

## 30. Gauss's Law

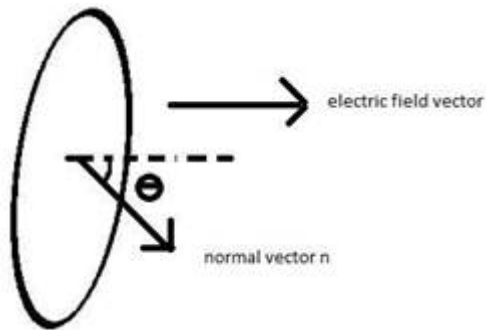
### Short Answer

#### 1. Question

A small plane area is rotated in an electric field. In which orientation of the area is the flux of the electric field through the area maximum? In which orientation is it zero?

#### Answer

The flux of electric field through an area is defined as the number of field lines that pass through that area in the direction of the electric field.



In mathematical terms,

$$\text{Flux, } \varphi = \int \vec{E} \cdot d\vec{s}$$

Where,

$\vec{E}$  = electric field vector,

$d\vec{s}$  = small area element.

Now,  $d\vec{s} = n da$ , where  $\mathbf{n}$  is the unit normal in the direction of the area, and  $da$  is a small area element.

By the law of dot product in vector calculus,

$$\vec{E} \cdot d\vec{s} = |\vec{E}| |d\vec{s}| \cos \theta,$$

Where  $\theta$  is the angle between the normal to the area and the electric field.

$|\vec{E}|$  = magnitude of electric field vector,

$|d\vec{s}|$  = magnitude of small area element.

Now, the flux is maximum when  $\cos \theta = 0$  or  $180^\circ$ , since  $\cos 0$  or  $\cos 180^\circ = +1$  or  $-1$ .

In this case, the flux will be  $|\mathbf{E}||\mathbf{ds}|$ . In all other cases, the flux will be less than this.

The flux is minimum when  $\cos 90^\circ$ , since  $\cos 90^\circ = 0$ .

$$\Phi = |\mathbf{E}||\mathbf{ds}| \cos(90^\circ) = 0$$

Thus, when the unit normal vector perpendicular to the direction of the electric field the flux is **ZERO**.

## 2. Question

A circular ring of radius  $r$  made of a non-conducting material is placed with its axis parallel to a uniform electric field. The ring is rotated about a diameter through  $180^\circ$ . Does the flux of electric field change? If yes, does it decrease or increase?

### Answer

No, the flux will remain the same.

$$\text{We know, Flux} = \Phi = \int \mathbf{E} \cdot \mathbf{ds}$$

Where

$\mathbf{E}$  = electric field vector,  $\mathbf{ds}$  = small area element.

Now,  $\mathbf{ds} = \mathbf{n} \cdot da$ ,

where  $\mathbf{n}$  is the unit normal in the direction of the area, and  $da$  is a small area element.

By the law of dot product in vector calculus,

$$\mathbf{E} \cdot \mathbf{ds} = |\mathbf{E}||\mathbf{ds}| \cos \theta$$

Where

$\theta$  is the angle between the normal to the area and the electric field.

$|\mathbf{E}|$  = magnitude of electric field vector,

$|\mathbf{ds}|$  = magnitude of small area element.

In the first case, since the axis of the ring is parallel to the electric field, the angle  $\theta$  between the normal to the surface is  $0^\circ$ , since the normal is parallel to the axis.

When  $\theta = 180^\circ$

$$\Phi = |\mathbf{E}||\mathbf{ds}| \cos(180^\circ) = |\mathbf{E}||\mathbf{ds}| (-1) = -|\mathbf{E}||\mathbf{ds}|$$

Therefore, the flux is just  $|\mathbf{E}||\mathbf{ds}|$ . When it is rotated about its diameter by  $180^\circ$ , the normal just rotates by  $180^\circ$ , and thus the angle changes to  $180^\circ$ . The flux becomes negative, but its value remains the same.

## 3. Question

A charge Q is uniformly distributed on a thin spherical shell. What is the field at the centre of the shell? If a point charge is brought close to the shell, will the field at the centre change? Do your answer depend on whether the shell is conducting or nonconducting?

### Answer

If we consider a thin spherical shell, then all its charge is concentrated in the centre. According to Gauss' law, the flux through a closed surface is  $\frac{1}{\epsilon_0}$  time the charge enclosed.

$$\text{flux through the closed surface, } \Phi = \int E \cdot ds = \frac{q}{\epsilon_0},$$

where

$\epsilon_0$  is the electric permittivity of vacuum,

$E$  = electric field,

$ds$  = surface element,

$q$  = charge enclosed.

Now in this case since all the charge is bound to the surface, the charge enclosed by the sphere (at the centre) is 0.

Hence,

$$\Phi = \int E \cdot ds = \frac{0}{\epsilon_0} = 0 \text{ and hence the electric field is also 0.}$$

If a point charge is brought close to the shell, the charge enclosed the sphere will still be 0. Hence the field inside the sphere will still be 0.

The flux will remain 0 if the shell is conducting, since in a conductor, the charge always remains on the surface.

Thus, for a non-conducting shell, there might be some charge induced inside, due to which the electric field might change.

### 4. Question

A spherical shell made of plastic, contains a charge Q distributed uniformly over its surface. What is the electric field inside the shell? If the shell is hammered to de-shape it without altering the charge, will the field inside be changed? What happens if the shell is made of a metal?

### Answer

The charge Q is uniformly distributed over its surface, according to Gauss' law,

$$\Phi = \int E \cdot ds = \frac{q}{\epsilon_0},$$

Where

$E$  = electric field,

$\Phi$  = electric flux flowing out through the surface

$ds$  = surface area element,  $q$  = charge enclosed,  $\epsilon_0$  = electric permittivity in vacuum.

Here, the charge enclosed inside the shell is 0, since the electric charge on the shell is accumulated on the surface of the shell and so the electric field inside the shell will also be 0 since  $q$  in Gauss' equation is 0.

Even if the shell is hammered to reshape it, the charge is not altered, it still there is no charge inside in the center of the shell. Hence, the electric field will remain zero inside the shell even after the reshaping the shell.

Even if the charge is made of metal, the field inside will remain 0 since the charge remains on the surface.

## 5. Question

A point charge  $q$  is placed in a cavity in a metal block. If a charge  $Q$  is brought outside the metal, will the charge  $q$  feel an electric force?

### Answer

Metals are conductors of electric charge, the charge placed in metal will always collect on the surface of the metal. However, it has an electrostatic shielding effect. That is, any charge in a cavity inside the metal block will be completely isolated from the outside world. This is because, if we take a Gaussian surface surrounding the cavity, the charge enclosed is zero. By Gauss' law,

$$\Phi = \int E \cdot ds = \frac{q}{\epsilon_0},$$

Where

$\Phi$  = electric flux,  $E$  = electric field,  $ds$  = area element,  $q$  = charge enclosed,  $\epsilon_0$  = electric permittivity of vacuum.

Here,  $q = 0$  since the charge inside a conductor is 0. This justifies why the cavity in an external electric field will be completely shielded or isolated.

There will be no lines of force that can penetrate into the cavity. Therefore, even if a charge  $Q$  is brought outside the metal, the charge  $q$  inside the cavity will experience no force. Since charge distributed on the surface hence total charge will be on the surface is  $q+Q$ .

## 6. Question

A rubber balloon is given a charge  $Q$  distributed uniformly over its surface. Is the field inside the balloon zero everywhere if the balloon does not have a spherical surface?

### Answer

If the balloon does not have a spherical surface, the charge enclosed by the surface still remains 0 since the charge  $Q$  is distributed only over its surface. Hence, by Gauss' law,

$$\Phi = E \cdot ds = \frac{q}{\epsilon_0},$$

where  $\epsilon_0$  is the electric permittivity of vacuum,

$\Phi$  = flux through closed surface,  $E$  = electric field,  $ds$  = surface element,  $q$  = charge enclosed.

Here,  $q$ (charge enclosed) = 0. Hence, the flux is 0, which give us that the electric field is also 0 everywhere.

## 7. Question

It is said that any charge given to a conductor comes to its surface. Should all the protons come to the surface? Should all the electrons come to the surface? Should all the free electrons come to the surface?

### Answer

A property of conductors is that all the free electrons come to its surface. We can explain this in the following way. If there was any charge inside the conductor, it would create an internal electric field. To compensate for that, an equal electric field would be created in the opposite direction to cancel out its effect. In this way, the inside of the conductor is absent of any free charges.

These are the free electrons that are responsible for all electrical phenomenon, so, if a conductor is a negative charge the free electron come to the surface of the conductor. If the conductor is given a positive charge, electron move away from the surface and leave a positive charge on the surface of the conductor.

## Objective I

### 1. Question

A charge  $Q$  is uniformly distributed over a large plastic plate. The electric field at a point  $P$  close to the centre of the plate is  $10 \text{ V m}^{-1}$ . If the plastic plate is replaced by a copper plate of the same geometrical dimensions and carrying the same charge  $Q$ , the electric field at the point  $P$  will become

- A. zero
- B.  $5 \text{ V m}^{-1}$
- C.  $10 \text{ V m}^{-1}$
- D.  $20 \text{ V m}^{-1}$

### Answer

A large plastic plate and copper plate is an example of infinite long plates.

As we know,

The electric field due to infinite long sheet is  $E = \frac{\sigma}{2\epsilon_0}$

Where

$\sigma$  is the surface charge density of the thin conducting sheet

$\epsilon_0$  is the permittivity of the free space

The electric field due to the plastic plate is  $10 \text{ Vm}^{-1}$

Thus, the electric field for the copper would be same as that of the plastic plate.

