CGPET 2019

Solved Paper

Question 1

A double convex lens has two surfaces of equal radii R and refractive index μ = 1.5. Then,

Options:

A. $f = \frac{R}{2}$

B. $f = R$

C. $f = -R$

D. $f = 2R$

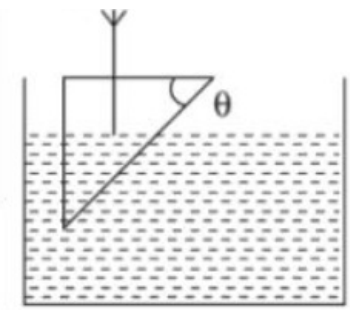
Answer: B

Solution:

Solution:
Given,
Refractive index, μ = 1.5
Radius of curvature, $R_1 = R, R_2 = -R$
As we know that, from the lens maker's formula,
$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
$\frac{1}{f} = (1.5 - 1)[\frac{1}{R} - \frac{1}{(-R)}]$
$\frac{1}{f} = 0.5[\frac{1}{R} + \frac{1}{R}]$
$\frac{1}{f} = 0.5[\frac{2}{R}] = \frac{1}{2}(\frac{2}{R})$
$\frac{1}{f} = \frac{1}{R} \Rightarrow f = R$

Question 2

The refractive indices of the material of the prism and liquid are 1.56 and 1.32, respectively. What will be the value of θ for the above refraction?



Options:

A. $\sin\theta \geq \frac{13}{11}$

B. $\sin\theta \geq \frac{11}{13}$

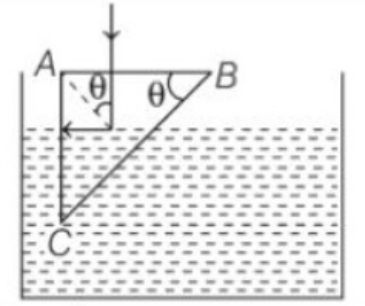
C. $\sin\theta \geq \frac{\sqrt{3}}{2}$

D. $\sin\theta \geq \frac{1}{\sqrt{2}}$

Answer: B

Solution:

Solution:



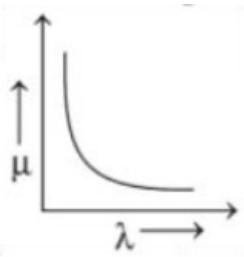
For total internal reflection from surface BC
$\theta \geq C \Rightarrow \sin\theta \geq \sin C$
$\Rightarrow \sin\theta \geq \left(\frac{1}{{}_{liquid}\mu_{prism}}\right) \Rightarrow \sin\theta \geq \left(\frac{\mu_{liquid}}{\mu_{prism}}\right)$
given, $[\mu_{liquid} = 1.32 \mu_{prism} = 1.56]$
$\sin\theta \geq (\frac{1.32}{1.56})$
or $\sin\theta \geq \frac{11}{13}$

Question 3

Which of the following graphs shows appropriate variation of refractive index μ with wavelength λ ?

Options:

A.



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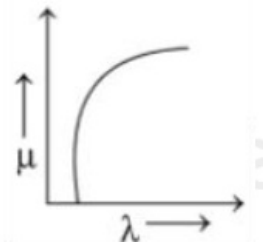
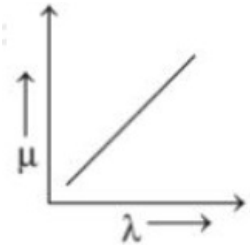
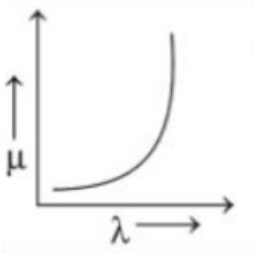


C.



D.



Answer: A

Solution:

Solution:
According to Cauchy's formula, $\mu = A + \frac{B}{\lambda^2}$
Where A and B are called Cauchy's constants for the prism.
$\mu \propto \frac{1}{\lambda^2}$
Hence, option (a) is represent the variation of refractive index μ with wavelength λ.

Question 4

A person cannot see objects clearly beyond $2.0m$. The power of lens required to correct his vision will be

Options:

A. $+2D$

B. $-1D$

C. $+1D$

D. $-0.5D$

Answer: D

Solution:

Solution:
For correcting far point.
$u = -\infty$
$v = -2m$
Applying len's formula
$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ .....(i)
Put the values in Eq.(i)
$\frac{1}{f} = \frac{1}{-2} + \frac{1}{\infty} \Rightarrow \frac{1}{f} = \frac{1}{-2} + 0$
$\Rightarrow \mathrm{f} = -2m$
So power of lens,
$P = \frac{1}{f} = \frac{1}{-2}$
$P = -0.5D$

Question 5

The image formed by the objective of a compound microscope is

Options:

A. virtual and enlarged

B. virtual and diminished

C. real and diminished

D. real and enlarged

Answer: D

Solution:

Solution:
The image formed by objective lens of compound microscope is real and enlarged, while final image formed by compound microscope is inverted, virtual, enlarged and at a distance D to infinite or from an eye, on same side of eye piece.

Question 6

As the intensity of incident light increases

Options:

A. photoelectric current increases

B. photoelectric current decreases

C. kinetic energy of photoelectron increases

D. kinetic energy of photoelectron decreases

Answer: A

Solution:

Solution:
If the intensity of light of a given frequency is increased, then the number of photons striking the surface per second will increase in the same ratio. This increased number of photons strike more electrons of metals and hence number of photoelectrons emitted through the surface increase and hence photoelectric current increases.

Question 7

In Bohr model of the hydrogen atom, the lowest orbit corresponds to

Options:

A. Infinite energy

B. The maximum energy

C. The minimum energy

D. Zero energy

Answer: C

Solution:

Solution:
In Bohr model, lowest orbits corresponds to minimum energy.
$E_n = -\frac{13.6}{n^2}$
for lowest orbit, $n = 1$
So, $E_n = \frac{-13.6}{(1)^2} = -13.6eV$

Question 8

In the nuclear reaction ${}_{6}C^{11} \rightarrow {}_{5}B^{11} + \beta^{+} + X^{-}$, what does X stand for?

Options:

A. An electron

B. A proton

C. A neutron

D. A neutrino

Answer: D

Solution:

Solution:
Given nuclear reaction, ${}_{6}C^{11} \rightarrow {}_{5}B^{11} + \beta^{+} + X$
[here X is stand for a neutrino, when β -particle emitted]

Question 9

In the given reaction

$${}_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} \rightarrow {}_{Z-1}K^{A-4} \rightarrow {}_{Z-1}K^{A-4}$$

Radioactive radiations are emitted in the sequence

Options:

A. α, β, γ

B. β, α, γ

C. α, γ, β

D. β, γ, α

Answer: A

Solution:

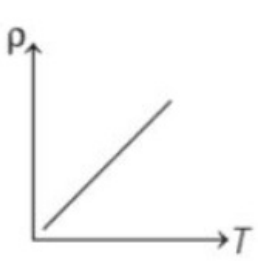
Solution:
In given reaction,
${}_{Z}X^{A} \xrightarrow{\alpha} {}_{Z-2}Y^{A-4} \xrightarrow{\beta} {}_{Z-1}K^{A-4} \xrightarrow{\gamma} {}_{Z-1}K^{A-4}$ In above reaction, in first atomic number z decreases by 2 and mass number A decreases by 4.
So, α -particle emitted.
In second step, atomic number $(z-2)$ increases by 1 and man number $(A-4)$ remains same hence β-particle emitted.
In third step, atomic number $(z-1)$ and mans number $(A-4)$ remain same.
Hence γ -particle emitted.
Hence, radioactive radiations are emitted in sequence of α, β, γ

Question 10

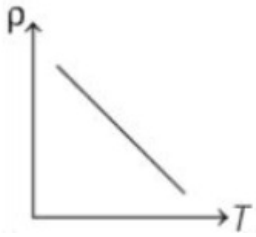
The temperature (T) dependence of resistivity (ρ) of a semiconductor is represented by

Options:

A.



B.



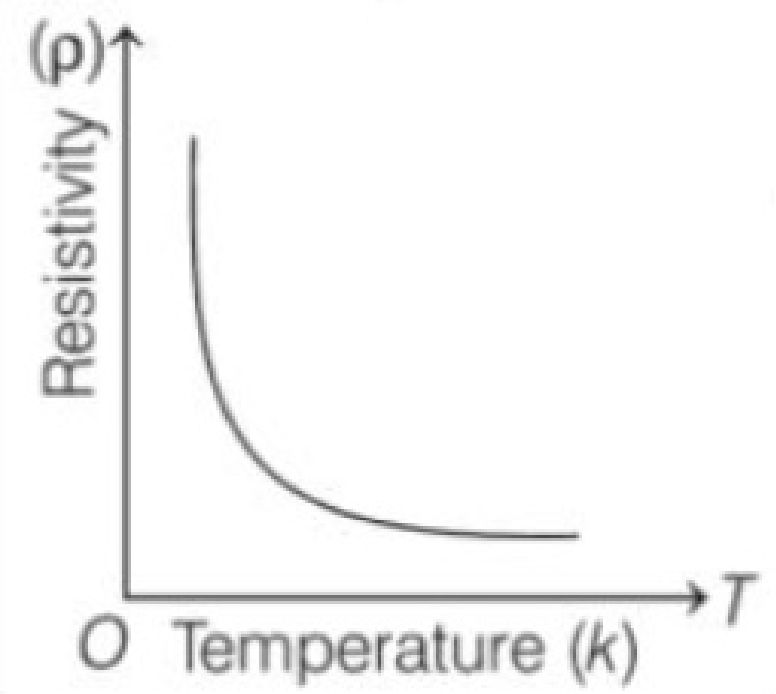
C.

D.

Answer: C

Solution:

Solution:
When the temperature is increased in semiconductor, the density of the charge carriers increase and the resistivity of semiconductor decreases.
The relation between resistivity and temperature is given by following equation.
$\rho_T = \rho_0[1 + \alpha(T - T_0)]$
Where ρ_0 is the resistivity at a reference temperature T_0 and ρ_T its value at temperature T.
The factor α is called the temperature coefficient of resistivity and the temperature coefficient is negative for semiconductor.
Hence the curve is non-linear for wide range of temperature as shown in the figure.



Hence, option (c) is represent the temperature (T) dependence on resistivity (ρ) of a semiconductor.

Question 11

If a full wave rectifier circuit is operated from 100 Hz mains, the fundamental frequency in the ripple will be

Options:

A. $100 Hz$

B. $200 Hz$

C. $50 Hz$

D. $150 Hz$

Answer: B

Solution:

Solution:
According to question, a full wave rectifier circuit is operated from $100Hz$ mains.
So, AC input frequency $= 100Hz$
In the full wave rectifier the frequency of ripple is double to that of AC input frequency.
Hence, Ripple frequency $= 2 \times$ AC input frequency
$= 2 \times 100 = 200Hz$
Hence, fundamental frequency in the ripple will be $200Hz$.

Question 12

A long magnetic needle of length 2l magnetic moment M and pole strength m is broken into two pieces at the middle. The magnetic moment and pole strength of each piece will be

Options:

A. $\frac{M}{2}, \frac{m}{2}$

B. $M, \frac{m}{2}$

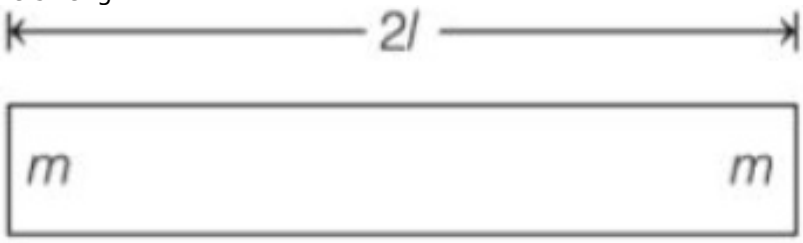
C. $\frac{M}{2}, m$

D. $M_1 m$

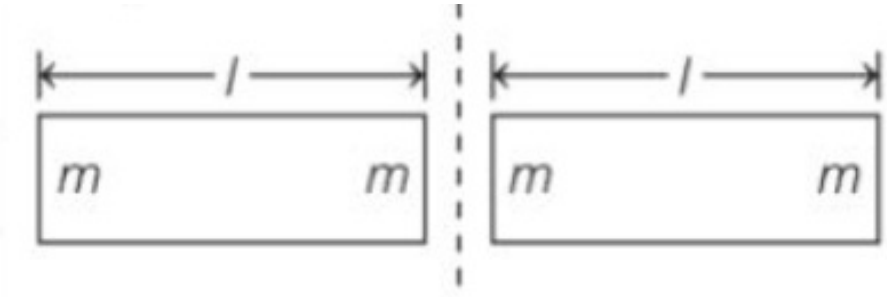
Answer: C

Solution:

Solution:
Given, Length of magnetic needle $= 2l$
Magnetic moment $= M$
pole strength $= m$



Magnetic moment, $M = m \times 2l$ ...(i)
According to the question, when magnetic needle is broken into two pieces at the middle as shown in the figure.



$M' = m \times l$
$M' = m \times 2l \times \frac{1}{2}$
From Eq. (i), we get
$M' = \frac{M}{2}$
Pole strength of magnetic needle cannot be changed.
Hence, the correct option is (c), i.e. $(\frac{M}{2}, m)$

Question 13

The sensitivity of tangent galvanometer is increased, if

Options:

A. number of turns decreases

B. number of turns increases

C. fields increases

D. None of the above

Answer: B

Solution:

Solution:
Current sensitivity of tangent galvanometer is given by $S_i = \frac{NBA}{c}$
where, $N =$ number of turns in coil,
$B =$ magnetic field
$A =$ area of coil.
So, $S_i \propto N$
Hence, the sensitivity of tangent galvanometer is increased, if number of turns in the coil increases.

Question 14

The ratio of forces between two small spheres with constant charge in air and in a medium of dielectric constant k is

Options:

A. $1:k$

B. $k:1$

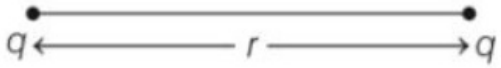
C. $1:k^2$

D. $k^2:1$

Answer: B

Solution:

Solution:
Let distance between two small spheres with constant charge is r as shown in figure.



According to the Coulomb's law, the force between the spheres in air is given by

$F_{\text{air}} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$ .....(i)

Force between the spheres in medium is given by

$F_{\text{medium}} = \frac{1}{4\pi\varepsilon_0 k}\frac{q^2}{r^2}$ .....(ii)

From Eqs. (i) and (ii), we get

$\frac{F_{\text{air}}}{F_{\text{medium}}} = \frac{k}{1}$

$\Rightarrow F_{\text{air}}:F_{\text{medium}} = k:1$

Question 15

The magnitude of electric field intensity E is such that, an electron placed in it would experience an electrical force equal to its weight, is given by

Options:

A. mge

B. $\frac{mg}{e}$

C. $\frac{e}{mg}$

D. $\frac{e^2}{m^2}g$

Answer: B

Solution:

Solution:
Given,
Magnitude of electric field intensity $= E$
Hence, the electric force on electron, $F = qE$ ..... (i)
Weight of electron, $w = mg$
where, $m =$ mass of electron and
$g =$ gravitational acceleration.
According to the question, Electrical force = weight of electron

$qE = mg \Rightarrow E = \frac{mg}{q}$
Here, charge on electron, $q = e$
$E = \frac{mg}{e}$

Question 16

N identical spherical drops charged to the same potential V are combined to form a big drop. The potential of the new drop will be

Options:

A. V

B. $\frac{V}{N}$

C. $V.N$

D. $V.N^{\frac{2}{3}}$

Answer: D

Solution:

Solution:
Let the charge of small drop $= q$
Potential on small drop, $V = \frac{Kq}{r}$ .....(i)
When N identical spherical drops are combined to form a big drop, then volume remains constant.
$N \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$
where, $R =$ radius of big drop.
$R = (N^{1/3})r$
Potential of the new drop, $V' = \frac{K(Nq)}{R}$
$V' = \frac{K(Nq)}{(N^{1/3})r}$
$V' = \frac{Kq}{r}N^{1-\frac{1}{3}}$
From Eq. (i), we get
$V' = V.N^{\frac{2}{3}}$

Question 17

A 6μ F capacitor is charged from $10V$ to $20V$. Increase in energy will be

Options:

A. $18 \times 10^{-4}J$

B. $9 \times 10^{-4}J$

C. $4.5 \times 10^{-4}J$

D. $9 \times 10^{-6}J$

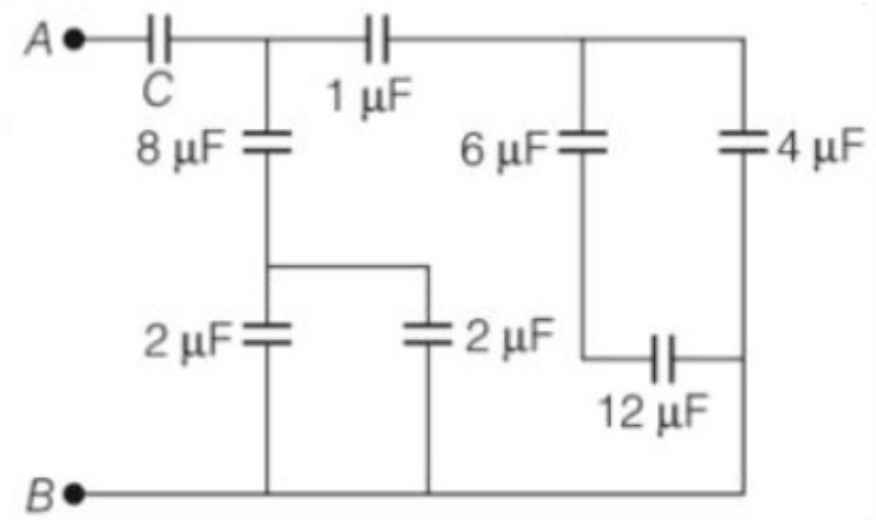
Answer: B

Solution:

Solution:
Given,
Capacitance of capacitor, $C = 6\mu F$
As we know that,
Electrical energy stored in capacitor, $U = \frac{1}{2}CV^2$
Hence, increase in energy.
$\Delta U = \frac{1}{2}CV_2^2 - \frac{1}{2}CV_1^2$
$\Delta U = \frac{C}{2}(V_2^2 - V_1^2)$
Here, $V_1 = 10V$ and $V_2 = 20V$
$\Delta U = \frac{6 \times 10^{-6}}{2}[(20)^2 - (10)^2]$
$\Delta U = \frac{6 \times 10^{-6}}{2} \times 300$
$\Delta U = 9 \times 10^{-4}J$

Question 18

In the following circuit the resultant capacitance between A and B is $1\mu F$. Then, the value of C is



Options:

A. $\frac{32}{11}\mu F$

B. $\frac{11}{32}\mu F$

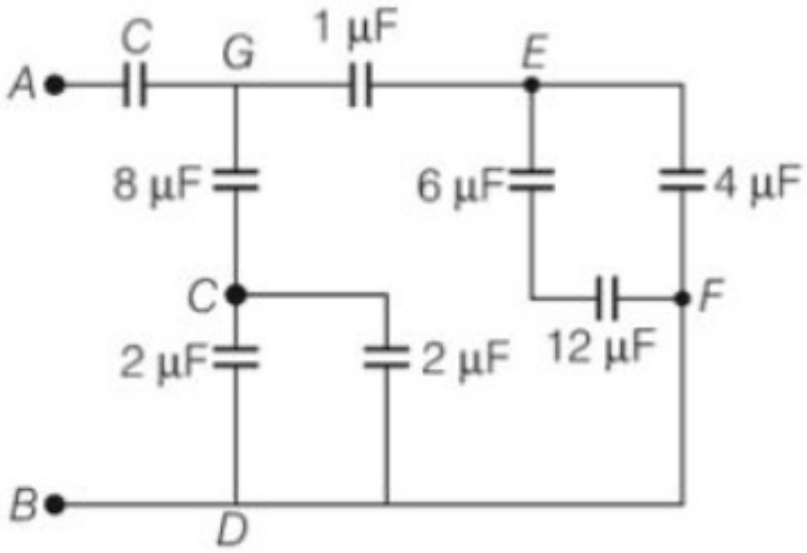
C. $\frac{23}{32}\mu F$

D. $\frac{32}{23}\mu F$

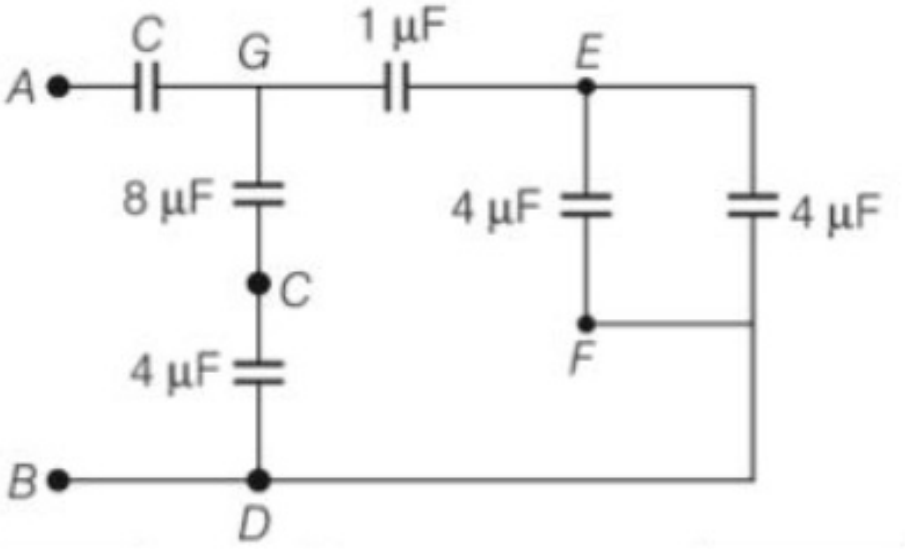
Answer: D

Solution:

Solution:
According to the question,



Given, capacitance between A and B, $C_{AB} = 1\mu F$.
$\therefore$ Capacitors between point C and D are connected in parallel. So, capacitance. $C_{CD} = 2\mu F + 2\mu F = 4\mu F$.
$\therefore$ Capacitors between points E and F connected in series, so the capacitance between points E and F, is
$\frac{1}{C_{EF}} = \frac{1}{6} + \frac{1}{12}$
or $\frac{1}{C_{EF}} = \frac{2+1}{12} = \frac{3}{12}$
or $C_{EF} = \frac{12}{3} = 4\mu F$



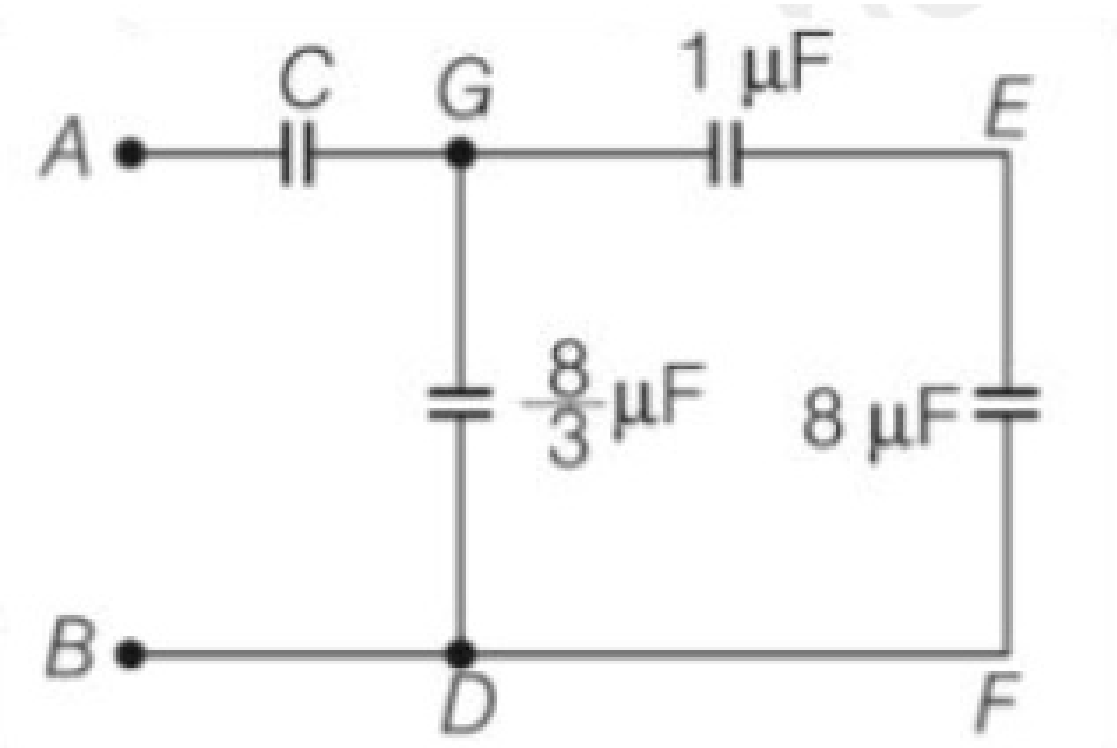
Capacitors between points G and D are connected in series, therefore

