Long Answer Type Questions – I

Q. 1. Write the first negative term of the sequence **[DDE-2017]**

$\mathbf{20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4} \ldots.}$

Sol. Given sequence is

$20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4} \ldots..$

Or $20\frac{77}{4}, \frac{37}{2}, \frac{71}{4} \ldots..$

Here, a=20, d=$\frac{77}{4} - 20 = \frac{37}{2} - \frac{77}{4} = -\frac{3}{4}$ (it is an A.P)

Let n^{th}term be its first negative term, i.e.

$T_n = a + (n-1)d < 0$

$\Rightarrow 20 + (n-1)(-\frac{3}{4}) < 0$

$\Rightarrow 80 - 3n + 3 < 0$

$\Rightarrow 83 - 3n < 0$

$\Rightarrow n > \frac{83}{3}$

$\Rightarrow n > 27\frac{2}{3}$

Hence, $n = 28$

Q. 2. Determine the number of terms in A. P.3,7,11 …. 407. Also, find its $\mathbf{11^{th}}$ term from the end. **[DDE-2017]**

Sol. Given A.P. is 3,7,11 … 440

Here, $a = 3, d = 4, I = 407$

Using formula, $T_n = 1 = a + (n-1)d,$ we get 407 = 3 + $(n-1)4$

$\Rightarrow 4n = 408$

$\Rightarrow n = 102$

We need 11^{th} termm from the end

Last term = 102^{th}

Second last term = $102 - 1 = 101^{th}$

Third last term = $102 - 2 = 100^{th}$

and so, on

So, 11^{th} term from the end = (102 – 10) term = 92^{th} term

$\therefore \quad T_{92} = a + (92 + 1)d$

$= 3 + 91 \times 4$

$= 3 + 364$

$= 367$

Or

11^{th} term from the end $= I + (n-1)(-d)$

$= 407 + (11 - 1)(-4)$

$= 407 - 40$

$= 367$

Q. 3. How many numbers are there between 200 and 500, which leave remainder 7 when divided by 9. [DDE-2017]

Sol. The first number lying between 200 and 500, which is divisible by 9 and leaves the remainder 7 is 205.

The last number lying between 200 and 500, which is divisible by 9 and leaves the remainder 7 is 493.

∴ The numbers lying between 200 and 500, which are divisible by 9 and leave the remainder 7 are: 205, 214, 223….493

It is an A.P. in which $a = 205$ and $d = 9$

$\therefore \quad T_n = I = a + (n-1)d$

$\Rightarrow \quad 493 = 205 + (n-1)9$

$\Rightarrow \quad 288 = 9n - 9$

$\Rightarrow \quad \frac{297}{9} = n$

$\Rightarrow \quad n = 33$

Q. 4. If in an A.P. $\frac{a_7}{a_{10}} = \frac{5}{7}$, find $\frac{a_4}{a_7}$ [DDE-2017]

Sol. Given, $\frac{a_7}{a_{10}} = \frac{5}{7}$

Let the first term and common difference of AP be $'A'$ and $'D'$, respectively.

$\therefore \quad \frac{A+6D}{A+9D} = \frac{5}{7}$

$\Rightarrow \quad 7A + 42D = 5A + 45D$

$\Rightarrow \quad 7A = 3D$

$\Rightarrow \quad A \, \frac{3}{2} D$ ….. (i)

Now, $\frac{a_4}{a_7} = \frac{A+3D}{A+6D}$

$= \frac{\frac{3}{2}D}{\frac{3}{2}D} = \frac{3D}{6D} \quad \ldots\ldots(\text{using(i)})$

$= \frac{9D}{15D}$

$= \frac{3}{5}$

Q. 5. If the $1^{st} 2^{nd}$ and last terms of an A.P are a, b and c, respectively then find the sum of all. terms of the A. P [DDE-2017]

Sol. Given A.P. is

a, b ... c

First term (A) = a

Common difference (d) = $b - a$

Last term i.e., n^{th} term (A_n) = c

Using formula, $A_n = A + (n-1)d,$ we get

$c = a + (n-1)(b-a)$

$\Rightarrow \; c - a + (n-1)(b-a)$

$\Rightarrow \; n - 1 = \frac{c-a}{b-a}$

$\Rightarrow \; n = \frac{c-a}{b-a} + 1$

$\Rightarrow \; n = \frac{b+c-2a}{b-a}$ ……(i)

