Digital Communication

Matched filter:

→ impulse response a(t) = P^* (T – t) . P(t) → i/p

→ Matched filter o/p will be max at multiples of 'T' . So, sampling @ multiples of 'T' will give max SNR (2nd point)

→ matched filter is always causal a(t) = 0 for t < 0

→ Spectrum of o/p signal of matched filter with the matched signal as i/p ie, except for a delay factor ; proportional to energy spectral density of i/p.

$$\emptyset_0(f) = H_{opt}(f)\, \emptyset(f) = \emptyset(f)\, \emptyset^*(f)\, e^{-2\pi f T}$$

$$\emptyset_0(f) = |\emptyset(f)|^2\, e^{-j2\pi f T}$$

→ o/p signal of matched filter is proportional to shifted version of auto correlation fine of i/p signal

$\emptyset_0(t) = R_{\emptyset}\,(t - T)$

At t = T $\emptyset_0(T) = R_{\emptyset}\,(0)$ → which proves 2nd point

Cauchy-Schwartz in equality :-

$$\int_{-\infty}^{\infty} |g_1^*(t)\, g_2(t)\, dt|^2 \;\leq\; \int_{-\infty}^{\infty} g_1^2(t)\, dt \;\; \int_{-\infty}^{\infty} |g_2(t)|^2\, dt$$

If $g_1(t) = c\, g_2(t)$ then equality holds otherwise '<' holds

Raised Cosine pulses :

$$P(t) = \frac{\sin(\frac{\pi t}{T})}{(\frac{\pi t}{T})} \cdot \frac{\cos\,(\frac{\pi \alpha t}{T})}{1 - 4\alpha^2 t\, T^2}$$

$$P(f) = \begin{cases} T, & |f| \leq \frac{1-\alpha}{2T} \\ T\;\cos^2\left(\frac{\pi t}{2\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right); & \frac{1-\alpha}{2T} \leq |f| \leq \frac{1+\alpha}{2T} \\ 0, & |f| > \frac{1+\alpha}{2T} \end{cases}$$

- Bamdwidth of Raised cosine filter $f_B = \frac{1+\alpha}{2T}$ ⇒ Bit rate $\frac{1}{T} = \frac{2f_B}{1+\alpha}$
  α → roll of factor
  T → signal time period

→ For Binary PSK $P_e = Q\left(\frac{d}{2\sigma}\right) = Q\left(\sqrt{\frac{2\varepsilon_s}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\varepsilon_s}{N_0}}\right)$.

→ 4 PSK $P_e = 2Q\left(\sqrt{\frac{2\varepsilon_b}{N_0}}\right)\left[1 - \frac{1}{2}\, Q\,\left(\sqrt{\frac{2\varepsilon_b}{N_0}}\right)\right]$

FSK:-
For BPSK

$P_e = Q\left(\frac{d}{2\sigma}\right) = Q\left(\sqrt{\frac{\varepsilon_s}{N_0}}\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{\varepsilon_s}{2N_0}}\right)$

→ All signals have same energy (Const energy modulation)

→ Energy & min distance both can be kept constant while increasing no. of points . But Bandwidth Compramised.
→ PPM is called as Dual of FSK .
→ For DPSK $P_e = \frac{1}{2}\, e^{-\varepsilon_b/N_0}$

→ Orthogonal signals require factor of ‘2’ more energy to achieve same P_e as anti podal signals

→ Orthogonal signals are 3 dB poorer than antipodal signals. The 3dB difference is due to distance b/w 2 points.
→ For non coherent FSK $P_e = \frac{1}{2}\, e^{-\varepsilon_b/N_0}$
→ FPSK & 4 QAM both have comparable performance .
→ 32 QAM has 7 dB advantage over 32 PSK.

- Bandwidth of Mary PSK = $\frac{2}{T_s} = \frac{2}{T_{b}\log_2^m}$; $S = \frac{\log_2^m}{2}$
- Bandwidth of Mary FSK = $\frac{M}{2T_s} = \frac{M}{2T_b\log_2^m}$; $S = \frac{\log_2^m}{m}$
- Bandwidth efficiency $S = \frac{R_b}{B.W}$.
- Symbol time $T_s = T_b \log_2^m$
- Band rate $= \frac{\text{Bit rate}}{\log_2^m}$