$\frac{1}{C_{GD}} = \frac{1}{8\mu F} + \frac{1}{4\mu F}$

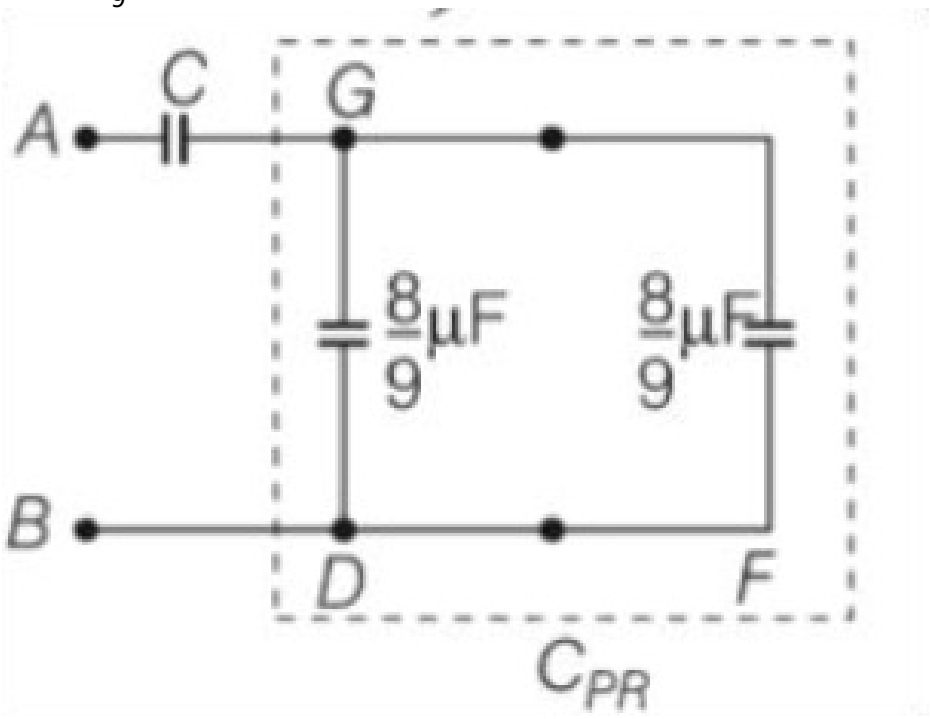
$\therefore \ = \frac{1+2}{8}$

or $\frac{1}{C_{GD}} = \frac{3}{8}$

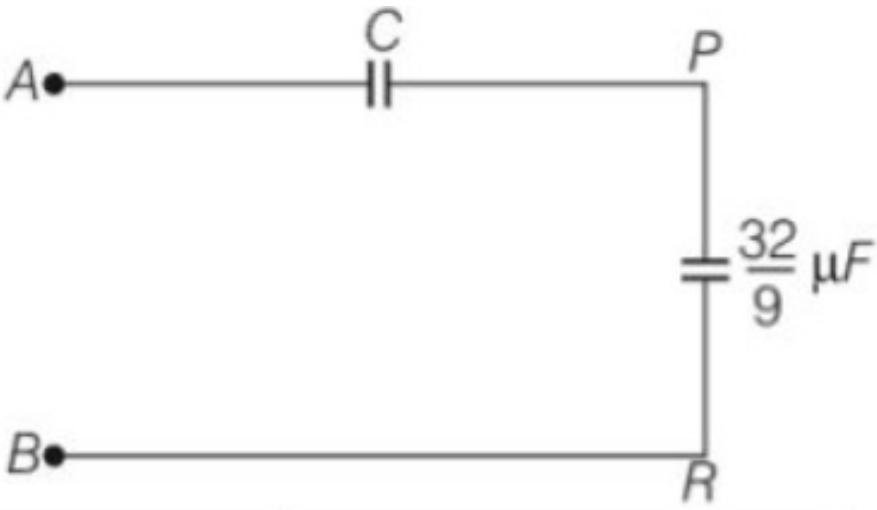
or $C_{GD} = \frac{8}{3}\mu F$



$\therefore$ Capacitance between points E and F, are connected in parallel, so
$\therefore C_{EF} = 4\mu F + 4\mu F = 8\mu F$
Capacitance between points G and F, are connected in series, therefore
$\frac{1}{C_{GF}} = \frac{1}{1} + \frac{1}{8}$
or $= \frac{8+1}{8} = \frac{9}{8}$
or $C_{GF} = \frac{8}{9}\mu F$



Now,
$\therefore C_{PR} = \frac{8}{9} + \frac{8}{3} = \frac{24+8}{9}$
$= \frac{32}{9}\mu F$



Capacitors are connected in series, then the total capacitance across points A and B,
$\therefore \frac{1}{C_{AB}} = \frac{1}{C} + \frac{1}{32/9}$
or $1 = \frac{1}{C} + \frac{9}{32}$
Here $C_{AB} = 1\mu F$
or $\frac{1}{C} = 1 - \frac{9}{32} = \frac{32-9}{32} = \frac{23}{32}$ or $\frac{1}{C} = \frac{23}{32}$
$C = \frac{32}{23}\mu F$

Question 19

The resistance of a wire is 10Ω. Its length is increased by 10% by stretching. The new resistance will be

Options:

A. 12Ω

B. 1.2Ω

C. 13Ω

D. 11Ω

Answer: A

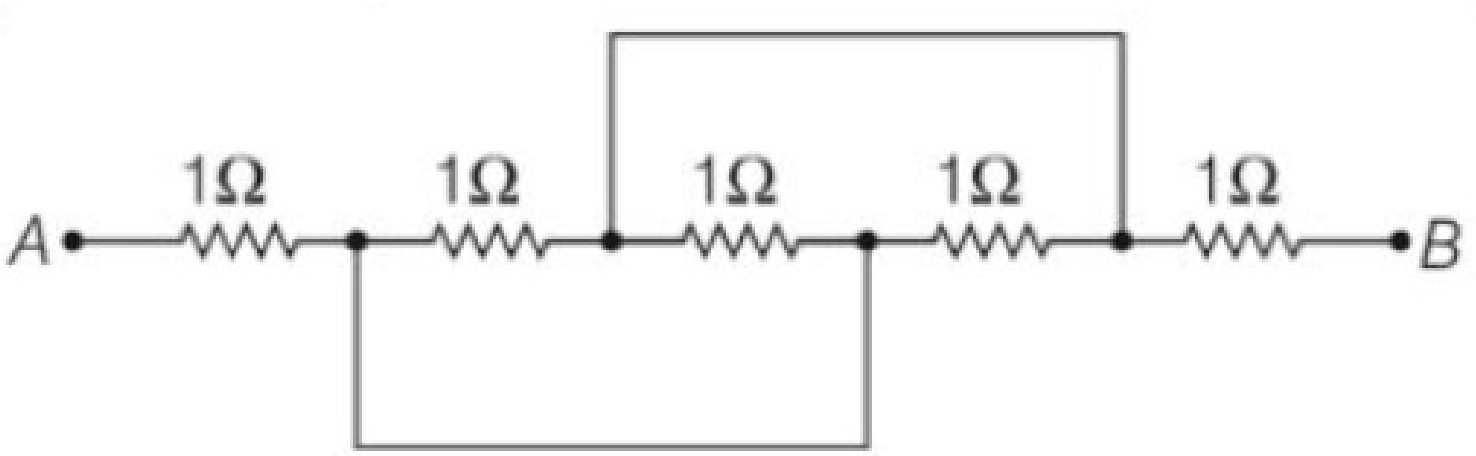
Solution:

Solution:
Resistance of wire, $R = 10\Omega$.
Let the initial length of wire $= l$
Resistance of wire, $R = \frac{\rho l}{A}$ .....(i)

According to the question,
New length of wire, $l' = l + \frac{l \times 10}{100}$
$l' = l + \frac{l}{10}$ or $l' = \frac{11l}{10}$
The volume of wire remains constant.
$\therefore$ Volume of wire before stretching = Volume of wire after stretching $Al = A'l'$
$Al = A'(\frac{11l}{10}) \Rightarrow A' = \frac{10}{11}A$
New resistance of wire,
$R' = \frac{\rho l'}{A'} \Rightarrow R' = \frac{\rho(\frac{11}{10}l)}{\frac{10}{11}A}$
$R' = \frac{\rho l}{A} \times \frac{11}{10} \times \frac{11}{10}$
From Eq. (i)
$R' = R \times (\frac{11}{10})^2 = 10 \times (\frac{11}{10})^2 = 12.1\Omega \approx 12\Omega$

Question 20

Equivalent resistance between the points A and B, is



Options:

A. $1\frac{1}{5}\Omega$

B. $1\frac{1}{4}\Omega$

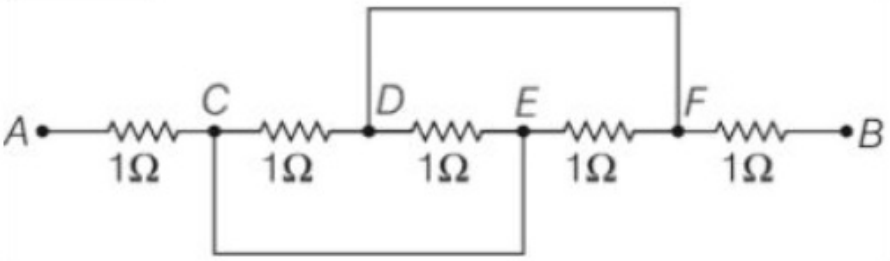
C. $2\frac{1}{3}\Omega$

D. $3\frac{1}{2}\Omega$

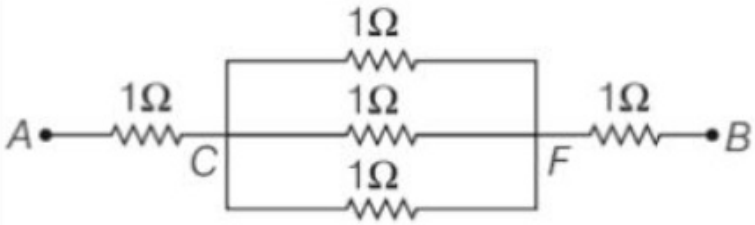
Answer: C

Solution:

Solution:
According to the question,



Above circuit simplified as,



The resistance between point C and F are connected in parallel combination. Hence, its equivalent resistance is given by

$\frac{1}{R_{CF}} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} \Rightarrow R_{CF} = \frac{1}{3}\Omega$

Now, circuit becomes

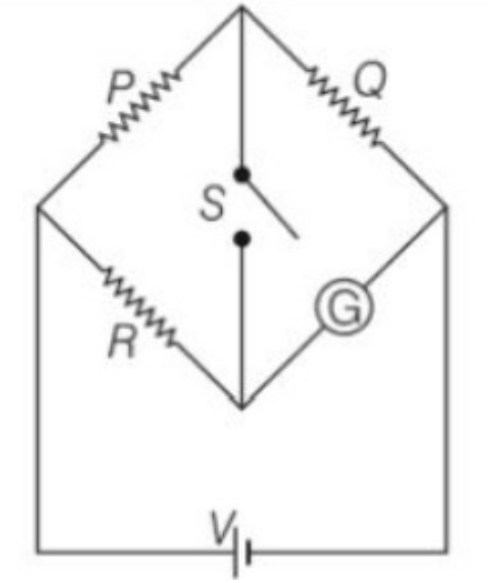
All the resistance in the above circuit are connected in the series combination.

Hence, $R_{AB} = 1 + 1 + \frac{1}{3}$

$= 2 + \frac{1}{3} = 2\frac{1}{3}\Omega$

Question 21

In the circuit shown $P \neq R$, the reading of the galvanometer is same with switch S open or closed, then



Options:

A. $I_R = I_G$

B. $I_P = I_G$

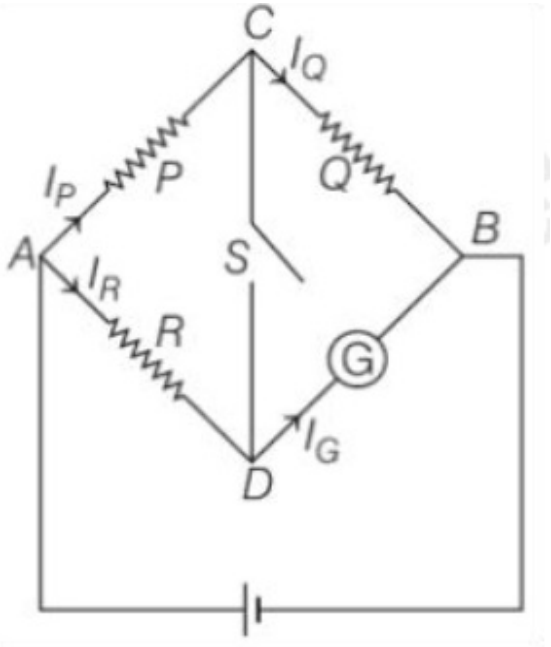
C. $I_Q = I_G$

D. $I_Q = I_R$

Answer: A

Solution:

Solution:
Given, $P \neq R$
The reading of the galvanometer is same with S open or closed i.e., the value of current in branch CD is zero. Hence, the potential at points C and D is same i.e., $V_C = V_D$



Hence, the value of current in branch ADB and ACB is same, i.e. $I_P = I_Q$
$I_R = I_G$
Hence, option (a) is correct.

Question 22

The resistance of 1 A ammeter is 0.018Ω. Toconvert it into 10A

ammeter, the shunt resistance required will be

Options:

A. 0.18Ω

B. 0.0018Ω

C. 0.002Ω

D. 0.12Ω

Answer: C

Solution:

Solution:
Given,
Resistance of ammeter, $G = 0.018\Omega$
$I_g = 1A$
$I = 10A$
As we know that,
Shunt resistance, $S = \frac{I_g G}{I - I_g}$
$= \frac{1 \times 0.018}{10 - 1} = \frac{0.018}{9}$
$= 0.002\Omega$

Question 23

Two electric bulbs whose resistances are in the ratio 1:2 are connected in parallel to a constant voltage source. The power dissipated in them have the ratio

Options:

A. 1:2

B. 1:1

C. 2:1

D. 1:4

Answer: C

Solution:

Solution:
Let the resistances of bulb are R_1 and R_2, respectively.
As we know that, $\frac{R_1}{R_2} = \frac{1}{2}$(given)
Power, $P = \frac{V^2}{R}$
Now, $\frac{P_1}{P_2} = \frac{V_1^2 / R_1}{V_2^2 / R_2}$
In the parallel combination potential remains same i.e. $V_1 = V_2$

$\frac{P_1}{P_2} = \frac{R_2}{R_1} = \frac{2}{1}$

Question 24

The current flowing in a copper voltmeter is $1.6A$. The number of Cu^{++} ions deposited at the cathode per minute is

Options:

A. 1.5×10^{20}

B. 3×10^{20}

C. 6×10^{20}

D. 1×10^{19}

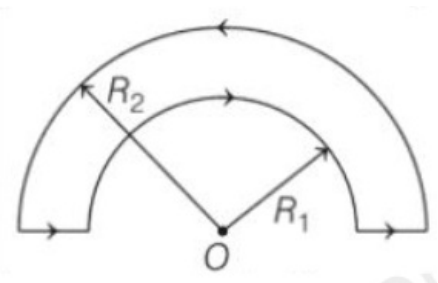
Answer: D

Solution:

Solution:
Given, the current flowing in a copper voltmeter, $I = 1.6A$
As we know that,
Charge, $Q =$ Total number of ions $\times$ Charge on electron (e)
$Q = n \times e$ .....(i)
Charge $Q = \frac{\text{Current}}{\text{Time}}$
$Q = \frac{I}{t}$
From Eq. (i), we get
$n \times e = \frac{I}{t}\left(\because t = 1\text{min given}\right)$
$n \times 1.6 \times 10^{-19} = 1.6(\because e = 1.6 \times 10^{-19}C)$
Now, the number of Cu^{++} ions deposited at the cathode per minute is $n = \frac{1.6}{1.6 \times 10^{-19}}$
$n = 1 \times 10^{19}$

Question 25

The magnetic induction at the centre O in the figure shown is



Options:

A. $\frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

B. $\frac{\mu_0 i}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

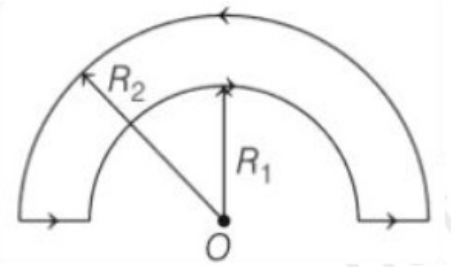
C. $\frac{\mu_0 i}{4}(R_1-R_2)$

D. $\frac{\mu_0 i}{4}(R_1+R_2)$

Answer: A

Solution:

Solution:
Magnetic field due to an arc of a circle at the centre is $B=(\frac{\mu_0}{4\pi})(\frac{i}{R})\theta$



Magnetic field due to straight wire at point O is zero and the magnetic field due to semicircular loop radius R_1 is given by $B_1=\frac{\mu_0 i}{4k_1}(\frac{\pi}{\pi})=\frac{\mu_0 i}{4R_1}$

Similarly, magnetic field due to semicircular loop of radius R_2 is given by $B_2=\frac{\mu_0 i}{4R_2}$

As, $R_2 > R_1$
Hence, net magnetic field
$B_{net}=B_1-B_2=\frac{\mu_0 i}{4R_1}-\frac{\mu_0 i}{4R_2}$
$=\frac{\mu_0 i}{4}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

Question 26

A 2 MeV proton is moving perpendicular to a uniform magnetic field of $2.5\,T$. The force applied on the proton is

Options:

A. $3\times10^{-10}N$

B. $8\times10^{-11}N$

C. $3\times10^{-11}N$

D. $8\times10^{-12}N$

Answer: D

Solution:

Solution:
Given,
Kinetic energy of proton, $K=2MeV=2\times10^6 eV$

$= 2 \times 10^6 \times 1.6 \times 10^{-19} J$
$= 3.2 \times 10^{-13} J$
Magnetic field, $B = 2.5 T$
and mass of proton, $m = 1.67 \times 10^{-27} kg$
Now, kinetic energy, $K = \frac{1}{2} m v^2$ .....(i)
Substitute given values in Eq. (i)
$\therefore 3.2 \times 10^{-3} = \frac{1}{2} \times 1.67 \times 10^{-27} \times v^2$
or $v^2 = \frac{2 \times 3.2 \times 10^{-13}}{1.67 \times 10^{-27}}$
or $v^2 = 3.83 \times 10^{14}$
or $v = 1.95 \times 10^7 m/s$
Force applied on proton in a uniform magnetic field is given by
$F = qvB \sin \theta$
Here, $\theta = 90°$, ($\because$ Proton is moving perpendicular to a uniform magnetic field).
$\therefore F = qvB$ [$\because \sin 90° = 1$]
or $F = 1.6 \times 10^{-19} \times 1.95 \times 10^7 \times 2.5$
or $F = 7.8 \times 10^{-12} N$
or $F \approx 8 \times 10^{-12} N$

Question 27

An arbitrary shaped closed coil is made of a wire of length L and a current I ampere is flowing through it. The plane of the coil is perpendicular to magnetic field B, the force on the coil is

Options:

A. zero

B. IBL

C. $2IBL$

D. $\frac{1}{2}IBL$

Answer: A

Solution:

Solution:
We know that, force on wire which length L and flowing current is I.
Force, $F = I(L \times B)$ .....(i)
But here arbitrary shaped is close. Magnetic field is zero is closed loop.
So, force on the coil is zero.

Question 28

The magnetic flux linked with a coil, in webers, is given by the equation $\phi = 3t^2 + 4t + 9$ Then, the magnitude of induced emf at $t = 2$ s will be

Options:

A. $2V$

B. $4V$

C. $8V$

D. $16V$

Answer: D

Solution:

Solution:
Given,
Magnetic flux, $\phi = 3t^2 + 4t + 9$
We know that, induced emf $|e| = \frac{d\phi}{dt}$
$\Rightarrow |e| = \left|\frac{d}{dt}(3t^2 + 4t + 9)\right| = |6t + 4|$
Induced emf at $t = 2s$
$|e| = |(6 \times 2) + 4| = 16V$

Question 29

An $L - R$ circuit has a cell of emf. E, which is switched ON a time $t = 0$. The current in the circuit after a long time will be

Options:

A. zero

B. $\frac{E}{R}$

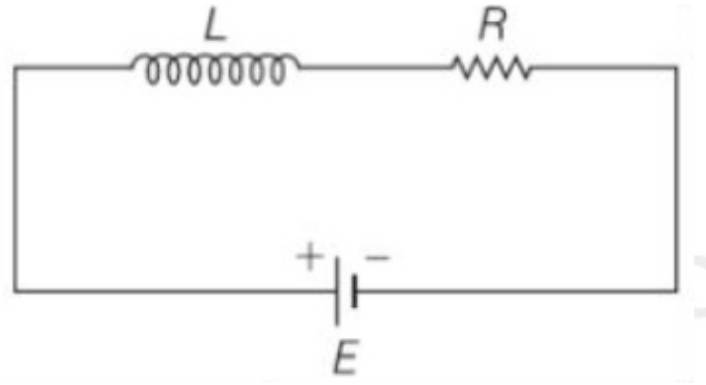
C. $\frac{E}{L}$

D. $\frac{E}{\sqrt{L^2 + R^2}}$

Answer: B

Solution:

Solution:
Given, situation can be shown in figure below R



Applying, kirchhoff's voltage law,
$\Rightarrow L\frac{dI}{dt} + IR = E \Rightarrow \frac{dI}{dt} + \frac{IR}{L} = \frac{E}{L}$
We know that, integration factor
$IF = e^{\int \frac{R}{L} t}$
or $e^{(\frac{R}{L})t} . I = \int e^{(\frac{R}{L})t} . \frac{E}{L}$
$\Rightarrow e^{\frac{R}{L}t} I = \frac{E}{L} . \frac{L}{R} e^{\frac{R}{L}t} \Rightarrow I = \frac{E}{L} . \frac{L}{R}$
So, the current in the circuit after a long time will be
$I = \frac{E}{R}$

Question 30

A body moving with uniform acceleration describes $12m$ in the $3s$ of its motion and $20m$ in the $5s$. The velocity after $10s$ is

Options:

A. $52 m/s$

B. $50 m/s$

C. $42 m/s$

D. $40 m/s$

Answer: C

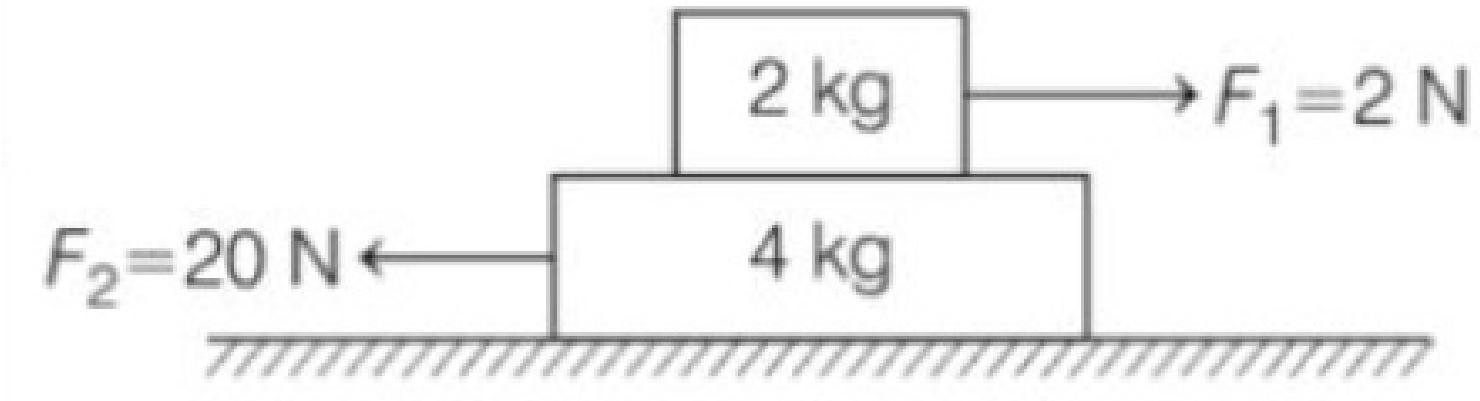
Solution:

Solution:
We know that, distance travelled in n th second

$s_n = u + \frac{a}{2}(2n-1)$
Case 1 Given, $n = 3, s = 12m$
So, $12 = u + \frac{a}{2}(6-1)$
$\Rightarrow 12 = u + 2.5a$ .....(i)
Case $2 s = 20m, n = 5$
So, $20 = u + \frac{a}{2}(10-1)$
$\Rightarrow 20 = u + 4.5a$ .....(ii)
from Eqs. (i) and (ii),
$8 = a(4.5-25)$
$\Rightarrow 8 = 2a$
$\Rightarrow a = 4m/s^2$ .....(iii)
From Eqs. (i),
$\Rightarrow 12 = u + 25 \times 4 \Rightarrow u = 2m/s$
From first equation of motion,
$v = u + at$
$v = 2 + 4 \times 10 = 42m/s$

Question 31

In the arrangement shown in figure, coefficient of friction between the two blocks, is $\mu = \frac{1}{2}$. The force of friction acting between the two blocks, is



Options:

A. $6N$

B. $8N$

C. $4N$

D. $9N$

Answer: B

Solution:

Solution:
Given, $\mu = \frac{1}{2}, m_1 = 2kg, m_2 = 4kg$
$F_1 = 2N, F_2 = 20N$
The maximum force of friction that can occur between the two blocks, is
$f_{max} = \mu mg$ .....(i)
Substitute given value in Eq. (i)
$\therefore f_{max} = \frac{1}{2} \times 2 \times 10 = 10N$

and acceleration of the block system,
$a = \frac{\text{total force}}{\text{total mass}} = \frac{20 + (-2)}{4 + 2} = \frac{+18}{6} = 3\ m/s^2$
The system of block has an acceleration in the left side. The force of friction opposing the motion of the $2kg$ block in the left side is
$ma +$ force acting in the direction of friction
$= [(2 \times 3) + 2N] = 8N$
Thus, the force of friction acting between the two blocks is $8N$.

Question 32

A ball of mass m is thrown upwards with a velocity v. If air exerts an average resisting force F, the velocity with which the ball returns to the thrower is

Options:

A. $v\sqrt{\frac{mg}{mg+F}}$

B. $v\sqrt{\frac{F}{mg+F}}$

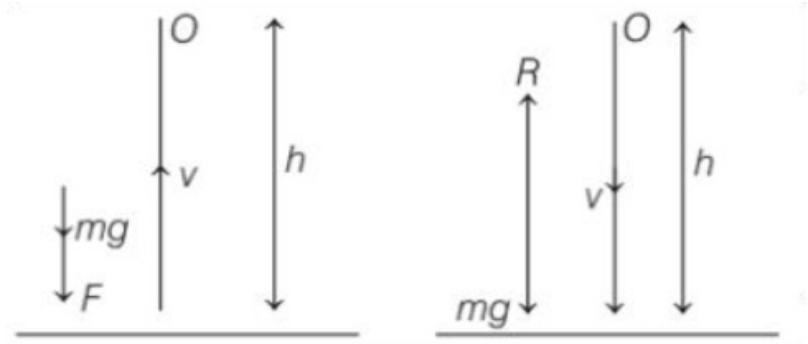
C. $v\sqrt{\frac{mg+F}{mg}}$

D. $v\sqrt{\frac{mg-F}{mg+F}}$

Answer: D

Solution:

Solution:
First of all we divide the journey of ball in two parts, upward and downward.



Case I For upward journey,
From work energy theorem,
Work done, $W = K_f - K_i = \Delta K$
$\Rightarrow -Fh - mgh = -\frac{1}{2}mv^2 - 0$

$\Rightarrow Fh + mgh = \frac{1}{2}mv^2$ .....(i)
Since, F is a resistance force.
Case II For downward journey,
From work energy theorem
$W = K_f - K_i = \Delta K$
$\Rightarrow -Fh + mgh = \frac{1}{2}mv_1^2$ .....(ii)
(here, $v_1 =$ final velocity)
Divide (i) and (ii), we get
$\frac{(Fh + mgh)}{(-Fh + mgh)} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv_1^2}$
$\Rightarrow \frac{h(F + mg)}{h(-F + mg)} = \frac{v^2}{v_1^2}$
$\Rightarrow v_1^2 = v^2\frac{(mg - F)}{(mg + F)}$
Hence, the velocity with which the ball returns to the thrower is
$v_1 = v\sqrt{\frac{(mg - F)}{(mg + F)}}$

Question 33

When work done by force of gravity is negative, then

Options:

A. kinetic energy decreases

B. kinetic energy increases

C. kinetic energy remains constant

D. potential energy remains constants

Answer: A

Solution:

Solution:
We know that, from work energy theorem,
Work done = Change in kinetic energy
$\Rightarrow W = \Delta K$
Here, work done is negative.
So. $-W = \Delta K$
It is possible when initial kinetic energy is greater than final kinetic energy.
i.e., kinetic energy will decrease.

