Ch-1 Basics of Control System.

What do you mean by system?

i/p → System → Proper o/p

FAN without blades: No air flow: No proper o/p: Not a system

FAN with blades: Air flow: Proper o/p: It is a system

System:- It is nothing but group of elements (or) physical components arranged in such a way that it ~~is~~ gives proper output to given i/p.

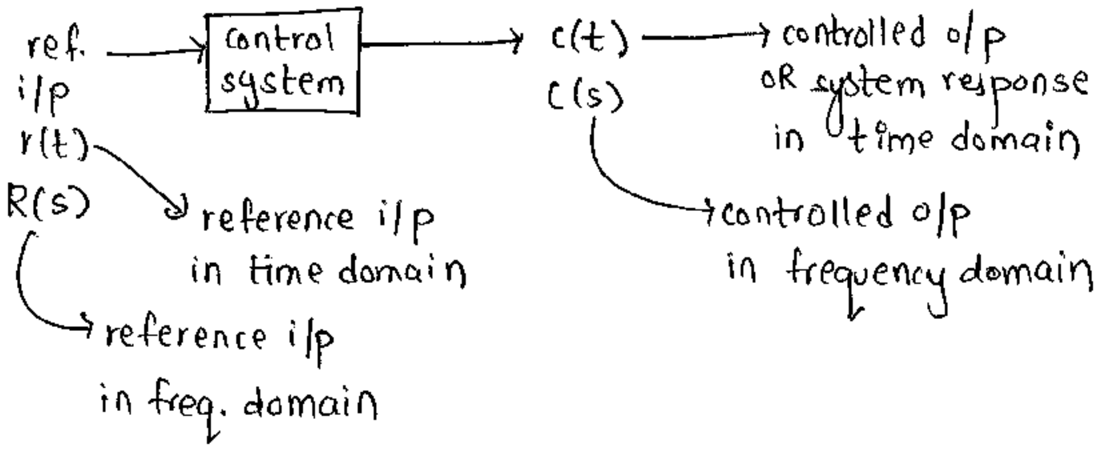
- The proper o/p may or may not be desired.

What is control system?

i/p → Controller → System / Plant / Process → Desired o/p.

FAN with blades & with regulator : Air flow : Desired o/p : Control system

Control system : It is nothing but group of elements (OR) physical components arranged in such a way that it gives desired o/p by means of control / command / regulate either direct or indirect method to given i/p.



$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

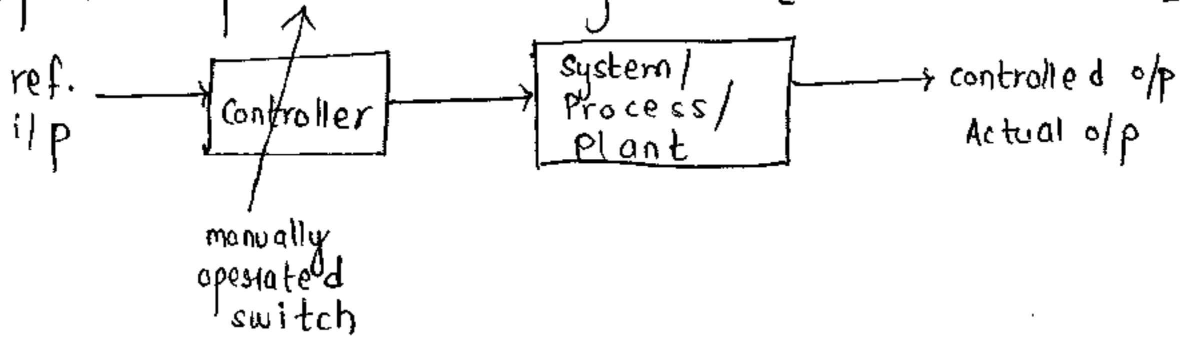
$$C(s) = R(s) \cdot G(s)$$

- Control system classified into two parts based on controlling action:-

(1) Open loop control system (OLCS)

(2) Closed loop control system (CLCS)

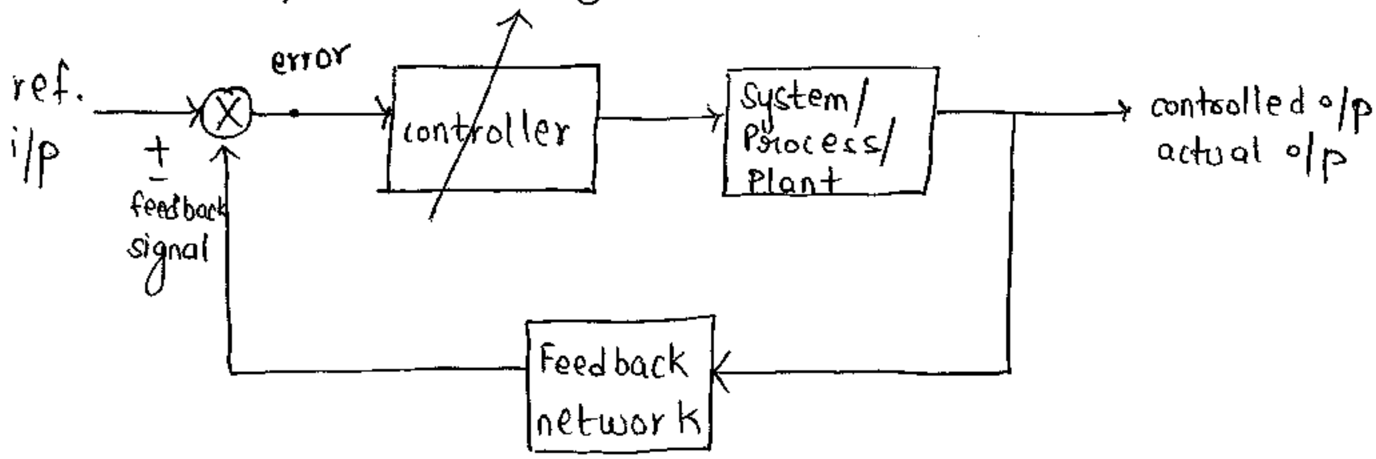
(1) Open loop control system [without feedback]



Eg:- FAN, tubelight, iron machine

- A control system in which controller action is not dependent on actual o/p of system is called open loop control system.

(2) Closed loop control system [with feedback]



Eg:- AC

- A control system in which controller action is dependent on actual o/p of system is called closed loop control system.

*What is feedback network?

- Feedback network is part of closed loop control system which brings actual o/p to the i/p & compares it with reference i/p according to comparison error will be generated and depending on that error controller action takes place.

Transfer function:

$$\underset{R(s)}{r(t)} \longrightarrow \boxed{\text{Control system}} \longrightarrow \underset{C(s)}{c(t)}$$

$$\boxed{\text{T.F.} = \left.\frac{L[o/p]}{L[i/p]}\right|_{I.C.=0}}$$

- The ratio of laplace transform of o/p to i/p considering all initial conditions are to be zero.

Initial conditions are to be zero means present o/p should not depend on past history of system. i.e. (past i/p & past o/p)

Eg:- If capacitor & inductors are used in system then both should be in relaxed condition.

$$\text{ref. i/p} \longrightarrow \boxed{\text{Controller}} \longrightarrow \boxed{\text{System/Process/Plant}} \longrightarrow \text{controlled o/p}$$

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

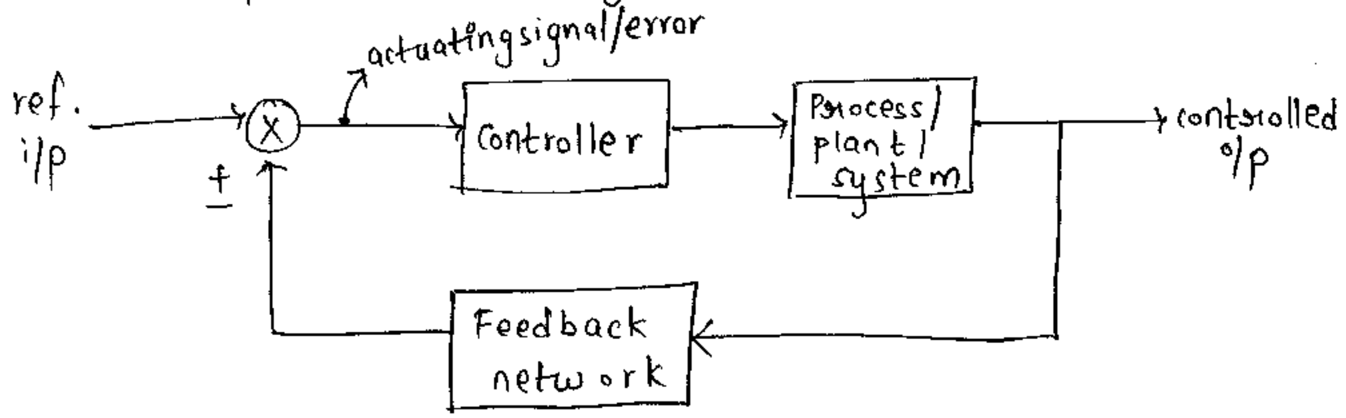
$$C(s) = R(s) \cdot G(s)$$

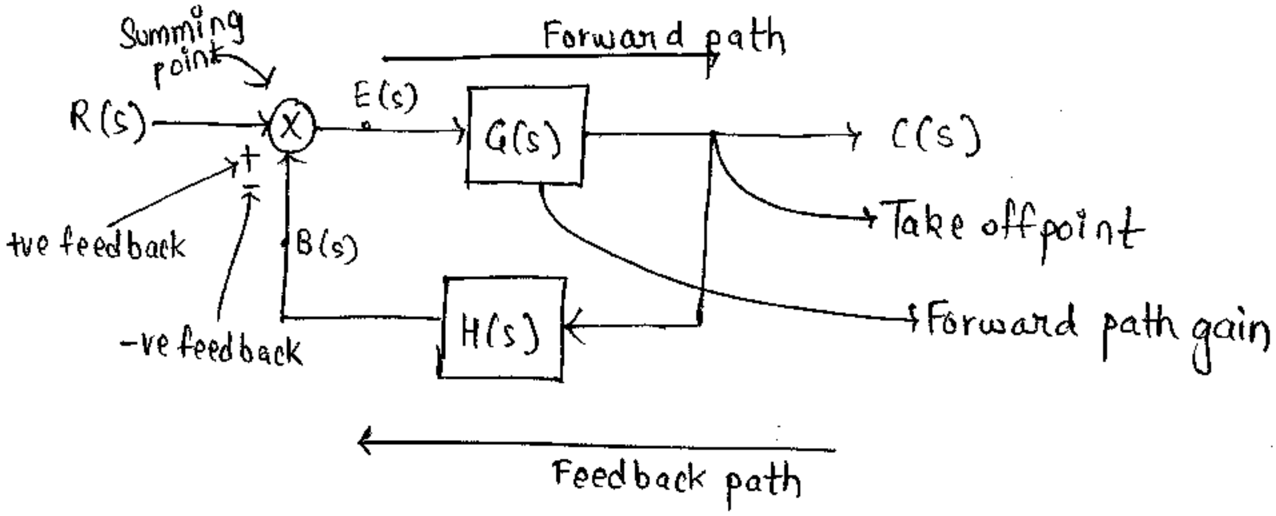
$$\text{T.F.} = \left.\frac{L[o/p]}{L[i/p]}\right|_{I.C.=0}$$

$$\text{O.L.T.F.} = \frac{C(s)}{R(s)} = G(s)$$

↳ gain without feedback

* Closed loop control system





$$T.F. = \left.\frac{L[o/p]}{L[i/p]}\right|_{I.C.=0}$$

$$T.F. = \frac{C[s]}{R[s]}$$

Here, $C(s) = G(s).E(s)$

$E(s) = R(s) \pm B(s)$

$B(s) = C(s) \cdot H(s)$

$\therefore \; C[s] = G[s] \cdot E[s]$

$= G[s] \cdot \left[R[s] \pm B[s] \right]$

$= G[s] \left[R[s] \pm C[s] H[s] \right]$

$C[s] = G[s] R[s] \pm G[s] C[s] H[s]$

$C[s] \left[1 \mp G[s] H[s] \right] = G[s] R[s]$

$$\boxed{\frac{C[s]}{R[s]} = \frac{G[s]}{1 \mp G[s]H[s]}}$$

→ +ve feedback

→ -ve feedback

(i) Positive feedback

$\uparrow E[s] = R[s] + B[s]$

Eg:- Schmitt trigger, oscillator, multivibrator

(ii) Negative feedback

$\downarrow E[s] = R[s] - B[s]$

Eg:- Amplifier

Advantage: Bandwidth is increased by same factors

Disadvantage: Overall gain is reduced by factor $1+G[s]H[s]$

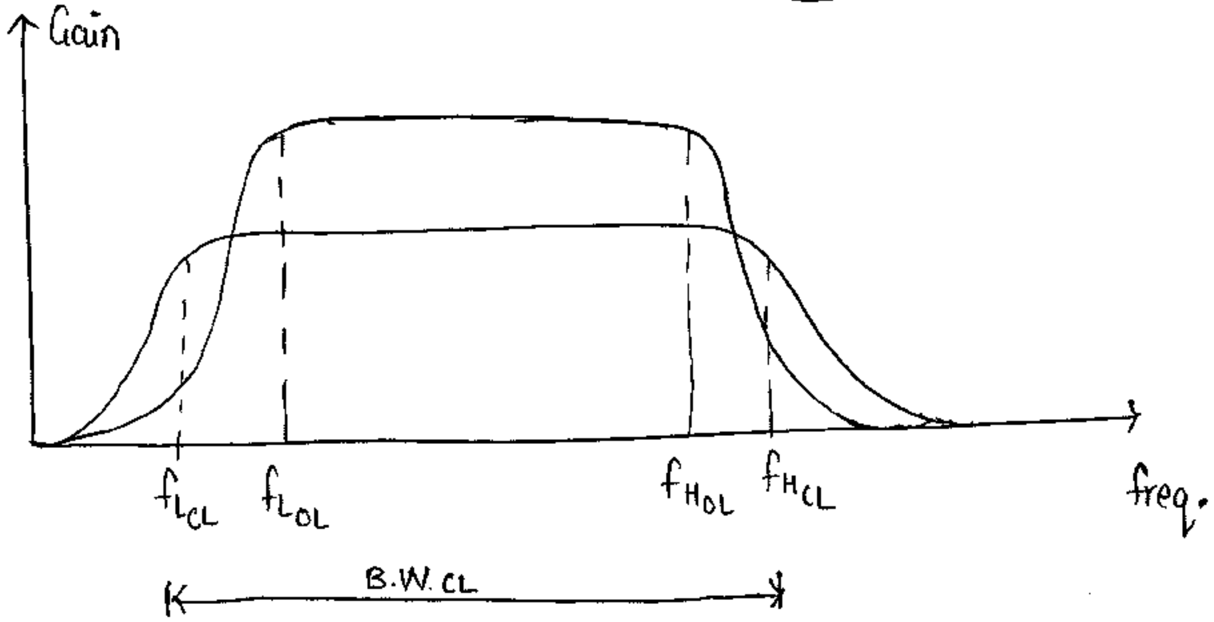
Gain with -ve $\dfrac{G[s]}{1+G[s]H[s]}$ = C.L.T.F. (Gain. Bandwidth = constant)

