9. Atomic and Nuclear Physics

9.1. A proton that has flown over a great distance hits a proton that is at rest. The impact parameter is zero, that is, the velocity of the incident proton is directed along the straight line connecting the centers of the protons. The mass of the proton is known, m, and the initial velocity of the incident proton is v_0. How close will the incidence proton get to the fixed proton?

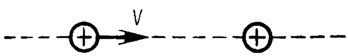

Fig. 9.1

9.2. Suppose that the energy required to ionize a hydrogen atom is W_1. Must the electron, the hydrogen ion, and the helium ion have the same initial kinetic energies for the hydrogen atom to become ionized?

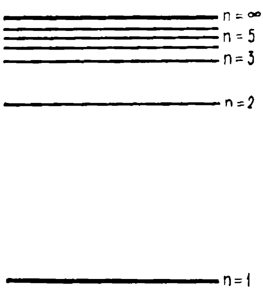

Fig. 9.3

9.3. The system of quantum levels of an atom is assumed to be like the one depicted in the figure. How will each of the energy components of the electron (the kinetic energy and the potential energy) vary if the electron moves from a lower level to a higher level?

9.4. The quantum levels of atoms of hydrogen and deuterium are only approximately the same (the difference between the two systems of levels is exaggerated in the figure). Which system of levels belongs to which atom? What is the reason for this discrepancy?

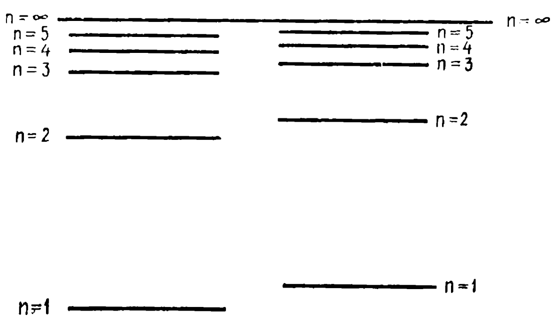

Fig. 9.4

9.5. Every other spectral line in one of the spectral series of an ionized helium atom (the Pickering series) closely resembles a line in the Balmer series for hydrogen. What is the principal quantum number of the level to which the electrons transfer when these lines are emitted?

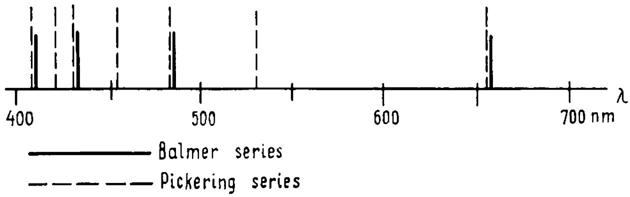

Fig. 9.5

Why don't the lines coincide exactly? What is the meaning of the lines that lie in between the lines of the Balmer series?

9.6. Four lines in the Balmer series lie in the visible part of the spectrum. What must the principal quantum number of the electron level in a doubly ionized lithium atom be for the lines emitted when electrons go over to this level to lie close to the lines of the Balmer series? What is the overall number of lines lying in this wavelength region?

9.7. An electron moving in an atom is acted upon by the Coulomb force of attraction generated by the nucleus. Can an external electric field be created that is capable of neutralizing the Coulomb force and ionizing, say, a hydrogen atom? Field strengths that can be created by modern devices are about 10^7 to 10^8 V/m.

9.8. In a He-Ne laser, the helium atoms are excited from the ground state to two sublevels, 2^1S and 2^3S, interact with Ne atoms, and give off their energy to Ne atoms, with the result that the latter are transferred to the $3S$ and $2S$ levels. The Ne atoms in these states emit radiation and go over to the $2P$ level. In the figure, the $3S$ and $2S$ levels, each consisting of four sublevels, and the $2P$ level, which consists of ten sublevels, are depicted by broad black bands. In addition to the above-mentioned transitions, a transition from the $3S$ state to the $3P$ level is possible, but we do not show this transition in the figure. From the $2P$ state, Ne atoms go over to the $1S$ state, and then gradually return to their ground state.

Why don't He atoms emit radiation during transitions from the 2^1S and 2^3S states directly to the ground state? What must be the relationship between the lifetimes of He atoms in states $3S$, $2S$, and $2P$ for continuous generation of radiation to be possible? It has been established that of the two transitions, $3S \rightarrow 2P$ and $2S \rightarrow 2P$, one is accompanied by radiation in the visible spectrum and the other, in the IR spectrum. Which transition corresponds to which spectrum?

9.9. The angular momentum of electrons in an atom and ts spatial orientations can be depicted schematically by

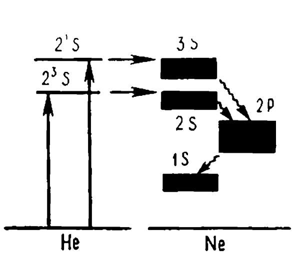

Fig. 9.8

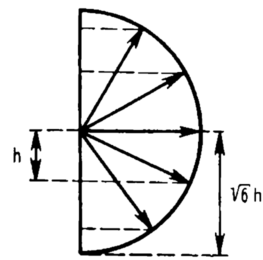

Fig. 9.9

a vector diagram where the length of the vector is proportional to the absolute value of the orbital angular momentum of an electron. What vectors in the diagram correspond to the minimal value of the principal quantum number n and what are the values of the quantum numbers l and m?

9.10. In the Stern-Gerlach experiment, which was conducted with the aim of discovering the spatial quantization of an atomic magnetic moment, a beam of silver atoms is sent through a nonuniform magnetic field generated by magnets whose configuration is shown in the figure. Why does the experiment require a nonuniform field?

9.11. The intensity distribution of X-ray radiation over wavelengths consists of a continuous spectrum, which is limited from the short-wave side by a limit wavelength λ_m, and a characteristic spectrum, which consists of separate peaks. In the figure (with an arbitrary scale) we depict such a distribution for a voltage U_1 applied to the X-ray tube. How will the distribution change if the voltage is decreased three-fold, that is, $U_2 = (1/3)\ U_1$?

9.12. An electron is inside a potential well with vertical walls. The electronic wave function is depicted in the figure. Is the depth of the well finite or infinite?

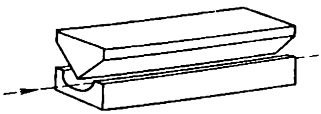

Fig. 9.10

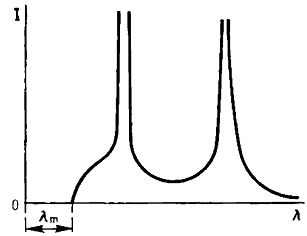

Fig. 9.11

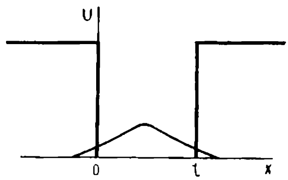

Fig. 9.12

9.13. An electron is in motion in a potential well of infinite depth. Depending on the electron kinetic energy, the electronic wave function has different configurations depicted in the figure. Which of these states is retained when the width of the potential well is decreased two-fold? By what factor will the minimal kinetic energy of the electron change in the process?

9.14. From the viewpoint of the optical analogy of the wave properties of an electron, the regions of space where it possesses different potential energies may be interpreted as regions with different refractive indices. In the figure two such regions are depicted, the regions are separated by a boundary where the potential energy P experiences a jump. In which of these regions is the refractive index greater? In which of the two cases, when the electron moves from left to right or when it moves from right to left, will the phase of the wave function be retained under reflection of the electron from the barrier, and in which will it change to its opposite?