Question 34

Power applied to a particle varies with time as $(P = 3t^2 - 2t + 1)W$, where t is in second. The change in its kinetic energy between $t = 2s$ and $t = 4s$ is

Options:

A. 32 J

B. 46J

C. 61J

D. 100J

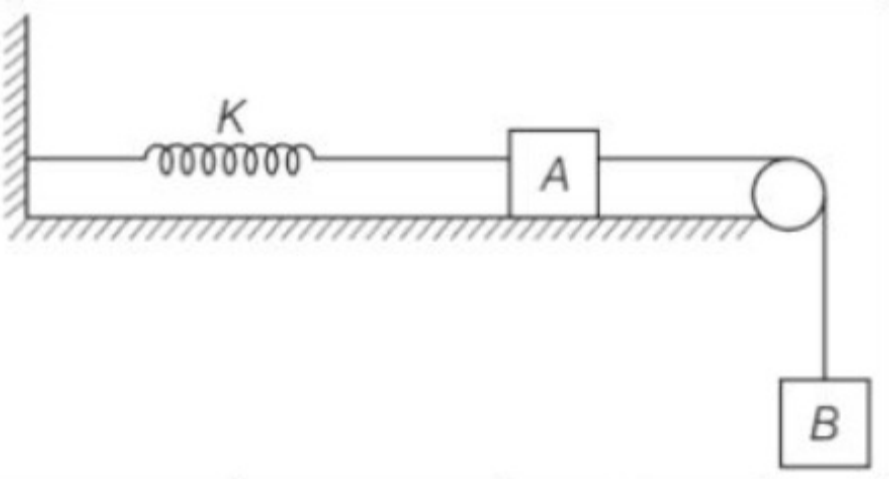
Answer: B

Solution:

Solution:
Given power, $P = 3t^2 - 2t + 1$
We know that, $P = \frac{dW}{dt}$ .....(i)
$\Rightarrow dW = Pdt$
and from work energy theorem,
Work down, $dW = K_f - K_i = \Delta K$ .....(ii)
From Eqs. (i) and (ii)
$\Delta K = Pdt$
$\Rightarrow \Delta K = \int_2^4 Pdt$
$= \int_2^4 (3t^2 - 2t + 1)\, dt$
$= \left[\frac{3t^3}{3} - \frac{2t^2}{2} + t\right]_2^4$
$= (t^3 - t^2 + t)_2^4 = [(4)^3 - (4)^2 + (4) \ -(2)^3 + (2)^2 - 2] \ = 64 - 16 + 4 - 8 + 4 - 2 = 46J$
So, the change in its kinetic energy between $t = 2s$ and $t = 4s$ is $46J$.

Question 35

In the figure, mass of A is m and that of B is $2m$. All the surfaces are smooth. System is released from rest with spring unstretched. Then, the maximum extension x_m in spring will be



Options:

A. $\frac{mg}{K}$

B. $\frac{2mg}{K}$

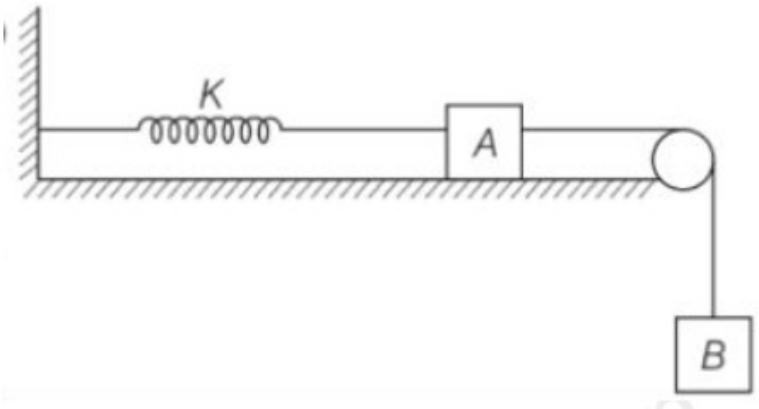
C. $\frac{3mg}{K}$

D. $\frac{4mg}{K}$

Answer: D

Solution:

Solution:



Given, mass of $A = m$
Mass of $B = 2m$
According to question, system is released from rest with spring unstretched.
Then, decrease in potential energy of B
= Increase in spring potential energy
$\Rightarrow \therefore 2mgx_m = \frac{1}{2}Kx_m^2$ .....(i)
Here, $x_m =$ maximum extension,
$K =$ force constant and
$g =$ gravitational acceleration
From Eq. (i)
$2mg = \frac{1}{2}Kx_m$
So, maximum extension x_m in spring,
$\Rightarrow x_m = \frac{4mg}{K}$

Question 36

What will be the effect on the weight of a body placed on the surface of earth, if earth suddenly starts rotating with half of its angular velocity

of rotation?

Options:

A. No effect

B. Weight will decrease

C. Weight will become zero

D. Weight will increase

Answer: D

Solution:

Solution:
The variation in gravitational acceleration is given by
$g' = g - \omega^2 R\cos^2\lambda$ .....(i)
where $g =$ gravitational acceleration at surface
omega = angualr velocity
R= Radius of earth
According to the question,
$g'' = g - \omega' R\cos^2\lambda$
Here$\omega' = \frac{\omega}{2}$
$g'' = g - (\frac{\omega}{2})^2 R\cos^2\lambda$ .....(ii)
From Eqs. (i) and (ii), it is clear that
$g'' > g$
As we know that
Weight of a body, $w = mg$
Here mass of body remains constant
Hence if earth suddenly starts rotating with half of its angular velocity of rotation, then the weight of body will increases.
Hence, $w \propto g$

Question 37

If the radius of the earth shrinks by 0.2% without any change in the mass, escape velocity from the surface of earth is

Options:

A. decreased by 0.1%

B. decreased by 0.4%

C. increased by 0.1%

D. increased by 0.4%

Answer: C

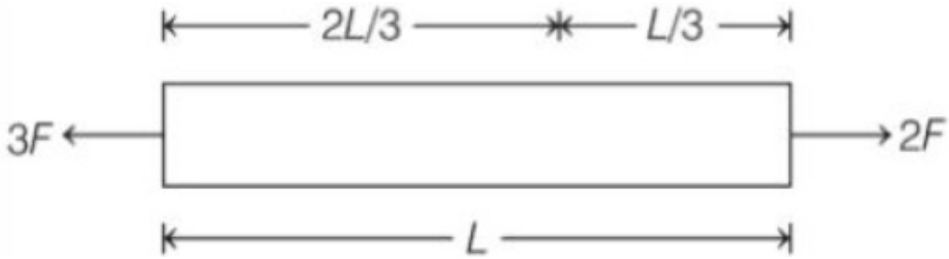
Solution:

Solution:
As, we know that,
Escape velocity, $v = \sqrt{2gR_e}$ .....(i)
Here, $R_e =$ radius of earth and

$g =$ gravitational acceleration.
Given, earth radius shrinks by $0.2\,\%$
So, $\frac{\Delta R_e}{R_e} \times 100 = 0.2$
From Eq. (i), $v^2 = 2gR_e$ .....(ii)
Differentiate, $2v\Delta v = 2g\Delta R_e$ .....(iii)
Dividing Eq. (iii) by Eq. (ii)
$2v\frac{\Delta v}{v^2} = \frac{2g\Delta R_e}{2gR_e}$
$\Rightarrow \frac{\Delta v}{v} \times 100 = \frac{1}{2}\frac{\Delta R_e}{R_e} \times 100 \ = \frac{1}{2} \times 0.2 = 0.1$
So, escape velocity from the surface of earth is increased by $0.1\,\%$.

Question 38

A uniform cylinder rod of length L. cross-sectional area A and Young's modulus Y is acted upon by the forces shown in the figure. The elongation of the rod is



Options:

A. $\frac{8}{3}\frac{FL}{AY}$

B. $\frac{3}{8}\frac{FL}{AY}$

C. $\frac{5}{3}\frac{FL}{AY}$

D. $\frac{3}{5}\frac{FL}{AY}$

Answer: A

Solution:

Solution:
First of all we divide the rod into 2 parts of lengths $\frac{2L}{3}$ and $\frac{L}{3}$ and where force F is acting.
Case I Force acting on both side of $\frac{2L}{3}$ is $3F$.
So, elongation x_1 will be
$x_1 = \frac{[3F \times \frac{2L}{3}]}{AY}$ .....(i)
Case II Force acting on both side of $\frac{L}{3}$ is $2F$.
So, elongation x_2 will be

$x_2 = \frac{[2F \times \frac{L}{3}]}{AY}$ .....(ii)
Now, total elongation, $x = x_1 + x_2$
$= \frac{(3F \times \frac{2L}{3})}{AY} + \frac{(2F \times \frac{L}{3})}{AY}$
$x = \frac{\frac{8}{3}FL}{AY} = \frac{8FL}{3AY}$

Question 39

Drop of liquid of density ρ is floating half immersed in a liquid of density ρ_0. If surface tension of liquid is s, the radius of drop is

Options:

A. $\sqrt{\frac{3s}{g(\rho - \rho_0)}}$

B. $\sqrt{\frac{3s}{g(2\rho - \rho_0)}}$

C. $\sqrt{\frac{3s}{g(3\rho - \rho_0)}}$

D. $\sqrt{\frac{3s}{g(4\rho - \rho_0)}}$

Answer: B

Solution:

Solution:
Given, density of liquid drop $= \rho$
Density of floating half immersed in liquid $= \rho_0$
Surface tension of liquid $= s$
Now, balancing the forces acting on the drops floating
$\therefore w = F_b + F_s$
where, w is the weight of the liquid drop, F_b is the Buoyant force acting on the drop and F_S is the surface tension acting on drop.
Then,
$w = F_b + F_s$
or $\frac{4}{3}\pi r^3 \rho g = \frac{2}{3}\pi r^3 \rho_0 g + s \times 2\pi r$
or $\frac{4}{3}\pi r^3 \rho g - \frac{2}{3}\pi r^3 \rho_0 g = s \times 2\pi r$
or $r^3[\frac{4}{3}\pi\rho g - \frac{2}{3}\pi\rho_0 g] = s \times 2\pi r$
or $r^2 \frac{[4\pi\rho g - 2\pi\rho_0 g]}{3} = s \times 2\pi$
or $r^2 \frac{2\pi g[2\rho - \rho_0]}{3} = s \times 2\pi$
or $r^2 \frac{g(2\rho - \rho_0)}{3} = s$
or $r^2 = \frac{3s}{g(2\rho - \rho_0)}$
or $r = \sqrt{\frac{3s}{g(2\rho - \rho_0)}}$.
So, the radius of drop is $r = \sqrt{\frac{3s}{g(2\rho - \rho_0)}}$.

Question 40

If the masses of all molecules of a gas are halved and their speeds doubled, then the ratio of initial and final pressure will be

Options:

A. $1:4$

B. $4:1$

C. $2:1$

D. $1:2$

Answer: D

Solution:

Solution:
Let mass of molecule m and speed v, then by kinetic theory of gasses,
$\therefore PV = \frac{1}{3}nmv^2$ .....(i)
Here, $P =$ initial pressure
and $V =$ initial volume.
According to question,
Mass of molecules of gas $= \frac{m}{2}$
and Speed of the gas molecules $= 2v$.
Then,
$P'V' = \frac{1}{3}n(\frac{m}{2})(2v)^2$
Here, P' is final pressure of gas molecules.
or, $P'V' = \frac{1}{3}n\frac{m}{2} \times 4v^2$
or $P'V' = \frac{1}{3}nm \times 2v^2$ .....(ii)
Now from equation (i) and (ii), we get
or, $\frac{PV}{P'V'} = \frac{\frac{1}{3}nmv^2}{\frac{1}{3}nmv^2 \times 2}$
or, $\frac{P}{P'} = \frac{1}{2}(\because v = v')$
So, the ratio of Initial and final pressure is $1:2$.

Question 41

1 mole of gas with $\gamma = \frac{7}{5}$ is mixed with l mole of gas with $\gamma = \frac{5}{3}$, then the value of γ for the resulting mixture is

Options:

A. $\frac{7}{5}$

B. $\frac{2}{5}$

C. $\frac{3}{2}$

D. $\frac{12}{7}$

Answer: C

Solution:

Solution:
According to the question,
1 mole of gas with $\gamma_1 = \frac{7}{5}$
Mixed with 1 mole of gas with $\gamma_2 = \frac{5}{3}$
Now, by mixture of non-reactive gases,
$\therefore \gamma = \frac{\text{Specific heat of constant pressure}(C_P)}{\text{Specific heat of constant volume}(C_V)}$
Number of gas molecules in mixture is
$n = n_1 + n_2 = 1 + 1 = 2 \left[\begin{matrix} \because n_1 = 1 \\ \text{and} n_2 = 1 \end{matrix} \right]$
$\therefore \gamma = \frac{C_P}{C_V}$
or $\frac{n}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$
Now, putting the values in above equation, we get
or $\frac{2}{\gamma - 1} = \frac{1}{\frac{7}{5} - 1} + \frac{1}{\frac{5}{3} - 1}$
or $\frac{2}{\gamma - 1} = \frac{5}{2} + \frac{3}{2}$ or $\frac{2}{\gamma - 1} = 4$
or $\gamma - 1 = \frac{1}{2}$ or $\gamma = \frac{1}{2} + 1 = \frac{3}{2}$
Then, the value of γ for resulting mixture is $\frac{3}{2}$.

Question 42

A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_1 (in kelvin) and the final temperature is αT_i, the value of α is

Options:

A. 1

B. 2

C. 3

D. 4

Answer: D

Solution:

Solution:
Let the initial volume of gas $= V_1$
Gas is compressed adiabatically then the volume.
$V_2 = \frac{V_1}{32}$

Initial temperature of gas $= T_i$ (in kelvin)
Final temperature of gas $= \alpha T_i$
For diatomic gas, $\gamma = \frac{7}{5}$
For adiabatic process,
$\Rightarrow TV^{\gamma - 1} =$ constant
When initial volume is V_1, then adiabatic process is given as
$\Rightarrow T_i V_1^{\gamma - 1} =$ constant .....(i)
When volume is $\frac{1}{32}$ of initial volume, then adiabatic process
$\therefore \alpha T_i \left(\frac{V_1}{32}\right)^{\gamma - 1} =$ constant .....(ii)
Now, Eqs. (i) and (ii), we get
or $T_i V_1^{\frac{7}{5} - 1} = \alpha T_i \left(\frac{V_1}{32}\right)^{\frac{7}{5} - 1}$
or $T_i V_1^{\frac{2}{5}} = \alpha T_i \left(\frac{V_1}{32}\right)^{\frac{2}{5}}$ or $1 = \alpha (\frac{1}{32})^{\frac{2}{5}}$
$\alpha = (32)^{2/5}$or$\alpha = 4$

Question 43

Which of the following is not true about the process?

Options:

A. For isothermal process, $dT = 0$

B. For isoboric process, $dP = 0$

C. For isochoric process, $dE = 0$

D. For adiabatic process, $dQ = 0$

Answer: C

Solution:

Solution:
Given, for isochoric process $dE = 0$ that is wrong because for isochoric process $dv = 0$.
For an isothermal process temperature remins constant, therefore, $dT = 0$ for an isobaric process pressure remains constant, therefore, $dp = 0$. For an adiabatic process, there is no exchange of heat between system and the surroundings, therefore, $dQ = 0$
Hence, option (c) is not true.

Question 44

A body cools from $60°C$ to $50°C$ in $10min$. If room temperature is $25°C$, then the temperature of body at the end of next 10 min will be

Options:

A. $38.5°C$

B. $42.8°C$

C. $45.5°C$

D. $40.8°C$

Answer: B

Solution:

Solution:
Given, $T_1 = 60°C, T_2 = 50°C$
$T_0 = 25°, t_2 = t_1 = 10min$
According to Newton's law of cooling,
$\frac{T_1 - T_2}{t} = k\left(\frac{T_1 + T_2}{2} - T_0\right)$
$\Rightarrow \frac{60-50}{10} = k(\frac{60+50}{2} - 25)$
$\Rightarrow 1 = k(55 - 25) \Rightarrow k = \frac{1}{30}$ .....(i)
Suppose if temperature of body θ in next $10min$, then $\frac{50-\theta}{10} = k(\frac{50+\theta}{2} - 25)$ .....(ii)
From Eqs. (i) and (ii)
$\frac{50-\theta}{10} = \frac{1}{30}(\frac{50+\theta}{2} - 25)$
$\Rightarrow \frac{50-\theta}{10} = \frac{1}{30}(\frac{50+\theta-50}{2})$
$\frac{50-\theta}{10} = \frac{1}{30} \times \frac{\theta}{2}$
$\Rightarrow 50 - \theta = \frac{\theta}{6}$
$\Rightarrow 300 - 6\theta = \theta$
$\Rightarrow 7\theta = 300 \Rightarrow \theta = \frac{300}{7} = 42.8°C$

Question 45

The rate of radiation of a black body at $0°C$ is E joule per sec. Then, the rate of radiation of this black body at $273°C$ will be

Options:

A. $16E$

B. $8E$

C. $4E$

D. $2E$

Answer: A

Solution:

Solution:
Given,
Rate of radiation of black body at $0°C, \frac{d\theta_1}{dt} = E$
Now, Stefan's law, rate at which heat is emitted is given as
$\frac{d\theta}{dt} = \sigma\varepsilon A T^4$
where, σ = Stefan constant and A = area of surface.
$\therefore \frac{d\theta_1}{dt} = (\sigma\varepsilon A) T_1^4$
or $E = (\sigma\varepsilon A)(0 + 273)^4$ .....(i)
Then, the rate of radiation of this black body at 273°C,

$\therefore \frac{d\theta_2}{dt} = (\sigma \varepsilon A)\, T_2^4$

Let, $\frac{d\theta_2}{dt} = X$

or $X = (\sigma \varepsilon A)\, T_2^4 (\because T_2 = 273°C)$

or $X = (\sigma \varepsilon A)(273° + 273°)$ .....(ii)

Dividing Eqs. (i) by (ii), we get

$\therefore \frac{E}{X} = \frac{(0° + 273)^4}{(273 + 273)^4} = \frac{(273)^4}{(546)^4} = \frac{(1)^4}{(2)^4} = \frac{1}{24} = \frac{1}{16}$

or $X = 16E$

Therefore, the rate of radiation of this black body at $273°C$ will be $16E$.

Question 46

A particle starts oscillating in simple harmonic motion from its equilibrium position with time period T. The ratio of KE and PE of the particle at time $t = \frac{T}{12}$ is

Options:

A. 3:1

B. 1:4

C. 4:1

D. 2:1

Answer: A

Solution:

Solution:

We know that, displacement at t is given by

$y = a \sin \omega t$ .....(i)

$\therefore$ Velocity, $v = a\omega \cos \omega t$ .....(ii)

Here, $a =$ amplitude

and $\omega =$ angular frequency $= \frac{2\pi}{T}$.

$\therefore \cos \omega t = \cos(\frac{360}{T} \times \frac{T}{12}) = \cos 30° = \frac{\sqrt{3}}{2}$

From Eq. (i), we get

Velocity, $v = \frac{\sqrt{3}}{2} a\omega$ .....(iii)

Kinetic energy $(KE) = \frac{1}{2} m v^2$

From Eq. (iii), we get

$= \frac{1}{2} m a^2 \frac{3}{4} \omega^2 = \frac{3}{8} m a^2 \omega^2$ .....(iv)

As we know that

Total energy $(TE) = \frac{1}{2} m a^2 \omega^2$ .....(v)

$\therefore$ Potential energy $= TE - KE$

$= \frac{1}{2} m a^2 \omega^2 - \frac{3}{4} m a^2 \omega^2$

$= \frac{1}{2} m a^2 \omega^2 [1 - \frac{3}{4}]$

$= \frac{1}{2} m a^2 \omega^2 \times \frac{1}{4} = \frac{1}{8} m a^2 \omega^2$ .....(iv)

Hence, $\frac{KE'}{PE} = \frac{\frac{3}{8} m a^2 \omega^2}{\frac{1}{8} m a^2 \omega^2} = 3:1$

Question 47

A simple harmonic motion has amplitude a and time period T. The maximums velocity will be

Options:

A. $\frac{4a}{T}$

B. $\frac{2a}{T}$

C. $2\pi\sqrt{\frac{a}{T}}$

D. $\frac{2\pi a}{T}$

Answer: D

Solution:

Solution:
Given,
Amplitude of a SHM = a
Time period of SHM $= T$
The displacement of a particle in case of SHM is given by,
$x = a\sin(\omega t + \phi)$
then the velocity of SHM is given as
$\therefore v = \frac{dx}{dt}$ or $v = \frac{d}{dt}[a\sin(\omega t + \phi)]$
or $v = a\omega[1 - \sin^2(\omega t + \phi)]^{1/2}$
or $= a\omega\left[1 - \frac{x^2}{a^2}\right]^{1/2}$
or $v = \omega\sqrt{a^2 - x^2}$
Now, at mean position $(x = 0)$ velocity is maximum.
i.e. $v_{max} = a\omega$ .....(i)
We know that, angular frequency oscillating particle is
$\omega = \frac{2\pi}{T}$ .....(ii)
Now, putting the value of omega is Eq. (i), we get
$v_{max} = a \times \frac{2\pi}{T}$
$v_{max} = \frac{2\pi a}{T}$
So, the maximum velocity of particle will be $\frac{2\pi a}{T}$.

Question 48

At what speed should a source of sound move, so that stationary observer finds the apparent frequency equal to half of the original frequency?

Options:

A. $\frac{v}{2}$

B. $2v$

C. $\frac{v}{4}$

D. v

Answer: D

Solution:

Solution:
Given,
The apparent frequency equal to half of the original frequency $v' = \frac{1}{2}v_0$
Now, the apparent frequency of observer is given as
$\therefore v' = \frac{v}{v + v_s} . v_0$
Here, $v' =$ apparent frequency of observer,
$v =$ speed of the sound,
$v_s =$ speed of the source of sound and
$v_0 =$ original frequency.
$\therefore v' = \frac{1}{2}v_0$
Then, $\frac{1}{2}v_0 = \frac{v}{v + v_S}v_0$ or $\frac{1}{2} = \frac{v}{v + v_s}$
or $v + v_s = 2v$
or $v_s = 2v - v$
or $v_s = v$
Therefore, speed of source of sound (v_s) more should be equal to the speed of sound (v), so that stationary observer finds the apparent frequency equal to half of the original frequency.

Question 49

In interference, the ratio of maximum intensity to the minimum intensity is 25 . The intensities of the sources are in the ratio

Options:

A. 25:1

B. 5:1

C. 9:4

D. 625:1

Answer: C

Solution:

Solution:
Given,
The ratio of maximum intensity to minimum intensity is $\frac{I_{\max}}{I_{\min}} = 25$
Now, in the interference,
$\therefore$ Intensities of two coherent sources is given by
$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

or $25 = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$
or $\sqrt{25} = \frac{(\sqrt{I_1} + \sqrt{I_2})}{(\sqrt{I_1} - \sqrt{I_2})}$
$5 = \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}$
or $5(\sqrt{I_1} - \sqrt{I_2}) = \sqrt{I_1} + \sqrt{I_2}$
or $5\sqrt{I_1} - \sqrt{I_1} = \sqrt{I_2} + 5\sqrt{I_2}$
or $\sqrt{I_1}(5-1) = \sqrt{I_2}(5+1)$
or $\frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{6}{4} \Rightarrow \frac{I_1}{I_2} = (\frac{6}{4})^2$
or $\frac{I_1}{I_2} = \frac{36}{16} \Rightarrow \frac{I_1}{I_2} = \frac{9}{4}$
Therefore, the intensities of the sources are in the ratio 9:4.

Question 50

Four light waves are represented by
(i) $y = a_1 \sin \omega t$
(ii) $y = a_2 \sin(\omega t + \phi)$
(iii) $y = a_1 \sin 2\omega t$
(iv) $y = a_2 \sin 2(\omega t + \phi)$
Interference fringes may be observed due to superimposition of

Options:

A. (i) and (ii)

B. (i) and (iii)

C. (ii) and (iii)

D. (ii) and (iv)

Answer: A

Solution:

Solution:
The interference fringes can be observed with coherent sources of light which have a constant initial phase difference and same frequency. Only the waves of option (a) satisfy this condition.

Question 51

Which of the following processes involves the roasting process?

Options:

A. $Fe_2O_3 + 3C \rightarrow 2Fe + 3CO$

B. $Al_2O_3.2H_2O \rightarrow Al_2O_3 + 2H_2O$

C. $ZnCO_3 \rightarrow ZnO + CO_2$

D. $2PbS + 3O_2 \rightarrow 2PbO + 2SO_2$

Answer: D

Solution:

Solution:
The process of heating the ore, strongly below its melting point $(m.p)$ in the presence of air (i.e. oxygen) is called roasting. Usually sulphide ores are converted to oxide of metal by this process.
Thus, PbS is converted to PbO by this method is the correct option.

Question 52

Match the following

I.	Bauxite	(a)	Cu
II.	Monazite	(b)	U
III.	Malachite	(c)	Th
IV.	Pitch blende	(d)	Al

Options:

A. I-d, II-c, III-a, IV-b

B. I-d, II-c, III-b, IV-a

C. I-b, II-c, III-d, IV-a

D. I-d, II-a, III-b, IV-c

Answer: A

Solution:

Solution:

I.	Bauxite	(d)	Al, aluminium is extracted from its ore $\left(Al_2O_3.xH_2O\right)$ (Bauxite)
II.	Monazite	(c)	Th, thorium is extracted from its ore (Monazite).
III.	Malachite	(a)	Cu, copper is extracted from its ore $\left[Cu_2CO_3\left(OH\right)_2\right]$
IV.	Pitch blende	(b)	U, uranium is extracted from its ore $\left(U_3O_8\right)$

Hence, (a) is the correct option.

Question 53

Which set of elements have nearly the same atomic radii?

Options:

A. Fe, Co, Ni, Cu

B. F, Cl, Br, I

C. Na, K, Rb, Cs

D. Li, Be, B, C

Answer: A

Solution:

Solution:
Due to lanthanide contraction, the inner electrons of $3d$ -orbitals offer poor shielding effect, which increases the effective nuclear charge and therefore, element belongs to $3d$ -series (i.e. Fe, Co, Ni and Cu) are nearly of same atomic radii (size).
Hence, option (a) is the correct answer.

Question 54

CO forms a volatile compound with

Options:

A. Cu

B. Al

C. Ni

D. Na

Answer: C

Solution:

Solution:
As Ni can form $Ni(CO)_4$ easily, which is of volatile nature and can give pure Ni as follows

$$Ni + 4CO \xrightarrow{350K} \underset{\text{(Nickel compound)}}{Ni(CO)_4} \xrightarrow{450-470K} Ni + CO$$

Hence, option (c) is the correct answer.