Gain without feedback	Gain with -ve feedback
O.L.T.F. $= \frac{C[s]}{R[s]} = G[s]$	C.L.T.F. $= \frac{C[s]}{R[s]} = \frac{G[s]}{1+G[s]H[s]}$
O.L.C.S.	-ve feedback CLCS
OL Gain $= G$ B.W. of OL CS $= B.W._{OL}$ Gain $\cdot B.W_{OL} =$ constant	CL gain $= \frac{G}{1+GH}$ CLCS B.W. $= B.W._{CL}$ $\frac{G}{1+GH} \cdot B.W._{CL} =$ constant

$$G \times B.W._{OL} = \frac{G}{1+GH} \cdot B.W._{CL}$$

$$\boxed{B.W._{CL} = B.W._{OL}\,(1+GH)}$$

Because we know that Gain $\cdot$ B.W. = Constant



- Any control system whether it is open loop or closed loop initially defined in terms of open loop controlsys i.e. given by

$$\boxed{OLCS = GH(s) = \frac{k(1+sT_1)(1+sT_2)(1+sT_3)\cdots\cdots}{s^n(1+sT_a)(1+sT_b)(1+sT_c)\cdots\cdots}}$$ → Time constant form

k, T : system parameters

k : system gain

T : system time constant

$s = \sigma + j\omega$

n : Type of system

*Zeros

$GH(s)\big|_{s=s_1} = 0$

s_1 is zeros of the system

$\boxed{N(s)=0}$ → zero of system

$(1+sT_1)(1+sT_2)(1+sT_3)\cdots = 0$

Zeros: $s = \frac{-1}{T_1}, \frac{-1}{T_2}, \frac{-1}{T_3}, \cdots$

*Poles

$OLTF = GH(s)\big|_{s=s_2} = \infty$

s_2 is poles of the system

$\boxed{D(s)=0}$ → poles of system

$s^n(1+sT_a)(1+sT_b)(1+sT_c)\cdots = 0$

Poles: $s = 0, 0, 0, \cdots$ n times, $\frac{-1}{T_a}, \frac{-1}{T_b}, \frac{-1}{T_c}, \cdots$

*Order

Total number poles in s-plane

*Type

Total number poles @ origin in s-plane

☆ OLTF and should be time-constant form ☆

*Gain (K)

TCF

time constant form

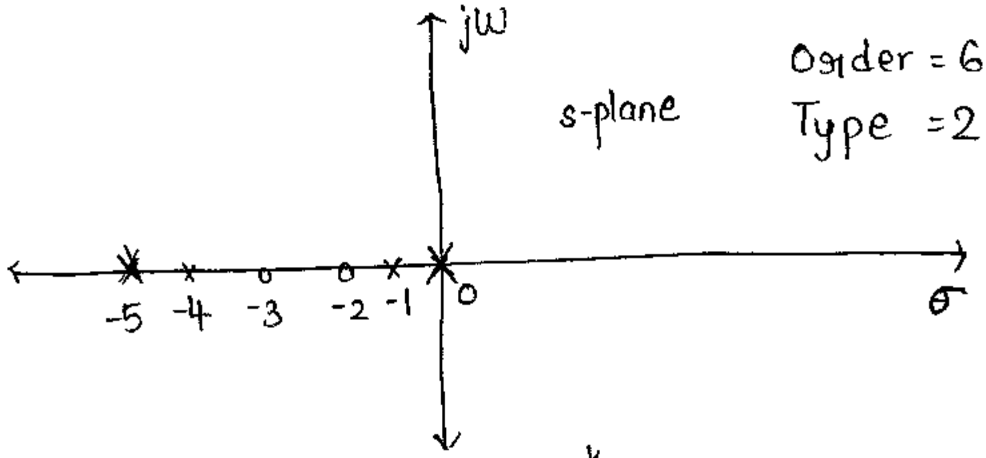
To find system gain (k) given OLTF should be in time-constant form

NOTE:-

- Type & order of system is defined only for open loop transfer function
- It is not defined for Closed loop T.F. If CLTF is given then first convert it into OLTF then find order and type

Q:- $GH(s) = \dfrac{10(s+2)(s+3)}{s^2(s+1)(s+4)(s+5)^2}$

Above is pole-zero s-plane form

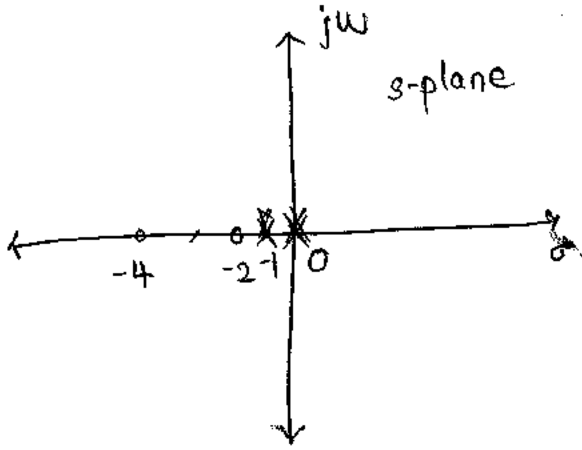


Order = 6
Type = 2

TCF: $GH(s) = \underbrace{\dfrac{\cancel{10}^{2}\times\cancel{2}\times 3}{\cancel{4}\times\cancel{5}\times 5}}_{k} \dfrac{(1+\frac{s}{2})(1+\frac{s}{3})}{s^2(1+s)(1+\frac{s}{4})(1+\frac{s}{5})^2}$

$\boxed{k = 3/5}$

Q:- $T.F. = \dfrac{15(s+2)(s+4)}{s^2(s^2+2s+1)}$. For given OLS (open loop system). Find system gain, type & order of system



Type: Total no. of poles at origin

Type = 2

Order = Total no. of poles in system

Order = 4

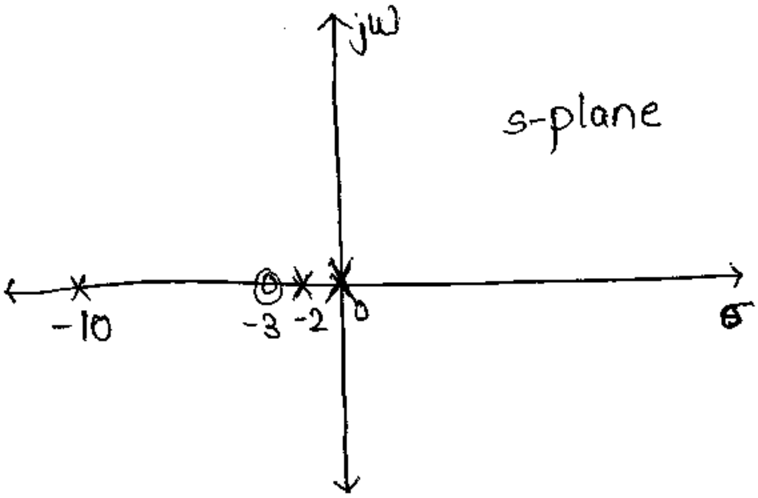
TCF: $GH(s) = \dfrac{15\times 2\times 4\,(1+\frac{s}{2})(1+\frac{s}{4})}{s^2(s+1)^2}$

(time constant form)

$\boxed{K = 120}$

Q:- For given OLCS find order, type & gain

$$\frac{C(s)}{R(s)} = \frac{10(s+3)^2}{s^3(s+2)^2(s+10)}$$



Order = 6

Type = 3

$$k = \frac{\cancel{10} \times 3 \times 3}{2 \times 2 \times \cancel{10}} \; \frac{(1+\frac{s}{3})^2}{s^3(1+\frac{s}{2})^2(1+\frac{s}{10})}$$

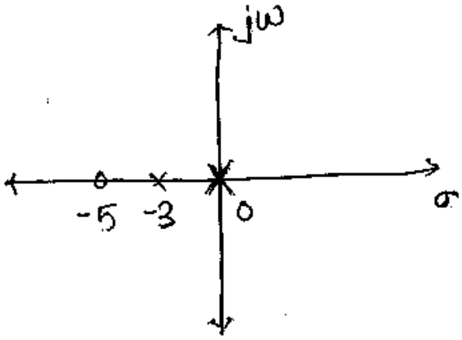
$$\boxed{k = \frac{9}{4}}$$

Q:- $$\frac{C(s)}{R(s)} = \frac{20(s+5)(s+2)}{s^2(s^3+5s^2+6s)}$$

$$\frac{C(s)}{R(s)} = \frac{20(s+5)(s+2)}{s^2 \cdot s\,(s^2+5s+6)} = \frac{20(s+5)(s+2)}{s^3(s^2+5s+6)} = \frac{20(s+5)\cancel{(s+2)}}{s^3\cancel{(s+2)}(s+3)}$$

Type : 3

Order: 4



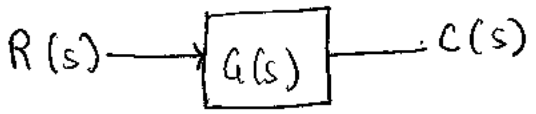
$$k = \frac{20 \times 5 \times \cancel{2}\,[1+\frac{s}{5}]\,\cancel{[1+\frac{s}{2}]}}{s^3\,\cancel{[1+\frac{s}{2}]}\,[1+\frac{s}{3}] \cdot \cancel{2} \times 3}$$

$$\boxed{k = \frac{100}{3}}$$

Any system is defined in form of open loop transfer fcn. (OLTF)

CONTROL SYSTEM

Open loop control system



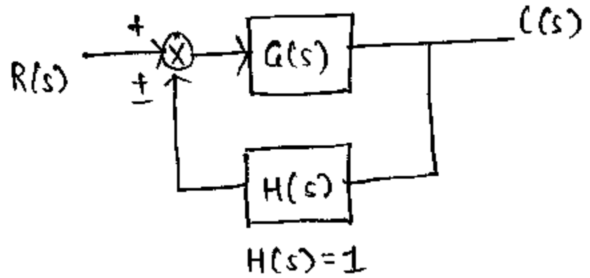
$$OLTF = \frac{C(s)}{R(s)} = G(s)$$

Closed loop control system

O.L.C.S. Gain is $G(s)H(s)$

Unity feedback system

$H(s) = 1$

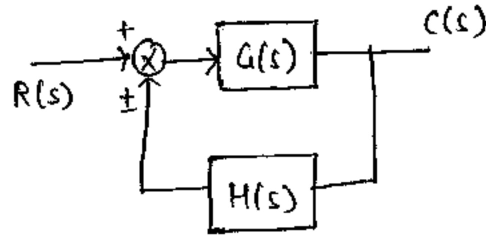


OLTF of UFCS $= G(s)$

$$C.L.T.F. = \frac{C(s)}{R(s)} = G(s)$$

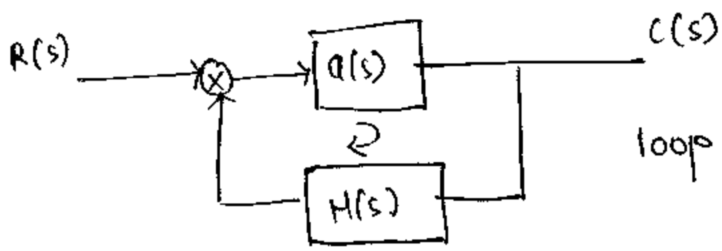
Non-unity feedback sys.

