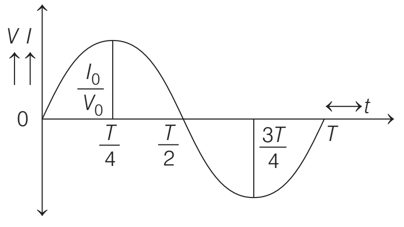
DAY TWENTY FOUR

Alternating Current

Learning & Revision for the Day

- Peak and RMS Values of Alternating Current/Voltage
- Different Types of AC Circuits
- Series AC Circuits
- Power in an AC Circuit
- AC Generator
- Transformer

An **alternating current** is the current (or voltage) whose magnitude keeps on changing continuously with time, between zero and a maximum value and its direction also reverses periodically.



Peak and RMS Values of Alternating Current/ Voltage

RMS value of alternating voltage is equal to $\frac{1}{\sqrt{2}}$ times of peak value. i.e. $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$

Similarly, $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$

Mean Value or Average Value

The steady current, which when passes through a circuit for half the time period of alternating current, sends the same amount of charge as done by the alternating current in the same time through the same circuit, is called mean or average value of alternating current. It is denoted by i_m or i_{av}

$$i_m \text{ or } i_{\text{av}} = \frac{2i_0}{\pi} = 0.636 i_0$$

Mean or average value of alternating current during a half cycle is 0.636 times (or 63.6% of) its peak value (i_0).

Similarly, mean or average value of alternating emf

$$V_m \text{ or } V_{av} = \frac{2V_0}{\pi} = 0.636$$

Peak Value

The maximum value (amplitude) of alternating current and voltage is called peak value.

RMS Value

The steady current, which when passes through a resistance for a given time will produce the same amount of heat as the alternating current does in the same resistance and in the same time, is called rms value of alternating current. It is denoted by i_{rms} or $i_v = \frac{i_0}{\sqrt{2}} = 0.707\, i_0$

where, i_0 = peak value of alternating current

Similarly, rms value of alternating emf

$$V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0$$

Reactance and Impedance

- The opposition offered by a pure inductor or capacitor or both to the flow of AC, through it, is called **reactance** (X). Its unit is ohm (Ω) and dimensional formula is $[ML^2T^{-3}A^{-2}]$.
- **Reactance is of two types**
  (i) Inductive reactance, $X_L = L\omega$ and
  (ii) Capacitive reactance, $X_C = \frac{1}{C\omega}$
- Reciprocal of reactance is known as **susceptance**.
  Thus, $S = \frac{1}{X}$
- Total opposition offered by an AC circuit to the flow of current through it, is called its **impedance** (Z). Its unit is ohm and dimensional formula is $[ML^2T^{-3}A^{-2}]$.
  For an AC circuit, $Z = \sqrt{X^2 + R^2} = \sqrt{(X_L - X_C)^2 + R^2}$
- Reciprocal of impedance is known as **admittance**.
  Thus, $Y = \frac{1}{Z}$. Its unit is Siemens (S).

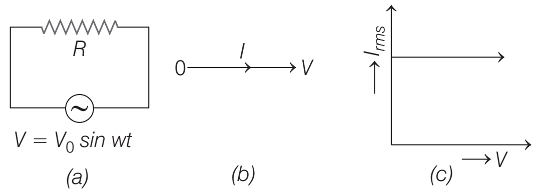
Different Types of AC Circuits

The circuit consists of only resistor or only capacitor or only inductor are called pure resistive, pure inductive and pure capacitive circuit.

1. Pure Resistive Circuit

Let an alternating voltage $V = V_0 \sin \omega t$ be applied across a pure resistance R. Then,

Current, $I = \frac{V}{R}$ or $I_{rms} = \frac{V_{rms}}{R}$



Current and voltage are in the same phase, i.e. current is given by $I = I_0 \sin \omega t$.

2. Pure Inductive Circuit

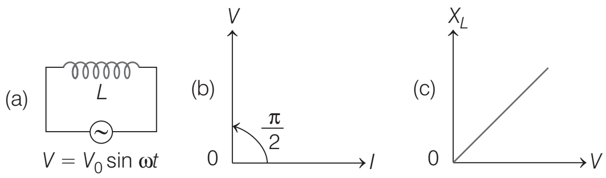
Let an alternating voltage $V = V_0 \sin \omega t$ be applied across a pure inductance L.

Then, the average power $= V_{rms} I_{rms} \cos \frac{\pi}{2} = 0$

The inductance offers some opposition to the flow of AC, known as **inductive reactance** $X_L = 2\pi\nu L = L\omega$.

Thus, a pure inductance does not oppose the flow of DC ($\omega = 0$) but opposes the flow of AC.

Current flowing, $I = \frac{V}{X_L}$



In pure inductive circuit, current decreases with an increase in frequency, it lags behind the voltage by $\frac{\pi}{2}$ (or voltage leads the current by $\frac{\pi}{2}$) and is thus given by

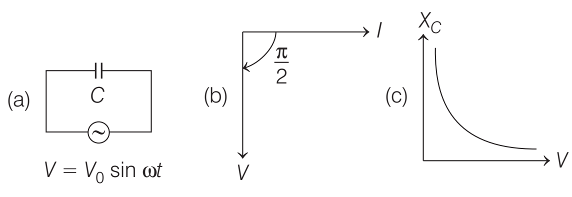
$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

3. Pure Capacitive Circuit

Let an alternating voltage $V = V_0 \sin \omega t$ be applied across a pure capacitance C. Then, the capacitance offers some opposition to the flow of current, but allows AC to pass through it. The opposition offered is known as the **capacitive reactance**.

$$X_C = \frac{1}{C\omega}\ \Omega$$
$$= \frac{1}{C \times 2\pi\nu}\ \Omega$$

Current flowing, $I = \frac{V}{X_C}$



In pure capacitive circuit, current increases with an increase in frequency and leads the voltage by $\frac{\pi}{2}$ (or voltage lags behind the current by $\frac{\pi}{2}$) and is thus, given by

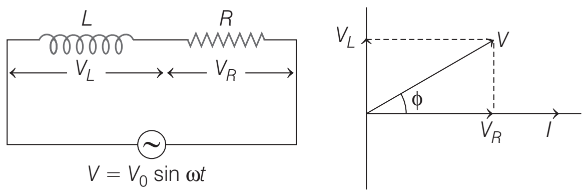
$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Series AC Circuits

Some of the series AC circuits are given below

1. Series *L*-*R* Circuit

The potential difference across the resistance in an AC circuit is in phase with current and it leads in phase by 90° with current across the inductor.



$$V = V_0 \sin \omega t \text{ and } I = \frac{V_0}{Z} \sin(\omega t - \phi)$$

where, $Z = \sqrt{R^2 + (\omega L)^2}$

Current lags behind the voltage by ϕ.

and $\tan \phi = \frac{\omega L}{R}$

$\therefore \quad V = \sqrt{V_R^2 + V_L^2}$

where, V_R = voltage across resistor R

and V_L = voltage across inductor.