9.15. An electron moving from left to right meets an obstacle, which in one case is a step (Figure (a)), and in the other a barrier (Figure (b)). What are the probabilities of the electron overcoming the step and the barrier according to the classical theory and the quantum theory in two separate cases, namely, when the electron kinetic energy E is lower than P and when it is higher than P?

9.16. An electron moving from left to right passes through three regions: I, II, and III. Its kinetic energy in regions I and III is the same, E. Assuming the potential energy in these two regions to be zero, find the relationship between the kinetic energy E and the potential energy P in region II if the electronic wave function has the configuration depicted in the figure.

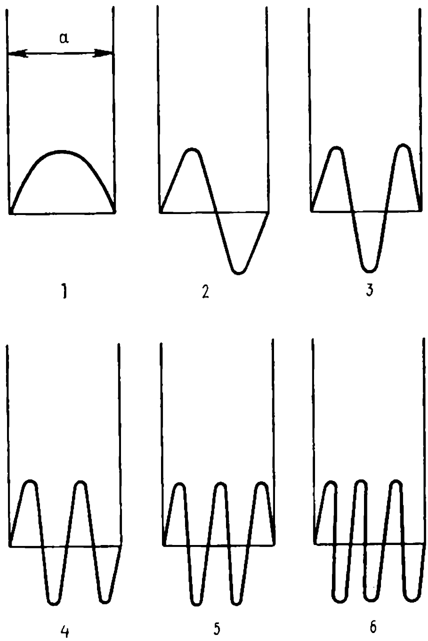

Fig. 9.13

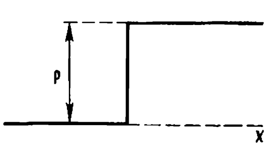

Fig. 9.14

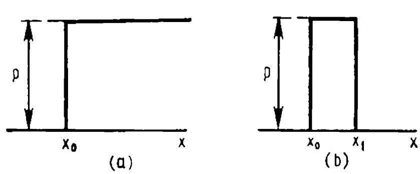

Fig. 9.15

9.17. According to classical kinetic theory, absolute zero is the temperature at which molecular motion ceases.

In relation to a solid body, this means that the thermal oscillatory motion of atoms or molecules forming the crystal lattice also ceases. Is the same conclusion valid from the standpoint of quantum mechanics?

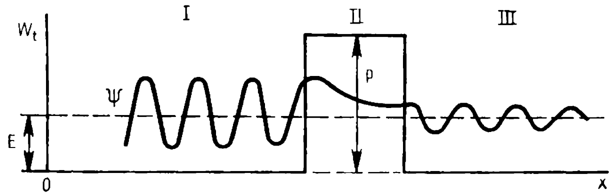

Fig. 9.16

9.18. In an experiment set up to study the diffraction of electrons, a beam of electrons whose energy can be varied by varying a potential difference is directed to a surface

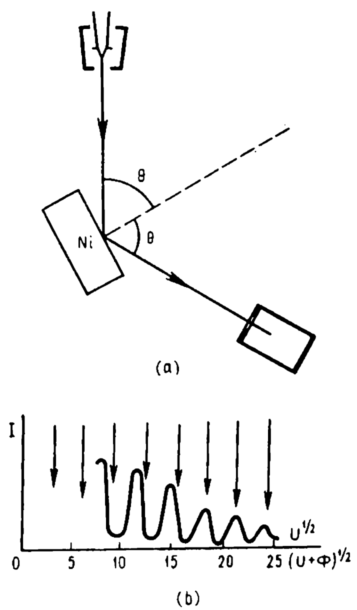

Fig. 9.18

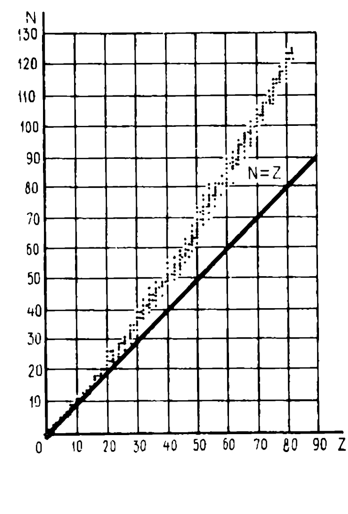

Fig. 9.19

of a single crystal at an angle θ. The diffracted (scattered) beam is analyzed by a detector positioned at the same angle θ (Figure (a)). In the experiment, the current of the scattered electrons was measured as a function of the applied potential difference used to accelerate the electrons. The results were plotted on a diagram, with the square root of the accelerating voltage laid off on the horizontal axis and the electron current, along the vertical axis. The curve consists of a number of alternating maxima and minima. As Figure (b) shows, the distance between the maxima at first is not the same, and the greater the voltage, the smaller the discrepancy. Explain the pattern of maxima and minima.

9.19. The number of protons and the number of neutrons in the nuclei of stable isotopes are laid off on the horizontal and vertical axes, respectively. Why does the fraction of neutrons in the overall number of nucleons increase with the mass number of the nuclei?

9.20. How many nucleons can there be in a nucleus on the lowest quantum level?

9.21. A counter registers the rate of radioactive decay, that is, the number of radioactive decay acts taking place

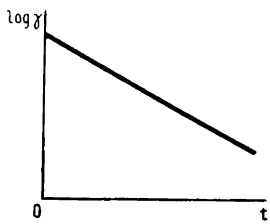

Fig. 9.21

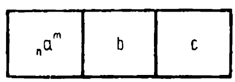

Fig. 9.22

Fig. 9.24

every second. The results obtained in such measurements are plotted in the form of a diagram in which the time interval from the beginning of counting is laid off on the horizontal axis and the logarithm of the decay rate, on the vertical axis. How to find the half-life of the radioactive element from such a diagram?

9.22. In the Periodic Table, we select three consecutive elements, say, a, b, and c. A radioactive isotope of element a whose proton and mass numbers are placed at the symbol of the element transforms into element b, which in turn transforms into element c. This last element transforms into an isotope of the initial element a. What processes cause the transformations $a \rightarrow b$, $b \rightarrow c$, and $c \rightarrow a$? What are the proton and mass numbers of the nuclei of elements b and c and those of the nucleus of element a after the final transformation is completed?

9.23. A number N_0 of atoms of a radioactive element are placed inside a closed volume. The radioactive decay constant for the nuclei of this element is λ_1. The daughter nuclei that form as a result of the decay process are assumed to be radioactive, too, with a radioactive decay constant λ_2. Determine the time variation of the number of such nuclei. Consider two limiting cases: $\lambda_1 \gg \lambda_2$ and $\lambda_1 \ll \lambda_2$.

9.24. The track of a beta particle (an electron) in a Wilson chamber has the shape of a limacon (a spiral). Where does the track begin and where does it end? How is the magnetic field that forces the beta particle to move in this manner directed?

9.25. In beta decay, the velocity of the nucleus that emits an electron is not directed along the line along which the electron velocity is directed. How can this phenomenon be explained?

9.26. The track of a proton in a Wilson chamber has a "knee", where the proton changes its trajectory by 45°.

Fig. 9.25

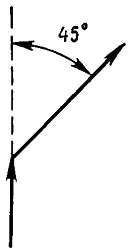

Fig. 9.26

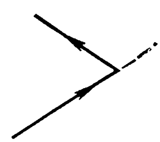

Fig. 9.27

Momentum and energy conservation implies that the proton has collided with a neutron. Which of the two particles has a higher energy if the neutron is considered to be initially at rest and free?

9.27. The track of an alpha particle in a Wilson chamber filled with a gas has a "knee", where the particle changes its direction of flight by an angle greater than 90°. Starting with what gas in the Periodic Table is such a track possible?

