrt{3}\times\dfrac{kq^2}{(CA)^2}$ ...(ii)

$[\,|\vec{F}_{CB}| = |\vec{F}_{CA}|$ and $\alpha = 60°\,]$

Let T make an angle θ with the vertical

$OC = \dfrac{2}{3}\sqrt{(0.03)^2-(0.015)^2} = 0.0173 \text{ m}$

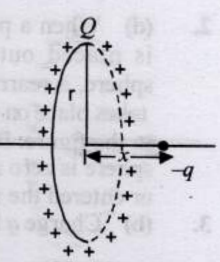
$\therefore OM = 0.9997$

$\therefore \tan\theta = \dfrac{OC}{OM} = \dfrac{0.0173}{0.9997}$ ....(iii)

From eq. (i), (ii) and (iii)

$\dfrac{0.0173}{0.9997} = \dfrac{\sqrt{3}\times9\times10^9\times q^2}{(0.03)^2\times10^{-3}\times9.8}$ $\therefore q = 3.16\times10^{-9}$ C.

12. Given : Mass of particle $m = 0.9\times10^{-3}$ kg
Charge on particle $q = -10^{-6}$ C
$r = 1$ m;
$Q = 10^{-5}$ C
The electric field due a uniformly charged ring of radius r at a point distant x from its center on its axis



$E = k\dfrac{Qx}{(r^2+x^2)^{3/2}}$

$\therefore$ Force on the negative charge
$qF = qE$

$\therefore F = \dfrac{-kQq}{(r^2+x^2)^{3/2}}\times x$ or, $ma = \dfrac{-kQq}{(r^2+x^2)^{3/2}}\times x$

or, $a = -k\dfrac{Qq}{m(r^2+x^2)^{3/2}}\times x$

For $x << r$ $a = -\frac{kQq}{r^3} \times x$

Hence motion is simple harmonic in nature.
Comparing the above equation with $a = -\omega^2 x$

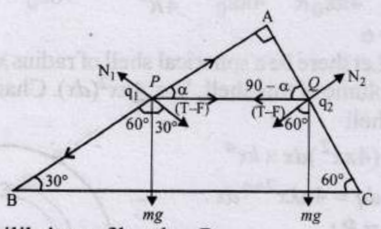
$$\therefore \omega^2 = \frac{kQq}{mr^3} \text{ or } \omega = \sqrt{\frac{kQq}{mr^3}}$$

$$\therefore \frac{2\pi}{T} = \sqrt{\frac{kQq}{mr^3}} \Rightarrow T = 2\pi\sqrt{\frac{mr^3}{kQq}}$$

$$T = 2 \times 3.14\left[\frac{0.9 \times 10^{-3} \times 1^3}{9 \times 10^9 \times 10^{-5} \times 10^{-6}}\right]^{1/2}$$

$$= 6.28\,[0.01]^{1/2} = 6.28\,[0.1] = 0.628\text{s}$$

13. If the charges q_1 and q_2 are similar in nature, then electric force $F = \frac{q_1 q_2}{l^2}$ will be repulsive and is opposite to tension force T'



For equilibrium of bead at P
$mg\cos 60° = (T - F)\cos\alpha$ ...(i)
$N_1 = mg\cos 30° + (T - F)\sin\alpha$ ...(ii)
Similarly, for the equilibrium of bead at Q
$mg\sin 60° = (T - F)\sin\alpha$ ...(iii)
$N_2 = mg\cos 60° + (T - F)\cos\alpha$ ...(iv)
(i) Dividing equation (iii) by (i), we get : $\tan 60° = \tan\alpha$
i.e. $\alpha = 60°$ ...(v)
Now (ii) from equation (iii) $T - F = mg$
or $T = F + mg = (q_1 q_2 / l^2) + mg$ ...(vi)
(iii) From equation (ii) $N_1 = mg\cos 30° + (T - F)\sin 60°$
$N_1 = mg\cos 30° + mg\sin 60°$
$\therefore N_1 = \sqrt{3}mg$ ...(vii)
From equation (iv) $N_2 = mg\cos 60° + mg\cos 60° = mg$...(viii)
Again if the chord is cut, then T becomes zero. Hence from equation (vi)
$q_1 q_2 = -mgl^2$ ... (ix)
This shows that q_1 and q_2 must have unlike charges for beads to remain stationary.

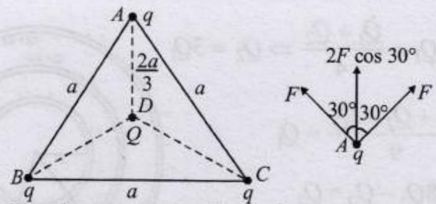
14. **(i)** Here three charges each of value q are placed at the corners of an equilateral triangle and a fourth charge Q is placed at the centre of the triangle as shown in the figure. The force on charge q kept at A due to charges kept at B and C
$F_1 = 2F\cos 30°$

$$F_1 = \sqrt{3} \times \left(9 \times 10^9 \frac{q^2}{a^2}\right)$$

The force on q due to charge $(-q)$ kept at D

$$F_2 = 9 \times 10^9 \frac{q^2}{(2a/3)^2} = \frac{9}{4} \times \left(9 \times 10^9 \times \frac{q^2}{a^2}\right)$$

Clearly the two forces are not equal. Also as $F_2 > F_1$ the charges will move towards the centre.