Question 55

The correct order of first electron affinity of O, S and Se is

Options:

A. $O > S > Se$

B. $Se > O > S$

C. Se $> S > 0$

D. $S > O > Se$

Answer: C

Solution:

Solution:
Electron affinity increases along the period from left to right and is decreases as we move down the group. The values of electron affinity for the given species are as follows:
S (sulphur) $= 200 kJ/mol$
Se (selenium) $= 195 kJ/mol$
O (oxygen) $= 141.1 kJ/mol$
Hence, correct order is : $S > Se > O$, but as electron affinity of $S \approx$ Se, we can also write it as $Se > S > O$ and option (c) will be the correct answer.
Note Electron affinity of $S > O$, due to very small size of oxygen having lone pair of electrons, which offer more repulsion for the incoming electron.

Question 56

The volume of ' 10 vol' of H_2O_2 required to liberate $500 mL\ O_2$ at NTP is

Options:

A. $125 mL$

B. $500 mL$

C. $50 mL$

D. $100 mL$

Answer: C

Solution:

Solution:
10 vol. H_2O_2 means 1 vol (in mL or L) of H_2O_2 will give $10L$ (or $10mL$) of oxygen O_2 at NTP/STP.
Now,
$\because$ 10 vol. of O_2 is given by -1 volume of H_2O_2
$\therefore$ 500 vol. of O_2 is given by $= \frac{500 \times 1}{10} = 50$ vol
Thus, $50mL$ of H_2O_2 is required to get $500mL$ of O_2 at NTP/STP and option (c) is the correct answer.

Question 57

The Nessler's reagent contains

Options:

A. $[HgI2]^{2-}$

B. $[Hg_2]^{2+}$

C. $[Hg]^{2+}$

D. $[HgI_4]^{2-}$

Answer: D

Solution:

Solution:
Nessler's reagent is K_2HgI_4. It is a basic solution of potassium mercuric iodide which gives reddish brown ppt. of millon's base $[H_2N-HgO-HgI]$ with NH_3, as follows
$NH_3 + 2K_2HgI_4 + 3KOH \longrightarrow \underset{(\text{Iodide of millon's base}) \text{ Brown ppt.}}{H_2N-HgO-HgI} + 7KI + 2H_2O$
Thus, it can give $[HgI_4]^{2-}$ ions so, option (d) is the correct answer.

Question 58

XeF_6 on complete hydrolysis gives

Options:

A. XeO_2

B. XeO_4

C. Xe

D. XeO_3

Answer: D

Solution:

Solution:
Complete hydrolysis of XeF_6 gives XeO_3 which occurs as follows
$XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$
Hence, (d) is the correct option.

Question 59

For which one of the following ions, the colour is not due to $d-d$ transition?

Options:

A. $[Cu(NH_3)_4]^{2+}$

B. CrO_4^{2-}

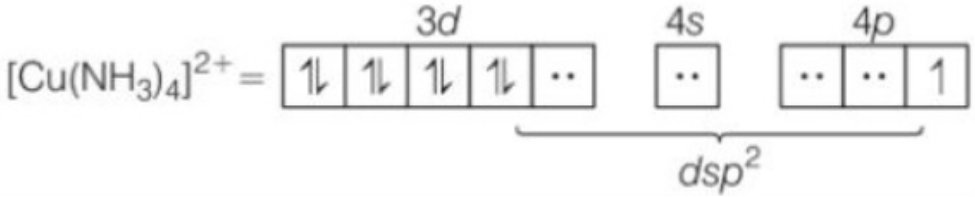
C. CoF_6^{3-}

D. $[Ti(H_2O)_6]^{3+}$

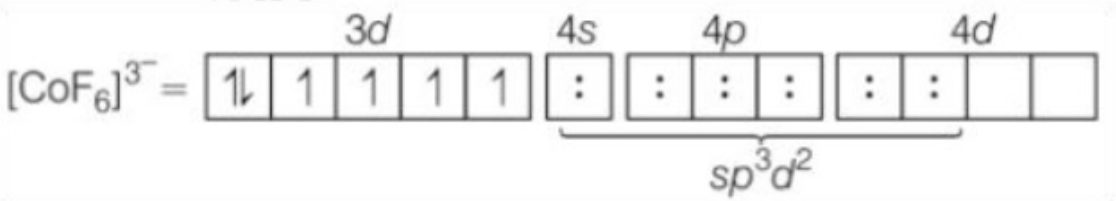
Answer: B

Solution:

Solution:
(a) In $[Cu(NH_3)_4]^{2+}$, in the presence of ligand NH_3, Cu^{2+} ions show the following electronic configuration.



Note In which one electron of $3d$ is transfered to $4p$.
The above complex show colour due to $d-d$ transition.
(b) In CrO_4^{2-}, Cr in compound CrO_4^{2-} offer colour due to charge transfer from metal to π^* orbitals of ligand and not due to $d-d$ transition.
(c) In $[CoF_6]^{3-}$, in the presence of weak ligand (i.e. fluorine), Co^{3+} ions show the following electronic configuration,



The above complex has unpaired electrons and can show colour due to $d-d$ transition.
(d) In $[Ti(H_2O)_6]^{3+}$

The presence of weak ligand i.e., (H_2O), Ti^{3+} ion show one unpaired electron and offer colour due to $d-d$ transition.
Hence, option (b) is the correct answer.

Question 60

Which is known as philosopher's wool?

Options:

A. ZnO

B. CdO

C. BaO

D. HgO

Answer: A

Solution:

Solution:
Zinc oxide (ZnO) is a white powder that is insoluble in water (H_2O) and is used as an additive in many products e.g. rubber.
It is also known as 'philosopher's wool.
Hence, option (a) is the correct answer.

Question 61

In the reaction, $2CuCl_2 + 2H_2O + SO_2 \longrightarrow A + H_2SO_4 + 2HCl$ A is

Options:

A. CuS

B. Cu

C. $CuSO_4$

D. Cu_2Cl_2

Answer: D

Solution:

Solution:
The reaction between $CuCl_2$ and SO_2 occurs as follows
$2CuCl_2 + 2H_2O + SO_2 \longrightarrow \underset{A}{Cu_2Cl_2} + H_2SO_4 + 2HCl$
Hence, (a) is Cu_2Cl_2 or $2CuCl$ so, option (d) is the correct answer.

Question 62

The hybridisation of $[Ni(CN)_4]^{2-}$ and $[Ni(NH_3)_6]^{2+}$ ions are respectively.

Options:

A. sp^3, d^2sp^3

B. dsp^2, sp^3d^2

C. dsp^2, d^2sp^3

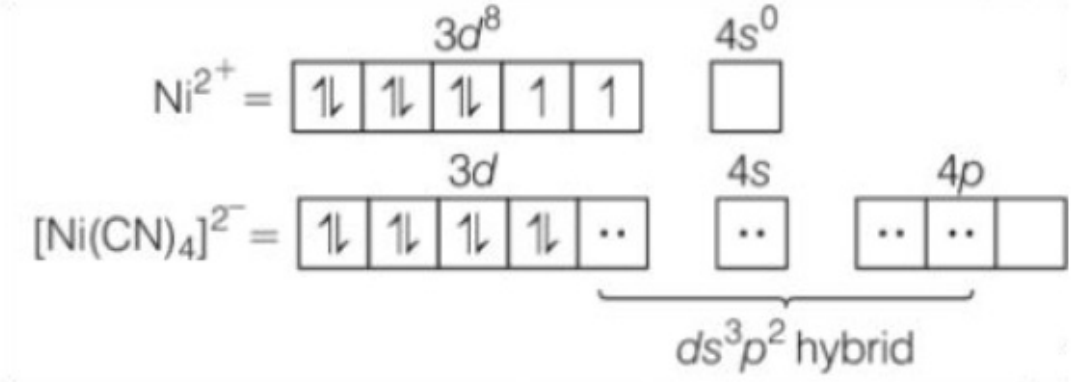
D. d^2sp, d^2sp^3

Answer: B

Solution:

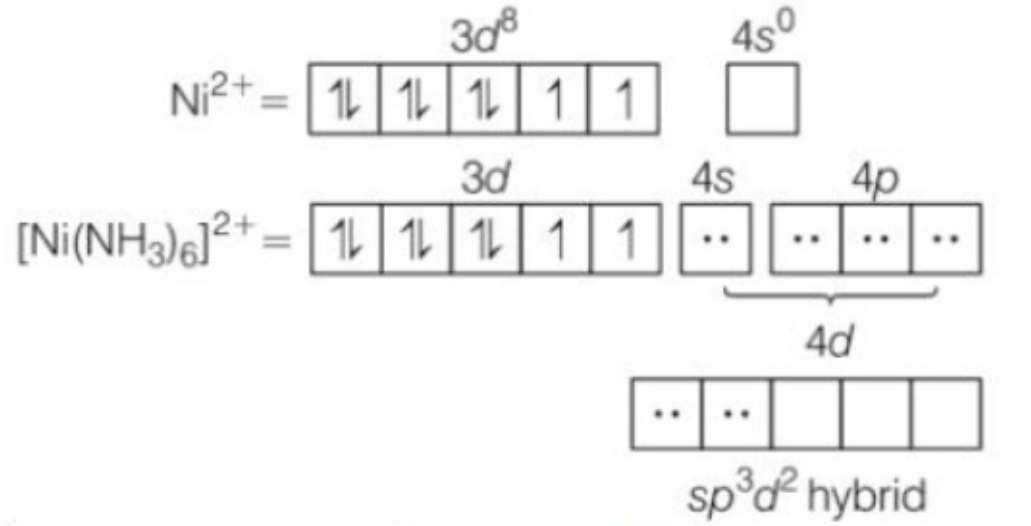
Solution:

Hybridisation of $\left[Ni(CN)_4\right]^{2}$ is calculated as follows



Hence, hybridisation of $\left[Ni(CN)_4\right]^{2}$ is dsp^2 (due to the presence of strong field ligand)

Also, hybridisation of $\left[Ni(NH_3)_6\right]^{2+}$ is shown as follows



Hence, hybridisation of $\left[Ni(NH_3)_6\right]^{2-}$ is sp^3d^2 $\left[(NH_3)\right.$. is a weak field ligand $\left.\right]$

Thus, option (b) is the correct answer.

Question 63

Aqua-regia reacts with Pt to yield

Options:

A. $Pt(NO_3)_4$

B. H_2PtCl_6

C. $PtCl_2$

D. $PtCl_4$

Answer: B

Solution:

Solution:
Aqua-regia is a mixture of HCl and HNO_3 in the ratio of $3:1$. When aqua-regia react with Pt (platinum), we get H_2PtCl_6 as follows:
$3Pt + 4HNO_3 + 18HCl \longrightarrow 3H_2PtCl_6(aq) + 4NO(g) + 8H_2O(l)$
Hence, option (b) is the correct.

Question 64

Which ion is paramagnetic?

Options:

A. $Ni(NH_3)_6]^{2+}$

B. $[Ni(CN)_4]^{2-}$

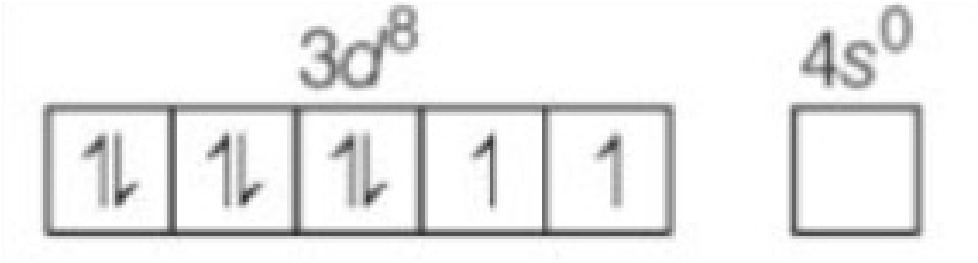
C. $Ni(CO)_4$

D. $[Co(NH_3)_6]^{3+}$

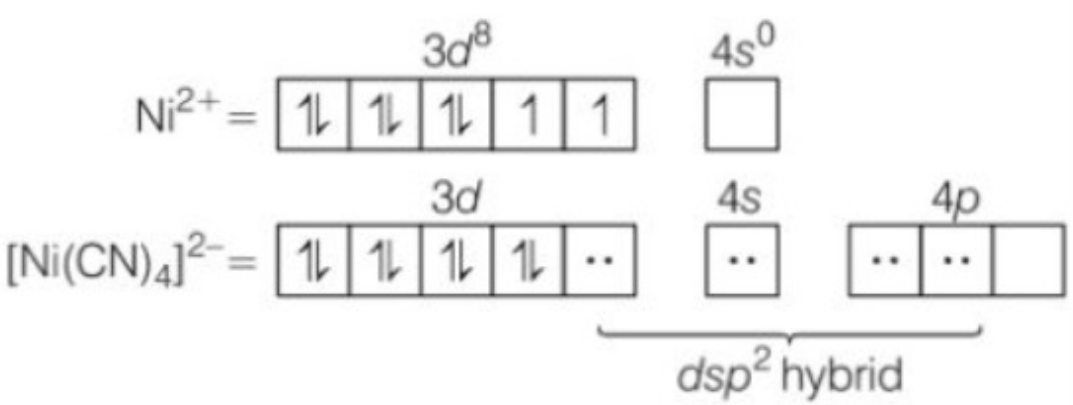
Answer: A

Solution:

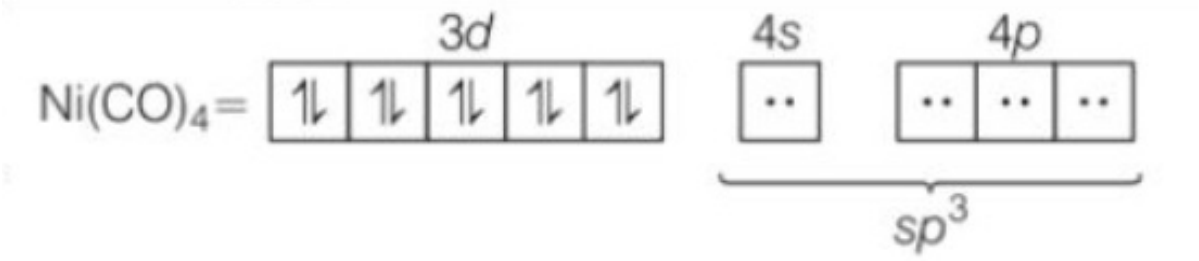
Solution:
(a) In $Ni(NH_3)_6]^{2+}$, Ni is present as Ni^{2+} ion having outer electronic configuration.



Here, NH_3 is regarded as a weak field ligand which can not go for pairing when present in d -subshell.
Due to presence of unpaired electrons it is paramagnetic in nature.
(b) In $[Ni(CN)_4]^{2-}$, Ni is present as Ni^{2+} ion and CN is a strong field ligand,
Thus, has the following electronic configuration.



The above configuration has no unpaired electron, thus, is diamagnetic in nature.
(c) In $[Ni(CO)_4]$, oxidation state of Ni is zero and show the following electronic configuration.



Note Here, electrons of 4 s migrate towards 3d and give sp^3 hybridisation.
As $Ni(CO)_4$ has no unpaired electron so, it is diamagnetic in nature.
(d) In $[Co(NH_3)_6]^{3+}$, Co is in Co^{3+} state and show the electronic configuration as follows
Due to presence of NH_3 as ligand, $[Co(NH_3)_6]^{3+}$ has no unpaired electron and therefore, is diamagnetic in nature.
Hence, option (a) is the correct.

Question 65

Which combines with Fe^{2+} to form brown complex?

Options:

A. NO

B. N_2O

C. N_2O_3

D. N_2O_5

Answer: A

Solution:

Solution:
When Fe^{2+} i.e., $[Fe(H_2O)]SO_4$ or $FeSO_4(aq)$
react with NO (nitric oxide), it gives brown ring complex as follows
$[Fe(H_2O)_6]SO_4 + NO \longrightarrow \underset{\text{brown complex}}{[Fe(H_2O)_5NO]SO_4} + H_2O$
The brown ring test is used to identify the presence of NO_3^- ions.
Hence, option (a) is the correct answer.

Question 66

How many isomers of $C_5H_{11}OH$ will be primary alcohols?

Options:

A. 2

B. 3

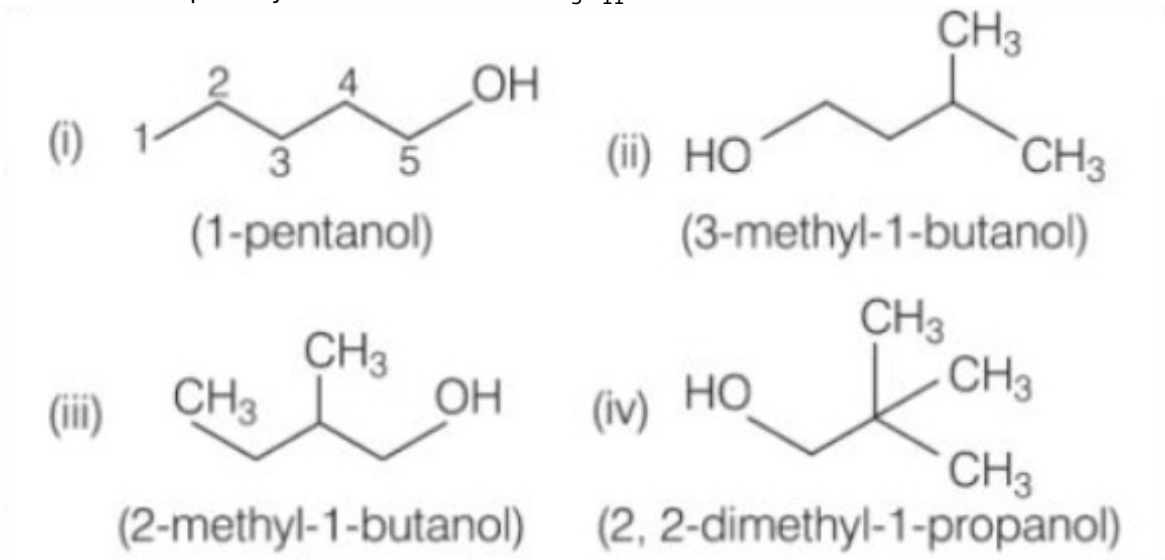
C. 4

D. 5

Answer: C

Solution:

Solution:
The isomers of primary alcohol with formula C_5H_{11}. OH are as follows



Hence, we have 4 (four) isomers and option (c) is the correct answer.

Question 67

IUPAC name of lowest molecular mass alkane containing chiral carbon is

Options:

A. 3-methylhexane

B. 2-methylhexane

C. 2, 4-dimethylpentane

D. 2-methylpentane

Answer: A

Solution:

Solution:
Key Point The carbon, which is bonded with four different atoms/groups is known as chiral carbon.
The structure of the given IUPAC names are as follows:
(a) 3 -methylhexane

$$\overset{6}{C}H_3-\overset{5}{C}H_2-\overset{4}{C}H_2-\underset{\displaystyle CH_3}{\overset{3}{C}H}-\overset{2}{C}H_2-\overset{1}{C}H_3$$

or $C_3H_7-\overset{H}{\underset{CH_3}{\overset{\bullet}{C}}}-C_2H_5$

(It has one chiral carbon atom, marked with dot.)
(b) 2 -methylhexane

$$\overset{6}{H_3}C-\overset{5}{C}H_2-\overset{4}{C}H_2-\overset{3}{C}H_2-\overset{2}{\underset{\displaystyle CH_3}{\overset{H}{C}}}-\overset{1}{C}H_3$$

It has no chiral carbon atom.
(c) 2, 4-dimethyl pentane

$$\overset{5}{H_3}-\underset{\displaystyle CH_3}{\overset{4}{C}H}-\overset{3}{C}H_2-\underset{\displaystyle CH_3}{\overset{2}{C}H}-\overset{1}{C}H_3$$

It has also no chiral carbon atom.
(d) 2 -methyl pentane

$$H_3\overset{5}{C}-\overset{4}{C}H_2-\overset{3}{C}H_2-\underset{\displaystyle CH_3}{\overset{2}{C}H}-\overset{1}{C}H_3$$

It has also no chiral carbon atom.
Thus, option (a) is the correct answer.

Question 68

64g of an organic compound contains 24g of carbon, 8 g of hydrogen and the rest oxygen. The empirical formula of the compound is

Options:

A. CH_2O

B. C_2H_4O

C. CH_4O

D. C_2H_8O

Answer: C

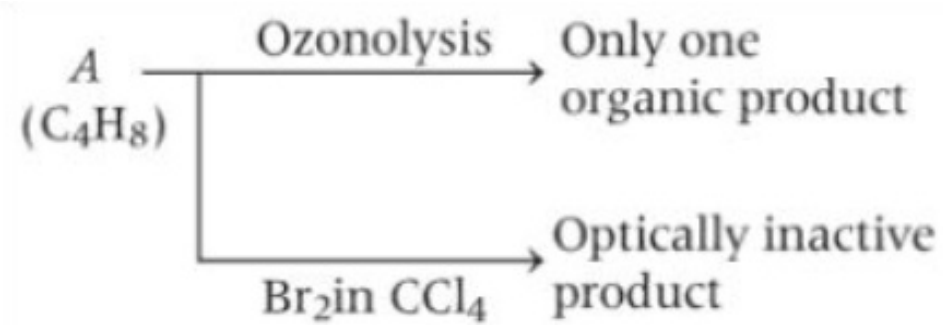
Solution:

Solution:
Key Point The simplest whole number ratio of the elements, present in any compound is called to empirical formula
Given
Total mass of an organic compound $= 64g$
Carbon $(C) = 24g$
Hydrogen $(H) = 8g$
Oxygen $(O) = 64 - (24 + 8) = 32g$
Thus, empirical formula is

Element	At weight (M)	Given weight (W)	No. of moles $(n)\left[=\frac{W}{M}\right]$
C	12	24	24/12=2
H	1	8	8/1=8
O	16	32	32/16=2

Hence, formula $= C_2H_8O_2$ and its empirical formula $= CH_4O$ and option (c) is the correct answer.

Question 69



A is

Options:

A. cis-but-2-ene

B. trans-but-2-ene

C. but-1-ene

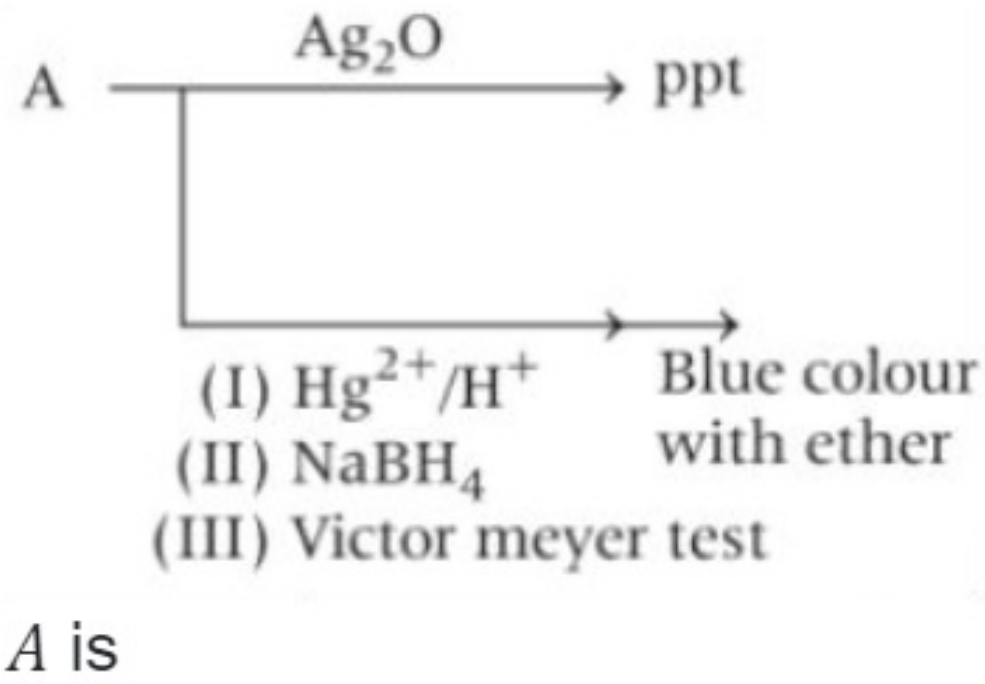
D. 2-methylpropene

Answer: B

Solution:

Solution:
(i) C_4H_8 (an alkene) on ozonolysis only give one organic product means it has the double bond as follows
$H_3C - CH = CH - CH_3$ i.e., show i.e., Symmetry about $C = C$
(ii) As trans C_4H_8
i.e., trans but-2-ene can give Meso compound with Br_2 in CCl_4, which is optically inactive.
Hence, option (b) is the correct answer.
Note cis but-2-ene give $\pm 2, 3$ dibromobutane on treatment with Br_2 in CCl_4.

Question 70



A is

Options:

A. $CH \equiv CH$

B. $CH_3 - C \equiv CH$

C. $CH_3 - C \equiv CH - CH_3$

D. $CH_2 = CH_2$

Answer: B

Solution:

Solution:
(i) $CH_3 - C \equiv CH$ can provide white ppt. with Ag_2O, as it can give aldehyde as intermediate product.
(ii) $CH_3 - C \equiv CH$ can give aldehyde on treatment with Hg^{2+} / H^+, which on reduction with $NaBH_4$ gives 2° -alcohol. The 2° -alcohol gives blue colour with ether during victor meyer test.
(i) $CH_3 - \underset{A}{C} \equiv CH \xrightarrow{Ag_2O} CH_3 - CH_2 - CHO$
(ii) (I) $CH_3 - C \equiv CH \xrightarrow{Hg^{2+} / H^+} CH_3 - CH_2 - CHO$
(II) $CH_3 - CH_2 - CHO \xrightarrow[\text{Reduction}]{NaBH_4} CH_3 - \underset{OH}{CH} - CH_3$
$(2° - alcohol)$
(III) $CH_3 - \underset{OH}{CH} - CH_3 \xrightarrow[\text{test}]{\text{Victor meyer}}$ blue colour with ether.
Hence, (b) is the correct option.

Question 71

Olefin 'A' gives 2 mol acetone and glyoxal on ozonolysis. The structure of 'A'.