Now, using formula $S_n = \frac{n}{2}(A + A_n)$for sum of n terms, we get

$S_n = \frac{n}{2}[a + c]$

$= \frac{(b+c-2a)(a+c)}{2}$ using (i)

$= \frac{(a+c)(b+c-2a)}{2}$

Q. 6. The ratio of the sum of n terms of two A.P.'s is $(7n-1):(3n+11)$, Find the ratio of their 10^{th}terms. **[DDE-2017]**

Sol. Let a_1, a_2 be the first terms and d_1, d_2 the common differences of the two given AP.'s. Then, the sums of the its n terms are given by

$S_n = \frac{n}{2}[2a_1 + (n-1)d_1]$ and $S_n = \frac{n}{2}[2a_2 + (n-1)d_2]$

$\therefore \frac{S_n}{S_n} = \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} \quad = \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2}$

It is given that

$\frac{S_n}{S_n} = \frac{7n-1}{3n+11}$

$\frac{S_n}{S_n} = \frac{7n-1}{3n+11}$

$\Rightarrow \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_1} = \frac{7n-1}{3n+11} \qquad (ii)$

To find the ratio of the 10^{th} ers of the two A.P.'s, we replace n by $(2 \times 10 - 1)$ i.e. "19' in (i) Replacing n ty "19' in (i), we get

$\therefore \frac{2a_1 + (19-1)d_1}{2a_2 + (19-1)d_2} = \frac{7(19)-1}{3(19)+11}$

$\Rightarrow \frac{a_1 + 9d_1}{a_2 + 9d_2} = \frac{133-1}{57+11} = \frac{132}{68}$

$\Rightarrow \frac{a_1 + 9d_1}{a_2 + 9d_2} = \Rightarrow \frac{a_1 + 9d_1}{a_2 + 9d_2} = \frac{33}{17}$

$\therefore$ Ratio of the 10^{th} terms of the two A.P.'s = $\frac{33}{17}$

Q. 7. Find the sum of the sequence **[DDE-2017]**

$-1\frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2} \ldots, \frac{10}{3}$

Sol. Given sequence in A.P.

$$-1\frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2} \ldots, \frac{10}{3}$$

where $a = -1, d = \frac{-5}{6} + 1 = \frac{1}{6} and\ I = \frac{10}{3}$

Using formula $I = a + (n - 1)d$, we get

$$\frac{10}{3} = -1 + (n - 1)\ \frac{1}{6}$$

$$\Rightarrow \frac{10}{3} + 1 = \frac{n}{6} - \frac{1}{6}$$

$$\Rightarrow \frac{13}{3} + \frac{1}{6} = \frac{n}{6}$$

$$\Rightarrow \frac{27}{3} = \frac{n}{6}$$

$\Rightarrow n =$ 27

Now, sum of Nh terms of an A.P. is

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\therefore \quad S_{15} = \frac{27}{2}\left[2(-1) + (26)\frac{1}{6}\right]$$

$$= \frac{27}{2}\left[-2 + \frac{26}{6}\right]$$

$$= \frac{27}{2}\left[\frac{14}{6}\right]$$

$$= \frac{63}{2}$$

Q. 8. Solve: $1 + 6 + 11 + 16 + \cdots + x = 148$ **[DDE-2017]**

Sol. Clearly, terms of the given series form an A.P. with first term $a = 1$ and common difference $d = 5$. Let there be n terms in this series. Then,

1 + 6 + 11 +16 +… + x 148

$\Rightarrow$ sum of n terms = 148

$$\Rightarrow \frac{n}{2}[2a + (n - 1)d)] = 148$$

$$\Rightarrow \frac{n}{2}[2 + (n - 1)5)] = 148$$

$$\Rightarrow 5n^2 - 3n - 296 = 0$$

$\Rightarrow (n-8)(5n+37)=0$

$\Rightarrow \ n=8$ [$\therefore n$ is not negative]

Now, $x = n^{th}$ term

$\Rightarrow \quad x = a + (n-1)d$

$\Rightarrow \quad x = a + (8-1) \times 5 = 36$

[$\therefore a = 1, d = 5, n = 8$]

Q. 9. log2, log$(2^n - 1)$ and log $(2^n + 3)$ are in A.P. Show that $n = \frac{log5}{\log 2}$ **[DDE-2017]**

Sol. Given, log2, log$(2^n - 1)$ and log $(2^n + 3)$ are in A.P.