(ii) For charges to remain stationary

$$2 \times K\frac{q^2}{a^2} \times \frac{\sqrt{3}}{2} = \frac{9}{4} \times K \times \frac{q^2 Q}{a^2} \Rightarrow \frac{4\sqrt{3}\,q}{9} = Q$$

The charge Q should be negative.
Potential energy of the system

$$= 3\left[K\frac{q^2}{a^2} + K\frac{q^2}{a^2}\right] + 3\left[K \times \frac{4\sqrt{3}}{9}\frac{q \times q}{(2a/3)^2}\right]$$

$$= 6K \times \frac{q^2}{a^2} + 3\sqrt{3}\,K\frac{q^2}{a^2} = 3(2 + \sqrt{3})\,K\frac{q^2}{a^2}$$

This is the amount of work needed to move the charges to infinity.

Topic-2: Electric Field and Field Lines

1. **(c)** $E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^2}$;

$$E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Q}{R^2};\; E_3 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q/2}{R^2}$$

Clearly $E_2 > E_1 > E_3$

2. **(d)** **When the electric field is on**
In equilibrium, force due to electric field = weight
$qE = mg$

$$\Rightarrow qE = \frac{4}{3}\pi R^3 \rho g \quad \therefore q = \frac{4\pi R^3 \rho g}{3E} \quad \text{...(i)}$$

When the electric field is switched off
Weight of the drop = viscous force on the drop
$mg = 6\pi\eta R v_t$

$$\frac{4}{3}\pi R^3 \rho g = 6\pi\eta R v_t \quad \therefore R = \sqrt{\frac{9\eta v_t}{2\rho g}} \quad \text{...(ii)}$$

From eq. (i) & (ii) $q = \frac{4}{3}\pi\left[\frac{9\eta v_t}{2\rho g}\right]^{\frac{3}{2}} \times \frac{\rho g}{E}$

$$= \frac{4}{3} \times \pi\left[\frac{9 \times 1.8 \times 10^{-5} \times 2 \times 10^{-3}}{2 \times 900 \times 9.8}\right]^{\frac{3}{2}} \times \frac{900 \times 9.8 \times 7}{81\pi \times 10^5}$$

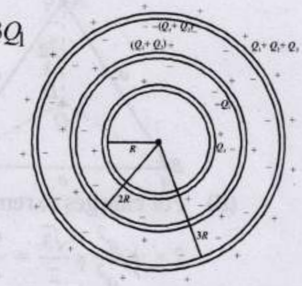
or, $q = 7.8 \times 10^{-19}$ C

3. **(b)** The charges on the surfaces of the metallic spheres are shown in the figure, charge density $\sigma = \frac{\text{charge}}{\text{area}}$ and as per question

$$\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi(2R)^2}$$

$$= \frac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2}$$

$\therefore Q_1 = \frac{Q_1 + Q_2}{4} \Rightarrow Q_2 = 3Q_1$



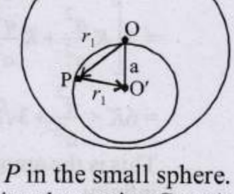
$\frac{Q_1 + Q_2 + Q_3}{9} = Q_1$

$\Rightarrow 8Q_1 - Q_2 = Q_3$

$\Rightarrow 8Q_1 - 3Q_1 = 5Q_1 = Q_3$

$\therefore Q_1 : Q_2 : Q_3 = 1 : 3 : 5$

4. **(b)** Let the charge per unit volume be σ and O be the centre of a uniforms charged solid sphere. Let us consider a uniformly charged sphere of negative charged density σ having its centre at O'. Also let OO' be equal to a.



Let us consider an arbitrary point P in the small sphere. The electric field due to charge on big sphere using Gauss's theorem $\vec{E}_1 = \frac{\sigma}{3\varepsilon_0}\vec{r}_1$

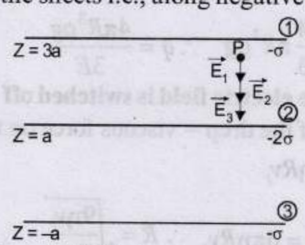
Also the electric field due to small sphere

$\vec{E}_2 = \frac{\sigma}{3\varepsilon_0}\vec{r}_2{}'$ $\quad\therefore$ The total electric field

$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{3\varepsilon_0}a$

Hence electric field will have a finite non-zero value which will be uniform.

5. **(c)** The direction of electric fields are according to the charge on the sheets i.e., along negative z-axis.



The total electric field at point P

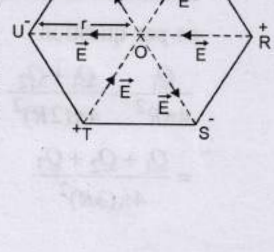
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = E_1(-\hat{k}) + E_2(-\hat{k}) + E_3(-\hat{k})$$

$$= \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right](-\hat{k}) = -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

6. **(c)** Electric field, $|\vec{E}| = \frac{Kq}{r^2}$

Electric field due to P on O is cancelled by electric field due to S on O.

Similarly Electric field due to Q on O is cancelled by electric field due to T and O.



The electric field due to R on O is in the same direction as that of U on O. Hence the net electric field $E + \vec{E} = 2\vec{E}$.

7. **(d)** The electric lines of force cannot enter the metallic sphere as E = 0 inside the solid metallic sphere. Also, the electric lines of force fall on the metallic surface normally.

8. **(c)** Electric field everywhere inside the metallic portion i.e., conductor of shell is zero.
Electric field lines are always normal to a surface.

9. **(c)** Electric lines of force do not form closed loops.