2. Series *R*-C Circuit

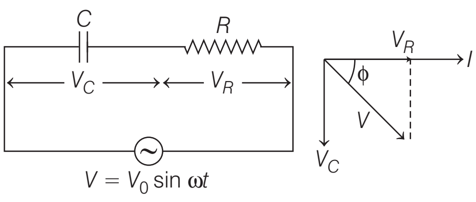
The potential difference across a resistance in AC circuit is in phase with current and it lags in phase by 90° with the current in the capacitor.

$$V = V_0 \sin \omega t \text{ and } I = \frac{V_0}{Z} \sin(\omega t + \phi)$$

where, $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

Current leads the voltage by ϕ.

and $\tan \phi = \frac{-1/\omega C}{R}$



$\therefore \quad V = \sqrt{V_R^2 + V_C^2}$

where, V_R = voltage across resistor R

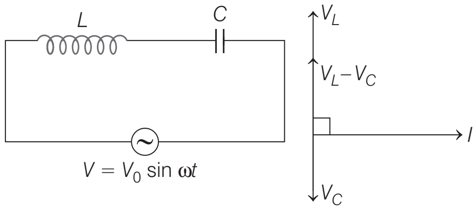
and V_C = voltage across capacitor.

3. Series *L*-*C* Circuit

The potential difference across a capacitor in AC lags in phase by 90° and leads in phase by 90° across inductor with the current in the circuit.

$$V = V_0 \sin \omega t, \; I = \frac{V_0}{Z} \sin(\omega t - \phi)$$

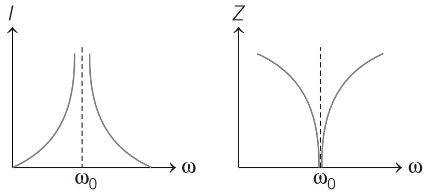
where, $Z = X_L - X_C$ and $\tan \phi = \frac{X_L - X_C}{0} = \infty$



For $X_L > X_C$, $\phi = \frac{\pi}{2}$ and for $X_L < X_C$, $\phi = -\frac{\pi}{2}$

If $X_L = X_C$ i.e. at $\omega = \frac{1}{\sqrt{LC}}$, $Z = 0$ and I_0 becomes infinity.

This condition is termed as the **resonant condition** and this frequency is termed as natural frequency of the circuit.



Variation of *I* with ω in *L*-*C* circuit

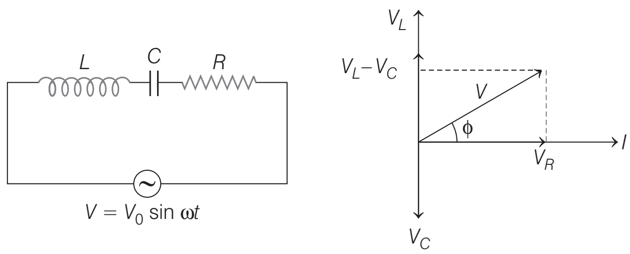
Variation of *Z* with ω in *L*-*C* circuit

4. Series *L*-*C*-*R* Circuit

For *L*-*C*-*R* circuit $V = V_0 \sin \omega t$, $I = \frac{V_0}{Z} \sin(\omega t - \phi)$

where, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

and $\tan \phi = \frac{X_L - X_C}{Z}$



Phasor diagram for *L*-*C*-*R* series circuit for $X_L > X_C$

For $X_L > X_C$, current lags voltage.

$X_L < X_C$, current leads voltage.

$X_L = X_C$, current and voltage are in phase.

If $X_L = X_C \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$, i.e. the natural frequency of the circuit is equal to the applied frequency, then the circuit is said to be in **resonance**.

At resonance, the current in the circuit is maximum and the impedance is minimum and equal to R.

Resonance frequency, $\nu = \frac{1}{2\pi\sqrt{LC}}$

Quality Factor

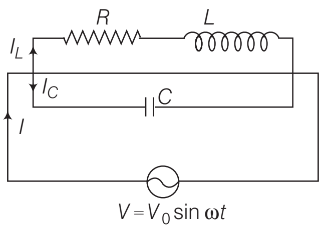
The Q-factor or quality factor of a resonant L-C-R circuit is defined as ratio of the voltage drop across inductor (or capacitor) to applied voltage. Thus,

$$Q = \frac{\text{voltage across } L \text{ (or } C)}{\text{applied voltage}}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Parallel Resonant Circuit

In this combination, a capacitor is connected in parallel with a series combination of inductor and resistor.



From the figure,

$$I = I_L + I_C$$

or $$\frac{V}{Z} = \frac{V}{R + j\omega L} + \frac{V}{-j/\omega C}$$

$\therefore$ $$\frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$\frac{1}{Z}$ is known as admittance (Y).

Thus, $$Y = \frac{1}{Z} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$\therefore$ $$Y = \frac{\sqrt{R^2 + (\omega CR^2 + \omega^3 L^2 C - \omega L)^2}}{R^2 + \omega^2 L^2}$$

The admittance will be minimum, when

$$\omega CR^2 + \omega^3 L^2 C - \omega L = 0 \text{ or } \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$\therefore$ $$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

It is known as **resonance frequency**.

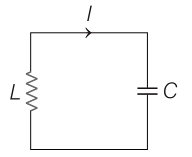
At resonance frequency admittance is minimum of the impedance is maximum. Thus, the parallel circuit does not allow this frequency from the source to pass in the circuit. Due to this reason, the circuit with such a frequency is known as **rejector circuit**.

- Dynamic resistance, $Z_{\max} = \frac{1}{Y_{\max}} = \frac{L}{CR}$
- Peak current through the supply $= \frac{V_0}{L/CR} = \frac{V_0 CR}{L}$
- The peak current through capacitor $= \frac{V_0}{1/\omega C} = V_0 \omega C$
- Q-factor $= \frac{V_0 \omega C}{V_0 CR/L} = \frac{\omega L}{R}$

This is basically the measure of current magnification.

L-*C* Oscillations

An L-C circuit also called a resonant circuit, tank circuit or tuned circuit. When connected together, they can act as an electrical resonator, storing energy oscillating at the circuits resonant frequency.



The energy oscillates back and forth between the capacitor and inductor until internal resistance makes the oscillations die out. The oscillation frequency is determined by the capacitance and inductance values,

$$f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

Power in an AC Circuit

Let a voltage $V = V_0 \sin \omega t$ be applied across an AC and consequently a current $I = I_0 \sin(\omega t - \phi)$ flows through the circuit. Then,

- **Instantaneous power** $= VI = V_0 I_0 \sin \omega t \sin(\omega t - \phi)$ and its value varies with time. Here, ϕ is known as **phase difference** between V and I.
- **Average power** over a full cycle of AC is

$$P_{av} = V_{rms} I_{rms} \cos\phi = \frac{1}{2} V_0 I_0 \cos\phi$$

  The term $V_{rms} I_{rms}$ is known as the **apparent** or **virtual power**, but $V_{rms} I_{rms} \cos\phi$ is called the **true power**.
- The term $\cos\phi$ is known as the **power factor** of the given circuit. Thus,

$$\cos\phi = \frac{R}{Z} = \text{power factor} = \frac{\text{true power}}{\text{apparent power}}$$

- For a pure resistive circuit, V and I are in phase ($\phi = 0°$), hence $\cos\phi = 1$ and average power $= V_{rms} I_{rms}$

  For a pure inductive or a pure capacitive circuit, current and voltage differ in phase by $\frac{\pi}{2}\left(\text{i.e. } \phi = \frac{\pi}{2}\right)$.

