Some Applications of Trigonometry

MATHEMATICS Olympiad Comprehensive Book

QUESTIONS

1. **If sin α + cos α = $\sqrt{3}$, then the value of tan α + cot α is equal to_____**

(a) 1 (b) 3

(c) $\sqrt{3}$ (d) 2

(e) None of these

2. **If $\text{Sin}\theta - \text{Cos}\theta = 0$, then the value of $\text{Cosec}^2\theta - \text{Sec}^2\theta$ is equal to _____**

(a) 1 (b) 2

(c) $\frac{1}{2}$ (d) 0

(e) None of these

3. **If PQR is an isosceles right angled triangle in which $\angle PRQ = 90°$, then the value of $Sec(R-Q)+CosP$ is equal to _____**

(a) 3 Sin P (b) 2 Sin P

(c) 2 Cos P (d) Sec P

(e) None of these

4. **If Sin $(\alpha+\beta)$ = 1 then Cos$(\alpha-\beta)$ can be reduced to _____**

(a) Sin 2β (b) Cos 2β

(c) Sin α (d) Cos β

(e) None of these

5. **Find the value of the expression given below:**

$$\frac{1}{8}\text{cosec}^2(52° - \theta) - \frac{1}{4}\tan^2(38° + \theta) + \frac{7}{8}\sec^2(38° + \theta) - \frac{3}{4}\cot^2(52° - \theta)$$

(a) 0 (b) 1

(c) -1 (d) 2

(e) None of these

6. **If cos A + $\cos^2$ A = 0 then the value of the expression $\sin^2$A − $\sin^4$A is _____**

(a) 0 (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 1

(e) None of these

7. **Which one among the following is incorrect statement for $0° \le \theta < 90°$?**

(a) The Value of tan θ increases as θ increases.

(b) The Value of tan θ increases faster than sin θ as θ increases.

(c) The Value of $\sin\theta + \cos\theta$ is always greater than 1.

(d) The Value of $\cos\theta$ increases as θ decreases.

(e) None of these

8. Find the value of

(a) 0 (b) 1

(c) 2 (d) 3

(e) None of these

9. Simplify $\tan\frac{\pi}{18}.\tan\frac{\pi}{9}.\tan\frac{\pi}{4}.\tan\frac{4\pi}{9}.\tan\frac{7\pi}{18}$

(a) 0 (b) 1

(c) 2 (d) $\tan\frac{\pi}{3}$

(e) None of these

10. If Sin 2α = 2 Sin α Cos α and Sin 40° = m, then the value of Cos 80°, Cos 160°, is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$

(c) $\frac{-1}{8}$ (d) $\frac{-1}{4}$

(e) None of these

11. Find the value of $\text{Cos}^2 0^\circ + \text{Cos}^2 5^\circ + \text{Cos}^2 10^\circ + \text{Cos}^2 15^\circ + \_\_\_\_ + \text{Cos}^2 90^\circ$

(a) 5 (b) 7

(c) 10 (d) 9

(e) None of these

12. The length of the shadow of a tower decreases by 30 m, when sun ray's inclination increased from 45° to 60°. Find the height of the tower (in m).

(a) 50.35 (b) 60.95

(c) 70.95 (d) 55

(e) None of these

13. Find the value of tan 15° by using identity tan 2θ = ————

(a) $2+\sqrt{3}$ (b) $2-\sqrt{3}$

(c) $\sqrt{3}+1$ (d) $\sqrt{3}-1$

(e) None of these

14. $\sqrt{-4+\sqrt{8+16\sec^4\theta+\cos^4\theta}} =$ _____

(a) $\cos\theta+\tan\theta$ (b) $\sec\theta+\cos\theta$

(c) $2\sec\theta-\cos\theta$ (d) $2\cos\theta+\tan\theta$

(e) None of these

15. Identify the correct statement among the following:

(a) $\text{Sin } 1° > 1$ (b) $\text{Sin } 1° > \text{Sin } 1$

(c) $\text{Sin } 1° < \text{Sin } 1$ (d) $\text{Sin } 1° = \text{Sin } 1$

(e) None of these

16. If $Cot^2\theta = a^2 - 1$, then the value of Cosec θ + Cot^3 θ Sec θ is _____

(a) $\left(a^2 - 1\right)^{3/2}$ (b) $a^2 - 1$

(c) a^3 (d) a^2

(e) None of these

17. If m Cot θ + n Cosec θ = a and n Cot θ + m Cosec θ = b, then $a^2 - b^2$ is equal to _____

(a) $n^2 - m^2$ (b) $m^2 + n^2$

(c) $m^2 - n^2$ (d) $(m - n)^2$

(e) None of these

18. In an acute angled triangle PQR, if $\sqrt{2}$ sin (P + Q - R) = 1 and tan (Q + R - P) = $\sqrt{3}$, find the values of P, Q and R.

