Work and Energy

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1. Introduction

The concept of work is related to the concept of energy.

The general ideas of work-energy can be applied to a wide range of phenomena in different fields of physics.

In our daily life, any physical or mental activity is termed as work done. However, in physics, the meaning of work is entirely different. In this chapter, we shall discuss the difference between the term 'work' used in our daily life and the term "work" defined in physics.

Work-energy approach is based on Newton's Laws and as such does not involve any new principles.

Further, from a practical point of view, it is important to know not only the work done on a particle but also the rate at which it is being done.

2. Definition of work

"Work is said to be done only when the force acting on a body produces motion in it, in the direction of the force applied.

Factors on which work done depends

(1) Magnitude of the force applied: More the force applied, the more is the work done provided the body is displaced.

Work done (W) = Force applied (F) provided the body is displaced W = F

(2) Displacement of the body: Work done by the force on a body is directly proportional to he displacement of the body in the direction of force applied.

Work done = Displacement of the body

Remember: No work is done if a body does not change its position on the application of force.

3. Measurement of work done by a constant force

(a) When the body moves in the direction of the applied force

OR

When a constant force is applied in the horizontal direction

When a force F acting on a body produces displacement s in it.

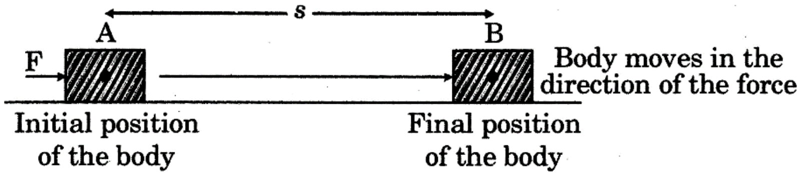


Fig.3.1.

Work done = Force x Displacement (in the direction of force)

$W = F \times s$

The work done on the block (or any other object) by a constant force is equal to the product of the magnitude of the applied force and the distance travelled by the body.

(b) When the body moves an angle to the direction of force.

In this case, whole of the force F is not utilised in pulling the body. If we resolve the force acting into two rectangular components, we observe that only the horizontal component of force is effective force which is pulling the body along the ground whereas, the vertical component of the force balances the weight of the body.

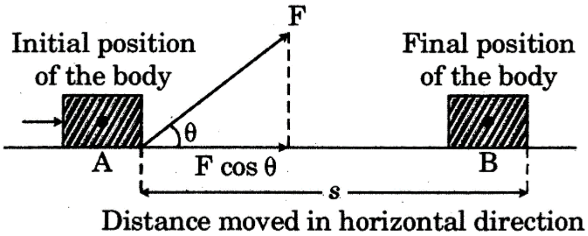


Fig.3.2

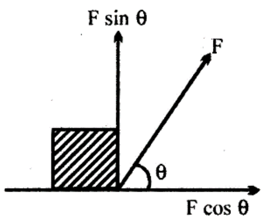


Fig.3.3

According to the definition of work done

W = Force applied $\times$ distance travelled by the body

$W = F\cos\theta \times s$

$W = Fs\cos\theta$

$W = \vec{F}.\vec{s}$

where $\vec{F}.\vec{s}$ is read as dot product of $\vec{F}$ and $\vec{s}$.

The product of the magnitude of the displacement and the force in the direction of the displacement of the body is called work done on a body.

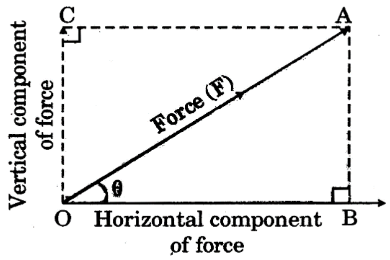


Fig. 3.4

In right angled ΔOBA,

Let $\angle AOB = \theta$

The trigonometric ratio cosine is written as

$$\cos\theta = \frac{Adjacent\ side}{Hypotenuse}$$

From the figure, $\cos\theta = \dfrac{Base}{Hypotenuse} = \dfrac{OB}{OA}$

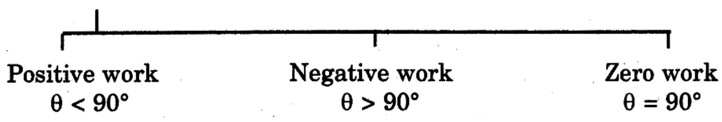
so $\cos\theta = \dfrac{Horizontal\ component\ of\ force}{Force\,(\mathrm{F})}$