  $\therefore \quad P_{avg} = 0$
- Power loss $= I^2 R = \dfrac{V^2}{R}$

Wattless Current

Average power is given by $P_{av} = E_{rms} I_{rms} \cos\phi$

The phase difference between E_{rms} and I_{rms} is ϕ. We can resolve I_{rms} into two components

$$I_{rms} \cos\phi \text{ and } I_{rms} \sin\phi$$

Here, the component $I_{rms} \cos\phi$ contributes towards power dissipation and the component $I_{rms} \sin\phi$ does not contribute towards power dissipation. Therefore, it is called **wattless current.**

Choke Coil

A low resistance inductor coil used to suppress or limit the flow of alternating current without affecting the flow of direct current is called **choke coil**.

Let us consider a choke coil (used in tube lights) of large inductance L and low resistance R. The power factor for such a coil is given by

$$\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \approx \frac{R}{\omega L} \qquad [\text{as, } R << \omega L]$$

As $R << \omega L$, $\cos\phi$ is very small. Thus, the power absorbed by the coil $V_{rms} I_{rms} \cos\phi$ is very small. On account of its large impedance $Z = \sqrt{R^2 + \omega^2 L^2}$, the current passing through the coil is very small. Such a coil is used in AC circuits for the purpose of adjusting current to any required value without wastage of energy.

The only loss of energy is due to hysteresis in the iron core, which is much less than the loss of energy in the resistance that can also reduce the current, if placed instead of the choke coil.

AC Generator

An electric generator or dynamo is a device used to produce electrical energy at the expense of mechanical/thermal energy.

It works on the principle of electromagnetic induction, when a coil is rotated in a uniform magnetic field, an induced emf is set up between its ends. The induced emf is given by

$$e = e_0 \sin\omega t = NBA\omega \sin\omega t.$$

The direction of the induced emf is alternating in nature.

Transformer

It is a device which works in AC circuits only and is based on the principle of mutual induction.

Transformer is used to suitably increase or decrease the voltage in an AC circuit. Transformer which transforms strong AC at low voltage into a weaker current at high alternating voltage is called a step-up transformer. A step-down transformer transforms weak current at a higher alternating voltage into a strong current at a lower alternating voltage.

For an **ideal transformer** $\dfrac{e_s}{e_p} = \dfrac{V_s}{V_p} = \dfrac{N_s}{N_p} = \dfrac{I_p}{I_s} = k$

where, k is known as the transformation ratio.

For a step-up transformer, $k > 1$ but for a step-down transformer, $k < 1$.

In a transformer, the input emf and the output emf differ in phase by π radians.

The **efficiency of a transformer** is given by

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{V_s I_s}{V_p I_p}$$

For an ideal transformer, $\eta = 100\%$ or 1. However, for practical transformers, $\eta \approx 85 - 90\%$.

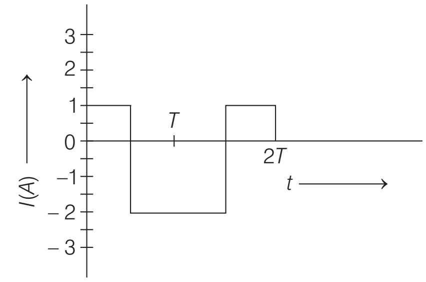
Possible causes of energy loss in transformer are

- Heating due to winding resistance
- Eddy current losses
- Magnetic flux leakage and
- Hysteresis loss. To minimise these losses, the transformer core is made up of a laminated soft iron strips.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 The alternating current in a circuit is described by the graph as shown in figure. The rms current obtained from the graph would be



(a) 1.414 A (b) 2.2 A
(c) 1.9 A (d) 2.6 A

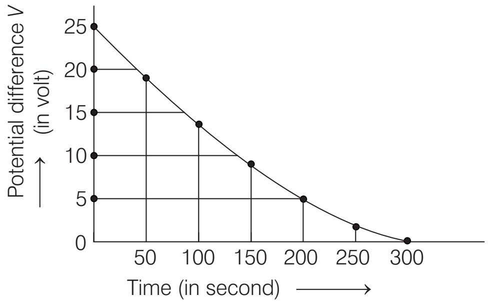
2 An alternating voltage (in volts) given by

$$V = 200\sqrt{2}\sin(100\,t)$$

is connected to a $1\mu F$ capacitor through an AC ammeter. The reading of the ammeter will be

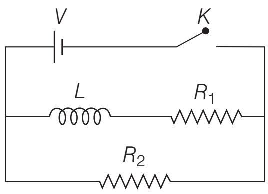
(a) 10 mA (b) 20 mA
(c) 40 mA (d) 80 mA

3 The figure shows an experimental plot discharging of a capacitor in an *R-C* circuit. The time constant τ of this circuit lies between **→ AIEEE 2012**



(a) 150 s and 200 s (b) 0 and 50 s
(c) 50 s and 100 s (d) 100 s and 150 s

4 In the circuit shown below, the key *K* is closed at $t = 0$. The current through the battery is **→ AIEEE 2010**



(a) $\dfrac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\dfrac{V}{R_2}$ at $t = \infty$

(b) $\dfrac{V}{R_2}$ at $t = 0$ and $\dfrac{V(R_1 + R_2)}{R_1R_2}$ at $t = \infty$

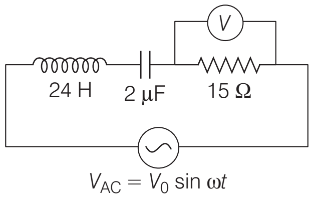
(c) $\dfrac{V}{R_2}$ at $t = 0$ and $\dfrac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

(d) $\dfrac{V(R_1 + R_2)}{R_1R_2}$ at $t = 0$ and $\dfrac{V}{R_2}$ at $t = \infty$

5 An *L-C* circuit is in a state of resonance. If $C = 0.1\mu F$ and $L = 0.25$ H, then neglecting the ohmic resistance of the circuit, find the frequency of oscillations.