$H(s) \neq 1$



OLTF of NUFCS $= G(s)H(s)$

$$\frac{C(s)}{R(s)} = C.L.T.F. = \frac{G(s)}{1 \pm G(s)H(s)}$$



loop gain $= G(s)H(s)$

Q:- Find type and order for CLTF of Unity gain feedback system.

$$\frac{C(s)}{R(s)} = \frac{2s+5}{s^5+s^4+6s^3+7s^2+2s+5}$$

⇒ Here CLTF is given

$$CLTF = \frac{G}{1+G} \quad \text{——— (1)}$$

But for order & type we require OLTF of UFS i.e. $G(s)$

$$OLTF = G(s)$$

Now, from eqn (1)

$$(2s+5)(1+G) = (s^5+s^4+6s^3+7s^2+2s+5)\,G$$

$$2s+5 + (2s+5)\,G = (s^5+s^4+6s^3+7s^2+2s+5)\,G$$

$$G\left(s^5+s^4+6s^3+7s^2+2s+5-(2s+5)\right) = 2s+5$$

$$G = \frac{2s+5}{s^5+s^4+6s^3+7s^2+2s+5-(2s+5)}$$

$$G = \frac{2s+5}{s^5+s^4+6s^3+7s^2} = \frac{2s+5}{s^2[s^3+s^2+6s+7]}$$

Type: 2

Order: 5

NOTE:

CLTF of UFS : $\frac{N(s)}{D(s)}$

$\downarrow$ Subtract N(s) from D(s)

OLTF of UFS : $\frac{N(s)}{D(s)-N(s)}$

$\downarrow$ Add N(s) into D(s)

CLTF of UFS : $\frac{N(s)}{D(s)-N(s)+N(s)}$

* Review of Laplace Transform

Only for causal system

$t > 0$

$u(t)$

t $\qquad\qquad$ s

① $x(t) \longleftrightarrow X[s] = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$

② $L[\delta(t)] \longleftrightarrow 1$

③ $L[u(t)] \longleftrightarrow 1/s$

$L[t^0 u[t]]$

④ $L[r(t)] \longleftrightarrow 1/s^2$

$L[t\, u(t)]$

⑤ $L[t^n u(t)] \longleftrightarrow \frac{n!}{s^{n+1}}$

⑥ $L[e^{-at}u(t)] \longleftrightarrow \frac{1}{s+a}$

⑦ $L[e^{at}u(t)] \longleftrightarrow \frac{1}{s-a}$

⑧ $L[e^{-at}x(t)] \longleftrightarrow X[s+a]$

⑨ $L[e^{at}x(t)] \longleftrightarrow X[s-a]$

⑩ $L[\sin bt] \longleftrightarrow \frac{b}{s^2+b^2}$

⑪ $L[\cos bt] \longleftrightarrow \frac{s}{s^2+b^2}$

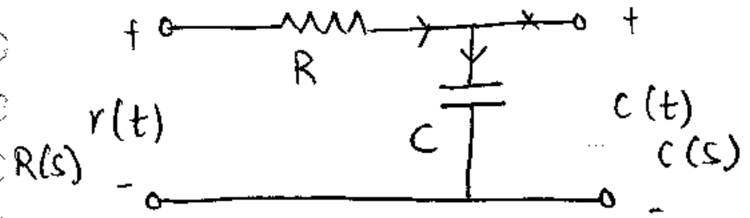
⑫ $L[e^{-at}\sin bt] \longleftrightarrow \frac{b}{(s+a)^2+b^2}$

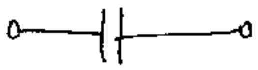
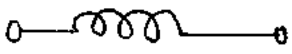
⑬ $L[e^{-at}\cos bt] \longleftrightarrow \frac{s+a}{(s+a)^2+b^2}$

⑭ $L[t \cdot e^{-at}u(t)] \longleftrightarrow \frac{1}{(s+a)^2}$

* Time constant of the system (IMP)

Consider given series RC circuit



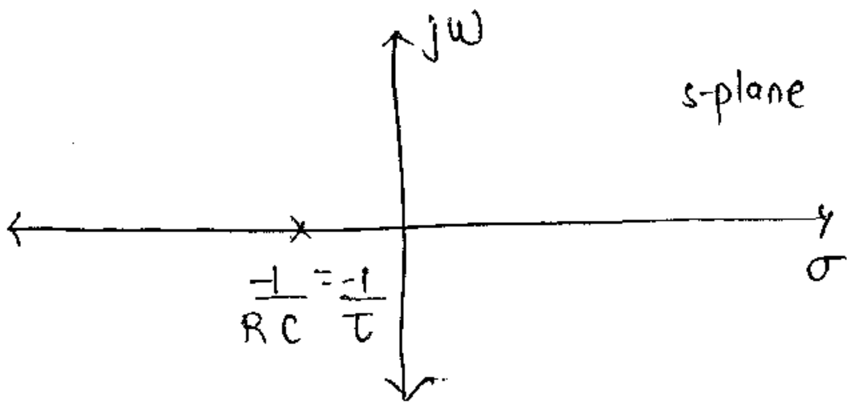
Elements	t	s
	$R\ (\Omega)$	$R\ (\Omega)$
	$C\ (F)$	$\frac{1}{Cs}\ (\Omega)$
	$L\ (H)$	$Ls\ (\Omega)$

$$T.F. = \left.\frac{L[o/p]}{L[i/p]}\right|_{I.C.=0}$$

$$C(s) = \frac{1/sC}{R + 1/sC} \cdot R(s)$$

$$T.F. = \frac{C(s)}{R(s)} = \frac{1}{1+sCR} = \frac{1}{s\tau+1} = \frac{1}{\tau\left(\frac{1}{\tau}+s\right)}$$

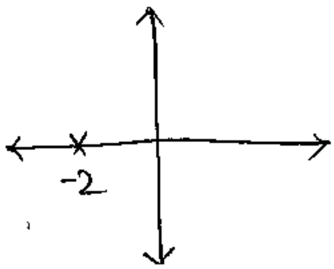
Let $\tau = RC$



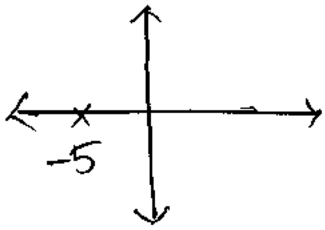
$$\boxed{\tau = \frac{-1}{\text{Real part of dominant pole}}}$$

Ex:- $\frac{1}{s+2} = \frac{1}{2\left(1+\frac{s}{2}\right)} = \frac{1}{(\tau s+1)}$

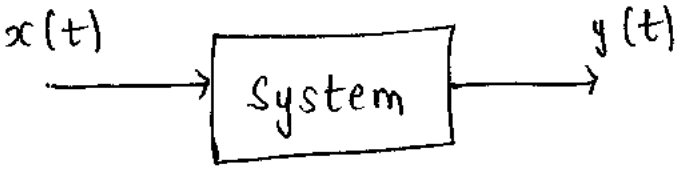
$\tau = 1/2$



Ex:- $\frac{1}{s+5}$ $\quad \tau = \frac{-1}{-5} = \frac{1}{5}$



$$C(s) = \frac{1}{1+s\tau} R(s)$$



Impulse $\longrightarrow$ Impulse response

Unit step $\longrightarrow$ Unit step response

Ramp $\longrightarrow$ Ramp response

Parabola $\longrightarrow$ Parabola response

* Impulse Response

$$r(t) = \delta(t)$$

$$R(s) = 1$$

$$C(s) = \frac{1}{1+s\tau} = \frac{1}{\tau}\cdot\frac{1}{(s+1/\tau)}$$

$$\downarrow$$

$$c(t) = \frac{1}{\tau}\left(e^{-t/\tau}\right) \longrightarrow \tau = RC = \text{time constant}$$

* Time constant

Time constant is defined only for stable system

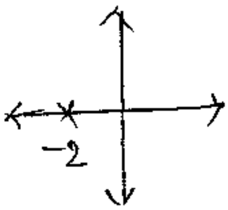
t s

$e^{-t/\tau}$ s-plane

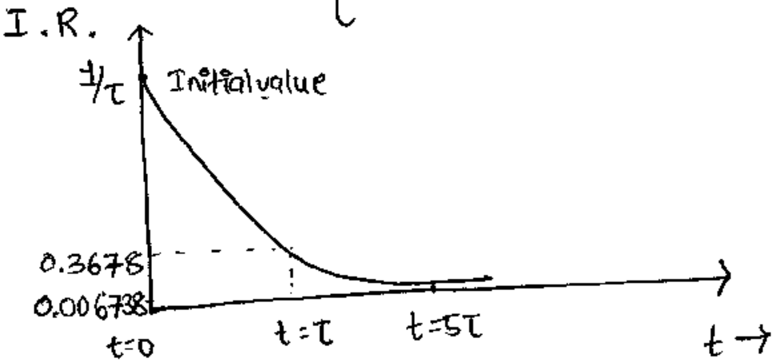
$$\tau = -\frac{1}{\text{Real part of dominant pole}}$$

$e^{-2t} \xrightarrow{\text{L.T.}} \frac{1}{s+2}$

$$\tau = \frac{+1}{+2} = \frac{1}{2}$$



Now, $c(t) = \frac{1}{\tau} e^{-t/\tau} \cdot u(t)$

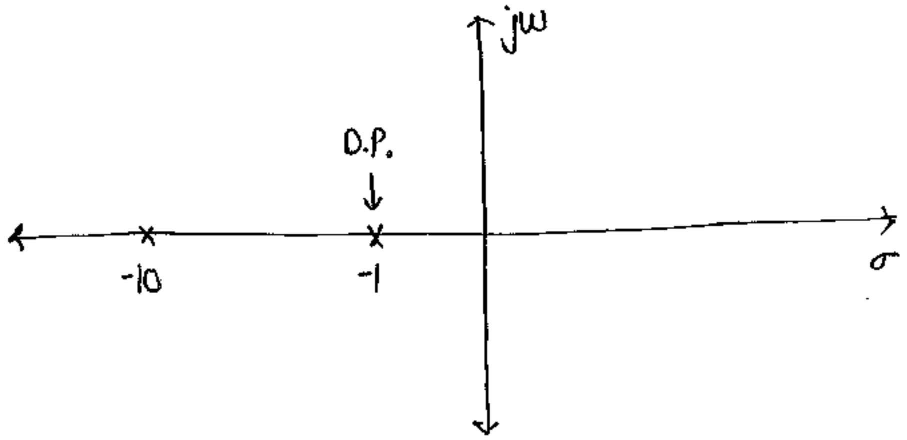


$$c(t)\big|_{t=\tau} = \frac{1}{\tau} e^{-1} = 0.3678 \cdot \frac{1}{\tau}$$

$$c(t)\big|_{t=5\tau} = \frac{1}{\tau} e^{-5} = 0.006738 \cong 0$$

* Dominant pole and Insignificant pole

$$GH(s) = \frac{1}{(s+1)(s+10)} = \frac{C(s)}{R(s)}$$



- The pole which is very close to imaginary axis is called as dominant pole.

- Dominant pole has large time constant. So it is not good pole. It affects the system so we have to compensate with by adding zero at the same location.

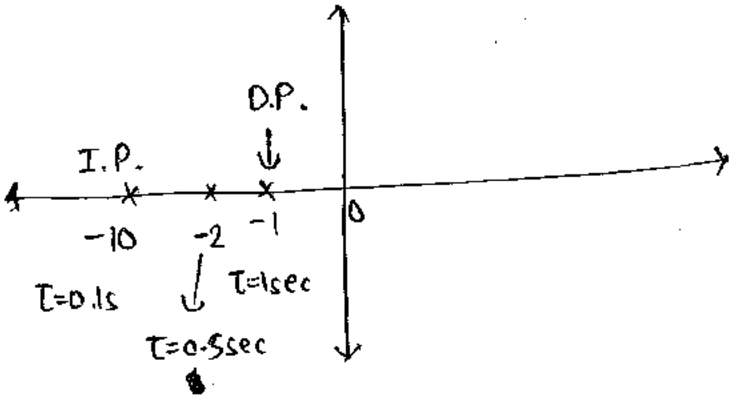
Insignificant pole :- The poles which lies on left hand most side in s-plane is nothing but insignificant pole.

- Insignificant pole has less time constant. So good performance and hence best pole.

$$\boxed{\tau_{D.P.} \geq 5\,\tau_{I.P.}}$$

$$\boxed{\text{I.P. Location} \leq 5\ (\text{Dominant Pole location})}$$

$$GH(s) = \frac{1}{(s+1)(s+10)} = \frac{C(s)}{R(s)}$$



$$\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)}$$

for I.R., $r(t) = \delta(t)$

$$R(s) = 1$$

$$C(s) = \frac{1}{(s+1)(s+10)}$$

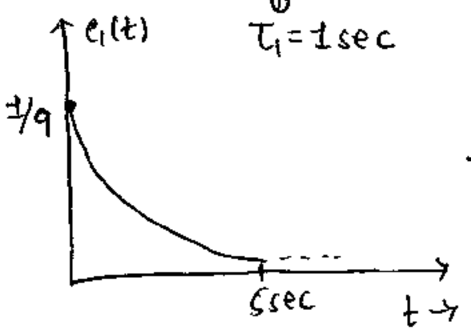
$$C(s) = \frac{A}{s+1} + \frac{B}{s+10}$$

$$= \left(\frac{1}{s+1} - \frac{1}{(s+10)}\right)\frac{1}{9}$$

$$c(t) = \left[\frac{1}{9}e^{-t} - \frac{1}{9}e^{-10t}\right]u(t)$$

$$c(t) = \frac{1}{9}e^{-t}u(t) - \frac{1}{9}e^{-10t}u(t)$$

$\tau_1 = 1\text{sec}$ $\quad\quad$ $\tau_2 = 0.1\text{sec}.$



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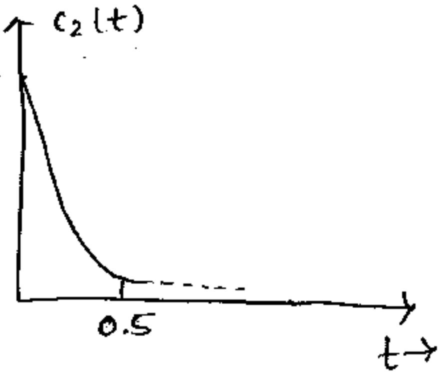
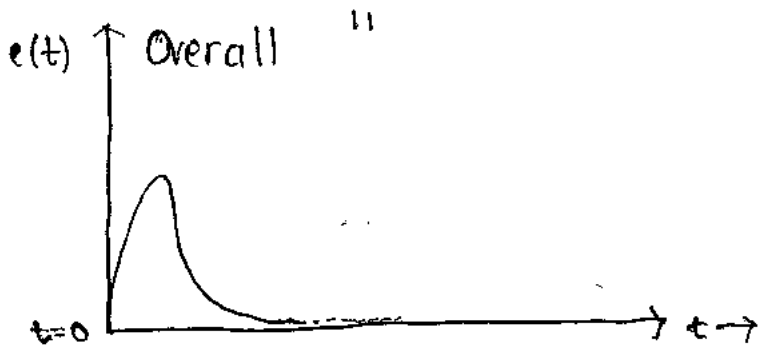




- The best pole is insignificant poles because it gives very quick response and more relatively stable because of the dominant pole pole system response becomes slow and system becomes less relatively stable.