so Horizontal component of force $= \mathrm{F}\cos\theta$

Value of $\cos\theta$

θ	0°	30°	45°	60°	90°	120°	180°
$\cos\theta$	$\cos 0$	$\cos 30^\circ$	$\cos 45^\circ$	$\cos 60^\circ$	$\cos 90^\circ$	$\cos 120^\circ$	$\cos 180^\circ$
value	1	$\frac{\sqrt{3}}{2} = 0.866$	$\frac{1}{\sqrt{2}} = 0.707$	$\frac{1}{2} = 0.500$	0.0	$\frac{-1}{2} = -0.500$	-1

4. Nature of work



(i) Positive work: Work done by a force on a body or an object is said to be positive work done when the body is displaced in the direction of applied force.

Condition: When the angle between the direction of force and the direction of displacement is acute $(\theta < 90^\circ)$, so $\cos\theta$ is positive.

Remember: W_{max} when $\theta = 0^\circ$

$$\cos 0^\circ = 1$$
$$W_{max} = F.s$$

Example: (i) When a body falls freely under the action of gravity $\theta = 0^\circ$ $\cos\theta = \cos 0^\circ = +1$, so work done by gravity on a body falling freely is positive.

(ii) When a coolie lifts a box from the ground to put it on his head, work done is said to be positive. In this case, coolie applies the force on the box in the upward direction and the box is also displaced in the upward direction.

(iii) When a spring is stretched, work done by the stretching force is positive.

(ii) Negative work: Work done by a force on a body is said to be negative work done when the body is displaced in a direction opposite to the direction of the force.

Condition: When the angle between the direction of force and direction of displacement is obtuse $(\theta > 90°)$, then θcos is negative. Hence, work done is negative.

Remember: When $\theta = 180°$

$\cos 180° = -1$

so $W = -F.s$

Example: (1) When a body is thrown up, its motion is opposed by the gravity. So, the work tone by the force of gravity on the object is negative. In this case, the force of gravity acts downwards on the object and the object is displaced upward so that the angle between the force of gravity and the displacement of the object is $180°$.

(2) When brakes are applied on a moving vehicle, work done by the breaking force is negative.

(3) Work done by friction is negative.

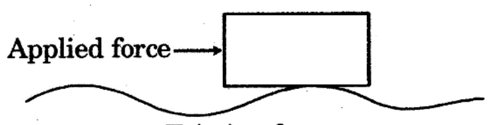


Fig. 4.1.

Demonstration of Positive and Negative work done by the gravity

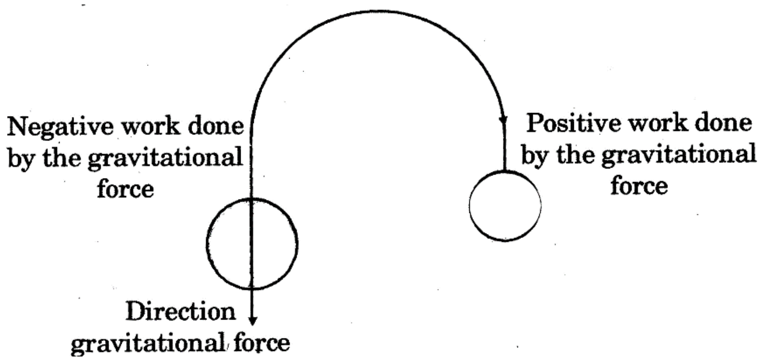


Fig. 4.2.

