# Chapter 4 Matrices and Determinants

## Ex 4.6

### Answer 1e.

For any two complex conjugates, the real parts have same magnitude and sign whereas the complex parts have same magnitude but different signs. Thus, the complex conjugate of $a - bi$ is $a + bi$.

### Answer 1gp.

Take the square root on each side.

$x = \pm\sqrt{-13}$

Write in terms of $i$. We know that $i = \sqrt{-1}$. Thus,

$$\pm\sqrt{-13} = \pm\sqrt{-1 \cdot 13}$$
$$= \pm i\sqrt{13}.$$

Therefore, we get $x = \pm i\sqrt{13}$.

The solutions are $i\sqrt{13}$ and $-i\sqrt{13}$.

### Answer 2e.

A complex number contains both real and imaginary numbers.
No, not every complex number is an imaginary number.
For example:
Consider

A complex number $1+i$

In the complex number there exists not only imaginary number but also real. The complex number $1+i$ is not an imaginary number.
The imaginary number is $i$ where the square of it is -1. But,

$$(1+i)^2 = 1^2 + 2i + i^2 = 2i \neq -1$$

### Answer 2gp.

Consider the equation

$$x^2 = -38$$

Solve the following quadratic equation

$x^2 = -38$ Write the original equation

$x = \pm\sqrt{-38}$ Apply Square root property

By the property of square root of a negative number

If $r$ is a positive number, then $\sqrt{-r} = i\sqrt{r}$,

$x = \pm\sqrt{-38}$

$x = \pm i\sqrt{38}$ Apply the property Square root of a negative number

Therefore, the roots of the quadratic equation are $\boxed{\pm i\sqrt{38}}$.

## Answer 3e.

First, take the square root on each side.

$x = \pm\sqrt{-28}$

Write in terms of $i$. We know that $i = \sqrt{-1}$. Thus,

$\pm\sqrt{-28} = \pm\sqrt{-1 \cdot 28}$

$= \pm i\sqrt{28}$.

Rewrite the radicand as a product of two factors such that one of them is a perfect square.

$\pm i\sqrt{28} = \pm i\sqrt{4 \cdot 7}$

Apply the product property. For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$\pm i\sqrt{4 \cdot 7} = \pm i\sqrt{4} \cdot \sqrt{7}$

Now, simplify.

$\pm i\sqrt{4} \cdot \sqrt{7} = \pm 2i\sqrt{7}$

Therefore, we get $x = \pm 2i\sqrt{7}$.

The solutions are $2i\sqrt{7}$ and $-2i\sqrt{7}$.

## Answer 3gp.

Subtract 11 from each side to bring the constants on one side.

$x^2 + 11 - 11 = 3 - 11$

$x^2 = -8$

Take the square root on each side.

$x = \pm\sqrt{-8}$

Write in terms of $i$. We know that $i = \sqrt{-1}$. Thus,

$\pm\sqrt{-8} = \pm\sqrt{-1 \cdot 8}$

$= \pm i\sqrt{8}$.

Rewrite the radicand as a product of two factors such that one of them is a perfect square.

$\pm i\sqrt{8} = \pm i\sqrt{4 \cdot 2}$

Apply the product property. For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$\pm i\sqrt{4 \cdot 2} = \pm i\sqrt{4} \cdot \sqrt{2}$

Now, simplify.

$\pm i\sqrt{4}\cdot\sqrt{2} = \pm 2i\sqrt{2}$

Therefore, we get $x = \pm 2i\sqrt{2}$.

The solutions are $2i\sqrt{2}$ and $-2i\sqrt{2}$.

## Answer 4e.

Consider the equation

$$r^2 = -624$$

Solve the quadratic equation.

By the property of square root of a negative number

If $r$ is a positive number, then $\sqrt{-r} = i\sqrt{r}$

| | |
|---|---|
| $r^2 = -624$ | Write the original equation |
| $r = \pm\sqrt{-624}$ | Apply Square root on each side |
| $r = \pm i\sqrt{624}$ | Write in terms of $i$ |
| $r = \pm i\sqrt{16\cdot 39}$ | Write in factor form under the Square root |
| $r = \pm i\left(4\sqrt{39}\right)$ | Simplify |

Therefore, the roots of the quadratic equation are $\boxed{\pm 4i\sqrt{39}}$.

## Answer 4gp.

Consider the equation

$$x^2 - 8 = -36$$

Solve the quadratic equation.

By the property of square root of a negative number

If $r$ is a positive number, then $\sqrt{-r} = i\sqrt{r}$

| | |
|---|---|
| $x^2 - 8 = -36$ | Write the original equation |
| $x^2 = 8 - 36$ | Add 8 on each side |
| $x^2 = -28$ | Add like terms |
| $x = \pm\sqrt{-28}$ | Apply Square root on each side |
| $x = \pm i\sqrt{28}$ | Write in terms of $i$ |
| $x = \pm i\left(2\sqrt{7}\right)$ | $\sqrt{28} = \sqrt{4.7} = 2\sqrt{7}$ |

Therefore, the roots of the quadratic equation are $\boxed{\pm 2i\sqrt{7}}$.

## Answer 5e.

Subtract 8 from each side to bring the constants on one side.

$z^2 + 8 - 8 = 4 - 8$

$z^2 = -4$

Take the square root on each side.

$z = \pm\sqrt{-4}$

Write in terms of $i$. We know that $i = \sqrt{-1}$. Thus,

$$\pm\sqrt{-4} = \pm\sqrt{-1 \cdot 4}$$
$$= \pm i\sqrt{4}.$$

Now, evaluate the radical.

$$\pm i\sqrt{4} = \pm 2i$$

Therefore, we get $x = \pm 2i$.

The solutions are $2i$ and $-2i$.

## Answer 5gp.

First, add 7 to each side to bring the constants on one side.

$$3x^2 - 7 + 7 = -31 + 7$$
$$3x^2 = -24$$

Divide each side by 3.

$$\frac{3x^2}{3} = \frac{-24}{3}$$
$$x^2 = -8$$

Now, take the square root on each side.

$$x = \pm\sqrt{-8}$$

Write in terms of $i$. We know that $i = \sqrt{-1}$. Thus,

$$\pm\sqrt{-8} = \pm\sqrt{-1 \cdot 8}$$
$$= \pm i\sqrt{8}.$$

Rewrite the radicand as a product of two factors such that one of them is a perfect square.

$$\pm i\sqrt{8} = \pm i\sqrt{4 \cdot 2}$$

Apply the product property. For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$\pm i\sqrt{4 \cdot 2} = \pm i\sqrt{4} \cdot \sqrt{2}$$

Now, simplify.

$$\pm i\sqrt{4} \cdot \sqrt{2} = \pm 2i\sqrt{2}$$

Therefore, we get $x = \pm 2i\sqrt{2}$.

The solutions are $2i\sqrt{2}$ and $-2i\sqrt{2}$.

## Answer 6e.

Consider the equation $s^2 - 22 = -112$.

Solving:

$$s^2 - 22 = -112$$

$$s^2 = -90 \quad \text{Add 22 on both sides}$$

$$s = \pm\sqrt{-90} \quad \text{Apply Square root}$$

By square root of a negative number property:
If $r$ is a positive number, then

$$\sqrt{-r} = i\sqrt{r}$$

This implies that

$$s = \pm i\sqrt{90}$$

$$s = \pm i\sqrt{9 \cdot 10}$$

$$s = \pm i\left(3\sqrt{10}\right)$$

Therefore $\boxed{s = \pm 3i\sqrt{10}}$ are the roots of the given quadratic equation.

## Answer 6gp.

Consider the equation

$$5x^2 + 33 = 3$$

Solve the quadratic equation.
By the property of square root of a negative number
If $r$ is a positive number, then $\sqrt{-r} = i\sqrt{r}$

$$5x^2 + 33 = 3 \quad \text{Write the original equation}$$

$$5x^2 = 3 - 33 \quad \text{Subtract 33 on each side}$$

$$5x^2 = -30 \quad \text{Add like terms}$$

$$x^2 = -6 \quad \text{Divide by 5 on each side}$$

$$x^2 = \pm\sqrt{-6} \quad \text{Apply Square root on each side}$$

$$x = \pm i\sqrt{6} \quad \text{Write in terms of } i$$

Therefore, the roots of the quadratic equation are $\boxed{\pm i\sqrt{6}}$.

## Answer 7e.

First, subtract 31 from each side to bring the constants on one side.

$$2x^2 + 31 - 31 = 9 - 31$$

$$2x^2 = -22$$

Divide each side by 2.

$$\frac{2x^2}{2} = \frac{-22}{2}$$

$$x^2 = -11$$

Now, take the square root on each side.

$x = \pm\sqrt{-11}$

Write in terms of $i$. We know that $i = \sqrt{-1}$. Thus,

$$\pm\sqrt{-11} = \pm\sqrt{-1 \cdot 11}$$
$$= \pm i\sqrt{11}.$$

Therefore, we get $x = \pm i\sqrt{11}$.

The solutions are $i\sqrt{11}$ and $-i\sqrt{11}$.

## Answer 7gp.

For adding two complex numbers, add their real parts and their imaginary parts separately.

$(9 - i) + (-6 + 7i) = [9 + (-6)] + (-1 + 7)i$

Simplify.

$[9 + (-6)] + (-1 + 7)i = 3 + 6i$

Thus, the given expression simplifies to $3 + 6i$.

## Answer 8e.

Consider the equation $9 - 4y^2 = 57$.

Solving:

| | |
|---|---|
| $9 - 4y^2 = 57$ | Given equation |
| $4y^2 = 9 - 57$ | Rewrite the given equation |
| $4y^2 = -48$ | Do subtraction |
| $y^2 = -12$ | Divide by 4 |
| $y = \pm\sqrt{-12}$ | Apply Square root |

By square root of a negative number property:

If $r$ is a positive number, then $\sqrt{-r} = i\sqrt{r}$

This implies that

| | |
|---|---|
| $y = \pm i\sqrt{12}$ | Write $\sqrt{12}$ as $\sqrt{4 \cdot 3}$ |
| $y = \pm i\sqrt{4 \cdot 3}$ | Since $\sqrt{4} = 2$ |
| $y = \pm i\left(2\sqrt{3}\right)$ | Simplify |

Therefore $\boxed{y = \pm 2i\sqrt{3}}$ are the roots of the given quadratic equation.

## Answer 8gp.

Consider the expression

$$(3+7i)-(8-2i)$$

Simplify the expression and write in standard form.
The difference of the complex numbers is

$$(a+bi)-(c+di)=(a-c)+(b-d)i$$

Here, $a=3, b=7, c=8, d=-2$

Therefore,

$$(3+7i)-(8-2i)=(3-8)+(7-(-2))i \quad \text{Use difference of complex numbers}$$

$$=(3-8)+(7+2)i \quad \text{Distribute the negative sign}$$

$$=-5+9i \quad \text{Standard form}$$

Therefore, the simplified expression in the standard form is $\boxed{-5+9i}$.

## Answer 9e.

First, subtract $2t^2$ and 5 from each side to bring the constants on one side.

$$6t^2 - 2t^2 + 5 - 5 = 2t^2 - 2t^2 + 1 - 5$$

$$4t^2 = -4$$

Divide each side by 4.

$$\frac{4t^2}{4} = \frac{-4}{4}$$

$$t^2 = -1$$

Now, take the square root on each side.

$$t = \pm\sqrt{-1}$$

We know that $i = \sqrt{-1}$. Thus,

$\pm\sqrt{-1} = \pm i$.

Therefore, we get $x = \pm i$.

The solutions are $i$ and $-i$.

## Answer 9gp.

By the definition of complex subtraction, group the real parts and the imaginary parts separately.

$$-4-(1+i)-(5+9i)=(-4-1-5)+(-1-9)i$$

Simplify.
$(-4-1-5)+(-1-9)i=-10+(-10)i$

Write in standard form.
$-10+(-10)i=-10-10i$

Thus, the given expression simplifies to $-10-10i$.

## Answer 10e.

Consider the equation $3p^2+7=-9p^2+4$.

Solving:

$$3p^2+7=-9p^2+4 \quad \text{Given equation}$$
$$9p^2+3p^2=4-7 \quad \text{Adding } 9p^2-7 \text{ on both sides}$$
$$12p^2=-3 \quad \text{Simplify}$$
$$p^2=-\frac{1}{4} \quad \text{Divide by 12}$$
$$p=\pm\sqrt{-\frac{1}{4}} \quad \text{Apply Square root}$$

By square root of a negative number property:
If $r$ is a positive number, then $\sqrt{-r}=i\sqrt{r}$

This implies that

$$p=\pm i\sqrt{\frac{1}{4}} \quad \text{Since } \sqrt{\frac{1}{4}}=\frac{1}{2}$$
$$p=\pm i\left(\frac{1}{2}\right) \quad \text{Simplify}$$

Therefore $\boxed{p=\pm i\left(\frac{1}{2}\right)}$ are the roots of the given quadratic equation.

## Answer 10gp.

We consider that the resistor has a resistance of 5 ohms, the inductor has a reactance of 3 ohms and the capacitor has a reactance of 7 ohms.
The resistor has a resistance of 5 ohms, so its impedance is 5 ohms.
The inductor has a reactance of 3 ohms, so its impedance is $3i$ ohms.
The capacitor has a reactance of 7 ohms, so its impedance is $-7i$ ohms.

Again, we know that the impedance for a series circuit is the sum of the impedances for the individual components.
Hence, the impedance of the circuit is

$$5+3i+(-7i)=5+3i-7i$$
$$=5-4i$$

So, the impedance of the circuit is $\boxed{5-4i \text{ ohms}}$ .

## Answer 11e.

First, divide each side by −5.

$$\frac{-5(n-3)^2}{-5} = \frac{10}{-5}$$
$$(n-3)^2 = -2$$

Now, take the square root on each side.

$$n-3 = \pm\sqrt{-2}$$

Write in terms of $i$. We know that $i = \sqrt{-1}$. Thus,

$$n-3 = \pm\sqrt{-1\cdot 2}$$
$$n-3 = \pm i\sqrt{2}$$

Add 3 to each side.

$$n-3+3 = \pm i\sqrt{2}+3$$
$$n = 3\pm i\sqrt{2}$$

The solutions are $3+i\sqrt{2}$ and $3-i\sqrt{2}$.

## Answer 11gp.

Apply the distributive property.

$$i(9-i) = i(9)-i(i)$$
$$= 9i - i^2$$

We know that $i^2=-1$. Thus,
$9i-i^2=9i-(-1)$.

Simplify.
$9i-(-1)=9i+1$

Write in standard form.
$9i+1=1+9i$

Thus, the given expression simplifies to $1+9i$.

## Answer 12e.

Consider the expression

$$(6-3i)+(5+4i)$$

Write the above expression as a complex number in standard form.

From the definition of complex addition,
To add two complex numbers, add their real parts and their imaginary parts separately.

**Sum of complex numbers**: $(a+bi)+(c+di)=(a+c)+(b+d)i$

Simplifying:

$(6-3i)+(5+4i)=(6+5)+(-3+4)i$ Definition of complex addition $(a+bi)+(c+di)=(a+c)+(b+d)i$

$=11+i$ Write in standard form

Therefore $\boxed{(6-3i)+(5+4i)=11+i}$.

## Answer 12gp.

Consider the expression

$$(3+i)(5-i)$$

Simplify the expression and write in standard form.

$(3+i)(5-i)=3\cdot5+(3)\cdot(-i)+i\cdot5+(i)(-i)$ Multiply by the use of FOIL method

$=15-3i+5i-i^2$ Multiply the terms

$=15-3i+5i-(-1)$ Use $i^2=-1$

$=15-3i+5i+1$ Add the like terms

$=16+2i$

Therefore, the simplified expression in the standard form is $\boxed{16+2i}$.

## Answer 13e.

For adding two complex numbers, add their real parts and their imaginary parts separately.

$(9+8i)+(8-9i)=(9+8)+[8+(-9)]i$

Simplify.

$(9+8)+[8+(-9)]i=17+(-i)$

Write in standard form.

$17+(-i)=17-i$

Thus, the given expression simplifies to $17-i$.

## Answer 13gp.

Multiply the numerator and the denominator by $1-i$, the complex conjugate of $1+i$.

$$\frac{5}{1+i} = \frac{5}{1+i}\cdot\frac{1-i}{1-i}$$
$$= \frac{5(1-i)}{(1+i)(1-i)}$$

Apply the distributive property in the numerator and the FOIL method in the denominator.

$$\frac{5(1-i)}{(1+i)(1-i)} = \frac{5-5i}{1-i+i-i^2}$$

We know that $i^2=-1$. Thus,

$$\frac{5-5i}{1-i+i-i^2} = \frac{5-5i}{1-i+i-(-1)}.$$

Simplify.

$$\frac{5-5i}{1-i+i-(-1)} = \frac{5-5i}{2+0i}$$
$$= \frac{5-5i}{2}$$

Write in standard form.

$$\frac{5-5i}{2} = \frac{5}{2}-\frac{5}{2}i$$

Thus, the given expression simplifies to $\frac{5}{2}-\frac{5}{2}i$.

## Answer 14e.

Consider the expression

$$(-2-6i)-(4-6i)$$

Write the above expression as a complex number in standard form.

From the definition of complex subtraction,
To subtract two complex numbers, subtract their real parts and their imaginary parts separately.

**Difference of complex numbers**: $(a+bi)-(c-di)=(a-c)+(b-d)i$

Simplifying:

$$(-2-6i)-(4-6i)=(-2-4)+(-6-(-6))i$$ Definition of complex subtraction $(a+bi)-(c-di)=(a-c)+(b-d)i$

$$=-6+0i$$ Write in standard form

$$=-6$$ Simplify

Therefore $\boxed{(-2-6i)-(4-6i)=-6}$.