- Insignificant pole (I.P.) is neglected because even if I.P. are neglected. There is no much change in system response.

- We can neglect I.P. only if OLTF and eq^n should be in T.C.F. (time constant form)

$$GH(s) = \frac{1}{(s+1)(s+10)}$$

$$GH(s) = \frac{1}{10\underbrace{(s+1)}_{D.P.}\underbrace{\left(1+\frac{s}{10}\right)}_{I.P.}}$$

Neglecting I.P.

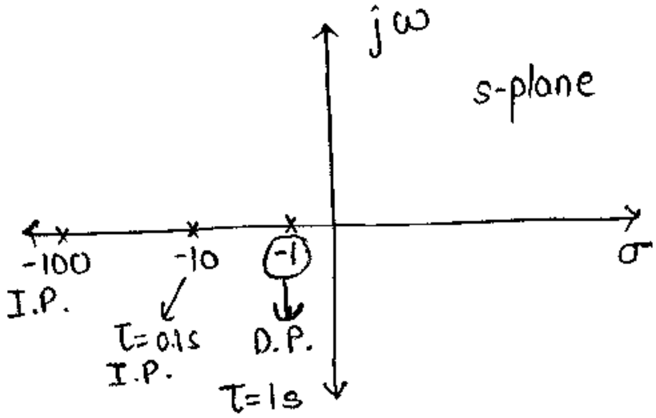
$$\boxed{GH(s) = \frac{0.1}{s+1}}$$

NOTE:- The I.P. must be neglected only in T.C.F. → time constant form

Q:- Find the eq^n transfer function of open loop system.

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)(s+100)}$$

(A) $\frac{1}{s+1}$ (B) $\frac{1}{(s+1)(s+10)}$ (C) $\frac{0.01}{(s+1)(s+10)}$ (d) $\frac{0.001}{(s+1)}$



$\tau_{D.P.} \geq 5\tau_{I.P.}$

$$\therefore \frac{C(s)}{R(s)} = \frac{1}{(s+1)\cdot 10\left(1+\frac{s}{10}\right)(100)\left(1+\frac{s}{100}\right)}$$

$$\frac{C(s)}{R(s)} = \frac{0.001}{s+1} \qquad \underline{(D)}$$

2nd definition of transfer function:

It is laplace transform of impulse response by considering all initial conditions are to be zero.

$r(t) \longrightarrow$ System $\longrightarrow c(t)$

Impulse   Impulse response

$r(t) = \delta(t)$

$R(s) = 1$

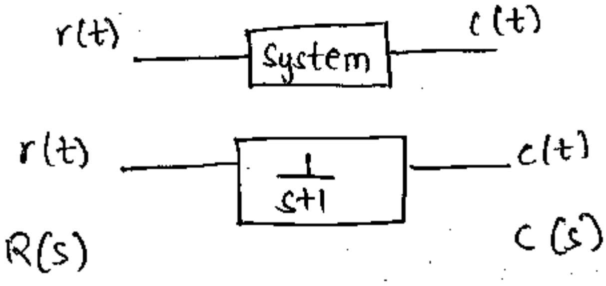
Def :- ① $\boxed{\text{T.F.} = \left.\frac{L[\text{o/p}]}{L[\text{i/p}]}\right|_{I.C.=0}}$

② $$\text{T.F.} = \left.\frac{L[\text{Impulse response}]}{L[\text{Impulse}]}\right|_{I.C.=0}$$

$$\boxed{\text{T.F.} = L[\text{Impulse response}]\big|_{I.C.=0}}$$

Impulse response gives system behaviour or characteristics of system bcz. impulse response consists only system parameters, no input term is present in impulse response

Hence impulse response is called system response or natural response OR free forced response



(1) Impulse response

$r(t) = \delta(t)$

$R(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1)}$$

$$C(s) = \frac{R(s)}{s+1}$$

$$C(s) = \frac{1}{s+1}$$

$$c(t) = e^{-t}u(t) \Rightarrow \text{I.R.}$$

(2) Unit step response

$r(t) = u(t)$

$R(s) = 1/s$

$$C(s) = \frac{R(s)}{s+1}$$

$$C(s) = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

$$c(t) = (1-e^{-t})u(t)$$

$$c(t) = \underbrace{u(t)}_{\text{input is there}} - \underbrace{e^{-t}u(t)}_{\text{USR}}$$

Therefore only I.R. is system response.

* Stability

- If the system response c(t) is finite value as $t \to \infty$ then system is said to be stable.

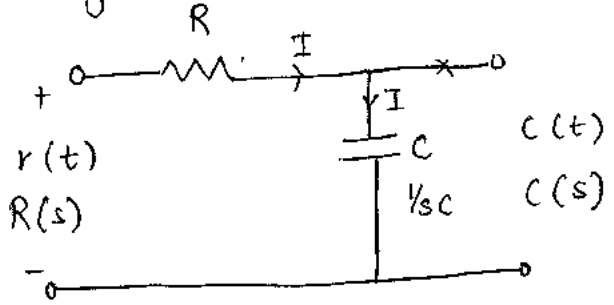
$$c(t)\big|_{t\to\infty} = \text{finite (including zero)} \Rightarrow \text{stable}$$
$$= \text{infinite } (\infty) \Rightarrow \text{unstable}$$

If the system response $c(t)$ is infinite value as $t \to \infty$ then system is said to be unstable.

If $c(t)\big|_{t\to\infty}$ = finite = stable

System Response = Impulse response

Q:- For the given RC circuit. Find the T.F. locate poles & zeros in s-plane. Find system response, and check system stability



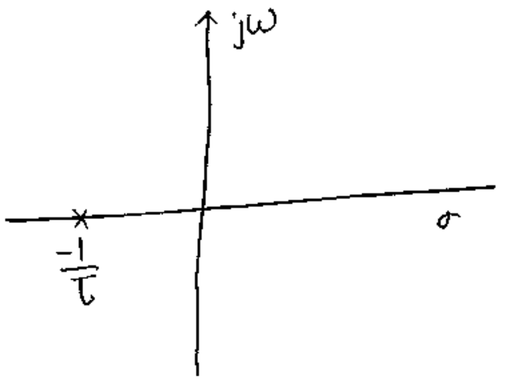
$$T.F. = \frac{L[o/p]}{L[i/p]}\bigg|_{I.C.=0}$$

$$C(s) = \frac{1/sC}{R + 1/sC} \cdot R(s)$$

$$T.F. = \frac{C(s)}{R(s)} = \frac{1}{1+sCR} = \frac{1}{1+s\tau}$$

$\tau = RC$ (sec.)

Pole: $s = -1/\tau$



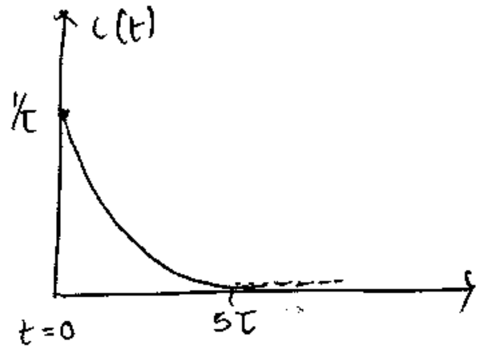
for system response

$r(t) = \delta(t)$

$R(s) = 1$

$$C(s) = \frac{1}{1+s\tau} = \frac{1}{\tau(s+\frac{1}{\tau})}$$

$$c(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$$



$$c(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$$

$$c(t)\big|_{t\to\infty} = \frac{1}{\tau} e^{-\infty} u(t) = \frac{1}{\tau} \cdot \frac{1}{e^{\infty}} u(t)$$

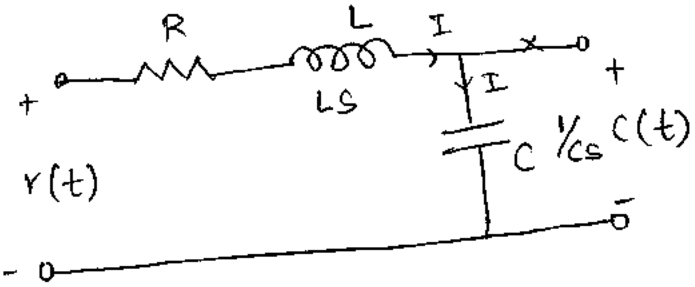
$= 0 \Rightarrow$ finite

$\boxed{\text{STABLE}}$

Q:- For the series RLC circuit. Find overall transfer function. Find system response, if $R = 0$, $L = 1H$ and $C = 1F$. Then check system stability locate poles & zeros in s-plane and also calculate system time constant

(b) If $R = 1\Omega$, $L = 1H$ & $C = 1F$ then find system response, T.F., locate poles & zeros, system time constant & stability.

Sol^n:-

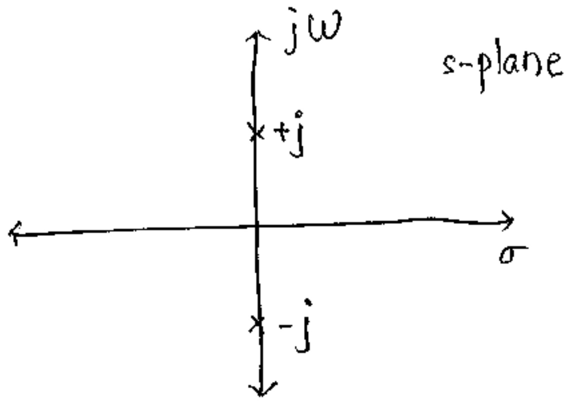


$$c(s) = \frac{1/sC}{R + Ls + 1/Cs} R(s)$$

$$\frac{C(s)}{R(s)} = \frac{1}{s^2 LC + RCs + 1}$$

$$\frac{C(s)}{R(s)} = \frac{1/LC}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

(a) If $R = 0$, $L = 1H$ & $C = 1F$



$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + 1}$$

Poles: $\pm j$

Time constant: $\frac{-1}{\text{Real part of dominant pole}} = \frac{-1}{0} = \infty$

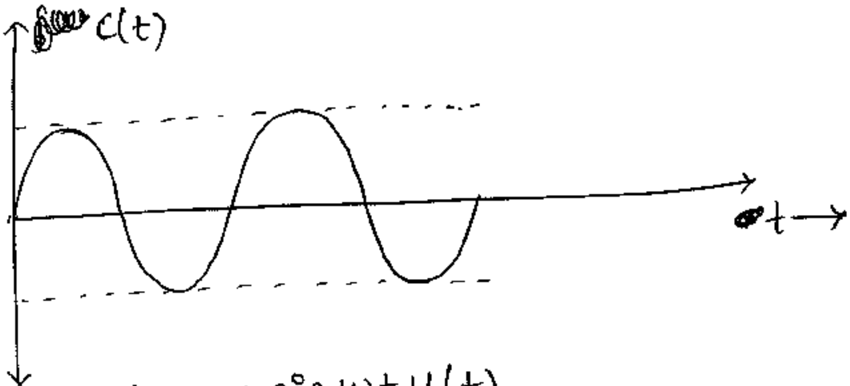
$$\therefore C(s) = \frac{1}{s^2 + 1} R(s)$$

for system response $\Rightarrow R(s) = 1$

$$C(s) = \frac{1}{s^2 + 1} \qquad \therefore c(t) = \sin t \cdot u(t)$$

Comparing it with $c(t) = \sin\omega t\, u(t)$

$\omega = 1$ rad/sec.



$c(t)\Big|_{t\to\infty} = \sin\omega t\, u(t)$

= value will be finite $[-1,1]$

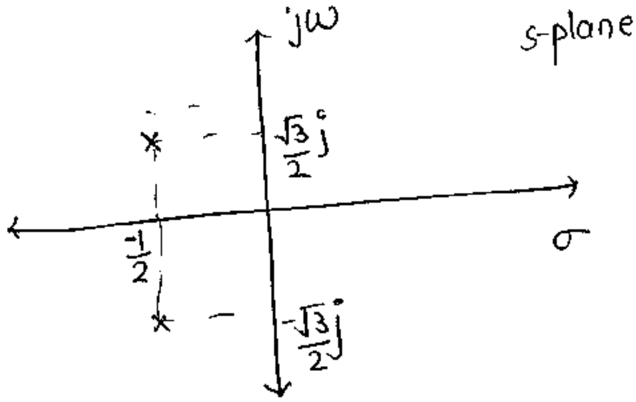
= stable

(b) $R = 1\,\Omega$, $L = 1\,H$ & $C = 1\,F$

$$\frac{C(s)}{R(s)} = \frac{1}{s^2+s+1}$$

$$\frac{C(s)}{R(s)} = \frac{1}{s^2+s+1}$$

Poles :- $\dfrac{-1 \pm \sqrt{3}j}{2}$



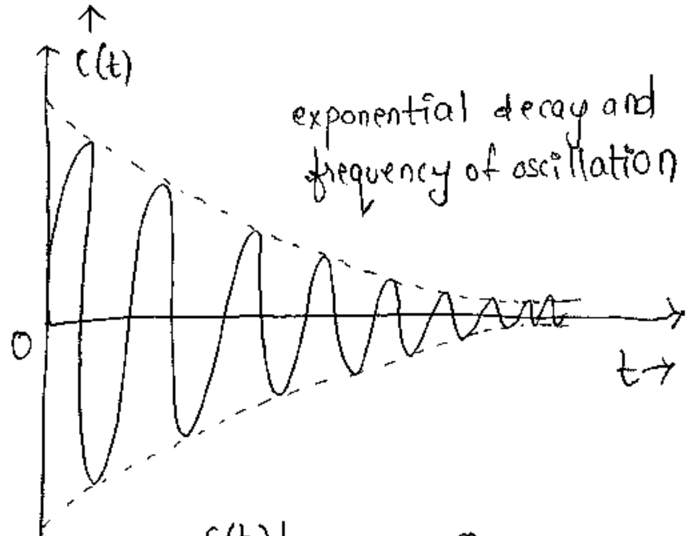
Non-repeated complex conjugate poles on L.H.S.