(3) No work done (Zero work done): No work is done by the force acting at right angle to the displacement of the body.

Condition: When the angle between the direction of force and the direction of displacement is $90°$.

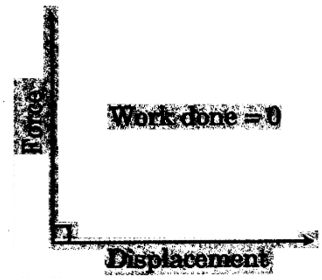


Fig. 4.3.

So, $\theta = 90°$

$\cos 90° = 0$

Hence work done is zero.

Remember: $W_{\min} = Fs\cos 90°$

$W_{\min} = 0$

5. Unit of work

In S.I., unit of work = Joule (J)

Since S.I. unit of force (F) is newton (N) and that of displacement *(s) is* metre (m).

$\therefore$ 1 J = 1 Nm

In C.G.S. system, unit of work = erg

1 erg = 1 dyne cm

$1N = 10^5$ dyne

$1J = 10^7$ erg.

Some bigger unit of work.

1 kilo Joule $= 10^3 J$

1 mega Joule $= 10^6 J$

1 giga Joule $= 10^9 J$

Definition of Joule (J): Work is said to be 1 Joule, if 1 Newton force acting on a body displaces the body through 1 metre in its own direction.

6. Energy

When a car runs, the engine of the car generates a force which displaces the car.

In other words, work is done by the car. This work is done on the expense of fuel. Fuel provides the energy needed to run the car. Had the petrol tank been empty car could not be run.

The conclusion is that, if there is no source of energy, no work will be done.

Anything which is capable of doing work has energy.

For example, the steam pushes up the lid placed on the boiling water container. It means the steam has the ability or capacity to do work. The work done by the steam on the lid is equal to the energy of the steam.

Definition of Energy: The capacity of doing work by the body or an object is known as the energy of the body or the object.

Energy is a scalar quantity.

S.I. unit of energy is Joule (J).

C.G.S. of energy is erg.

1 J (Nm) = 10^7 erg (dyne cm)

Some other units of energy:

$$1eV = 1.6 \times 10^{-19} J, 1kJ = 1000J$$

$$1Cal = 4.186J$$

$$1kWh = 3.6 \times 10^6 J$$

7. Different forms of energy

(1) Chemical energy: Petrol, diesel, kerosene oil, coal, charcoal, CNG, LPG are fuels which are rich in chemical energy. The energy released during chemical reactions is known as chemical energy.

(2) Heat or Thermal energy: The energy possessed by a body due to its temperature is called heat energy. This energy is related to the kinetic energy of the constituent particles (atoms, molecules) of the body.

(3) Light energy: The energy which produces sensation of vision in our eyes is called light energy.

(4) Electrical energy: The energy of moving electrons in a conductor connected with a battery is known as electrical energy.

(5) Nuclear energy: The energy released by the nuclei during fission or fusion process is known as nuclear energy.

(6) Mechanical energy: The energy possessed by a body because of its speed or position or change in shape is called the mechanical energy. It is the sum of kinetic energy and potential energy of a body.

8. Kinetic Energy

A moving body is capable of doing work *e.g.* when a moving ball hits another stationary object, like small wooden block, then the wooden block is displaced from its position. Hence, moving object has energy which is known as kinetic energy.

Definition: The energy possessed by a body by virtue of its motion is called kinetic energy.

Example: (i) A moving car or bus or a running train.

(ii) Flowing water.

(iii) A moving ball or anything which moves has kinetic energy.

Expression for kinetic energy:

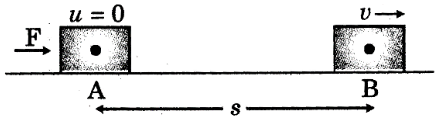


Fig. 8.1.

Suppose initially the body A is at rest $u = 0$. Let a force F be applied on the body, so that the body attains a velocity v after travelling a distance s.