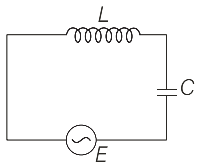
(a) 1007 Hz (b) 100 Hz (c) 109 Hz (d) 500 Hz

6 An *L-C-R* circuit as shown in the figure is connected to a voltage source V_{AC} whose frequency can be varied. The frequency, at which the voltage across the resistor is maximum, is **→ JEE Main (Online) 2013**



(a) 902 Hz (b) 143 Hz (c) 23 Hz (d) 345 Hz

7 In the circuit shown here, the voltage across *L* and *C* are respectively, 300 V and 400 V. The voltage *E* of the AC source is **→ JEE Main Online 2013**



(a) 400 V (b) 500 V (c) 100 V (d) 700 V

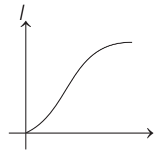
8 A fully charged capacitor *C* with initial charge q_0 is connected to a coil of self-inductance *L* at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is **→ AIEEE 2011**

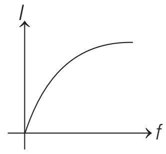
(a) $\dfrac{\pi}{4}\sqrt{LC}$ (b) $2\pi\sqrt{LC}$ (c) $\sqrt{LC}$ (d) $\pi\sqrt{LC}$

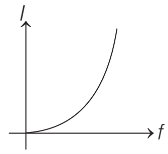
9 In an *L-C-R* circuit, if *V* is the effective value of the applied voltage, V_R is the voltage across *R*, V_L is the effective voltage across *L*, V_C is the effective voltage across *C*, then

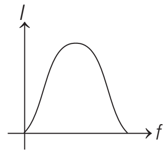
(a) $V = V_R + V_L + V_C$ (b) $V^2 = V_R^2 + V_L^2 + V_C^2$
(c) $V^2 = V_R^2 + (V_L - V_C)^2$ (d) $V^2 = V_L^2 + (V_R - V_C)^2$

10 An AC circuit of variable frequency f is connected to an L-C-R series circuit. Which one of the graphs in the figure, represents the variation of current I in the circuit with frequency f?

(a) 

(b) 

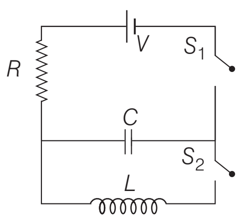
(c) 

(d) 

11 In a series L-C-R circuit, $C = 10^{-11}$ F, $L = 10^{-5}$ H and $R = 100\ \Omega$, when a constant DC voltage E is applied to the circuit, the capacitor acquires a charge 10^{-9} C. The DC source is replaced by a sinusoidal voltage source in which the peak voltage E_0 is equal to the constant DC voltage E. At resonance, the peak value of the charge acquired by the capacitor will be **→ JEE Main (Online) 2013**

(a) 10^{-15} C (b) 10^{-6} C
(c) 10^{-10} C (d) 10^{-8} C

12 In a L-C-R circuit as shown below, both switches are open initially. Now, switch S_1 and S_2, kept open (q is charge on the capacitor and $\tau = RC$ is capacitance time constant). Which of the following statement is correct? **→ JEE Main 2013**



(a) Work done by the battery is half of the energy dissipated in the resistor

(b) At $t = \tau$, $q = \frac{CV}{2}$

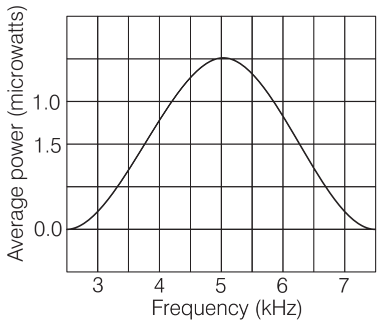
(c) At $t = 2\tau$, $q = CV\,(1 - e^{-2})$

(d) At $t = \frac{\tau}{2}$, $q = CV\,(1 - e^{-1})$

13 In a series resonant L-C-R circuit, the voltage across R is 100 V and $R = 1\ \text{k}\Omega$ with $C = 2\ \mu$F. The resonant frequency ω is 200 rad/s. At resonance, the voltage across L is

(a) 2.5×10^{-2} V (b) 40 V
(c) 250 V (d) 4×10^{-3} V

14 The plot given below is of the average power delivered to an L-R-C circuit *versus* frequency. The quality factor of the circuit is **→ JEE Main (Online) 2013**



(a) 5.0 (b) 2.0
(c) 2.5 (d) 0.4

15 For an R-L-C circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$, the current exhibits resonance. The quality factor Q is given by **→ JEE Main 2018**

(a) $\frac{\omega_0 L}{R}$ (b) $\frac{\omega_0 R}{L}$

(c) $\frac{R}{\omega_0 C}$ (d) $\frac{CR}{\omega_0}$

16 In a series L-C-R circuit, $R = 200\ \Omega$ and the voltage; and the frequency of the main supply is 220 V and 50 Hz, respectively. On taking out the capacitance from the circuit, the current lags behind the voltage by 30°. On taking out the inductor from the circuit, the current leads the voltage by 30°. The power dissipated in the L-C-R circuit is **→ AIEEE 2010**

(a) 305 W (b) 210 W (c) zero (d) 242 W

17 In an AC circuit, the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by

(a) $P = \frac{E_0 I_0}{\sqrt{2}}$ (b) $P =$ zero

(c) $P = \frac{E_0 I_0}{2}$ (d) $P = \sqrt{2} E_0 I_0$

18 Which of the following components of an L-C-R circuit, with an AC supply, dissipates energy?

(a) L (b) R (c) C (d) All of these

19 An AC circuit consists of a 220 Ω resistance and a 0.7 H choke. The power absorbed from 220 V and 50 Hz source connected in this circuit, if the resistance and choke are joined in series is

(a) 110 W (b) 50 W (c) 220 W (d) 440 W

20 The output of a step-down transformer is measured to be 24 V when connected to a 12 W light bulb. The value of the peak current is

(a) $1/\sqrt{2}$A (b) $\sqrt{2}$A (c) 2A (d) $2\sqrt{2}$A

21 A transformer has turn ratio 2 and input power 3600 W. Load current is 20 A. Efficiency $\eta = 90\%$. The internal resistance is

(a) 1 Ω (b) 0.9 Ω (c) 1.9 Ω (d) 3 Ω

Direction (Q. Nos. 22-24) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes* (*a*), (*b*), (*c*), (*d*) *given below.*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

22 **Statement I** Two identical heaters are connected to two different sources one DC and other AC having same potential difference across their terminals.The heat produced in heater supplied with AC source is greater.

Statement II The net impedance of an AC source is greater than resistance.

23 **Statement I** In a series *L*- *C*-*R* circuit, the resonance can take place.

Statement II Resonance takes place, if the inductive and capacitive reactances are equal and opposite.

24 **Statement I** In a series *R*-*L*-*C* circuit, the voltage across the resistor, inductor and capacitor are 8V, 16V and 10V, respectively. The resultant emf in the circuit is 10V.

Statement II Resultant emf of the circuit is given by the relation $E = \sqrt{V_R^2 + (V_L - V_C)^2}$.