The electric field of the copper plate is  $10 \text{ Vm}^{-1}$

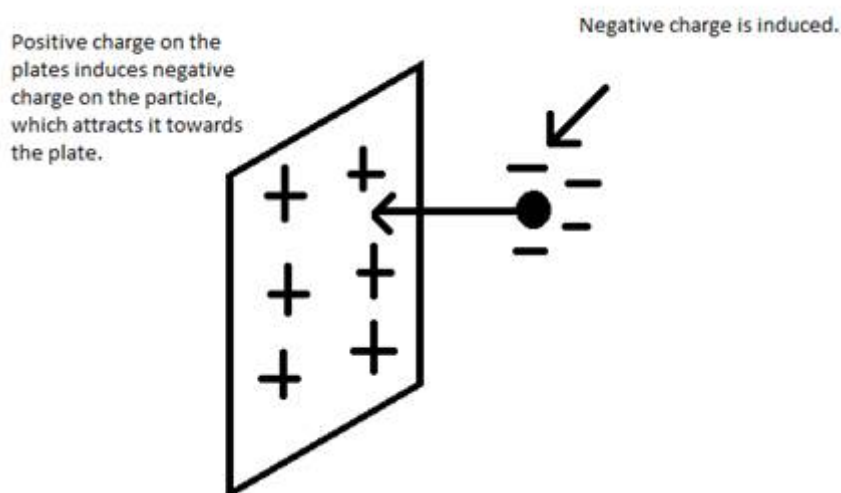
## 2. Question

A metallic particle having no net charge is placed near a finite metal plate carrying a positive charge. The electric force on the particle will be

- A. towards the plate
- B. away from the plate
- C. parallel to the plate
- D. zero

## Answer

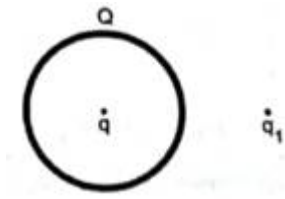
The metal plate carries a positive charge; it will induce an equal and opposite negative charge on the metallic particle following the law of electric induction. Hence, since electric field lines move from negative charge to positive charge, the force will pull the particle towards the plate.



Thus, option A is the correct option.

## 3. Question

A thin, metallic spherical shell contains a charge  $Q$  on it. A point charge  $q$  is placed at the center of the shell, and another charge  $q_1$  is placed outside it as shown in the figure. All three charges are positive. The force on the charge at the centre is:



- A. towards left
- B. towards right
- C. upward
- D. zero

**Answer**

Charge  $q_1$  will only affect the outside charge  $Q$ . However, a charge can only induce charge on the surface of the conductor, but not on any charge inside it. Hence, there will be no force on the charge inside the conductor,  $q$ .

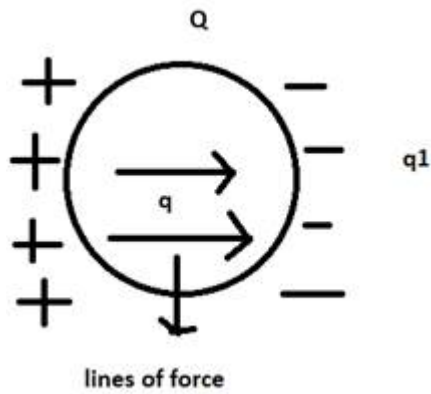
**4. Question**

Consider the situation of the previous problem. The force on the central charge due to the shell is

- A. towards left
- B. towards right
- C. upward
- D. zero.

**Answer**

We know that electric field lines originate from a positive charge and terminate towards a negative charge. Now, the charge  $Q$  outside will try to nullify the field lines that emerge from charge  $q_1$ , and hence, negative charge will be induced on the face nearer to  $q_1$ . This will induce positive charge on the farther face of the sphere. Hence, a force acts on the central charge from left to right, that is, from negative to positive.



### 5. Question

Electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 25 V m. The flux over a concentric sphere of radius 20 cm will be

- A. 25 V m
- B. 50 V m
- C. 100 V m
- D. 200 V m.

### Answer

Gauss' law states that the electric flux through a closed surface is equal to  $1/\epsilon_0$  times the total charge enclosed by the surface.

$$\Phi = \int E \cdot ds = \frac{q}{\epsilon_0},$$

where

$\Phi$  = electric flux

E = electric field

ds = area element

q = charge enclosed

$\epsilon_0$  = electric permittivity of vacuum

Hence, the flux depends on the charge enclosed.

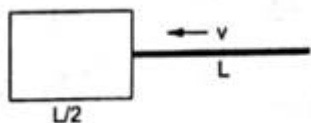
In this case, even though the volume of the sphere enclosing the charge is increasing, the net charge enclosed inside it is remaining the same.

Hence, the electric field will remain the same = 25 V m. (Option a)

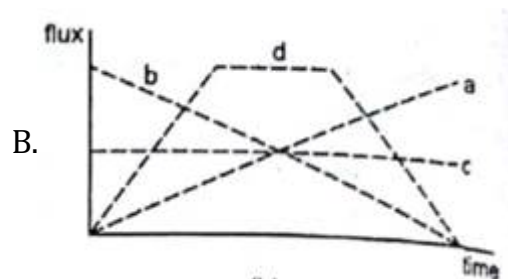
### 7. Question



Figure shows an imaginary cube of edge  $L/2$ . A uniformly charged rod of length  $L$  moves towards left at a small but constant speed  $v$ . At  $t = 0$ , the left end just touches the centre of the face of the cube opposite it. Which of the graphs shown in figure represents the flux of the electric field through the cube as the rod goes through it?



A.



**Answer**

Graph d

According to Gauss' law,

We know, Electric flux =  $\mathbf{E} \cdot d\mathbf{s} = q/\epsilon_0$ ,

Where

$q$  = total charge enclosed by the surface,

$\epsilon_0$  = electric permittivity of vacuum.

Now at  $t=0$ , the left end just touches the centre of the face of the cube opposite the left face.

Initially, charge inside the cube = 0.

Hence, the graph starts from the origin.

Now when the rod is introduced inside, the charge inside the cube slowly increases linearly, since the length of the rod going inside the cube increases, and  $q_{\text{inside}} = \lambda l$ , where  $\lambda$  = linear charge density of the rod, and  $l$  = length of rod inside the cube. This indicates the rise in the graph.

When a length of  $L/2$  is inside the cube, that is its maximum length is filled, then the charge inside the cube is maximum, which will be  $L/2$  times the total charge carried by the rod.

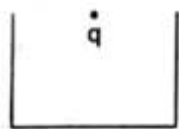
As long as the inside of the cube stays filled with a portion of length  $L/2$ , the flux remains constant, indicating the horizontal line in the graph.

Now when the length of the rod inside slowly keeps decreasing as the rod moves out of the, the charge inside it slowly decreases which also leads to a linear decrease in the flux.

Hence, the graph is completed with the straight line with negative slope until the flux becomes 0(rod is completely out of the cube).

### 8. Question

A charge  $q$  is placed at the centre of the open end of a cylindrical vessel. The flux of the electric field through the surface of the vessel is



- A. zero
- B.  $q/\epsilon_0$
- C.  $q/2\epsilon_0$
- D.  $2q/\epsilon_0$

### Answer

Gauss' law states the electric flux through an enclosed surface is  $1/\epsilon_0$  times the charge enclosed by the surface.

$$\Phi = \int E \cdot ds = \frac{q}{\epsilon_0}$$

where

$\Phi$  = electric flux,

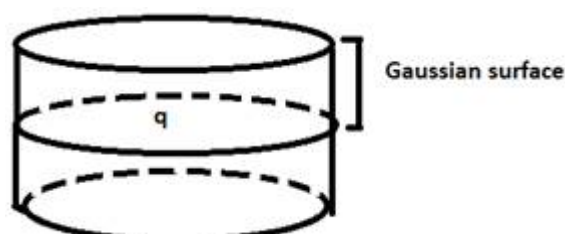
$E$  = electric field,

$ds$  = area element,

$q$  is the charged enclosed,

$\epsilon_0$  is the electric permittivity of vacuum.

In this case, the surface is not closed, since we are considering the open end of a cylindrical vessel. We construct another identical cylindrical vessel on top of it.



Now, the charge enclosed by two of the surfaces is  $q$ .

Hence, the charge enclosed by only of the surfaces will be  $q/2$ .

Therefore, the net flux will be half of the total =  $q/2\epsilon_0$ .

## Objective II

### 1. Question

Mark the correct options:

- A. Gauss's law is valid only for symmetrical charge distributions.
- B. Gauss's law is valid only for charges placed in vacuum.
- C. The electric field calculated by Gauss's law is the field due to the charges inside the Gaussian surface.
- D. The flux of the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface.

### Answer

Gauss' law states that the flux through a surface is equal to  $1/\epsilon$  times the total charge enclosed by the surface (if the surface enclosing is in a medium other than vacuum)

$$\Phi = \int E \cdot ds = \frac{q}{\epsilon_0},$$

where  $\epsilon$  is the electric permittivity of that medium.

Hence, the flux through the Gaussian surface will be due to charges enclosed by it (option d)

### 2. Question

A positive point charge  $Q$  is brought near an isolated metal cube.

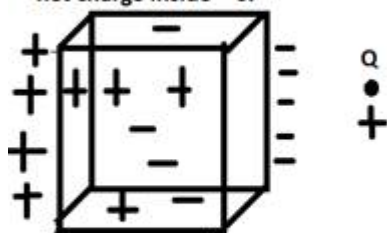
- A. The cube becomes negatively charged.
- B. The cube becomes positively charged.
- C. The interior becomes positively charged and the surface becomes negatively charged.
- D. The interior remains charge free and the surface gets non-uniform charge distribution.

### Answer

The face of the metal cube closest to the point charge, when brought near to it, had negative charge induced on it. This negative charge induces an equal and opposite positive charge on its immediate adjacent surface in the interior. In this way, the

faces of the cube induce charges in the interior such that the total charge in the interior becomes zero. The net charge is on the surface of the cube, and it has a non-uniform charge distribution, consisting of both positive and negative charges.

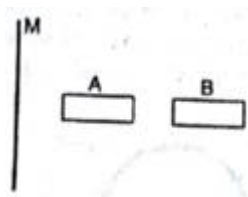
Equal amount of positive and negative charge on the inside of the cube neutralises it, making the net charge inside = 0.



The surviving charge remains on the surface, having a non-uniform distribution.

### 3. Question

A large non-conducting sheet M is given a uniform charge density. Two uncharged small metal rods A and B are placed near the sheet as shown in figure.



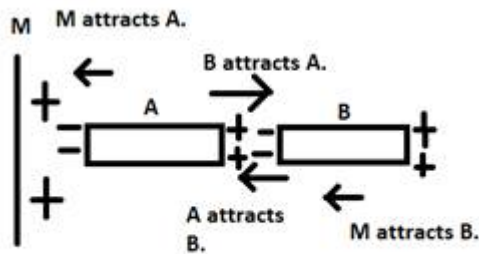
- A. M attracts A
- B. M attracts B
- C. A attracts B
- D. B attracts A

### Answer

The face of rod A which faces M will develop an induced charge on its surface, equal and opposite to the charge on M. Hence it will attract A(option a).