From the equation of motion

$$v^2 - u^2 = 2as$$

$$v^2 - (0)^2 = 2as$$

$$v^2 = 2as$$

$$s = \frac{v^2}{2a} \qquad \text{...(1)}$$

According to Newton's second law of motion, the force applied on the body

$$F = ma \qquad \text{...(2)}$$

The work done by this force on the body is

$$W = F \times s \qquad \text{...(3)}$$

or Work = force $\times$ displacement

Substituting the value of F and s in equation (3),

$$\text{Work (W)} = \text{ma} \times \frac{v^2}{2a}$$

$$\text{Work} = \frac{1}{2}mv^2$$

This work done is equal to the kinetic energy of the body

$$E_k = \frac{1}{2}mv^2$$

$$E_k = \frac{1}{2}\text{(mass of the body) (speed of the body)2}$$

Some Important conclusions from the equation

$$K.E. = \frac{1}{2}mv^2$$

(a) The kinetic energy of a moving body is directly proportional to its mass when the velocity of the body is kept constant:

$$E_k = m$$

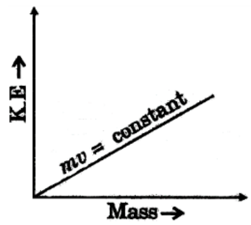


Fig. 8.2.

(b) The kinetic energy of a body is directly proportional to the square of its velocity wheat the mass of the body is kept constant:

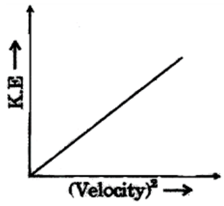


Fig. 8.2

Remember: Kinetic energy of a body is always positive, as mass (m) can never be negative and square of a real value is always positive.

9. Relation between Kinetic Energy and Linear Momentum

$\because$ $$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2}\frac{m^2v^2}{m} = \frac{(mv)^2}{2m}$$

$\because$ $mv = p$ **(linear momentum)**

$$E_k = \frac{p^2}{2m}$$

$$p = \sqrt{2mE_k}$$

10. Potential Energy

Let a small mass m be released from a smooth inclined plane. Another mass M is kept at a rough horizontal plane at rest. The mass m will move along the inclined plane and strike the mass M. Both the masses will move along the horizontal surface for some distance and come forest. The mass M moves a distance s by the force applied by m. Thus, m does work for which it requires energy.

This energy is possessed by m at A as it was at a height h from the horizontal surface. This energy due to position is called potential energy.

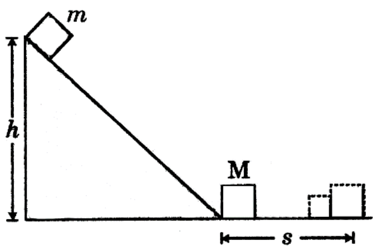


Fig. 10.1

Activity: Consider a block attached to a spring. The and release it. What do you observe? When the compressed spring is released, it does the work of pushing the block. This means the compressed spring has the ability to do work.

In the above situation the compressed spring has energy in it because of its configuration,

This energy gets stored in the spring due to the work done on stretching or compressing it.

The energy transferred to the stretched or the compressed spring is stored as potential energy. This form of potential energy is the elastic potential energy.

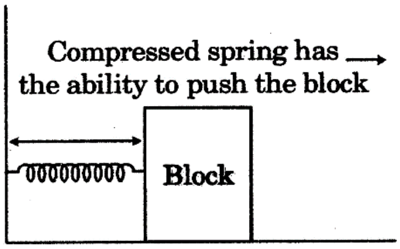


Fig. 10.2

11. Definition of Potential Energy

The energy possessed by an object by virtue of its position or shape or configuration is known as potential energy.

Examples: (i) Water stored in a dam has potential energy due to its position.

(ii) A stretched or compressed spring has potential energy due to its configuration.

(iii) A stretched bow and arrow has potential energy due to its shape.