10. **(6)** The magnitude of the electric field at the point P which is at a distance $2R$ from the axis of the cylinder

$E = E_{total} - E_{cavity}$

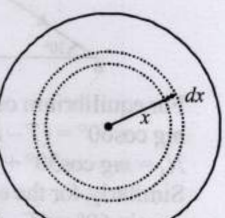
$$= \frac{\gamma}{2\pi\varepsilon_0(2R)} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)^2}$$

$$Q_{sphere} = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\rho = \frac{\pi R^3\rho}{6};\ \lambda_{cylinder} = \pi R^2\rho$$

$$\therefore E = \frac{\pi R^2\rho}{4\pi\varepsilon_0 R} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{\pi R^3\rho/6}{4R^2} = \frac{23\rho R}{96\varepsilon_0} = \frac{23\rho R}{16\times 6\times\varepsilon_0}$$

$\therefore k = 6$

11. **(2)** Let there be a spherical shell of radius x and thickness dx. Volume of this shell, $V = 4\pi x^2(dx)$. Charge enclosed in this shell



$dq = (4\pi x^2)dx \times kx^a$

or, $dq = 4\pi k x^{2+a}dx$.

For $r = R$:
Total charge enclosed in the sphere of radius R

$$Q = \int_0^R 4\pi k\, x^{2+a}\, dx = 4\pi k\frac{R^{3+a}}{3+a}.$$

$\therefore$ Electric field at $r = R$

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k R^{3+a}}{(3+a)R^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}R^{1+a}$$

For $r = R/2$:
Total charge enclosed in the sphere of radius $R/2$.

$$Q' = \int_0^{R/2} 4\pi k\, x^{2+a}dx = \frac{4\pi k(R/2)^{3+a}}{3+a}$$

$\therefore$ The electric field at $r = R/2$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}\frac{(R/2)^{3+a}}{(R/2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}\left(\frac{R}{2}\right)^{1+a}$$

$\because E_2 = \frac{1}{8}E_1$

$$\therefore \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{(3+a)}\left(\frac{R}{2}\right)^{1+a} = \frac{1}{2^3}\times\frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}R^{1+a}$$

$\Rightarrow 1 + a = 3 \therefore a = 2$

12. **(–qEA)** Since electrostatic field is a conservative field

$\therefore (W_{PQ} + W_{QR} + W_{RS}) + W_{SP} = 0$

or $W_{PS} + W_{SP} = 0$ or $W_{PS} = -W_{SP}$

$\therefore$ Work done $= q\vec{E}.\overrightarrow{SP}(\because F = qE)$

$W_{SP} = qE\hat{i}(\vec{r}_P - \vec{r}_S)(\because E 11\times\text{–axis})$

$W_{SP} - qE\hat{i}\left(a\hat{i} + b\hat{j}\right)$

$W_{SP} = qEa$

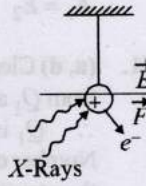
$\therefore\ W_{PS} = -W_{SP} = -qEa$

$\therefore$ Work done by field along process $PQRS = -qEa$

13. **(False)** For a particle to move in circular motion, we need a centripetal force which is not available.

14. **(True)** The electric field produced between the parallel plate capacitor is uniform. The force acting on charged particle placed in an electric field is given by $F = qE$. Hence, the forces on the two protons are identical being equal charge and uniform electric field.

15. **(True)** When high energy *X*-ray beam falls, it will knock out electrons from the small metal ball due to photoelectric effect, making it positively charged and an electrostatic force in the direction of electric field acts. Therefore the ball will be deflected in the direction of electric field.



16. **(b, c)** Given: electric field, $E_y = -400\sqrt{3}\ NC^{-1}$

Initial speed of charged particle, $u = 2\sqrt{10} \times 10^6$ m/s

Range, R = 5 m

$F = ma$ and $F = qE \therefore a_y = \frac{q}{m}E_y = -400\sqrt{3} \times 10^{10}$

$$\left[\because \frac{q}{m} = 10^{10}\,\text{Ckg}^{-1}\ \text{(given)}\right]$$

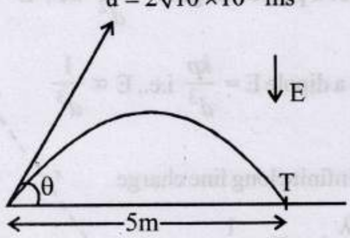
$$\text{Range}, R = \frac{u^2 \sin 2\theta}{a_y} \Rightarrow 5 = \frac{40 \times 10^{12} \sin 2\theta}{400\sqrt{3} \times 10^{10}}$$

$$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$$

$\Rightarrow 2\theta = 60^\circ$ or, $120 \Rightarrow \theta = 30^\circ$ or, 60°

Particle hits the target, if $\theta = 30^\circ$ or $\theta = 60^\circ$

$u = 2\sqrt{10} \times 10^6\ \text{ms}^{-1}$



$$\text{Time of flight } T_1 = \frac{2u \sin\theta}{a_y} = \frac{2 \times 2\sqrt{10} \times 10^6 \times \frac{1}{2}}{400\sqrt{3} \times 10^{10}}$$

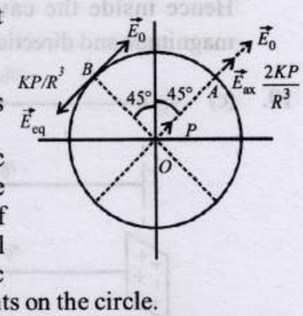
$$= \sqrt{\frac{5}{6}}\mu\text{s (for } \theta = 30^\circ)$$

$$\text{Time of flight } T_2 = \frac{2u \sin\theta}{a_y} = \frac{2 \times 2\sqrt{10} \times 10^6 \times \frac{\sqrt{3}}{2}}{400\sqrt{3} \times 10^{10}}$$

$$= \sqrt{\frac{5}{6}}\mu\text{s (for } \theta = 60^\circ)$$

17. **(b, d)** Given dipole moment of electric dipole

$\vec{P} = \frac{P_0}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$ and circle is equipotential. Also electric field is normal to such a line that is the direction of electric field is either radial or the magnitude of electric field should be zero at points on the circle.



Now considering point A, the electric field due to dipole $\frac{2Kp}{R^3}$ (directed from O to A) as point A lies on the axial line of electric dipole. The external electric field E_0 should also be in the direction of O to A.