Now on the far end of A, charge equal and opposite to the nearer end of A will be induced, and this will induce charge opposite to M as before, on the nearer end of B. Hence, B will have charge of same polarity as that on the nearer end of A, and will be attracted by M(Option b).

A and B have opposite polarities of charges on the sides facing each other, and hence they will attract each other as well(Options c and d).



The attraction between M, A and B if M had positive charge.

#### 4. Question

If the flux of the electric field through a closed surface is zero.

- A. the electric field must be zero everywhere on the surface.
- B. the electric field may be zero everywhere on the surface.
- C. the charge inside the surface must be zero.
- D. the charge in the vicinity of the surface must be zero.

#### Answer

Gauss' law states that electric flux through an enclosed surface is equal to  $1/\epsilon_0$  times the charge enclosed by the surface.

$$\Phi = \int E \cdot ds = \frac{q}{\epsilon_0},$$

where,

$\Phi$  = electric flux,

ds = area element,

q = charge enclosed,

E = electric field,

$\epsilon_0$  = electric permittivity of vacuum

Now, this law does not include the flux which is located on the surface. So it may or may not be zero, even though the flux through the surface is zero(option b).

Since the flux through the surface is zero, by Gauss' law, the charge inside the surface must be zero(option c).

#### 5. Question

An electric dipole is placed at the centre of a sphere. Mark the correct options:

- A. The flux of the electric field through the sphere is zero.
- B. The electric field is zero at every point of the sphere.

C. The electric field is not zero anywhere on the sphere.

D. The electric field is zero on a circle on the sphere.

### Answer

A dipole consists of an equal positive and negative charge separated by a distance. Hence, the net charge due to a dipole is 0. Since the sphere encloses the dipole, by Gauss' law,

$$\Phi = \int E \cdot ds = \frac{q}{\epsilon_0}$$

where

$\Phi$  = electric flux,

ds = area element,

q = charge enclosed,

E = electric field,

$\epsilon$  = electric permittivity of vacuum.

q(enclosed charge) = 0. Hence, flux = 0(option a).

At any point on the sphere, the electric field is given by

$$E = \frac{p}{4\pi\epsilon_0 r^3} \sqrt{1 + \cos^2 \theta}, \text{ where}$$

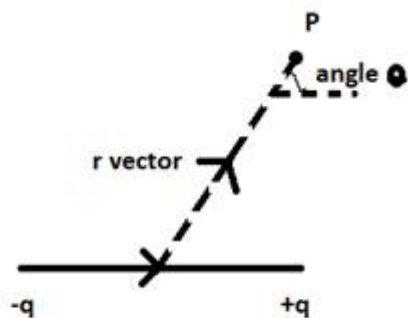
p = dipole moment

r = distance of the point from the centre of the dipole

E = electric field

$\theta$  = angle between the dipole and the line joining the dipole with that point

$\epsilon$  = electric permittivity of vacuum

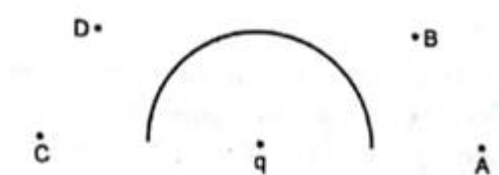


dipole with dipole moment p

Hence, the electric field anywhere on the sphere is not 0, since  $\cos^2\theta$  cannot be equal to -1(option c).

### 6. Question

Figure shows a charge  $q$  placed at the centre of a hemisphere. A second charge  $Q$  is placed at one of the positions A, B, C and D. In which position(s) of this second charge, the flux of the electric field through the hemisphere remains unchanged?



- A. A
- B. B
- C. C
- D. D

### Answer

As we know, the electric flux through a surface element  $ds$  is

$$\Phi = \vec{E} \cdot \Delta\vec{S} = |E||\Delta S \cos \theta$$

Where,

$\vec{E}$  is the electric field passing through the area

$\Delta S$  is the area element

$\theta$  is the angle between the area element and electric field

The point A and C lie in the straight line with the charge  $q$ ,

While the points B and D make an angle  $\theta$  with the area element,

Thus, Electric flux at point A and C is

$$\varphi = \vec{E} \cdot \Delta\vec{S} = |E||\Delta S| \cos \theta \quad (\theta=0^\circ \text{ for the point in the straight line})$$

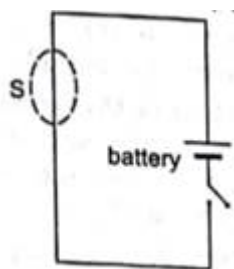
$$\varphi = \vec{E} \cdot \Delta\vec{S} = |E||\Delta S| \cos \theta = |E||\Delta S|$$

Hence, the flux of the electric field through the hemisphere remains unchanged when second charge is placed at point A and C.

### 7. Question

A closed surface  $S$  is constructed around a conducting wire connected to a battery and a switch. As the switch is closed, the free electrons in the wire start moving

along the wire. In any time interval, the number of electrons entering the closed surface S is equal to the number of electrons leaving it. On closing the switch, the flux of the electric field through the closed surface.



- A. is increased
- B. is decreased
- C. remains unchanged
- D. remains zero

**Answer**

At any interval of time, the net charge enclosed by the surface S is zero, since an equal number of electrons enter and leave the surface. So the charge enclosed is 0.

According to Gauss' law, electric flux through a closed surface is equal to  $1/\epsilon_0$  times the charge enclosed by the surface.

$$\Phi = \int E \cdot ds = \frac{q}{\epsilon_0},$$

Where

$\Phi$  = electric flux

E = electric field

ds = area element

q = charge enclosed.

$\epsilon_0$  = electric permittivity of vacuum

Here,

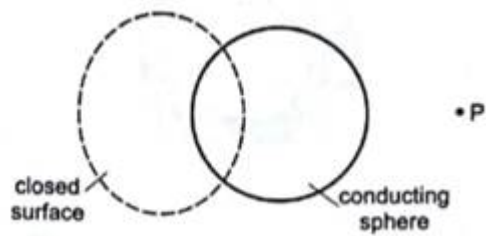
Q (total charge enclosed by the surface) = 0.

Hence, flux of electric field is zero and remains unchanged.

**8. Question**

Figure shows a closed surface which intersects a conducting sphere. If a positive charge is placed at point P, the flux of the electric field through the closed surface.





- A. will remain zero
- B. will become positive
- C. will become negative
- D. will become undefined.

**Answer**

The conducting sphere will have a negative charge induced on the surface which is closer to the point P. However, it will have equal positive charge induced on the farther end by electric induction. Hence, by Gauss' law, the flux through a closed surface is equal to  $1/\epsilon_0$  times the total charge enclosed by it.

$$\Phi = \int E \cdot ds = \frac{q}{\epsilon_0}$$

E = electric field,

q = charge enclosed.

The charge enclosed by the closed surface will not be zero, but will consist of the charge in the intersecting region of the sphere and the surface, which is positive. Hence, the flux of electric field will be positive.

**Exercises**

**1. Question**

The electric field in a region is given by  $\vec{E} = \frac{3}{5} E_0 \vec{i} + \frac{4}{5} E_0 \vec{j}$  with  $E_0 = 2.0 \times 10^3 \text{ N C}^{-1}$ .

Find the flux of this field through a rectangular surface of area  $0.2 \text{ m}^2$  parallel to the y-z plane.

**Answer**

Given:

Electric field :  $\frac{3}{5} E_0 i + \frac{4}{5} E_0 j$

$E_0 = 2.0 \times 10^3 \text{ N/C}$

Surface area of plane =  $0.2 \text{ m}^2$

The plane is parallel to y-z plane. The normal to this plane is parallel to x-axis. Therefore, area vector for this plane is given by  $0.2\text{m}^2\hat{i}$

$$\therefore \Delta S = 0.2\text{m}^2\hat{i}$$

We know that,

Flux of electric field through a surface of area  $\Delta S$  is given by dot product of electric field  $\vec{E}$  with surface area  $\Delta\vec{S}$

$$\phi = \vec{E} \cdot \Delta\vec{S}$$

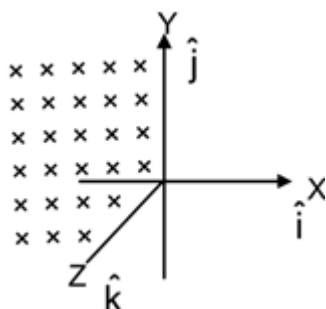
$$\Rightarrow \phi = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right) \cdot (0.2\hat{i})\text{Nm}^2/\text{C}$$

$$\Rightarrow \phi = \frac{3}{5} \times 0.2 \times E_0 \text{ Nm}^2/\text{C}$$

$$\Rightarrow \phi = \frac{3}{5} \times 0.2 \times 2 \times 10^3 \text{ Nm}^2/\text{C} \text{ (putting the value of } E_0\text{)}$$

$$\phi = 240 \text{ Nm}^2/\text{C}$$

**Therefore the flux of this electric field through the given plane surface is  $240\text{Nm}^2/\text{C}$ .**



## 2. Question

A charge  $Q$  is uniformly distributed over a rod of length  $\ell$ . Consider a hypothetical cube of edge  $\ell$  with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.

### Answer

Given:

Length of rod = edge of cube =  $l$

Portion of rod inside cube =  $l/2$

Total charge =  $Q$

Linear charge density of rod =  $Q/l$  of rod =  $\lambda$

We know that,

By Gauss's law, flux of net electric field through a closed surface equals the net charge enclosed by the surface divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0} \dots (i)$$

Where  $q_{in}$  is the net charge enclosed by the surface through which the flux is calculated.

$E \rightarrow$  =net electric field at the surface

$dS \rightarrow$  =area of differential surface element

Using gauss law flux through the cube is given by  $Q_{in}/\epsilon_0$  where

$Q_{in}$  is the charge enclosed by the cube

Charge enclosed by the cube is given by charge density  $\times$  length of rod inside cube

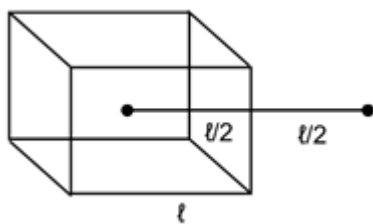
$$Q_{in} = \frac{Q}{l} \times \frac{l}{2}$$

$$\Rightarrow Q_{in} = \frac{Q}{2}$$

$\therefore$  by (i)

$$\text{Flux } \phi = \frac{Q}{2\epsilon_0}$$

**Therefore flux of electric field through the entire surface of cube is given by  $Q/2\epsilon_0$**

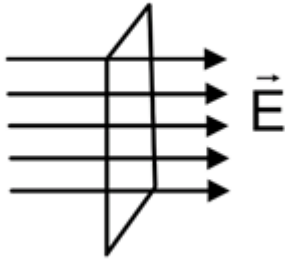


### 3. Question

Show that there can be no net charge in a region in which the electric field is uniform at all points.