(iv) It is due to the potential energy of the compressed spring in a loaded pistol that the bullet is released with a greater velocity on firing the pistol.

12. Types of Potential Energy of a system

(1) Gravitational Potential Energy: The work done against the gravitational force gets stored in the form of gravitational potential energy.

The gravitational potential energy of an object at a point above the ground is defined as the work done in raising it from the ground to that point against gravity *e.g.*, water stored at a height in a dam.

(2) Elastic Potential Energy: Elastic potential energy of body is the energy possessed k the body by virtue of its configuration.

13. Potential energy of an object at a height

Consider an object of mass m raised to a height h against gravity.

Minimum force required to lift the object

$$F = \text{Weight of body} = mg$$

$$W = F \times s = mgh$$

This work done against force of gravity is equal to the potential energy or gravitational potential energy of the object.

Potential energy $(E_p) = \text{mgh}$

Important observations from the formula of Potential energy = mgh

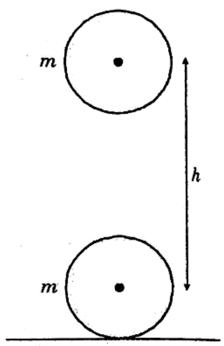


Fig.13.1

(i) The gravitational potential energy of an object is directly proportional to the mass (m) of the body when other parameters are kept constant *i.e.*

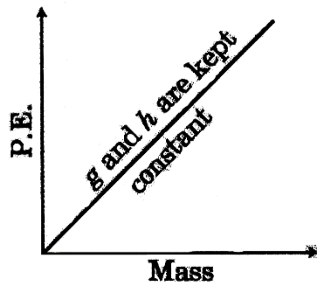


Fig. 13.2

(ii) The gravitational potential energy of an object is directly proportional to the height A to which the body is raised when other parameters are kept constant.

$$E_p = h$$

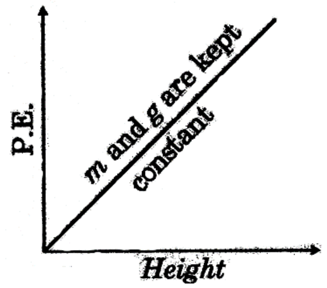


Fig. 13.3

(iii) The potential energy of a body is directly proportional to the acceleration due to the gravity h when other parameters are kept constant.

Potential energy = g

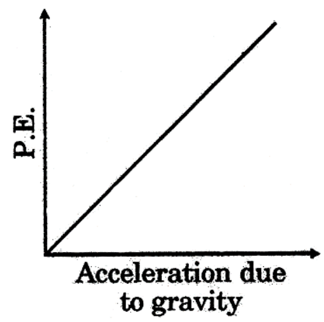


Fig. 13.4

Gravitational Potential energy depends upon the difference in heights of the initial position and final position of a body but is independent path followed by the body while going from initial position to final position

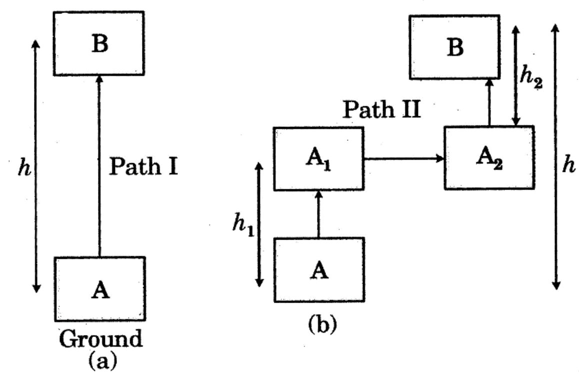


Fig. 13.5.

Consider a ball of mass *m* raised through height *h* from position A to position B along path I.
Potential energy at height $h = mgh$ ...(1)
Now let the ball is raised through height *h* along path II.
Total Potential energy of the ball at position $B = mgh_1 + mgh_2$

$$= mg(h_1 + h_2) = mgh \qquad ...(2) \qquad (\because h_1 + h_2 = h)$$

so from equation (1) and (2) we conclude that:
(i) Gravitational potential energy of a body depends upon the difference in height of initial and final position of the body.
(ii) Gravitational potential energy of the body does not depend upon the path followed by the body in going from initial and final positions.