## Answer 14gp.

Consider the expression

$$\frac{5+2i}{3-2i}$$

Simplify the expression and write in standard form.

$$\frac{5+2i}{3-2i}=\frac{(5+2i)(3+2i)}{(3-2i)(3+2i)}$$ Multiply the numerator and denominator by $3+2i$

$$=\frac{5\cdot 3+5\cdot 2i+2i\cdot 3+2i\cdot 2i}{3\cdot 3+3\cdot 2i+(-2i)\cdot(3)+(-2i)\cdot 2i}$$ Use FOIL method

$$=\frac{15+10i+6i+4i^2}{9+6i-6i-4i^2}$$ Simplify

$$=\frac{15+10i+6i+4(-1)}{9+6i-6i-4(-1)}$$ Use $i^2=-1$

$$=\frac{15+16i-4}{9+4}$$ Add the terms in the denominator

$$=\frac{11+16i}{13}$$ Subtract the terms in the numerator

$$=\frac{11}{13}+\frac{16}{13}i$$ Simplify

Therefore, the simplified expression in the standard form is $\boxed{\frac{11}{13}+\frac{16}{13}i}$.

## Answer 15e.

For subtracting two complex numbers, subtract their real parts and their imaginary parts separately.

$(-1+i)-(7-5i)=(-1-7)+[1-(-5)]i$

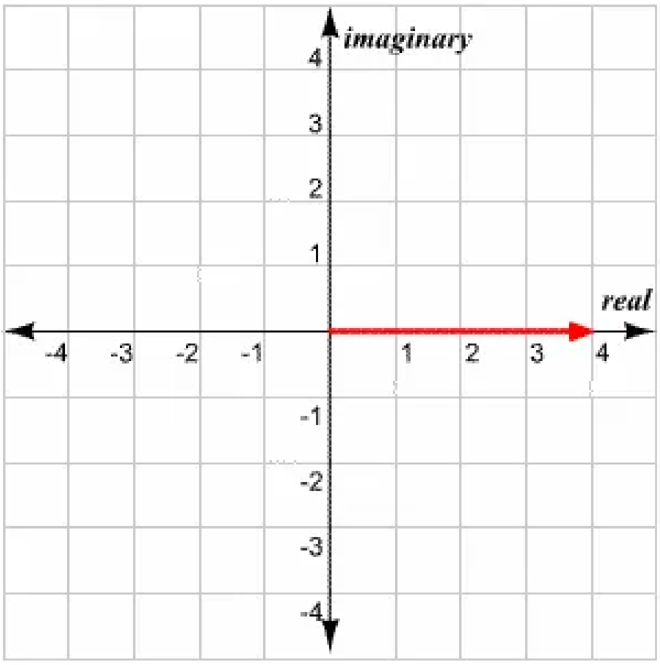
Simplify.

$(-1-7)+[1-(-5)]i=-8+6i$

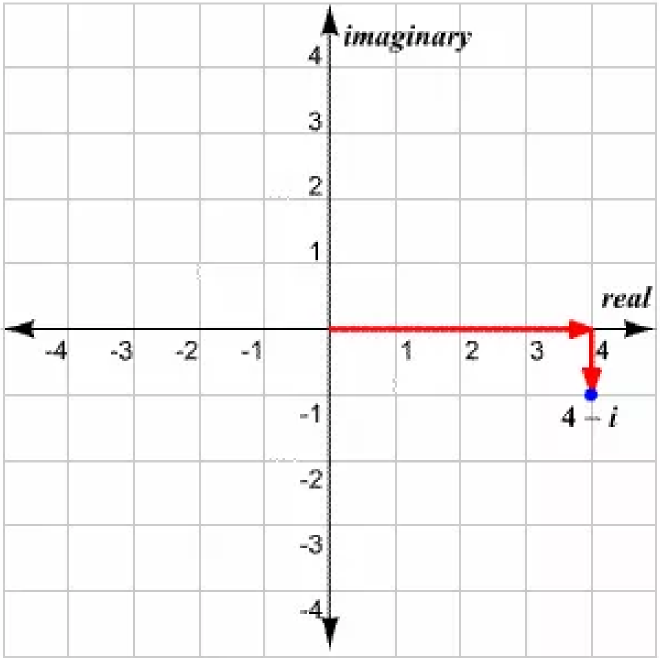
Thus, the given expression simplifies to $-8+6i$.

## Answer 15gp.

First, draw a complex plane. In order to plot $4-i$, start at the origin and move 4 units to the right.




Now, move 1 unit down and mark the point where we end up.




For any complex number $z$ of the form $a+bi$, its absolute value is defined as $|z| = \sqrt{a^2+b^2}$. Thus,

$$|4-i| = \sqrt{4^2+(-1)^2}.$$

Evaluate the powers and then add.

$$\begin{aligned}\sqrt{4^2+(-1)^2} &= \sqrt{16+1}\\ &= \sqrt{17}\end{aligned}$$

Therefore, the absolute value of the given complex number is $\sqrt{17}$.

**Answer 16e.**

Consider the expression

$$(8+20i)-(-8+12i)$$

Write the above expression as a complex number in standard form.

From the definition of complex subtraction,
To subtract two complex numbers, subtract their real parts and their imaginary parts separately.

**Difference of complex numbers**: $(a+bi)-(c-di)=(a-c)+(b-d)i$

Simplifying:

$$(8+20i)-(-8+12i)=(8-(-8))+(20-12)i$$ Definition of complex subtraction $(a+bi)-(c-di)=(a-c)+(b-d)i$

$$=(8+8)+(8)i$$ Simplify

$$=16+8i$$ Write in standard form

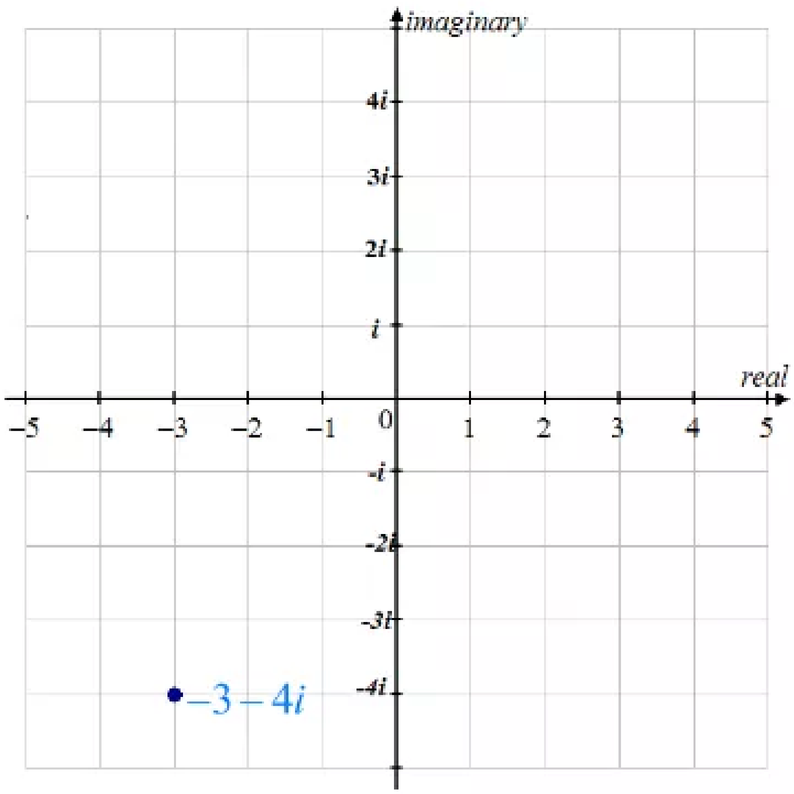
Therefore $\boxed{(8+20i)-(-8+12i)=16+8i}$.

**Answer 16gp.**

Consider the complex number

$$-3-4i$$

Plot the point in the complex plane and find its absolute value.
To plot the point start at the origin and then move 3 units to the left and then move 4 units down.
Sketch the complex plane by plotting the complex number.
The diagram contains the complex number in the complex plane.



The absolute value of a complex number $z = a + bi$ is denoted by $|z|$ and is non-negative number given by $|z| = \sqrt{a^2 + b^2}$ .

Compare $-3-4i$ with $a+bi$

Here, $a=-3, b=-4$

Therefore, the absolute value of the given complex number is

$$|-3-4i| = \sqrt{(-3)^2 + (-4)^2}$$ Use the definition of absolute value

$$= \sqrt{9+16}$$

$$= \sqrt{25}$$

$$= \boxed{5}$$ Simplify

## Answer 17e.

For subtracting two complex numbers, subtract their real parts and their imaginary parts separately.

$(8 - 5i) - (-11 + 4i) = [8 - (-11)] + (-5 - 4)i$

Simplify.

$[8 - (-11)] + (-5 - 4)i = 19 + (-9)i$

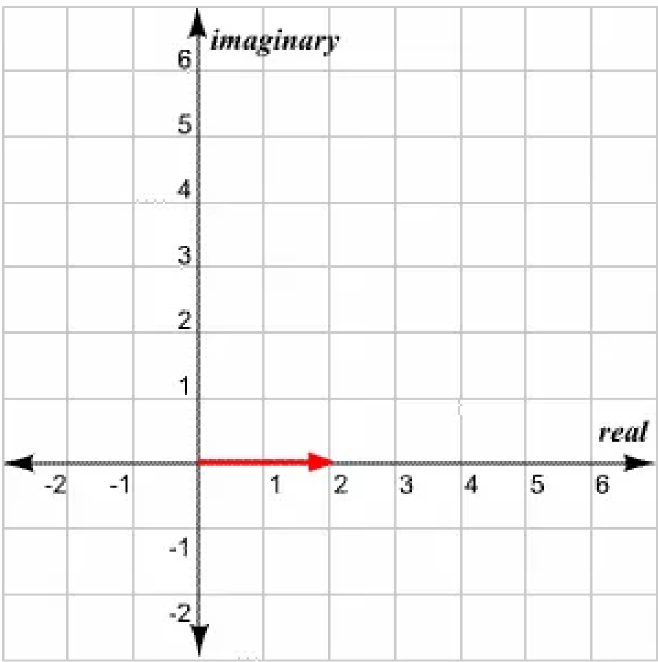
Write in standard form.

$19 + (-9)i = 19 - 9i$

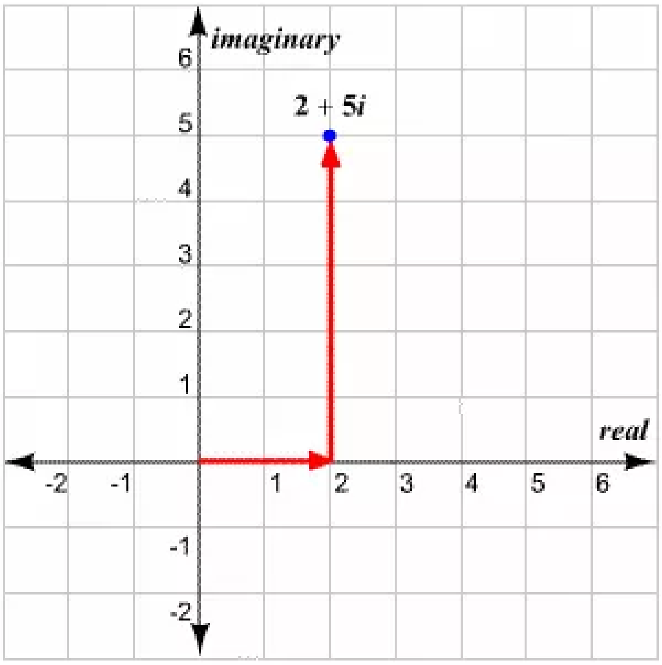
Thus, the given expression simplifies to $19 - 9i$.

## Answer 17gp.

First, draw a complex plane. In order to plot $2 + 5i$, start at the origin and move 2 units to the right.



Now, move 5 units up and mark the point where we end up.




For any complex number $z$ of the form $a + bi$, its absolute value is defined as

$|z| = \sqrt{a^2 + b^2}$. Thus,

$|2 + 5i| = \sqrt{2^2 + (5)^2}$.

Evaluate the powers and then add.

$$\begin{aligned}\sqrt{2^2 + (5)^2} &= \sqrt{4 + 25} \\ &= \sqrt{29}\end{aligned}$$

Therefore, the absolute value of the given complex number is $\sqrt{29}$.

## Answer 18e.

Consider the expression

$$(10 - 2i) + (-11 - 7i)$$

Write the above expression as a complex number in standard form.

From the definition of complex addition,

To add two complex numbers, add their real parts and their imaginary parts separately.

**Sum of complex numbers**: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Simplifying:

$$(10-2i)+(-11-7i)=(10-11)+(-2+(-7))i$$ Definition of complex addition $(a+bi)+(c+di)=(a+c)+(b+d)i$

$$=-1+(-9i)$$ Simplify

$$=-1-9i$$ Write in standard form

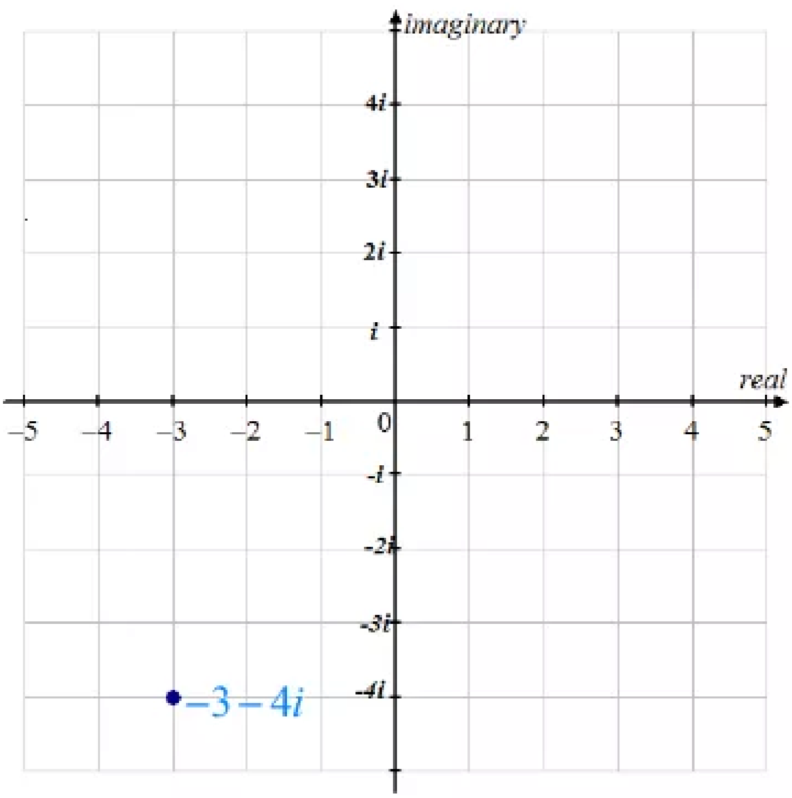
Therefore $\boxed{(10-2i)+(-11-7i)=-1-9i}$.

**Answer 18gp.**

Consider the complex number

$-4i$

Plot the point in the complex plane and find its absolute value.
To plot the point starts at the origin and then move 4 units down.
Sketch the complex plane by plotting the complex number.
The diagram contains the complex number in the complex plane.



The absolute value of a complex number $z = a + bi$ is denoted by $|z|$ and is non-negative number given by $|z| = \sqrt{a^2 + b^2}$ .

Compare $-4i$ with $a + bi$

Here, $a = 0, b = -4$

Therefore, the absolute value of the given complex number is

$$
\begin{aligned}
|-4i| &= \sqrt{(0)^2 + (-4)^2} && \text{Use the definition of absolute value} \\
&= \sqrt{0 + 16} \\
&= \sqrt{4} \\
&= \boxed{2} && \text{Simplify}
\end{aligned}
$$

## Answer 19e.

For adding two complex numbers, add their real parts and their imaginary parts separately.

$(14 + 3i) + (7 + 6i) = (14 + 7) + (3 + 6)i$

Simplify.

$(14 + 7) + (3 + 6)i = 21 + 9i$

Thus, the given expression simplifies to $21 + 9i$.

## Answer 20e.

Consider the expression

$$(-1 + 4i) + (-9 - 2i)$$

Write the above expression as a complex number in standard form.

From the definition of complex addition,

To add two complex numbers, add their real parts and their imaginary parts separately.

**Sum of complex numbers**: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Simplifying:

$$
\begin{aligned}
(-1 + 4i) + (-9 - 2i) &= (-1 + (-9)) + (4 + (-2))i && \text{Definition of complex addition} \\
& && (a + bi) + (c + di) = (a + c) + (b + d)i \\
&= (-1 - 9) + ((4 - 2)i) && \text{Simplify} \\
&= -10 + 2i && \text{Write in standard form}
\end{aligned}
$$

Therefore $\boxed{(-1 + 4i) + (-9 - 2i) = -10 + 2i}$.

## Answer 21e.

For subtracting two complex numbers, subtract their real parts and their imaginary parts separately.
$(2+3i)-(7+4i)=(2-7)+(3-4)i$

Simplify.
$(2-7)+(3-4)i=-5+(-1)i$

Write in standard form.
$-5+(-1)i=-5-i$

Thus, the given expression simplifies to $-5-i$. The correct answer is choice **C**.

## Answer 22e.

Consider the expression

$$6i(3+2i)$$

Write the above expression as a complex number in standard form.

In order to multiplying two complex numbers, use the distributive property or the FOIL method.

Simplifying:

| | |
|---|---|
| $6i(3+2i)=6i(3)+6i(2i)$ | Using distributive property |
| $=18i+12i^2$ | Simplify |
| $=18i+12(-1)$ | Use $i^2=-1$ |
| $=-12+18i$ | Write in standard form |

Therefore $\boxed{6i(3+2i)=-12+18i}$.

## Answer 23e.

Apply the distributive property.
$$-i(4-8i) = -i(4)-(-i)(8i)$$
$$= -4i+8i^2$$

We know that $i^2=-1$. Thus,
$-4i+8i^2=-4i+8(-1)$.