#### Answer

Given the electric field is uniform, if we consider a plane perpendicular to electric field we can say that this plane is an **equipotential surface** (as equipotential surfaces are perpendicular to the electric field lines.)



Equipotential surface: A surface on which all points on the surface have the same electric potential.

That means by definition of potential difference (between two points) which is defined as work done in moving a unit test charge from one point to another point,

We can say that zero work is done in moving a unit test charge on this surface.

Let us assume that some charge is present in this region where electric field is uniform. Now due to this charge our test charge when introduced experiences a force (repulsive or attractive) due to which non-zero work needs to be done to move our test charge from one point to another.

And if non zero work is done that means the potential difference between these two points is finite (and not zero) and the surface can't be considered as equipotential.

This is a contradiction to the fact that the surface is equipotential.

**Therefore net charge in the region of uniform electric field is zero.**

#### 4. Question

The electric field in a region is given by  $\vec{E} = \frac{E_0 x}{l} \hat{i}$ . Find the charge contained inside a cubical volume bounded by the surfaces  $x = 0$ ,  $x = a$ ,  $y = 0$ ,  $y = a$ ,  $z = 0$  and  $z = a$ . Take  $E_0 = 5 \times 10^3 \text{ NC}^{-1}$ ,  $l = 2 \text{ cm}$  and  $a = 1 \text{ cm}$ .

#### Answer

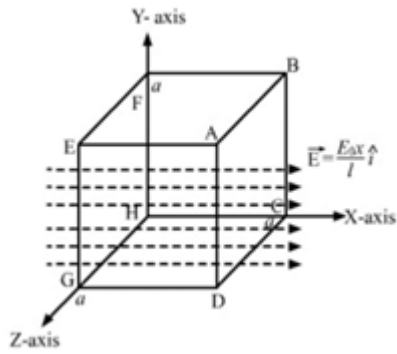
Given:

$$\vec{E} = \frac{E_0 x}{l} \hat{i}$$

$$E_0 = 5 \times 10^3 \text{ N/C}$$

$$l = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$

$$a = 1 \text{ cm} = 1 \times 10^{-2} \text{ m}$$



We know that,

Flux of electric field through a surface of area  $\Delta S$  is given by dot product of electric field  $\vec{E}$  with surface area  $\Delta \vec{S}$

$$\phi = \vec{E} \cdot \Delta \vec{S}$$

From the fig. we can see that electric field lines are parallel to the normal of the surfaces ABCD and EFGH therefore flux is nonzero through these surfaces, whereas for other faces of the cube electric field is perpendicular to normal of faces therefore flux through these surfaces is zero

For face ABCD:

$$\text{Area vector } \Delta \vec{S} = a^2 \hat{i}$$

$$\text{Electric field } \vec{E} = \frac{E_0 (x=a)}{l} \hat{i} = \frac{E_0 a}{l} \hat{i}$$

$\therefore$  flux from (i) is given by

$$\phi_{ABCD} = \frac{E_0 a}{l} \hat{i} \cdot (a^2 \hat{i})$$

$$\Rightarrow \phi_{ABCD} = \frac{E_0 a^3}{l}$$

For face EFGH

$$\text{Area vector } \Delta \vec{S} = -a^2 \hat{i}$$

$$\text{Electric field } \vec{E} = \frac{E_0 (x=0)}{l} \hat{i} = 0 \hat{i}$$

$\therefore$  flux from (i) is given by

$$\phi_{EFGH} = 0$$

$$\therefore \text{Total flux} = \frac{E_0 a^3}{l}$$

$$\Rightarrow \phi = 5 \times 10^3 \times \frac{(1 \times 10^{-2})^3}{2 \times 10^{-2}} \text{ Nm}^2/\text{C}$$

$$\Rightarrow \phi = 2.5 \times 10^{-1} \text{ Nm}^2/\text{C}$$

We know that,

By Gauss's law, flux of net electric field through a closed surface equals the net charge enclosed by the surface divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

$$\Rightarrow q_{in} = \phi \times \epsilon_0$$

$$\Rightarrow q_{in} = 2.5 \times 10^{-1} \times 8.85 \times 10^{-12} \text{ C}$$

$$\therefore q_{in} = 2.2125 \times 10^{-12} \text{ C}$$

**Therefore the net charge contained inside the cubical volume is given by  $2.2125 \times 10^{-12} \text{ C}$**

### 5. Question

A charge Q is placed at the centre of a cube. Find the flux of the electric field through the six surfaces of the cube.

### Answer

Given:

Total charge in the cube = Q at the center

We know that,

By Gauss's law, flux of net electric field through a closed surface equals the net charge enclosed by the surface divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

From Gauss's law we can conclude that total flux enclosed by the cube is given by

$$\phi = \frac{Q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

Since charge is placed at centre it is placed symmetrically with respect to all the faces of the cube . therefore we can say that equal flux passes through all the six surfaces.

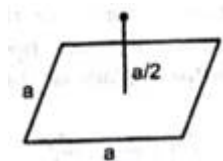
$$\therefore \text{flux passing through each surface } \phi_0 = \frac{\phi}{6}$$

$$\Rightarrow \phi_0 = \frac{Q}{6\epsilon_0}$$

**Therefore flux of electric field passing through each of the six surfaces are equal and is given by  $Q/6\epsilon_0$**

## 6. Question

A charge  $Q$  is placed at a distance  $a/2$  above the centre of a horizontal, square of edge  $a$  as shown in figure. Find the flux of the electric field through the square surface.



## Answer

Given:

Length of square edge= $a$

Distance of charge  $Q$  above the center= $a/2$

**Concept: Imagine the square to be one face of a cube of side  $a$  with the center at a point where charge  $Q$  is placed.**

Since charge is placed at centre it is placed symmetrically with respect to all the faces of the cube. therefore we can say that equal flux passes through all the six surfaces.

$\therefore$  flux passing through each surface  $\phi_0 = \frac{\phi}{6\epsilon_0}$

We know that,

By Gauss's law, flux of net electric field through a closed surface equals the net charge enclosed by the surface divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

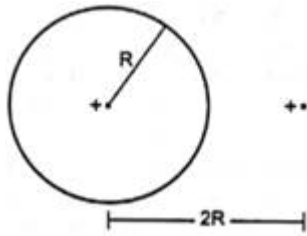
$$\Rightarrow \phi_0 = \frac{Q}{6\epsilon_0}$$

(as  $q_{in}=Q$ )

**Therefore flux of electric field through square surface is given by  $Q/6\epsilon_0$**

## 7. Question

Find the flux of the electric field through a spherical surface of radius  $R$  due to a charge of  $10^{-7}$  C at the centre and another equal charge at a point  $2R$  way from the centre.



### Answer

Given:

Magnitude of charges =  $10^{-7} \text{C}$

One at centre and another at a point  $2R$  from centre.

We know that,

By Gauss's law, flux of net electric field ( $\vec{E}$ ) through a closed surface  $S$  equals the net charge enclosed ( $q_{in}$ ) by the surface divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Here,

$$Q_{in} = q = 10^{-7} \text{C}$$

(Note that Gauss's law includes only those charges which are enclosed by the closed surfaces and not the charges outside the surfaces)

Therefore flux through the sphere is given by

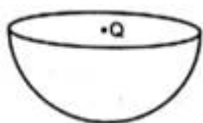
$$\phi = \frac{q}{\epsilon_0} = \frac{10^{-7}}{8.85 \times 10^{-12}} \text{Nm}^2/\text{C}$$

$$\Rightarrow \phi = 1.1 \times 10^4 \text{Nm}^2/\text{C}$$

**Therefore flux of electric field through a spherical surface of radius  $R$  is given by  $1.1 \times 10^4 \text{Nm}^2/\text{C}$**

### 8. Question

A charge  $Q$  is placed at the centre of an imaginary hemispherical surface. Using symmetry arguments and Gauss's law, find the flux of the electric field due to this charge through the surface of the hemisphere (figure).



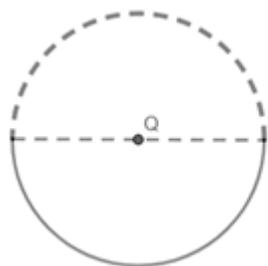
### Answer

Given:



Charge Q placed at centre of imaginary hemispherical surface

**Concept: Consider the hemisphere to be one half of an imaginary sphere in which charge Q is at the centre**



We know that,

By Gauss's law, flux of net electric field ( $E$ ) through a closed surface S equals the net charge enclosed ( $q_{in}$ ) by the surface divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Net charge enclosed by the sphere  $Q_{in}=Q$

Therefore from Gauss's law flux of electric field through this sphere

$$\phi = \frac{Q}{\epsilon_0}$$

Since charge is symmetrically located with respect to sphere equal flux pass through each part of sphere of equal area. Therefore flux passing through both the hemispherical surfaces at the top and bottom are the same

So the flux passing through the hemispherical surface  $= \frac{Q}{2\epsilon_0}$

**Therefore flux passing of electric field due to this charge through the surface of the hemisphere is  $Q/2\epsilon_0$**

## 9. Question

A spherical volume contains a uniformly distributed charge of density  $2.0 \times 10^{-4} \text{ C m}^{-3}$ . Find the electric field at a point inside the volume at a distance 4.0 cm from the centre.

**Answer**

Given:

Volume charge density  $\rho = 2.0 \times 10^{-4} \text{ C/m}^3$

We have to find the electric field at a point at a distance 4cm from the centre.

Assume a spherical gaussian surface inside the sphere of radius  $4\text{cm}=r$

Volume of this spherical surface  $= \frac{4}{3} \pi r^3$

Charge enclosed by this gaussian spherical surface = (volume charge density)  $\times$  (volume of gaussian surface containing charge)

$$Q_{in} = \rho \times \frac{4}{3} \pi r^3 \dots\dots(i)$$

Surface area of this spherical gaussian surface  $= 4\pi r^2$

All the points on this surface are equivalent and by symmetry we can say that field at every point on this surface is equal in magnitude and radial in direction.