$\therefore$ log$\left(2^n + 3\right) -$ log$(2^n - 1)$ = log$(2^n - 1)$ - log2

$\Rightarrow$ log $\left(\frac{2^n+3}{2^n-1}\right) = \log\left(\frac{2^n-1}{2}\right)$

Taking antilog, we get

$$\Rightarrow \frac{2^n+3}{2^n-1} = \frac{2^n-1}{2}$$

$\Rightarrow \ 2^n - 4.2^n - 5 = 0$

$\Rightarrow \ y^2 - 4y - 5 = 0$ where $\ y = 2^n$

$\Rightarrow \ y - 5 \text{ and } y = -1 \text{ (reject)}$

Hence, $2^n = 5$

or n log2 = log5 or $n = \frac{log5}{\log 2}$

Hence proved

Q. 10. If a, b, c are in A.P show that following are also in A.P.

(i) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$

(ii) $b+c, c+a, a+b$ **[DDE-2017]**

Sol. (i) Given a, b, c are in A.P

$\Rightarrow \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are also in A.P.

[On dividing each term by a, b, c]

$\Rightarrow \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are also in A.P.

(ii) $b+c, c+a, a+b$ will be in A.P.

If $(c+a)-(b+c)=(a+b)-(c+a)$

i.e if $a-b=b-c$

i.e if $2b=a+c$

I.e if a, b, c are in A.P

Thus, a, b, c are in A.P $\Rightarrow b+c, c+a, a+b$ are in A.P.

Q. 11. If a, b, c are in A.P. show that

$a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are also in A.P. **[DDE-2017]**

Sol. a, b, c are in A.P.

$\Rightarrow \frac{a}{abc}, \frac{b}{abc}, \frac{c}{abc}$ are in A.P. [on dividing each term by abc]

$\Rightarrow \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A.P.

$\Rightarrow \frac{ab+bc+ca}{bc}, \frac{ab+bc+ca}{ac}, \frac{ab+bc+ca}{ab}$ are in A.P.

[on multiplying each term by $ab+bc+ca$]

$\Rightarrow \frac{ab+bc+ca}{bc}-1, \frac{ab+bc+ca}{ca}-1, \frac{ab+bc+ca}{ab}$

are also in A.P.

[On adding -1 to each term]

$\Rightarrow \frac{ab+ac}{bc}, \frac{ab+bc}{ca}, \frac{bc+ca}{ab}$ are also in A.P.

$\Rightarrow a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

Hence proved

Q. 12. If the numbers a^2, b^2, c^2 are given to be in A.P

Show that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P. **[DDE-2017]**

Sol. $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ will be in A.P.

if $\frac{1}{c+a}-\frac{1}{b+c}=\frac{1}{c+a}$

i.e if $\frac{b-a}{(c+a)(b+c)}=\frac{c-b}{(c+a)(b+c)}$

i.e if $\frac{b-a}{b+c} = \frac{c-b}{a+b}$

i.e if $b^2 - a^2 = c^2 - b^2$

i.e if $2b^2 = a^2 + c^2$

i.e. So $a^2,\ b^2, c^2$ are in A.P

Thus, $a^2,\ b^2, c^2$ are in A.P. $\Rightarrow \frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

Q. 13. If $\frac{b+c-2a}{a}, \frac{c+a-2b}{b}, \frac{a+b-2c}{c}$ are in A.P., then show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

[DDE-2017]

Sol. Given, $\frac{b+c-2a}{a}, \frac{c+a-2b}{b}, \frac{a+b-2c}{c}$ are in A.P

$\Rightarrow \left\{\frac{b+c-2a}{a} + 3\right\}, \left\{\frac{c+a-2a}{b} + 3\right\}, \left\{\frac{a+b-2a}{c} + 3\right\}$

are in A.P. [on adding 3 to each tech]

$\Rightarrow\ \frac{b+c+a}{a}, \frac{c+a+b}{b}, \frac{a+b+c}{a}$ are in A.P.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

[Dividing each term by $a + b + c$]

Hence proved

Q. 14. Show that if the positive number a, b, c are in A.P. Show that the numbers $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ will be in A. P.