Now considering point B which is a point on the equitorial line of the electric dipole. The electric field here due to dipole is $\frac{Kp}{R^3}$ in a direction opposite to the dipole. The external electric field should cancel out this field. i.e., $E_B = 0$

$$\text{Further } \vec{E}_0 = \frac{-K\vec{p}}{R^3} \qquad ...(i)$$

The electric field at A

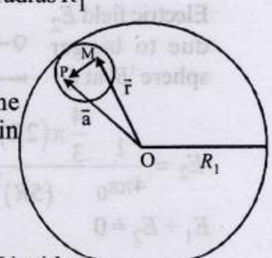
$$\vec{E}_A = \frac{2K\vec{p}}{R^3} + \vec{E}_0 = -2\vec{E}_0 + \vec{E}_0 = -\vec{E}_0$$

$$\text{Again from eq. (i) } E_0 = \frac{1}{4\pi\epsilon_0}\frac{p_0}{\sqrt{2}(R^3)} \times \sqrt{2}$$

$$\therefore\ R^3 = \frac{p_0}{4\pi\epsilon_0 E_0} \quad \therefore\ R = \left[\frac{p_0}{4\pi\epsilon_0 E_0}\right]^{1/3}$$

18. **(d)** Let us consider a point M inside the cavity where electric field has to be calculated. Assume the cavity to contain similar charge distribution of positive and negative charge as the rest of sphere. Electric field at *M* due to uniformly distributed charge of the whole sphere of radius R_1

$$\vec{E}_1 = \frac{\rho}{3\varepsilon}\vec{r}$$



Electric field at M due to the negative charge distribution in the cavity

$$\vec{E}_2 = \frac{\rho}{3\varepsilon}\overrightarrow{MP}$$

$\therefore$ The total electric field at M inside the cavity,

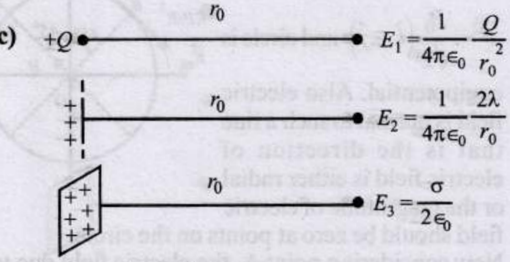
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon}\vec{r} + \frac{\rho}{3\varepsilon}\overrightarrow{MP}$$

$$\therefore\ \vec{E} = \frac{\rho}{3\varepsilon}\vec{r} + \frac{\rho}{3\varepsilon}\left(\vec{a} - \vec{r}\right)\left[\because\ \vec{r} + \overrightarrow{MP} = \vec{a}\right]$$

$\therefore \quad \vec{E} = \frac{\rho}{3\varepsilon}\vec{a}$

Hence inside the cavity $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$.

19. (c)



$E_1(r_0) = E_2(r_0) = E_3(r_0)$ (Given)

(a) $E_1(r_0) = E_3(r_0) \therefore \frac{Q}{4\pi\epsilon_0 r_0^2} = \frac{\sigma}{2\epsilon_0} \Rightarrow Q = 2\pi\sigma r_0^2$

(b) $E_2(r_0) = E_3(r_0)$

$\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r_0} = \frac{\sigma}{2\epsilon_0} \Rightarrow r_0 = \frac{\lambda}{\sigma\pi}$

(c) $E_1(r_0/2) = \frac{1}{4\pi\epsilon_0}\frac{4Q}{r_0^2}$

$= \frac{1}{4\pi\epsilon_0} \times \frac{4\times 2\lambda r_0}{r_0^2} = \frac{1}{4\pi\epsilon_0}\frac{8\lambda}{r_0}$

$\because \quad E_1(r_0) = E_2(r_0) \quad \therefore Q = 2\lambda r_0$

And $2E_2(r_0/2) = 2\left[\frac{1}{4\pi\epsilon_0}\frac{4\lambda}{r_0}\right] = \frac{1}{4\pi\epsilon_0}\frac{8\lambda}{r_0}$

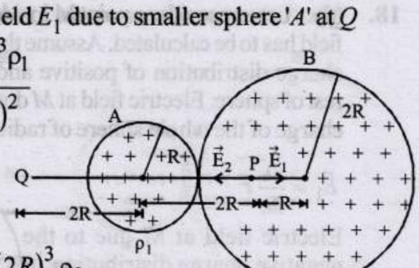
(d) $E_2(r_0/2) = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r_0/2} = \frac{1}{4\pi\epsilon_0}\frac{4\lambda}{r_0} = \frac{\lambda}{\pi\epsilon_0 r_0}$

$4E_3(r_0/2) = \frac{4\sigma}{2\epsilon_0} = \frac{2\sigma}{\epsilon_0} = \frac{2}{\epsilon_0} \times \frac{\lambda}{\pi r_0}$

20. (b,d) Electric field E_1 due to smaller sphere 'A' at Q

$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3\rho_1}{(2R)^2}$

Electric field E_2 due to bigger sphere 'B' at Q



$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi(2R)^3\rho_2}{(5R)^2}$

$E_1 + E_2 = 0$

$\frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3\rho_1}{(2R)^2} + \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi(2R)^3\rho 2}{(5R)^2} = 0$

$\therefore \quad \frac{\rho_1}{\rho_2} = -\frac{32}{25}$

Electric field E_1 due to smaller sphere 'A' at P

$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{\rho_1 \times \frac{4}{3}\pi R^3}{(2R)^2} = \frac{1}{4\pi\epsilon_0} \times \frac{\rho_1 \pi R}{3} = \frac{\rho_1 R}{4\epsilon_0 \times 3}$

Electric field E_2 due to bigger sphere 'B' at P

$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi(2R)^3\rho_2 R}{(2R)^3} = \frac{\rho_2 R}{3\varepsilon_0}$

$E_1 = E_2 \qquad \therefore \frac{\rho_1 R}{4\epsilon_0 \times 3} = \frac{\rho_2 R}{3\epsilon_0} \Rightarrow \frac{\rho_1}{\rho_2} = 4$

21. (a, d) Clearly from figure electric field lines are orginating from Q_1 and terminating on Q_2.