9.28. Two radioactive ions are emitted by an accelerator in the same direction with the same velocity **v** whose absolute value is close to the speed of light. Following this event, the nuclei of the ions emit electrons (each nucleus emits one electron). The velocity of one electron coincides in direction with v while the velocity of the other electron is in opposition to v. With respect to the nuclei the electron velocities (their absolute values, that is) are the

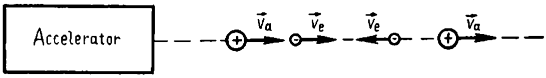

Fig. 9.28

same, v. Find the electron velocities with respect to the (fixed) accelerator and the velocity of one electron with respect to the other.

9.29. Within the framework of the "classical" Bohr theory, an excited atom is an atom one electron of which moves along an orbit that is farther from the nucleus

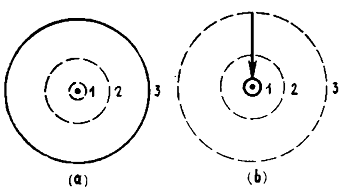

Fig. 9.29

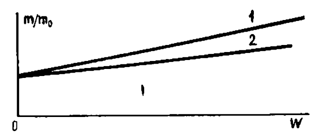

Fig. 9.30

than in the ground state (Figure (a)). When the atom goes over to its ground state (Figure (b)), the atom emits a photon. In the literature, especially popular-science literature, the common way to describe this process is to say that mass has transformed into energy. Is this actually the case?

9.30. Two charged particles acquire equal energy when moving in an accelerator. The dependence of the mass of each particle on the energy acquired is depicted by curves *1* and *2* in the figure. Which of the two particles has a greater rest mass?

9.31. The principle of operation of a linear accelerator is illustrated in the figure accompanying the problem. A charged particle is emitted by a source and is accelerated by a potential difference U between source S and cylinder *1*. During the time it takes the particle to fly through cylinder *1*, the potential difference between *1* and the next cylinder, *2*, changes its sign and, leaving cylinder *2*, the particle again finds itself in an accelerating field with the same potential difference U. The length of cylinder *2* is selected such that when the particle leaves this cylinder, the field will again change sign, so that the particle is accelerated anew, and so on. If the particle has

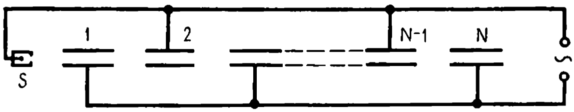

Fig. 9.31

passed N gaps between the cylinders, the energy it acquires is $W = eUN$ (it is assumed that the particle is singly charged). Since as the particle is accelerated the path it traverses in the course of a single change in polarity between the cylinders increases, each subsequent cylinder must be longer than the previous one. However, at a certain high energy the size of cylinders ceases to grow. What determines the maximal length of a cylinder if the frequency of variation of the voltage between the cylinders is ν?

9.32. Why cyclotrons are not employed to accelerate electrons? What generated a need for building more complex accelerators such as the synchrocyclotron and the synchrophasotron?

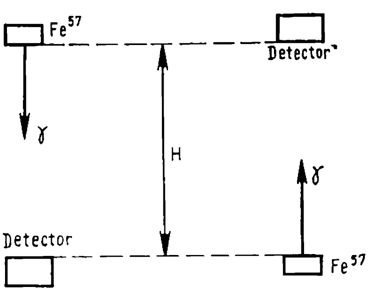

Fig. 9.33

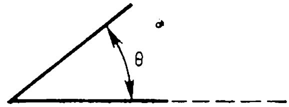

Fig. 9.34

9.33. Two samples of radioactive iron ^{57}Fe emit gamma-ray quanta. One sample is placed at an altitude H above sea level and the appropriate detector at sea level, while the second sample is placed at sea level and the appropriate detector at altitude H. Which of the two detected quanta has a higher frequency?

9.34. In observing Cerenkov radiation it was found that light propagates at an angle θ to the direction of electron motion. Find the refractive index of the substance in which the radiation is excited.

9. Atomic and Nuclear Physics

9.1. The protons move toward each other until their relative velocity becomes equal to zero. When the velocity is zero, the incident proton slows down and the immobile proton begins to accelerate, so that the distance between the two protons starts to increase. According to momentum conservation, when this happens, mv_0 becomes equal to $2mv$, where v is the velocity of both protons at the moment when the distance between the protons is minimal. At this moment both the velocities and, hence, the kinetic energies of the two protons are the same. The difference between the initial kinetic energy of the incident proton and the total kinetic energies of the two protons is equal to the energy associated with the interaction between the protons:

$$\frac{mv_0^2}{2} - 2\,\frac{m\,(v_0/2)^2}{2} = \frac{e^2}{4\pi\varepsilon_0 r},$$

whence

$$r = \frac{e^2}{\pi\varepsilon_0 m v_0^2}.$$

9.2. Assuming that ionization occurs as a result of a completely inelastic collision, we can write

$$mv_0 = (m + m_{\mathrm{H}})\,u,$$

where m is the mass of the incident particle, m_{H} the mass of a hydrogen atom, v_0 the initial velocity of the incident particle, and u the final common velocity of the particle after collision. Prior to collision, the kinetic energy of the incident particle was

$$W_0 = mv_0^2/2.$$

The total kinetic energy after collision is

$$W = \frac{(m+m_{\mathrm{H}})\,u^2}{2} = \frac{m^2v^2}{2\,(m+m_{\mathrm{H}})}.$$

The decrease in kinetic energy must be equal to the ionization energy:

$$W_0 - W = W_i = \frac{m_{\mathrm{H}}}{m+m_{\mathrm{H}}}\,W_0.$$

The greater the mass of the incident particle, the smaller the fraction of the initial kinetic energy that can be used for ionization. When an electron is used as the ionization agent, the initial kinetic energy of the electron is almost completely used for ionization. When an accelerated ion of hydrogen is used for ionization, the initial kinetic energy must double that of the electron, and when ionization is initiated by a helium atom, the energy must be five times that of the electron. This estimate explains why in a gas-discharge plasma, ionization is initiated almost exclusively by electrons, while ionization by the proper ions plays practically no role.

9.3. The kinetic energy of the electron in a hydrogen-like atom is

$$W_{\mathrm{kin}} = \frac{me^4Z^2}{8\varepsilon_0^2 n^2 h^2},$$

while the potential energy is

$$W_{\mathrm{pot}} = -\frac{me^4Z^2}{4\varepsilon_0^2 n^2 h^2}.$$

As n grows (i.e. as the electron moves to higher levels), W_{kin} decreases in inverse proportion to n^2, while W_{pot} grows, tending to the maximal value of $W_{\mathrm{pot}} = 0$ as $n \to \infty$. The total energy,

$$W = -\frac{me^4Z^2}{8\varepsilon_0^2 n^2 h^2},$$

also tends to zero as $n \to \infty$. The minimal value of the total energy is

$$W_{\mathrm{min}} = -\frac{me^4Z^2}{8\varepsilon_0^2 h^2}.$$

Obviously, to detach the electron from the atom, the following work must be performed:

$$A = W_{\mathrm{max}} - W_{\mathrm{min}} = 0 - \left(-\frac{me^4Z^2}{8\varepsilon_0^2 h^2}\right) = \frac{me^4Z^2}{8\varepsilon_0^2 h^2}.$$

The ratio of this quantity to the elementary charge e is known as the ionization potential. This is the minimal potential difference that a particle of infinitely small mass and carrying the elementary charge (practically an electron) must pass for the given atom to become ionized.