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 In an AC circuit, the instantaneous emf and current are given by

$$e = 100 \sin 30\, t,\ i = 20 \sin\left(30\, t - \frac{\pi}{4}\right)$$

In one cycle of AC, the average power consumed by the circuit and the wattless current are respectively,

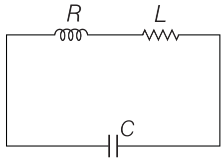
→ **JEE Main 2018**

(a) 50 W, 10 A (b) $\frac{1000}{\sqrt{2}}$ W, 10 A
(c) $\frac{50}{\sqrt{2}}$ W, 0 A (d) 50 W, 0 A

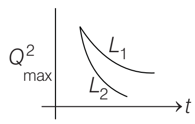
2 An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to → **JEE Main 2016**

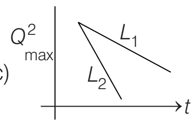
(a) 80 H (b) 0.08 H (c) 0.044 H (d) 0.065 H

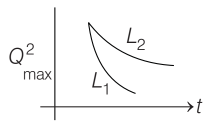
3 An *L*-*C*-*R* circuit is equivalent to a damped pendulum. In an *L*-*C*-*R* circuit, the capacitor is charged to Q_0 and then connected to the *L* and *R* as shown below

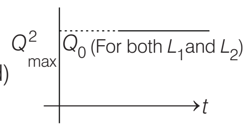


If a student plots graphs of the square of maximum charge (Q_{max}^2) on the capacitor with time (*t*) for two different values L_1 and L_2 ($L_1 > L_2$) of *L*, then which of the following represents this graph correctly? (Plots are schematic and not drawn to scale) → **JEE Main 2015**

(a)


(c)


(b)


(d)


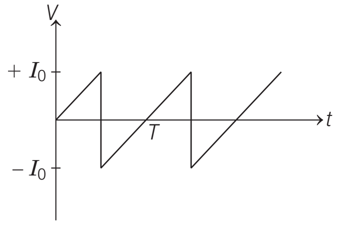
4 A resistor *R* and 2 μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of *R* to make the bulb light up 5 s after the switch has been closed (Take, $\log_{10} 2.5 = 0.4$) → **AIEEE 2011**

(a) $1.7 \times 10^5\ \Omega$ (b) $2.7 \times 10^6\ \Omega$
(c) $3.3 \times 10^7\ \Omega$ (d) $1.3 \times 10^4\ \Omega$

5 Let *C* be the capacitance of a capacitor discharging through a resistor *R*. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then, the ratio $\frac{t_1}{t_2}$ will be → **AIEEE 2010**

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 2

6 The average current in terms of I_0 for the waveform as shown is



(a) I_0 (b) $\frac{I_0}{3}$ (c) $\frac{I_0}{2}$ (d) $\frac{I_0}{4}$

7 A bulb is rated 55 W/110 V. It is to be connected to a 220 V/50 Hz with inductor in series. The value of inductance, so that bulb gets correct voltage is

(a) 200 Ω (b) 110 Ω (c) 50 Ω (d) 220 Ω

8 A coil of 0.01 H inductance and 1 Ω resistance is connected to 200 V, 50 Hz AC supply. The impedance of the circuit and time lag between maximum alternating voltage and current would be

(a) 3.3 Ω and $\frac{1}{250}$ s (b) 3.9 Ω and $\frac{1}{160}$ s

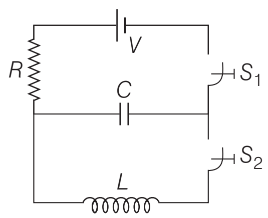
(c) 4.2 Ω and $\frac{1}{100}$ s (d) 2.8 Ω and $\frac{1}{120}$ s

9 The bandwidth in a series *L*-*C*-*R* circuit is

(a) $\frac{LC}{2\sqrt{R^2C^2+4LC}}$ (b) $\frac{2LC}{\sqrt{R^2C^2+4LC}}$

(c) $\frac{\sqrt{R^2C^2+4LC}}{LC}$ (d) zero

10 An *L*-*C*-*R* circuit, consists of an inductor, a capacitor and a resistor driven by a battery and connected by two switches S_1 and S_2, as shown in the figure.



At time $t = 0$, when the charge on the capacitor plates is q, switch S_1 is opened and S_2 is closed. The maximum charge the capacitor can hold, is q_0. Choose the correct equation

(a) $q = q_0 \cos\left(\frac{t}{\sqrt{LC}} + \frac{\pi}{2}\right)$ (b) $q = q_0 \cos\left(\frac{t}{\sqrt{LC}} - \frac{\pi}{2}\right)$

(c) $q = -LC\frac{d^2q}{dt^2}$ (d) $q = -\frac{1}{\sqrt{LC}}\frac{d^2q}{dt^2}$

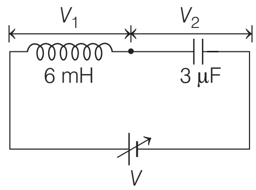
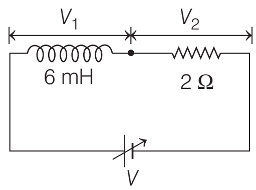
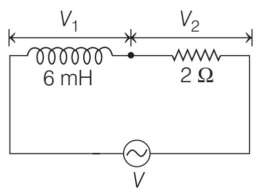
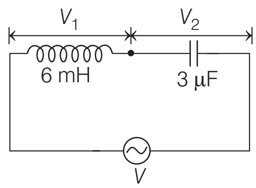
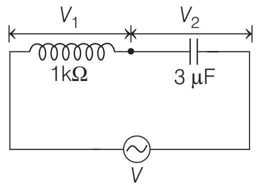
11 An alternating emf of angular frequency ω is applied across an inductance. The instantaneous power developed in the circuit has an angular frequecy

(a) ω/4 (b) ω/2 (c) ω (d) 2ω

12 An inductor of reactance 1 Ω and a resistor of 2Ω are connected in series to the terminal of a 6V(rms) AC source.The power dissipated in the circuit is

(a) 8 W (b)12 W (c)14.4 W (d)18 W

13 You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50Hz frequency (the next three circuits)in different ways as shown in Column II. When a current *I* (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 (indicated in circuits) are related as shown in Column I. Match the Column I with Column II and mark the correct option from the codes given below.

	Column I		Column II
A.	$I \neq 0, V_1$ is proportional to I	1.	
B.	$I \neq 0, V_2 > V_1$	2.	
C.	$V_1 = 0, V_2 = V$	3.	
D.	$I \neq 0, V_2$ is proportional to I	4.	
		5.	

Codes

	A	B	C	D
(a)	(2,3,4,5)	(1,2)	(2,3,4,5)	(3,4,5)
(b)	(3,4,5)	(2,3,4,5)	(1,2)	(2,3,4,5)
(c)	(1,2)	(3,4,5)	(2,3,4,5)	(2,3,4,5)
(d)	(3,4,5)	(1,2)	(2,3,4,5)	(2,3,4,5)

Direction (Q. Nos. 14 *and* 15) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes* (*a*), (*b*), (*c*), (*d*) *given below.*

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14 **Statement I** A sinusoidal AC current flows through a resistance R. If the peak current is I_0, then the power dissipated is $\frac{RI_0^2}{2}$.

Statement II For a purely resistive circuit, the power factor, $\cos \phi = 1$.

15 **Statement I** The nature of the impedance of L-C-R circuit, at resonance is pure inductive.

Statement II The phase angle between E and I in a R-L-C circuit at resonance, is zero.