Simplify.
$-4i+8(-1)=-4i-8$

Write in standard form.
$-4i-8=-8-4i$

Thus, the given expression simplifies to $-8-4i$.

## Answer 24e.

Consider the expression

$$(5-7i)(-4-3i)$$

Write the above expression as a complex number in standard form.

In order to multiplying two complex numbers, use the distributive property or the FOIL method.

Simplifying:

$$(5-7i)(-4-3i) = -20-15i+28i+21i^2 \quad \text{Multiply using FOIL}$$

$$= -20-15i+28i+21(-1) \quad \text{Simplify and Use } i^2 = -1$$

$$= -20-15i+28i-21 \quad \text{Simplify}$$

$$= -41+13i \quad \text{Write in standard form}$$

Therefore $\boxed{(5-7i)(-4-3i) = -41+13i}$.

## Answer 25e.

Apply the FOIL method and multiply.

$$(-2+5i)(-1+4i) = (-2)(-1)+(-2)(4i)+(5i)(-1)+(5i)(4i)$$
$$= 2-8i-5i+20i^2$$

We know that $i^2 = -1$. Thus,
$2-8i-5i+20i^2 = 2-8i-5i+20(-1)$.

Simplify.
$2-8i-5i+20(-1) = -18-13i$

Thus, the given expression simplifies to $-18-13i$.

## Answer 27e.

Apply the FOIL method and multiply.

$$(8-3i)(8+3i) = (8)(8)+(8)(3i)+(-3i)(8)+(-3i)(4i)$$
$$= 64+24i-24i-12i^2$$

We know that $i^2 = -1$. Thus,
$64+24i-24i-12i^2 = 64+24i-24i-12(-1)$.

Simplify.
$64+24i-24i-12(-1) = 76+0i$

Write in standard form.
$76+0i = 76$

Thus, the given expression simplifies to 76.

## Answer 28e.

Consider the expression,

$$\frac{7i}{8+i}$$

Write in standard form:

Now

$$\frac{7i}{8+i}=\frac{7i(8-i)}{(8+i)(8-i)}$$ Multiplying both numerator and denominator by $8-i$

$$=\frac{7i\cdot 8-7i\cdot i}{8\cdot 8+8\cdot(-i)+i\cdot 8+i\cdot(-i)}$$ Multiplying using FOIL

$$=\frac{56i-7i^2}{64-8i+8i-i^2}$$ Simplify

$$=\frac{56i-7(-1)}{64-(-1)}$$ Simplify and use $i^2=-1$

$$=\frac{7+56i}{65}$$ Simplify

$$=\frac{7}{65}+\frac{56}{65}i$$ Divide by 65

Therefore,

$$\frac{7i}{8+i}=\boxed{\frac{7}{65}+\frac{56}{65}i}.$$

## Answer 29e.

Multiply the numerator and the denominator by $3+i$, the complex conjugate of $3-i$.

$$\frac{6i}{3-i}=\frac{6i}{3-i}\cdot\frac{3+i}{3+i}$$

$$=\frac{6i(3+i)}{(3-i)(3+i)}$$

Apply the distributive property in the numerator and the FOIL method in the denominator.

$$\frac{6i(3+i)}{(3-i)(3+i)}=\frac{18i+6i^2}{9+3i-3i-i^2}$$

We know that $i^2 = -1$. Thus,

$$\frac{18i + 6i^2}{9 + 3i - 3i - i^2} = \frac{18i + 6(-1)}{9 + 3i - 3i - (-1)}.$$

Simplify.

$$\frac{18i + 6(-1)}{9 + 3i - 3i - (-1)} = \frac{18i - 6}{10 + 0i}$$
$$= \frac{18i - 6}{10}$$

Write in standard form.

$$\frac{18i - 6}{10} = -\frac{6}{10} + \frac{18}{10}i$$
$$= -\frac{3}{5} + \frac{9}{5}i$$

Thus, the given expression simplifies to $-\frac{3}{5} + \frac{9}{5}i$.

## Answer 30e.

Consider the expression,

$$\frac{-2-5i}{3i}$$

Write in standard form:

Now,

$$\frac{-2-5i}{3i} = \frac{(-2-5i)(-i)}{3i(-i)}$$ Multiplying both numerator and denominator by $-i$

$$= \frac{(-2)(-i)+(-5i)(-i)}{-3i^2}$$ Multiplying using FOIL

$$= \frac{2i+5i^2}{-3i^2}$$ Simplify

$$= \frac{2i+5(-1)}{-3(-1)}$$ Use $i^2 = -1$

$$= \frac{2i-5}{3}$$ Simplify

$$= -\frac{5}{3}+\frac{2}{3}i$$ Write in standard form

Therefore,

$$\frac{-2-5i}{3i} = \boxed{-\frac{5}{3}+\frac{2}{3}i}.$$

## Answer 31e.

Multiply the numerator and the denominator by $-12i$, the complex conjugate of $12i$.

$$\frac{4+9i}{12i} = \frac{4+9i}{12i} \cdot \frac{-12i}{-12i}$$
$$= \frac{-12i\left(4+9i\right)}{12i\left(-12i\right)}$$

Apply the distributive property in the numerator and multiply.

$$\frac{-12i\left(4+9i\right)}{12i\left(-12i\right)} = \frac{-48i-108i^2}{-144i^2}$$

We know that $i^2 = -1$. Thus,

$$\frac{-48i-108i^2}{-144i^2} = \frac{-48i-108\left(-1\right)}{-144\left(-1\right)}.$$

Simplify.

$$\frac{-48i-108\left(-1\right)}{-144\left(-1\right)} = \frac{-48i+108}{144}$$

Write in standard form.

$$\frac{-48i+108}{144} = \frac{108}{144} - \frac{48}{144}i$$
$$= \frac{3}{4} - \frac{1}{3}i$$

Thus, the given expression simplifies to $\frac{3}{4} - \frac{1}{3}i$.

## Answer 32e.

Consider the expression,

$$\frac{7+4i}{2-3i}$$

Write in standard form:

Now,

$$\frac{7+4i}{2-3i} = \frac{(7+4i)(2+3i)}{(2-3i)(2+3i)}$$ Multiplying numerator and denominator by $2+3i$, using the complex conjugate of $2-3i$

$$= \frac{7\cdot 2+7\cdot 3i+4i\cdot 2+4i\cdot 3i}{2\cdot 2+2\cdot 3i+(-3i)\cdot 2+(-3i)(3i)}$$ Multiplying using FOIL

$$= \frac{14+21i+8i+12i^2}{4+6i-6i-9i^2}$$ Simplify

$$= \frac{14+29i+12(-1)}{4-9(-1)}$$ Simplify using $i^2=-1$

$$= \frac{2+29i}{13}$$ Simplify

$$= \frac{2}{13}+\frac{29}{13}i$$ Divide by 13, to get standard form

Therefore,

$$\frac{7+4i}{2-3i} = \boxed{\frac{2}{13}+\frac{29}{13}i}.$$

## Answer 33e.

Multiply the numerator and the denominator by $5-9i$, the complex conjugate of $5+9i$.

$$\frac{-1-6i}{5+9i} = \frac{-1-6i}{5+9i}\cdot\frac{5-9i}{5-9i}$$

$$= \frac{(-1-6i)(5-9i)}{(5+9i)(5-9i)}$$

Apply the FOIL method.

$$\frac{(-1-6i)(5-9i)}{(5+9i)(5-9i)} = \frac{-5+9i-30i+54i^2}{25-45i+45i-81i^2}$$

We know that $i^2=-1$. Thus,

$$\frac{-5+9i-30i+54i^2}{25-45i+45i-81i^2} = \frac{-5+9i-30i+54(-1)}{25-45i+45i-81(-1)}.$$

Simplify.

$$\frac{-5+9i-30i+54(-1)}{25-45i+45i-81(-1)} = \frac{-59-21i}{106}$$

Write in standard form.

$$\frac{-59-21i}{106} = -\frac{59}{106} - \frac{21}{106}i$$

Thus, the given expression simplifies to $-\frac{59}{106} - \frac{21}{106}i$.

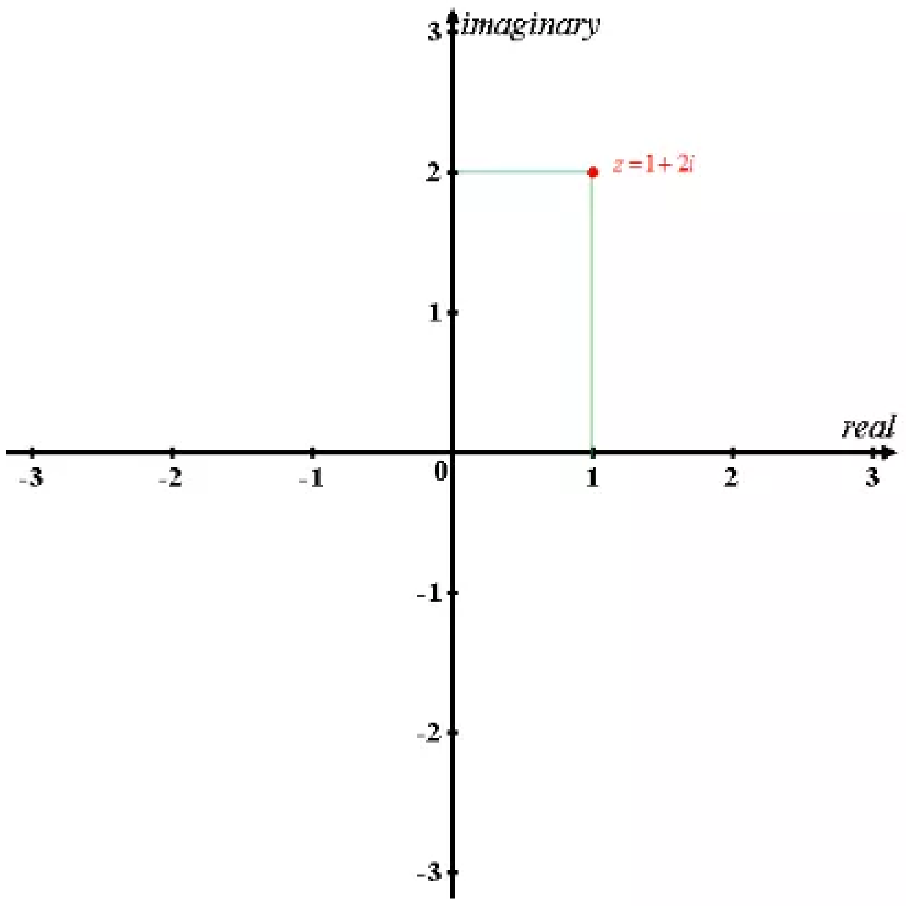
## Answer 34e.

Consider the complex number,

$$1+2i$$

Plot the complex number:

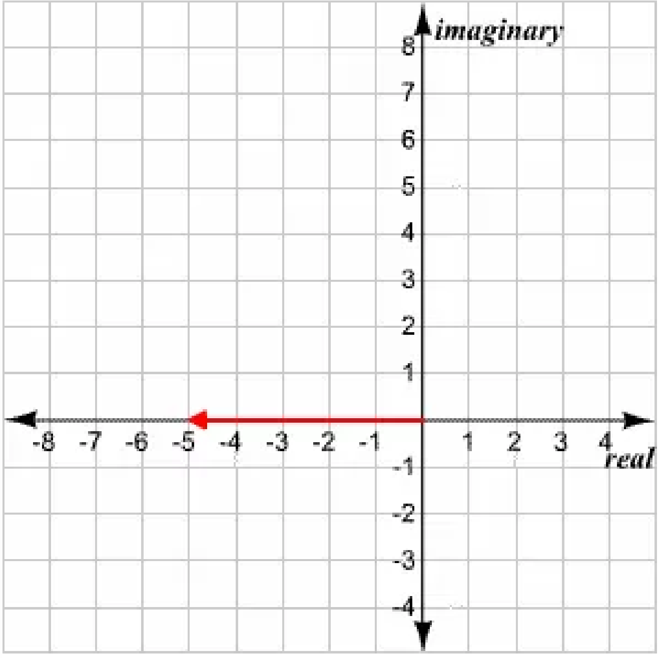
The following diagram contains the above complex number in the complex plane.



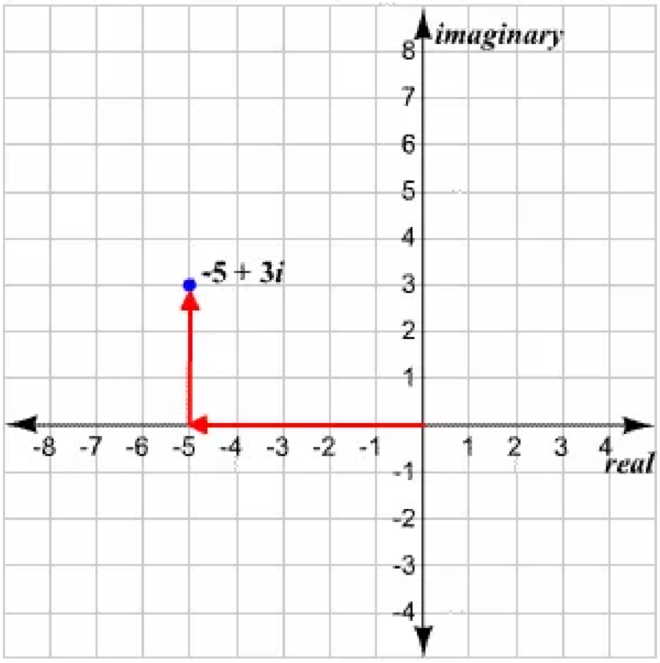

From the graph the point is located at 1 unit right and 2 units up in the complex plane.

## Answer 35e.

First, draw a complex plane. In order to plot $-5 + 3i$, start at the origin and move 5 units to the left.




Now, move 3 units up and mark the point where we end up.



## Answer 36e.

Consider the complex number,

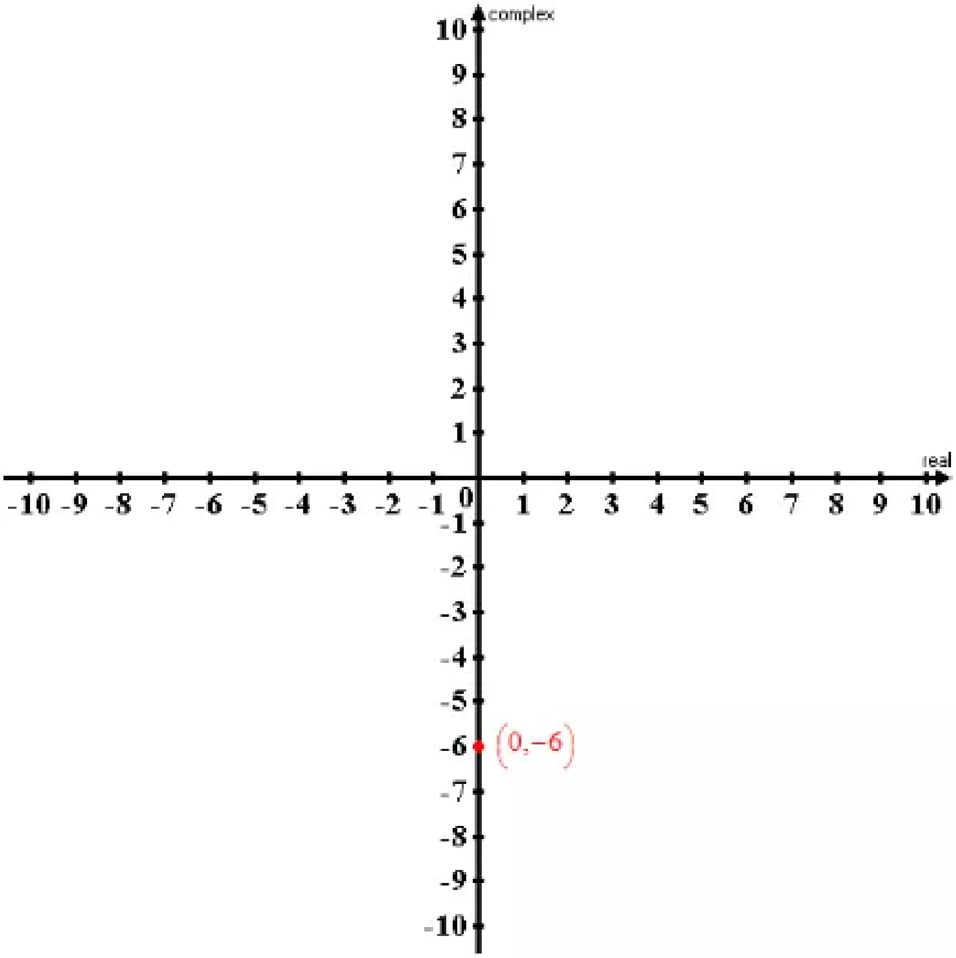
$-6i$

Rewrite the complex number as,

$0-6i$

Plot the number in complex plane:

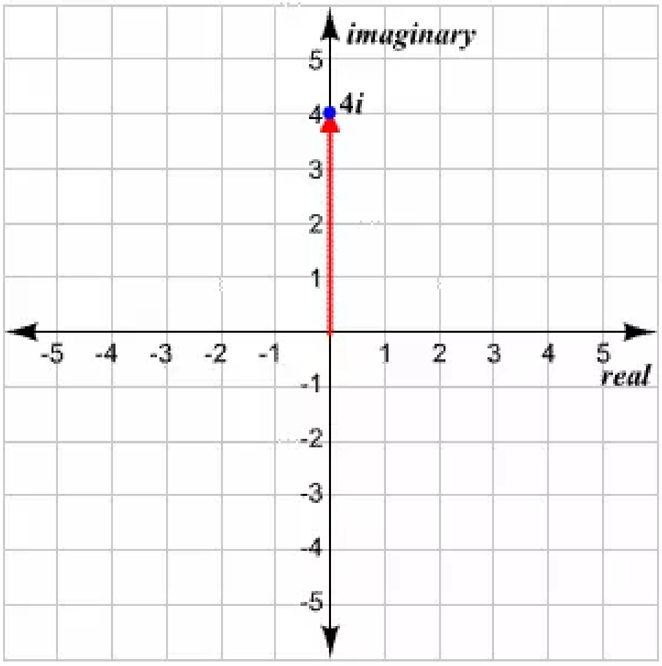
The following diagram contains the above complex number in the complex plane.




