Quadratic Equations

- **Quadratic equation:** An equation of the form $ax^2+bx+c=0$ where a, b and $c \in R$ and $a \neq 0$ is called a quadratic equation.

  If p(x) is a quadratic polynomial, then p(x) = 0 is called a quadratic equation.

  Note: (i) An equation of degree 2 is called a quadratic equation.

  (ii) The quadratic equation of the form $ax^2+bx+c=0$.

  Solution or roots of a quadratic equation: If p(x) = 0 is a quadratic equation, then the zeros of the polynomial p(x) are called the solutions or roots of the quadratic equation p(x)=0.

  Note: (i) Since the degree of a quadratic equation is 2, it has 2 roots or solutions.

  (ii) x = a is the root of p(x) = 0, if p(a) = 0.

  (iii) Finding the roots of a quadratic equation is called solving the quadratic equation.

- Methods of solving a quadratic equation: There are different methods of solving a quadratic equation.

  (a) Factorization method

  (i) Splitting the middle term

  (ii) Completing the square

  (b) Formula method

  (a) (i) Splitting the middle term: Consider the quadratic equation $ax^2+bx+c=0$.

  Step 1: Find the product of the coefficient of x^2 and the constant term i.e., ac.

  Step 2: (a) If ac is positive, then choose two factors of ac, whose sum is equal to b (the coefficient of the middle term).

  (b) If ac is negative, then choose two factors of ac, whose difference is equal to b (the coefficient of the middle term).

  Step 3: Express the middle terms as the sum (or difference) of the two factors obtained in step 2. [Now the quadratic equation has 4 terms]

- Step 4: Separate the term common to the first two terms and then write the first two terms as a product. Take the common term (binomial) out of the last two terms and get another factor so that the last two terms are written as a product.

Step 5: Express the given quadratic equation as a product of two binomials, and solve them.

Step 6: The two values obtained in step 5 are the roots of the given quadratic equation.

Note: An important property used in solving a quadratic equation by splitting the middle term.

"If ab = 0, then either a = 0, or b = 0 or both a and b are 0, where 'a' and 'b' are real numbers"

(ii) Completing the square: In some cases where the given quadratic equation can be solved by factorization, a suitable term is added and subtracted. Then terms are regrouped in such a manner that a square is completed by three of the terms. The equation is then solved using factorization method.

Note: Usually, the term added and subtracted is the square of half the coefficient of x.

b. Formula method: The roots of a quadratic equation $ax^2+bx+c=0$ are given by $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ provided $b^2-4ac\geq 0$. This formula for finding the roots of a quadratic equation is called the quadratic formula.

Note: The roots of the quadratic equation using the quadratic formula are

$x=\frac{-b\sqrt{b^2-4ac}}{2a}$ and $x=\frac{-b-\sqrt{b^2-4ac}}{2a}$

Nature of roots: For a quadratic equation $ax^2+bx+c=0$, b^2-4ac (denoted by D) is called the discriminate.

Value of Discriminate	**Roots**	**Nature of roots**	**No. of Roost**
Greater than (positive)	$\frac{-b+\sqrt{b^2-4ac}}{2a}$ **and** $\frac{-b-\sqrt{b^2-4ac}}{2a}$	**Real and distinct**	**2**
Lesser than 0 (Negative)	$\frac{-b+i\sqrt{\lvert D\rvert}}{2a}$ **and** $\frac{-b-i\sqrt{\lvert D\rvert}}{2a}$	**Complex or imaginary**	**0**
Equal to 0	$\frac{-b}{2a},\frac{-b}{2a}$	**Real and**	**2 coincident roots or repeated roots**

If $b^2 - 4ac > 0$, then $\sqrt{b^2 - 4ac}$ is a real number and the quadratic equation $ax^2 + bx + c = 0$ has two real roots a and p.

If $b^2 - 4ac < 0$, then $\sqrt{b^2 - 4ac}$ is not a real number and the quadratic equation $ax^2 + bx + c = 0$ has no real root.

- Quadratic equations can be applied to solve word problems.

Note: Any root of the quadratic equation that does not satisfy the condition of the problem is discarded.