Therefore flux through this surface can be written as

$$\phi = \oint \vec{E} \cdot d\vec{S} = E \oint dS = E \cdot 4\pi r^2 \dots\dots(ii)$$

We know that,

By Gauss's law, flux of net electric field ( $\vec{E}$ ) through a closed surface S equals the net charge enclosed ( $q_{in}$ ) by the surface divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Using gauss's law and eqn.(i) and eqn.(ii)

$$E \cdot 4\pi r^2 = \rho \times \frac{4\pi r^3}{\epsilon_0}$$

$$\Rightarrow E = \frac{\rho r}{\epsilon_0}$$

Putting the value of  $\rho$ ,  $r$  and  $\epsilon_0$

$$E = (2 \times 10^{-4}) \times \frac{4 \times 10^{-2}}{8.85 \times 10^{-12}} \text{N/C}$$

$$E = 3 \times 10^5 \text{N/C}$$

**Therefore electric field at a point inside volume at a distance 4cm from the centre is given by  $3 \times 10^5 \text{N/C}$**

### 10. Question

The radius of a gold nucleus ( $Z = 79$ ) is about  $7.0 \times 10^{-15}$  m. Assume that the positive charge is distributed uniformly throughout the nuclear volume. Find the strength of the electric field at

- (a) the surface of the nucleus and
- (b) at the middle point of a radius.

Remembering that gold is a conductor, is it justified to assume that the positive charge is uniformly distributed over the entire volume of the nucleus and does not come to the outer surface?

### Answer

Given:

Radius of gold nucleus =  $7.0 \times 10^{-15} \text{ m} = r$

Charge on the nucleus =  $79 \times 1.6 \times 10^{-19} \text{ C} = q$

(a)

Consider a gaussian spherical surface of radius of the nucleus =  $r$

By symmetry all points on this surface are equivalent and electric field at all points have same magnitude and is in radial direction

Therefore flux passing through this surface is given by

$$\phi = \oint \vec{E} \cdot d\vec{S} = E \oint dS = E \cdot 4\pi r^2 \dots (i)$$

We know that,

By Gauss's law, flux of net electric field ( $E \vec{r}$ ) through a closed surface  $S$  equals the net charge enclosed ( $q_{in}$ ) by the surface divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Charge enclosed by this sphere = total charge on the nucleus =  $q$

Using gauss's law and eqn.(i)

$$E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi r^2 \epsilon_0}$$

Putting the values of  $r, q$  and  $\epsilon_0$

$$E = 79 \times 1.6 \times \frac{10^{-19}}{4 \times 3.14 \times (7 \times 10^{-15})^2 \times 8.85 \times 10^{-12}}$$

$$E = 2.31 \times 10^{21} \text{ N/C}$$

**Therefore the strength of electric field at the surface of nucleus is  $2.31 \times 10^{21} \text{ N/C}$**

(b) for calculating electric field at middle point of radius consider similar gaussian surface with radius half of that of nucleus =  $r/2$

Charge enclosed by volume  $\frac{4}{3}\pi r^3 = Q$

Charge enclosed by volume  $\frac{4}{3\pi\left(\frac{r}{2}\right)^3} = Q'$

We get

$$Q' = \frac{Q}{8}$$

Using gauss law and eqn.(i)

$$E \cdot 4\pi r^2 = \frac{Q}{8\epsilon_0}$$

$$E = \frac{Q}{32\pi r^2 \epsilon_0}$$

$$E = 2.31 \times \frac{10^{21}}{8} \text{N/C}$$

$$E = 1.16 \times 10^{21} \text{N/C}$$

**Therefore electric field at middle point of radius of nucleus is given by  $1.16 \times 10^{21} \text{N/C}$**

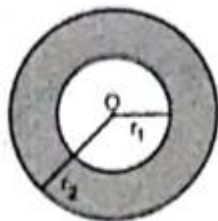
We know that when electric charge is given to a conductor it comes on its surface

But nucleons are bound by strong nuclear force inside nucleus which holds them and prevents them from coming out of conductor.

**Therefore it is justified to assume that electric charge is uniformly distributed in its entire volume.**

### 11. Question

A charge  $Q$  is distributed uniformly within the material of a hollow sphere of inner and outer radii  $r_1$  and  $r_2$  figure. Find the electric field at a point  $P$  a distance  $x$  away from the centre for  $r_1 < x < r_2$ . Draw a rough graph showing the electric field as a function of  $x$  for  $0 < x < 2r_2$  figure.



### Answer

Given:

Amount of charge distributed within the hollow sphere =  $Q$

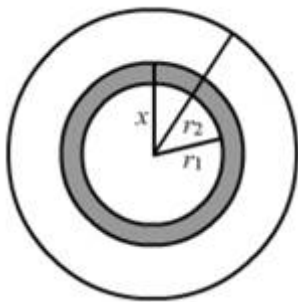
Inner and outer radius of hollow sphere= $r_1$  and  $r_2$

Volume charge density of hollow sphere is given by

$$\rho = \frac{\text{total charge}}{\text{volume of charge}}$$

$$\rho = \frac{Q}{\frac{4}{3}\pi(r_2^3 - r_1^3)} \dots(i)$$

To find the electric field at a point P at a distance  $x$  away from the centre consider a spherical gaussian surface of radius  $x$  ( $r_1 < x < r_2$ )



By symmetry all points on this surface are equivalent and electric field at all points have same magnitude and is in radial direction

Therefore flux through this surface is given by

$$\oint \vec{E} \cdot d\vec{S} = E \oint d\vec{S} = E \cdot 4\pi x^2 \dots(ii)$$

Charge enclosed by this surface is given by

$$Q_{in} = \text{volume charge density} \\ \times \text{volume of gaussian surface containing charge}$$

$$Q_{in} = \frac{Q}{\frac{4}{3}\pi(r_2^3 - r_1^3)} \times \frac{4}{3}\pi(x^3 - r_1^3)$$

$$Q_{in} = \frac{x^3 - r_1^3}{r_2^3 - r_1^3} Q \dots(iii)$$

We know that,

By Gauss's law, flux of net electric field ( $\vec{E}$ ) through a closed surface  $S$  equals the net charge enclosed ( $q_{in}$ ) by the surface divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Using gauss's law and eqn.(ii) and (iii)

$$E \cdot 4\pi x^2 = \frac{x^3 - r_1^3}{r_2^3 - r_1^3} Q$$

$$E = \frac{(x^3 - r_1^3)Q}{(r_2^3 - r_1^3) 4\pi x^2}$$

This electric field is directly proportional to  $x$  for  $r_1 < x < r_2$

For  $r_2 < x < 2r_2$

Considering a similar gaussian spherical surface of radius  $x$  such that  $r_2 < x < 2r_2$

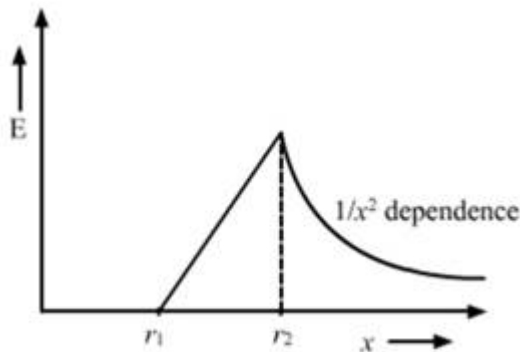
Charge enclosed by this surface =  $Q$

Total electric flux through this surface =  $E \cdot 4\pi x^2$

Using gauss law we can write

$$E = \frac{Q}{4\pi x^2 \epsilon_0}$$

The graph showing electric field as a function of  $x$  is shown as follows



## 12. Question

A charge  $Q$  is placed at the centre of an uncharged, hollow metallic sphere of radius  $a$ .

(a) Find the surface charge density on the inner surface and on the outer surface.

(b) If a charge  $q$  is put on the sphere, what would be the surface charge densities on the inner and the outer surfaces?

(c) Find the electric field inside the sphere at a distance  $x$  from the centre in the situations (a) and (b).

## Answer

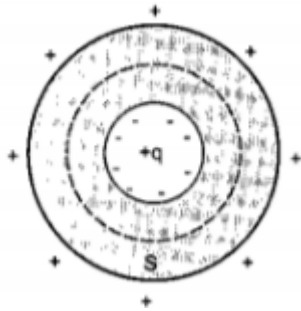
Given:

Charge present at the centre of hollow metallic sphere =  $Q$

Radius of sphere =  $a$

Surface area of sphere =  $4\pi a^2$

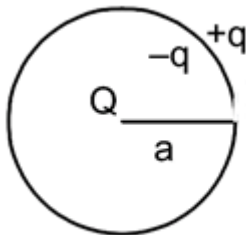
We know that,



Electric fields at all points inside the conductor is zero

So,

If a charge  $q$  is placed within the cavity of a metallic sphere then taking the Gaussian surface  $S$  as shown in fig. electric field  $E = 0$  at all points on this surface and hence  $\oint \vec{E} \cdot d\vec{S} = 0$  which ensures that charge contained in  $S$  is zero (by Gauss's law)



And if a charge  $+q$  is placed in the cavity, there must be a charge  $-q$  on the inner surface of the conductor. If the conductor is neutral i.e. no charge is placed on it, a charge  $+q$  will appear on the outer surface.