From the figure, we notice that the complex number $-6i$ situated at 6 units below from the complex plane. Here real part is equal to zero. So the point lies on complex line.

## Answer 37e.

First, draw a complex plane. In order to plot $4i$, start at the origin and move 4 units up. Mark the point where we end up.




## Answer 38e.

Consider the complex number,

$$-7-i$$

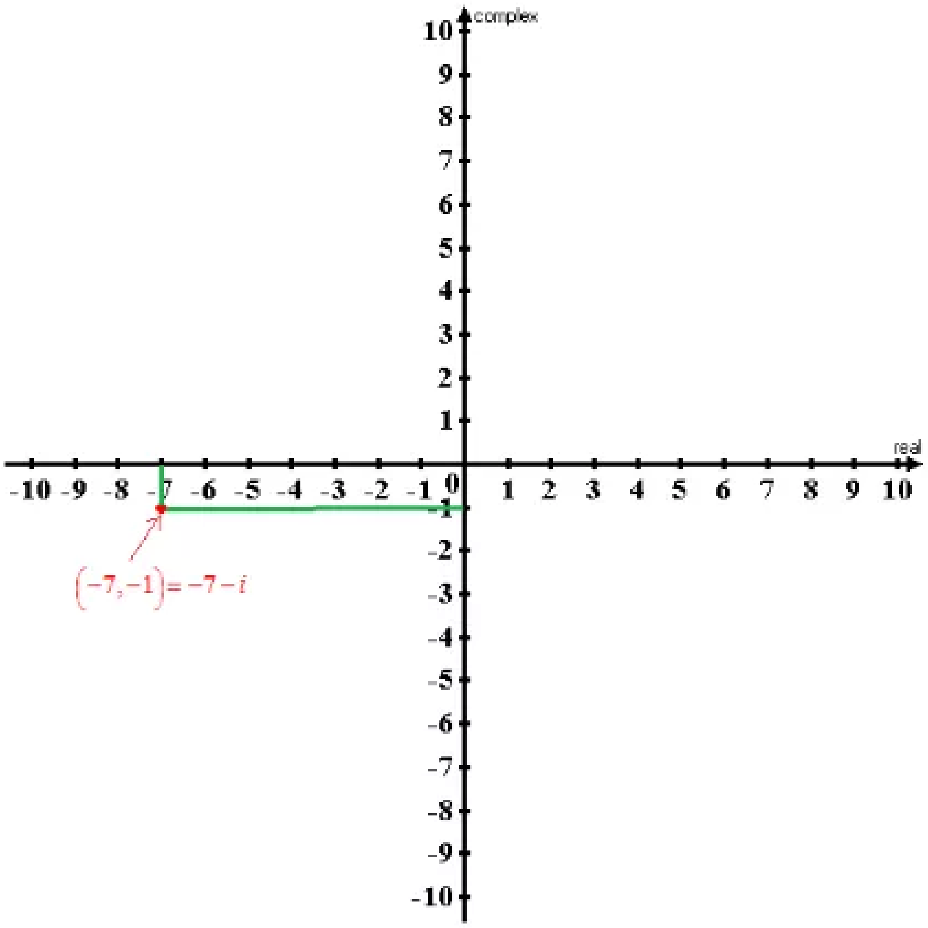
Rewrite the number as,

$$(-7)+(-1)i=(-7,-1)$$

Plot the point in complex plane:

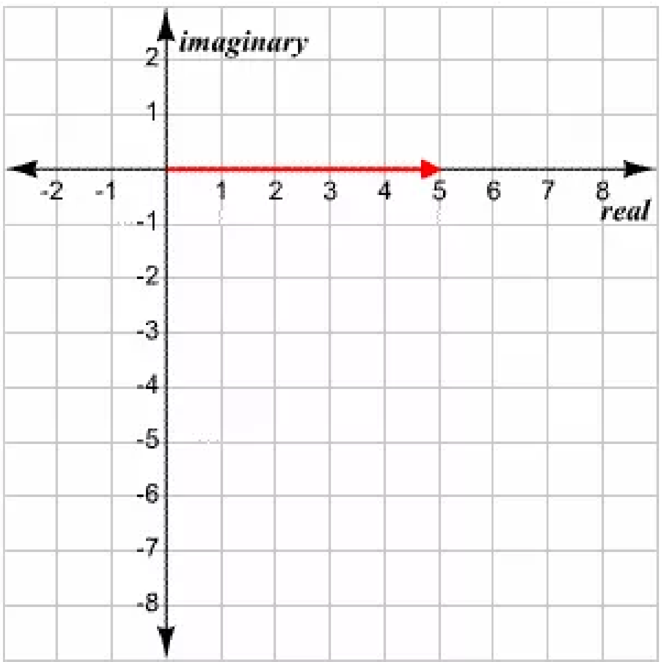
The point is situated at 7units left and 1 unit below the complex plane.

The following diagram contains the above complex number in the complex plane.

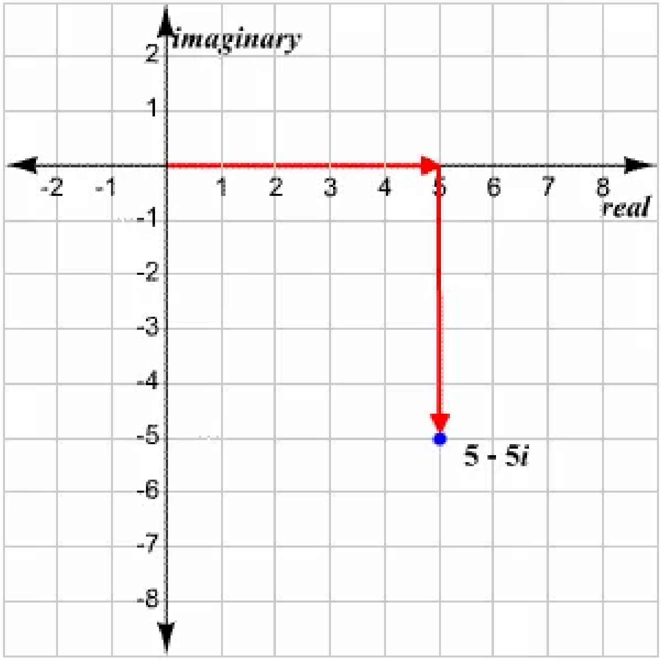



## Answer 39e.

First, draw a complex plane. In order to plot $5 - 5i$, start at the origin and move 5 units to the right.



Now, move 5 units down and mark the point where we end up.




## Answer 40e.

Consider the number,

$$7$$

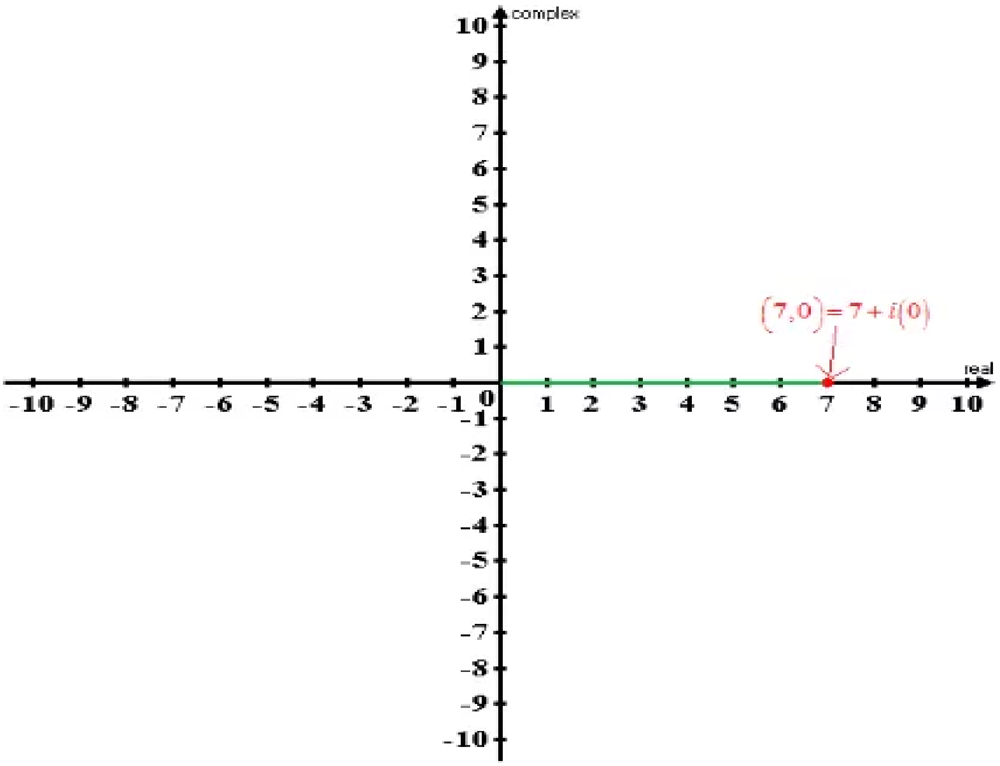
Rewrite the number as,

$$7+0\cdot i=(7,0)$$

Plot the point in the complex plane.

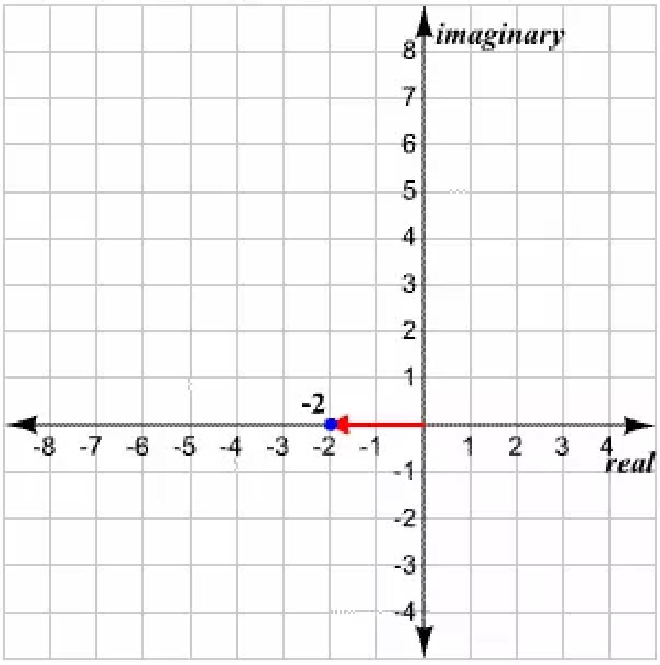
The number is situated at 7 unit's right on the real line.

The diagram contains the above complex number in the complex plane.



## Answer 41e.

First, draw a complex plane. In order to plot $-2$, start at the origin and move 2 units to the left. Mark the point where we end up.




## Answer 42e.

Consider the complex number,

$$4+3i$$

To find the absolute value of the complex number

Since the absolute value of $z = x + iy$ is denoted by $|z|$ and defined as

$$|z| = \sqrt{x^2 + y^2}$$

On comparing the complex number $4+3i$ with $z = x + iy$, we obtain

$$x = 4, y = 3$$

Plug the values $x = 4, y = 3$ in $|z| = \sqrt{x^2 + y^2}$, we get

$$\begin{aligned} |4+3i| &= \sqrt{4^2 + 3^2} \\ &= \sqrt{16+9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Hence,

$$|4+3i| = \boxed{5}.$$

## Answer 43e.

For any complex number $z$ of the form $a + bi$, its absolute value is defined as $|z| = \sqrt{a^2 + b^2}$. Thus,

$$|-3 + 10i| = \sqrt{(-3)^2 + 10^2}.$$

Evaluate the powers and then add.

$$\sqrt{(-3)^2 + 10^2} = \sqrt{9 + 100}$$
$$= \sqrt{109}$$

Therefore, the absolute value of the given complex number is $\sqrt{109}$.

## Answer 44e.

Consider the complex number,

$$10 - 7i$$

To find the absolute value of the complex number $10 - 7i$

Since the absolute value of $z = x + iy$ is denoted by $|z|$ and defined as

$$|z| = \sqrt{x^2 + y^2}$$

On comparing the complex number $4 + 3i$ with $z = x + iy$, we obtain

$$x = 4, y = 3$$

Plug the values $x = 4, y = 3$ in $|z| = \sqrt{x^2 + y^2}$, we get

$$|10 - 7i| = \sqrt{10^2 + (-7)^2}$$
$$= \sqrt{100 + 49}$$
$$= \sqrt{149}$$

Hence,

$$|10 - 7i| = \boxed{\sqrt{149}}.$$

## Answer 45e.

For any complex number $z$ of the form $a + bi$, its absolute value is defined as $|z| = \sqrt{a^2 + b^2}$. Thus,

$$|-1 - 6i| = \sqrt{(-1)^2 + (-6)^2}.$$

Evaluate the powers and then add.

$$\sqrt{(-1)^2+(-6)^2} = \sqrt{1+36}$$
$$= \sqrt{37}$$

Therefore, the absolute value of the given complex number is $\sqrt{37}$.

## Answer 46e.

Consider the complex number $-8i$.
Need to find the absolute value.
The absolute value of the complex number $-8i$ is

$$\begin{aligned}|-8i| &= |0+(-8i)| \\ &= \sqrt{0^2+(-8)^2} \\ &= \sqrt{0+64} \\ &= \sqrt{64} \\ &= \boxed{8}\end{aligned}$$

## Answer 47e.

For any complex number $z$ of the form $a + bi$, its absolute value is defined as $|z| = \sqrt{a^2+b^2}$. Thus,

$|4i| = |0+4i| = \sqrt{0^2+4^2}$.

Evaluate the powers and then add.

$$\sqrt{0^2+4^2} = \sqrt{0+16}$$
$$= 4$$

Therefore, the absolute value of the given complex number is 4.

## Answer 48e.

Consider the complex number $-4+i$.
Need to find the absolute value.
The absolute value of the complex number $-4+i$ is

$$\begin{aligned}|-4+i| &= \sqrt{(-4)^2+1^2} \\ &= \sqrt{16+1} \\ &= \boxed{\sqrt{17}}\end{aligned}$$

## Answer 49e.

For any complex number $z$ of the form $a + bi$, its absolute value is defined as $|z| = \sqrt{a^2 + b^2}$. Thus,

$|7 + 7i| = \sqrt{7^2 + 7^2}$.

Simplify.

$\sqrt{7^2 + 7^2} = 7\sqrt{2}$

Therefore, the absolute value of the given complex number is $7\sqrt{2}$.

## Answer 50e.

Consider the complex number $9+12i$.
Need to find the absolute value.
The absolute value of the complex number $9+12i$ is

$$
\begin{aligned}
|9+12i| &= \sqrt{9^2+12^2} \\
&= \sqrt{81+144} \\
&= \sqrt{225} \\
&= 15
\end{aligned}
$$

Therefore, the answer is

$\boxed{(B)}$

## Answer 51e.

Apply the distributive property to remove the parentheses.
$-8 - (3 + 2i) - (9 - 4i) = -8 - 3 - 2i - 9 + 4i$

Combine the like terms.

$$
\begin{aligned}
-8 - 3 - 2i - 9 + 4i &= (-8 - 3 - 9) + (-2 + 4)i \\
&= -20 + 2i
\end{aligned}
$$

Thus, the given expression in simplified form is $-20 + 2i$.

## Answer 52e.

Consider the expression $(3+2i)+(5-i)+6i$.

Need to write the following expression as a complex number in standard form.

$$
\begin{aligned}
&(3+2i)+(5-i)+6i \\
&=((3+5)+(2+(-1))i)+6i \qquad \text{(Since } (a+bi)+(c+di)=(a+c)+(b+d)i\text{)} \\
&=(8+i)+6i \\
&=8+(6+1)i \qquad \text{(Since } (a+bi)+(c+di)=(a+c)+(b+d)i\text{)} \\
&=\boxed{8+7i}.
\end{aligned}
$$

## Answer 53e.

Apply the FOIL method to multiply $3 + 2i$ and $8 + 3i$.
$5i(3 + 2i)(8 + 3i) = 5i(24 + 9i + 16i + 6i^2)$

We know that $i^2 = -1$. Thus,
$5i(24 + 9i + 16i + 6i^2) = 5i[24 + 9i + 16i + 6(-1)]$.

Simplify within the brackets.
$5i[24 + 9i + 16i + 6(-1)] = 5i(18 + 25i)$

Apply the distributive property.
$5i(18 + 25i) = 90i + 125i^2$

Substitute $-1$ for $i^2$ and simplify.

$$\begin{aligned} 90i + 125i^2 &= 90i + 125(-1) \\ &= 90i - 125 \end{aligned}$$

Write in standard form.
$90i - 125 = -125 + 90i$

Thus, the given expression simplifies to $-125 + 90i$.

## Answer 54e.

Consider the expression $(1-9i)(1-4i)(4-3i)$.