14. **Transformation of Energy**
Transformation of energy plays an important role in our day to day activities. In fact, life exists on earth because of energy transfers.
The process of changing or converting one form of energy into another form is known as transformation of energy.

Examples of Transformation of Energy
(A) Conversion of potential energy into kinetic energy:
(1) A stretched bow and arrow has potential energy. When arrow is released potential energy is converted into kinetic energy of the moving arrow.
(2) Let the pendulum be displaced from 0 to A where it is at rest. At position A, the pendulum has potential energy *(mgh)*. When the pendulum is released from position A, it begins to move towards position 0.
The speed of the bob of the pendulum increases and its height decreases. So potential energy of the pendulum is changed into its kinetic energy.

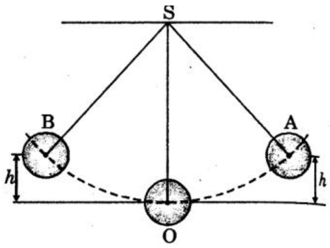
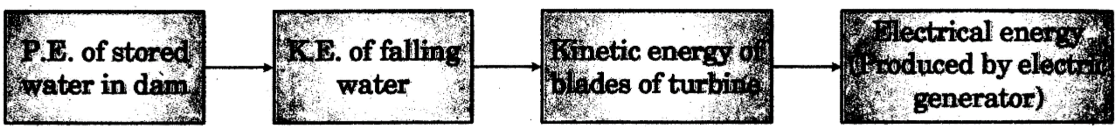


Fig. 14.1

At position O whole of the potential energy of the pendulum is converted into kinetic energy.
We find that the potential energy is converted into kinetic energy and vice versa during the swinging of a pendulum.
But the total energy of the pendulum at any position during swinging remains the same.

15. Conversation of Potential energy into Kinetic energy and then in Electrical energy
Conversion of chemical energy into light and heat energy:



(1) When a torch is switched on, chemical energy of the torch cell is converted into electrical energy which is then converted into light and heat energy.
(2) The explosion of a fire cracker convert chemical energy into sound, light and heat I energy.

Conversion of heat energy into mechanical energy: The heat energy produced due to the burning of coal changes water into steam. This steam is used to run the steam engine. Thus, chemical energy of coal is converted into heat energy and then heat energy is converted into mechanical energy (used to run the steam engine).
Conversion of elastic potential energy into kinetic energy: When the spring of a watch is wound *i.e.* the spring of the watch is coiled, work done to do so is stored in the form of elastic potential energy,
When the watch operates and shows time, the spring is uncoiled and the elastic potential energy is converted into kinetic energy of the hands of the watch.
Conversion of mechanical energy into heat energy:
(1) When we rub our hands, they become warm. In this case, mechanical energy is converted into heat energy.
A log of wood cut by a saw becomes hot. In this case mechanical energy is converted into heat energy.
Conversion of muscular energy into kinetic energy: A boy riding a bicycle. The muscular energy used for pedaling a bicycle is converted into the kinetic energy of the moving bicycle.
Conversion of chemical energy into electrical energy: In a thermal power plant, coal is burnt to produce electricity. Thus, chemical energy of coal is converted into electrical energy.

16. Devices and used to transform energy from one form to another
(1) Heat engine converts heat energy into mechanical energy.
(2) A thermal power plant converts chemical energy of the coal into electrical energy.
(3) Microphone converts sound energy into electrical energy.
(4) Dry cell converts chemical energy into electrical energy.
(5) Generator converts mechanical energy into electrical energy.
(6) Electric motor converts electrical energy into mechanical energy.

17. Lau of Conservation of Energy
According to this law, "'Energy can neither be created nor be destroyed, but can be changed from one form to another form."