Time constant :- $\dfrac{-1}{1/2} = 2$ sec.

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1/2)^2 + (\sqrt{3}/2)^2}$$

$$c(t) = \frac{2}{\sqrt{3}}\underbrace{e^{-t/2}}_{\text{exp. decay}}\underbrace{\sin\frac{\sqrt{3}}{2}t}_{\text{sinusoidal function}} \cdot u(t)$$

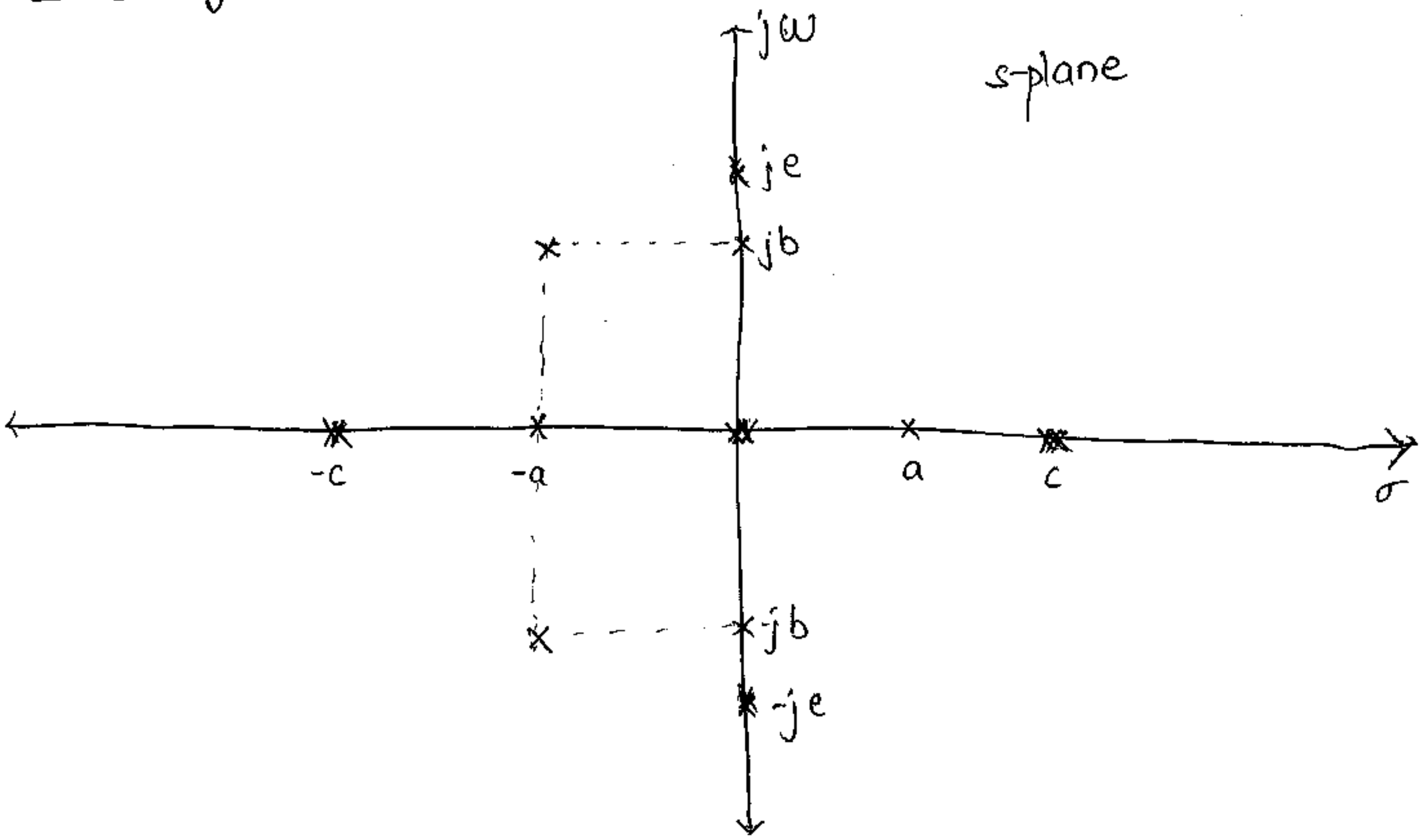
$w = \sqrt{3}/2$ rad/sec.



$c(t)\Big|_{t\to\infty} = 0$
= finite

stable

*Stability



① $\dfrac{1}{s+a}$

② $\dfrac{1}{s-a}$

③ $\dfrac{1}{(s+c)^2}$

④ $\dfrac{1}{(s-c)^2}$

⑤ $\dfrac{1}{s}$

⑥ $\dfrac{1}{s^2}$

⑦ $\dfrac{1}{(s+a)^2+b^2}$

⑧ $\dfrac{1}{(s-a)^2+b^2}$

⑨ $\dfrac{1}{s^2+b^2}$

⑩ $\dfrac{1}{(s^2+e^2)^2}$

⑪ $\dfrac{1}{(s+a)(s+b)}$

⑫ $\dfrac{1}{(s-a)(s-b)}$

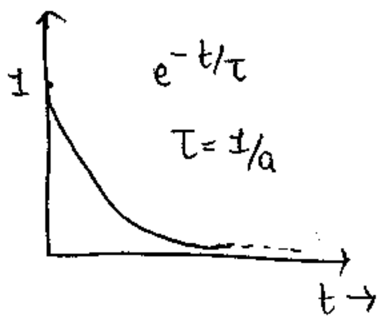
Non-repeated pole or simple real pole:

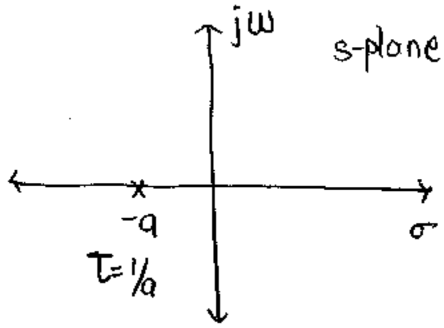
(1) $\frac{C(s)}{R(s)} = \frac{1}{s+a}$

For system response,

$R(s) = 1 \quad , \quad C(s) = \frac{1}{s+a}$

$c(t) = e^{-at} \cdot u(t)$





w = No imaginary pole = 0

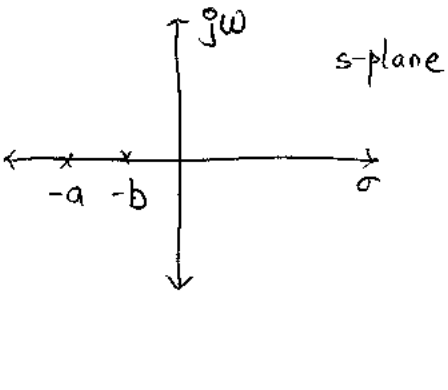
Stability

$c(t)|_{t \to \infty} = e^{-at} \cdot u(t) = 0$ = finite - stable

(2) $\frac{C(s)}{R(s)} = \frac{1}{(s+a)(s+b)}$; $a > b$

For system response,

$R(s) = 1 \quad ; \quad C(s) = \frac{1}{(s+a)(s+b)}$

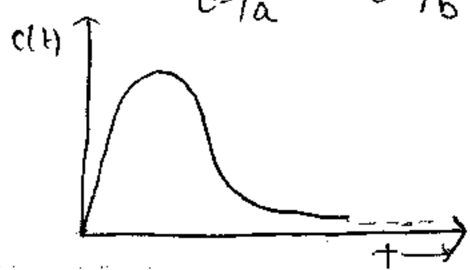


$C(s) = \frac{k_1}{s+a} + \frac{k_2}{s+b}$

$c(t) = (k_1 e^{-at} + k_2 e^{-bt}) u(t)$

$\downarrow \qquad \downarrow$

$\tau = 1/a \qquad \tau = 1/b$



Stability

$c(t)|_{t \to \infty} = (k_1 e^{-at} + k_2 e^{-bt}) u(t)$

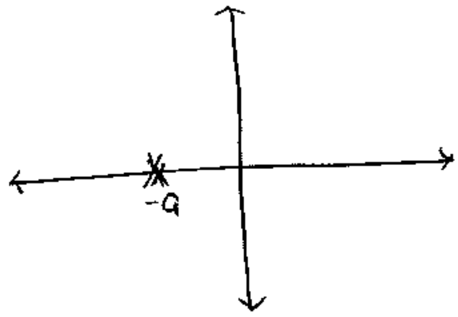
= Finite $\Rightarrow$ stable

Repeated real poles:-

(1) $\frac{C(s)}{R(s)} = \frac{1}{(s+a)^2}$

For system response,

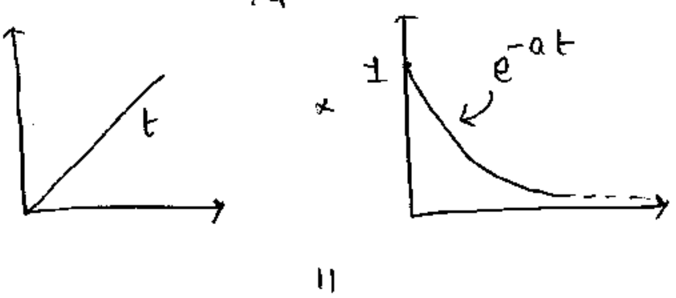
$$C(s) = \frac{1}{(s+a)^2}$$

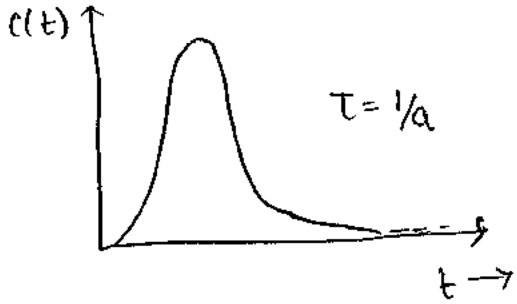


$$c(t) = t \cdot e^{-at} u(t)$$

$\downarrow$

$\tau = 1/a$





Stability:-

$$c(t)\Big|_{t\to\infty} = t \cdot e^{-at} \cdot u(t)$$

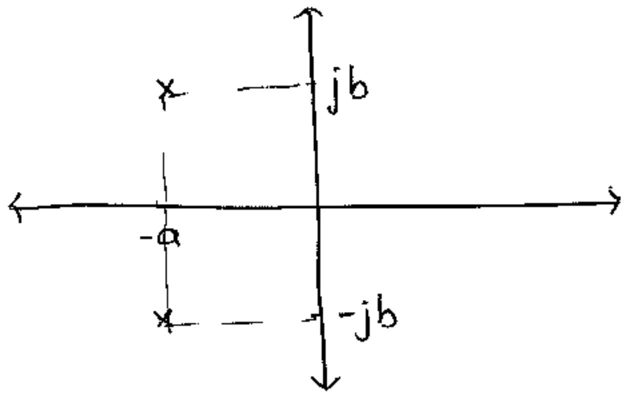
$$= \infty \cdot 0 \, u(t)$$

$$= 0$$

$$= \text{finite} \Rightarrow \text{stable}$$

Repeated complex conjugate pole:-

$$\frac{C(s)}{R(s)} = \frac{1}{(s+a)^2+b^2}$$



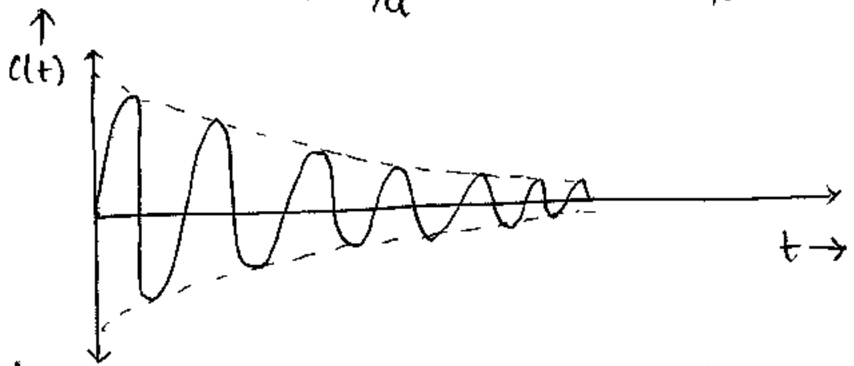
For system response,

$R(s) = 1$

$$C(s) = \frac{1}{b} \cdot \frac{b}{(s+a)^2+b^2}$$

$$c(t) = \frac{1}{b} e^{-at} \sin bt \, u(t)$$

$\tau = 1/a$, $\omega = b$ rad/sec.