Need to write the expression as a complex number in standard form.

$$\begin{aligned} &(1-9i)(1-4i)(4-3i) \\ &= (1-4i-9i+36i^2)(4-3i) && \text{Using } (a+ib)(c+id) = ac+adi+bci+bd(i)^2 \\ &= (1-13i+36(-1))(4-3i) && \text{Using } (a+bi)+(c+di) = (a+c)+(b+d)i \\ & && \text{And } i^2 = -1) \\ &= (-35-13i)(4-3i) \\ &= -140+105i-52i+39i^2 \\ &= -140+53i+39(-1) && (\text{Since } i^2 = -1) \\ &= \boxed{-179+53i} \end{aligned}$$

## Answer 55e.

For adding (or subtracting) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

$$\begin{aligned} \frac{(5-2i)+(5+3i)}{(1+i)-(2-4i)} &= \frac{(5+5)+(-2+3)i}{(1-2)+[1-(-4)]i} \\ &= \frac{10+i}{-1+5i} \end{aligned}$$

Multiply the numerator and the denominator by $-1-5i$, the complex conjugate of $-1+5i$.

$$\frac{10+i}{-1+5i} = \frac{10+i}{-1+5i}\cdot\frac{-1-5i}{-1-5i}$$
$$= \frac{(10+i)(-1-5i)}{(-1+5i)(-1-5i)}$$

Apply the FOIL method.

$$\frac{(10+i)(-1-5i)}{(-1+5i)(-1-5i)} = \frac{-10-50i-i-5i^2}{1+5i-5i-25i^2}$$

We know that $i^2=-1$. Thus,

$$\frac{-10-50i-i-5i^2}{1+5i-5i-25i^2} = \frac{-10-50i-i-5(-1)}{1+5i-5i-25(-1)}.$$

Simplify.

$$\frac{-10-50i-i-5(-1)}{1+5i-5i-25(-1)} = \frac{-5-51i}{26}$$

Write in standard form.

$$\frac{-5-51i}{26} = -\frac{5}{26}-\frac{51}{26}i$$

Thus, the given expression simplifies to $-\frac{5}{26}-\frac{51}{26}i$.

## Answer 56e.

Consider the expression $\frac{(10+4i)-(3-2i)}{(6-7i)(1-2i)}$.

Need to write the expression as a complex number in standard form.

$$\frac{(10+4i)-(3-2i)}{(6-7i)(1-2i)}$$

$$=\frac{(10-3)+(4-(-2))i}{(6-7i)(1-2i)}$$ Using $(a+bi)-(c-di)=(a-c)+(b-d)i$

$$=\frac{7+6i}{6-12i-7i+14i^2}$$ Using $(a+ib)(c+id)=ac+adi+bci+bd(i)^2$ and $i^2=-1)$

$$=\frac{7+6i}{6-19i+14(-1)}$$

$$=\frac{7+6i}{-8-19i}$$

$$=\frac{-7-6i}{8+19i}$$

$$=\frac{(-7-6i)(8-19i)}{(8+19i)(8-19i)}$$ Multiplying numerator and denominator with $(8-19i)$

$$=\frac{-56+133i-48i+114i^2}{8^2-(19i)^2}$$ Using $(a+ib)(c+id)=ac+adi+bci+bd(i)^2$

$$=\frac{-56+85i+114(-1)}{64-361(-1)}$$ (Since $i^2=-1$)

$$=\frac{-170+85i}{425}$$

$$=\frac{-2+i}{5}$$

$$=-\frac{2}{5}+\frac{1}{5}i$$

## Answer 57e.

The standard form of a complex number is $a+bi$, where $a$ and $b$ are real numbers. The complex number $-2i^2+7i+4$ is not in standard form. For writing it in standard form, we have to use $i^2=-1$. The error is that $i$ is considered as a variable and not as an imaginary unit.

In order to correct the error, substitute $-1$ for $i^2$ in $-2i^2+7i+4$.
$-2i^2+7i+4=-2(-1)+7i+4$

Simplify.
$-2(-1)+7i+4=6+7i$

Therefore, the simplified form of the given expression is $6+7i$.

## Answer 58e.

Consider $|2-3i|=\sqrt{2^2-3^2}$

It is given that $|2-3i|=\sqrt{2^2-3^2}$ which is wrong.

And the correct answer is

$$\begin{aligned}|2-3i| &= \sqrt{2^2+(-3)^2}\\ &= \sqrt{4+9}\\ &= \boxed{\sqrt{13}}\end{aligned}$$

## Answer 59e.

It is given that $z+z_a=0$ and $z\cdot z_m=1$, where $z_a$ is the additive inverse and $z_m$ is the multiplicative inverse of $z$. Thus,

$z_a=-z \cdots\cdots(eq.1)$

and

$z_m=\dfrac{1}{z}\cdots\cdots(eq.2)$

**a)** For finding the additive inverse of $2+i$, substitute $2+i$ for $z$ in $(eq.1)$ and simplify.

$$\begin{aligned}z_a &= -(2+i)\\ &= -2-i\end{aligned}$$

The additive inverse of $2+i$ is $-2-i$.

For finding the multiplicative inverse of $2+i$, first substitute $2+i$ for $z$ in $(eq.2)$.

$$z_m=\frac{1}{2+i}$$

Multiply the numerator and the denominator by $2-i$, the complex conjugate of $2+i$.

$$\frac{1}{2+i}=\frac{1}{2+i}\cdot\frac{2-i}{2-i}$$

Apply the distributive property in the numerator and the FOIL method in the denominator.

$$\frac{1}{2+i}\cdot\frac{2-i}{2-i}=\frac{2-i}{4-2i+2i-i^2}$$

We know that $i^2=-1$. Thus,

$$\frac{2-i}{4-2i+2i-i^2}=\frac{2-i}{4-2i+2i-(-1)}.$$

Simplify.

$$\frac{2-i}{4-2i+2i-(-1)}=\frac{2-i}{5}$$

Write in standard form.

$$\frac{2-i}{5}=\frac{2}{5}-\frac{1}{5}i$$

The multiplicative inverse of $z$ is $\frac{2}{5}-\frac{1}{5}i$.

**b)** For finding the additive inverse of $5-i$, substitute $5-i$ for $z$ in $(eq.1)$ and simplify.

$$\begin{aligned}z_a&=-(5-i)\\&=-5+i\end{aligned}$$

The additive inverse of $5-i$ is $-5+i$.

For finding the multiplicative inverse of $5-i$, first substitute $5-i$ for $z$ in $(eq.2)$.

$$z_m=\frac{1}{5-i}$$

Multiply the numerator and the denominator by $5+i$, the complex conjugate of $5-i$.

$$\frac{1}{5-i}=\frac{1}{5-i}\cdot\frac{5+i}{5+i}$$

Apply the FOIL method and use $i^2=-1$.

$$\begin{aligned}\frac{1}{5-i}\cdot\frac{5+i}{5+i}&=\frac{5+i}{25-i^2}\\&=\frac{5+i}{26}\end{aligned}$$

Write in standard form.

$$\frac{5+i}{26}=\frac{5}{26}+\frac{1}{26}i$$

The multiplicative inverse of $z$ is $\frac{5}{26}+\frac{1}{26}i$.

**c)** For finding the additive inverse of $-1+3i$, substitute $-1+3i$ for $z$ in $(eq.1)$ and simplify.

$$z_a = -(-1+3i)$$
$$= 1-3i$$

The additive inverse of $-1+3i$ is $1-3i$.

For finding the multiplicative inverse of $-1+3i$, first substitute $-1+3i$ for $z$ in $(eq.2)$.

$$z_m = \frac{1}{-1+3i}$$

Multiply the numerator and the denominator by $-1-3i$, the complex conjugate of $-1+3i$.

$$\frac{1}{-1+3i} = \frac{1}{-1+3i}\cdot\frac{-1-3i}{-1-3i}$$

Apply the FOIL method and use $i^2 = -1$.

$$\frac{1}{-1+3i}\cdot\frac{-1-3i}{-1-3i} = \frac{-1-3i}{1-9i^2}$$
$$= \frac{-1-3i}{10}$$

Write in standard form.

$$\frac{-1-3i}{10} = -\frac{1}{10}-\frac{3}{10}i$$

The multiplicative inverse of $z$ is $-\frac{1}{10}-\frac{3}{10}i$.

## Answer 60e.

Consider the complex numbers $1+i$ and $2-i$

Need to give an example for two imaginary numbers whose sum is real.

Therefore consider the sum of the complex numbers $1+i$ and $2-i$

That is

$$(1+i)+(2-i) = (1+2)+(1+(-1))i$$ Using the formula $(a+bi)+(c+di) = (a+c)+(b+d)i$

$$= 3+0i$$
$$= 3$$

And the complex numbers have imaginary parts with equal magnitude but opposite signs.

## Answer 61e.

Multiply the numerator and the denominator by $c-di$, the complex conjugate of $c+di$.

$$\frac{a+bi}{c+di} = \frac{a+bi}{c+di}\cdot\frac{c-di}{c-di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)}$$

Apply the FOIL method.

$$\frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{ac-adi+bci-bdi^2}{c^2-cdi+dci-d^2i^2}$$

We know that $i^2=-1$. Thus,

$$\frac{ac-adi+bci-bdi^2}{c^2-cdi+dci-d^2i^2} = \frac{ac-adi+bci-bd(-1)}{c^2-cdi+dci-d^2(-1)}.$$

Simplify.

$$\frac{ac-adi+bci-bd(-1)}{c^2-cdi+dci-d^2(-1)} = \frac{ac+bd+(bc-ad)i}{c^2+d^2}$$

Write in standard form.

$$\frac{ac+bd+(bc-ad)i}{c^2+d^2} = \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i$$

Thus, the given expression simplifies to $\frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i$.

## Answer 62e.

Consider the expression

$$\frac{a-bi}{c-di}$$

To write the expression as a complex number:

$$\frac{a-bi}{c-di}$$

$$= \frac{(a-bi)(c+di)}{(c-di)(c+di)} \quad \text{Multiplying numerator and denominator with } (c+di)$$

$$= \frac{ac+adi-bci-bdi^2}{c^2-(di)^2}$$

$$= \frac{ac+(ad-bc)i-bd(-1)}{c^2-d^2(-1)} \quad \text{Using } i^2=-1$$

$$= \frac{(ac+bd)+(ad-bc)i}{c^2+d^2}$$

$$= \boxed{\left(\frac{ac+bd}{c^2+d^2}\right)+\left(\frac{ad-bc}{c^2+d^2}\right)i}$$

## Answer 63e.

Multiply the numerator and the denominator by $c + di$, the complex conjugate of $c - di$.

$$\frac{a+bi}{c-di} = \frac{a+bi}{c-di} \cdot \frac{c+di}{c+di}$$
$$= \frac{(a+bi)(c+di)}{(c-di)(c+di)}$$

Apply the FOIL method.

$$\frac{(a+bi)(c+di)}{(c-di)(c+di)} = \frac{ac+adi+bci+bdi^2}{c^2+cdi-dci-d^2i^2}$$

We know that $i^2 = -1$. Thus,

$$\frac{ac+adi+bci+bdi^2}{c^2+cdi-dci-d^2i^2} = \frac{ac+adi+bci+bd(-1)}{c^2+cdi-dci-d^2(-1)}.$$

Simplify.

$$\frac{ac+adi+bci+bd(-1)}{c^2+cdi-dci-d^2(-1)} = \frac{ac-bd+(ad+bc)i}{c^2+d^2}$$

Write in standard form.

$$\frac{ac-bd+(ad+bc)i}{c^2+d^2} = \frac{ac-bd}{c^2+d^2} + \frac{ad+bc}{c^2+d^2}i$$

Thus, the given expression simplifies to $\frac{ac-bd}{c^2+d^2} + \frac{ad+bc}{c^2+d^2}i$.

## Answer 64e.

Consider the expression

$$\frac{a-bi}{c-di}$$

To write the expression as a complex number:

$$\frac{a-bi}{c+di}$$

$$= \frac{(a-bi)(c-di)}{(c+di)(c-di)}$$ Multiplying numerator and denominator with $(c-di)$

$$= \frac{ac-adi-bci+bdi^2}{c^2-(di)^2}$$

$$= \frac{ac-(ad+bc)i+bd(-1)}{c^2-d^2(-1)}$$ Using $i^2 = -1$

$$= \frac{(ac-bd)-(ad+bc)i}{c^2+d^2}$$

$$= \boxed{\left(\frac{ac-bd}{c^2+d^2}\right) - \left(\frac{ad+bc}{c^2+d^2}\right)i}$$

## Answer 65e.

The resistor has a resistance of 4 ohms, so its impedance is 4 ohms. The inductor has a reactance of 6 ohms. Thus, its impedance is $6i$ ohms. Since the capacitor has a reactance of 9 ohms, its impedance is $-9i$ ohms.

For a series circuit, the impedance is the sum of the impedances for the individual components.

$$\begin{aligned}\text{Impedence of circuit} &= 4 + 6i + (-9i)\\ &= 4 - 3i\end{aligned}$$

Therefore, the impedance of the given circuit is $4 - 3i$ ohms.

## Answer 66e.

We consider that the resistor has a resistance of 14 ohms, the inductor has a reactance of 7 ohms and the capacitor has a reactance of 8 ohms.
The resistor has a resistance of 14 ohms, so its impedance is 14 ohms.
The inductor has a reactance of 7 ohms, so its impedance is $7i$ ohms.
The capacitor has a reactance of 8 ohms, so its impedance is $-8i$ ohms.

Again, we know that the impedance for a series circuit is the sum of the impedances for the individual components.
Hence, the impedance of the circuit is

$$\begin{aligned}14+7i+(-8i) &= 14+7i-8i\\ &= 14-i\end{aligned}$$

So, the impedance of the circuit is $\boxed{14-i \text{ ohms}}$ .

## Answer 67e.

The resistor has a resistance of 12 ohms, so its impedance is 12 ohms. The inductor has a reactance of 8 ohms. Thus, its impedance is $8i$ ohms. Since the capacitors have reactance of 6 ohms and 10 ohms, their impedances are $-6i$ ohms and $-10i$ ohms.

For a series circuit, the impedance is the sum of the impedances for the individual components.

$$\begin{aligned}\text{Impedence of circuit} &= 12 + 8i + (-6i) + (-10i)\\ &= 12 - 8i\end{aligned}$$

Therefore, the impedance of the given circuit is $12 - 8i$ ohms.

**Answer 68e.**

(a)

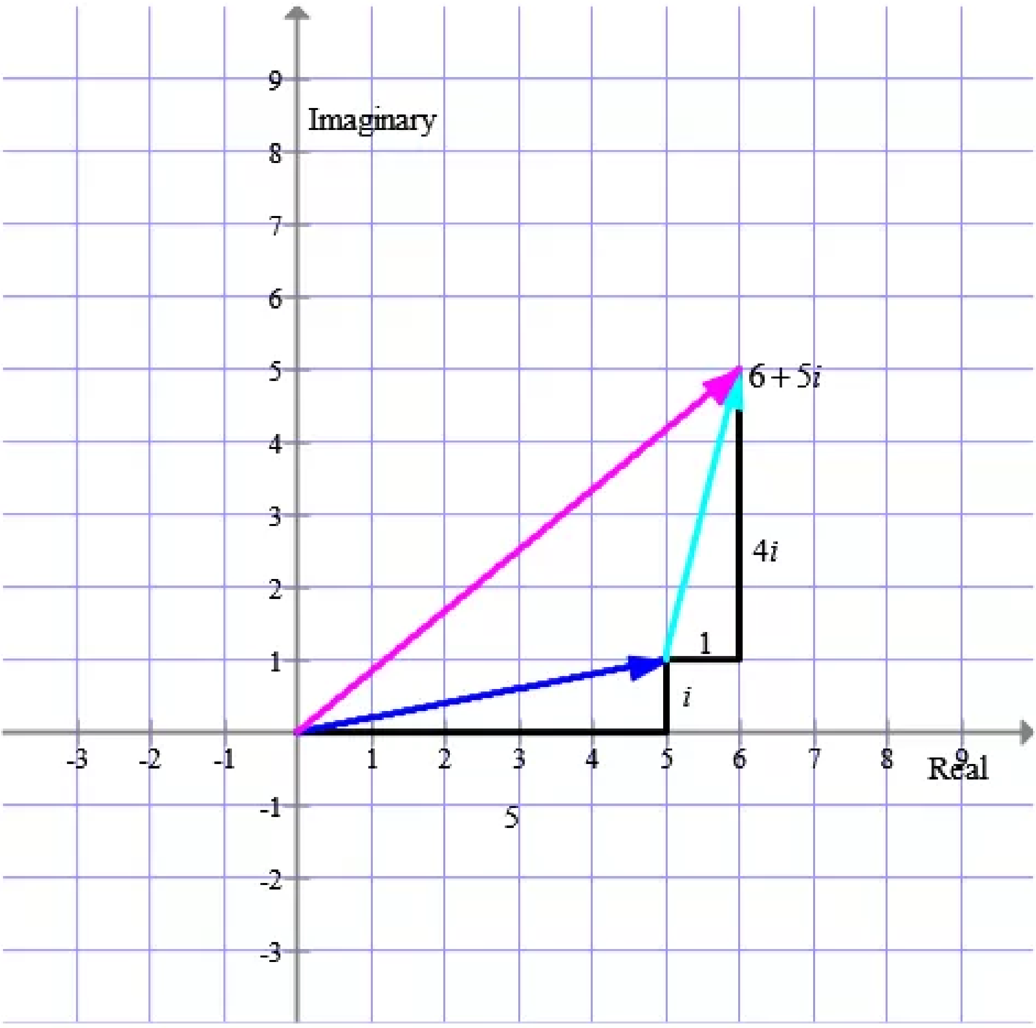
We consider the two complex numbers $5+i$ and $1+4i$ .

So, the sum of the two complex numbers $5+i$ and $1+4i$ is as given below.

$$(5+i)+(1+4i)=(5+1)+(1+4)i \quad \text{[Definition of complex addition]}$$

$$=6+5i \quad \text{[Write in standard form]}$$

The graph given below shows how we can geometrically add two complex numbers $5+i$ and $1+4i$ to find their sum $6+3i$ .