OR

When one form of energy is changed or transformed into other forms of energy.
The total energy before transformation = The sum of the different energies transformed.
Verification of Law of Conservation of Energy: Consider a body of mass *m* at a height *h* above the ground. Suppose this position of the body is A, at A the body is at rest i.e. $v = 0$.
At position A: Potential energy of the body, $E_p = mgh$

Kinetic energy of the body, $E_k = 0$

Total energy of the body at $A = E_k + E_p = mgh + 0 = mgh$ ...(1)

Let the body falls freely under the action of gravity to position B through a height *x*. Now, the height of the body from the ground = *(h* - x).
At position B: Potential energy of the body, $E_p = mg(\text{h}-\text{x})$ ...(2)

Kinetic energy of the body, $E_k = \frac{1}{2}mv^2$ ...(3)

where v is the velocity of the body at position B.
Calculation of u,

$\because$ $v^2 - u^2 = 2as$ ...(4)

$u = 0$ (body at A is at rest)

$a = g$ and $s = x$

From eq. (4), $v^2 - 0 = 2gx$ or $v^2 = 2gx$

Put this value of v^2 in equation (3), we get

$$E_k = \frac{1}{2}m \times 2gx = mgx$$

$\therefore$ Total energy of the body at $B = E_P + E_K$

$$= mg(h - x) + mgx$$

$$= mgh - mgx + mgx$$

$$= mgh \qquad ...(5)$$

Finally, let the body touches the ground at C, so that the distance through which it falls = h.

At position C: $E_p = mg(0) = 0$ (h = 0 at ground level)

$$E_k = \frac{1}{2}mv^2 \qquad ...(6)$$

where v is the velocity of the body just at position C.

Calculation of u: $v^2 - u^2 = 2as$ ...(7)

$u = 0$ ($\because$ body is at rest at position A)

$a = g$ and s = h

From equation (7), $v^2 - 0 = 2gh$ or $v^2 = 2gh$

Put this value in eq. (6), we get

Kinetic energy of the body at $E_K = \frac{1}{2}m \times 2gh = mgh$

$\therefore$ Total energy of the body at $C = E_P + E_K = 0 + mgh$

$$= mgh \qquad ...(8)$$

From equation (1), (5) and (8) it is clear that the total energy of a body at any instant during, free fall of the body remains constant. Hence, the law of conservation of energy is verified.

18. Power

Power measures the speed of work done, that is, how fast or slow work is done.

Power is defined as the rate of doing work by a body or the rate of transfer of energy.

$$P = \frac{Work\,done}{Time\,taken} = \frac{W}{t}$$

Power depends on two factors *i.e. on* the amount of work done and the time taken.

(1) $P = \frac{1}{t}$ (for the same amount of work)

(2) P = W (for a constant time)

19. Average Power

It is the ratio of total work done to the total time taken

$$P = \frac{Total\,work\,done}{Total\,time}$$

Since, Total work done = Energy supplied

$$P = \frac{Energy\,supplied}{Total\,time}$$

20. Power in terms of Energy

Work done = Energy supplied (transferred)

$$Power = \frac{Energy\,supplied}{Time\,taken} = \frac{E}{t} = \frac{F \times s}{t}$$

$$= F \times \frac{s}{t} = F \times v$$

$$P = F \times v$$

Where v is the velocity of the body.

Units of Power: S.I. unit of power is watt (W)

Since $$P = \frac{W}{t}$$

$\therefore$ $$1 watt (\mathrm{W}) = \frac{1\,Joule}{1\,Second} = \frac{1J}{1s}$$

Power of a machine is 1 watt if it does 1 joule work in 1 second.
The power of machines is usually expressed in horse power (h.p.). So, the practical unit of power is horse power (h.p.)
1 horse power (h.p.) = 746 W

Remember:

1. $1\,kilowatt(\mathrm{kW}) = 1000\,\mathrm{W} = 10^3 W$

2. $1\,Megawatt\,(\mathrm{MW}) = 10^6 W$

3. $1 Gigawatt\,(\mathrm{GW}) = 10^9 W$.