9.4. The wave number of the emission lines of a hydrogen-like atom (when an electron "travels" from one quantum level to another) is given by the formula

$$\tilde{\nu} = RZ^2 \left(\frac{1}{k^2} - \frac{1}{n^2} \right),$$

where R is the Rydberg constant. For a nucleus of infinite mass,

$$R_\infty = \frac{me^4}{8\varepsilon_0^2 h^2 c}.$$

For a nucleus that has a finite mass M we must substitute the reduced mass

$$\mu = \frac{m}{1 + m/M}$$

for the electron mass m. Assuming that the electron energy is zero at infinity, we arrive at the following formula for the energy level with the principal quantum number n:

$$W_n = -R_\infty \frac{ch}{n^2} \frac{1}{1 + m/M}.$$

This formula shows that the greater the mass of the nucleus, the deeper are the levels of the nucleus and the greater the separation of the levels and the higher the frequency of the spectral line reflecting the transition between levels with the same initial quantum numbers and the same final quantum numbers. Of course, since $m \ll M$, the difference between the corresponding values is small, but for hydrogen and deuterium it is sufficiently high. The aforesaid implies that system *1* belongs to deuterium and system *2*, to hydrogen.

9.5. An ionized helium atom belongs to the class of atoms known as hydrogen-like, for which the following general series formula is valid:

$$\tilde{\nu} = R_M Z^2 \left(\frac{1}{k^2} - \frac{1}{n^2} \right),$$

where Z is the proton number. The Rydberg constant for an atom whose mass is M is

$$R_M = R_\infty \frac{1}{1 + m/M}. \tag{9.5.1}$$

If we ignore the difference between the Rydberg constants for hydrogen and a helium ion, then it can be assumed that the lines of the first coincide with those of the second. This occurs if

$$4\left(\frac{1}{k_{\mathrm{He}}^2}-\frac{1}{n_{\mathrm{He}}^2}\right)=\frac{1}{k_{\mathrm{H}}^2}-\frac{1}{n_{\mathrm{H}}^2}.$$

In the Balmer series, $k = 2$. We set $n_{\mathrm{H}} = n_{\mathrm{He}} = \infty$. Then

$$k_{\mathrm{He}} = 4 \quad \text{and} \quad n_{\mathrm{He}} = 6, 8, 10, 12, \ldots .$$

In the spectrum of a helium ion, between these lines are the lines for which $n_{\mathrm{He}} = 5, 7, 9, 11, \ldots$. These lines are also shown in the figure accompanying the problem. We note, in connection with formula (9.5.1), that since $R_{\mathrm{He}} > R_{\mathrm{H}}$, the lines of a helium atom correspond to slightly higher frequencies than the corresponding lines in the Balmer series.

9.6. For a doubly ionized lithium atom, $Z = 3$. For this reason the spectral lines of the lithium ion are described by the general series formula

$$\tilde{\nu}=9R\left(\frac{1}{k_{\mathrm{Li}}^2}-\frac{1}{n_{\mathrm{Li}}^2}\right).$$

For the Balmer series we have $k_{\mathrm{H}} = 2$, whereby only the lines of lithium that obey the relationship $9/k_{\mathrm{Li}}^2 = 1/4$ can be found in the visible spectrum. Hence

$$k_{\mathrm{Li}} = 6.$$

The last line in the Balmer series corresponds to a value of the principal quantum number n_{H} being equal to 6. The corresponding line for lithium exists at $9/n_{\mathrm{Li}}^2 = 1/6^2$, that is, at

$$n_{\mathrm{Li}} = 18.$$

Thus, in the spectral region of the first four lines of the Balmer series the overall number of lines is 12 ($n_{\mathrm{Li}} =$ 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18). The lines with $n_{\mathrm{Li}} = 9, 12, 15, 18$ lie close to the lines in the Balmer series with $n_{\mathrm{H}} = 3, 4, 5, 6$. Since there is a small difference in the values of the Rydberg constant, these lines do not coincide exactly. The difference is somewhat greater than in the case of the Pickering series.

9.7. The electric field in which the electron is moving is

$$E=\frac{e}{4\pi\varepsilon_0 r^2}, \tag{9.7.1}$$

where r is the radius of the electron orbit according to the "classical" Bohr theory. In the ground state of the hydrogen atom, the radius of the orbit is $r_1 = 5.29 \times 10^{-11}$ m. Formula (9.7.1) then yields the following value for the electric field strength:

$$E = 5.15 \times 10^{11} \text{ V/m},$$

which exceeds all practically attainable field strengths by several orders of magnitude. However, if an electron is moving along a circular orbit which corresponds to a value of the principal quantum number that differs from unity, the radius of such an orbit is

$$r = r_1 n^2,$$

and the electric field strength proves to be inversely proportional to n^2. If, say, $n = 10$, the electric field lies within the limits of practically attainable fields. Indeed, the ionization of highly excited states of the hydrogen atom by an electric field was actually observed in experiments.

9.8. Optical transitions between the ground state of helium and the 2^1S and 2^3S states are forbidden by selection rules. Although the selection rules that forbid such transitions are not absolute, they nevertheless permit defining the 2^1S and 2^3S states as metastable with lifetimes of the order of 10^{-3} s, which is an extremely large time interval on the scale of atomic processes. Excitation to such levels is possible in a discharge almost exclusively due to electron impact. What is needed for continuous generation of radiation is inverted population of levels. This becomes possible if the lifetime on the higher level exceeds considerably the lifetime on the lower level, with the result that the lower level has time to "get rid" of the electrons before new electrons arrive. Indeed, the lifetime of the $2S$ and $3S$ atomic states is of the order of 10^{-6} s, while the lifetime of state $2P$ is of the order of 10^{-8} s. In the first of the two transitions $3S \rightarrow 2P$ and $2S \rightarrow 2P$ the energy changes by a larger amount; hence a quantum of a higher frequency corresponds to this transition, and this frequency lies in the visible spectrum ($\lambda = 632.8$ nm), while the second transition corresponds to a quantum with a lower frequency, $\lambda = 1153$ nm, which lies in the IR region.

9.9. Since the length of all the vectors is the same, the absolute values of the angular momenta in all the states are the same, too. If the orbital quantum number is l, the magnetic quantum number m may assume $2l + 1$ different values. The figure accompanying the problem shows five different states. Hence, $l = 2$. The value of l cannot exceed $n - 1$, whereby the minimal value of the principal quantum number is 3. The values -2, -1, 0, $+1$, $+2$ of the magnetic quantum number correspond to different orientations of the angular momentum vector.

9.10. In a uniform magnetic field, a magnetic dipole, which is an object possessing a magnetic moment, experiences only a torque. For a force to act on a magnetic dipole, the field must be nonuniform. For an atomic magnetic moment this force is defined by the expression

$$F = \mu \frac{dB}{dz}, \qquad (9.10.1)$$

where μ is the magnetic moment of the atom. In formula (9.10.1) we assume that the vector of magnetic induction of the magnetic field generated by the atom is oriented along the lines of force of the external magnetic field and its direction coincides with that of the induction $\mathbf{B}$ of the external magnetic field or is opposite. In the first case the atom is pulled into the region where the field is stronger, while in the second case it is pushed out of that region. In the Stern-Gerlach experiment, the beam of silver atoms is sent through the (nonuniform) magnetic field and splits into two beams in accordance with two possible directions of the magnetic moment of a silver atom. If there was no spatial quantization, the silver atom would be oriented at random and the beam would spread in all directions. The silver atoms in the beam are in the ground state, whereby the difference in orientation is due to the different directions of the magnetic moment of outer electrons in silver atoms.