(b)

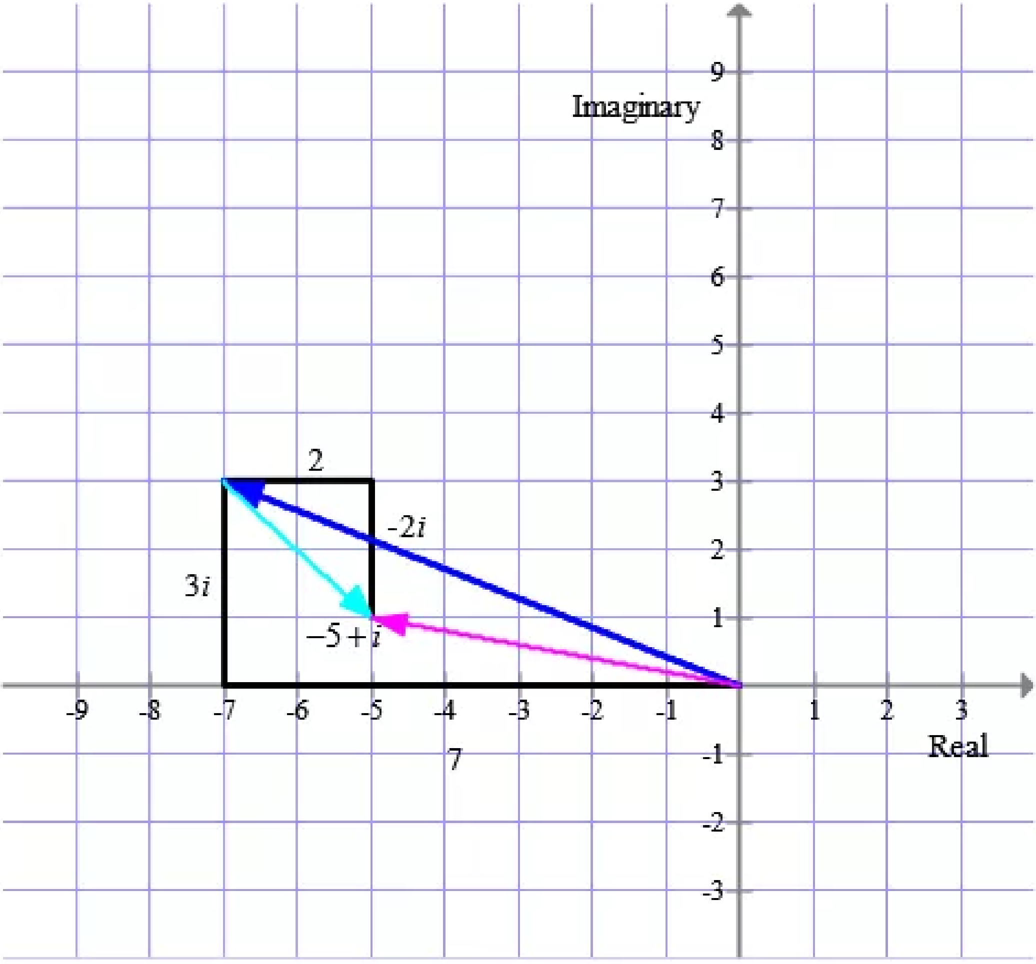
We consider the two complex numbers $-7+3i$ and $2-2i$ .

So, the sum of the two complex numbers $-7+3i$ and $2-2i$ is as given below.

$$(-7+3i)+(2-2i)=(-7+2)+(3-2)i \qquad \text{[Definition of complex addition]}$$

$$=-5+i \qquad \text{[Write in standard form]}$$

The graph given below shows how we can geometrically add two complex numbers $-7+3i$ and $2-2i$ to find their sum $-5+i$.




(c)

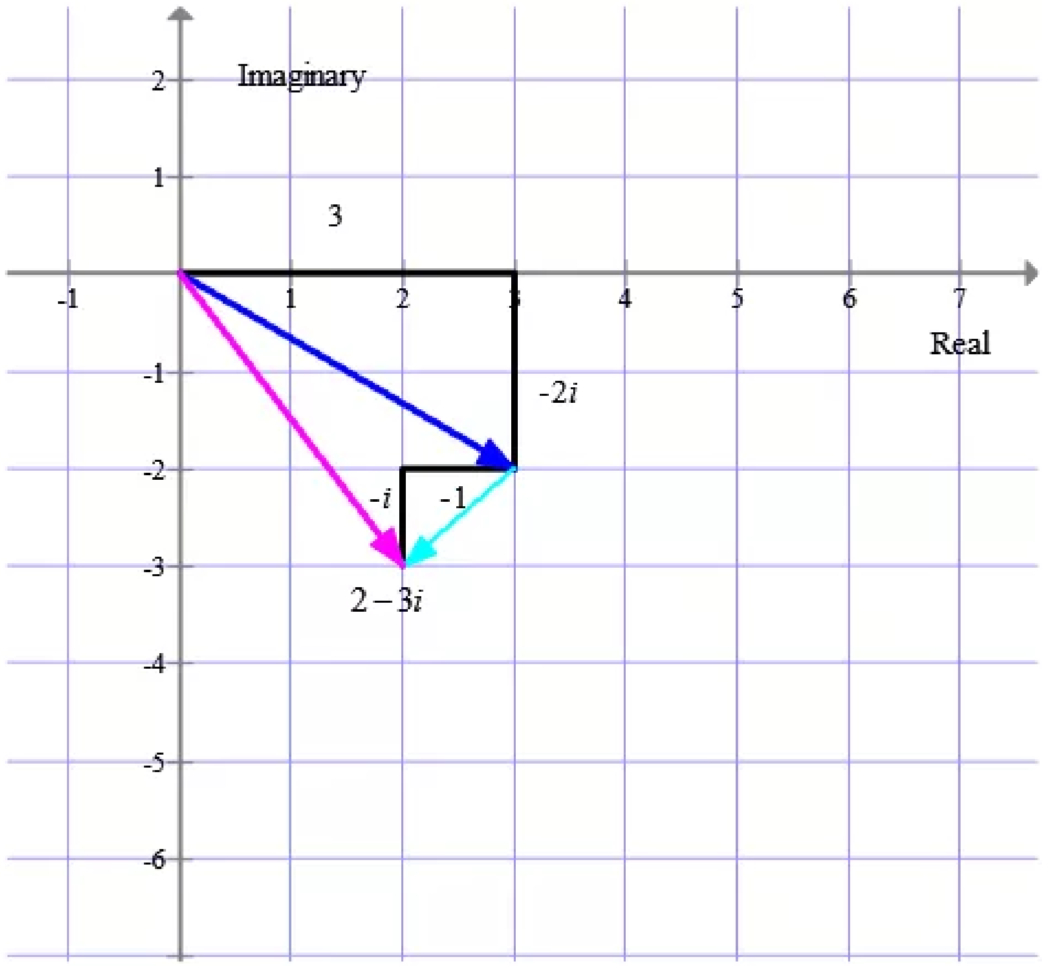
We consider the two complex numbers $3-2i$ and $-1-i$ .

So, the sum of the two complex numbers $3-2i$ and $-1-i$ is as given below.

$$(3-2i)+(-1-i)=(3-1)+(-2-1)i \qquad \text{[Definition of complex addition]}$$

$$=2-3i \qquad \text{[Write in standard form]}$$

The graph given below shows how we can geometrically add two complex numbers $3-2i$ and $-1-i$ to find their sum $2-3i$.



Imaginary
Real
3
-2i
-i
-1
2 − 3i

(d)

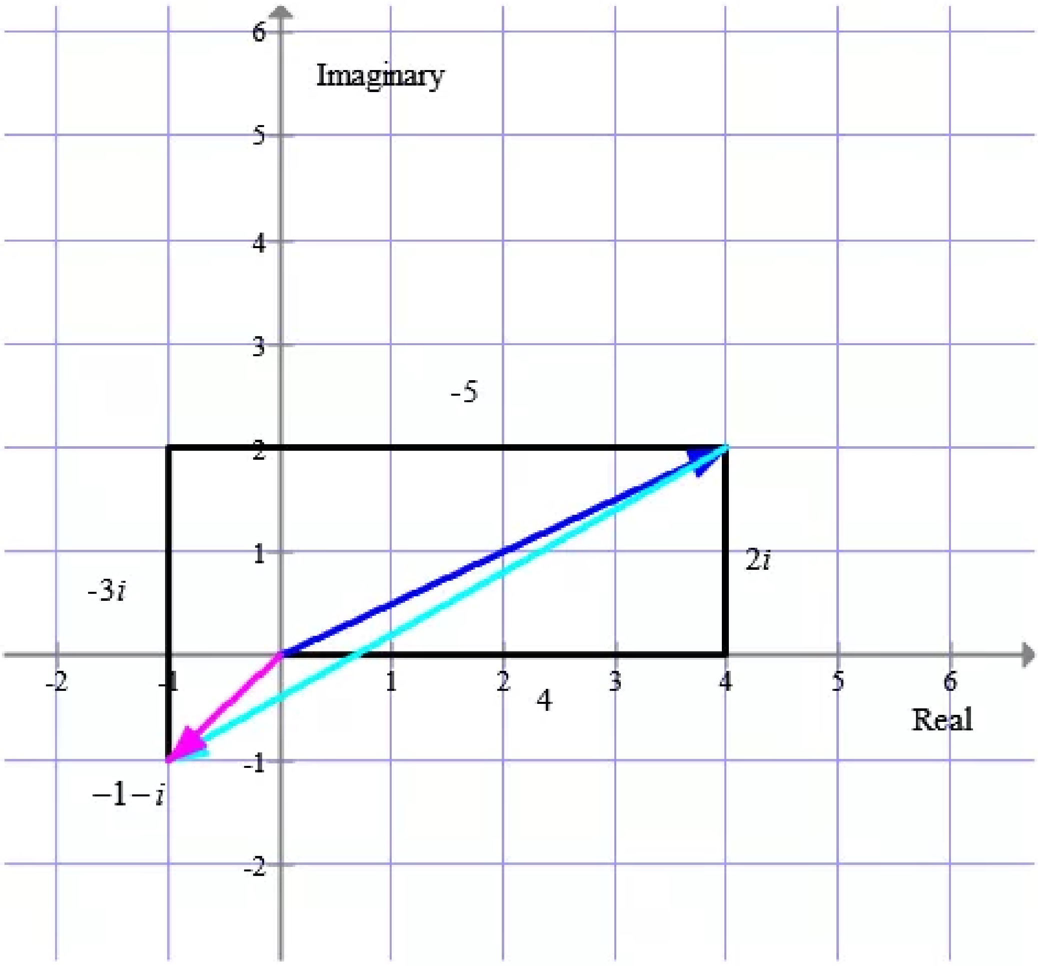
We consider the two complex numbers $4+2i$ and $-5-3i$ .

So, the sum of the two complex numbers $4+2i$ and $-5-3i$ is as given below.

$$(4+2i)+(-5-3i)=(4-5)+(2-3)i \quad \text{[Definition of complex addition]}$$

$$=-1-i \quad \text{[Write in standard form]}$$

The graph given below shows how we can geometrically add two complex numbers $4+2i$ and $-5-3i$ to find their sum $-1-i$ .




## Answer 69e.

The first power of any number equals that number. Thus, $i^1=i$. Since $i = \sqrt{-1}$, $i^2=-1$. The third power $i^3$ can be written as a product of $i^2$ and $i$.

$$\begin{aligned} i^3 &= i^2 \cdot i \\ &= -1 \cdot i \\ &= -i \end{aligned}$$

Similarly, evaluate the powers of $i$ from $i^4$ to $i^8$.

$i^4 = \left(i^2\right)^2 = (-1)^2 = 1$

$i^5 = i^4 \cdot i = 1 \cdot i = i$

$i^6 = i^4 \cdot i^2 = 1 \cdot (-1) = -1$

$i^7 = i^6 \cdot i = -1 \cdot i = -i$

$i^8 = \left(i^4\right)^2 = (1)^2 = 1$

Organize the results in a table as shown.

| Powers of $i$ | $i$ | $i^2$ | $i^3$ | $i^4$ | $i^5$ | $i^6$ | $i^7$ | $i^8$ |
|---|---|---|---|---|---|---|---|---|
| Simplified form | $i$ | $-1$ | $-i$ | $1$ | $i$ | $-1$ | $-i$ | $1$ |

From the table, we can see that the pattern repeats every four powers of $i$.

Evaluate the next four powers of $i$, from $i^9$ to $i^{12}$.

$i^9 = i^8 \cdot i = 1 \cdot i = i$

$i^{10} = \left(i^5\right)^2 = i^2 = -1$

$i^{11} = i^{10} \cdot i = -1 \cdot i = -i$

$i^{12} = \left(i^6\right)^2 = (-1)^2 = 1$

Thus, it is verified that the pattern continues for the next four powers of $i$.

## Answer 70e.

Consider $c = i$.

Determine, whether $c = i$ belongs to the Mandelbrot set or not.

Let $f(z) = z^2 + i$

$z_0 = 0$ then $|z_0| = 0$

$$\begin{aligned} z_1 &= f(z_0) \\ &= f(0) \\ &= 0^2 + i \\ &= i \end{aligned} \quad \text{and} \quad \begin{aligned} |z_1| &= |i| \\ &= 1 \end{aligned}$$

$$\begin{aligned} z_2 &= f(z_1) \\ &= f(i) \\ &= i^2 + i \\ &= -1 + i \end{aligned} \quad \text{and} \quad \begin{aligned} |z_2| &= |-1 + i| \\ &= \sqrt{2} \end{aligned}$$

Continue the above steps, we have

$$\begin{aligned} z_3 &= f(z_2) \\ &= f(-1+i) \\ &= (-1+i)^2 + i \\ &= -i \end{aligned} \quad \text{and} \quad \begin{aligned} |z_3| &= |-i| \\ &= 1 \end{aligned}$$

$$\begin{aligned} z_4 &= f(z_3) \\ &= f(-i) \\ &= (-i)^2 + i \\ &= -1+i \end{aligned} \quad \text{and} \quad \begin{aligned} |z_4| &= |-1+i| \\ &= \sqrt{2} \end{aligned}$$

According to the above results, we conclude that

$$|z_0| = 0, \quad |z_n| = \begin{cases} 1 & \text{if } n \text{ is odd} \\ \sqrt{2} & \text{if } n \text{ is even} \end{cases}$$

$$|z_n| < 2, \quad \forall n \in {}^{-}$$

Therefore $c = i$ belongs to the Mandelbrot set.

## Answer 71e.

For the function $f(z) = z^2 + c$, if the absolute values of $z_0, z_1, z_2, \ldots$ are all less than some fixed number $N$, then $c$ belongs to the Mandelbrot set.

Let $f(z) = z^2 + (-1+i)$. Since $z_0 = 0$, $|z_0| = 0$.

Evaluate $z_1$.

$$\begin{aligned} z_1 &= f(0) \\ &= 0^2 + (-1+i) \\ &= -1+i \end{aligned}$$

Find the absolute value of $z_1$.

$$\begin{aligned} |z_1| &= \sqrt{(-1)^2 + 1^2} \\ &= \sqrt{2} \\ &\approx 1.41 \end{aligned}$$

Evaluate $z_2$.

$$z_2 = f(-1+i)$$
$$= (-1+i)^2 + (-1+i)$$
$$= -1-i$$

Find the absolute value of $z_2$.

$$|z_2| = \sqrt{(-1)^2 + (-1)^2}$$
$$= \sqrt{2}$$
$$\approx 1.41$$

Evaluate $z_3$.

$$z_3 = f(-1-i)$$
$$= (-1-i)^2 + (-1+i)$$
$$= -1+3i$$

Find the absolute value of $z_3$.

$$|z_3| = \sqrt{(-1)^2 + 3^2}$$
$$= \sqrt{10}$$
$$\approx 3.16$$

Evaluate $z_4$.

$$z_4 = f(-1+3i)$$
$$= (-1+3i)^2 + (-1+i)$$
$$= -9-5i$$

Find the absolute value of $z_3$.

$$|z_4| = \sqrt{(-9)^2 + (-5)^2}$$
$$= \sqrt{106}$$
$$\approx 10.30$$

On comparing the five absolute values, we find that the absolute values are becoming infinitely large. Therefore, the given complex number does not belong to the Mandelbrot set.

## Answer 72e.

Consider : $c=-1$

Determine, whether $c=-1$ belongs to the Mandelbrot set or not.

Let

$$f(z)=z^2+c$$
$$=z^2-1 \quad \text{Since } c=-1$$

Let $z_0=0$, then $|z_0|=0$

$$z_1=f(z_0)$$
$$=f(0)$$
$$=0-1$$
$$=-1$$

then

$$|z_1|=|-1|$$
$$=1$$

Continue the above steps, we have

$$z_2=f(z_1)$$
$$=f(-1)$$
$$=(-1)^2-1$$
$$=0$$

then $|z_2|=0$

Continue the above steps, we have

$$z_3=f(z_2)$$
$$=f(0)$$
$$=0^2-1$$
$$=-1$$

then

$$|z_3|=|-1|$$
$$=1$$

Similarly we can find the other values $|z_4|=0$, $|z_5|=1$, and so on.

Simply we can write

$$|z_n|=\begin{cases}0 & \text{if } n \text{ is even}\\ 1 & \text{if } n \text{ is odd}\end{cases}$$

Therefore, $|z_n|<2$, $\forall n\in\_$ .

Hence $c=-1$ belongs to the Mandelbrot set.

## Answer 73e.

For the function $f(z) = z^2 + c$, if the absolute values of $z_0, z_1, z_2, \ldots$ are all less than some fixed number $N$, then $c$ belongs to the Mandelbrot set.

Let $f(z) = z^2 + (-0.5i)$. Since $z_0 = 0$, $|z_0| = 0$.