Options:

A. 2, 4-dimethylhex-2, 4 -diene

B. 3,4 -dimethylhex-2, 4 -diene

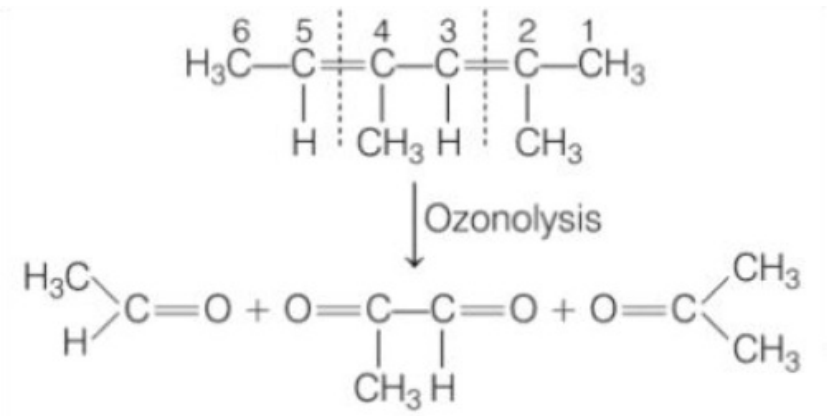
C. 3,4 -dimethylhex-1, 5 -diene

D. 2, 5-dimethylhex-2, 4-diene

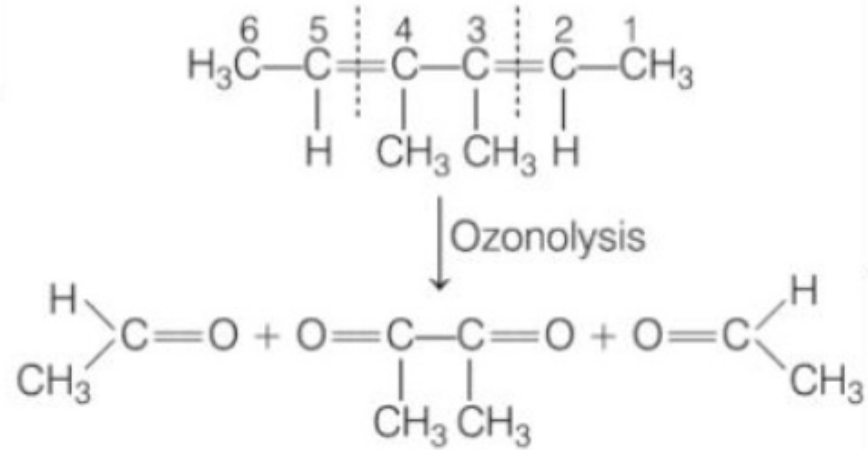
Answer: D

Solution:

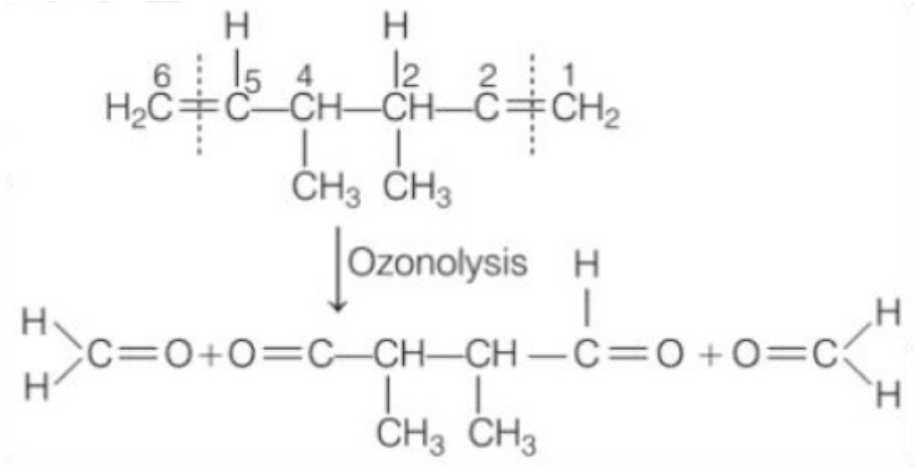
Solution:
Key Point Olefins (alkenes) give respective aldehydes/ketones on ozonolysis. To find the parent alkene that gives related aldehyde/ketone (i.e., moles of acetone and one mole of glyoxal), we remove the double bond from the given structure and add = 0 in the obtained fragment (i.e., in place of double bond), to get the required aldehyde/ketone.
The products formed on the ozonolysis of alkene are as follows:
(a) 2, 4 dimethylhex 2,4 -diene



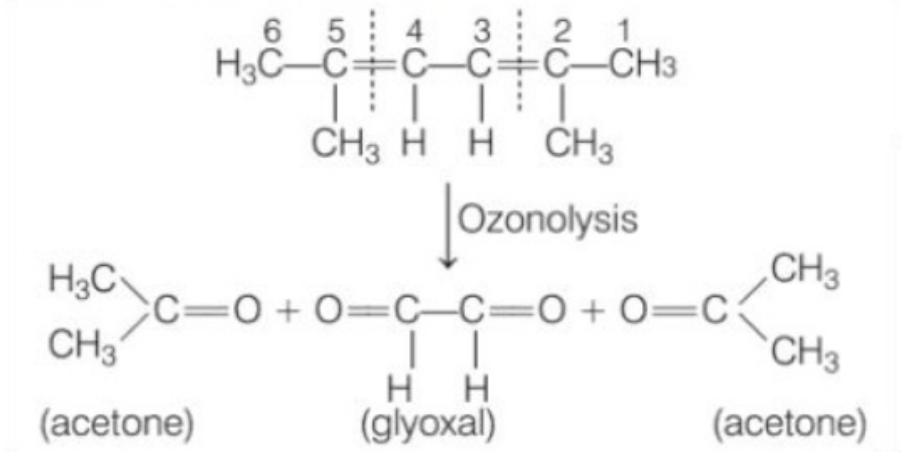
(b) 3,4 -dimethylhex 2,4 diene



(c) 3, 4 -dimethylhex 1,5 diene



(d) 2, 5 dimethylhex 2,4 , diene



Hence, solution (d) is the correct option.

Question 72

Which of the following compound does not show geometrical isomerism?

Options:

A. Crotonic acid

B. Citraconic acid

C. Cis-platin

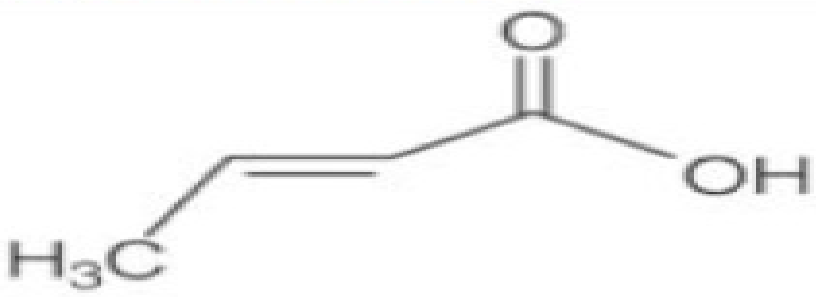
D. Butanone-2

Answer: C

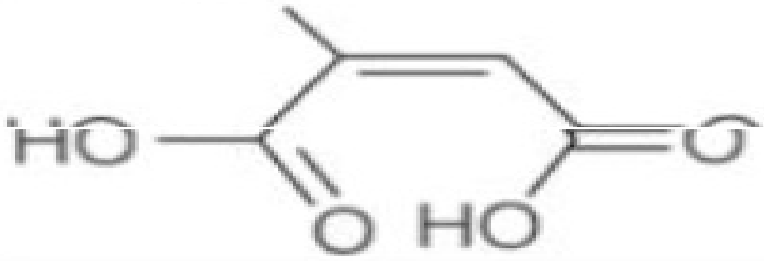
Solution:

Solution:
Conditions of geometrical isomerism:
In order to represent the geometrical isomerism by the compound, the two groups should be different lies at each stereo centre, in which geometrical isomerism is observed.

(a) Crotonic acid

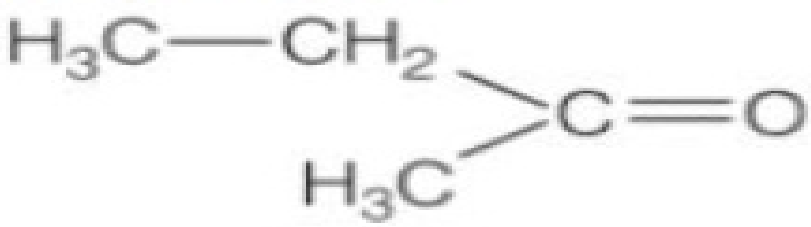


(b) Citraconic acid



(c) Cis-platin

(d) Butanone-2



In these compounds, Cis-platin does not show geometrical isomerism. Thus, the option (c) is correct.

Question 73

Butene- 2 $\xrightarrow[\text{2. alc.}KOH,\Delta]{\text{1.}Br_2}[A]\xrightarrow[H_3^+O]{O_3}[B]$

Compounds A and B are

Options:

A. but-1, 3-diene, formaldehyde

B. allene, formaldehyde

C. butyne-2, acetic acid

D. butyne-1, formic acid

Answer: C

Solution:

Solution:

Given reaction.

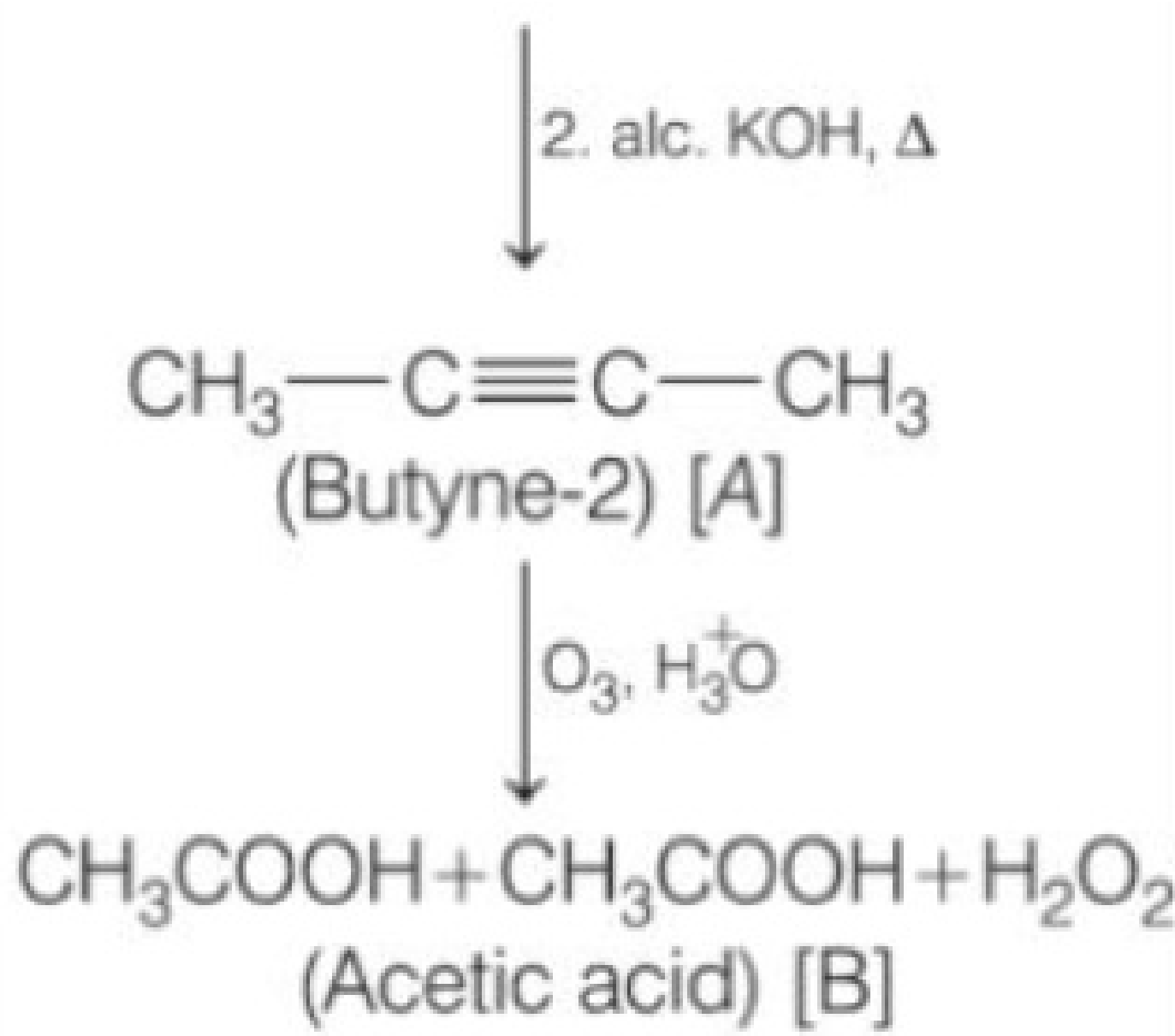
$$\text{Butene- 2}\xrightarrow[\text{2. alc.}KOH,\Delta]{\text{1.}Br_2}[A]\xrightarrow[H_3^+O]{O_3}[B]$$

$CH_3-CH=CH-CH_3$

(Butene-2)

$\downarrow$ 1. Br_2

$CH_3-CH(Br)-CH(Br)-CH_3$



So, in this reaction the product [A] is butyne- 2 and product [B] is acetic acid.

Question 74

1.2g of Mg is treated with 100mL of 1M H_2SO_4. Molar concentration of the H_2SO_4solution after complete reaction will be

Options:

A. 0.20M

B. 0.005M

C. 0.10M

D. 0.5M

Answer: D

Solution:

Solution:

Given,
mass of $Mg(W) = 1.2g$
molar mass of $Mg(M) = 24g$
volume of $1M H_2SO_4 = 100mL$
Thus,
$\because$ moles (n) of $Mg = \frac{W}{M}$
$= \frac{1.2}{24} = 0.05$ mole
and moles (n) of H_2SO_4 in $100mL$
solution of $1MH_2SO_4 = M \times V_{(L)}$
$= \frac{1 \times 100}{1000} = 0.1$ moles.
Where, ($V =$ volume of solution)
Therefore, H_2SO_4 left after treated with $100mL$ of $1MH_2SO_4 = 0.1 - 0.05$
i.e., $n' = 0.05$ moles
Thus, concentration of H_2SO_4 (left)
$= \frac{n' \times 1000}{V} = \frac{0.05 \times 1000}{100} = 0.5M$
Hence, option (d) is the correct answer.

Question 75

Diethyl ether $\xrightarrow[\text{2. KCN}]{\text{1. HI},\Delta}$ [A] $\xrightarrow{\text{LiAlH}_4}$ [B]

Compound [B] gives carbylamine test. What are the structures of [A] and [B] ?

Options:

A. Ethane nitrile, ethylamine

B. Ethane amine, acid

C. Propane nitrile, propanol

D. Propane nitrile, propyl amine

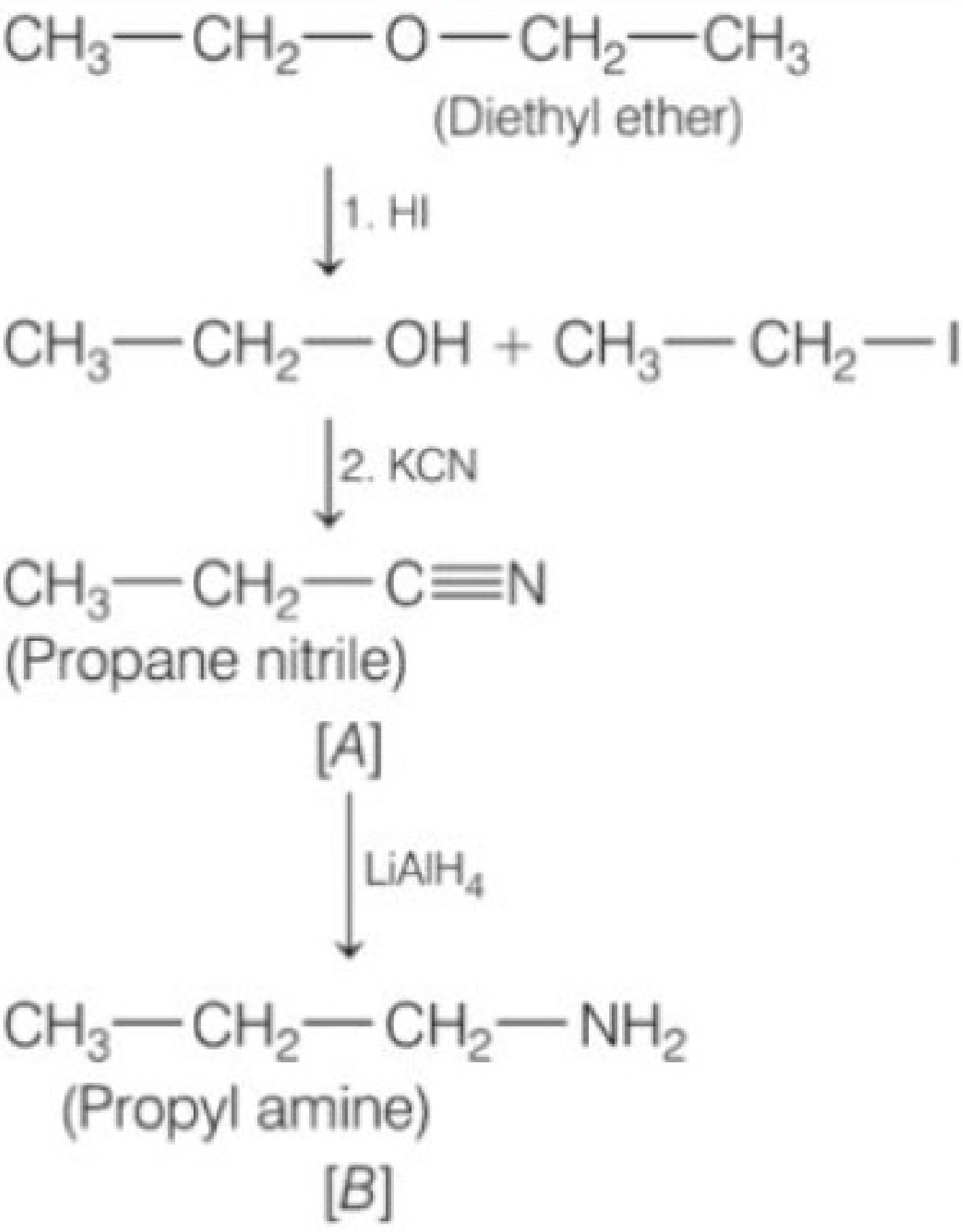
Answer: D

Solution:

Solution:

Given reaction,

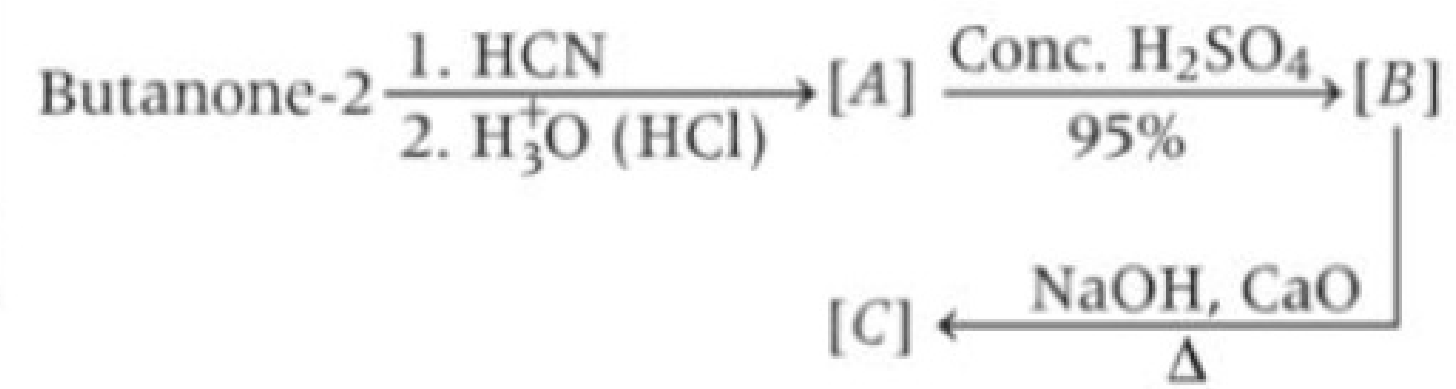
$$\text{Diethyl ether} \xrightarrow[2.\ KCN]{1.\ HI,\Delta} [A] \xrightarrow{LiAlH_4} [B]$$



In this reaction the product [A] is propane nitrile and product [B] is propyl amine.

Question 76

Complete the following reaction. The structure of final product [C] is



Options:

A. pentene-2

B. butene-2

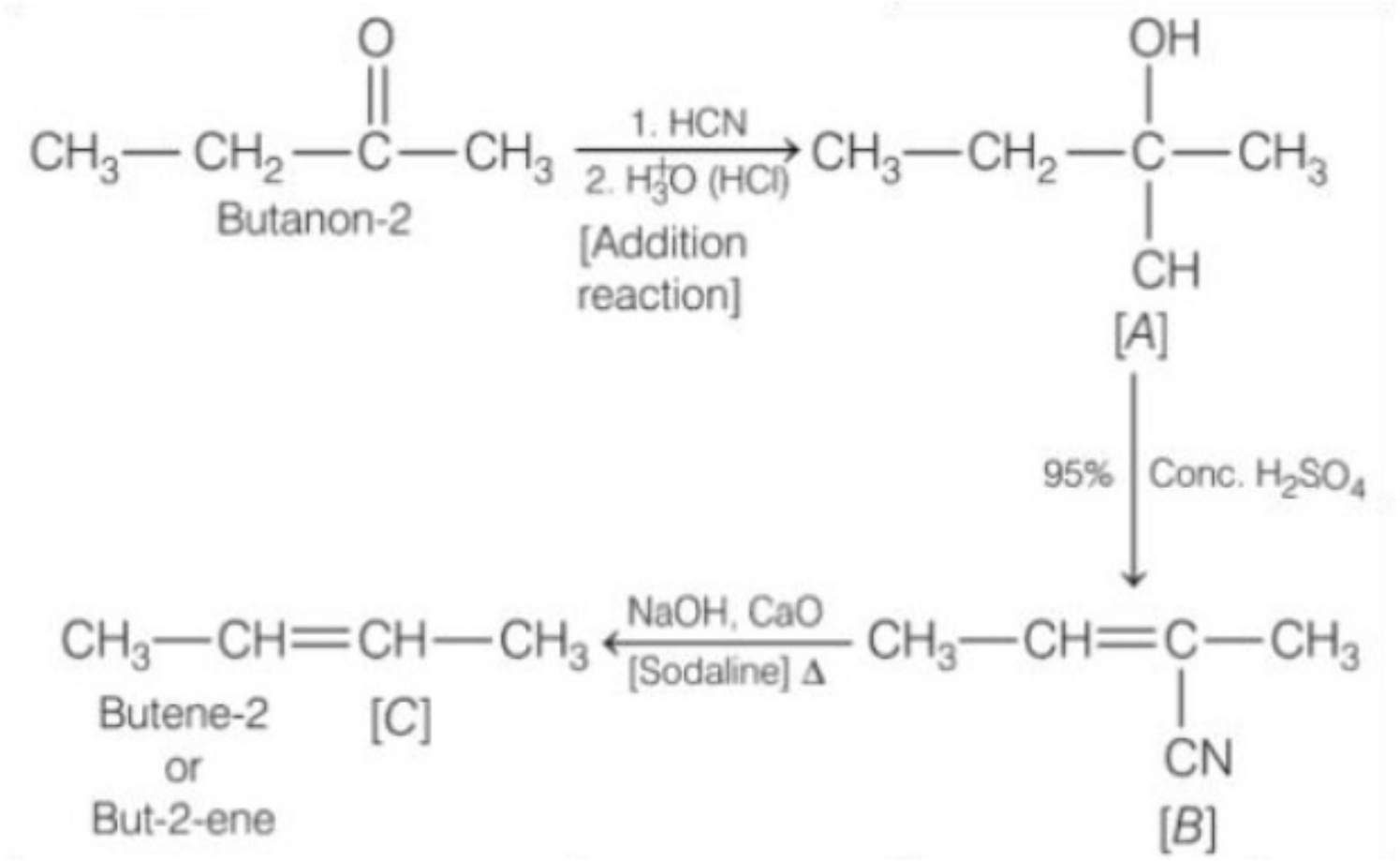
C. pentene-1

D. butene-1

Answer: B

Solution:

Solution:



From the given reaction, final product [*C*] is butene- 2 or but-2-ene.

Question 77

Which pair of following compounds gives intra cannizzaro's reaction?

Options:

A. Glyoxal, phthaldehyde

B. Formaldehyde, acetone

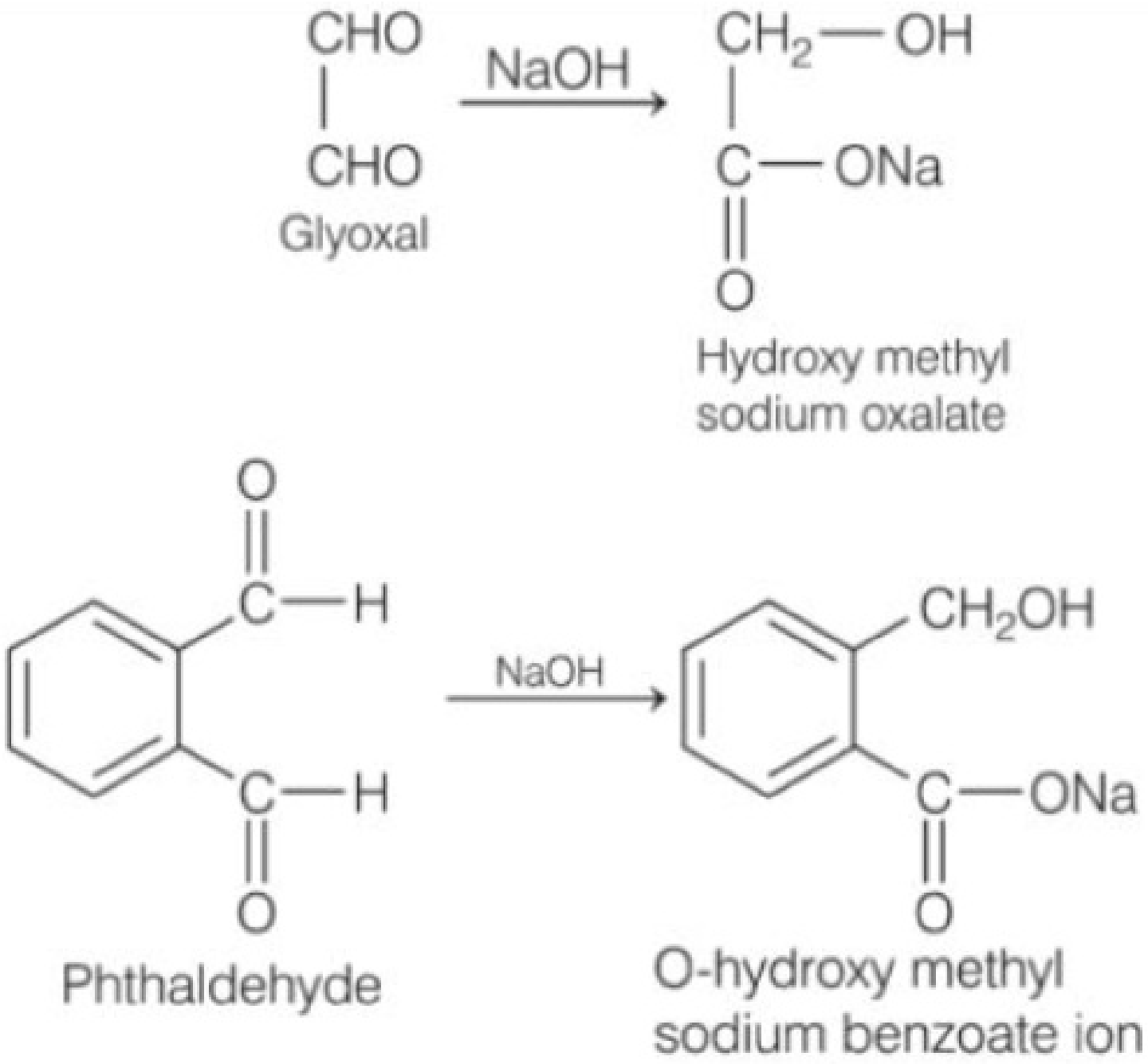
C. Benzaldehyde, formaldehyde

D. Benzaldehyde, glyoxal

Answer: A

Solution:

Solution:
Intra cannizzaro's reaction: Glyoxal and phthaldehyde undergoes an intra molecular cannizzaro reaction in aqueous base.