9.11. The minimal wavelength in the X-ray spectrum is determined by the maximal energy which a bombarding electron may transfer to the anode. This energy is eU and, hence,

$$\lambda_{\min} = \frac{ch}{eU}.$$

If the voltage is decreased three-fold, the minimal wavelength increases three-fold, too. As the figure accompanying the problem shows, as a result of such an increase in the wavelength the short-wave peak, which is one of the characteristics of the material of the anode, disappears. Separate characteristic peaks may disappear even when the wavelength corresponding to these peaks is longer than λ_{min} if to excite the quantum level from which the transition that generates the radiation with the wavelength of a particular peak begins an energy higher than eU_0 is required.

9.12. In infinitely deep potential well, the wave function at the boundary of the well ($x = 0$ and $x = l$) is zero. Since the figure accompanying the problem clearly shows that the wave function does not vanish at the boundary, we conclude that the well is of finite depth.

9.13. In a potential well of infinite depth the wave function at the "walls" of the well must vanish. This means that only states labeled by even numbers, e.g. 2, 4, 6, etc., may remain. The distance between the nodes of a standing wave function is equal to one-half of the de Broglie wavelength:

$$\frac{\lambda}{2} = \frac{h}{2mv}.$$

The maximal value of λ is a, which means that the electron velocity has a minimal value $v = h/2ma$, and hence the minimal value of the electron energy is $W_{min} = h^2/8ma^2$. If the width of the well decreases two-fold, the minimal kinetic energy of the electron in the well increases four-fold.

9.14. If the initial kinetic energy of the electron in the motion from left to right is E, to the right of the barrier it will be $E - P$. In the first case the de Broglie wavelength is

$$\lambda_1 = h/\sqrt{2mE},$$

while in the second it is

$$\lambda_2 = h/\sqrt{2m(E-P)}.$$

The wavelength ratio is in inverse proportion to the refractive index ratio:

$$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E-P}{E}}.$$

The right region can be considered as being less optically dense, whereby when the electron is moving from left to right the phase is retained, while when the electron is moving from right to left, the phase changes to its opposite.

9.15. From the viewpoint of classical mechanics, for $E < P$ this probability is zero in both cases, while for $E > P$ it is equal to unity ("step" *1* in Figures (a) and (b) accompanying the answer). From the viewpoint of quantum mechanics, however, in the first case for $E < P$

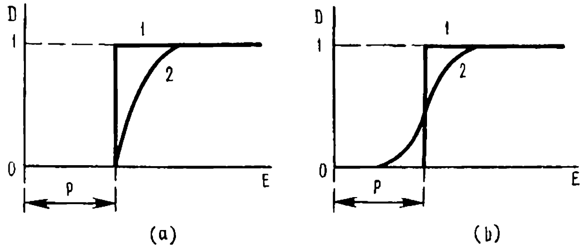

Fig. 9.15

the probability is also zero, whereas for $E > P$ the probability is lower than unity (curve *2* in Figure (a) accompanying the answer), since there is a nonzero probability of the electrons being reflected from the step, in other words, a fraction of the electrons moving from left to right begins to move in the opposite direction. Partial reflection takes place even when the potential energy to the left of x_0 is greater than the potential energy to the right of x_0 rather than lower. For the potential barrier depicted in Figure (b) accompanying the problem there is a nonzero probability of the electrons tunneling through the barrier even when $E < P$, but this probability does not become equal to unity even when $E > P$ (curve *2* in Figure (b) accompanying the answer). The passage of electrons through the potential barrier when $E < P$ under the conditions that the barrier has a finite width and that the potential energy to the right of the barrier is equal to or less than to the left of the barrier became known as the tunneling effect. This effect is encountered in many atomic and nuclear processes and in the field emission of electrons by metals and semiconductors. The probability of electrons passing through the barrier for $E < P$ is the higher the lower and narrower the barrier.

9.16. In region II, the wave function does not obey the sinusoidal law; it falls off exponentially. This happens within the framework of classical mechanics when a negative kinetic energy is assigned to the electron, or $E < P$. The passage of the electron into region III, which is forbidden from the classical standpoint, can be observed in experiments if the width of region II is sufficiently small (of the order of the electron wavelength in region I) and if the difference between P and E is not too great (see Problem 9.15). This phenomenon (the tunneling effect) resembles the partial passage of light across a narrow gap between two prisms (see the figure accompanying the answer) with the incident light experiencing total internal reflection in the first prism.

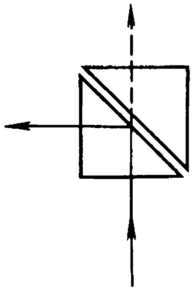

Fig. 9.16

9.17. The statement that the energy of the vibrational motion of atoms or molecules in a crystal lattice is nil at absolute zero contradicts one of the main principles of quantum mechanics, the uncertainty principle. If the kinetic energy is zero, so is the momentum. But if an atom or a molecule is at rest, its position is fixed. In other words, each coordinate and the projection of momentum on the respective coordinate axis are known with absolute accuracy. Meanwhile the wave properties of particles permit determining the collection of a coordinate and the respective projection of momentum within the intervals Δp_x and Δx, where in accordance with the uncertainty principle

$$\Delta p_x \Delta x \geqslant h/2\pi.$$

For this reason the energy of the atoms or molecules of a crystal is not nil at absolute zero. The motion of these objects is vibrational (zero-point vibrations), and the energy associated with this motion is the zero-point energy

$$E_0 = \frac{1}{2} h\nu = \frac{h}{4\pi}\,\omega,$$

where ω is the natural cyclic frequency of the vibration of a particle in the lattice. The existence of zero-point vibrations has been proved in experiments. They manifest themselves in light scattering in crystals at temperatures close to absolute zero.

9.18. The diffraction of electrons by a crystal obeys the same Bragg law as X-ray diffraction does:

$$2d \sin \theta = k\lambda.$$

In this formula $\lambda = h/mv$ is the de Broglie wavelength. Substituting the necessary constants (the electron mass and charge and the Planck constant) and transforming the units of measurement, we arrive at the following formula*:

$$\lambda = \sqrt{1.5/U} \text{ nm.} \qquad (9.18.1)$$

According to this formula, diffraction maxima are observed for the following wavelengths: λ_0 ($k = 1$), $(1/2)\lambda_0$ ($k = 2$), $(1/3)\lambda_0$ ($k = 3$), etc., with the voltages that determine the electron energy being U_0, $U_0\sqrt{2}$, $U_0\sqrt{3}$, etc. If on the horizontal axis we lay off the square roots of the values of the accelerating voltage, as is done in Figure (b) accompanying the problem, the current maxima must be spaced by equal distances. In experiments, however, this condition is not met exactly, and the smaller the voltage the greater the deviation from this pattern. The reason for this is that formula (9.18.1) contains the energy (in electron volts) of an electron inside the metal, and this quantity is the sum of the energy acquired by the electron in passing the potential difference and the difference in potential energies of the electron inside and outside the metal. Therefore, along the horizontal axis in Figure (b) accompanying the problem we must lay off $\sqrt{U + \Phi}$ rather than $\sqrt{U}$, where Φ is the internal potential in the metal. The quantity measured in experiments is, of course, U. Electron diffraction patterns obtained as a result of electron scattering on a metal lattice make it possible to obtain Φ.

* Here U is the potential difference through which the electron travels and, hence, the electron energy expressed in electron volts.

9.19. The stability of a nucleus is ensured by the fact that the Coulomb repulsive force experienced by each proton in the nucleus is equal to the force of nuclear attraction (the nuclear force). The Coulomb force falls off with distance relatively slowly (in inverse proportion to the square of the distance), while the nuclear force falls off very rapidly. For this reason the protons are held in the nucleus only by the closest neutrons, while experiencing the repulsive action of all the protons in the nucleus, even those farthest from a given proton. Thus, as the general number of nucleons grows, more and more neutrons are required so as to compensate for the growing action of the Coulomb repulsive forces.