So here,

Charge induced at inner surface =  $-Q$

Charge induced at outer surface =  $+Q$

Surface charge density is given by charge/total surface area

$$\therefore \text{surface charge density on inner surface} = -\frac{Q}{4\pi a^2}$$

$$\text{Surface charge density on outer surface} = +\frac{Q}{4\pi a^2}$$

**Therefore surface charge densities on inner and outer surfaces are given by  $-Q/4\pi a^2$  and  $Q/4\pi a^2$  respectively.**

(b) If a charge  $q$  is put on the sphere all the charge will move to the outer surface of the sphere (since, total charge enclosed by the sphere is zero)

Therefore inner charge density will remain same and outer charge density will increase as the new outer charge is  $Q+q$

$$\therefore \text{surface charge density on inner surface} = -\frac{Q}{4\pi a^2}$$

$$\therefore \text{surface charge density on outer surface} = \frac{Q+q}{4\pi a^2}$$

**Therefore surface charge densities of inner and outer surfaces after putting charge  $q$  on the sphere is given by  $-Q/4\pi a^2$  and  $(Q+q)/4\pi a^2$  respectively**

(c)

To find the electric field inside the sphere at a distance  $x$  from the sphere consider a spherical Gaussian surface of radius  $x$

Applying Gauss law on this sphere

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{in}}{\epsilon_0}$$

$$E \oint d\vec{S} = E \cdot 4\pi x^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 x^2}$$

Where  $q$  is charge enclosed by the sphere

For situation(b) the charge enclosed by the Gaussian surface remains the same as the charge provided to the metallic sphere it will move to the outer surface of the sphere

$$\therefore E = \frac{Q}{4\pi\epsilon_0 x^2}$$

**Therefore for situations (a) and (b) the electric field inside the sphere at a distance  $x$  from the centre is same and given by  $Q/4\pi\epsilon_0 x^2$**

### 13. Question

Consider the following very rough model of a beryllium atom. The nucleus has four protons and four neutrons confined to a small volume of radius  $10^{-15}$  m. The two 1s electrons make a spherical charge cloud at an average distance of  $1.3 \times 10^{-11}$  m from the nucleus, whereas the two 2s electrons make another spherical cloud at an average distance of  $5.2 \times 10^{-11}$  m from the nucleus. Find the electric field at

(a) a point just inside the 1s cloud and (b) a point just inside the 2s cloud.

**Answer**

Given:



Radius of nucleus= $10^{-15}\text{m}$

Radius of 1-s charge cloud= $1.3 \times 10^{-11}\text{m}=r_1$

Radius of 2-s charge cloud = $5.2 \times 10^{-11}\text{m}=r_2$

(a)for calculating electric field at a point just inside 1s cloud

Consider a gaussian spherical surface of radius just equal to the radius of 1s cloud= $r_1$

By symmetry all points on this surface are equivalent and electric field at all points have same magnitude and is in radial direction

Therefore flux passing through this surface is given by

$$\phi = \oint \vec{E} \cdot d\vec{S} = E \oint d\vec{S} = E \cdot 4\pi r_1^2 \dots(i)$$

We know that,

By Gauss's law, flux of net electric field ( $\vec{E}$ ) through a closed surface S equals the net charge enclosed ( $q_{in}$ ) by the surface divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Charge enclosed by this sphere =total charge on the nucleus= $q$

Now,

$$q = (\text{charge of 4 protons inside nucleus}) = (4) \times 1.6 \times 10^{-19}\text{C}$$

$$q = 4 \times 1.6 \times 10^{-19}\text{C}$$

Using gauss's law and eqn.(i)

$$E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi r_1^2 \epsilon_0}$$

Putting the values of  $r_1$ ,  $q$  and  $\epsilon_0$

$$E = \frac{2 \times 1.6 \times 10^{-19}}{4 \times 3.14 \times (1.3 \times 10^{-11})^2 \times 8.85 \times 10^{-12}} \text{N/C}$$

$$E = 3.4 \times 10^{13}\text{N/C}$$

**Therefore electric field at a point just inside the 1s cloud is given by  $3.4 \times 10^{13}\text{N/C}$**

(b)for calculating electric field at a point just inside the 2s cloud

Consider a gaussian spherical surface of radius just equal to the radius of 2s cloud= $r_2$

By symmetry all points on this surface are equivalent and electric field at all points have same magnitude and is in radial direction

Therefore flux passing through this surface is given by

$$\phi = \oint \vec{E} \cdot d\vec{S} = E \oint dS = E \cdot 4\pi r_2^2 \dots (i)$$

We know that,

By Gauss's law, flux of net electric field ( $E \vec{r}$ ) through a closed surface S equals the net charge enclosed ( $q_{in}$ ) by the surface divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Charge enclosed by this sphere =total charge on the nucleus+ charge of 2 1s electrons= $q$

Now,

$$q = \text{(charge of 4 protons inside nucleus)} + \text{(charge of two electrons inside 1s cloud)} = (4 - 2) \times 1.6 \times 10^{-19} \text{ C}$$

$$q = 2 \times 1.6 \times 10^{-19} \text{ C}$$

Using gauss's law and eqn.(i)

$$E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi r_2^2 \epsilon_0}$$

Putting the values of  $r_1$ ,  $q$  and  $\epsilon_0$

$$E = \frac{2 \times 1.6 \times 10^{-19}}{4 \times 3.14 \times (5.2 \times 10^{-11})^2 \times 8.85 \times 10^{-12}} \text{ N/C}$$

$$E = 1.1 \times 10^{12} \text{ N/C}$$

**Therefore electric field at a point just inside the 2s cloud is given by  $1.1 \times 10^{12} \text{ N/C}$**

#### 14. Question

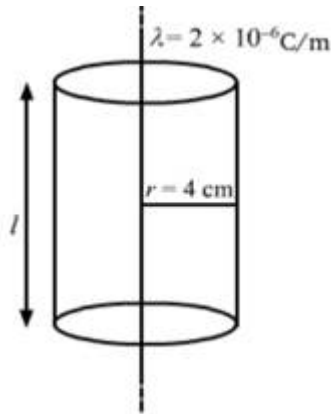
Find the magnitude of the electric field at a point 4 cm away from a line charge of density  $2 \times 10^{-6} \text{ C m}^{-1}$ .

## Answer

Given:

Charge density of line charge =  $2 \times 10^{-6} \text{ C/m} = \lambda$

Distance of point from line charge =  $4 \text{ cm} = 4 \times 10^{-2} \text{ m} = r$



Consider a Gaussian cylindrical surface around the line charge of radius 4cm and height  $l$ .

All the points on the curved part of this gaussian surface are at the same perpendicular distance from the line charge. Therefore all these points are equivalent. The electric field at all these points will have the same magnitude  $E$  and direction of field at any point on the curved surface is normal to the line and hence normal to the cylindrical surface.

We know that,

By Gauss's law, flux of net electric field through a closed surface equals the net charge enclosed by the surface divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0} \dots (i)$$

Through the circular faces at the top and bottom the angle between electric field and area vector is  $90^\circ$  therefore flux through these surfaces is zero.

By applying gauss's law for the curved cylindrical surface,

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Where  $\vec{E}$  = electric field vector

$d\vec{S}$  = area of differential surface element

$q_{in}$  = charge enclosed by Gaussian surface

curved surface area of cylinder =  $2\pi r l$

flux through the curved part is given by

$$\phi = \oint \vec{E} \cdot d\vec{S} = E \oint dS = E \cdot 2\pi r l$$

The total flux through the gaussian surface is given by  $2\pi r l$

The charge enclosed by the gaussian surface is ( line charge density  $\times$  height of cylindrical surface )

$$\Rightarrow \lambda \times l$$

Using gauss's law(i)

$$E \cdot 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$\therefore E = \frac{\lambda}{2\pi r \epsilon_0}$$

$$\Rightarrow E = 2 \times \frac{10^{-6}}{2 \times 3.14 \times 4 \times 10^{-2} \times 8.85 \times 10^{-12}} \text{N/C (putting the values of } \lambda, r, \pi \text{ and } \epsilon_0)$$

$$\Rightarrow E = 8.99 \times 10^5 \text{N/C}$$

**Therefore the magnitude of electric field at a distance 4cm away from the line charge is given by  $8.99 \times 10^5 \text{N/C}$**

### 15. Question

A long cylindrical wire carries a positive charge of linear density  $2.0 \times 10^{-6} \text{ C m}^{-1}$ . An electron revolves around it in a circular path under the influence of the attractive electrostatic force. Find the kinetic energy of the electron. Note that it is independent of the radius.

### Answer

Given:

Linear charge density of wire =  $2.0 \times 10^{-6} \text{ C} = \lambda$

We know that,

Electric field  $E$  due to a linear charge distribution of linear charge density  $\lambda$  at a distance  $r$  from the line is given by

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \dots(i)$$

Now,

Magnitude of Force experienced by a charge  $q$  in an electric field of intensity  $E$  is given by

$$F = q \times E$$

Here the charge particle is electron so the charge =  $q = e$

**Since this electron revolves around the wire in a circular path under the influence of this electrostatic force this force is equal to the centripetal force experienced by the electron**

$$\therefore q \times E = \frac{mv^2}{r} \dots(ii)$$

Where,

m=mass of electron= $9.1 \times 10^{-31}$ kg

r=radius of orbit in which electron revolves

q=e=charge of electron= $1.6 \times 10^{-19}$ C

E=electric field due to line charge

V=velocity of electron

We know that ,

Kinetic energy of electron is given by

$$K = \frac{1}{2}mv^2 \dots(iii)$$

Where,

m=mass of electron

v=velocity of electron

from eqn,(ii) ,

$$\frac{1}{2} \times mv^2 = \frac{1}{2} \times q \times E \times r$$

Therefore kinetic energy of electron is given by

$$K = \frac{qEr}{2}$$

Putting value of E from eqn.(i)

$$K = \frac{qr}{2} \times \frac{\lambda}{2\pi r \epsilon_0}$$

$$K = \frac{q\lambda}{4\pi\epsilon_0}$$

**Thus kinetic energy is independent of radius(r)**

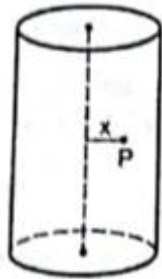
$$\Rightarrow K = \frac{1.6 \times 10^{-19}}{2} \times \frac{2 \times 10^{-6}}{2 \times 3.14 \times 8.85 \times 10^{-12}} \text{ J}$$

$$\Rightarrow K = 2.88 \times 10^{-17} \text{ J}$$

Therefore kinetic energy of electron while revolving around a cylindrical charge is given by  $2.88 \times 10^{-17} \text{J}$  and it is independent of radius.

### 16. Question

A long cylindrical volume contains a uniformly distributed charge of density  $\rho$ . Find the electric field at a point P inside the cylindrical volume at a distance  $x$  from its axis.