(a) $P = 60°,\ Q = 52\frac{1^\circ}{2},\ R = 67\frac{1^\circ}{2}$ (b) $P = 52\frac{1^\circ}{2},\ Q = 67\frac{1^\circ}{2},\ R = 60°$

(c) $P = 37\frac{1^\circ}{2},\ Q = 90°,\ R = 52\frac{1^\circ}{2}$ (d) $P = 90°,\ Q = 37\frac{1^\circ}{2},\ R = 52\frac{1^\circ}{2}$

(e) None of these

19. $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} =$ _____

(a) $\dfrac{1 + \sin\theta}{\cos\theta}$ (b) $\dfrac{1 - \sin\theta}{\cos\theta}$

(c) $1 + \tan\theta$ (d) $\sec\theta + \text{cosec}\,\theta$

(e) None of these

20. If $\text{Cos}\theta - \text{Sin}\theta = \sqrt{2}\sin\theta$, then $\text{Cos}\theta + \text{Sin}\theta$ is equal to _____

(a) $\text{Cos}\,\theta$ (b) $2\,\text{Cos}\,\theta$

(c) $\sqrt{2}\,\text{Cos}\,\theta$ (d) $2\sqrt{2}\,\text{COS}\,\theta$

(e) None of these

21. If $\cot\alpha + \tan\alpha = m$ and $\sec\alpha - \cos\alpha = n$, then $\left(m^2 n\right)^{2/3} - \left(m\,n^2\right)^{2/3}$ is equal to _____

(a) 0 (b) 1

(c) 2 (d) $\sec\theta + \cos\theta$

(e) None of these

22. **If** $\text{Cosec}\,\alpha - \text{Sin}\,\alpha = a$ **and** $\text{Sec}\,\alpha - \text{Cos}\,\alpha = b$, **then** $\left(\mathbf{a^2 + b^2 + 3}\right)$ **is equal to** _____

(a) $\frac{1}{ab}$ (b) $a^2 + b^2$

(c) $\frac{1}{a^2 + b^2}$ (d) $a^2b^2 + 1$

(e) None of these

23. **In a** Δ **PQR, Sin** $\left(\frac{\mathbf{P+R}}{\mathbf{2}}\right)$ **is equal to ——**

(a) $-Cos\frac{Q}{2}$ (b) $Cos\frac{Q}{2}$

(c) $Sin\frac{Q}{2}$ (d) $-Sin\frac{Q}{2}$

(e) None of these

24. **If tan 82° = K, then** $\frac{\mathbf{tan164^\circ + cot8^\circ}}{\mathbf{K + cot106^\circ}}$ **is equal to————**

(a) K (b) $K^2 + 1$

(c) $\frac{k^2 - 1}{k^2 + 1}$ (d) 1

(e) None of these

25. **If tan 18 = K then** $\frac{\mathbf{tan162^\circ + tan72^\circ}}{\mathbf{k + tan108^\circ}}$ **is equal to**

(a) $\frac{k^2 + 1}{k^2 - 1}$ (b) $\frac{k^2 - 1}{k^2 + 1}$

(c) 1 (d) -1

(e) None of these

26. **If** $m = k\,\text{Sin}\phi\,\text{Cos}\theta$, $n = k\,\text{Sin}\phi\,\text{Sin}\theta$ **and** $r = k\,\text{Cos}\phi$ **then** $m^2 + n^2 + r^2 =$ _____

(a) 1 (b) 2

(c) k^2 (d) $\frac{1}{k^2}$

(e) None of these

27. **If** $\frac{\mathbf{sin\,P}}{\mathbf{sin\,Q}}$ **= K and** $\frac{\mathbf{tan\,P}}{\mathbf{tan\,Q}}$ **= I then** $\mathbf{sin^2\,p}$ **is equal to** _____

(a) $\frac{I^2 - k^2}{I}$ (b) $\frac{I^2 - k^2}{k}$

(c) $\frac{k^2 - I^2}{I}$ (d) $\frac{k^2 - 1}{I^2 - 1}$

(e) None of these

28. **If** $k\sec\theta + I\tan\theta + m = 0$

and $x\sec\theta + y\tan\theta + z = 0$

then $(Iz - ym)^2 - (xm - kz)^2 =$ _____

(a) $(ky - Ix)^2$ (b) $(kx - Iy)^2$

(c) $(Ix + ky)^2$ (d) $(kz - Ix)^2$

(e) None of these

29. **A spherical balloon of radius 3cm subtends an angle of** $60°$ **at the eye of the observer. If the angle of elevation of its centre is** $45°$**, then the height of the centre of the balloon is __**

(a) $6\sqrt{2}$ cm (b) $3\sqrt{2}$ cm

(c) 9cm (d) 3cm

(e) None of these

30. **If p Cos** θ **- q Sin** θ **=** $\sqrt{p^2 + q^2 - r^2}$ **then the value of** $p\,Sin\theta + q\,Cos\theta$ **is equal to**