9.20. According to the Pauli exclusion principle, a single quantum level can carry no more than two identical particles with half-integral spin. The directions of the spins must be opposite. In a nucleus such particles are the nucleons, protons and neutrons. Since these are distinct particles, there can be not more than four nucleons on the lowest level—two neutrons and two protons.

9.21. If N_0 is the number of radioactive atoms in the radioactive sample at the beginning of counting and λ

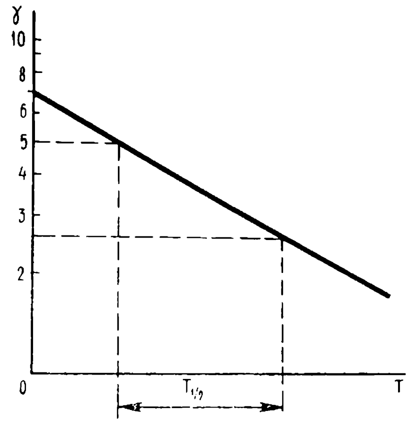

Fig. 9.21

is the decay constant, then at time t after the beginning of counting the number of atoms will be

$$N = N_0 e^{-\lambda t}. \qquad (9.21.1)$$

The rate with which this number changes is

$$\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} = -\lambda N.$$

A counter registers only the radioactive particles that fly in its direction. The fraction of such particles in the overall number of radioactive particles emitted by the sample depends on the size and position of the counter and can be characterized by a factor a (with $a < 1$). Thus, the counting rate can be expressed in the form

$$\gamma = a \left| \frac{dN}{dt} \right| = aN_0\lambda e^{-\lambda t}.$$

Taking logs, we get

$$\log \gamma = \log (aN_0\lambda) - \lambda t.$$

To determine the half-life of the radioactive element, there is no need to measure the slope and find the λ vs. t dependence and, using the well-known formula, to calculate $T_{1/2}$. Suffice it to lay off in any place on the vertical axis a segment equal to the logarithm of two (irrespective of what logarithms are laid off on the vertical axis, base-10 or base-e) and draw through the end points of this segments straight lines parallel to the horizontal axis. The points at which these straight lines intersect the experimental straight line that represents the variation in the rate of counting determine the boundaries of the time interval in the course of which the counting rate decreases by a factor of 2. Since the experimental law representing the decrease in the counting rate with the passage of time coincides with the law representing the decrease in the number of radioactive atoms (9.21.1), this time interval is the sought half-life.

9.22. A shift to the right by one place in the Periodic Table occurs as a result of a beta decay act. The mass number does not change in this act while proton number increases by unity. Hence,

$$_{n}a^{m} \rightarrow {}_{n+1}b^{m} + {}_{-1}\beta^{0}, \quad {}_{n+1}b^{m} \rightarrow {}_{n+2}c^{m} + {}_{-1}\beta^{0}.$$

A shift to the left by two places occurs in alpha decay. The mass number decreases by four, while the proton number decreases by two:

$$_{n+2}c^{m} \rightarrow {}_{n}a^{m-4} + {}_{2}^{4}\mathrm{He}.$$

The mass number of the resulting isotope of atom a differs from the initial number by four units.

Examples of such radioactive transformations are the chains of transformations in the ${}^{238}_{92}U$ and ${}^{232}_{90}Th$ families:

$$ {}^{238}_{92}U \to {}^{234}_{90}Th \to {}^{234}_{91}Ra \to {}^{234}_{92}U \to {}^{230}_{90}Th, $$
$$ {}^{232}_{90}Th \to {}^{228}_{88}Ra \to {}^{228}_{89}Ac \to {}^{228}_{90}Th \to {}^{224}_{88}Ra. $$

9.23. In the course of the time interval dt the number of nuclei of the new element (the "daughter" nuclei) changes thanks to the emergence of new nuclei as a result of the decay of initial (or "parent") nuclei and the departure of new nuclei as a result of their decay:

$$ dN_2 = N_1\lambda_1 dt - N_2\lambda_2 dt. $$

Here N_1 is the number of parent nuclei and N_2 is the number of the daughter nuclei at the given moment. According to the law of radioactive decay,

$$ N_1 = N_0 e^{-\lambda_1 t}. $$

Thus,

$$ dN_2 = \lambda_1 N_0 e^{-\lambda_1 t}\, dt - \lambda_2 N_2\, dt, $$

or

$$ \frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_0 e^{-\lambda_1 t}. \qquad (9.23.1) $$

We start by considering the limiting cases.

(1) $\lambda_1 \gg \lambda_2$. If we rewrite (9.23.1) in the form

$$ \frac{d\,(N_2/N_0)}{dt} + \lambda_2 \frac{N_2}{N_0} = \lambda_1 e^{-\lambda_1 t} $$

and assume that after a small time interval we can set $e^{-\lambda_1 t} = 0$, we obtain

$$ \frac{N_2}{N_0} = \frac{N_{20}}{N_0} e^{-\lambda_2 t}. $$

With $\lambda_1 \gg \lambda_2$ we can assume that $N_{20} = N_0$, so that

$$ N_2 = N_0 e^{-\lambda_2 t}. $$

Physically this means that parent nuclei practically instantly transform into daughter nuclei, which then decay according to the law of radioactive decay with a certain decay constant.

(2) $\lambda_1 \ll \lambda_2$. In this case the number of parent nuclei can be assumed to remain constant over a sizable time interval and is equal to N_0. This transforms (9.23.1) into

$$ \frac{dN_2}{dt} = -(\lambda_2 N_2 - \lambda_1 N_0), $$

which after integration yields

$$ N_2 = \frac{\lambda_1}{\lambda_2} N_0 (1 - e^{-\lambda_2 t}). $$

The number of daughter nuclei tends to a constant (saturation) value (see Figure (a) accompanying the answer):

$$N_2 = \frac{\lambda_1}{\lambda_2} N_0. \tag{9.23.2}$$

Of course, over a long time interval this number will decrease in accord with the decrease of the number of parent nuclei, whereby a more exact form of (9.23.2) is

$$N_2 = \frac{\lambda_1}{\lambda_2} N_0 e^{-\lambda_1 t}.$$

An example of the case with $\lambda_1 \ll \lambda_2$ is the radioactive decay of radium $^{226}_{88}Ra$ with a decay constant equal to 1.354×10^{-11} s^{-1} (a half-life of 1622 years). Its product

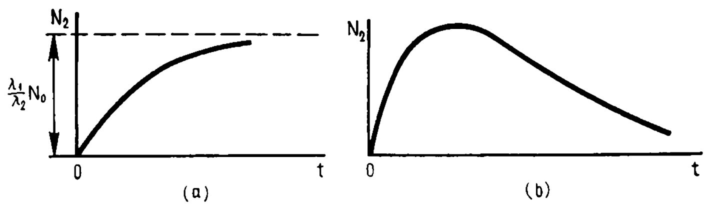

Fig. 9.23

is radon $^{226}_{86}Rn$ with a decay constant equal to 2.097×10^{-6} s^{-1} (a half-life of 3.825 days). If radium is placed inside a closed vessel, already after one month the amount of radon in the vessel will be only 0.4% less than the equilibrium amount, while the equilibrium amount, as shown by (9.23.2), constitutes only 6.46 parts to a million of the initial number of radium atoms.

To find the overall dependence of N_2 on t, we must integrate Eq. (9.23.1). The solution has the form*

$$N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}).$$

This expression has a maximum at a value of t equal to t_m, which can be found if we nullify the derivative dN_2/dt:

$$t_m = \frac{\ln \lambda_2 - \ln \lambda_1}{\lambda_2 - \lambda_1}.$$

The N_2 vs. t curve is depicted in Figure (b) accompanying the answer.