Evaluate $z_1$.

$$\begin{aligned} z_1 &= f(0) \\ &= 0^2 + (-0.5i) \\ &= -0.5i \end{aligned}$$

Find the absolute value of $z_1$.

$$\begin{aligned} |z_1| &= \sqrt{0^2 + (-0.5)^2} \\ &= \sqrt{0.25} \\ &= 0.5 \end{aligned}$$

Evaluate $z_2$.

$$\begin{aligned} z_2 &= f(-0.5i) \\ &= (-0.5i)^2 + (-0.5i) \\ &= -0.25 - 0.5i \end{aligned}$$

Find the absolute value of $z_2$.

$$\begin{aligned} |z_2| &= \sqrt{(-0.25)^2 + (-0.5)^2} \\ &= \sqrt{0.3125} \\ &\approx 0.56 \end{aligned}$$

Evaluate $z_3$.

$$\begin{aligned} z_3 &= f(-0.25 - 0.5i) \\ &= (-0.25 - 0.5i)^2 + (-0.5i) \\ &= -0.1875 - 0.25i \end{aligned}$$

Find the absolute value of $z_3$.

$$\begin{aligned} |z_3| &= \sqrt{(-0.1875)^2 + (-0.25)^2} \\ &= 0.3125 \end{aligned}$$

Evaluate $z_4$.

$$z_4 = f(-0.1875 - 0.25i)$$
$$= (-0.1875 - 0.25i)^2 + (-0.5i)$$
$$= -0.02734375 - 0.40625i$$

Find the absolute value of $z_3$.

$$|z_4| = \sqrt{(-0.02734375)^2 + (-0.40625)^2}$$
$$\approx 0.41$$

On comparing the five absolute values, we find that the absolute values are less than 1. Therefore, the given complex number belongs to the Mandelbrot set.

## Answer 74e.

Consider the expressions : $\sqrt{-4}\cdot\sqrt{-25}$ and $\sqrt{100}$.
We need to evaluate $\sqrt{-4}\cdot\sqrt{-25}$ and $\sqrt{100}$.

$$\sqrt{-4}\cdot\sqrt{-25} = \sqrt{4(-1)}\cdot\sqrt{25(-1)} \quad \text{writing as a product of numbers}$$
$$= \sqrt{4}\sqrt{-1}\cdot\sqrt{25}\sqrt{-1}$$
$$= 2i\cdot 5i \quad \text{Since } i=\sqrt{-1},\ \sqrt{4}=2 \text{ and } \sqrt{25}=5$$
$$= 10i^2$$
$$= 10(-1) \quad \text{Since } i^2=-1$$
$$= -10$$

Now the second expression is

$$\sqrt{100} = \sqrt{10^2}$$
$$= 10$$

Therefore from the above two results, $\sqrt{-4}\cdot\sqrt{-25} \neq \sqrt{100}$.

Hence the product property of square roots, $\sqrt{ab} = \sqrt{a}\cdot\sqrt{b}$ is not true, when $a$ and $b$ are both negative.

## Answer 75e.

**a)** First, we have to find $Z_1$. Consider the pathway. The resistor has a resistance of 4 ohms, so its impedance is 4 ohms. The inductor has a reactance of 5 ohms. Thus, its impedance is $5i$ ohms.

For a series circuit, the impedance is the sum of the impedances for the individual components.

$Z_1 = 4 + 5i$

Now, consider the pathway $Z_2$ and find its impedance.
The impedance of the resistor is 7 ohms. Since the capacitor has a reactance of 3 ohms, its impedance is $-3i$ ohms.

Add the impedances.
$Z_2 = 7 - 3i$

Substitute for $Z_1$ and $Z_2$ in the given formula to find $Z$.

$$Z = \frac{(4+5i)(7-3i)}{(4+5i)+(7-3i)}$$

Apply the FOIL method in the numerator and the definition of complex addition in the denominator.

$$\frac{(4+5i)(7-3i)}{(4+5i)+(7-3i)} = \frac{28 - 12i + 35i - 15i^2}{11+2i}$$
$$= \frac{28 + 23i - 15i^2}{11+2i}$$

Use $i^2 = -1$ and simplify.

$$\frac{28 + 23i - 15i^2}{11+2i} = \frac{28 + 23i - 15(-1)}{11+2i}$$
$$= \frac{43 + 23i}{11+2i}$$

Multiply the numerator and the denominator by $11 - 2i$, the complex conjugate of $11 + 2i$.

$$\frac{43+23i}{11+2i} = \frac{43+23i}{11+2i} \cdot \frac{11-2i}{11-2i}$$

Apply the FOIL method and simplify.

$$\frac{43+23i}{11+2i} \cdot \frac{11-2i}{11-2i} = \frac{473 - 86i + 253i - 46i^2}{121 - 4i^2}$$
$$= \frac{473 - 86i + 253i - 46(-1)}{121 - 4(-1)}$$
$$= \frac{519 + 167i}{125}$$

Simplify and write in standard form.

$$\frac{473 - 86i + 253i - 46i^2}{121 - 4i^2} = \frac{473 - 86i + 253i - 46(-1)}{121 - 4(-1)}$$
$$= \frac{519 + 167i}{125}$$
$$= \frac{519}{125} + \frac{167}{125}i$$

Therefore, the impedance of the parallel circuit is $\frac{519}{125} + \frac{167}{125}i$.

**b)** Find $Z_1$ and $Z_2$.

$Z_1 = 6 + 8i$

$Z_2 = 10 - 11i$

Substitute for $Z_1$ and $Z_2$ in the given formula to find $Z$.

$$Z = \frac{(6 + 8i)(10 - 11i)}{(6 + 8i) + (10 - 11i)}$$

Write the numerator and the denominator in simplified form.

$$\frac{(6 + 8i)(10 - 11i)}{(6 + 8i) + (10 - 11i)} = \frac{60 - 66i + 80i - 88i^2}{16 - 3i}$$
$$= \frac{60 + 14i - 88(-1)}{16 - 3i}$$
$$= \frac{148 + 14i}{16 - 3i}$$

Multiply the numerator and the denominator by $16 + 3i$, the complex conjugate of $16 - 3i$.

$$\frac{148 + 14i}{16 - 3i} = \frac{148 + 14i}{16 - 3i} \cdot \frac{16 + 3i}{16 + 3i}$$

Simplify and write in standard form.

$$\frac{148 + 14i}{16 - 3i} \cdot \frac{16 + 3i}{16 + 3i} = \frac{2368 + 444i + 224i - 42}{256 + 9}$$
$$= \frac{2326 + 668i}{265}$$
$$= \frac{2326}{265} + \frac{668}{265}i$$

The impedance of the parallel circuit is $\frac{2326}{265} + \frac{668}{265}i$.

**c)** Find $Z_1$ and $Z_2$.

$Z_1 = 3 + i$

$Z_2 = 4 - 6i$

Substitute for $Z_1$ and $Z_2$ in the given formula to find $Z$.

$$Z = \frac{(3+i)(4-6i)}{(3+i)+(4-6i)}$$

Write the numerator and the denominator in simplified form.

$$\frac{(3+i)(4-6i)}{(3+i)+(4-6i)} = \frac{12-18i+4i-6i^2}{7-5i}$$
$$= \frac{12-14i-6(-1)}{7-5i}$$
$$= \frac{18-14i}{7-5i}$$

Multiply the numerator and the denominator by $7+5i$, the complex conjugate of $7-5i$.

$$\frac{18-14i}{7-5i} = \frac{18-14i}{7-5i} \cdot \frac{7+5i}{7+5i}$$

Simplify and write in standard form.

$$\frac{18-14i}{7-5i} \cdot \frac{7+5i}{7+5i} = \frac{126+90i-98i+70}{49+25}$$
$$= \frac{196-8i}{74}$$
$$= \frac{98}{37} - \frac{4}{37}i$$

The impedance of the parallel circuit is $\frac{98}{37} - \frac{4}{37}i$.

## Answer 76e.

Let $f(z) = z^2 + (1+i)$.

(a)

We consider the infinite list of complex numbers as given below.

$$z_0 = i, z_1 = f(z_0), z_2 = f(z_1), z_3 = f(z_2), \dots$$

So, we have

$$z_0 = i$$

Now, we find the absolute value of $z_0$.

The absolute value of a complex number $z = a + ib$ , denoted $|z|$ is a nonnegative real number defined as $|z| = \sqrt{a^2 + b^2}$ .

Hence, the absolute value of $z_0$ is as given below.

$$|z_0| = |i| \quad \text{[Substitute } i \text{ for } z_0\text{]}$$
$$= \sqrt{0^2 + 1^2} \quad \left[\text{Substitute 0 and 1 for } a \text{ and } b \text{ respectively in } \sqrt{a^2 + b^2}\right]$$
$$= 1 \quad \text{[Simplify]}$$

We have

$$z_1 = f(z_0)$$
$$= f(i) \quad \text{[Substitute } i \text{ for } z_0\text{]}$$
$$= i^2 + (1+i) \quad \left[\text{Substitute } i \text{ for } z \text{ in } f(z) = z^2 + (1+i)\right]$$
$$= -1 + 1 + i \quad \left[i^2 = -1\right]$$
$$= i$$

We find the absolute value of $z_1$.

Hence, the absolute value of $z_1$ is as given below.

$$|z_1| = |i| \quad \text{[Substitute } i \text{ for } z_1\text{]}$$
$$= \sqrt{0^2 + 1^2} \quad \left[\text{Substitute 0 and 1 for } a \text{ and } b \text{ respectively in } \sqrt{a^2 + b^2}\right]$$
$$= 1 \quad \text{[Simplify]}$$

We have

$$z_2 = f(z_1)$$
$$= f(i) \quad \text{[Substitute } i \text{ for } z_1\text{]}$$
$$= i^2 + (1+i) \quad \left[\text{Substitute } i \text{ for } z \text{ in } f(z) = z^2 + (1+i)\right]$$
$$= -1 + 1 + i \quad \left[i^2 = -1\right]$$
$$= i$$

We find the absolute value of $z_2$.

Hence, the absolute value of $z_2$ is as given below.

$$|z_2| = |i| \quad \text{[Substitute } i \text{ for } z_2\text{]}$$
$$= \sqrt{0^2 + 1^2} \quad \left[\text{Substitute 0 and 1 for } a \text{ and } b \text{ respectively in } \sqrt{a^2 + b^2}\right]$$
$$= 1 \quad \text{[Simplify]}$$

We have

$$z_3 = f(z_2)$$
$$= f(i) \quad \text{[Substitute } i \text{ for } z_2\text{]}$$
$$= i^2 + (1+i) \quad \text{[Substitute } i \text{ for } z \text{ in } f(z) = z^2 + (1+i)\text{]}$$
$$= -1+1+i \quad [i^2 = -1]$$
$$= i$$

We find the absolute value of $z_3$.

Hence, the absolute value of $z_3$ is as given below.

$$|z_3| = |i| \quad \text{[Substitute } i \text{ for } z_3\text{]}$$
$$= \sqrt{0^2 + 1^2} \quad \left[\text{Substitute 0 and 1 for } a \text{ and } b \text{ respectively in } \sqrt{a^2+b^2}\right]$$
$$= 1 \quad \text{[Simplify]}$$

And so on.

Because the absolute values are remain same the given number $z_0 = i$ belongs to the Julia set associated with the function $f(z) = z^2 + (1+i)$ .

(b)

We consider the infinite list of complex numbers as given below.

$$z_0 = 1, z_1 = f(z_0), z_2 = f(z_1), z_3 = f(z_2), \ldots$$

So, we have

$$z_0 = 1$$

Now, we find the absolute value of $z_0$.

Hence, the absolute value of $z_0$ is as given below.

$$|z_0| = |1| \quad \text{[Substitute 1 for } z_0\text{]}$$
$$= \sqrt{1^2 + 0^2} \quad \left[\text{Substitute 1 and 0 for } a \text{ and } b \text{ respectively in } \sqrt{a^2+b^2}\right]$$
$$= 1 \quad \text{[Simplify]}$$

We have

$$z_1 = f(z_0)$$

$= f(1)$ [Substitute 1 for $z_0$]

$= 1^2 + (1+i)$ [Substitute $i$ for $z$ in $f(z) = z^2 + (1+i)$]

$= 1 + 1 + i$ [$1^2 = 1$]

$= 2 + i$

We find the absolute value of $z_1$.

Hence, the absolute value of $z_1$ is as given below.

$|z_1| = |2+i|$ [Substitute $2+i$ for $z_1$]

$= \sqrt{2^2 + 1^2}$ [Substitute 2 and 1 for $a$ and $b$ respectively in $\sqrt{a^2+b^2}$]

$= \sqrt{5}$ [Simplify]

⌟ 2.24

We have

$$z_2 = f(z_1)$$

$= f(2+i)$ [Substitute $2+i$ for $z_1$]

$= (2+i)^2 + (1+i)$ [Substitute $2+i$ for $z$ in $f(z) = z^2 + (1+i)$]

$= 4 + 4i + i^2 + 1 + i$

$= 4 + 4i - 1 + 1 + i$ [$i^2 = -1$]

$= 4 + 5i$ [Combine like terms]

We find the absolute value of $z_2$.

Hence, the absolute value of $z_2$ is as given below.

$|z_2| = |4+5i|$ [Substitute $4+5i$ for $z_2$]

$= \sqrt{4^2 + 5^2}$ [Substitute 4 and 5 for $a$ and $b$ respectively in $\sqrt{a^2+b^2}$]

$= \sqrt{41}$ [Simplify]

∟ 6.40

We have

$$z_3 = f(z_2)$$

$$= f(4+5i) \qquad [\text{Substitute } 4+5i \text{ for } z_2]$$

$$= (4+5i)^2 + (1+i) \qquad \left[\text{Substitute } 4+5i \text{ for } z \text{ in } f(z) = z^2 + (1+i)\right]$$

$$= 16 + 40i + 25i^2 + 1 + i$$

$$= 16 + 40i - 25 + 1 + i \qquad [i^2 = -1]$$

$$= -8 + 41i \qquad [\text{Combine like terms}]$$

We find the absolute value of $z_3$.

Hence, the absolute value of $z_3$ is as given below.

$$|z_3| = |-8+41i| \qquad [\text{Substitute } -8+41i \text{ for } z_3]$$

$$= \sqrt{(-8)^2 + (41)^2} \qquad \left[\begin{array}{l}\text{Substitute } -8 \text{ and } 41 \text{ for } a \text{ and } b \text{ respectively} \\ \text{in } \sqrt{a^2+b^2}\end{array}\right]$$

$$= \sqrt{1745} \qquad [\text{Simplify}]$$

$$\approx 41.77$$

We have

$$z_4 = f(z_3)$$

$$= f(-8+41i) \qquad [\text{Substitute } -8+41i \text{ for } z_3]$$

$$= (-8+41i)^2 + (1+i) \qquad \left[\text{Substitute } -8+41i \text{ for } z \text{ in } f(z) = z^2 + (1+i)\right]$$

$$= 64 - 656i + 1681i^2 + 1 + i$$

$$= 64 - 656i - 1681 + 1 + i \qquad [i^2 = -1]$$

$$= -1616 - 655i \qquad [\text{Combine like terms}]$$

We find the absolute value of $z_4$.

Hence, the absolute value of $z_4$ is as given below.

$$|z_4| = |-1616-655i| \qquad [\text{Substitute } -1616-655i \text{ for } z_3]$$

$$= \sqrt{(-1616)^2 + (-655)^2} \qquad \left[\begin{array}{l}\text{Substitute } -1616 \text{ and } -655 \text{ for } a \text{ and } b \text{ respectively} \\ \text{in } \sqrt{a^2+b^2}\end{array}\right]$$

$$= \sqrt{3040481} \qquad [\text{Simplify}]$$

$$\approx 1744.45$$

And so on.

Because the absolute values are becoming infinitely large the given number $z_0 = 1$ **does not belongs** to the Julia set associated with the function $f(z) = z^2 + (1+i)$ .

(c)

We consider the infinite list of complex numbers as given below.

$$z_0 = 2i, z_1 = f(z_0), z_2 = f(z_1), z_3 = f(z_2), \ldots$$

So, we have

$$z_0 = 2i$$

Now, we find the absolute value of $z_0$.

Hence, the absolute value of $z_0$ is as given below.

$$|z_0| = |2i| \quad [\text{Substitute } 2i \text{ for } z_0]$$
$$= \sqrt{0^2 + 2^2} \quad \left[\text{Substitute 0 and 2 for } a \text{ and } b \text{ respectively in } \sqrt{a^2+b^2}\right]$$
$$= 2 \quad [\text{Simplify}]$$

We have

$$z_1 = f(z_0)$$
$$= f(1) \quad [\text{Substitute 1 for } z_0]$$
$$= 1^2 + (1+i) \quad [\text{Substitute } i \text{ for } z \text{ in } f(z) = z^2 + (1+i)]$$
$$= 1 + 1 + i \quad [1^2 = 1]$$
$$= 2 + i$$

We find the absolute value of $z_1$.

Hence, the absolute value of $z_1$ is as given below.

$$|z_1| = |2+i| \quad [\text{Substitute } 2+i \text{ for } z_1]$$
$$= \sqrt{2^2 + 1^1} \quad \left[\text{Substitute 2 and 1 for } a \text{ and } b \text{ respectively in } \sqrt{a^2+b^2}\right]$$
$$= \sqrt{5} \quad [\text{Simplify}]$$
$$\approx 2.24$$

We have

$$z_2 = f(z_1)$$
$$= f(2+i) \quad [\text{Substitute } 2+i \text{ for } z_1]$$
$$= (2+i)^2 + (1+i) \quad [\text{Substitute } 2+i \text{ for } z \text{ in } f(z) = z^2 + (1+i)]$$
$$= 4 + 4i + i^2 + 1 + i$$
$$= 4 + 4i - 1 + 1 + i \quad [i^2 = -1]$$
$$= 4 + 5i \quad [\text{Combine like terms}]$$

We find the absolute value of $z_2$.