(a) $\sqrt{p^2 + q^2 + r^2}$ (b) $\sqrt{p^2 - q^2 + r^2}$

(c) $\sqrt{p^2 + q^2}$ (d) r

(e) None of these

31. **The angle of elevation of a jet plane from a point Q on the ground is 60° . After a flight of 20 seconds, the angle of elevation changes to 45° . If the Jet plane is flying at a constant height of 3000** $\sqrt{3}$ **m, then find the speed of the jet plane.**

(a) 295.18 km/hr (b) 395.28 km/hr

(c) 255.20 km/hr (d) 345.38 km/hr

(e) None of these

32. **The angle of elevation of a cloud from a point 90m above a lake is** $30°$ **and the angle of depression of the reflection of cloud in the lake is 60° . Find the height of the cloud.**

(a) 90m (b) 120m

(c) 180m (d) 45m

(e) None of these

33. **The angle of elevation of a cliff from a fixed point is 45° . On up a distance of 300 m towards the top of the cliff at an angle of** $30°$**, it is that the angle of elevation is 60°. The height of the cliff is _____**

(a) 93.42 m (b) 97.12 m

(c) 102.18 m (d) 112.39 m

(e) None of these

34. **A ladder rests against a wall at an angle θ to the horizontal. Its foot is away from the wall through a distance 'm', so that it slides a distance 'n' down the wall making an angle ϕ with the horizontal then $\frac{n}{m}$ is equal to**

(a) $\frac{\cos\theta - \cos\phi}{\sin\phi - \sin\theta}$ (b) $\frac{\sin\phi - \sin\theta}{\cos\theta - \cos\phi}$

(c) $\frac{\cos\theta + \cos\phi}{\sin\phi + \sin\theta}$ (d) $\frac{\sin\phi + \sin\theta}{\cos\theta + \cos\phi}$

(e) None of these

35. **If 15 Sin A + 8 Cos A = 8 then find the value of 8 Sin A - 15 Cos A.**

(a) 8 (b) 12

(c) 15 (d) 64

(e) None of these

36. **The lower window of a house is at a height of 3 m above the ground and its upper window is 5m vertically above the lower window. At certain instant it is found that the angles of elevation of a balloon from these windows are $60°$ and $30°$ respectively. Find the height of the balloon above the ground.**

(a) 8.5 m (b) 10.5 m

(c) 7.5 m (d) 8 m

(e) None of these

37. **Simplify $\text{Sin}^{12}\theta$ - $\text{Cos}^{12}\theta$.**

(a) $\text{Cos}2\theta(1 - \text{Sin}^2\theta\text{Cos}^2\theta)\ (1 - 3\text{Sin}^2\theta\text{Cos}^2\theta)$

(b) $\text{Cos}^2\theta\ (1 - \text{Sin}^2\theta\text{Cos}^2\theta)\ (1 - 2\ \text{Sin}^2\theta\text{Cos}^2\theta)$

(c) $\text{Cos}2\theta(1 - 2\ \text{Sin}^2\theta\text{Cos}^2\theta)(1 - 3\ \text{Sin}^2\theta\text{Cos}^2\theta)$

(d) $\text{Sin}^2\theta(1 - 2\ \text{Sin}^2\theta\text{Cos}^2\theta)\ (1 - 3\ \text{Sin}^2\theta\text{Cos}^2\theta)$

(e) None of these

38. **If $4\sqrt{2}\,\text{Sin}\theta - 2\sqrt{6}\,\text{Cos}\theta = \sqrt{6}$ then value of θ can be _____**

(a) $0°$ (b) $30°$

(c) $45°$ (d) $60°$

(e) None of these

39. **If $\text{cosec}\,\theta$ + $\cot\theta$ = 2 where $0° < \theta < 90°$ then the value of $\tan\theta$ + $\sec\theta$ is equal to _____**

(a) 0 (b) 1

(c) 2 (d) 3

(e) None of these

40. **The angle of elevation of the top of a tower from a point A due south of the tower is and from a point B due east of the tower is 45°. If AB = 160 metre then the height of the tower is _____**

(a) $\frac{160}{\sqrt{3}}$ m

(b) $75\sqrt{3}$ m

(c) $80\sqrt{3}$ m

(d) 80 m

(e) None of these

ANSWER - KEY				
1. (a)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (b)
6. (a)	**7.** (c)	**8.** (b)	**9.** (b)	**10.** (c)
11. (c)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (c)
16. (c)	**17.** (a)	**18.** (a)	**19.** (a)	**20.** (c)
21. (b)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (d)
26. (c)	**27.** (a)	**28.** (a)	**29.** (b)	**30.** (d)
31. (b)	**32.** (c)	**33.** (b)	**34.** (b)	**35.** (c)
36. (b)	**37.** (a)	**38.** (d)	**39.** (d)	**40.** (d)