* To integrate Eq. (9.23.1), we introduce a new variable, $z = N_2 e^{\lambda_2 t}$. This yields

$$\frac{dz}{dt} = \left(\frac{dN_2}{dt} + \lambda_2 N_2\right) e^{\lambda_2 t}, \quad e^{-\lambda_2 t}\frac{dz}{dt} = \lambda_1 N_0 e^{-\lambda_1 t},$$

$$dz = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t}\, dt, \quad z = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}\left(e^{-(\lambda_2-\lambda_1)t} - 1\right),$$

$$N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right).$$

9.24. As the electron moves in the Wilson chamber, it gradually loses its energy to ion formation, and it is on these ions that drops of mist form, which make visible the track of the electron. This loss of energy results in a loss of speed, which means that the radius of curvature of the electron trajectory in the external magnetic field becomes smaller, since

$$R = mv/eB.$$

The wider part of the spiral corresponds to the beginning of the track, and the narrower part corresponds to the end of the track. If we take into account the negativity of the electron charge and the direction of its motion in the chamber, we can conclude that the magnetic field is directed toward the reader.

9.25. According to Pauli's hypothesis, which was verified in experiments, simultaneously with the escape of an electron the nucleus emits a neutrino (more precisely, an antineutrino), which is the particle that carries off a fraction of the energy released in beta decay and which has a momentum whose vector sum with the nucleus momentum and the electron momentum is zero:

$$_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} + {}_{-1}\beta^{0} + \tilde{\nu}.$$

9.26. The proton and neutron masses can be considered practically equal. When the proton and the neutron collide, the scattering angle after collision will be 90°, whereby after collision the direction of the neutron velocity will also make an angle of 45° with the initial direction of the proton velocity. Thus, after collision the proton and neutron energies are practically the same.

9.27. A change in the direction of motion (following a collision act) by an angle greater than 90° is possible if the mass of the incident particle is smaller than that of the particle that initially was at rest (in the laboratory system). The mass of the atom and molecule of hydrogen is smaller than the mass of an alpha particle, while the mass of helium is equal to the mass of an alpha particle. The gas closest to helium in the Periodic Table that has a mass greater than that of the alpha particle is nitrogen.

9.28. The relative velocity, according to the relativistic formula for velocity addition, is

$$v_{rel} = \frac{v_1 + v_2}{1 + v_1 v_2/c^2}.$$

The velocity of the electron flying away from the accelerator with respect to the accelerator is

$$V_a = \frac{2v}{1 + v^2/c^2},$$

while the velocity of the electron flying toward the accelerator is

$$V_b = 0.$$

The relative velocities of the electrons with respect to each other are:

$$V_{ab} = \frac{2v}{1 + v^2/c^2}$$

for the electron moving away from the accelerator, and

$$V_{ba} = -\frac{2v}{1 + v^2/c^2}$$

for the electron moving toward the accelerator, that is, they are equal in absolute value.

For the sake of an example we assume that $v = 0.9c$. In this case the velocity of the electron flying away from the accelerator and the relative velocities of the electrons are related through the following formula:

$$V_a = V_{ab} = -V_{ba} = \frac{1.8}{1 + 0.81} c = 0.9945c.$$

9.29. The statement carries no physical meaning whatsoever. First, there is not a single physical quantity that can transform into another physical quantity (time cannot transform into area, field strength into length, and so on). Second, for processes in relation to which this statement is usually made, the common conservation laws, the energy conservation law and the mass conservation law, are valid, that is, if isolated systems are considered. In the case at hand, these balance equations are as follows:

$$E^* = E + h\nu$$

for energy (here E^* is the energy of the excited atom, E is the energy of the atom in the ground state, and $h\nu$ is the photon energy), and

$$m^* = m + \mu_0$$

for mass (here m^* is the mass of the excited atom, m is the mass of the atom in the ground state, and $\mu_0 = h\nu/c^2$ is the photon "mass").

The first balance equation expresses the law of energy conservation and the second, the law of mass conservation (for the same process).

9.30. The ratio of the mass of a moving particle to the rest mass of that particle is

$$\frac{m}{m_0} = \frac{1}{\sqrt{1-v^2/c^2}}.$$

The kinetic energy acquired by a particle in an accelerator is determined by the following difference:

$$W_{\text{kin}} = mc^2 - m_0c^2 = m_0c^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right),$$

whence

$$\frac{m}{m_0} = \frac{W_{\text{kin}}}{m_0c^2} + 1.$$

For a fixed value of W_{kin}, the ratio m/m_0 is the smaller the greater m_0 is and, hence, curve *2* corresponds to the particle with the greater rest mass.

9.31. If the kinetic energy of the particle is W_{kin}, its velocity can be found from the equation

$$W_{\text{kin}} = m_0c^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right),$$

with the result that

$$\frac{v^2}{c^2} = 1 - \left(\frac{m_0c^2}{W_{\text{kin}}+m_0c^2}\right)^2.$$

If $W_{\text{kin}} \ll m_0c^2$, we arrive at an expression for the velocity that is identical to the one following from classical mechanics and electrodynamics:

$$v = \sqrt{2eU_0N/m_0}.$$

The voltage across the cylinders changes its sign in the course of a half-period $T/2 = 1/2\nu$, whereby the length of the cylinders must increase according to the law

$$l_N = \frac{1}{2\nu}\sqrt{\frac{2eU_0}{m_0}}\,N^{1/2}.$$

However, as W_{kin} grows, the velocity grows slower and slower. For instance, for $v = 0.87c$, $v = 0.89c$, and $v = 0.90c$ we have, respectively, $W_{\text{kin}} = m_0c^2$, $2m_0c^2$, and $3m_0c^2$. For sufficiently high energies the velocity of the particle approaches that of light and the length of the cylinders does not change any more: $l = c/\nu$.

9.32. The operation of a cyclotron is based on the fact that the time a charged particle takes to perform a full circle in a magnetic field does not depend on the particle's velocity. The time it takes the particle to complete one-half of a full circle, that is, the time in the course of which the electric field between the Dees reverses its direction, is $\pi mv/Be$. As the particle is accelerated, its mass grows according to the formula

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}.$$

The particle moving inside a Dee will gradually begin to get out of step with oscillatory electric field between the Dees.

The electron mass is doubled already at an energy equal to 0.51 MeV, whereby the discrepancy between the time it takes the electron to make a half-circle and the period of reversal of direction of the field between the Dees becomes noticeable already at accelerating voltages of the tens of kiloelectronvolts. This, naturally, limits the possibility of accelerating to high energies electrons in cyclotrons.

For ions, whose rest mass is greater than the electron rest mass by a factor of 10^3, 10^4 or even 10^5, the effect of increase of mass with velocity manifests itself at much higher energies. But here, too, there is a limit of acceleration of such particles in a cyclotron. To overcome this difficulty, other types of accelerators have been designed, in which the frequency of the electric field or the magnetic field is varied in the proper manner (separately or together).

9.33. The energy of the quantum that flies upward decreases while that of the quantum flying downward increases, as a result of which the frequency of the first gets lower and that of the second, increases. The difference proves to be so small, that could be detected only after a discovery made by Mössbauer, whose name was later given to this effect. An experiment in "weighing" the photon was conducted later by Pound. The results of these experiments are in full agreement with the theory of relativity. The present problem constitutes a simplified and schematized version of the idea of Pound's experiment.