* stability

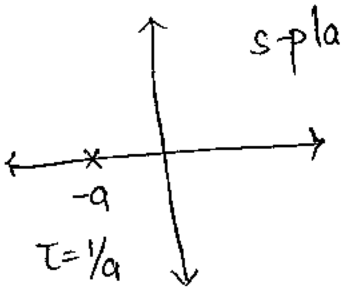
$$c(t)\big|_{t\to\infty} = \frac{1}{b} e^{-\infty} \sin b\infty u($$

$= 0 =$ Finite

stable

*Non-repeated pole OR simple real pole: X

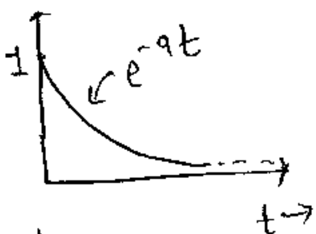
1. $$\frac{C(s)}{R(s)} = \frac{1}{s+a}$$



For system response,

$R(s) = 1$, $C(s) = \frac{1}{s+a}$

$c(t) = e^{-at} u(t)$



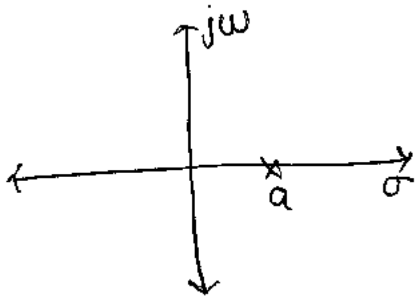
$\omega =$ No imaginary pole $= 0$

* Stability

$c(t)\big|_{t\to\infty} = e^{-\infty} u(t) = 0 =$ Finite stable

Right hand side pole

1. $\frac{C(s)}{R(s)} = \frac{1}{s-a}$



For system response, $R(s)=1$

$C(s) = \frac{1}{s-a}$

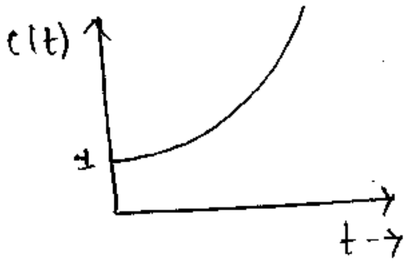
$c(t) = e^{at} \cdot u(t)$

$\omega = 0$ = No imaginary poles

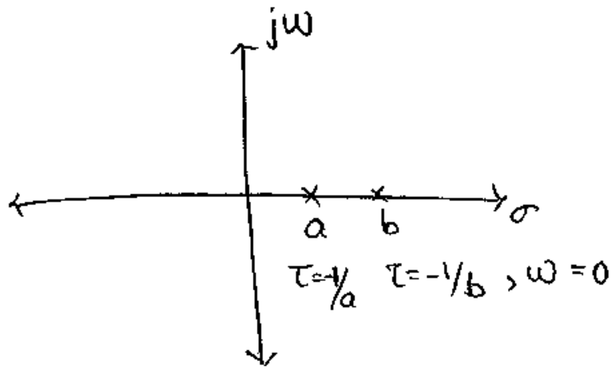
$\tau = \frac{-1}{a}$

* Stability

$c(t)|_{t\to\infty} = e^{\infty} u(\infty) = \infty \Rightarrow$ Not stable



2. $\frac{C(s)}{R(s)} = \frac{1}{(s-a)(s-b)}$



$C(s) = \frac{K_1}{s-a} + \frac{K_2}{s-b}$

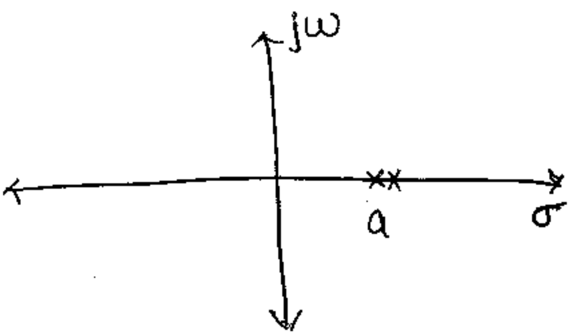
$c(t) = \left(K_1 e^{at} + K_2 e^{bt}\right) u(t)$

* stability

$c(t)|_{t\to\infty} = (K_1 e^{\infty} + K_2 e^{\infty}) u(\infty)$

$= \infty \Rightarrow$ Not stable

3. $C(s) = \frac{1}{(s-a)^2}$



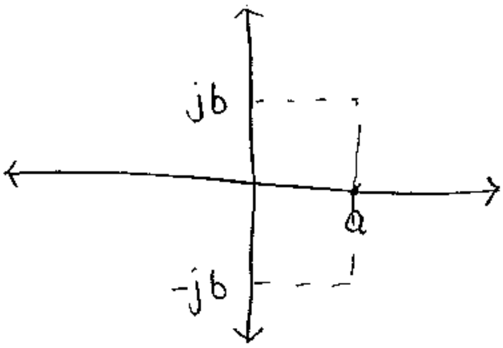
$c(t) = t \cdot e^{at} \cdot u(t)$

$\tau = -1/a$, $\omega = 0$

* stability

$c(t)|_{t\to\infty} = \infty \Rightarrow$ Not stable

4. $\frac{C(s)}{R(s)} = \frac{1}{(s-a)^2+b^2}$

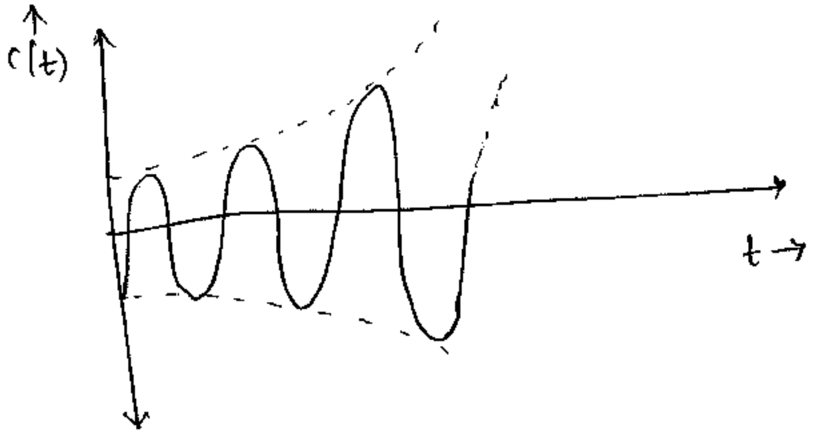


$c(t) = \frac{1}{b} e^{at} \sin bt$

$\tau = -1/a$, $\omega = b$ rad/sec.

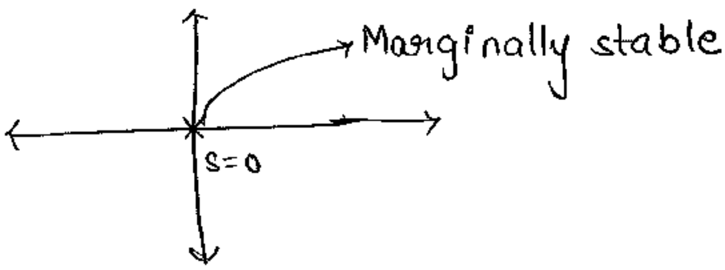
*stability

$c(t)\big|_{t\to\infty} = \infty \Rightarrow$ Not stable



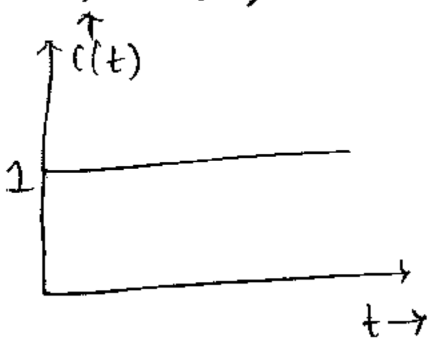
Non-repeated poles at origin

$\frac{C(s)}{R(s)} = \frac{1}{s}$



$C(s) = \frac{1}{s}$

$c(t) = u(t)$



⇒ steady state

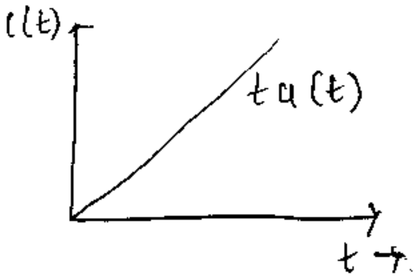
Marginally stable

$\tau = \infty$

$\omega = 0$

Repeated poles at origin

$\frac{C(s)}{R(s)} = \frac{1}{s^2}$

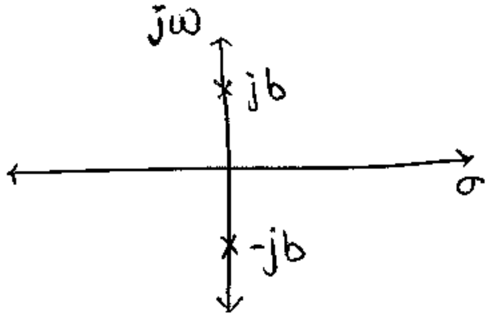


*Stability

$c(t)\big|_{t\to\infty} = \infty \Rightarrow$ Not stable

$c(t) = t\,u(t)$

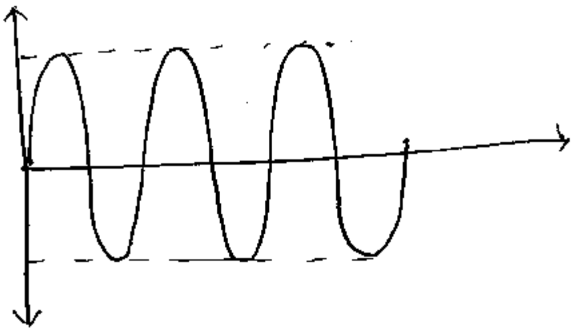
$$\frac{C(s)}{R(s)} = \frac{1}{s^2+b^2}$$



$$c(t) = \frac{1}{b}\sin bt$$

$\tau = \infty$

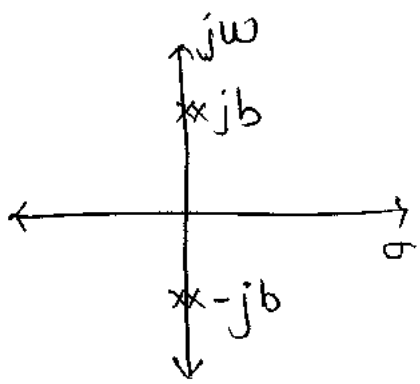
$\omega = b$ rad/sec.



constant amplitude

Marginally stable

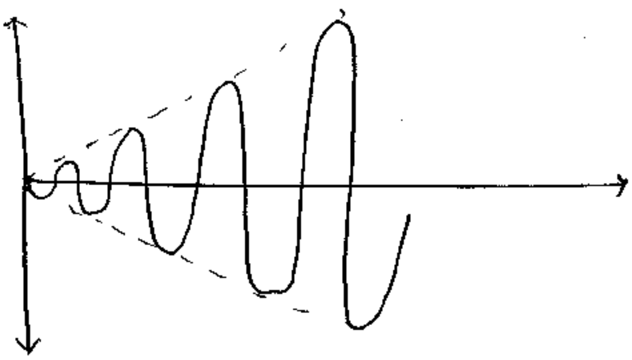
$$\frac{C(s)}{R(s)} = \frac{1}{(s^2+b^2)^2}$$



$$c(t) = \frac{1}{b}\cdot t\sin bt\ u(t)$$

$\tau = \infty$

$\omega = b$ rad/sec.



linearly increasing

Unstable

*Observation from above cases:-

1. Left hand side poles [repeated or non-repeated] system is always STABLE.

2. Right hand side pole [repeated or non-repeated] system is always UNSTABLE.

3. Non-repeated poles at origin, system is MARGINALLY STABLE.

4. Repeated pole at origin, system becomes UNSTABLE

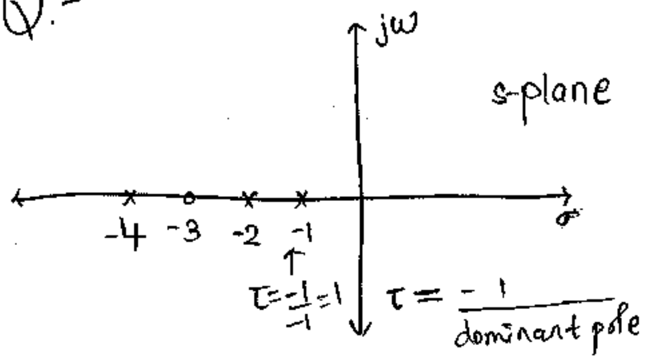
5. Non-repeated pole on imaginary axis, system becomes MARGINALLY UNSTABLE

6. Repeated poles on imaginary axis, system becomes UNSTABLE.

---

s Term		Time Response
1) Real part	$\longrightarrow$	expon. function
2) Img. part	$\longrightarrow$	sine function
3) Real + Imag.	$\longrightarrow$	exp. x sine function
4) Repeated	$\longrightarrow$	Ramp

Q:-



$$\frac{C(s)}{R(s)} = \frac{s+3}{(s+1)(s+2)(s+4)}$$

For system response,

Take $R(s) = 1$

$$C(s) = \frac{s+3}{(s+1)(s+2)(s+4)}$$

$$C(s) = \frac{k_1}{s+1} + \frac{k_2}{s+2} + \frac{k_3}{s+4}$$

$$c(t) = \left(k_1 e^{-t} + k_2 e^{-2t} + k_3 e^{-4t}\right)u(t) = \left(\frac{2}{3}e^{-t} - \frac{1}{2}e^{-2t} - \frac{1}{6}k_3 e^{-4t}\right)u(t)$$

For stability,

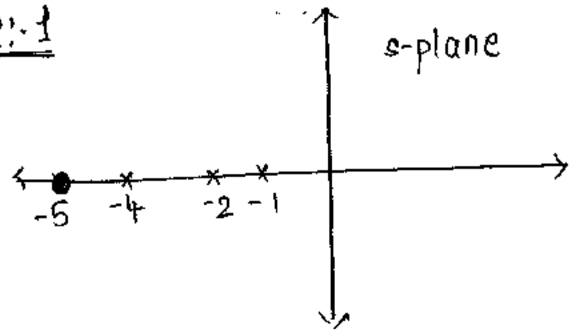
$$c(t)\big|_{t\to\infty} = 0 = \text{finite} \Rightarrow \text{stable}$$

NOTE:-

To find: system response, system time constant, system stability → We are considering any poles but not zeros.