[DDE-2017]

Sol. $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ will be in A. P.

If $\frac{1}{\sqrt{c}+\sqrt{a}} - \frac{1}{\sqrt{b}+\sqrt{c}} = \frac{1}{\sqrt{a}+\sqrt{b}} - \frac{1}{\sqrt{c}+\sqrt{a}}$

i.e if $\frac{\sqrt{b}-\sqrt{a}}{(\sqrt{c}+\sqrt{a})(\sqrt{b}+\sqrt{c})} = \frac{\sqrt{c}-\sqrt{b}}{(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{a})}$

i.e if $\frac{\sqrt{b}-\sqrt{a}}{\sqrt{b}+\sqrt{c}} = \frac{\sqrt{c}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

i.e if $b - a = c + b$

i.e if $2b = a + c$

i.e if a, b, c are in A.P.

Thus, a, b, c are in A.P. $\Rightarrow \frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in A. P.

Hence proved

Q. 15. The product of first three terms of G.P. is 1000.If 6 is added to its second term and 7 is added to its third term, the terms become in A.P. Find G.P. [DDE-2017]

Sol. Let the numbers in G.P. be $\frac{a}{r}, a, ar$ ... (i)

Product $\frac{a}{r}, a. ar = 1000$

$\Rightarrow a^3 = 1000$

$\Rightarrow a = 10$

According to question

A.P. $a_1 = \frac{a}{r} = \frac{10}{r}$

$a_2 = a + 6 = 10 + 6 = 16$

$a_3 = ar + 7 = 10r + 7$

Also, $a_3 = a_1 + 2(a_2 - a_1)$ [$\therefore a, b, c$ are in A.P.]

$\Rightarrow 10r + 7\frac{10}{r} + 2\left[10 - \frac{10}{r}\right]$

$\Rightarrow 10r^2 + 7r = 10 + 32r - 20$

$\Rightarrow 10r^2 - 25r + 10 = 0$

$\Rightarrow (r - 2)(10r - 5) = 0$

$\Rightarrow r = 2\ or\ \frac{1}{2}$

Substituting the value of a and r in eq (i), we get G.P. : 5,10,20… when $r = 2$

and G.P.: 20,10,5... whenr $r = \frac{1}{2}$

Q. 16. If the continued product of three numbers in G.P. is 216 and the sum of their products in pair is 156, find numbers. [DDE-2017]

Sol. Let three numbers in G.P. be $\frac{a}{r}, a, ar$ ... (i)

Given, $\frac{a}{r}. a. ar = 216$

$\Rightarrow a^3 = 216 = 6^3$

$\Rightarrow a = 6$

Therefore sum of their product in pair are:

$\frac{a}{r} \times a + a \times ar \times \frac{a}{r} = 156$

$\Rightarrow \frac{36}{r} + 36r + 36 = 156$ $(\therefore a = 6)$

$\Rightarrow \frac{36}{r} + 36r = 120$

$\Rightarrow \frac{3}{r} + 3r = 10$

$\Rightarrow 3r^2 - 10r + 3 = 3$

$\Rightarrow (3r - 1)(r - 3) = 0$

$\Rightarrow r = \frac{1}{3}, 3$

Substituting value of a and r in eq (i), we get

18,6,2 when $r = \frac{1}{3}$

and 2,6,18 when when $r = 3$

Q. 17. If A=1 + $r^a + r^{2a} + \cdots \infty$,then express r in terms of $'a'$ and $'A'$. **[DDE-2017]**

Sol. Given series is in G.P. with common ratio r^a

$\therefore$ Sum of infinite terms of G.P. = $\frac{x}{1-r}$

$\therefore \; A = \frac{1}{1 - r^a}$ $[x = 1]$

$\Rightarrow A\,(1 - r^a) = 1$

$\Rightarrow A - Ar^a = 1$

$\Rightarrow A - 1 = Ar^a$

$\Rightarrow r^a = \frac{A - 1}{A}$

$\Rightarrow r = \left(\frac{A - 1}{A}\right)^{1/a}$

Q. 18. If $x = a + \frac{a}{r} + \frac{a}{r^2} + \cdots + \infty,\; y = b - \frac{b}{r} + \frac{b}{r^2} - \cdots + \infty$ and $z = c + \frac{c}{r^2} + \frac{c}{r^4} + \cdots + \infty$ prove that $\frac{x.y}{z} = \frac{ab}{c}$ **[DDE-2017]**