9.34. Cerenkov radiation appears when the speed of light in the given medium is lower than the electron velocity. From the figure accompanying the answer we can see how the light wave is formed. In the time that it takes the electron to cover a path AB the light covers a distance AC, with

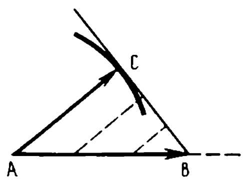

Fig. 9.34

$$\frac{|AC|}{|AB|}=\frac{c'}{v},$$

where $c' = c/n$ is the speed of light in the given medium. The envelope of the waves emitted by different points constitutes the wave front BC. The figure accompanying the answer shows that

$$\frac{|AC|}{|AB|}=\cos\theta.$$

The refractive index is

$$n=\frac{c}{v\cos\theta}.$$

Postface

Solution of the concluding problems in this *Collection* falls on the period when you are completing the general physics course in your college. It would be a mistake, however, to think that your studies in physics have come to an end. Physics will "pursue" you all your life unless, of course, you change your profession as engineer to that of opera singer or sports commentator.

Today numerous fields of human activity require a knowledge of physics, from astronautics to microbiology and from radio engineering to archeology.

But what portion of the physics studied in college will you find most needed in your future work? The laws? Naturally, one must know the main laws of physics, but I would not call this the most important aspect of your knowledge. The expression of a law or its mathematical formulation can be found in a reference book. This is even truer of the many specific formulas, such as the Poiseuille formula for viscous flow or the formula for the capacitance of a cylindrical capacitor.

Of course, the more formulas and laws that you remember the less frequently will you have to look into reference books and the more productive your work. And yet among the qualities that an engineer must have I would put first the ability to grasp the method required for a project. The aim of this book is to inculcate in the reader a taste for the physical method of thinking.

Solution of the majority of physical problems can be divided into four stages.

The first deals with the physical model of the phenomenon in question. A qualitative picture of the phenomenon is formulated, allowing for the factors that could be important. The second involves a mathematical model. An equation is set up that in accordance with an assumed law connects the factors introduced in the first stage. In the third stage mathematics steps in, so to say. By solving algebraic, trigonometric, or differential equations one can obtain the sought quantity in the form of an explicit function. The difficulties that arise in the third stage are more easily surmounted if the student has mastered the respective sections of mathematics. Mathematics for the engineer is what a cutting tool is for the lathe operator or a soldering iron for the assembler of electronic circuits.

Once the problem is solved, the very important fourth stage comes into the picture, namely, interpretation of the result obtained. The fourth stage is an analysis of the effect of the various parameters on the quantity of interest to the investigator.

To illustrate what has been said, let us examine damped oscillations, a common phenomenon known to everyone but not simple, nonetheless.

For instance, after performing several free oscillations, a pendulum finally stops; so does a load on a spring. The forces acting on the load are the elastic force exerted by the spring and the drag exerted by the surrounding medium (air). We assume that the elongation of the spring is small and, hence, the elastic force obeys Hooke's law. We also assume that the drag is proportional to the rate of motion of the load. All this constitutes the physical model of the phenomenon. Its mathematical model can be built by writing Newton's second law of motion: the mass of the load multiplied by the acceleration equals the sum of the projections of the forces acting on the load. This is a second-order differential equation, which can be solved (or integrated) if we consider the existence of two constants that depend on the initial data (the third stage). The resulting rather cumbersome formula expresses the time dependence of the load's displacement. The parameters in the formula are the mass of the load, the elasticity of the spring, and the resistance coefficient of the medium.

The analysis of the solution (the fourth stage) shows that a certain ratio of the parameters may produce periodic damped oscillations while another ratio may lead to aperiodic motion.

Such an analysis is given in a number of problems in this *Collection*. Take a careful look at their solution and pinpoint the four stages mentioned earlier. Give special consideration to the drawings accompanying the problems. Unfortunately, many students perceive a diagram as a simple illustration to be memorized and later drawn when necessary. As a result one sometimes gets a drawing that resembles a cartoon more than a physical diagram.

Often a student constructs the necessary curve more or less correctly but does not know the quantities that must be laid off on the axes. It is also difficult to overestimate the importance of knowing how to interpret a diagram. This requires, among other things, the skill of knowing how to "read" a diagram in the mathematical sense of the word, that is, understand that the derivative is positive where the curve goes up and negative where it goes down, and is zero at points of maxima and minima. In segments where the curve is convex downward the second derivative is positive; where it is convex upward the second derivative is negative. At inflection points the second derivative vanishes.

One must not forget that physics is an experimental science. In some cases an experiment helps one to find a sought law, discover a new phenomenon, or clarify certain aspects of a known effect; in others it serves as strict judge of the validity of a theory. Therefore, one must always prepare an experiment with care, understand the workings of the various devices involved, and analyze the results.

I believe that if you have solved or studied the solution of a large number of problems, the basics of the physical method of thinking have become clearer.

In conclusion I would like to hope that after you have finished college, far from being forgotten, physics will prove to be the real basis of your further development as an all-round person in this age of scientific and technical progress.

Some Fundamental Constants*

Quantity	Symbol	Numerical Value
Gravitational constant	G	6.672×10^{-11} N·m²·kg⁻²
Speed of light in vacuum	c	299 792 458 $\mathrm{m \cdot s^{-1}}$ (exact**)
Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$ $\mathrm{H \cdot m^{-1}}$ (exact***) $= 1.256\,637\,06144$ $\mathrm{H \cdot m^{-1}}$
Permittivity of vacuum	ε_0	$8.854\,1878 \times 10^{-12}$ $\mathrm{F \cdot m^{-1}}$
Planck constant	h	6.62618×10^{-34} J·s
Planck-Dirac constant	$\hbar$	1.05459×10^{-34} J·s
Atomic mass unit	amu	1.66057×10^{-27} kg
Energy equivalent of 1 amu		931.502 MeV
Electron rest mass	m_e	9.10953×10^{-31} kg $= 5.48580 \times 10^{-4}$ amu
Energy equivalent of m_e		0.511 003 MeV
Proton rest mass	m_p	1.67265×10^{-27} kg $= 1.007\,2765$ amu
Energy equivalent of m_p		938.28 MeV
Neutron rest mass	m_n	$1.674\,954 \times 10^{-27}$ kg $= 1.008\,665$ amu
Energy equivalent of m_n		939.57 MeV
Elementary charge (electron charge)	e	1.60219×10^{-19} C $= 4.80324 \times 10^{-10}$ esu
Avogadro constant	N_A	6.02204×10^{23} $\mathrm{mol^{-1}}$
Faraday constant	F	9.64846×10^{-4} $\mathrm{C \cdot mol^{-1}}$
Molar gas constant	R	8.3144 $\mathrm{J \cdot mol^{-1} \cdot K^{-1}}$
Molar volume of ideal gas at S.T.P.	V_m	22.4138×10^{-3} $\mathrm{m^3 \cdot mol^{-1}}$
Boltzmann constant	k	1.38066×10^{-23} $\mathrm{J \cdot K^{-1}}$
Stefan-Boltzmann constant	σ	5.6703×10^{-8} $\mathrm{W \cdot m^{-2} \cdot K^{-4}}$
Wien constant	b	2.8978×10^{-3} m·K
Rydberg constant	R_∞	$1.097\,3731 \times 10^{7}$ $\mathrm{m^{-1}}$
Compton wavelength of the electron	λ	$2.426\,309 \times 10^{-12}$ m
	$\bar{\lambda} = \lambda/2\pi$	$0.386\,159 \times 10^{-12}$ m
Bohr radius	a_0	$0.529\,177 \times 10^{-10}$ m

* The numerical values of the constants are given with an accuracy such that corrections may occur only by several units in the last digit.

** According to definition.

*** According to the resolution of the Seventeenth General Conference on Weights and Measures, the value of this constant is defined as not subject to further refinement.