Number System and Codes

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Introduction

A digital system is a combination of devices designed to manipulate physical quantities or information that are represented in digital form, i.e. they can take on only discrete values. Examples of digital system are digital computer, calculator, telephone etc.

Advantages of Digital Techniques

1. Size and cost is less.
2. Power dissipation is less.
3. Digital circuits are less affected by noise.
4. Accuracy and precision are greater.
5. Information storage is easy.

Digital Number System

Many number systems are used in digital technology. The most common are the binary, octal, decimal and hexadecimal system.

Note:

A number system with base 'b' will have 'b' different digits from 0 to (b-1).

Number Representation

$$(N_b) = \underbrace{d_{n-1}\, d_{n-2} \cdots d_i \cdots d_1\, d_0}_{\text{Integral Portion}} \;\bullet\; \underbrace{d_{-1}\, d_{-2} \cdots d_{-f} \cdots d_{-m}}_{\text{Fraction Portion}}$$

(Radix point)

where N = number

b = base or radix

- $d_{n-1}\, d_{n-2} \ldots d_i \ldots d_1\, d_0$ represents integral portion of number $(N)_b$, $d_{-1}\, d_{-2} \ldots d_{-f} \ldots d_{-m}$ represents fraction portion of number and between these two there is a radical sign.

Weighted number system

- It is a positional weightage system.
- **Ex:** Binary, octal, hexadecimal, BCD, 2421 etc.

Unweighted number system

- It is non positional weightage system.
- **Ex:** Gray code, Excess 3-code etc.

Number System	Base (b)	Digits
Binary	2	0, 1
Octal	8	0, 1, 2, 3, 4, 5, 6, 7
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9 A, B, C, D, E, F

- In binary number system, a group of "Four bits" is known as "Nibble" and group of "Eight bits" is known as "Byte".

Therefore,
4 bits = 1 Nibble
8 bits = 1 Byte

Decimal	Hexadecimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Note:

- In positional weightage system, the position of each digit of a number has some positional weightage.
- In non-positional weightage system, a digit of a number does not indicate any significance in a position and weight.

Codes

Binary Coded Decimal Code (BCD)

- Each digit of a decimal number is represented by binary equivalent.

In 4-bit binary formats

Total number of possible representation = 2^4 = 16
Valid BCD codes = 10
Invalid BCD codes = 6

In 8-bit binary formats

Valid BCD codes = 100
Invalid BCD codes = 256 – 100 = 156

Excess-3 code

- It is a 4-bit code.
- It can be derived from BCD code by adding "3" to each coded number.
- It is a "self-complementing code".

Gray code

- Also called "minimum change code" or "unit distance code" in which only one bit in the code group changes when going from one step to the next.
- Gray code is a minimum error code.

Binary-to-Gray conversion:

- 'MSB' in the gray code is same as corresponding digit in binary number.
- Starting from "Left to Right", add each adjacent pair of binary digits to get next and gray code digit. (Discard the carry if generated).

Gray-to-Binary conversion:

- "MSB" of Binary is same as that of gray code .
- Add each binary digit to the generated gray digit in the next adjacent position (discard the carry if generated).

Remember:

- In self-complemented code, the sum of weightage = 9.
- The largest number that can be represented by using N-bits is $(2^N - 1)_{10}$.

Conversion of Number System

(a) Decimal Number System to Other Number System Conversion

To convert decimal to any other base 'r' divide integer part and multiply fractional part with 'r'.

(b) Other Number System to Decimal Number System Conversion

Any 'r' base number can be converted to decimal equivalent by multiplying each digit by its positional weightage and summing the products.

$(x_2\, x_1\, x_0 \cdot y_1\, y_2)_r \quad \rightarrow \quad (\quad)_{10}$

$(x_2 \times r^2 + x_1\, r^1 + x_0\, r^0 \times y_1\, r^{-1} + y_2\, r^{-2})_{10}$

(c) Octal to Binary

Each digit is represented with 3 bit binary equivalent.

(d) Hexadecimal to Binary

Each digit is represented with 4 bit binary equivalent.

(e) Octal to Hexadecimal

(i) First convert octal to binary and then binary to Hexadecimal.

(ii) From Hexadecimal to octal conversion first convert Hexadecimal to binary and then binary to octal number.

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