21. Commercial Unit of Energy

For commercial purposes, we use a bigger unit of electrical energy which is called 'kilowatt hour'.
One kilowatt hour (kWh) is the amount of electrical energy consumed by an electrical appliance of power 1 kilowatt in one hour.
kWh is also known as "Board of Trade unit'(B.O.T .) or simply a unit.
Relation between kWh & Joule.

$$1kWh = 1000Wh \qquad [\because \text{ k W=1000W}]$$
$$1W = 1Js^{-1} \text{ and } 1h = (60 \times 60)s = 3600s$$
$$1kWh = 1000\ Js^{-1} \times 3600\ s = 3600000\ J = 3.6 \times 106\ J$$
$$1kWh = 3.6 \times 106\ J$$

How to calculate the electricity bill?

Suppose electric gadget of house have consumed 200 kWh of electricity in a month and the cost **of one unit is R 2.**
Then, total bill for a month $= 200 \times 2 = Rs.400$ (Here 1 kWh = 1 unit)
Formula used to calculate the number of units

Electricity bill $= \dfrac{P \times 4 \times D}{1000}$

where P = Power in watt, H s Time in hour, D = Number of days

- **Work:** Work is said to be done whenever a force acts on a body and the body moves in the direction of the force.
- Work done = Force $\times$ Distance
  or $W = Fs\cos\theta$.
- Work done on an object by a force would be zero, if the displacement of the object is zero.
- **Positive and negative work:** Work done is positive when the force is in the direction of displacement. Work done is negative when the force acts opposite to the direction of displacement.
- **Joule:** It is the S.I. unit of work. One joule of work is said to be done, whenever a force of one newton displaces a body through a distance of 1 metre in its own direction.
  1 joule = 1 newton $\times$ 1 metre or 1 J = 1 Nm.
- **Energy:** Energy of a body is defined as its capacity to do work. It is a scalar quantity.
- **Unit of energy:** As the energy is measured by the amount of work that a body can do, so the unit of energy is same as that of work. The S.I. unit of energy is joule (J). One joule of energy is the energy required to do 1 J of work.

1 kilojoule = 1 kJ = 1000 J.

- **Kinetic energy:** The energy possessed by a body by virtue of its motion is called its 'kinetic energy". An object of mass m moving with velocity v has a kinetic energy.

  $K.E. = \frac{1}{2}mv^2$

- **Relation between work done and kinetic energy:**

  Work done = Change in $K.E. = \frac{1}{2}m(v^2 - u^2)$

- **Potential energy:** The energy possessed by a body by virtue of its position or shape configuration is called its "potential energy.'
- **Gravitational potential energy:** The gravitational potential (P.E.) of an object at a point above the ground is defined as the work done in raising it from the ground to that point against gravity. The gravitational potential energy of an object of mass m raised through a height h above the earth's surface is given by

  $$E_p = mgh$$

- **Law of conservation of energy:** Energy can neither be created nor destroyed. It can be transformed from one form to another. The total energy before and after the transformation always remains constant.
- **Power:** The rate of doing work is called power.

  $Power = \frac{Work}{Time}$ or $P = \frac{W}{t}$

- The work, power and energy are all scalar quantities.
- **Watt:** It is the S.I. unit of power. The power of an agent is one watt if it does work at the rate of 1 joule per second.

  $$1watt = \frac{1\,joule}{1\sec ond} \text{ or } 1W = 1Js^{-1}$$

  1 kilowatt = 1000 watt or 1 kW = 1000 W

  1 horse power == 746 watt or 1 H.P. = 746 W.

- **Kilowatt-hour (kWh):** It is the commercial unit of electric energy. It is defined as the electric energy consumed by an appliance of power 1000 watt in one hour.

  $1kWh = 3.6 \times 10^6 J$