Sol. Given series, $x = a + \frac{a}{r} + \frac{a}{r^2} + \cdots + \infty$ is in G.P with common ratio $\frac{1}{r}$.

and series $y = b - \frac{b}{r} + \frac{b}{r^2} - \cdots + \infty$ is in G.P with common ratio $-\frac{1}{r}$.

and series $z = c + \frac{c}{r^2} + \frac{c}{r^4} + \cdots + \infty$ is in G.P with common ratio $-\frac{1}{r^2}$

Sum of infinite terms of series

$x = a + \frac{a}{r} + \frac{a}{r^2} + \cdots + \infty$ is $x = \dfrac{a}{1 - \frac{1}{r}}$

Sumofinfinite term of series $y = b - \frac{b}{r} + \frac{b}{r^2} - \cdots + \infty$ is $y = \frac{b}{1-\left(-\frac{1}{r}\right)} = \frac{b}{1+\frac{1}{r}}$

Sum of infinite terms of series $z = c + \frac{c}{r^2} + \frac{c}{r^4} + \cdots + \infty$ is $z = \frac{c}{1-\frac{1}{r^2}}$

Now, LHS = $= \frac{xy}{z}$

$$= \frac{\left\{\dfrac{a}{1 - \frac{1}{r}}\right\}\left\{\dfrac{b}{1 + \frac{1}{r}}\right\}}{\dfrac{c}{1 - \frac{1}{r^2}}}$$

$$= \frac{\dfrac{ab}{\left(1 - \frac{1}{r^2}\right)}}{\dfrac{c}{\left(1 - \frac{1}{r^2}\right)}}$$

$$= \frac{ab}{c} = \mathrm{R.H.S}$$

Hence proved

Q. 19. The sum of first three terms of a G.P. is 15 and sum of next three terms is 120. Find the sum of first n terms. [DDE-2017]

Sol. Let the G.P. be $a, ar, ar^2, ar^3, ..$

Acording to the gven condition,

$a + ar + ar^2 = 15$ and $ar^3 + ar^4 + ar^5 = 120$

$\Rightarrow a(1 + r + r^2) = 15 \;\; \ldots(i)$and

$ar^3(1 + r + r^2) = 120 \;\; \ldots(ii)$

Dividing eq. (ii) by (i), we obtain

$$\frac{ar^3(1+r+r^2)}{a(1+r+r^2)} = \frac{120}{15}$$

$\Rightarrow r^3 = 8$

$\therefore r = 2$

Substituting $r = 2$ in (i), we get $a(1+2+4) = 15$

$$\therefore a = \frac{15}{7}$$

$$Now, S_n = \frac{a(2^n - 1)}{r - 1} \qquad [\because r > 1]$$

$$\therefore S_n = \frac{\frac{15}{7}(2^n - 1)}{2 - 1}$$

$$= \frac{15(2^n - 1)}{7}$$

Q. 20. Prove that : $0.3\overline{56} = \frac{353}{990}$ **[DDE-2017]**

Sol. We have,

$0.0.3\overline{56} = 0.3 + 0.056 + 0.00056 + 0.0000056 + \cdots\infty$

$$\Rightarrow 0.3\overline{56} = 0.3 + \left[\frac{56}{10^3} + \frac{56}{10^5} + \frac{56}{10^7} + \cdots\infty\right]$$

$$\Rightarrow 0.3\overline{56} = \frac{3}{10} + \frac{\frac{56}{10^3}}{1 - \frac{1}{10^2}} = \frac{3}{10} + \frac{56}{990} = \frac{353}{990}$$

Q. 21. Find the sum of first $'n'$ terms of the series $0.7 + 0.77 + 0.777 + \cdots$ **[DDE-2017]**

Sol. We have, $0.7 + 0.77 + 0.777 + \cdots \text{to } n \text{ terms} = 7 \times 0.1 + 7 \times 0.11 + 7 \times 0.111 +$ to n terms