ANSWERS

SESSION 1	1 (a)	2 (b)	3 (d)	4 (b)	5 (a)	6 (c)	7 (c)	8 (a)	9 (c)	10 (d)
	11 (c)	12 (c)	13 (c)	14 (d)	15 (a)	16 (d)	17 (b)	18 (b)	19 (a)	20 (a)
	21 (b)	22 (a)	23 (a)	24 (a)						
SESSION 2	1 (b)	2 (d)	3 (a)	4 (b)	5 (c)	6 (c)	7 (d)	8 (a)	9 (c)	10 (c)
	11 (d)	12 (c)	13 (a)	14 (b)	15 (d)					

Hints and Explanations

SESSION 1

1 $I_{\text{rms}} = \sqrt{\frac{I_1^2 + I_2^2 + I_3^2}{3}}$

$= \sqrt{\frac{1^2 + 2^2 + 1^2}{3}} = \sqrt{\frac{6}{3}}$

$= \sqrt{2} = 1.414\text{ A}$

2 Given, $V = 200\sqrt{2}\sin(100\,t)$.

Comparing this equation with $V = V_0 \sin \omega t$, we have

$V_0 = 200\sqrt{2}$ V and $\omega = 100\text{ rad s}^{-1}$

The current in the capacitor is

$$I = \frac{V_{\text{rms}}}{Z_C} = V_{\text{rms}} \times \omega C \qquad \left(\because Z_C = \frac{1}{\omega C}\right)$$

$$= \frac{V_0}{\sqrt{2}} \times \omega C$$

$$= \frac{200\sqrt{2}}{\sqrt{2}} \times 100 \times 1 \times 10^{-6}$$

$$= 20 \times 10^{-3}\text{ A} = 20\text{ mA}$$

3 Time constant τ is the duration when the value of potential drops by 63% of its initial maximum value (i.e. V_0/e). Here, 37% of 25 V = 9.25 V which lies between 100 s to 150 s in the graph.

4 At $t = 0$, inductor behaves like an infinite resistance.

So, at $t = 0$, $I = \frac{V}{R_2}$ and at $t = \infty$, inductor behaves like a conducting wire $I = \frac{V}{R_{\text{eq}}} = \frac{V(R_1 + R_2)}{R_1 R_2}$

5 At the resonance, $\nu = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2 \times 3.14 \times \sqrt{0.25 \times 0.1 \times 10^{-6}}}$$

$$= 1007\text{ Hz}$$

6 As voltage across resistance is maximum, therefore a power is maximum which is at the resonance frequency.

At resonance,

$$\frac{1}{\text{frequency}} = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi}\frac{1}{\sqrt{24 \times 2 \times 10^{-6}}}$$

$$= \frac{1}{2\pi}\frac{1000}{\sqrt{48}}$$

$$= \frac{1}{2\pi}\frac{1000}{6.9V}$$

$$= \frac{1000}{2 \times 3.14 \times 6.92}$$

$$= \frac{1000}{43.45} = 23\text{ Hz}$$

7 Since, reactances produced by inductor and capacitor in opposite direction. So, voltage in these elements are distributed at 180°, i.e. out of phase.

Net voltage = 400 V − 300 V

= 100 V

8 As initially charge is maximum,

$$q = q_0 \cos \omega t$$

$$\Rightarrow \quad I = \frac{dq}{dt} = -\omega q_0 \sin \omega t$$

Given, $\frac{1}{2} L I^2 = \frac{q^2}{2C}$

$$\Rightarrow \quad \frac{1}{2} L (\omega q_0 \sin \omega t)^2 = \frac{(q_0 \cos \omega t)^2}{2C}$$

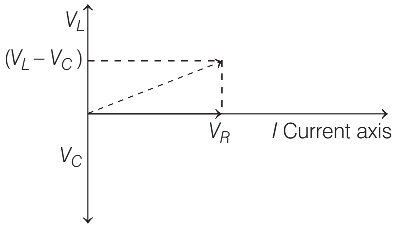
But $\quad \omega = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \quad \tan \omega t = 1$$

$$\omega t = \frac{\pi}{4}$$

$$\Rightarrow \quad t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC}$$

9 Phaser diagram of L-C-R series circuit is shown in figure.



$$V^2 = V_R^2 + (V_L - V_C)^2$$

10 The current in an L-C-R circuit is given by

$$I = \frac{V}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$$

where, $\omega = 2\pi f$

Thus, I increases with an increase in ω upto a value given by $\omega = \omega_c$, i.e. at $\omega = \omega_c$, we have

$$\omega L = \frac{1}{\omega C}$$

$$\Rightarrow \qquad \omega_c = \frac{1}{\sqrt{LC}}$$

where, I is maximum.

At $\omega > \omega_c$, I again starts decreasing with an increase in ω.

11 As energy stored in capacitor $= \frac{1}{2}\frac{q^2}{C}$

Now, when AC is connected to the circuit energy speed $= \frac{1}{2} LI^2$

By equating the energies, we get

$$\frac{1}{2}\frac{q^2}{C} = \frac{1}{2} LI^2$$

$$\frac{(10^{-9})^2}{10^{-11}} = \frac{1}{2} \times 10^{-5} I^2$$

$$\Rightarrow \qquad I = \frac{1}{10} \text{ A}$$

Now, $\qquad V = IR$

$$= \frac{1}{10} \times 100 = 10 \text{ V}$$

Therefore,

$$Q = CV$$

$$= 10^{-11} \times 10 = 10^{-10} \text{C}$$

12 For charging of capacitor,

$q = CV\,(1 - e^{-t/\tau})$ at $t = 2\tau$,

$q = CV\,(1 - e^{-2})$

13 At resonance, $\omega L = 1/\omega C$

Current flowing through the circuit,

$$I = \frac{V_R}{R} = \frac{100}{1000} = 0.1 \text{ A}$$

So, voltage across L is given by

$$V_L = I X_L = I\omega L$$

But $\omega L = 1/\omega C$

$$\therefore \qquad V_L = \frac{I}{\omega C} = V_C$$

$$= \frac{0.1}{200 \times 2 \times 10^{-6}} = 250 \text{ V}$$

14 As quality factor, $Q = \frac{\omega_0}{B}$

where, ω_0 = resonant frequency

and $\qquad B$ = bandwidth.

From the graph, $B = 2.5$ kHz,

$Q = 0.4$

(by observing the curve)

15 Sharpness of resonance of a resonant L-C-R circuit is determined by the ratio of resonant frequency with the selectivity of circuit. This ratio is also called "quality factor" or Q-factor.

$$Q\text{-factor} = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$$

16 The given circuit is under resonance as $X_L = X_C$

Hence, power dissipated in the circuit is

$$P = \frac{V^2}{R} = 242 \text{ W}$$

17 For given circuit, current is lagging the voltage by $\frac{\pi}{2}$, so circuit is purely inductive and there is no power consumption in the circuit. The work done by battery is stored as magnetic energy in the inductor.

18 In an AC circuit, only resistor R dissipates energy. L and C do not dissipate energy, because for both of them current is wattless ($\phi = 90°$).