$\therefore \quad Q_1$ is positive and Q_2 is negative.

Number of electric field lines originating from Q_1 is more than terminating at Q_2.

$\therefore |Q_1| > |Q_2|$.

Since at a finite distance to the right of Q_2, the electric field is zero. The electric field created by Q_2 at a particular point will cancel out the electric field created by Q_1. But at a finite distance to the left of Q_1 the electric field is non-zero.

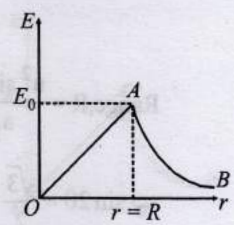
22. (a, c) The electric field inside the sphere

$(r<R)\,E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{R^3}r$

or, $E \propto r$ Outside the sphere

$(R<r<\infty)\,E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$

or, $E \propto \frac{1}{r^2}$



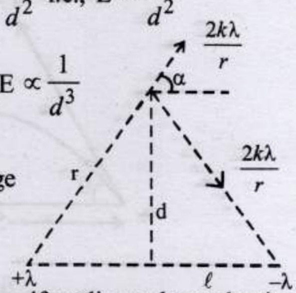
Hence, E increases for $r<R$ and decreases for $R<r<\infty$.

23. (b) For a point charge $E = \frac{kQ}{d^2}$ i.e., $E \propto \frac{1}{d^2}$

and for a dipole $E = \frac{kp}{d^3}$ i.e., $E \propto \frac{1}{d^3}$

For an infinite long line charge

$E = \frac{2k\lambda}{d}$ i.e., $E \propto \frac{1}{d}$



For two infinite wires carrying uniform linear charge density.

$E = \frac{2k\lambda}{r}\cos\alpha = \frac{2k\lambda}{\sqrt{d^2+\ell^2}} \times \frac{\ell}{\sqrt{d^2+\ell^2}} = \frac{2k\lambda\ell}{d^2+\ell^2}$

or, $E \propto \frac{1}{d^2} \because 2l << d$

For infinite plane charge $E = \frac{\sigma}{2\epsilon_0}$ i.e., E is in dependent of d

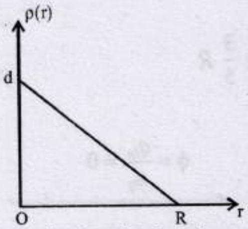
24. (a) At $r=R$, area $A = 4\pi R^2$ and total enclosed charge, $q_{in} = ze$

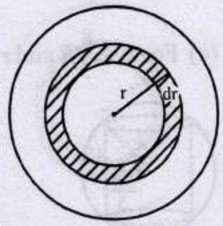
From Gauss's theorem $\phi = \frac{q_{in}}{\varepsilon_0}$

or, $E.A = \frac{q_{in}}{\varepsilon_0}$ or, $E(4\pi R^2) = \frac{ze}{\varepsilon_0}$ $\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{ze}{R^2}$

Clearly electric field E is independent of A.

25. (b) For a = 0, the equation for the graph line $r = d - \frac{r}{R}dr$





Charge in the shaded element

$dq = r \times 4\pi r^2 dr$

$\therefore dq = \left(d - \frac{d}{R}r\right)4pr^2 dr$ or, $Ze = \int_0^R 4\pi d r^2 dr - \int_0^R \frac{4\pi d}{R} r^3 dr$

$\Rightarrow Ze = 4\pi d\frac{R^3}{3} - \frac{4\pi d}{R}\frac{R^4}{4}$

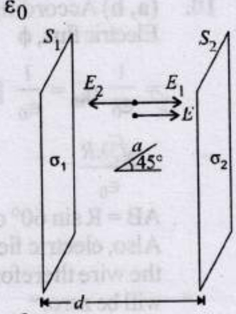
$\therefore \frac{Ze.}{4\pi dR^3} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$ $\Rightarrow d = \frac{3Ze}{\pi R^3}$

26. (c) If the volume charge density is constant then $E \propto r$.

27. Electric field due to sheet $S_1, E_1 = \frac{\sigma_1}{\varepsilon_0}$

Electric field due to sheet

$S_2, E_2 = \frac{\sigma_2}{\varepsilon_0}$



$\therefore$ Net electric field, $E = E_1 - E_2$

$= \frac{\sigma_1 - \sigma_2}{\varepsilon_0}$ $(\because \sigma_1 > \sigma_2)$

Work done by electric field

$W = qv = (q_0 E)a\cos 45° = q_0 E \times \frac{a}{\sqrt{2}}$

$\therefore W = \frac{q_0(\sigma_1 - \sigma_2)a}{\sqrt{2}\,\varepsilon_0}$

Topic-3: Electric Dipole, Electric Flux and Gauss's Law

1. (b, d) The electric field inside sphere is zero, so dipole will oscillate when r > R so option (b) is correct (a) is incorrect.