Hence, the absolute value of $z_2$ is as given below.

$$|z_2| = |4+5i| \quad [\text{Substitute } 4+5i \text{ for } z_2]$$
$$= \sqrt{4^2 + 5^2} \quad \left[\text{Substitute 4 and 5 for } a \text{ and } b \text{ respectively in } \sqrt{a^2+b^2}\right]$$
$$= \sqrt{41} \quad [\text{Simplify}]$$
$$\approx 6.40$$

We have

$$z_3 = f(z_2)$$

$$= f(4+5i) \quad [\text{Substitute } 4+5i \text{ for } z_2]$$

$$= (4+5i)^2 + (1+i) \quad [\text{Substitute } 4+5i \text{ for } z \text{ in } f(z) = z^2 + (1+i)]$$

$$= 16 + 40i + 25i^2 + 1 + i$$

$$= 16 + 40i - 25 + 1 + i \quad [i^2 = -1]$$

$$= -8 + 41i \quad [\text{Combine like terms}]$$

We find the absolute value of $z_3$.

Hence, the absolute value of $z_3$ is as given below.

$$|z_3| = |-8+41i| \quad [\text{Substitute } -8+41i \text{ for } z_3]$$

$$= \sqrt{(-8)^2 + (41)^2} \quad \left[\begin{array}{l}\text{Substitute } -8 \text{ and } 41 \text{ for } a \text{ and } b \text{ respectively} \\ \text{in } \sqrt{a^2+b^2}\end{array}\right]$$

$$= \sqrt{1745} \quad [\text{Simplify}]$$

$$\approx 41.77$$

We have

$$z_4 = f(z_3)$$

$$= f(-8+41i) \quad [\text{Substitute } -8+41i \text{ for } z_3]$$

$$= (-8+41i)^2 + (1+i) \quad [\text{Substitute } -8+41i \text{ for } z \text{ in } f(z) = z^2 + (1+i)]$$

$$= 64 + 656i + 1681i^2 + 1 + i$$

$$= 64 + 656i - 1681 + 1 + i \quad [i^2 = -1]$$

$$= -1616 + 657i \quad [\text{Combine like terms}]$$

We find the absolute value of $z_4$.

Hence, the absolute value of $z_4$ is as given below.

$$|z_4| = |-1616+657i| \quad [\text{Substitute } -1616+657i \text{ for } z_3]$$

$$= \sqrt{(-1616)^2 + (657)^2} \quad \left[\begin{array}{l}\text{Substitute } -1616 \text{ and } 657 \text{ for } a \text{ and } b \text{ respectively} \\ \text{in } \sqrt{a^2+b^2}\end{array}\right]$$

$$= \sqrt{3043105} \quad [\text{Simplify}]$$

$$\approx 1744.45$$

And so on.

Because the absolute values are becoming infinitely large the given number $z_0 = 1$ does not belongs to the Julia set associated with the function $f(z) = z^2 + (1+i)$.

(d)

We consider the infinite list of complex numbers as given below.

$$z_0 = i, z_1 = f(z_0), z_2 = f(z_1), z_3 = f(z_2), \ldots$$

So, we have

$$z_0 = i$$

Now, we find the absolute value of $z_0$.

The absolute value of a complex number $z = a + ib$, denoted $|z|$ is a nonnegative real number defined as $|z| = \sqrt{a^2 + b^2}$.

Hence, the absolute value of $z_0$ is as given below.

$$\begin{aligned} |z_0| &= |i| && [\text{Substitute } i \text{ for } z_0] \\ &= \sqrt{0^2 + 1^1} \\ &= 1 && [\text{Simplify}] \end{aligned}$$

We have

$$\begin{aligned} z_1 &= f(z_0) \\ &= f(i) && [\text{Substitute } i \text{ for } z_0] \\ &= i^2 + (1+i) && [\text{Substitute } i \text{ for } z \text{ in } f(z) = z^2 + (1+i)] \\ &= -1 + 1 + i && [i^2 = -1] \\ &= i \end{aligned}$$

We find the absolute value of $z_1$.

Hence, the absolute value of $z_1$ is as given below.

$$\begin{aligned} |z_1| &= |i| && [\text{Substitute } i \text{ for } z_0] \\ &= \sqrt{0^2 + 1^1} \\ &= 1 && [\text{Simplify}] \end{aligned}$$

We have

$$\begin{aligned} z_2 &= f(z_1) \\ &= f(i) && [\text{Substitute } i \text{ for } z_1] \\ &= i^2 + (1+i) && [\text{Substitute } i \text{ for } z \text{ in } f(z) = z^2 + (1+i)] \\ &= -1 + 1 + i && [i^2 = -1] \\ &= i \end{aligned}$$

We find the absolute value of $z_2$.

Hence, the absolute value of $z_2$ is as given below.

$$\begin{aligned} |z_2| &= |i| && [\text{Substitute } i \text{ for } z_0] \\ &= \sqrt{0^2 + 1^1} \\ &= 1 && [\text{Simplify}] \end{aligned}$$

We have

$$\begin{aligned} z_3 &= f(z_2) \\ &= f(i) && [\text{Substitute } i \text{ for } z_2] \\ &= i^2 + (1+i) && [\text{Substitute } i \text{ for } z \text{ in } f(z) = z^2 + (1+i)] \\ &= -1+1+i && [i^2 = -1] \\ &= i \end{aligned}$$

We find the absolute value of $z_3$.

Hence, the absolute value of $z_3$ is as given below.

$$\begin{aligned} |z_3| &= |i| && [\text{Substitute } i \text{ for } z_0] \\ &= \sqrt{0^2 + 1^1} \\ &= 1 && [\text{Simplify}] \end{aligned}$$

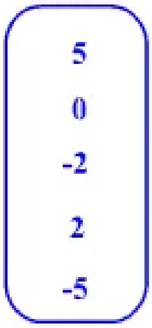
And so on.

Because the absolute values are remain same the given number $z_0$ belongs to the Julia set associated with the function $f(z) = z^2 + (1+i)$ .

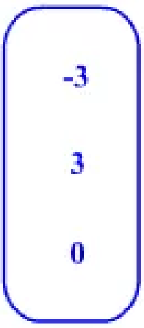
## Answer 77e.

We know that a relation is a mapping of input values with the corresponding output values. A function is said to be a relation when each input has exactly one output.

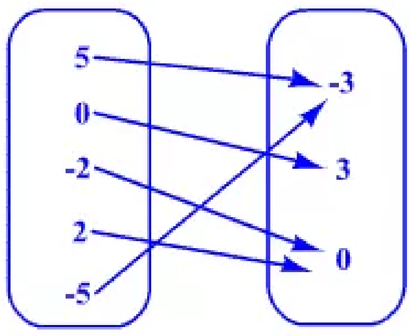
The input values are 5, 0, −2, 2, and −5. List the points in a rounded rectangle.




Similarly, the output values are −3, 3, 0, 0, and −3. List the points in a rounded rectangle. The repeated values will be listed only once.



Now, map the input and output values.

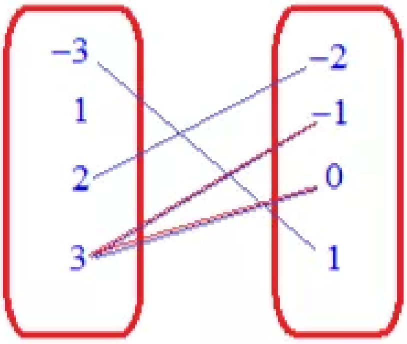



We can see that each input is mapped to exactly one output. Therefore, the relation is a function.

## Answer 78e.

Consider the relation : $\{(-3,1),(2,-2),(3,-1),(1,-1),(3,0)\}$.

We can sketch the mapping diagram as show below :




In this diagram, the number 3 mapped with both 0 and −1..

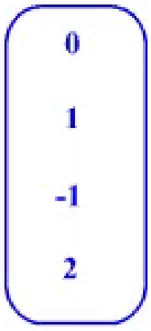
That is, the element 3 has more than one image.

Hence the relation $\{(-3,1),(2,-2),(3,-1),(1,-1),(3,0)\}$ is not a function.

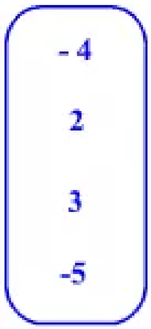
## Answer 79e.

We know that a relation is a mapping of input values with the corresponding output values. A function is said to be a relation when each input has exactly one output.

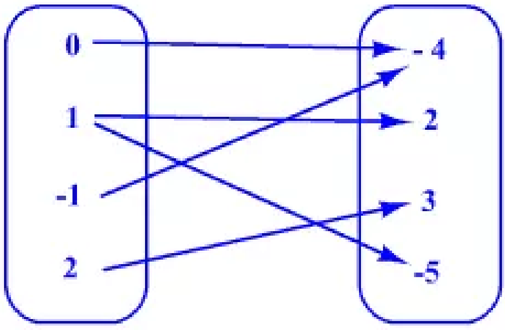
The input values are 0, 1, −1, 2, and 1. List the points in a rounded rectangle. The repeated values will be listed only once.



Similarly, the output values are –4, 2, –4, 3, and –5. List the points in a rounded rectangle.




Now, map the input and output values

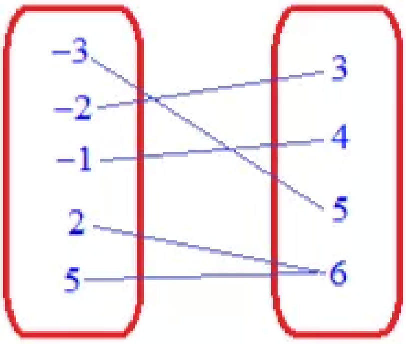



We can see that input 1 is mapped to both 2 and –5. Therefore, the relation is not a function.

## Answer 80e.

Consider the relation $\{(2,6),(5,6),(-1,4),(-3,5),(-2,3)\}$.

We can sketch the mapping diagram as show below :




In this diagram, every element has a unique image.

Hence the relation $\{(2,6),(5,6),(-1,4),(-3,5),(-2,3)\}$ is a function.

## Answer 81e.

We can add two matrices by adding the elements in the corresponding positions. The two matrices should have the same dimensions.

$$\begin{bmatrix} 5 & -4 \\ -2 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 9 \\ 1 & -8 \end{bmatrix} = \begin{bmatrix} 5+0 & -4+9 \\ -2+1 & 6-8 \end{bmatrix}$$

Simplify.

$$\begin{bmatrix} 5+0 & -4+9 \\ -2+1 & 6-8 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ -1 & -2 \end{bmatrix}$$

Therefore, the resulting matrix is $\begin{bmatrix} 5 & 5 \\ -1 & -2 \end{bmatrix}$.

## Answer 82e.

Consider $\begin{bmatrix} 6 & 3 \\ -5 & -1 \end{bmatrix} - \begin{bmatrix} 9 & -3 \\ 4 & -1 \end{bmatrix}$.

We need to compute the following.

$$\begin{bmatrix} 6 & 3 \\ -5 & -1 \end{bmatrix} - \begin{bmatrix} 9 & -3 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 6-9 & 3-(-3) \\ -5-4 & -1-(-1) \end{bmatrix}$$ Subtract corresponding elements

$$= \begin{bmatrix} -3 & 6 \\ -9 & 0 \end{bmatrix}$$ Simplify

Hence the result is $\boxed{\begin{bmatrix} -3 & 6 \\ -9 & 0 \end{bmatrix}}$.

## Answer 83e.

Two matrices can be multiplied when the number of columns of first matrix is equal to the number of rows of the second matrix. The dimensions of the two matrices are $2 \times 2$.
This means that the product of the resulting matrix is $2 \times 2$.

***STEP 1*** Multiply the elements in the first row of the first matrix by the elements in the first column of the second matrix. Add the products and put the result in the first row, first column of the product.

$$\begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} 2(0)+(-1)(3) & \\ & \end{bmatrix}$$

***STEP 2*** Multiply the elements in the first row of the first matrix by the elements in the second column of the second matrix. Add the products and put the result in the first row, second column of the product.

$$\begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} 2(0)+(-1)(3) & 2(4)+(-1)(-5) \\ & \end{bmatrix}$$

***STEP 3*** Multiply the elements in the second row of the first matrix by the elements in the first column of the second matrix. Add the products and put the result in the second row, first column of the product.

$$\begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} 2(0)+(-1)(2) & 2(4)+(-1)(-5) \\ 0(0)+4(3) & \end{bmatrix}$$

***STEP 4*** Multiply the elements in the second row of the first matrix by the elements in the second column of the second matrix. Add the products and put the result in the second row, second column of the product.

$$\begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} 2(0)+(-1)(3) & 2(4)+(-1)(-5) \\ 0(0)+4(3) & 0(4)+4(-5) \end{bmatrix}$$

***STEP 5*** Simplify the elements in the matrix.

$$\begin{bmatrix} 2(0)+(-1)(3) & 2(4)+(-1)(-5) \\ 0(0)+4(3) & 0(4)+4(-5) \end{bmatrix} = \begin{bmatrix} -3 & 13 \\ 12 & -20 \end{bmatrix}$$

Therefore, the resulting matrix is $\begin{bmatrix} -3 & 13 \\ 12 & -20 \end{bmatrix}$.

## Answer 84e.

Consider $\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} -3 \\ 2 \end{bmatrix}$.

We need to compute the following.

$$\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1\times(-3)+0\times(2) \\ 4\times(-3)+(-1)\times 2 \end{bmatrix}$$ Using the rule of product of two matrices

$$= \begin{bmatrix} -3+0 \\ -12-2 \end{bmatrix}$$ Simplify

$$= \begin{bmatrix} -3 \\ -14 \end{bmatrix}$$

Hence the result is $\boxed{\begin{bmatrix} -3 \\ -14 \end{bmatrix}}$.

## Answer 85e.

The left side of the expression has a common factor 3. Factor out 3 from the left side.

$3(x^2 - x - 12) = 0$

Divide both the sides by 3 and simplify.

$$\frac{3}{3}\left(x^2 - x - 12\right) = \frac{0}{3}$$
$$x^2 - x - 12 = 0$$

The expression on the left side of the equation is of the form $x^2 + bx + c$. We have to find two numbers such that their sum is $-1$ and product is $-12$. Two such numbers as $-4$ and 3. Thus,

$x^2 - x - 12 = (x - 4)(x + 3)$.

Rewrite the equation.

$(x - 4)(x + 3) = 0$

Apply Zero product property. According to this property, if the product of two expressions is zero, then one or both the expression is equal to zero.

Thus,

either $x - 4 = 0$ or $x + 3 = 0$.

Add 4 to both the sides of the first equation, and subtract 3 from both the sides of the second equation.

$x - 4 + 4 = 0 + 4$ or $x + 3 - 3 = 0 - 3$

$x = 4$ or $x = -3$

Therefore, the solutions for the equation are $-3$, and 4.

## Answer 86e.

Consider the equation : $2x^2 - 9x + 4 = 0$.
We need to solve the quadratic equation $2x^2 - 9x + 4 = 0$.

| | |
|---|---|
| $2x^2 - 9x + 4 = 0$ | Write original equation |
| $2x^2 - 8x - x + 4 = 0$ | Splitting the equation to write as factors |
| $(x-4)(2x-1) = 0$ | Factorise |
| $(x-4) = 0$ or $(2x-1) = 0$ | Zero product propert |
| $x = 4$ or $x = \frac{1}{2}$ | Solve for $x$ |

Hence the solution of the equation $2x^2 - 9x + 4 = 0$ is $\boxed{\frac{1}{2} \text{ and } 4}$.

## Answer 87e.

Divide both sides of the equation by 6.

$$\frac{6}{6}x^2 = \frac{96}{6}$$

$$x^2 = 16$$

Take square root of both the sides.

$$x = \sqrt{16}$$

$$= \pm 4$$

Therefore, the solutions for the equation are 4 and –4.

## Answer 88e.

Consider the equation : $14x^2 = 91$.
Solve the quadratic equation $14x^2 = 91$.

| | |
|---|---|
| $14x^2 = 91$ | Write original equation |
| $x^2 = \frac{91}{14}$ | Divide both sides with 14 |
| $x = \sqrt{\frac{91}{14}}$ | Squaring on both sides |
| $= \pm\sqrt{\frac{7 \times 13}{7 \times 2}}$ | Write as factors |

Continue the above steps,

$$= \pm\sqrt{\frac{13}{2}}$$ Simplify

$$= \pm\frac{\sqrt{13}}{\sqrt{2}}$$ Quotient property of square roots : $a > 0, b > 0$ then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Therefore, $x = \pm\frac{\sqrt{13}}{\sqrt{2}}$ are the roots of the given quadratic equation.

Hence the solution is $\boxed{x = \pm\frac{\sqrt{13}}{\sqrt{2}}}$.

## Answer 89e.

Add 8 to both sides of the equation.

$$2x^2 - 8 + 8 = 42 + 8$$

$$2x^2 = 50$$

Divide both the sides by 2.

$$\frac{2}{2}x^2 = \frac{50}{2}$$

$$x^2 = 25$$

Take square root of both the sides.

$$x = \sqrt{25}$$

$$= \pm 5$$

Therefore, the solutions for the expression are 5 and $-5$.

## Answer 90e.

Consider the equation : $3x^2 + 13 = 121$.

To find the value of x, Solve the equation $3x^2 + 13 = 121$.

$$3x^2 + 13 = 121$$ Write original equation

$$3x^2 + 13 - 13 = 121 - 13$$ Add -13 on both sides

$$3x^2 = 108$$ Simplify

$$x^2 = \frac{108}{3}$$ Divide with 3 on both sides

$$x^2 = 36$$

$$x = \pm 6$$ Taking square root on both sides

Therefore, $x = \pm 6$ are the roots of the given quadratic equation.

Hence the solution of the equation $3x^2 + 13 = 121$ is $\boxed{\pm 6}$.