Thus, the option (a) is correct.

Question 78

Compound [X] M.F C_7H_7 NO reacts with $Br_2 - KOH$ to yield compound [Y] which gives mustard oil reaction. The structures of [X] and [Y] are

Options:

A. acetamide, ethylamine

B. benzamide, aniline

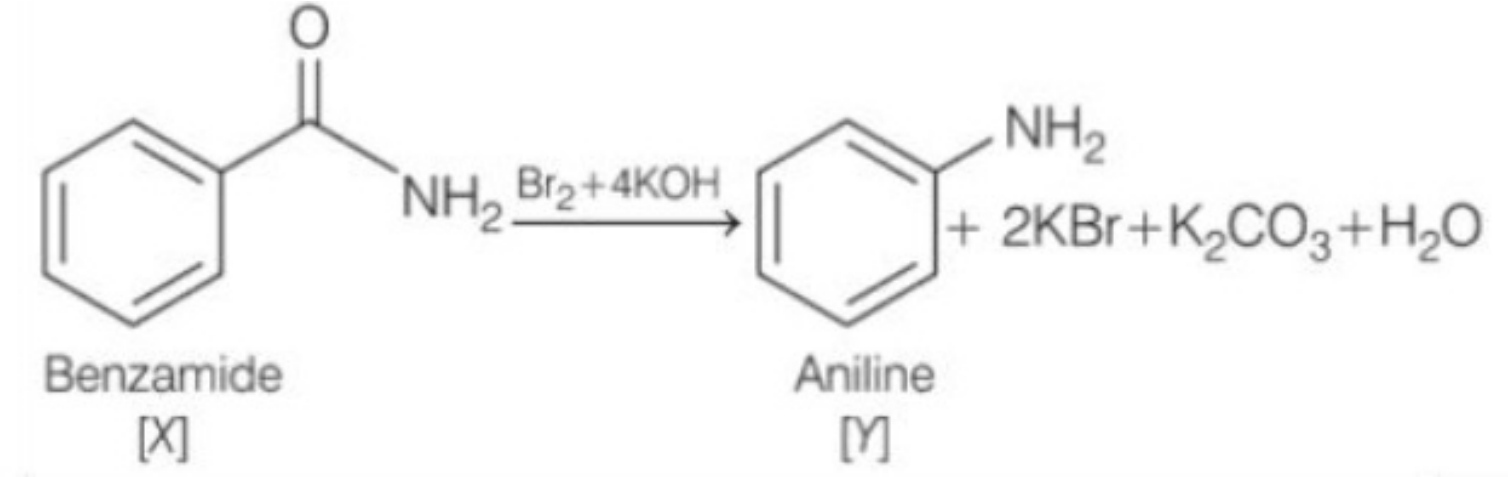
C. benzamide, anicidine

D. nitrosobenzene, aniline

Answer: B

Solution:

Solution:
When compound $[X]$ C_7H_7 NO reacts with $Br_2 - KOH$ to yield compound $[Y]$ which gives mustured oil reaction.
The structures of $[X]$ and $[Y]$ are



Question 79

Which one of the following is biopolymer?

Options:

A. Dextron

B. P C T F E

C. Nylon-6, 6

D. Neoprene

Answer: A

Solution:

Solution:
Biopolymers are polymers produced by living organisms. Biopolymers contain monomeric units that are covalently bonded to form larger structures. These are three main classes of biopolymers, that is monomers, polypeptides and polynucleotides.
Other examples of biopolymers include suberin, melanin and lignin. Dextron is a copolymer of glycolic acid and lactic acid.

Question 80

Which polymer is used in electrical insulator?

Options:

A. Buna-N

B. Dextron

C. PHBV

D. Styrene

Answer: D

Solution:

Solution:
Polymer insulator is an electrical device consisting of insulation section made of polymer materials and metal fittings.
There are many polymers which used for insulation.
For example, epoxy resins, polyster, polyethylene, styrene and phenol formaldehyde (bakelite) etc.
So, option (d) is correct.

Question 81

Which compound gives saccharic acid when it reacts with conc. HNO_3 ?

Options:

A. Sucrose

B. Starch

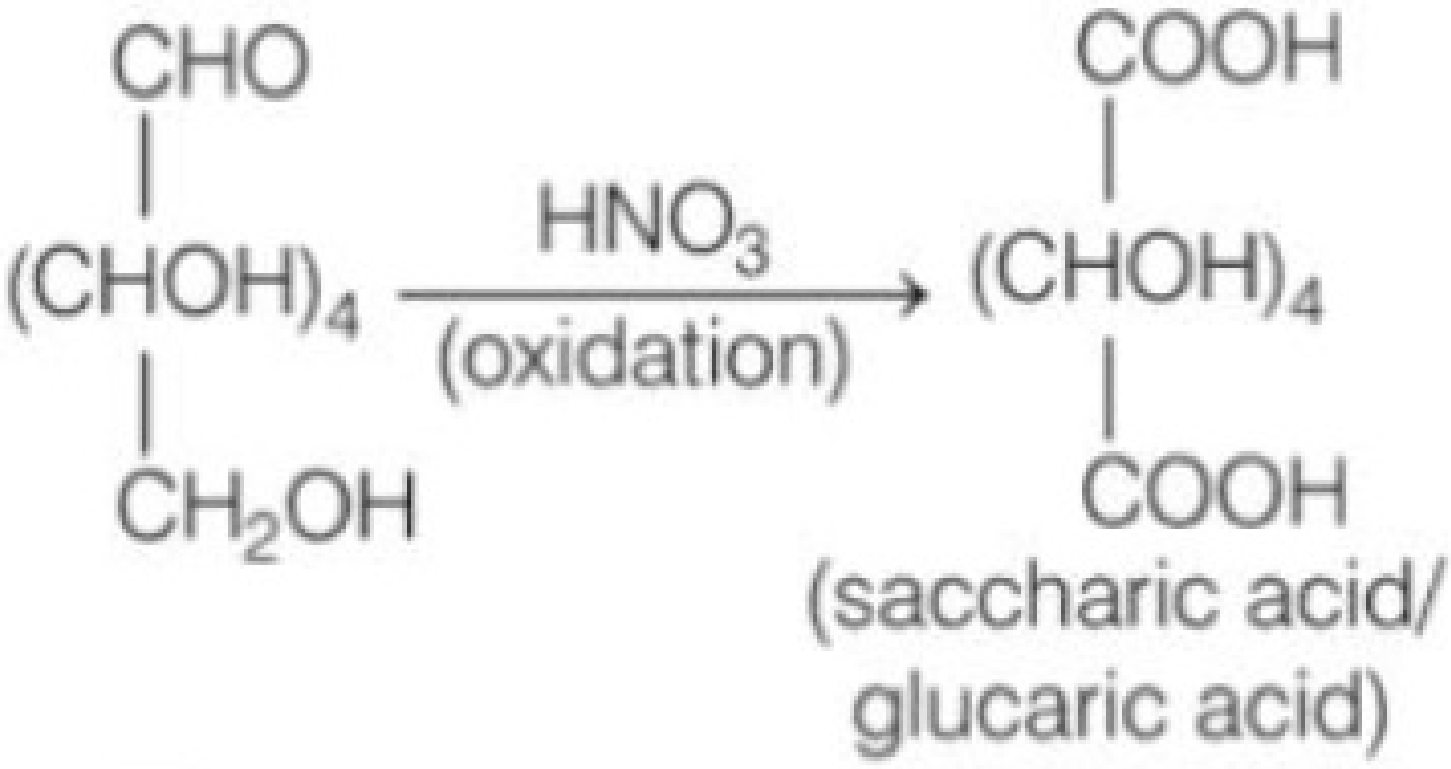
C. Maltose

D. Glucose

Answer: D

Solution:

Solution:
Glucose when reacts with HNO_3 (conc.), it gives saccharicacid/gluceric acid as main product, the reaction occurs as follows



Hence, option (d) is the correct answer.

Question 82

Which statements are correct?
1. Lysozyme present in bacterial cell, on hydrolysis gives NAM and NAG.
2. Hemoglobin S has two α and two β chain.
3. Sangar reagent is used to protect C-terminus of protein.
4. Vitamin E, Vitamin K and coenzyme Q are derived lipids

Options:

A. $1, 2, 4$

B. $2, 3, 4$

C. $1, 3, 4$

D. $1, 2, 3$

Answer: D

Solution:

Solution:
Correct statements are 1,2 and 3 .
1. Lysozyme is a special enzyme found in tears, saliva, sweat and other body fluids. It is a glycoside hydrolase that catalyses the hydrolysis of 1,4 -beta-linkage between N-acetylmuramic acid (NAM) and N-acetyl-D-glucosamine (NAG) residues, which is the major component of gram positive bacterial cell wall.
2. The hemoglobin S molecule is made up of four polypeptide chains that is two alpha and two beta chain.
3. Sanger's reagent is 1-fluoro-2,4-dinitrobenzene. It reacts with N-terminal amino acid of polypeptides. This can be helpful for sequencing proteins.
4. Derived lipids are the substances derived from simple and compound lipids by hydrolysis. The most common derived lipids are steroids, terpenes and carotenoids. So, vitamin mathrm K and coenzyme Q are derived lipids.

Question 83

If uncertainty of position and momentum are equal then uncertainty of velocity is

Options:

A. $\sqrt{\frac{h}{\pi}}$

B. $\sqrt{\frac{h}{2\pi}}$

C. $\frac{1}{2m}\sqrt{\frac{h}{\pi}}$

D. $\frac{h}{2\pi}$

Answer: C

Solution:

Solution:
According to uncertainty principle, It is impossible to determine simultaneously, the exact position and momentum (or velocity) of an moving electron.
Mathematically $\Delta x . \Delta p \geq \frac{h}{4\pi}$
Given, when,
$\Delta x = \Delta p$
where, $\Delta x =$ uncertainty in position.
$\Delta p =$ uncertainty in momentum and
$\Delta p = m \Delta v$
where, $\Delta v =$ uncertainty in velocity and
$m =$ mass of electron
$m . \Delta = \Delta x$
or, $m^2 \Delta v^2 = \frac{h}{4\pi}$
or $\Delta v^2 = \frac{1}{2m} \sqrt{\frac{h}{\pi}}$
Hence, option (c) is the correct answer.

Question 84

The introduction of a neutron into the nucleus of an atom would lead to a change in

Options:

A. atomic number

B. atomic mass

C. chemical nature of the atom

D. number of electron

Answer: B

Solution:

Solution:
(b) Mass number is the sum of masses of all protons and neutrons present in the nucleus of an atom while atomic number is the sum of all protons present in the nucleus.
Electrons are negatively charges particles having negligible mass moving around the nucleus.
When a neutron is added to the nucleus of an atom only atomic mass of the atom changes, as neutron has no-charge on it.
Hence, option (b) is the correct answer.

Question 85

The number of lone pair of electron and hybridisation in $XeOF_4$ is

Options:

A. $0, sp^3$

B. $1, sp^3d$

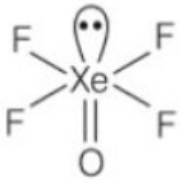
C. $1, sp^3d^2$

D. 2, sp^3d^2

Answer: C

Solution:

Solution:
Key Point Only one sigma bond is formed by each surrounding atom and each atom try to complete its octave and only orbitals involved in sigma bond formation or have lone pair of electrons can so for hybridisation.
The structure of $XeOF_4$ is as follows:



In the structure of $XeOF_4$, we have five (5) sigma(σ) bonds and one lone pair of electron associated with Xe-atom.
Thus, has 5 sigma bonds and 1 lone pair of electron with $sp^3d^2(5+1=6)$ hybridisation.
Hence, option (c) is the correct answer.

Question 86

Match List-I with List-II and select the correct answer using following codes

List-I	List-II
A. XeF_2	1. Regular tetrahedron
B. XeO_3	2. Distorted octahedron
C. XeO_4	3. Linear
D. XeF_6	4. Square planar
E. XeF_4	5. Pyramidal

Options:

A. A-3, B-5, C-1, D-2, E-4

B. A-5, B-3, C-2, D-4, E-1

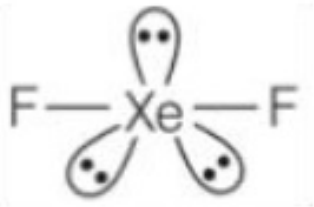
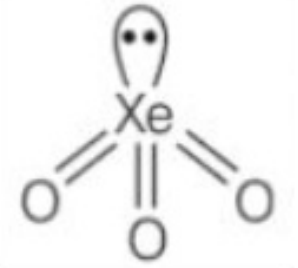
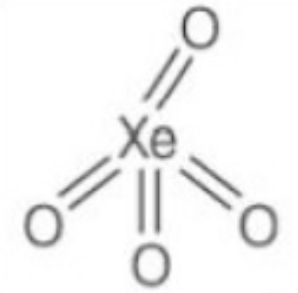
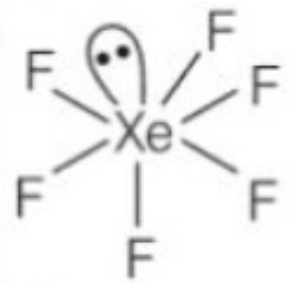
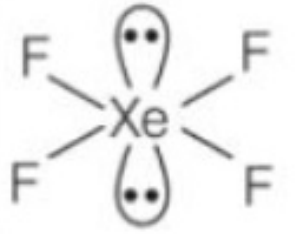
C. A-1, B-3, C-5, D-2, E-4

D. A-4, B-5, C-2, D-1, E-3

Answer: A

Solution:

Solution:

a.	XeF_2	(3)	Linear i.e.	
b.	XeO_3	(5)	Pyramidal i.e.,	
c.	XeO_4	(1)	Regular Tetrahedron i.e.,	
d.	XeF_6	(2)	Distorted octahedron i.e.	
e.	XeF_4	(4)	Square planar i.e.	

Hence, option (a) is the correct answer.

Question 87

Number of unit cells present in a cubic shaped ideal crystal of NaCl of mass 1.00g is [Molecular mass = 58.5]

Options:

A. 1.28×10^{21} unit cells

B. 1.71×10^{21} unit cells

C. 5.14×10^{21} unit cells

D. 2.57×10^{21} unit cells

Answer: D

Solution:

Solution:
NaCl has fcc structure, therefore we have 4 units of $NaCl$ in one unit cell of $NaCl$.
and
6.02×10^{23} units of $NaCl = 1mol$
$= 23 + 35.5 = 58.5g$
and molar mass of $NaCl(23 + 35.5 = 58.5)$
$= 58.5g$ of $NaCl$ permol
$= 1mol = 6.02 \times 10^{23}$ units of $NaCl$
$\because 58.5g$ of $NaCl$ contain $= 6.02 \times 10^{23}$ units of $NaCl$
$\therefore 1g$ of $NaCl$ contain
$= \frac{6.02 \times 10^{23}}{58.5}$ unit cells of $NaCl$
$= 0.1029 \times 10^{23}$ units of $NaCl$
Also,
$\because$ Each unit cell contain 4 units of $NaCl$.
$\therefore$ Number of $NaCl$ unit cells $= 0.0257 \times 10^{23}$
unit cells of $NaCl = 2.5 \times 10^{21}$ unit cells.
Hence, option (d) is the correct answer.

Question 88

A match box exhibits

Options:

A. cubic geometry

B. monoclinic geometry

C. orthorhombic geometry

D. tetragonal geometry

Answer: C

Solution:

Solution:

The diamentions and angles between the edges of the match box are as follows
$\alpha = \beta = \gamma = 90$
where, α, β and γ are the bond angles between the edges
and $a \neq b \neq c$, where, a, b and c are the bond lengths of match box.
Hence, it has orthorhombic geometry and option (c) is the correct answer.

Question 89

If the quantity of a radioactive element is doubled, the rate of disintegration will

Options:

A. be halved

B. be doubled

C. become four times

D. remain unaffected

Answer: D

Solution:

Solution:
The radioactive elements follow first order disintegration, which are not attacted by the initial concentration i.e., rate of disintegration remain unaffected.
Hence, option (d) is the correct answer.

Question 90

In the radioactive decay process

$$A \xrightarrow{-\alpha} B \xrightarrow{-\alpha} C \xrightarrow{-4\beta} D$$

1. A and B are isobars
2. A and D are isotopes
3. C and D are isobars
4. A and C are isotopes
The correct answer is

Options:

A. 1 and 2

B. 2 and 3

C. 3 and 4

D. 1 and 4

Answer: B

Solution:

Solution:
On emission of each α -particle, atomic number of an element decreased by 2 units and atomic mass by 4 units, while on emission of each β -particle, atomic number is increased by one unit and atomic mass remain unaffected.
Let initial atomic number of radioactive species $(A) = Z$ and atomic mass of $(A) = M$, then,
$A_Z^M \xrightarrow{-\alpha} B_{Z-2}^{M-4} \xrightarrow{-\alpha} C_{Z-4}^{M-8} \xrightarrow{-4\beta} D_Z^{M-8}$
Also,
Isotopes are the species which have same atomic number but have different atomic masses while isobers have same atomic mass and different atomic numbers.
The species with same number of neutrons but have different number of protons are called isotones.
Thus, species A and $'B'$ are isotopes while species C and D are isobars.
Hence, option (b) is the correct answer.

Question 91

If solubility of $M(OH)_3$ is S, its solubility product will be:

Options:

A. $108S^5$

B. $27S^3$

C. $4S^4$

D. $27S^4$

Answer: D

Solution:

Solution:
The relation between solubility (S) and solubility product (K_{SP}) is as follows
$K_{SP} = x^x . y^9 . S^{x+y}$
where, x and y are the number of atoms/radicals of the given species.
For $M(OH)_3$ type species
$x = 1$
$y = 3$
thus, $K_{SP} = 1^1 \times 3^3 S^{3+1} = 27S^4$
Hence, solution (d) is the correct answer.

Question 92

For the reaction $2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$ $K_c = 1.8 \times 10^{-6}$ at $184°C$ $R = 0.0831 kJK^{-1}mol^{-1}$ The relationship between K_p and K_c at $184°C$ is

Options:

A. $K_p > K_c$

B. $K_p < K_c$

C. $K_p = K_c$

D. K_p is independent of K_c

Answer: A

Solution:

Solution:
Given, $2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$
$K_c = 1.8 \times 10^{-6}$ at $184°C$
$R = 0.0831 kJK^{-1}mol^{-1}$
$K_p = K_c(RT)^{\Delta n(g)}$
where,
$\Delta n(g) =$ (Gaseous products-gaseous reactants)
$= 3 - 2 = 1$
$K_p = 1.8 \times 10^{-6}(0.0831 \times 457)$
$= 6.835806 \times 10^{-6}$
So, there have, $K_p > K_c$

Question 93

For a chemical reaction, ΔG is always less than zero ($\Delta G < 0$) if

Options:

A. ΔH and $T\Delta S$ both are positive

B. ΔH and $T\Delta S$ both are negative

C. ΔH is negative and $T\Delta S$ is positive

D. ΔH is positive and $T\Delta S$ is negative

Answer: C

Solution:

Solution:
For a chemical reaction, ΔG is always less than zero i.e., $(\Delta G < 0)$ for a spontaneous process.
The reaction can be expressed as follows
$\Delta G = \Delta H - T\Delta S = (-)$ ve
Thus, for the ΔG to be $(-)$ ve at all temperature i.e., under all conditions.
ΔH must be $(-)$ ve and
T\Δ S must be $(+)$ ve so that
ΔG remain $(-)$ ve
Hence, option (c) is the correct answer.

Question 94

Equal volumes of 1$MHCl$ and 1MH_2SO_4 are neutralised by dil. $NaOH$ solution separately. If X kcal and Y kcal of heat are liberated respectively, then which of the following relation is correct?

Options:

A. $X = Y$

B. $X = \frac{1}{2}Y$

C. $X = 2Y$

D. $X = \frac{1}{3}Y$

Answer: B

Solution:

Solution:
$\because$ Equal volume of $1MHCl$ and $1MH_2SO_4$ are used to neutralised by NaOH (dil.) separately.
HCl will give 1 mol of H^+ ions per mole while.
Given,
H_2SO_4 will give 2 moles of H^+ ions per mole.
Given:
$1MHCl +$ dil $NaOH, \Delta H = Xk.cal$
$1MH_2SO_4 +$ dil $NaOH$,
$\Delta H = Y$ k.cal
$\because H_2SO_4$ provide 2 moles of H^+ ions per mole of H_2SO_4
Therefore, $2X = Y$
$\therefore X = \frac{1}{2}Y$
Hence, option (b) is the correct answer.

Question 95

The Arrhenius equation gives the dependence of the rate constant of a chemical reaction on the absolute temperature and may be expressed as $k = Ae^{-\frac{Ea}{RT}}$. If the unit of k is s^{-1}, the unit of A will be

Options:

A. $kJmol^{-1}$

B. mol^{-1}

C. s^{-1}

D. K (kelvin)

Answer: C

Solution:

Solution:
According to Arrhenius equation where,
$k = A.e^{-E_0/RT}$
where, $k =$ rate constant
$A =$ Arrhenius constant
$E_0 =$ Activation energy
$R =$ Gas constant
$T =$ Temperature

Here, A can be related to the frequency, whose unit is s^{-1}.
Hence, option (c) is the correct answer.

Question 96

Consider the following reaction

$A \underset{k_1}{\overset{k_1}{\rightleftharpoons}} M + C; M + B \xrightarrow{k_2} k_{-1}$ **products**

The rate can be expressed as

Options:

A. Rate $= k_2[A][B][M]$

B. Rate $= \frac{k_1 k_2}{k_{-1}} \frac{[A][B]}{[C]}$

C. Rate $= \frac{k_1 k_2}{k_{-1}} \frac{[A][B]}{[C][M]}$

D. Rate $= \frac{k_1 k_{-1}}{k_2} \frac{[A][M]}{[B]}$

Answer: B

Solution:

Solution:
frac $d(P)\ dt = k_2[M][B]$.....(i)
Here, $P =$ Product
Reaction-rate-for Intermediate
$\frac{d(A)}{dt} = k_1[A] - k_{-1}[M][C]$
According to law of study state
$\frac{d[A]}{dt} = k_1[A] - k_{-1}[M][C] = 0$
$\therefore k_{-1}[M][C] = k_1[A]$
$[M] = \frac{k_1[A]}{k_{-1}[C]}$
$\therefore \frac{d(P)}{dt} = \frac{k_2 \times k_1[A][B]}{k_{-1}[C]}$
Thus (b) is the correct option

Question 97

The $E°$ values of the following half cells are given: $(25°C)$

$Fe^{3+}(aq) + e^- \xrightarrow{E°} Fe^{2+}(aq)\ +0.771V$
$Fe^{2+}(aq) + 2e^- \longrightarrow Fe(s)\ -0.447V$
The $\Delta G°$ **of** $Fe^{3+}(aq) + 3e^- \longrightarrow Fe(s)$
will be, $(F = 96485 Cmol^{-1})$

Options:

A. $160.65 kJmol^{-1}$

B. $-74.39 kJmol^{-1}$

C. $86.26 kJmol^{-}$

D. $11.87 kJmol^{-1}$

Answer: C

Solution:

Solution:
Given,
$Fe^{3+}(aq) + e^{-} \longrightarrow Fe^{2+}(aq)\ \ E° = +0.771V$ .....(i)
$Fe^{2+}(aq) + 2e^{-} \longrightarrow Fe(s)\ \ -0.447V$ .....(ii)
$(F = 96485 Cmol^{-1})$
To find $\Delta G°$ of $Fe^{3+}(aq) + 3e^{-} \longrightarrow\ \ Fe(s)$
$\because \Delta G_{(f)}° = \Delta G_1° + \Delta G_2°$
$= -nFE_1° + -nFE_2°(\because \Delta G° = -nFE°)$
for eq. no. $(1), n = 1$ and for eq. no. $(2), n = 2$
$E_1° = 0.771V$ and $E_2° = (-)0.447V$
on adding Eq. (i) and (ii), we have
$\Delta G_{(f)}° = (-nFE_1°) + (-nFE_2°)$
$= F[-0.771 + (-)2 \times 0.447]$
$= F[-0.771(-) - (0.894)]$
$= 96485 \times 0.123 = 11867.655 kJ/mol$
$= 11.87 kJ/mol$
Hence, option (d) is the correct answer.

Question 98

If 50×10^{-3} amp of current is passed through copper coulometer for $60 min$, calculate the amount of copper deposited ($F = 96500 Cmol^{-1}$)

Options:

A. $0.118g$

B. $0.180g$

C. $50g$

D. $1.18g$

Answer: A

Solution:

Solution:
Given,
Current $(i) = 50 \times 10^{-3}$ amp.
Time $(t) = 60 min = 3600 sec$
$(F) = 96500 C$
$\because$ charge $(Q) = i \times t$
$Q = i \times t = 50 \times 10^{-3} \times 3600 = 180 C$
Now, molar mass of copper $(Cu) = 63.5g$ and to convert to $Cu(s)$
$\because 96500 C$ charge is required to deposit $= 63.5g$ of copper (s) and to convert $Cu^{+} \rightarrow Cu(s)$:
$\because 1C$ charge will deposit $= \frac{63.5}{96500} g \text{ of } Cu(s)$

$\therefore 180C$ charge will deposit
$= \frac{63.5 \times 180}{96500} = 0.118 g \text{of} Cu$
Hence, option (a) is the correct answer.

Question 99

The critical micelle concentration of a surfactant depends on

Options:

A. hydrophobic part

B. hydrophilic part

C. counterion

D. All of the above

Answer: B

Solution:

Solution:
Surfactant contain charged groups, which are hydrophillic in nature and the maximum concentration of the surfactant above which they form micelles are called critical micelle concentration (CMC)
Thus, critical micelle concentration depends on hydrophillic part of the surfactant and option (b) is the correct answer.

Question 100

Formula of diaspore is

Options:

A. Al_2O_3

B. $Al_2O_3 . H_2O$

C. $Al_2O_3 . 2H_2O$

D. $Al_2O_3 . 3H_2O$

Answer: B

Solution:

Solution:
Diaspore is one of the three component minerals of the economically important aluminium ore bauxite.
The chemical formula of diaspore is $AlO(OH)$.
$Al_2O_3 . H_2O$ is just a general way of writing the formula of the aluminium oxy-hydroxides that occur in bauxites.