Above note can be concluded by taking following three cases:-

Case:-1



Change zero location to s=-3 to s=-5

$$\frac{C(s)}{R(s)} = \frac{(s+5)}{(s+1)(s+2)(s+4)}$$

For system response

$$R(s) = 1$$

$$C(s) = \frac{(s+5)}{(s+1)(s+2)(s+4)}$$

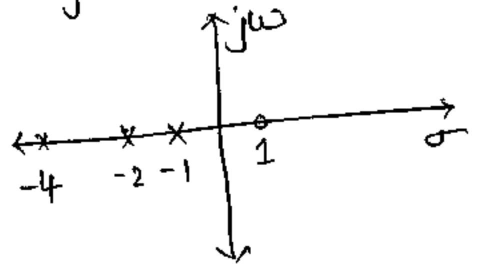
$$C(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$c(t) = (k_1 e^{-t} + k_2 e^{-2t} + k_3 e^{-4t}) u(t)$$

$$c(t)\Big|_{t\to\infty} = 0 \Rightarrow \text{finite}$$ stable

Case:-2

Change zero from -3 to $+1$



$$\frac{C(s)}{R(s)} = \frac{(s-1)}{(s+1)(s+2)(s+4)}$$

$R(s) = 1$

$$\therefore C(s) = \frac{s-1}{(s+1)(s+2)(s+4)}$$

$$C(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+4}$$

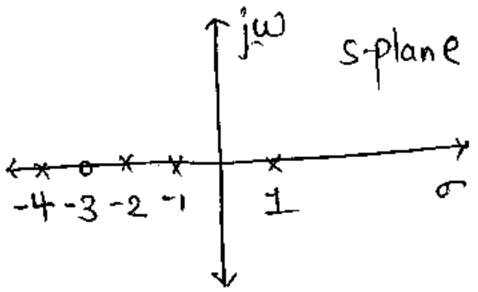
$$\therefore c(t) = (k_1 e^{-t} + k_2 e^{-2t} + k_3 e^{-4t}) u(t)$$

* For stability

$c(t)|_{t\to\infty} = 0 \Rightarrow$ finite

stable

Case:-3 Add one single pole at $s = 1$



$$\frac{C(s)}{R(s)} = \frac{s+3}{(s+1)(s+2)(s+4)(s-1)}$$

$R(s) = 1$

$$C(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+4} + \frac{D}{s-1}$$

$$c(t) = (k_1 e^{-t} + k_2 e^{-2t} + k_3 e^{-4t} + k_4 e^{t}) u(t)$$

* Stability

$c(t)|_{t\to\infty} = \infty \Rightarrow$ unstable

$\tau =$ not defined

While finding system response, time constant & stabili
we are considering only poles but not zeros
because system response consists only poles response
term. There is no zeros response term exists
in the system response. $\boxed{\text{NOTE}}$

Addition of zeros in s-plane [at any location], will not affect system response, system time constant and system stability, it will change only amplitude of response.

Addition of poles in s-plane will affect system stability, system response and system time constant
If addition of poles in L.H.S. of s-plane then system becomes stable.

But if addition of poles is R.H.S. it will make system unstable.

* Characteristics eqⁿ of system

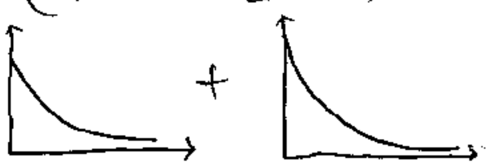
Ex:- $\dfrac{C(s)}{R(s)} = \dfrac{s-10}{(s+1)(s+5)}$

For system response:-

$R(s) = 1$

$$C(s) = \frac{s-10}{(s+1)(s+5)}$$

$$c(t) = \left(k_1 e^{-t} + k_2 e^{-5t}\right) u(t)$$

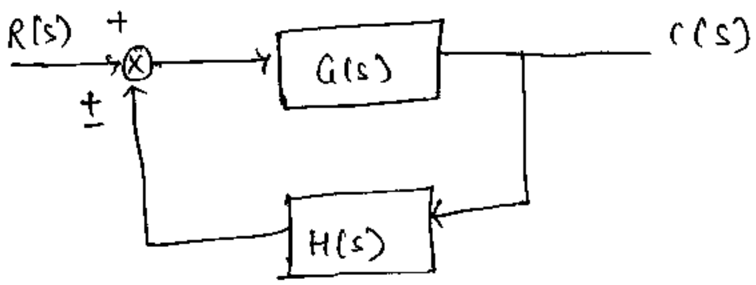


- Denominator terms decides the characteristics of system not numerator terms so for characteristics eq^n we are considering denominator term = 0.

- The denominator of the transfer function makes = 0 then it is called <u>characteristic eq^n</u>.

- Characteristics eq^n gives system behaviour or characteristics of system.

- Consider closed loop control system,



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)} = \frac{\text{C.L. zeros}}{\text{C.L. poles}}$$

$\boxed{1 + G(s)H(s) = 0} \longrightarrow$ Roots of C.E. = C.L. poles
closed loop

characteristics eq^n of C.L.T.F.

O.L.T.F. of CLCS = $GH(s) = \frac{KN(s)}{D(s)} = K \frac{\text{OL zeros}}{\text{OL poles}}$

$\boxed{1 \pm GH(s) = 0} \rightarrow$ CL poles

Considering -ve feedback .

$1 + GH(s) = 0$

$\boxed{1 + K \frac{\text{OL zeros}}{\text{OL poles}} = 0} \longrightarrow$ C.L. poles

$$\boxed{\text{O.L. poles} + k\cdot\text{O.L. zeros} = 0} \rightarrow \text{C.L. poles}$$

Open loop system

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

$$\text{O.L.T.F.} = \frac{C(s)}{R(s)} = G(s) = \frac{\text{OL zeros}}{\text{OL poles}}$$

NOTE:-

- O.L. zeros will not affect open loop system stability
- O.L. poles will affect open loop system stability
- C.L. zeros will not affect closed loop sys. stability
- C.L. poles will affect closed loop sys. stability.
- O.L. zeros will affect closed loop system stability.
- O.L. poles will affect closed loop system stability

IMP points to be remembered:-

1. In the system response exponential terms are real part of the poles

2. Sine or cosine function are pore imaginary part of the pole

3. t - terms in the response represent repeated nature of the pole.

4. To get the time ~~response~~ constant for the response compare the response with $e^{-t/\tau}$.

5. The system time constant is nothing but dominant pole time constant and it should have largest value.

6. If one or more poles lies in the left of s-plane at different location then the system response is exponential decay irrespective of position of zeros and system is stable.

7. When poles lies on imaginary axis which are non-repeated then system response is constant amplitude & constant frequency of oscillation.

8. Any system which produces constant amplitude and frequency of oscillation then it is called marginally stable system or undamped system.

9. In complex conjugate poles, the real part will give time constant and imaginary part will give frequency of oscillation.

10. Whenever the poles are complex conjugate in the left of s-plane then the system response is exponential decay and frequency of oscillation, which are called damped oscillation.

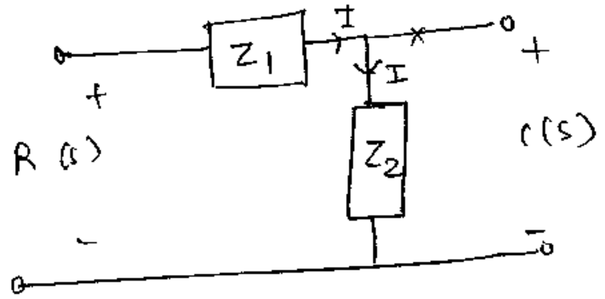
11. Any system which produces damped oscillation is stable system.

12. Whenever more than one poles lies in Right of s-plane at any location at different location on real axis then the system responce is exponential rise to infinity & system becomes unstable.

13. Whenever the poles are complex conjugate on right of s-plane then the system response is exponential increasing & frequency of oscillation and system becomes unstable.

* Transfer function of electorical networks:-

Q:- Find transfer function to given electrical network



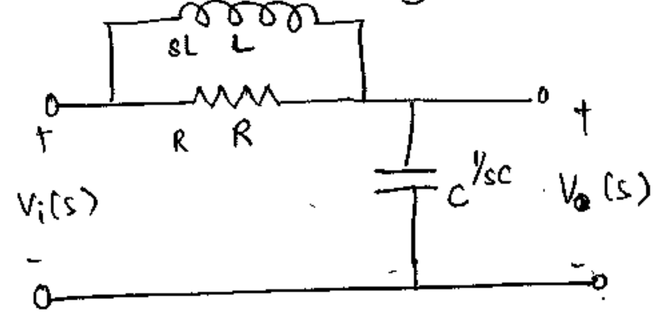
$$C(s) = \frac{Z_2}{Z_1+Z_2} R(s)$$

$$T.F. = \left.\frac{L[o/p]}{L[i/p]}\right|_{I.C.=0}$$

$$T.F. = \frac{C(s)}{R(s)} = \frac{V_o(s)}{V_i(s)} = \frac{Z_2}{Z_1+Z_2}$$

$R \rightarrow Z = R\ \Omega$
$L \rightarrow Z = sL\ \Omega$
$C \rightarrow Z = 1/sC\ \Omega$

Q:- Find T.F. to given ckt:-



$Z_1 = R \parallel L$

$Z_1 = \dfrac{R \cdot sL}{R+sL}$ & $Z_2 = \dfrac{1}{sC}$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{Z_2}{Z_1+Z_2} = \cancel{\frac{1\,(R+sL)}{sC\,(R\cdot sL)}} \quad \frac{1/sC}{1/sC + \frac{R\cdot sL}{R+sL}}$$

$$= \cancel{\frac{R+sL}{s^2 RCL}} \quad \frac{R+sL}{R+sL+s^2RLC}$$

$$= \frac{R+sL}{s^2RLC+sL+R}$$

$$s^2RLC+sL+R = 0$$

$$s = \frac{-L \pm \sqrt{L^2-4R^2LC}}{2RLC}$$

$$s = \frac{-L \pm \sqrt{L^2-4R^2LC}}{2RLC}$$ = complex conjugate pole

For DC gain

$\downarrow$

$f=0$

$\downarrow$

$\omega = 2\pi f = 0$

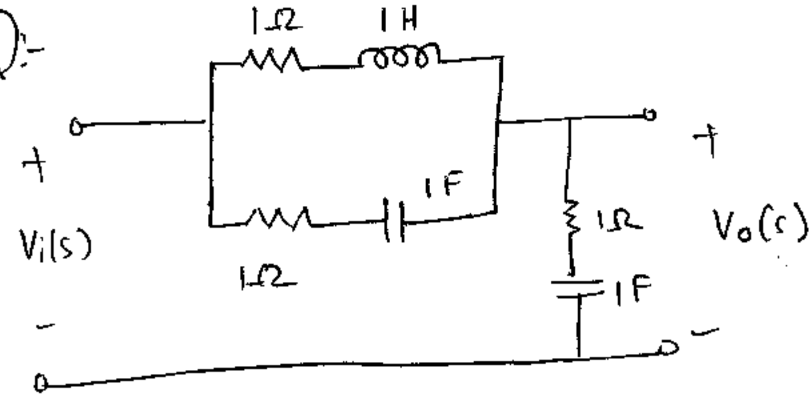
$\downarrow$

$s = j\omega = 0$

for DC gain put $s=0$ in given T.F.

$\therefore$ Here, DC gain $= \frac{R}{R} = 1$

Q:-



$Z_1 = 1+s \,||\, 1+ 1/s$ $\qquad\qquad$ $Z_2 = 1 + 1/s$

$$= \frac{(1+s)(1+1/s)}{1+s+1+1/s}$$

$$= \frac{1+1/s+s+1}{2+s+1/s}$$

$$= \frac{2+\cancel{s+}1/s}{2+\cancel{s+}1/s}$$

$Z_1 = 1$

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{1+1/s}{1+1+1/s} = \frac{s+1}{2s+1}$$

For DC. gain

put $s=0$

DC. gain = 1

* Transfer function of differential eqn:-

Q:- Find the T.F. of the given system where x is input and y is output

$$\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 9y = 2\frac{dx}{dt} + x(t-\tau)$$

Solⁿ:- Taking laplace transform:-

$$s^3Y(s) + 5s^2Y(s) + 7sY(s) + 9Y(s) = 2sX(s) + e^{-\tau s}X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{2s + e^{-\tau s}}{s^3+5s^2+7s+9}$$

Q:- Write differential eqⁿ to given transfer function

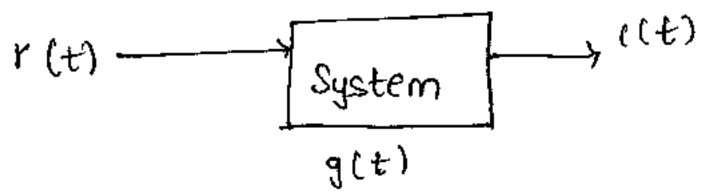
$$T.F. = \frac{Y(s)}{X(s)} = \frac{2s+3}{s^2+5s+6}$$

$$Y(s)s^2 + 5sY(s) + 6Y(s) = 2sX(s) + 3X(s)$$

Taking inverse laplace transform,

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 2\frac{dx}{dt} + 3x$$

* Transfer function to the ~~system~~ signal response.



S.R. = I.R.