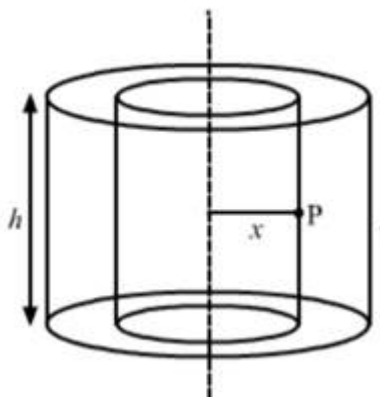
### Answer

Given:

Volume charge density inside cylinder =  $\rho$

We need to find the electric field at point P which is at a distance  $x$  from the axis of the cylinder

Consider a cylindrical gaussian surface of radius  $x$  and height  $h$



We know that,

By Gauss's law, flux of net electric field ( $\vec{E}$ ) through a closed surface  $S$  equals the net charge enclosed by the surface ( $q_{in}$ ) divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0} \quad \text{..(i)}$$

Volume of this gaussian surface =  $\pi x^2 h$

Curved surface area of this surface =  $2\pi x h$

$\therefore$  charge enclosed by this surface =

$$q_{in} = (\text{volume charge density}) \times \text{volume of surface}$$

$$\rightarrow q_{in} = \rho \times \pi x^2 h \dots(ii)$$

Now,

All the points on the curved part of this gaussian surface are at the same perpendicular distance from the line charge. Therefore all these points are equivalent. The electric field at all these points will have the same magnitude  $E$  and direction of field at any point on the curved surface is normal to the line and hence normal to the cylindrical surface.

Therefore flux through this surface can be written as,

$$\oint \vec{E} \cdot d\vec{S} = E \cdot \oint d\vec{S} = E \cdot 2\pi x h \dots(iii)$$

From gauss law(i) and using eqns (ii) and (iii)

$$E \cdot 2\pi x h = \rho \times \frac{\pi x^2 h}{\epsilon_0}$$

$$\Rightarrow E = \frac{\rho x}{2\epsilon_0}$$

**Therefore electric field at a point P inside cylindrical volume at a distance  $x$  from its axis is given by  $\rho x / 2\epsilon_0$**

### 17. Question

A nonconducting sheet of large surface area and thickness  $d$  contains uniform charge distribution of density  $\rho$ . Find the electric field at a point P inside the plate, at a distance  $x$  from the central plane. Draw a qualitative graph of  $E$  against  $x$  for  $0 < x < d$ .

### Answer

Given:

Thickness of sheet= $d$

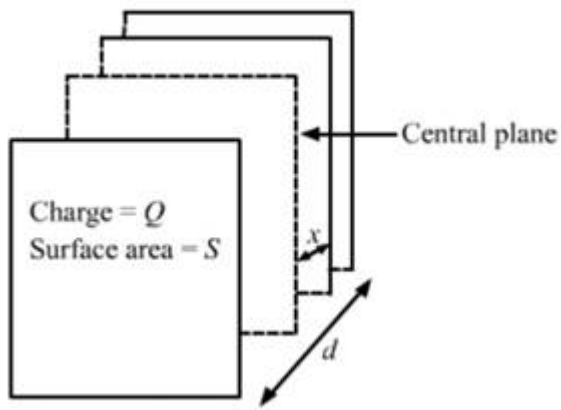
Volume charge density= $\rho$

Let the surface area of sheet be  $A$

Consider a cuboidal Gaussian surface of width  $x$  from the central plane such that the central plane becomes one face of the cuboidal surface.

Volume of this cuboidal surface is given by

$$V = A \times x$$



Charge enclosed by this gaussian surface = (volume charge density)  $\times$  (volume of gaussian surface containing charge)

$$q_{in} = \rho \times A \times x \text{ ..(i)}$$

We know that,

By Gauss's law, flux of net electric field ( $\vec{E}$ ) through a closed surface  $S$  equals the net charge enclosed by the surface ( $q_{in}$ ) divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0} \text{ .....(i)}$$

By symmetry arguments we can say that electric field is normal to the plane and has same magnitude at all points which are at same distance from the central plane.

Here flux only pass through the face which is parallel to the central plane which is at distance  $x$  from the central plane . for remaining faces of the gaussian surfaces the angle between the electric field vector and area vector =90.

Therefore no flux pass through these faces .

Therefore total flux passing through the gaussian surface is given by

$$\oint \vec{E} \cdot d\vec{S} = E \oint d\vec{S} = E \cdot A \text{ .....(ii)}$$

Using gauss's law(i) and eqn.(ii) we can write

$$E \cdot A = \frac{\rho \times A \times x}{\epsilon_0}$$

$$\Rightarrow E = \frac{\rho x}{\epsilon_0} \text{ .....(iii)}$$

**Therefore , the electric field at a point P inside the plate at a distance  $x$  from the central plane is given by  $\rho x / \epsilon_0$**

Eqn(iii) is valid only inside the sheet i.e. when  $x < d/2$

(as  $x$  is measured from the central plane )

Now for outside the sheet (i.e.  $x > d/2$ )



Consider again a similar Gaussian surface of width  $x$  where  $x$  is measured from the central plane

Charge enclosed by this gaussian surface =(volume charge density  $\times$  volume of gaussian surface containing charge)

$$q_{in} = \rho \times A \times \frac{d}{2} \dots(iv)$$

Using gauss's law(i) and eqn(ii) and (iv)

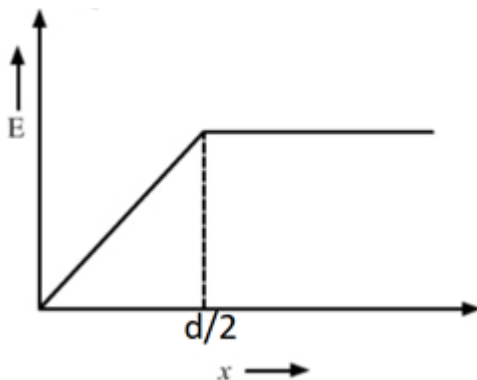
$$E \cdot A = \frac{\rho \times A \times \frac{d}{2}}{\epsilon_0}$$

$$E = \frac{\rho \times d}{2\epsilon_0}$$

Which is independent of  $x$

**Therefore electric field at a point P outside the sheet at a distance  $x$  is constant and given by  $\rho d/2\epsilon_0$**

Graph of electric field for  $0 < x < d$  is given by



### 18. Question

A charged particle having a charge of  $-2.0 \times 10^{-6}$  C is placed close to a nonconducting plate having a surface charge density  $4.0 \times 10^{-6}$  C  $m^{-2}$ . Find the force of attraction between the particle and the plate.

### Answer

Given:

Charge of the particle =  $-2.0 \times 10^{-6}$  C =  $q$

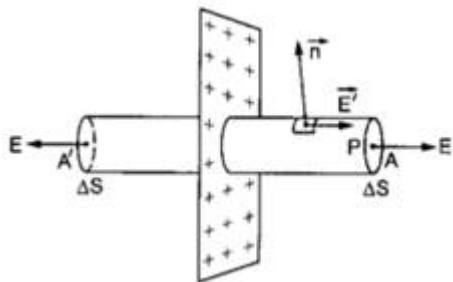
Surface charge density =  $4.0 \times 10^{-6}$  C/ $m^2$  =  $\sigma$

The electric field due to a plane thin sheet of charge density  $\sigma$  is given by

$$E = \frac{\sigma}{2\epsilon_0}$$

Proof:

To calculate the electric field at P we choose a cylindrical Gaussian surface as shown in the fig. in which the cross section A and A' are at equal distance from the plane.



The electric field at all points of A have equal magnitude E. and direction along positive normal. The flux of electric field through A is given by

$$\phi = E. \Delta S$$

Since A and A' are at equal distance from sheet the electric field at any point of A' is also equal to E and flux of electric field through A' is also given by

$$E.\Delta S$$

At the points on curved surface the field and area make an angle of 90° with each other and hence  $\vec{E} \cdot \Delta \vec{S} = 0$

The total flux through the closed surface is given by

$$\phi = E. \Delta S + E. \Delta S = 2E. \Delta S \dots(i)$$

The area of sheet enclosed in the cylinder is given by ΔS

So the charge contained in the cylinder is given by

$$q_{in} = \sigma. \Delta S \dots(ii)$$

We know that,

By Gauss's law, flux of net electric field ( $E'$ ) through a closed surface S equals the net charge enclosed by the surface ( $q_{in}$ ) divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Using gauss law and eqns(i) and (ii)

$$2E. \Delta S = \sigma. \frac{\Delta S}{\epsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{2\epsilon_0} \dots(iii)$$

Now,

Force  $\vec{F}$  on a charge particle of charge  $q$  in presence of electric field  $\vec{E}$  is given by

$$\vec{F} = q\vec{E}$$

Using eqn(iii),we get

$$F = \frac{q\sigma}{2\epsilon_0}$$

Putting the values of  $\sigma$  and  $q$  we get

$$F = (-2.0 \times 10^{-6}) \times \frac{4.0 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} \text{N}$$

$$F = -0.45 \text{N} \text{ (-ve sign indicates that the force is attractive in nature)}$$

**Therefore force of attraction between the particle and the plate is given by 0.45N**

### 19. Question

One end of a 10 cm long silk thread is fixed to a large vertical surface of a charged nonconducting plate and the other end is fastened to a small ball having a mass of 10 g and a charge of  $4.0 \times 10^{-6}$  C. In equilibrium, the thread makes an angle of  $60^\circ$  with the vertical. Find the surface charge density on the plate.

### Answer

Given:

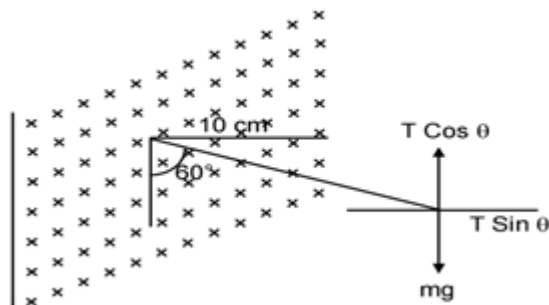
Length of silk thread=10cm

Mass of ball=10g

Charge of ball= $4.0 \times 10^{-6}$ C

Equilibrium angle of thread with vertical= $60^\circ$

Let the surface charge density on the plate be  $\sigma$



The forces acting on the ball are

- Weight of the ball downwards ( $W=mg$ )

Where  $m$ =mass of ball

$g$ =acceleration due to gravity

- Electric force due to non-conducting plate producing electric field  $E$  ( $F=qE$ )

Where  $q$ =charge of the ball

$E$ =electric field intensity

- Tension force ( $T$ ) due to string at an angle of  $60^\circ$  from vertical

Electric field due to a thin non-conducting plate of surface charge density  $\sigma$  is given by

$$E = \frac{\sigma}{2\epsilon_0} \dots\dots(i)$$

The tension force  $T$  due to string is divided into horizontal and vertical components given by  $T\sin 60^\circ$  and  $T\cos 60^\circ$

Since the ball is in equilibrium, the net horizontal and vertical force on the ball is zero

Applying equilibrium along horizontal direction, we get

$$T\sin 60^\circ = qE \dots(ii)$$

Similarly, applying equilibrium along vertical direction, we get

$$T\cos 60^\circ = mg \dots(iii)$$

Dividing eqn. (ii) by (iii)

$$\tan 60^\circ = \frac{qE}{mg}$$

$$E = \frac{mg\tan 60^\circ}{q}$$

Putting the value of  $E$  from eqn.(i), we get,

$$\frac{\sigma}{2\epsilon_0} = \frac{mg\tan 60^\circ}{q}$$

$$\Rightarrow \sigma = \frac{2\epsilon_0 \times mg\tan 60^\circ}{q}$$

Putting the values of  $\epsilon_0, m, g$  and  $q$  in the above equation

$$\sigma = \frac{2 \times 8.85 \times 10^{-12} \times 10 \times 10^{-3} \times 9.8 \times \sqrt{3}}{4 \times 10^{-6}}$$

$$\sigma = 7.5 \times 10^{-7} \text{C/m}^2$$

**Therefore the surface charge density on the plate is given by  $7.5 \times 10^{-7} \text{C/m}^2$**

## 20. Question

Consider the situation of the previous problem.