19 In series impedance of circuit is

$$Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi f L)^2}$$

$$= \sqrt{(220)^2 + (2 \times 3.14 \times 50 \times 0.7)^2}$$

$$= 311\,\Omega$$

$$\therefore \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$$

$$= \frac{220}{311} = 0.707 \text{ A} = 0.707 \text{ A}$$

and $\cos\phi = \frac{R}{Z} = \frac{220}{311} = 0.707$

Now, power absorbed in the circuit is

$$P = V_{\text{rms}} I_{\text{rms}} \cos\phi$$

$$= (220)(0.707)(0.707)$$

$$= 109.96 \approx 110 \text{ W}$$

20 Secondary voltage, $V_S = 24$ V

Power associated with secondary

$P_S = 12$ W

$$\therefore \quad I_S = \frac{P_S}{V_S} = \frac{12}{24} = \frac{1}{2} \text{ A} = 0.5 \text{ A}$$

Peak value of the current in the secondary,

$$I_0 = I_S\sqrt{2}$$

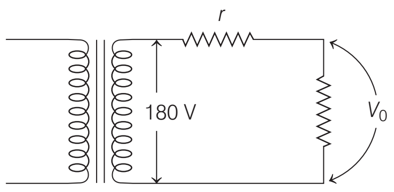
$$= (0.5)(1.414)$$

$$= 0.707 = \frac{1}{\sqrt{2}} \text{ A}$$

21 As, $V_2 = \frac{3600}{20} = 180$ V

$\therefore \quad V_0 = V_2 \times \eta = 180 \times 0.9 = 162$ V

Now, $V_0 = V_2 - I_2 r$



$$\therefore \qquad r = \frac{V_2 - V_0}{I} = \frac{18}{20} = 0.9\,\Omega$$

22 For the case of DC, the frequency is zero and the net impedance is equal to the resistance.For the case of AC, the impedance of the AC circuit is given by

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

where, R = resistance,

ω = angular frequency

and $\quad L$ = inductance.

23 At a particular value of angular frequency, the inductive reactance and capacitive reactance will becomes just equal to each other and opposite in value. So that, the impedance of circuit is minimum, i.e. equal to R.

$$X_L = X_C$$

$$\Rightarrow \qquad \omega_0 L = \frac{1}{\omega_0 C}$$

$$\Rightarrow \qquad \omega = \frac{1}{\sqrt{LC}}$$

24 In a series R-L-C circuit, the resultant emf is

$$E = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$= \sqrt{8^2 + (16-10)^2}$$
$$= \sqrt{64+36} = \sqrt{100}$$
$$= 10\ \text{V}$$

SESSION 2

1 Given, $e = 100\sin 30t$

and $\quad i = 20\sin\left(30t - \dfrac{\pi}{4}\right)$

$\therefore$ Average power,

$$P_{av} = e_{rms} I_{rms} \cos\phi$$
$$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \cos\frac{\pi}{4}$$
$$= \frac{1000}{\sqrt{2}}\ \text{W}$$

Wattless current is

$$I = I_{rms}\sin\phi$$
$$= \frac{20}{\sqrt{2}} \times \sin\frac{\pi}{4}$$
$$= \frac{20}{2} = 10\ \text{A}$$

$\therefore \quad P_{av} = \dfrac{1000}{\sqrt{2}}\ \text{W}$

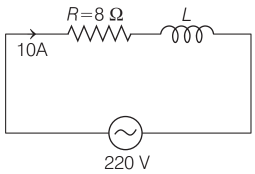
and $I_{wattless} = 10$ A

2 Given, $I = 10\,\text{A}, V = 80\,\text{V}$

$\therefore \quad R = \dfrac{V}{I} = \dfrac{80}{10} = 8\,\Omega$

and $\quad \omega = 50\,\text{Hz}$

For AC circuit, we have



$$I = \frac{V}{\sqrt{8^2 + X_L^2}}$$

$\Rightarrow \quad 10 = \dfrac{220}{\sqrt{64 + X_L^2}}$

$\Rightarrow \quad \sqrt{64 + X_L^2} = 22$

Squaring on both sides, we get

$$64 + X_L^2 = 484$$

$\Rightarrow \quad X_L^2 = 484 - 64 = 420$

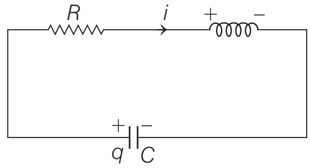
$$X_L = \sqrt{420}$$

$\Rightarrow \quad 2\pi \times \omega L = \sqrt{420}$

Series inductor on an arc lamp,

$$L = \frac{\sqrt{420}}{(2\pi \times 50)} = 0.065\,\text{H}$$

3 Consider the L-C-R circuit at any time t



Now, applying KVL, we have

$$\frac{q}{C} - iR - \frac{Ldi}{dt} = 0$$

As current is decreasing with time we can write $i = -\dfrac{dq}{dt}$

$\Rightarrow \quad \dfrac{q}{C} + \dfrac{dq}{dt}R + \dfrac{Ld^2q}{dt^2} = 0$

or $\quad \dfrac{d^2q}{dt^2} + \dfrac{R}{L}\dfrac{dq}{dt} + \dfrac{q}{LC} = 0$

This equation is equivalent to that of a damped oscillator.

Thus, we can write the solution as

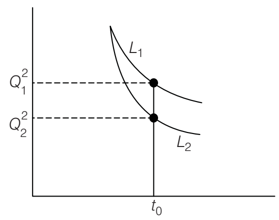
$$Q_{\max}(t) = Q_0 \cdot e^{-Rt/2L}$$

or $\quad Q_{\max}^2 = Q_0^2\, e^{-\frac{Rt}{L}}$

As $L_1 > L_2$ damping is faster for L_2.

Aliter Inductance is inertia of circuit. It means inductance opposes the flow of charge, more inductance means decay of charge is slow.

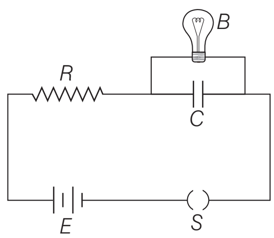
In option (a), in a given time to, $Q_1^2 > Q_2^2$.



So, $L_1 > L_2$.

Hence, option (a) is correct.

4 Neon bulb is filled with gas, so its resistance is infinite, hence no current flows through it.



Now, $\quad V_C = E\,(1 - e^{-t/RC})$

$\Rightarrow \quad 120 = 200\,(1 - e^{-t/RC})$

$\Rightarrow \quad e^{-t/RC} = \dfrac{2}{5}$

$\Rightarrow \quad t = RC \ln 2.5$

$\Rightarrow \quad R = \dfrac{t}{C\ln 2.5}$

$$= \frac{t}{2.303\,C\,\log 2.5}$$
$$= 2.7 \times 10^6\,\Omega$$

5 As, $U = \dfrac{1}{2}\dfrac{q^2}{C} = \dfrac{1}{2C}(q_0 e^{-t/\tau})^2$

$$= \frac{q_0^2}{2C}e^{-2t/\tau} \qquad \text{(where, } \tau = CR\text{)}$$

$$U = U_I e^{-2t/\tau}$$

$\dfrac{1}{2}U_I = U_I e^{-2t_1/\tau}, \ \dfrac{1}{2} = e^{-2t_1/\tau}$

$\Rightarrow \quad t_1 = \dfrac{\tau}{2}\ln 2$

Now, $\quad q = q_0 e^{-t/\tau}, \ \dfrac{1}{4}q_0 = q_0 e^{-t_2/\tau},$

$$t_2 = \tau \ln 4 = 2\tau \ln 2$$

$\therefore \quad \dfrac{t_1}{t_2} = \dfrac{1}{4}$

6 As, $I = 2I_0 \dfrac{t}{T_0}$

where, $0 < t < \dfrac{T_0}{2}$

and $\quad I = 2I_0\left(\dfrac{t}{T_0} - 1\right)$

where, $\dfrac{T_0}{2} < t < T_0$

$\therefore \quad I_{av} = \dfrac{2}{T}\displaystyle\int_0^{T_0} I\,dt$

$$= \frac{2}{T_0}\left[\int_0^{\frac{T}{2}} \frac{2I_0 t}{T_0}\,dt\right]$$
$$= \frac{2}{T_0^2}\left[\frac{2I_0 T_0^2}{2\times 4}\right] = \frac{I_0}{2}$$