$= 7\{0.1 + 0.11 + 0.111 + \cdots \text{to } n \text{ terms}\}$

$$= \frac{7}{9}\{0.9 + 0.99 + 0.999 + \cdots \text{to } n \text{ terms}\}$$

$$= \frac{7}{9}\left\{\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \cdots \text{to } n \text{ terms}\right\}$$

$= \frac{7}{9}\left\{\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\left(1-\frac{1}{1000}\right)+\cdots \text{to } n \text{ terms}\right\}$

$= \frac{7}{9}\left\{\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10^2}\right)+\left(1-\frac{1}{10^3}\right)+\cdots\left(1-\frac{1}{10^n}\right)\right\} 1$

$= \frac{7}{9}\left\{n-\left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+\cdots+\frac{1}{10^n}\right)\right\}$

$= \frac{7}{9}\left\{n-\frac{1}{10}\frac{\left\{1-\left(\frac{1}{10}\right)^n\right\}}{\left(1-\frac{1}{10}\right)}\right\}$

$= \frac{7}{9}\left\{n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right\}$

$= \frac{7}{81}\left\{9n-1-\frac{1}{10^n}\right\}$

Q. 22. Find the sum of following sequence up to n terms 7, 77, 777, 7777

[KVS 2017 Agra]

Sol. Let,S = 7 + 77 + 777 + …77 … 7 (upto n terms)

S = 7[1+ 11 + 111 + …11 … 1](upto n terms)

$s = \frac{7}{9}[9+99+999+\cdots 99 \ldots 9]$

$s = \frac{7}{9}[10-1+10^2-1+10^3-1+\cdots 10^n-1]$

$s = \frac{7}{9}[10^1+10^2+\cdots 10^n-n]$

$s = \frac{7}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]$

$s = \frac{7}{9}\left[\frac{10}{9}(10^n-1)-n\right]$

Q. 23. Find the least value of n for which **[DDE-2017]**

$\mathbf{1+3+3^2+\cdots+3^{n-1}>1000}$

Sol. We have,

$1+3+3^2+\cdots+3^{n-1}>1000$

$\Rightarrow 3^0+3^1+3^1+\cdots 3^{n-1}>1000$

$\Rightarrow 3^0\left(\frac{3^n-1}{3-1}\right) > 1000$

$\Rightarrow 3^n - 1 > 2000$

$\Rightarrow 3^n > 2001$

Least value of n, which satisfies this inequality is n = 7 ($\because 3^7 = 2187$)

Hence, least value of n = 7.

Q. 24. Find the sum of the series $\mathbf{1 + (1+x) + (1+x+x^2) + (1+x+x^2+x^3) + \cdots}$ **[DDE-2017]**

Sol. Let

$S_n = 1 + (1+x) + (1+x+x^2) + (1+x+x^2+x^3) + \cdots$

to n terms

Hence, $(1+x)\, S_n = (1+x) + (1-x)(1+x)$

$+(1-x) + (1+x+x^2)$

$+(1-x) + (1+x+x^2+x^3) + \cdots$

to n terms

or $(1-x)\, S_n = (1-x) + (1-x^2) + (1-x^3)$to n terms $+(1-x^4) + \cdots$

$= n - (x + x^2 + x^3 + x^4 + \cdots)$ to n terms

$= n - \frac{x(1-x^n)}{(1-x)}$

Hence, $S_n = \frac{n}{1-x} - \frac{x(1-x^n)}{(1-x)^2}$

Q. 25. If a, b, c are in G.P., then the following are also in G.P.

(i) a^2, b^2, c^2

(ii) a^3, b^3, c^3 **[DDE-2017]**

Sol. (i) Given, a, b, c are in G.P.

$\therefore b^2 = ac$

On squaring both side

$(b^2)^2 = (ac)^2$

$\Rightarrow (b^2)^2 = a^2c^2$

$\Rightarrow a^2 b^2 c^2$ are in G.P.

(ii) Given a, b, c are in G.P.

$\therefore b^2 = ac$

$(b^2)^3 = (ac)^3$

$\Rightarrow (b^3)^2 = a^3 c^3$

$\Rightarrow a^3, b^3, c^3$ are in G.P.