For r > R $E = \frac{\sigma R^2}{\varepsilon_0 r^2}$

$\omega = \sqrt{\frac{PE}{1}} = \sqrt{\frac{P_0\sigma R^2}{I\varepsilon_0 r^2}}$

when r = 2R; $\omega = \sqrt{\frac{P_0\sigma R^2}{I\varepsilon_0(2R)^2}}$

or, $\omega = \sqrt{\frac{P_0\sigma}{4I\varepsilon_0}}$

so option (c) is incorrect.

when r = 10R; $\omega = \sqrt{\frac{P_0\sigma R^2}{I\varepsilon_0(10R)^2}}$

$\therefore \omega = \sqrt{\frac{P_0\sigma}{100I\varepsilon_0}}$

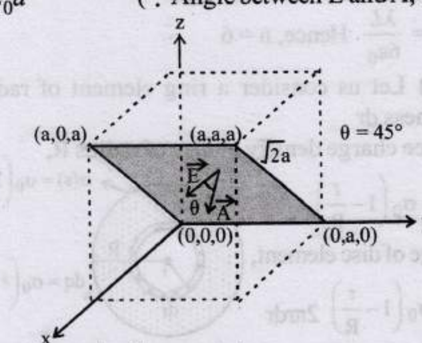
so option (d) is correct.

2. (c) Given $\vec{E} = E_0\hat{x}$

i.e, electric field $\vec{E}$ acts along $+x$ direction and is a constant. Therefore the electric flux through the shaded portion whose area $\vec{A} = a \times \sqrt{2}a = \sqrt{2}a^2$

$\phi = \vec{E}.\vec{A} = EA\cos\theta = E_0(\sqrt{2}a^2)\cos 45° = E_0(\sqrt{2}a^2) \times \frac{1}{\sqrt{2}}$

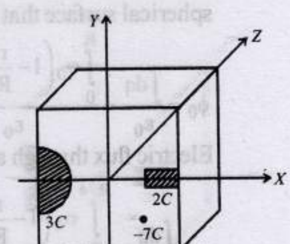
$= E_0 a^2$ ($\because$ Angle between E and A, Q = 45°)



3. (a) From the figure total charge enclosed in the cubical surface $q_{in} = 3C + 2C - 7C = -2C$.

According to Gauss's theorem the electric flux through the cube

$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{-2C}{\varepsilon_0}$



4. (d) The flux through the Gaussian surface is due to the charges inside the Gaussian surface. But the electric field on the Gaussian surface is the vector sum of electric fields due to all the charges *i.e.*, $q_1, -q_1$ and q_2.

5. (3) Solid angle subtended at centre by plane surface

$\Omega = 2 \times 2\pi(1 - \cos\theta)$

$= 4\pi - 4\pi\cos\theta$

So solid angle made by curved surface $= 4\pi - \Omega$

$= 4\pi - (4\pi - 4\pi\cos\theta) = 4\pi\cos\theta$

Flux through curved surface $= \frac{Q}{\varepsilon_0}\cos\theta$

$\phi_{30°} = \phi = \frac{4\pi\cos 30°}{4\pi}\frac{Q}{\varepsilon_0} = \cos 30°\frac{Q}{\varepsilon_0}$

$\phi_{60} = \frac{4\pi\cos 60°}{4\pi}\frac{Q}{\varepsilon_0} = \cos 60°\frac{Q}{\varepsilon_0}$

$\frac{\phi_{30}}{\phi_{60}} = \frac{\cos 30°}{\cos 60°} = \sqrt{3}$

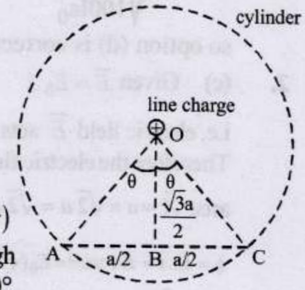
$\frac{\phi}{\phi_{60}} = \sqrt{3} \Rightarrow \phi_{60} = \frac{\phi}{\sqrt{3}} \therefore n = 3$

6. **(3)** $\phi_{cone} = \frac{q}{2\epsilon_0} = \frac{3q}{6\epsilon_0}$. So, n = 3

7. **(6)** From figure $\tan\theta = \frac{BC}{OB} = \frac{a/2}{\sqrt{3}a/2} = \frac{1}{\sqrt{3}}$

$\therefore \quad \theta = 30°$

Electric flux through the complete cylinder by Gauss's theorem



$\phi \text{ cylinder} = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$

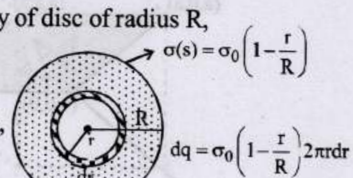
(Where L = length of cylinder)

$\therefore$ Electric flux passing through cylindrical surface i.e., for 60° angle $= \frac{\lambda L}{6\varepsilon_0}$ Hence, n = 6

8. **(6.40)** Let us consider a ring element of radius r and thickness dr

Surface charge density of disc of radius R,

$\sigma(r) = \sigma_0\left(1 - \frac{r}{R}\right)$



Charge of disc element,

$dq = \sigma_0\left(1 - \frac{r}{R}\right) 2\pi r dr$

Now from Gauss's theorem, Electric flux, through a large spherical surface that encloses the charged disc completely.

$$\phi_0 = \frac{\int dq}{\varepsilon_0} = \frac{\int_0^R \sigma_0\left(1 - \frac{r}{R}\right)2\pi r dr}{\varepsilon_0}$$

Electric flux through another spherical surface of radius R/4

$$\phi = \frac{\int dq}{\varepsilon_0} = \frac{\int_0^{R/4} \sigma_0\left(1 - \frac{r}{R}\right)2\pi r dr}{\varepsilon_0}$$

$$\therefore \frac{\phi_0}{\phi} = \frac{\sigma_0 2\pi \int_0^R \left(r - \frac{r^2}{R}\right)dr}{\sigma_0 2\pi \int_0^{R/4}\left(r - \frac{r^2}{R}\right)dr} = \frac{\frac{R^2}{2} - \frac{R^2}{3}}{\frac{R^2}{32} - \frac{R^2}{3 \times 64}} = \frac{32}{5} = 6.40$$