7 We have, $110 = \dfrac{V_{app}\ R}{\sqrt{R^2 + L^2\omega^2}}$

$\Rightarrow \quad 110 = \dfrac{220\,R}{\sqrt{R^2 + L^2\omega^2}}$

$\Rightarrow \quad 4R^2 = R^2 + L^2\omega^2$

$\Rightarrow \quad L\omega = \sqrt{3}\ R$

$\Rightarrow \quad L(100\pi) = 220\sqrt{3}$

$\Rightarrow \quad L = \dfrac{2.2\sqrt{3}}{3.14}$

$$= \frac{2.2 \times (1.732)}{3.14} = 1.2\,\text{H}$$

$\Rightarrow \quad R = \dfrac{110 \times 110}{55} = 220\,\Omega$

8 Given, inductance, $L = 0.01\,\text{H}$, resistance, $R = 1\Omega$, voltage, $V = 200\,\text{V}$ and frequency, $f = 50\,\text{Hz}$

Impedance of the circuit,

$$Z = \sqrt{R^2 + X_L{}^2} = \sqrt{R^2 + (2\pi f L)^2}$$
$$= \sqrt{1^2 + (2\times 3.14\times 50\times 0.01)^2}$$

or $Z = \sqrt{10.86} = 3.3\Omega$

$$\therefore\ \tan\phi = \frac{\omega L}{R} = \frac{2\pi f L}{R}$$
$$= \frac{2\times 3.14\times 50\times 0.01}{1}$$
$$= 3.14$$
$$\phi = \tan^{-1}(3.14) = 72^\circ$$

Phase difference, $\phi = \frac{72\times\pi}{180}$ rad

Time lag between alternating voltage and current,

$$\Delta t = \frac{\phi}{\omega} = \frac{72\pi}{180\times 2\pi\times 50} = \frac{1}{250}\,\text{s}$$

9 At cut-off frequency, $Z = \sqrt{2}\,R$

$$R^2 + \left(L\omega - \frac{1}{C\,\omega}\right)^2 = 2R^2$$

$$\Rightarrow\quad L\omega - \frac{1}{C\omega} = R$$

$$\Rightarrow\quad LC\omega^2 - RC\omega - L = 0$$

$$\Rightarrow\quad \omega = \frac{RC \pm \sqrt{R^2C^2 + 4LC}}{2LC}$$

$$\Delta\omega = \omega_{01} - \omega_{02}$$
$$= \frac{2\sqrt{R^2C^2 + 4LC}}{2LC}$$
$$= \frac{\sqrt{R^2C^2 + 4LC}}{LC}$$

10 When S_2 is closed and S_1 is open, the charge oscillates in the L-C circuit at an angular frequency is given by

$$\omega = \frac{1}{\sqrt{LC}} \quad \text{...(i)}$$

Now, $q \neq 0$ at $t = 0$. Hence, options (a) and (b) are wrong. The charge q varies with time t as

$$q = q_0\cos(\omega t + \phi) \quad \text{...(ii)}$$

where, ϕ is not equal to $\pi/2$.

Differentiating Eq. (ii) twice with respect to t, we get

$$\frac{d^2q}{dt^2} = -\omega^2 q_0\cos(\omega t + \phi) = -\omega^2 q$$

$$q = -\frac{1}{\omega^2}\frac{d^2q}{dt^2}$$
$$= -LC\frac{d^2q}{dt^2} \quad \text{[using Eq. (i)]}$$

11 The instantaneous value of emf and current in inductive circuit are given by $E = E_0\sin\omega t$ and $I = I_0\sin\left(\omega t - \frac{\pi}{2}\right)$ respectively.

So, $P_{\text{inst}} = EI$

$$= E_0\sin\omega t\times I_0\sin\left(\omega t - \frac{\pi}{2}\right)$$
$$= E_0 I_0\sin\omega t\left(\sin\omega t\cos\frac{\pi}{2} - \cos\omega t\sin\frac{\pi}{2}\right)$$
$$= E_0 I_0\sin\omega t\cos\omega t$$
$$= \frac{1}{2}E_0 I_0\sin 2\omega t$$
$$(\because \sin 2\omega t = 2\sin\omega t\cos\omega t)$$

Hence, angular frequency of instantaneous power is 2ω.

12 Given, $X_L = 1\Omega, R = 2\Omega$

$$E_{\text{rms}} = 6\,\text{V}, P_{\text{av}} = ?$$

Average power dissipated in the circuit,

$$P_{\text{av}} = E_{\text{rms}}\cos\phi$$
$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{E_{\text{rms}}}{Z}$$
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{4+1} = \sqrt{5}$$
$$\therefore\quad I_{\text{rms}} = \frac{6}{\sqrt{5}}\,\text{A}$$
$$\cos\phi = \frac{R}{Z} = \frac{2}{\sqrt{5}}$$
$$P_{\text{av}} = 6\times\frac{6}{\sqrt{5}}\times\frac{2}{\sqrt{5}} \quad \text{[from Eq.(i)]}$$
$$= \frac{72}{\sqrt{5}\sqrt{5}} = \frac{72}{5} = 14.4\,\text{W}$$

13 In circuit 1,

In steady state, $I = 0$

So, no option matches for circuit 1.

In circuit 2,

$V_1 = 0$ and $V_2 = 2I = V$

$\therefore$ B, C, D $\rightarrow$ 2

In circuit 3,

$$V_1 = X_L I = 2\pi f LI$$
$$= 2\pi\times 50\times 6\times 10^{-3} = 1.88I$$

and $V_2 = 2I$

$\therefore$ A, B, D $\rightarrow$ 3

In circuit 4,

$$V_1 = X_L I = 1.88I$$
$$V_2 = X_C I = 1061I$$

$\therefore$ A, B, D $\rightarrow$ 4

In circuit 5,

$$V_1 = IR = 1000I$$
$$V_2 = X_C I = 1016I$$

$\therefore$ A, B, D $\rightarrow$ 5

14 Power dissipated is given by $P = E_{\text{rms}} I_{\text{rms}}\cos\phi$

We know that for a purely resistive circuit, the power factor,

$$\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$$

Hence, $P = E_{\text{rms}}\times I_{\text{rms}}$

$$= (RI_{\text{rms}})\times I_{\text{rms}}$$
$$= R(I_{\text{rms}})^2$$
$$= R\left(\frac{I_0}{\sqrt{2}}\right)^2 = \frac{RI_0^2}{2}$$

15 Since, at resonance,

$$\omega L = \frac{1}{\omega C}$$

and impedance,

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

or $$Z = \sqrt{R^2} = R$$

Hence, nature of impedance at resonance is resistive.

Also, in a L-C-R circuit, phase angle between the emf and the alternating current I, is zero at resonance.