Question 101

The value of $\left(r.\hat{i}\right)^2+\left(r.\hat{j}\right)^2+\left(r.\hat{k}\right)^2$

Options:

A. 0

B. 1

C. $3r^2$

D. r^2

Answer: D

Solution:

Solution:
Let $r=x\hat{i}+y\hat{j}+z\hat{k}$
$(r.\hat{i})=(x\hat{i}+y\hat{j}+z\hat{k}).\hat{i}$
$(r.\hat{i})=x\Rightarrow(r.\hat{i})^2=x^2$ .....(i)
$(r.\hat{j})=(x\hat{i}+y\hat{j}+z\hat{k}).\hat{j}$
$(r.\hat{j})=y\Rightarrow(r.\hat{j})^2=y^2$ .....(ii)
And $(r.\hat{k})=(x.\hat{i}+y.\hat{j}+z.\hat{k}).\hat{k}$
$(r.\hat{k})=z\Rightarrow(r.\hat{k})^2=z^2$ .....(iii)
Now, $(r.\hat{i})^2+(r.\hat{j})^2+(r.\hat{k})^2$
Put the value of Eqs.(i), (ii) and (iii), we get
$=x^2+y^2+z^2=r^2$

Question 102

The component of $\hat{i}+\hat{j}$ along $\hat{j}+\hat{k}$ be

Options:

A. $\frac{1+\hat{j}}{2}$

B. $\frac{\hat{j}+\hat{k}}{2}$

C. $\frac{\hat{k}+\hat{i}}{2}$

D. None of these

Answer: B

Solution:

Solution:
Let $a = \hat{i} + \hat{j}$ and $b = \hat{j} + \hat{k}$
We know that, the component of a along b $= \frac{(a \,.\, b)\, b}{b \,.\, b}$
$= \frac{[(\hat{i} + \hat{j}) \,.\, (\hat{j} + \hat{k})](\hat{j} + \hat{k})}{(\hat{i} + \hat{k}) \,.\, (\hat{i} + \hat{k})} = \frac{1(\hat{j} + \hat{k})}{1 + 1} = \frac{\hat{j} + \hat{k}}{2}$

Question 103

The range of the function $f(x) = x^2$

Options:

A. R

B. $(0, \infty)$

C. $(0, -\infty)$

D. $(1, \infty)$

Answer: B

Solution:

Solution:
Since, $\forall x \in R$
$x^2 \geq 0 \Rightarrow x^2 \in [0, \infty)$
Now, from the options, the range of function
$f(x) = x^2$is$(0, \infty)$

Question 104

The range of the function $f(x) = \sec^{-1} x + \sin^{-1} x$ is

Options:

A. $[0, \pi] - \{\frac{\pi}{4}\}$

B. $[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$

C. $\{\frac{\pi}{2}\}$

D. None of these

Answer: C

Solution:

Solution:
$f(x) = \sec^{-1}x + \sin^{-1}x$
domain of $\sec^{-1}x = (-\infty, -1] \cup [1, \infty)$
and domain of $\sin^{-1}x = [-1, 1]$
So, domain of f is $\{-1, 1\}$
Since, $f(-1) = \sec^{-1}(-1) + \sin^{-1}(-1)$
$= \pi - \frac{\pi}{2} = \frac{\pi}{2}$
and $f(1) = \sec^{-1}(1) + \sin^{-1}(1)$
$= \pi + \frac{\pi}{2} = \frac{\pi}{2}$
Hence, the range of $f(x)$ is $\{\frac{\pi}{2}\}$.

Question 105

$\lim_{x \to 0^+} x^7(\log x)^8$ is equal to

Options:

A. 0

B. $\frac{7}{8}$

C. $\frac{8}{7}$

D. None of these

Answer: A

Solution:

Solution:
$\lim_{x \to 0^+} x^7(\log x)^8$
$= \lim_{h \to 0} (0 + h)^7[\log(0 + h)]^8$ [$\because$ for $h > 0$, $\lim_{h \to 0} f(x) = \lim_{h \to 0} f(0 + h)$]
On applying successive L 'Hospital Rule in
$\lim_{h \to 0} \frac{(\log h)^8}{1/h^7}$, we get
$\lim_{h \to 0} h^7[\log h]^8 = 0$

Question 106

If $m, n \in R$ and $f: R \to R$ is a continuous function given by

$$f(x) = \begin{cases} m^2\cos^2 x + n^2\sin^2 x & , \text{ if } x \leq 0 \\ e^{mx+n} & , \text{ if } x > 0 \end{cases}$$

Options:

A. $n^2 = \log|m|$

B. $n = 2\log|m|$

C. $2n = \log|m|$

D. $n = \log|2m|$

Answer: B

Solution:

Solution:
$m, n \in R$ and $f: R \to R$ is a continuous function.

$$f(x) = \begin{cases} m^2\cos^2 x + n^2\sin^2 x & , \text{ if } x \le 0 \\ e^{mx+n} & , \text{ if } x > 0 \end{cases}$$

$f(0) = m^2\cos^2 0 + n^2\sin^2 0$
$= m^2 . 1 + n^2 . 0 = m^2$
$f(0^+) = \lim_{x \to 0^+}(e^{mx+n}) = e^{m(0)+n} = e^n$
$f(0^-) = \lim_{x \to 0^-}(m^2\cos^2 x + n^2\sin^2 x) = m^2$
$\because f$ is continuous function.
$\therefore f(0) = f(0^+) = f(0^-), m^2 = e^n$
Taking log both sides, we get
$\log(m^2) = \log(e^n)$
$\Rightarrow n = 2\log|m|$

Question 107

If $f(x) = |x|^{|\sin x|}$, then $f'(-\frac{\pi}{6})$ is equal to

Options:

A. $\sqrt{\frac{\pi}{6}}(-\frac{\sqrt{3}}{2}\log\frac{\pi}{6} + \frac{3}{\pi})$

B. $\sqrt{\frac{\pi}{6}}(-\frac{\sqrt{3}}{2}\log\frac{\pi}{6} - \frac{3}{\pi})$

C. $\sqrt{\frac{\pi}{6}}(\frac{\sqrt{3}}{2}\log\frac{\pi}{6} - \frac{3}{\pi})$

D. None of the above

Answer: B

Solution:

Solution:
Since, $f(x) = |x|\sin x|$
$y = |x|^{|\sin x|}$[Let $f(x) = y$]
Taking log both sides, we get
$\log y = |\sin x|\log|x|$[$\because \log^M A = M\log A$]
Differentiate w.r. to $'x'$, we get

$$\frac{1}{y}\frac{dy}{dx} = \left[\frac{|\sin x|}{\sin x}\cos x . \log|x| + \frac{|\sin x|}{|x|}.\frac{|x|}{x}\right]$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{|\sin x|}{\sin x}\cos x . \log\left|x\right| + \frac{|\sin x|}{x}\right]$$

$$\Rightarrow f'(x) = |x|^{|\sin x|}\left[\frac{|\sin x|}{\sin x}\cos x \ . \log|x| + \frac{|\sin x|}{x}\right]$$

$$\Rightarrow f'(\frac{-\pi}{6}) = \left|\frac{-\pi}{6}\right|^{|\sin(-\pi/6)|}$$

$$\left[\frac{\left|\sin(-\frac{\pi}{6})\right|}{\sin(-\frac{\pi}{6})}\cos(-\frac{\pi}{6}).\log\left|\frac{-\pi}{6}\right|+\frac{|\sin(-\pi/6)|}{(-\pi/6)}\right]$$

$$\Rightarrow f'(\frac{-\pi}{6})=(\frac{\pi}{6})^{1/2}[\frac{-\sqrt{3}}{2}\log\frac{\pi}{6}+\frac{1/2}{-\pi/6}]$$

$$\Rightarrow f'(\frac{-\pi}{6}) \ =\sqrt{\frac{\pi}{6}}[\frac{-\sqrt{3}}{2}\log\frac{\pi}{6}-\frac{3}{\pi}]$$

Question 108

The difference between the greatest and the least values of the function $f(x)=\cos x+\frac{1}{2}\cos 2x-\frac{1}{3}\cos 3x$ is

Options:

A. $\frac{4}{9}$

B. $\frac{9}{4}$

C. $\frac{8}{7}$

D. $\frac{7}{8}$

Answer: B

Solution:

Solution:

Since, $f(x)=\cos x+\frac{1}{2}\cos 2x-\frac{1}{3}\cos 3x$

$f'(x)=-\sin x-\frac{1}{2}(2)\sin 2x+\frac{1}{3}(3)\sin 3x$

$f'(x)=-\sin x-\sin 2x+\sin 3x$

$f'(x)=-(\sin x+\sin 2x)+\sin 3x$

$f'(x)=-(2\sin\frac{3x}{2}.\cos\frac{x}{2})+\sin\frac{2}{2}(3x)$

$\{\because \sin C+\sin D=2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\}$

$f'(x)=-2\sin\frac{3x}{2}.\cos\frac{x}{2}+2\sin\frac{3x}{2}.\cos\frac{3x}{2}$ $[\because \sin 2\theta=2\sin\theta\cos\theta]$

$f'(x)=2\sin\frac{3x}{2}[\cos\frac{3x}{2}-\cos\frac{x}{2}]$

$f'(x)=2\sin\frac{3x}{2}[-2\sin x\sin\frac{x}{2}]$

$f'(x)=-4\sin\frac{x}{2}\sin x\sin\frac{3x}{2}$

For greatest and least value, $f'(x)=0$

$4\sin\frac{x}{2}\sin x.\sin\frac{3x}{2}=0$

$\sin x=0$

$x=n\pi, n\in Z$

$\sin\frac{x}{2}=0$

$\frac{x}{2}=n\pi$

$x=2n\pi, n\in Z;$

$\sin\frac{3x}{2}=0$

$\frac{3x}{2}=n\pi$

$x=\frac{2n\pi}{3}, n\in Z$

Now, For $x = 0, \pi, \frac{2\pi}{3}$
$\because f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$
Now, at $x = 0$
$f(0) = \cos 0 + \frac{1}{2}\cos 2(0) - \frac{1}{3}\cos 3(0)$
$= 1 + \frac{1}{2} \times 1 - \frac{1}{3} = \frac{7}{6}$
$f(\pi) = \cos \pi + \frac{1}{2}\cos 2\pi - \frac{1}{3}\cos 3\pi$
$f(\pi) = -1 + \frac{1}{2} \times 1 - \frac{1}{3}(-1) = -1/6$
and $f(\frac{2\pi}{3}) = \cos \frac{2\pi}{3} + \frac{1}{2}\cos 2(\frac{2\pi}{3}) - \frac{1}{3}\cos 3(\frac{2\pi}{3})$
$= -\frac{1}{2} + \frac{1}{2}(-\frac{1}{2}) - \frac{1}{3} = -\frac{13}{12}$
Hence, greatest value of $f(x) = 7/6$
Least value of $f(x) = -\frac{13}{12}$
Difference between the greatest and least value
$= \frac{7}{6} - (-\frac{13}{12}) = \frac{7}{6} + \frac{13}{12} = \frac{27}{12} = \frac{9}{4}$

Question 109

The function $2x^7 + 7x^4 + 14x + 28$ is increasing

Options:

A. for all x

B. for $x < 0$

C. for $x > 0$

D. None of these

Answer: A

Solution:

Solution:
Given, $f(x) = 2x^7 + 7x^4 + 14x + 28$
Differentiate w.r.t. x, we get
$f'(x) = 14x^6 + 28x^3 + 14$
$= 14(x^6 + 2x^3 + 1) = 14(x^3 + 1)^2$
For $f(x)$ to be increasing
$f'(x) \geq 0$
$\therefore 14(x^3 + 1)^2 \geq 0$
Hence, $f(x)$ is increasing for all x

Question 110

Rolle's theorem is not applicable for which of the following functions?

Options:

A. $f(x) = \sqrt{4 - x^2}$, on $[-2, 2]$

B. $f(x) = \log(x^2 + 2) - \log 6$, on $[-2, 2]$

C. $f(x) = e^{1-x^2}$, on $[-1, 1]$

D. $f(x) = 3 + (x-1)^{\frac{2}{3}}$, on $[0, 3]\, x$

Answer: D

Solution:

Solution:

We have, $f(x) = 3 + (x-1)^{\frac{2}{3}}$

$\Rightarrow f'(x) = \dfrac{2}{3(x-1)^{1/3}}$

Clearly, $f(x)$ is not derivable at $x = 1$

So, Rolle's theorem is not applicable on $[0, 3]$

Question 111

$\int e^{\frac{x}{2}} \sec 2x(1 + 4\tan 2x)\, dx$ is equal to

Options:

A. $\dfrac{1}{2}e^{\frac{x}{2}} \sec 2x + C$

B. $e^{\frac{x}{2}} \sec 2x \tan 4x + C$

C. $2e^{\frac{x}{2}} \sec 2x + C$

D. None of these

Answer: D

Solution:

Solution:

$I = \int e^{\frac{x}{2}} \sec 2x(1 + 4\tan 2x)\, dx$

$= \int e^{\frac{x}{2}} (\sec 2x + 4\tan 2x . \sec 2x)\, dx$

Let $\dfrac{x}{2} = t, x = 2t, dx = 2dt$

$\therefore I = 2\int e^t (\sec 4t + 4\tan 4t . \sec 4t)\, dt$ .....(i)

We know, $\int e^x (f(x) + f'(x))\, dx = e^x f(x) + C$ .....(ii)

and $\dfrac{d}{dx}(\sec 4x) = 4\sec 4x \tan 4x$

From Eqs. (i) and (ii), we get

$= 2[e^t \sec 4t + C'] = 2e^t \sec 4t + 2C'$

Put the value of t, we get

$= 2e^{x/2} \sec 4 . (\dfrac{x}{2}) + 2c' = 2e^{x/2} \sec 2x + C$

[Here $2C' = C$]

Question 112

$\int_0^3 \frac{4x^3+17x-4}{4x^2+25}dx$ is equal to

Options:

A. $\frac{x^2}{2}-\log(4x^2+25)+\frac{4}{5}\tan^{-1}\frac{4x}{5}+C$

B. $\frac{x^2}{2}-\log(4x^2+25)-\frac{4}{5}\tan^{-1}\frac{4x}{5}+C$

C. $\frac{x^2}{2}+\log(4x^2+25)-\frac{4}{5}\tan^{-1}\frac{4x}{5}+C$

D. None of the above

Answer: D

Solution:

Solution:

$\int_0^3 \frac{4x^3+17x-4}{4x^2+25}dx$

$= \int(x-\frac{8x+4}{4x^2+25})dx$

$= \int x dx - \int\frac{8x}{4x^2+25}dx - \frac{4}{2}\int\frac{2}{4x^2+25}dx$

$= \frac{x^2}{2}-\log(4x^2+25) - \frac{2}{5}[\tan^{-1}\frac{2x}{5}]+C$

Question 113

$\int_0^{2\pi}(\cos x+|\cos x|)dx$ is equal to

Options:

A. 0

B. 1

C. 4

D. None of these

Answer: C

Solution:

Solution:

$\int_0^{2\pi}(\cos x+|\cos x|)dx$

$= \int_0^{2\pi}\cos x\,dx + \int_0^{2\pi}[|\cos x|]dx$

$= [\sin x]_0^{2\pi} + 2\int_0^{\pi}|\cos x|dx$

$= \sin 2\pi + \sin 0 + 2\left[\int_0^{\pi/2} \cos x\, dx - \int_{\pi/2}^{\pi} \cos x\, dx\right]$

$= 0 + 0 + 2\left[\int_0^{\frac{\pi}{2}} \cos x\, dx - \int_{\pi/2}^{\pi} \cos x\, dx\right]$

$= 2[[\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi}]$

$= 2[(\sin \pi/2 - \sin 0) - (\sin \pi - \sin \pi/2)]$

$= 2[(1-0) - (0-1)]$

$= 2[1+1] = 4$

Question 114

Half of the area bounded by the curve $xy^2 = a^2(a-x)$ and Y-axis is

Options:

A. πa^2

B. $\frac{\pi a^2}{2}$

C. $3\pi a^2$

D. None of these

Answer: B

Solution:

Solution:

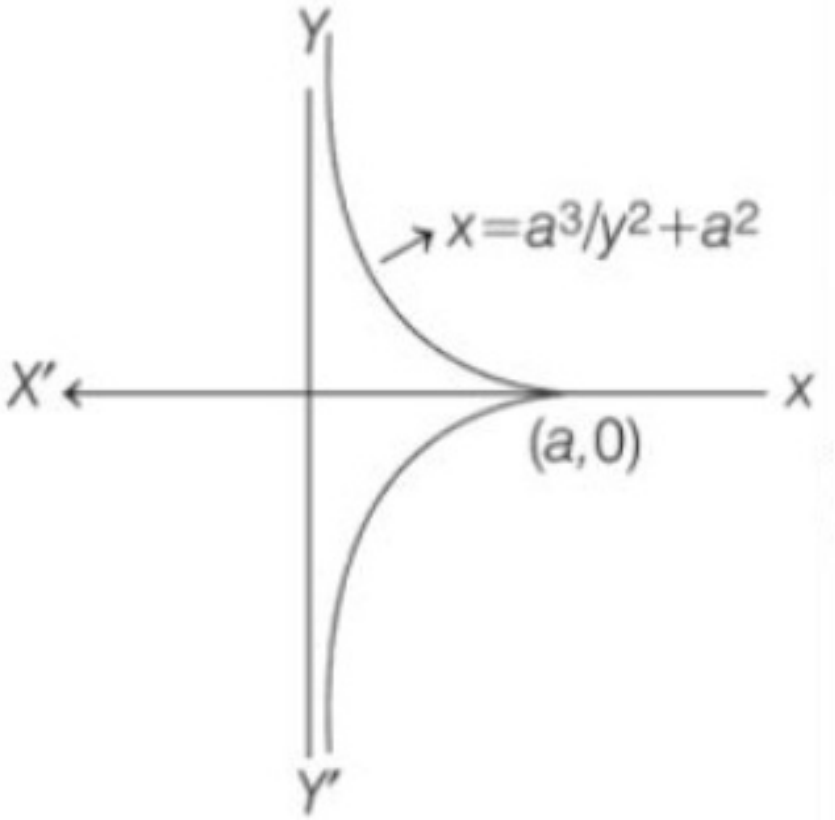
$xy^2 = a^2(a-x)$

$xy^2 = a^3 - a^2x$

$\Rightarrow x(y^2 + a^2) = a^3$

$\therefore x = \frac{a^3}{y^2 + a^2}$

At $x = a, a = \frac{a^3}{y^2 + a^2}$



$\Rightarrow y^2 + a^2 = \frac{a^3}{a} = a^2 \Rightarrow y^2 = a^2 - a^2 = 0$

$\Rightarrow y = 0$

Required area $= 2 \times \int_0^\infty x\,dy$

$= 2 \times \int_0^\infty \frac{a^3}{y^2 + a^2} dy$

$= 2a^2[\tan^{-1}\frac{y}{a}]_0^\infty = 2a^2[\frac{\pi}{2}] = \pi a^2$

Half of the area bounded of the given curve $= \frac{1}{2}\pi a^2$

Question 115

Order of the differential equation of all parabolas whose axes are parallel to Y -axis is

Options:

A. 0

B. 1

C. 3

D. 2

Answer: C

Solution:

Solution:
The general equation of parabola whose axes are parallel to Y -axis.
$y = ax^2 + bx + c$ .....(i)
Differentiate w.r to 'x', we get
$\frac{dy}{dx} = 2ax + b$
Again, differentiate. W.r. to 'x', we get
$\frac{d^2y}{dx^2} = 2a$
And again, differentiate. W.r to 'x', we get
$\frac{d^3y}{dx^3} = 0$
This is required differential equation.
The order of this equation is 3 .

Question 116

The solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ is

Options:

A. $cy^2 = e^{\frac{x^2}{y^2}}$

B. $cy = e^{\frac{x}{y}}$

C. $y = e^{x^2} + y + c$

D. $y = e^x + e^y + c$

Answer: A

Solution:

Solution:
$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ .....(i)
Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$
From Eq. (i)
$v + x\frac{dv}{dx} = \frac{(x)(vx)}{x^2 + v^2x^2} = \frac{x^2v}{x^2 + v^2x^2} = \frac{x^2v}{x^2(1 + v^2)}$
$\Rightarrow x\frac{dv}{dx} = \frac{v}{1 + v^2} - v = \frac{v - v - v^3}{1 + v^2} = \frac{-v^3}{1 + v^2}$
$\Rightarrow x\frac{dv}{dx} = \frac{-v^3}{1 + v^2} \Rightarrow \frac{1 + v^2}{v^3}dv = -\frac{dx}{x}$
On integration both sides

$\Rightarrow \int \frac{(1+v^2)}{v^3} dv = -\int \frac{dx}{x}$

$\Rightarrow \int \frac{1}{v^3} dv + \int \frac{1}{v} dv = -\int \frac{dx}{x}$

$\Rightarrow -\frac{1}{2y^2} + \log v = -\log x - \log c'$

$\Rightarrow \frac{-x^2}{2y^2} + \log \frac{y}{x} = -\log x - \log c'$

$\Rightarrow -x^2 2y^2 + \log \frac{y}{x} + \log x = -\log c'$

$\Rightarrow \frac{-x^2}{2y^2} + \log(\frac{y}{x} . x) = -\log c' \ \{ \because \log m + \log n = \log m n\}$

$\Rightarrow \log y + \log c' = \frac{x^2}{2y^2} \Rightarrow \log(y . c') = \frac{x^2}{2y^2}$

$\Rightarrow y c' = e^{x^2/2y^2} \Rightarrow y^2 c'^2 = e^{x^2/y^2} \Rightarrow y^2 c = e^{x^2/y^2}$

[Let $c'^2 = c$]

Question 117

The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$ is

Options:

A. $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$

B. $2xe^{\tan^{-1} y} = e^{\tan^{-1} y} + k$

C. $xe^{\tan^{-1} y} = e^{\tan^{-1} y} + k$

D. $xe^{\tan^{-1} y} = 3e^{\tan^{-1} y} + k$

Answer: A

Solution:

Solution:

$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$

$\Rightarrow (1 + y^2) = (e^{\tan^{-1} y} - x)\frac{dy}{dx}$

$\Rightarrow \frac{dx}{dy} = \frac{e^{\tan^{-1} y} - x}{(1 + y^2)} \Rightarrow \frac{dx}{dy} = \frac{e^{\tan^{-1} y}}{1 + y^2} - \frac{x}{1 + y^2}$

$\Rightarrow \frac{dx}{dy} + \frac{x}{1 + y^2} = \frac{e^{\tan^{-1} y}}{1 + y^2}$ .....(i)

This is first order linear diff. equation in x.

$IF = \int \frac{1}{1 + y^2} dy = e^{\tan^{-1} y}$

The general solution of Eq. (i)

$x . e^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1 + y^2} . e^{\tan^{-1} y} dy + c$

Put $\tan^{-1} y = t$

$\frac{1}{1 + y^2} dy = dt$

$\therefore x . e^t = \int e^t . e^t dt + c$

$\Rightarrow x . e^t = \int e^{2t} dt + c \ \Rightarrow x . e^t = \frac{e^{2t}}{2} + c$

Put the values of t

$\Rightarrow x . e^{\tan^{-1} y} = \frac{e^{2\tan^{-1} y}}{2} + c$

$\Rightarrow 2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + 2c$

$\Rightarrow 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$ [Let $k = 2c$]

Question 118

The differential equation whose solution is $ax^2 + by^2 = 1$, where a and b are arbitraryconstants, is of

Options:

A. Second order and second degree

B. First order and second degree

C. First order and first degree

D. Second order and first degree

Answer: D

Solution:

Solution:
$ax^2 + by^2 = 1$
Differentiate w.r. to ' x', we get
$a(2x) + b(2y)\frac{dy}{dx} = 0 \Rightarrow 2[ax + by\frac{dy}{dx}] = 0$
$\Rightarrow ax + by\frac{dy}{dx} = 0$
Again differentiate w.r. to $'x'$, we get
$\left[a + b(\frac{dy}{dx})^2 + \text{ by } \frac{d^2y}{dx^2}\right] = 0$
$\Rightarrow a = -b(\frac{dy}{dx})^2 - by\frac{d^2y}{dx^2}$
Put the value of a in the eq. (i), we get
$\left[-b(\frac{dy}{dx})^2 - by\frac{d^2y}{dx^2}\right]x + by\frac{dy}{dx} = 0$
$\Rightarrow -b\left[\{(\frac{dy}{dx})^2 + y\frac{d^2y}{dx^2}\}x - y\frac{dy}{dx}\right] = 0$
$\Rightarrow \left[(\frac{dy}{dx})^2 + y\frac{d^2y}{dx^2}\right]x - y\frac{dy}{dx} = 0$
This is the required differential equation and it is second order and first degree differential equation.

Question 119

A card is drawn at random from a pack of cards. What is the probability that the drawn card is neither a heart nor a king?

Options:

A. $\frac{13}{26}$

B. $\frac{9}{13}$

C. $\frac{4}{13}$

D. $\frac{9}{52}$

Answer: B

Solution:

Solution:
Total number of all possible outcomes = 52
i.e., $n(S) = 52$
Let E be the favourable outcomes of getting neither a heart nor a king, then $n(E) = 36$.
Therefore, P (neither a heart nor a king)
$= \frac{36}{52} = \frac{9}{13}$.