9. **(a, b, c)** (a) For $h > 2R$ and $r = \frac{3R}{5}$

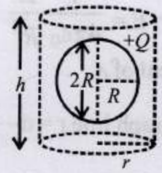
$\sin\theta = \frac{3R/5}{R} = \frac{3}{5} = 37°$

$q_{in} = Q[1 - \cos 37°] = Q\left[1 - \frac{4}{5}\right] = \frac{Q}{5}$

From Gauss's theorem $Q = \frac{q_{in}}{\varepsilon_0}$

$\therefore \quad \phi = \frac{Q}{5\epsilon_0}$

(b) For $h > 2R$ and $r > R$

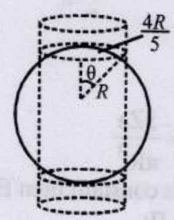


$\phi = \frac{q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0}$

(c) For $h < \frac{8}{5}R$ and $r = \frac{3}{5}R$

$\phi = \frac{q_{in}}{\epsilon_0} = 0$

(d) For $h > 2R$ and $r > \frac{4}{5}R$

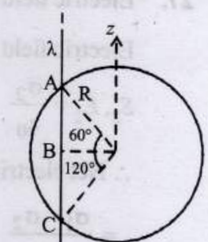


$\sin\theta = \frac{4R/5}{R} = \frac{4}{5} = 0.8 \Rightarrow \theta = 53°$

$q_{in} = Q[1 - \cos\theta] = Q\left[1 - \frac{3}{5}\right] = \frac{2Q}{5}$

$\therefore \quad \phi = \frac{2Q}{5\epsilon_0}$

10. **(a, b)** According to Gauss's Law, Electric flux, ϕ



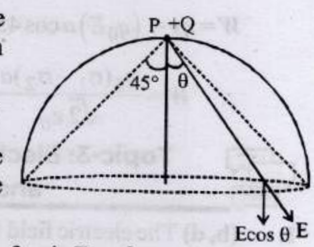
$= \frac{1}{\epsilon_0} q_{in} = \frac{1}{\epsilon_0}[\lambda \times 2R\sin 60°]$

$= \frac{\sqrt{3}\lambda R}{\epsilon_0}$

AB = R sin 60° or AC = 2R sin 60°

Also, electric field is perpendicular to the wire therefore its z-com ponent will be zero.

11. **(a, d)** The circumference of the flat surface is an equipotential $V = \frac{KQ}{\sqrt{2}R}$ because the circumference is equidistant from +Q. The component of electric field per-pendicular to the flat surface is $E\cos\theta$.



Here E as well as θ changes for different point on the flat surface. The total flux through the curved and flat surface should be less than $\frac{Q}{\varepsilon_0}$.

The solid angle subtended by the flat surface at P

$= 2\pi(1 - \cos\theta) = 2\pi(1 - \cos 45°) = 2\pi\left(1 - \frac{1}{\sqrt{2}}\right)$

$\therefore$ Flux passing through curved surface

$$= -\frac{Q'}{\varepsilon_0}\frac{2\pi\left(1 - \frac{1}{\sqrt{2}}\right)}{4\pi} = -\frac{Q}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right).$$

12. **(a, c, d)** Due to symmetry, the net electric flux passing through $x=+\frac{a}{2}$,

$x=-\frac{a}{2}, z=+\frac{a}{2}$ is same

According to Gauss's theorem, net electric flux net electric flux crossing through any closed surface $\phi=\frac{q_{in}}{\varepsilon_0}$

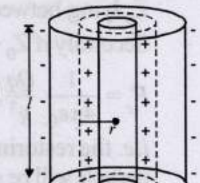
$=\frac{-q+3q-q}{\varepsilon_0}=\frac{q}{\varepsilon_0}$.

13. **(c, d)** The potential of all points lie on the conductor is same. Thus potential at A = Potential at B

Total electric flux through cavity $=\frac{q}{\epsilon_0}$ this according to Gauss's theorem.

Option (a) and (b) are dependent on the curvature which is different at points A and B because cavity is elliptical.

14. **(c)** Electric field at a distance r from the axis of cylinder



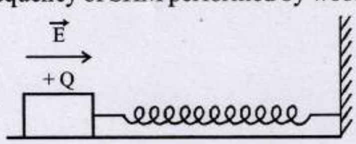
$E=\frac{\lambda}{2\pi\varepsilon_0 r}$ (λ= charge per unit length)

$\therefore\ E\propto\frac{1}{r}$

Topic-4: Miscellaneous (Mixed Concepts) Problems

1. **(a)** Here frequency of SHM performed by wooden block

$\nu_0=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$



This value of frequency will remain unchange because when electric field is switched on, the value of k and m is not affected the mean position of SHM shifts towards right by $l=\frac{QE}{k}$

due to force acting $F=qE$.

2. **(a)** The electrostatic force per unit area i.e., electrostatic pressure at a point on the surface of a uniformly charged sphere $=\frac{1}{2}\varepsilon_0 E^2=\frac{\sigma^2}{2\epsilon_0}$

$\therefore$ The force on a hemispherical shell $F=\frac{\sigma^2}{2\epsilon_0}\times\pi R^2$

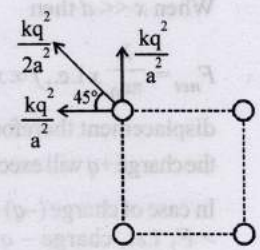
or, $F\propto\frac{\sigma^2 R^2}{\varepsilon_0}$

3. **(3)** Net electrostatic force on one charge due to remaining three charges,

$$F_{electro}=\frac{kq^2}{2a^2}+2\left[\frac{kq^2}{a^2}\times\frac{1}{\sqrt{2}}\right]=\frac{q^2}{a^2}\times\text{constant}$$