1) Impulse ⟶ ① Impulse response

2) Unit step ⟶ ② Unit step response

3) Ramp ⟶ ③ Ramp response

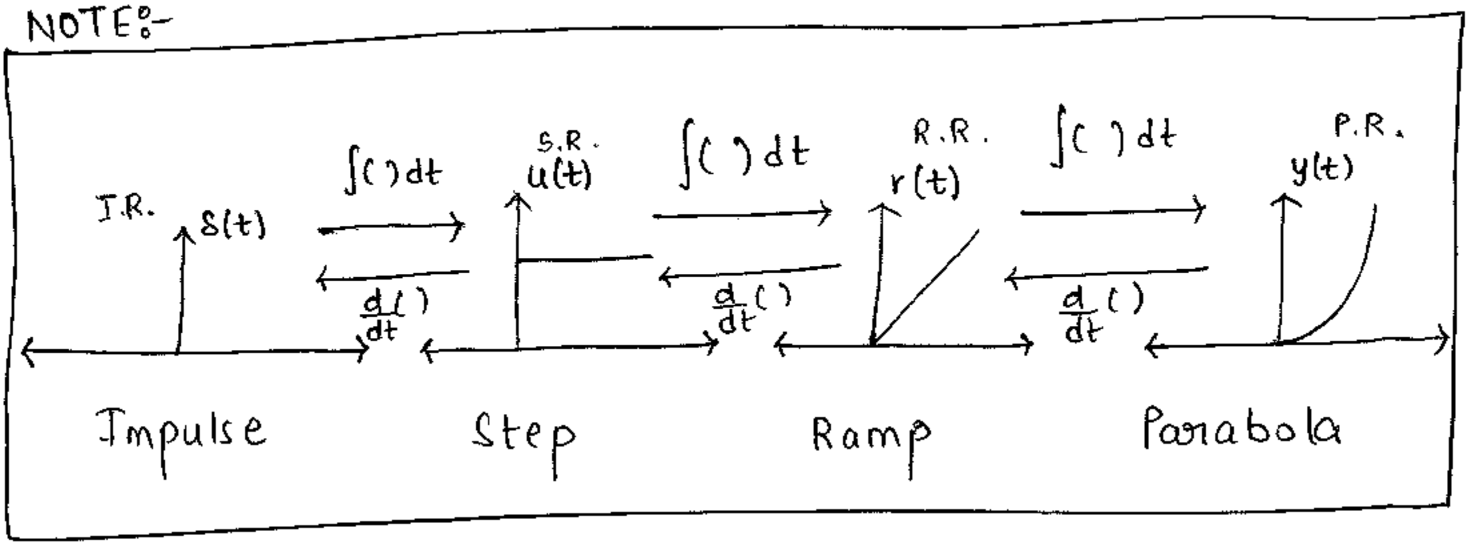
4) Parabola ⟶ ④ Parabola response

$$T.F. = \left.\frac{L[o/p]}{L[i/p]}\right|_{I.C.=0}$$

$$c(t) = r(t) * g(t)$$

$$\boxed{C[s] = R[s] \cdot G[s]}$$

NOTE:-



Q:- Unit step response of the system is $y(t) = \frac{5}{2} - \frac{5}{2}e^{-2t} + 5$ ..., $t \geq$ ... its T.F. is ______.

Sol:- $y(t) = \frac{5}{2} - \frac{5}{2}e^{-2t} + 5t \; ; \; t \geq 0$

U.S. → system → y(t)

$x(t) = u(t)$ U.S.R.

$$T.F. = \left.\frac{L[\text{unit step response}]}{L[\text{unit step}]}\right|_{I.C.=0}$$

$$= \frac{\frac{5}{2s} - \frac{5}{2(s+2)} + \frac{5}{s^2}}{1/s}$$

$$= \frac{\frac{5s(s+2) - 5s^2 + 5(s+2)}{2s^2(s+2)}}{1/s} = \frac{10s + 5(s+2)}{2(s+2)s} = \frac{10(s+1)}{s(s+2)}$$

Q:- The I.R. of the system is $c(t) = -4e^{-t} + 6e^{-2t}$; $t \geq 0$ equivalent S.R. is ______.

Sol:-

Impulse → system → Impulse response

$$S.R. = \int_{-\infty}^{t} (I.R.)\, dt$$

$$= \int_{-\infty}^{t} (-4e^{-t} + 6e^{-2t})\, dt = \int_{0}^{t} (-4e^{-t} + 6e^{-2t})\, dt$$

$$= \left[+4e^{-t} + \frac{6e^{-2t}}{-2} \right]_0^t$$

$$= 4e^{-t} - 3e^{-2t} - [4 - 3]$$

$$S.R. = (4e^{-t} - 3e^{-2t} - 1)\, u(t)$$

2nd Method:-

$R(s)$ → $G(s)$ → $C(s)$

$$C[s] = R[s] \cdot G[s]$$

For S.R. $\Rightarrow R[s] = 1/s$

$$\boxed{C[s] = \frac{1}{s} \cdot G[s]} \quad \text{—(1)}$$

Now, for I.R. $\Rightarrow R(s) = 1$

$$C[s] = G[s]$$

$$G[s] = \frac{-4}{(s+1)} + \frac{6}{(s+2)} \quad \text{—(2)}$$

$$C[s] = \frac{-4}{s(s+1)} + \frac{6}{s(s+2)}$$

$$\frac{-4}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$-4 = A(s+1) + Bs$$

At $s=0$ $\quad \boxed{-4 = A}$

At $s=-1$ $\quad -4 = B(-1)$ $\quad \boxed{B=4}$

$$\frac{6}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$6 = A(s+2) + Bs$$

At $s=0$ $\quad \frac{6}{2} = \boxed{A = \cancel{4}3}$

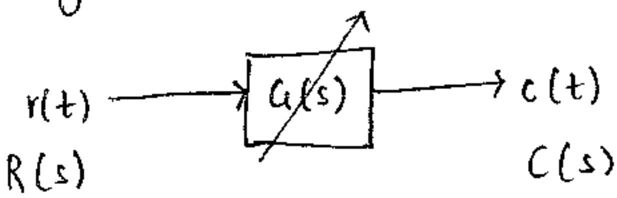
At $s=-2$ $\quad 6 = B(-2$ $\quad \boxed{B=-3}$

$$c[s] = \frac{-4}{s} + \frac{4}{s+1} + \frac{3}{s} - \frac{3}{s+2} = \frac{4}{s+1} - \frac{3}{s+2} - \frac{1}{s}$$

$$c(t) = (4e^{-t} - 3e^{-2t} - 1)u(t)$$

* Sensitivity (Imp)

- Sensitivity is nothing but variation in system output or system response due to parameter variations in (i) $g(s)$ & (ii) $H(s)$



$$G[s] = \frac{C[s]}{R[s]}$$

$T_1 = 20^\circ C$

$T_2 = 25^\circ C$

Change in Temp. $= T_2 - T_1 = \Delta T$

% Change in Temp. $= \frac{\Delta T}{T_1} \times 100\%$

Let T.F. = T

% Change in T.F. $= \frac{dT}{T} \times 100\%$

% Change in G $= \frac{dG}{G} \times 100\%$

% Change in H $= \frac{dH}{H} \times 100\%$

% Change in K $= \frac{dK}{K} \times 100\%$

$$S_G^T = \frac{\%\text{ change in T.F.}}{\%\text{ change in G}}$$

$$S_G^T = \frac{\frac{dT}{T}}{\frac{dG}{G}} = \frac{dT}{dG} \times \frac{G}{T}$$

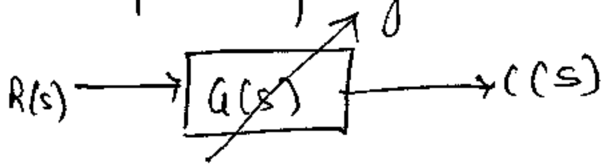
$$\%\ S_G^T = \frac{dT}{dG} \times \frac{G}{T} \times 100\%$$

$$S_H^T = \frac{dT}{dH} \times \frac{H}{T}$$

$$S_K^T = \frac{dT}{dK} \times \frac{K}{T}$$

$$S_G^T = \frac{dT}{dG} \times \frac{G}{T}$$

* Open loop system



$$T.F. = T = \frac{C}{R} = G$$

$$\Rightarrow T = G$$

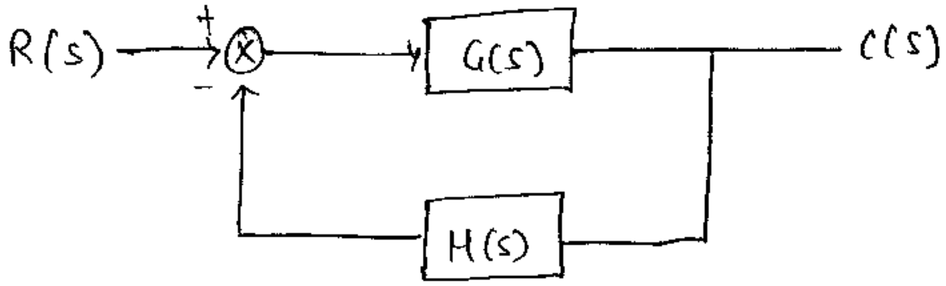
$$\frac{dT}{dG} = 1$$

① $S_G^T = \frac{dT}{dG} \times \frac{G}{T}$

$$\boxed{S_G^T = 1 \times \frac{G}{G} = 1}$$

$$\% S_G^T = 100\%$$

* Closed loop system



$$T = \frac{G}{1+GH} \Rightarrow \frac{dT}{dG} = \frac{(1+GH)\cdot 1 - G(H)}{(1+GH)^2} = \frac{1}{(1+GH)^2}$$

(1) $S_G^T = \frac{dT}{dG} \times \frac{G}{T}$

$$= \frac{1}{(1+GH)^2} \times \frac{G}{T}$$

$$= \frac{1}{(1+GH)^{\cancel{2}}} \times \frac{\cancel{G}\,(1+\cancel{G}H)}{\cancel{G}}$$

$$\boxed{S_G^T = \frac{1}{1+GH}} \Rightarrow S_G^T(\%) = \frac{1}{1+GH} \times 100\%$$

(2) $S_H^T = \frac{dT}{dH} \times \frac{H}{T}$

$$\frac{dT}{dH} = \frac{(1+GH)0 - G(G)}{(1+GH)^2}$$

$$\frac{dT}{dH} = \frac{-G^2}{(1+GH)^2}$$

$$S_H^T = \frac{-G^{\cancel{2}}}{(1+GH)^{\cancel{2}}} \times \frac{H(\cancel{1+GH})}{\cancel{G}}$$

$$\boxed{S_H^T = \frac{-GH}{1+GH}}$$

$$S_H^T (\%) = \frac{-GH}{1+GH} \times 100\%$$

NOTE:-

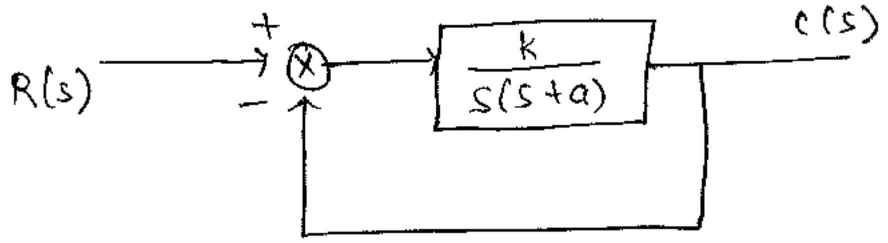
Open loop	Close loop
① $S_G^T = 100\%$	① $S_G^T = \frac{1}{1+GH} \times 100\%$
	② $S_H^T = \frac{-GH}{1+GH} \times 100\%$

- Close loop system is less sensitivity than open loop system due to parameter variation in $G(s)$ [i.e. forward path gain]

- Close loop system sensitivity is decreased by $1+GH$ factor w.r.t. G

- In close loop system, system response is more sensitivity due to parameter variation in feedback path gain compare to forward path gain G

Q:- Find the sensitivity of closed loop system w.r.t. variation in (1) K (2) a



Sol^n:- $T.F. = T = \dfrac{G}{1+GH}$

$$T = \frac{\dfrac{k}{s(s+a)}}{1+\dfrac{k}{(s+a)s}} = \frac{k}{s(s+a)+k}$$

(1) $S_K^T = \dfrac{dT}{dK} \times \dfrac{k}{T}$

$$= \frac{(s(s+a)+k)1 - k\cdot 1}{(s(s+a)+k)^2} \times \frac{\cancel{k}\,(s(s+a)+k)}{\cancel{k}}$$

$$= \frac{s(s+a)}{(s(s+a)+k)^2}$$

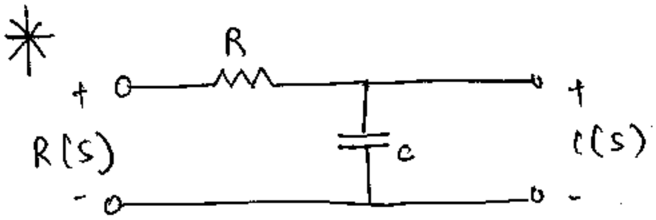
$$S_K^T = \frac{s(s+a)}{s(s+a)+k}$$

(2) $S_a^T = \dfrac{dT}{da} \times \dfrac{a}{T}$

$$= \frac{-\cancel{k}\cdot s}{(s(s+a)+k)^2} \times \frac{a(s(s+a)+k)}{\cancel{k}}$$

$$S_a^T = \frac{-as}{s^2+as+k}$$

*



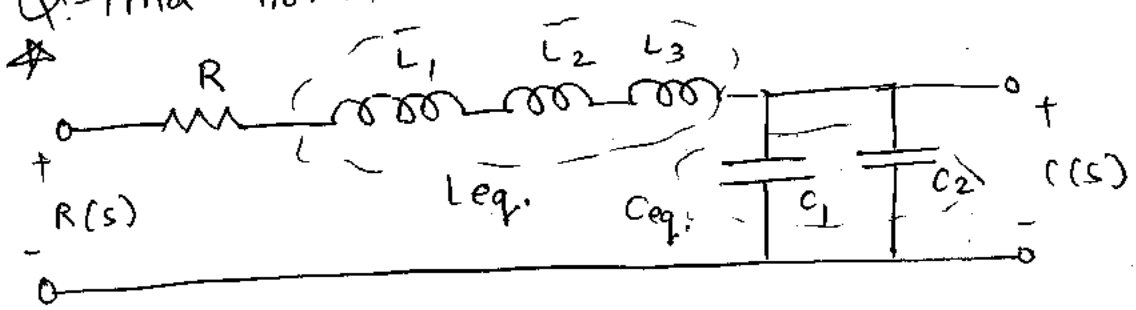
$$T.F. = \frac{1}{1+sCR}$$

Let $\tau = RC$

$$T.F. = \frac{1}{1+s\tau}$$

No. of time constant = 1 = No. of memory storage element

Q:- Find no. of time constant



No. of time constant = 2

- First find equivalent when there is more than one inductor and capacitance present and find time constant.