Question 120

For two events A and B, if $P(A) = P(\frac{A}{B}) = \frac{1}{4}$ and $P(\frac{B}{A}) = \frac{1}{2}$, then

Options:

A. A and B are independent

B. $P\left(\frac{A^1}{B}\right) = \frac{3}{4}$

C. $P\left(\frac{B^1}{A^1}\right) = \frac{1}{2}$

D. All of these

Answer: D

Solution:

Solution:
Since, $P(A) = P(\frac{A}{B}) = \frac{1}{4}$ and $P(\frac{B}{A}) = \frac{1}{2}$
$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{4}$ and $\frac{P(A \cap B)}{P(A)} = \frac{1}{2}$
So $P(A \cap B) = \frac{1}{2}P(A) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$
and $P(B) = 4P(A \cap B) = 4 \times \frac{1}{8} = \frac{1}{2}$
$\because P(A \cap B) = \frac{1}{8} = P(A)P(B)$
$\therefore$ Events A and B are independent.
$\because P(\frac{A'}{B}) = 1 - P(\frac{A}{b}) = 1 - \frac{1}{4} = \frac{3}{4}$
and $P(\frac{B'}{A'}) = \frac{P(A' \cap B')}{P(A')} = \frac{P((A \cup B)')}{P(A^1)}$
$= \frac{1 - P(A \cup B)}{1 - P(A)} = 1 - \frac{\{P(A) + P(B) - P(A \cap B)\}}{1 - P(A)}$
$= 1 - \{\frac{1}{4} + \frac{1}{2} - \frac{1}{8}\} 1 - \frac{1}{4} = \frac{1 - \frac{5}{8}}{\frac{3}{4}} = \frac{\frac{3}{8}}{\frac{3}{4}} = \frac{1}{2}$

Question 121

Three fair coins are tossed all together. The probability of getting at least two heads is

Options:

A. $\frac{1}{8}$

B. $\frac{3}{8}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

Answer: C

Solution:

Solution:
Total number of all possible outcomes $= 8$
i.e., $n(S) = 8$
Let E be the favourable outcomes of getting at least two heads, the $n(E) = 3 + 1 = 4$
$\therefore P($ at least two heads$) = \frac{4}{8} = \frac{1}{2}$

Question 122

If the two lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$, then the means of x and y are

Options:

A. $-\frac{4}{7}, \frac{11}{7}$

B. $-\frac{4}{7}, -\frac{11}{7}$

C. $\frac{4}{7}, -\frac{11}{7}$

D. $\frac{4}{7}, \frac{11}{7}$

Answer: B

Solution:

Solution:
$\because$ Regression lines passes through the $(\overline{x}, \overline{y})$.
$\therefore 4\overline{x} + 3\overline{y} + 7 = 0$ .....(i)

$3\overline{x} + 4\overline{y} + 8 = 0$ .....(ii)
Eq. (i) multiplying by 4 and Eq. (ii) multiplying by 3 and Eq. (ii) subtract from Eq. (i), we get
$16\overline{x} + 28 - (9\overline{x} + 24) = 0$
$7\overline{x} + 4 = 0$
$\overline{x} = -\frac{4}{7}$
Value of $\overline{x}$, put the Eq. (i), we get
$4(-\frac{4}{7}) + 3\overline{y} + 7 = 0$
$\Rightarrow 3\overline{y} = -7 + \frac{16}{7} \Rightarrow \overline{y} = -\frac{33}{7} \times \frac{1}{3} = -\frac{11}{7}$

Question 123

For the solution of equation $f(x) = 0$ by the Newton-Raphson method, the value of x tends to root of the equation highly when $f'(x_n)$ is

Options:

A. very big

B. zero

C. very small

D. None of these

Answer: A

Solution:

Solution:
For the solution of equation $f(x) = 0$ by the Newton-Raphson method, the value of x tends to root of the equation highly when $f'(x_n)$ is very big.

Question 124

By Simpson's rule, the value of $\int_1^2 \frac{dx}{x}$ dividing the interval $(1, 2)$ into four parts is

Options:

A. 0.6932

B. 0.6692

C. 0.6753

D. 0.6720

Answer: A

Solution:

Solution:

As we know that,

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\{f(x_0)+f(x_0+nh)\} +4\{f(x_0+h)+f(x_0+3h)+.....\} +2\{f(x_0+2h)+f(x_0+4h)+.....\}]$$

.....(i)

Put $x_0 = a, n = 2n, x_0 + 2nh = x_{2n} = b$

and $h = \frac{b-a}{2n}$ in Eq. (i)

$$\int_a^b f(x)\,dx = \frac{b-a}{6n}\ [\{f(x_0)+f(x_{2n})\} + 4\{f(x_1)+f(x_3)+.....\} +2\{f(x_2)+f(x_4)+.....\}]$$

$$= \frac{b-a}{6n}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3) +.....+f(x_{2n})]$$

Now, divide the range $[1, 2]$ into four equal parts

$x_0 = 1, x_1 = 1.25, x_2 = 1.5,\ x_3 = 1.75, x_4 = 2$

$$\int_1^2 \frac{dx}{x} = \int_{x_0}^{x_4} f(x)\,dx$$

$$= \frac{b-a}{6n}[f(x_0)+4f(x_1)+2f(x_2) +4f(x_3)+f(x_4)]$$

$$= \frac{2-1}{6\times 2}[\frac{1}{1}+\frac{4}{1.25}+\frac{2}{1.5}+\frac{4}{1.75}+\frac{1}{2}]$$

$$= 0.6932$$

Question 125

Simpson's rule for evaluation of $\int_a^b f(x)\,dx$ requires the interval (a, b) to be divided into

Options:

A. $2n + 1$ intervals

B. $3n$ intervals

C. $2n$ intervals

D. any number of intervals

Answer: C

Solution:

Solution:

Simpson's rule for evaluation of $\int_a^b f(x)\,dx$ requires the interval $[a, b]$ to be divided into an even number of sub intervals of equal width i.e. divided by $2n$ intervals.

Question 126

Secondary storage memory is basically

Options:

A. Volatile memory

B. non-volatile memory

C. backup memory

D. impact memory

Answer: B

Solution:

Solution:
Secondary storage memory is basically non-volatile memory.

Question 127

When was the first email sent?

Options:

A. 1963

B. 1969

C. 1971

D. 1974

Answer: C

Solution:

Solution:
In 1971 , the first email sent.

Question 128

Which of the following controls the process of interaction between the user and the operating system?

Options:

A. User interface

B. Language translator

C. Platform

D. Screen saver

Answer: A

Solution:

Solution:
User interface controls the process of interaction between the user the operating system.

Question 129

The value of $(1 + i)^8 + (1 - i)^8$ is

Options:

A. 2^8

B. 2^5

C. $2^4 \cos \frac{\pi}{4}$

D. $2^8 \cos \frac{\pi}{8}$

Answer: B

Solution:

Solution:
We have, $1 + i = \sqrt{2}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$
and $1 - i = \sqrt{2}(\cos \frac{\pi}{4} - i \sin \frac{\pi}{4})$
$\therefore (1 + i)^8 + (1 - i)^8$
$= 2^4(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})^8 + 2^4(\cos \frac{\pi}{4} - i \sin \frac{\pi}{4})^8$
$= 2^4(\cos 2\pi + i \sin 2\pi) \ + 2^4(\cos 2\pi - i \sin 2\pi)$
$= 2^4(2 \cos 2\pi) = 2^5$

Question 130

If ω is a non-real cube root of unity, then the expression $(1 - \omega)(1 - \omega^2)(1 + \omega^4)(1 + \omega^8)$ is equal to

Options:

A. 0

B. 3

C. 1

D. 2

Answer: B

Solution:

Solution:
Given, $(1 - \omega)(1 - \omega^2)(1 + \omega^4)(1 + \omega^8)$
$= (1 - \omega)(1 - \omega^2)(1 + \omega)(1 + \omega^2)[\because \omega^3 = 1]$

$= (1-\omega)(1+\omega)(1-\omega^2)(1+\omega^2)$
$= (1-\omega^2)(1-\omega^4)$ [$\because (a+b)(a-b) = a^2 - b^2$]
$= (1-\omega^2)(1-\omega) = 1 - \omega - \omega^2 + \omega^3$
$= 1 - \omega - \omega^2 + 1 = 2 - (\omega + \omega^2)$ [$\because 1 + \omega + \omega^2 = 0$]
$= 2 - (-1) = 3$

Question 131

If $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = 0$, then

Options:

A. $z_1 = z_2$

B. $z_1 = \overline{z}_2$

C. $z_1 z_2 = 1$

D. $\frac{z_1}{z_2} = 1$

Answer: B

Solution:

Solution:
Let $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$
Now, $|z_1| = |z_2| \Rightarrow r_1 = r_2 = r$ (let)
Also, given that
$\arg(z_1) + \arg(z_2) = 0 \Rightarrow \theta_1 + \theta_2 = 0$
$\Rightarrow z_1 z_2 = r^2 e^{i(\theta_1 + \theta_2)} = r^2$
Hence, they are conjugate
$\therefore z_1 = \overline{z}_2$

Question 132

If z is a point on the argand plane such that $|z - 1| = 1$, then $\frac{z-2}{z}$ is equal to

Options:

A. $\tan(\arg z)$

B. $\cot(\arg z)$

C. $i\tan(\arg z)$

D. $i\cot(\arg z)$

Answer: C

Solution:

Solution:

Let $z = x + iy$

$\therefore |z - 1| = 1$

$\Rightarrow |(x - 1) + iy| = 1$

$\Rightarrow \sqrt{(x-1)^2 + y^2} = 1$

$\Rightarrow x^2 + 1 - 2x + y^2 = 1$[on squaring both sides]

$\Rightarrow x^2 + y^2 = 2x$ .....(i)

Now, $\frac{z-2}{z} = \frac{(x-2)+iy}{x+iy}$

$= \frac{(x-2)+iy}{(x+iy)} \times \frac{(x-iy)}{(x-iy)}$

$= \frac{x(x-2) - y(x-2)i + xyi + y^2}{x^2 + y^2}$

$= \frac{x^2 - 2x - xyi + 2yi + xyi + y^2}{x^2 + y^2}$

$= \frac{2x - 2x + 2yi}{2x}$ [By Eq. (i)]

$= \frac{2yi}{2x} = \frac{y}{x}i = i\tan(\tan^{-1}\frac{y}{x})$ $= i\tan(\arg z)$

Question 133

If $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\ldots\infty}}}$ then

Options:

A. x is a irrational number

B. $2 < x < 3$

C. $x = 3$

D. $2 > x > 3$

Answer: C

Solution:

Solution:

Given, $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\ldots\infty}}}$

Squaring both sides, we get

$x^2 = 6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\infty}}}$

$\Rightarrow x^2 = 6 + x$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow x(x-3) + 2(x-3) = 0$

$\Rightarrow (x-3)(x+2) = 0 \Rightarrow x = 3$ or $x = -2$

$\therefore x = 3$ [$\because x$ is positive]

Question 134

The sum of factors of 8! which are odd and are of the form $3m + 2$, where m is a natural number, is

Options:

A. 40

B. 8

C. 45

D. 35

Answer: A

Solution:

Solution:
We have, $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 2^7 \times 3^2 \times 5 \times 7$
Since factors must be odd and of the form $3m + 2$.
So, they must not be multiple of 2 and 3 .
$\therefore$ Required factors are 5 and 35 .
$\therefore$ Required sum $= 5 + 35 = 40$

Question 135

The value of $\frac{2}{1}.\frac{1}{3}+\frac{3}{2}.\frac{1}{9}+\frac{4}{3}.\frac{1}{27}+\frac{5}{4}.\frac{1}{81}+.....\infty$ is

Options:

A. $\frac{1}{2}-\log_e\frac{2}{3}$

B. $-\log_e\frac{2}{3}$

C. $\frac{1}{2}+\log_e\frac{2}{3}$

D. $\log_e\frac{2}{3}$

Answer: A

Solution:

Solution:
Let $S = \frac{2}{1}.\frac{1}{3}+\frac{3}{2}.\frac{1}{9}+\frac{4}{3}.\frac{1}{27}+\frac{5}{4}.\frac{1}{81}+.....+\frac{n+1}{n}.\frac{1}{3^n}+.....\infty$
Where, $T_n = \frac{n+1}{n}.\frac{1}{3^n} = (1+\frac{1}{n})\frac{1}{3^n} = \frac{1}{3^n}+\frac{1}{n.3^n}$

$$\Rightarrow S = \sum T_n = \sum\frac{1}{3^n}+\sum\frac{1}{n.3^n}$$

$= \frac{\frac{1}{3}}{1-\frac{1}{3}} + \{-\log_e(1-\frac{1}{3})\} = \frac{1}{2}-\log_e(\frac{2}{3})$

Question 136

Assume that $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$, then the statement true for A is

Options:

A. $A^2 = I$

B. $A = (-1)I$, where I is a unit matrix

C. A^{-1} does not exist

D. A is a null matrix

Answer: A

Solution:

Solution:

Given, $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$\therefore A^2 = A \cdot A$

$= \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+0+1 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 1+0+0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^2 = I$

Hence, option (a) is correct.

Question 137

The equation $\sqrt{3}\sin x + \cos x = 4$ has

Options:

A. only one solution

B. two solutions

C. infinitely many solutions

D. No solution

Answer: D

Solution:

Solution:

Given equation is, $\sqrt{3}\sin x + \cos x = 4$
Let $\sin x = t$
$\because \cos x = \sqrt{1 - \sin^2 x} \quad \therefore \cos x = \sqrt{1 - t^2}$
Now, $\sqrt{3}t + \sqrt{1 - t^2} = 4, \sqrt{1 - t^2} = 4 - \sqrt{3}t$
Squaring both sides, we get
$1 - t^2 = 16 + 3t^2 - 8\sqrt{3}t$
$\Rightarrow 4t^2 - 8\sqrt{3}t + 15 = 0$
Discriminant $(D) = (8\sqrt{3})^2 - 4(4)15$
$= 192 - 240 = -48 < 0$
So, the equation has imaginary roots, So, there are no solutions.

Question 138

A ΔABC is such that $\sin(2A + B) = \frac{1}{2}$. If A, B and C are in AP, then the values of A, B and C are

Options:

A. $\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{12}$

B. $\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{6}$

C. $\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{4}$

D. $\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{6}$

Answer: A

Solution:

Solution:
Given, $\sin(2A + B) = \frac{1}{2}$
$\Rightarrow \sin(2A + B) = \sin\frac{5\pi}{6} \quad \Rightarrow 2A + B = \frac{5\pi}{6}$ .....(i)
$A + B + C = \pi$ [angle sum property] .....(ii)
As, A, B and C are in A.P.

So, $B - A = C - B \quad \Rightarrow A - 2B + C = 0$ .....(iii)
From Eqs. (ii) and (iii), we get
$B = \frac{\pi}{3}$
On putting the value of angle B in Eq. (i), we get
$2A + \frac{\pi}{3} = \frac{5\pi}{6} \Rightarrow 2A = \frac{\pi}{2} \quad \Rightarrow A = \frac{\pi}{4}$
On putting thevalue of angles A and B in Eq. (ii), we get
$\frac{\pi}{4} + \frac{\pi}{3} + C = \pi \Rightarrow C = \frac{5\pi}{12}$
Hence, (A, B, C) i.e.. $(\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{12})$.

Question 139

If $\tan(\cos^{-1}x) = \sin(\cot^{-1}\frac{1}{2})$, then the value of x is

Options:

A. $\frac{1}{\sqrt{5}}$

B. $\frac{2}{\sqrt{5}}$

C. $\frac{3}{\sqrt{5}}$

D. $\frac{\sqrt{5}}{3}$

Answer: D

Solution:

Solution:

Given, $\tan(\cos^{-1}x) = \sin(\cot^{-1}\frac{1}{2})$

$\Rightarrow \tan\{\sec^{-1}(\frac{1}{x})\} = \sin(\tan^{-1}2)$

$\Rightarrow \tan\{\tan^{-1}\frac{\sqrt{1-x^2}}{x}\} = \sin(\sin^{-1}\frac{2}{\sqrt{5}})$

$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow \frac{1-x^2}{x^2} = \frac{4}{5}$

$\Rightarrow 5-5x^2 = 4x^2 \Rightarrow 9x^2 = 5$

$\Rightarrow x^2 = \frac{5}{9} \Rightarrow x = \pm\frac{\sqrt{5}}{3} \Rightarrow x = \frac{\sqrt{5}}{3}$

Question 140

A tree is broken by wind, its upper part touches the ground at a point $10m$ from the foot of the tree and makes an angle of 45° with the ground. The entire length of the tree is

Options:

A. $15m$

B. $20m$

C. $10(1+\sqrt{2})\,m$

D. $10(1+\frac{\sqrt{3}}{2})\,m$

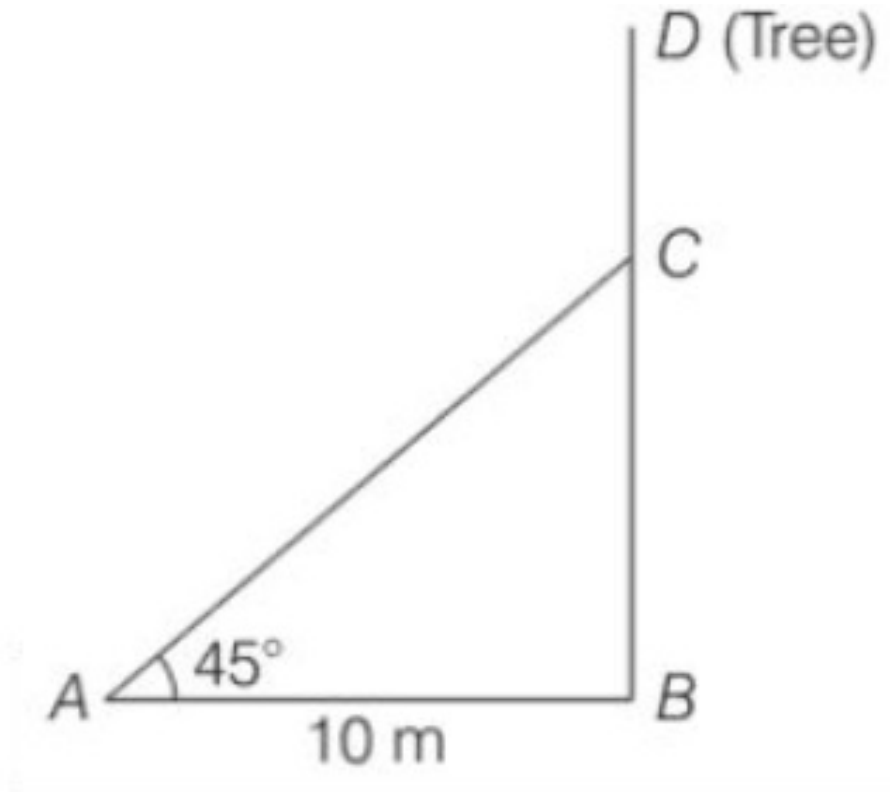
Answer: C

Solution:

Solution:

Let BD be the entire length of the tree.

The tree breaks at point C and makes ΔABC as given into the figure such that $AC = CD$



In triangle ABC,
$\tan 45° = \frac{BC}{AB} \Rightarrow 1 = \frac{BC}{AB} \Rightarrow BC = AB \Rightarrow BC = 10m$
Also, $\cos 45° = \frac{AB}{AC} = \frac{10}{AC}$
$\Rightarrow \frac{1}{\sqrt{2}} = \frac{10}{AC} \Rightarrow AC = 10\sqrt{2}m$
$\therefore$ Total height of tree $= BC + AC$
$= 10\sqrt{2} + 10 = 10(1 + \sqrt{2})m$

Question 141

The vertices of a triangle are $(3, -5)$ and $(-7, 4)$. If its centroid is $(2, -1)$, then the third vertex is

Options:

A. $(10, 2)$

B. $(10, -2)$

C. $(2, 2)$

D. $(10, 10)$

Answer: B

Solution:

Solution:
Let points are $A(3, -5), B(-7, 4)$ and $C(x, y)$
Given centroid is $G(2, -1)$
As, centroid $G = (\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3})$
$\Rightarrow 3 - 7 + x3 = 2$ and $\frac{-5 + 4 + y}{3} = -1$
$\Rightarrow -4 + x = 6$ and $-1 + y = -3$
$\Rightarrow x = 6 + 4$ and $y = -3 + 1$
$\Rightarrow x = 10$ and $y = -2$
$\therefore (x, y) = (10, -2)$
Hence, third vertex is $(10, -2)$

Question 142

The area enclosed by the curve $(x) + (y) = 1$ is

Options:

A. 1 sq.unit

B. 2 sq.unit

C. $\sqrt{2}$ sq.unit

D. $\sqrt{3}$ sq.unit

Answer: B

Solution:

Solution:
Assuming, $|x| + |y| = 1$
The straight lines are
$x + y = 1, x - y = 1, -x - y = 1, -x + y = 1$
Which form a square withe each side
$= \sqrt{(1 - 0)^2 + (0 - 1)^2}$
$= \sqrt{1 + 1} = \sqrt{2}$
Or form four equal right isosceles triangle with each equal side 1 unit.
So, each triangle's area $= \frac{1}{2} . 1 . 1 = \frac{1}{2}$ sq. unit
Therefore, required area $= \frac{1}{2} \times 4 = 2sq.$ unit.

Question 143

The angle between $x = 2$ and $x - 3y = 6$ is

Options:

A. ∞

B. $\tan^{-1}(3)$

C. $\tan^{-1}(\frac{1}{3})$

D. None of these

Answer: B

Solution:

Solution:
Since, $x = 2$ is parallel to Y-axis.
So, the angle between the line $x = 2$ and $x - 3y = 6$ is the angle between $x - 3y = 6$ and Y-axis which is given by $\tan\theta = \left|\frac{1}{m}\right|$, where m is the slope of the line
$x - 3y = 6$
Consider the line $x - 3y = 6, -3y = 6 - x$
$\Rightarrow y = \frac{x-6}{3} = \frac{x}{3} - 2$
Slope of the line $x - 3y = 6$ is $\frac{1}{3}$
Therefore, $\tan\theta = \left|\frac{\frac{1}{3}}{\frac{1}{3}}\right| = |3| \Rightarrow \theta = \tan^{-1}(3)$

Question 144

Line passing through $(1, 2)$ and $(2, 5)$ is

Options:

A. $3x - y + 1 = 0$

B. $3x + y + 1 = 0$

C. $y - 3x + 1 = 0$

D. $3x + y - 1 = 0$

Answer: C

Solution:

Solution:
We know that, the line passing through points (x_1, y_1) and (x_2, y_2) is
$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
$\therefore$ Line passing through $(1, 2)$ and $(2, 5)$ is
$y - 2 = \frac{5-2}{2-1}(x - 1)$
$\Rightarrow y - 2 = 3(x - 1) \Rightarrow y - 2 = 3x - 3$
$\Rightarrow 3x - y - 3 + 2 = 0 \Rightarrow 3x - y - 1 = 0$
$\Rightarrow y - 3x + 1 = 0$

Question 145

The sum of the square of the direction cosine of a line is

Options:

A. 0

B. 1

C. Constant

D. Variable

E.

Answer: B

Solution:

Solution:
The sum of the squares of the direction cosine of a line is 1 .
i.e., $l^2 + m^2 + n^2 = 1$

Question 146

The angle between two diagonals of a cube will be

Options:

A. Constant

B. Variable

C. $\sin^{-1}(\frac{1}{3})$

D. $\cos^{-1}(\frac{1}{3})$

Answer: D

Solution:

Solution:
Let O, one vertex of a cube be the origin and three edges through O along the co-ordinate axes. The four diagonals are OP, AA', BB' and CC'. Let ' a' be the length of each edge. Then the coordinates of P, A, A' are
$(a, a, a), (a, 0, 0), (0, a, a)$
The direction ratios of OP are a, a, a.
The direction cosines of OP are
$\frac{a}{a\sqrt{3}}, \frac{a}{a\sqrt{3}}, \frac{a}{a\sqrt{3}}$ i.e., $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$
Similarly, direction cosines of AA' are
$(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$
Let θ be the angle between the diagonals OP and AA', then
$\cos\theta = \frac{1}{\sqrt{3}}(-\frac{1}{\sqrt{3}}) + \frac{1}{\sqrt{3}}(\frac{1}{\sqrt{3}}) + \frac{1}{\sqrt{3}}(\frac{1}{\sqrt{3}})$

$= -\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{1}{3} \Rightarrow \cos\theta = \frac{1}{3}$
$\Rightarrow \theta = \cos^{-1}(\frac{1}{3})$
Thus, the angle between any two diagonals of a cube is $\cos^{-1}(\frac{1}{3})$

Question 147

The perpendicular distance of the point $(2, 4, -1)$, from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ is

Options:

A. 3

B. 5

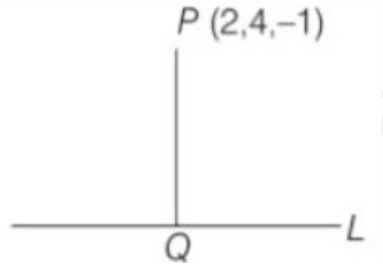
C. 7

D. 9

Answer: C

Solution:

Solution:
The given line l is $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$
Any point Q on the line l is given by
$\Rightarrow \frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$
$\Rightarrow x = \lambda - 5, y = 4\lambda - 3, z = -9\lambda + 6$
Direction ratios of PQ are
$\lambda - 5 - 24\lambda - 3 - 4, -9\lambda + 6 + 1$
or $\lambda - 7, 4\lambda - 7, -9\lambda + 7$



Since, $PQ \perp L$
$\therefore (\lambda - 7) + 4(4\lambda - 7) - 9(-9\lambda + 7) = 0$
$\Rightarrow \lambda - 7 + 16\lambda - 28 + 81\lambda - 63 = 0$
$\Rightarrow 98\lambda = 98 \Rightarrow \lambda = 1$
$\therefore$ Coordinates of Q are $(1 - 5, 4 - 3, -9 + 6)$
i.e., $(-4, 1, -3)$
Now, Required distance
$= \sqrt{(2+4)^2 + (4-1)^2 + (-1+3)^2}$
$= \sqrt{36 + 9 + 4} = \sqrt{49} = 7$ unit.

Question 148

The locus of the equation $x^2 + y^2 + z^2 + 1 = 0$ is

Options:

A. An empty set

B. A sphere

C. A pair of lines

D. None of these

Answer: A

Solution:

Solution:
We have,
$x^2 + y^2 + z^2 + 1 = 0 \Rightarrow x^2 + y^2 + z^2 = -1$
Since, sum of squares of real number is always be non-negative.
$\therefore x^2 + y^2 + z^2 \neq -1$
$\therefore$ Locus of the equation is an empty set.

Question 149

If $u = \hat{i} \times (a \times \hat{i}) + \hat{j} \times (a \times \hat{j}) + \hat{k} \times (a \times \hat{k})$, then

Options:

A. $u = 0$

B. $u = \hat{i} + \hat{j} + \hat{k}$

C. $u = 2a$

D. $u = a$

Answer: C

Solution:

Solution:
Let, $a = x\hat{i} + y\hat{j} + z\hat{k}$
$\because \hat{i} \times \hat{i} = 0$
$\hat{i} \times \hat{k} = -\hat{j}$
$\hat{i} \times \hat{j} = \hat{k}$
Similarly for $\hat{j}$ and $\hat{k}$
$u = \hat{i} \times (a \times \hat{i}) + \hat{j} \times (a \times \hat{j}) + \hat{k} \times (a \times \hat{k})$
$= \hat{i} \times (-y\hat{k} + z\hat{j}) + \hat{j} \times (x\hat{k} + - z\hat{i}) + \hat{k} \times (-x\hat{j} + y\hat{i})$
$= y\hat{j} + z\hat{k} + x\hat{i} + z\hat{k} + x\hat{i} + y\hat{k}$
$= 2(x\hat{i} + y\hat{j} + z\hat{k}) = 2a$

Question 150

The vector $2\hat{i} + \hat{j} - \hat{k}$ is perpendicular to $\hat{i} - 4\hat{j} + \lambda\hat{k}$, then λ is equal to

Options:

A. -2

B. -1

C. -3

D. 0

Answer: A

Solution:

Solution:

Let $a = 2\hat{i} + \hat{j} - \hat{k}$

And $b = \hat{i} - 4\hat{j} + \lambda\hat{k}$

If two vectors a and b are perpendicular,

Then, $a \,.\, b = 0$

$(2\hat{i} + \hat{j} - \hat{k}) \,.\, (\hat{i} - 4\hat{j} + \lambda\hat{k}) = 0$

$\Rightarrow 2 - 4 - \lambda = 0 \Rightarrow -2 - \lambda = 0$

$\Rightarrow \lambda = -2$