Surface tension force $F_{st}=\gamma a$

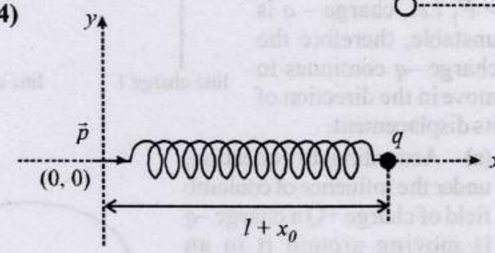
In equilibrium,



$\frac{q^2}{a^2}\times\text{constant}=\gamma a$

$\therefore\quad a=k\left(\frac{q^2}{\gamma}\right)^{1/3}\qquad\therefore\ N=3$

4. **(3.14)**



Original frequency, $f=\frac{2}{\delta}\sqrt{\frac{K}{m}}$

If dipole appears at equilibrium,

$\frac{2KP}{(\ell+x_0)^3}q=Kx_0$ ...(i)

When displaced towards right by length x_0

$f_{net}=\frac{2KP}{(\ell+x_0+x)^3}q-K(x_0+x)$

$ma=\frac{2KPq}{(\ell+x_0)^3}\left[1+\frac{x}{\ell-x_0}\right]^3-K(x_0+x)$

$=\frac{2KPq}{(\ell+x_0)^3}\left[1-\frac{3x}{\ell+x_0}\right]^3-K(x_0+x)$

$=\frac{6KPqx}{(\ell+x_0)^4}Kx=\frac{3x}{\ell+x_0}Kx_0-Kx=-Kx\left[\frac{3x_0}{\ell+x_0}+1\right]$

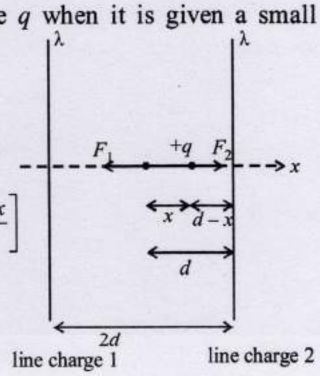
As 'ℓ' is negative, $ma=-Kx4\Rightarrow a=\frac{-4K}{m}x$

New frequency, $f'=\frac{1}{2\pi}\sqrt{\frac{4K}{m}}=2f=\frac{1}{\pi}\sqrt{\frac{K}{m}}$

$\therefore\ \delta=\pi=3.14$

5. **(False)** Electrostatic force is conservative in nature, Hence work done is path independent.

6 **(c)** Net force on charge q when it is given a small displacement x



$F_{net}=F_1-F_2$

$=\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d-x}-\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d+x}$

$\therefore\ F_{net}=\frac{\lambda}{2\pi\varepsilon_0}\left[\frac{d+x-d+x}{d^2-x^2}\right]$

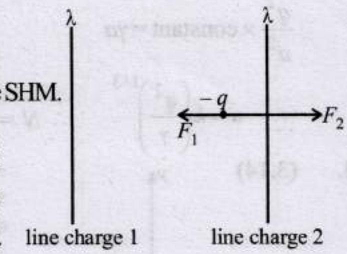
$\therefore\ F_{net}=\frac{\lambda}{2\pi\varepsilon_0}\frac{2x}{d^2-x^2}$

When $x << d$ then

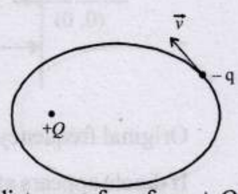
$F_{net} = \frac{\lambda}{\pi\varepsilon_0} x$ i.e., $f \propto x$

displacement therefore the charge $+q$ will execute SHM.

In case of charge $(-q)$ $F_2 > F_1$ *i.e.*, charge $-q$ is unstable, therefore the charge $-q$ continues to move in the direction of its displacement.

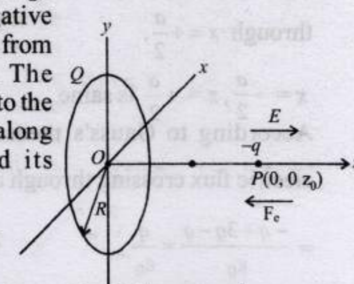


7. **(a)** According to question, under the influence of coulomb field of charge +Q a charge $-q$ is moving around it in an elliptical orbit and this situation is shown in the figure which is similar to a planet revolving around sun. The distance of $-q$ from $+Q$ is changing, therefore, force between the charges will change. The speed of the charge $-q$ will be greater when the charge is nearer to $+Q$ as compared to when it is far. Hence, the angular velocity of charge $-q$ is not constant. The direction of the velocity changes continuously, therefore, linear momentum is not constant. The angular momentum of charge $(-q)$ about Q is constant because the torque about charge $+Q$ is zero.



8. **(a, c)** Let Q be the charge on the ring, the negative charge $-q$ is released from point P $(0, 0, Z_0)$. The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qz_0}{(R^2 + z_0^2)^{3/2}}$$



Therefore, force on charge P will be towards centre as shown, and its magnitude is

$$F_e = qE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{(R^2 + Z_0^2)^{3/2}} \cdot Z_0 \quad ...(1)$$

Similarly, when it crosses the origin, the force is again towards centre O.

Thus the motion of the particle is periodic for all values of Z_0 lying between 0 and ∞.

Secondly if $Z_0 << R$, $(R^2 + Z_0^2)^{3/2} \approx R^3$

$$F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{R^3} \cdot Z_0 \quad \text{[From equation 1]}$$

i.e. the restoring force $F_e \propto -Z_0$. Hence the motion of the particle will be simple harmonic. (Here negative sign implies that the force is towards its mean position).