(a) Find the tension in the string in equilibrium.

(b) Suppose the ball is slightly pushed aside and released. Find the time period of the small oscillations.

## Answer

Given:

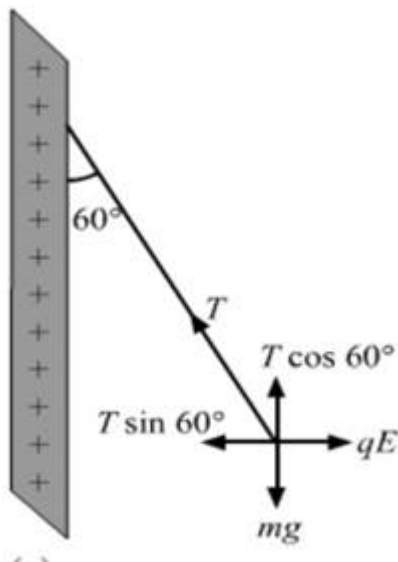
Length of silk thread=10cm

Mass of ball=10g

Charge of ball= $4.0 \times 10^{-6}\text{C}$

Equilibrium angle of thread with vertical= $60^\circ$

Let the surface charge density on the plate be  $\sigma$



The forces acting on the ball are

- Weight of the ball downwards ( $W=mg$ )

Where  $m$ =mass of ball

$g$ =acceleration due to gravity

- Electric force due to non-conducting plate producing electric field  $E$  ( $F=qE$ )

Where  $q$ =charge of the ball

$E$ =electric field intensity

- Tension force ( $T$ ) due to string at an angle of  $60^\circ$  from vertical

Electric field due to a thin non-conducting plate of surface charge density  $\sigma$  is given by

$$E = \frac{\sigma}{2\epsilon_0} \dots (i)$$

The tension force  $T$  due to string is divided into horizontal and vertical components given by  $T\sin 60^\circ$  and  $T\cos 60^\circ$

Since the ball is in equilibrium, the net horizontal and vertical force on the ball is zero

Applying equilibrium along horizontal direction, we get

$$T\sin 60^\circ = qE \dots (ii)$$

Similarly, applying equilibrium along vertical direction, we get

$$T\cos 60^\circ = mg \dots (iii)$$

Dividing eqn. (ii) by (iii)

$$\tan 60^\circ = \frac{qE}{mg}$$

$$E = \frac{mg \tan 60^\circ}{q}$$

Putting the value of  $E$  from eqn.(i), we get,

$$\frac{\sigma}{2\epsilon_0} = \frac{mg \tan 60^\circ}{q}$$

$$\Rightarrow \sigma = \frac{2\epsilon_0 \times mg \tan 60^\circ}{q}$$

Putting the values of  $\epsilon_0, m, g$  and  $q$  in the above equation

$$\sigma = \frac{2 \times 8.85 \times 10^{-12} \times 10 \times 10^{-3} \times 9.8 \times \sqrt{3}}{4 \times 10^{-6}}$$

$$\sigma = 7.5 \times 10^{-7} \text{C/m}^2$$

(a) Now to find the tension in the string in equilibrium condition we can use eqn. (iii)

$$T\cos 60^\circ = mg$$

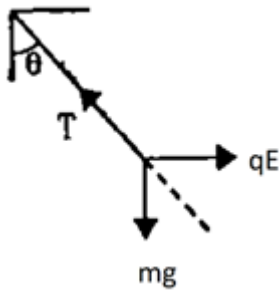
$$T = 2mg$$

$$T = 2 \times (10 \times 10^{-3}) \times 9.8$$

$$T = 0.196 \text{N}$$

Therefore tension in the string in equilibrium is given by 0.196N

(b) when the ball is slightly pushed aside and released, the ball will undergo small oscillations due to the restoring forces. When the ball will come in mean position, tension, weight and electric force will balance.



The forces acting on the ball are  $mg\vec{}$  in vertical direction and  $qE\vec{}$  in horizontal direction

From the fig, tension is given by

$$T = \sqrt{(mg)^2 + (qE)^2}$$

So the net acceleration  $g_{eff}$  is given by

$$g_{eff} = \frac{T}{m}$$

$$g_{eff} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$$

Therefore time period of small oscillations T is given by

$$T = 2\pi \sqrt{\frac{l}{g_{eff}}}$$

$$T = 2\pi \sqrt{\frac{l}{\left(g^2 + \left(\frac{qE}{m}\right)^2\right)^{0.5}}}$$

Putting the value of E from eqn.(i) we get

$$T = 2\pi \sqrt{\frac{l}{\left(g^2 + \left(\frac{q\sigma}{2\epsilon_0 m}\right)^2\right)^{0.5}}}$$

Putting the values of g, l, q,  $\sigma$ ,  $\epsilon_0$  and m and solving we get,

$$T = 0.45\text{sec}$$

**Therefore the time period of small oscillations of ball is given by 0.45sec**

### 21. Question

Two large conducting plates are placed parallel to each other with a separation of 2.00 cm between them. An electron starting from rest near one of the plates reaches the other plate in 2.00 microseconds. Find the surface charge density on the inner surfaces.

### Answer

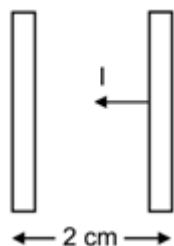
Given:

Distance travelled by the electron = 2 cm =  $2 \times 10^{-2}$  m = s

Time taken by the electron =  $2 \mu\text{s} = 2 \times 10^{-6}$  s = t

Let the surface charge density of the plate be  $\sigma$

Let the acceleration of electron be a



Using 2<sup>nd</sup> law of motion

$$s = ut + \frac{1}{2}at^2$$

Where,

u = initial velocity

t = time taken to travel

a = acceleration of particle

s = displacement of particle

it is given that electron starts from rest  $\therefore u = 0$

$$s = \frac{1}{2}at^2$$

$$a = \frac{2s}{t^2}$$

**This acceleration is provided by the force due to electric field between plates**

Force applied to the particle is given by



$$F = m \times a = m \times \frac{2s}{t^2} \dots(i)$$

Where,

$$m = \text{mass of electron} = 9.1 \times 10^{-31} \text{kg}$$

this force is provided by the electric field (E) between the plates which is given by

$$F = qE \dots(ii)$$

Equating eqns.(i) and (ii)

$$qE = m \times \frac{2s}{t^2} \dots(iii)$$

We know that

Electric field due to a conducting plate of surface charge density  $\sigma$  is given by

$$E = \frac{\sigma}{\epsilon_0}$$

Putting this value of E in eqn.(iii)

$$\frac{q\sigma}{\epsilon_0} = \frac{2ms}{t^2}$$

$$\sigma = \frac{2ms\epsilon_0}{qt^2}$$

Putting values of s, m, t and q

$$\sigma = 2 \times 9.1 \times 10^{-31} \times 8.85 \times \frac{10^{-12}}{(1.6 \times 10^{-19}) \times (2 \times 10^{-6})^2} \text{C/m}^2$$

$$\sigma = 0.503 \times 10^{-12} \text{C/m}^2$$

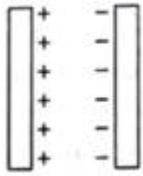
**Therefore the surface charge density of the plate is given by**

$$0.503 \times 10^{-12} \text{C/m}^2$$

## 22. Question

Two large conducting plates are placed parallel to each other and they carry equal and opposite charges with surface density  $\sigma$  as shown in figure. Find the electric field

- (a) at the left of the plates.
- (b) in between the plates and
- (c) at the right of the plates.



### Answer

Given:

Surface charge density of both plate= $\sigma$

Both plates carry equal and opposite charges

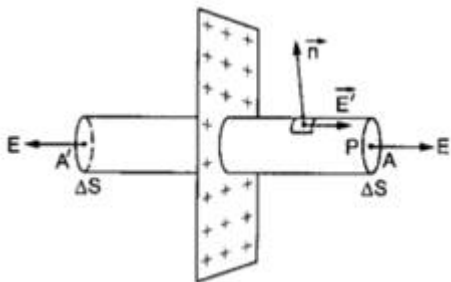
We know that,

The electric field due to a plane thin sheet of charge density  $\sigma$  is given by

$$E = \frac{\sigma}{2\epsilon_0}$$

Proof:

To calculate the electric field at P we choose a cylindrical Gaussian surface as shown in the fig. in which the cross section A and A' are at equal distance from the plane.



The electric field at all points of A have equal magnitude E. and direction along positive normal. The flux of electric field through A is given by

$$\phi = E \cdot \Delta S$$

Since A and A' are at equal distance from sheet the electric field at any point of A' is also equal to E and flux of electric field through A' is also given by

$$E \cdot \Delta S$$

At the points on curved surface the field and area make an angle of  $90^\circ$  with each other and hence  $\vec{E} \cdot \Delta \vec{S} = 0$

The total flux through the closed surface is given by

$$\phi = E \cdot \Delta S + E \cdot \Delta S = 2E \cdot \Delta S \dots(i)$$

The area of sheet enclosed in the cylinder is given by  $\Delta S$

So the charge contained in the cylinder is given by

$$q_{in} = \sigma \cdot \Delta S \dots(ii)$$

We know that,

By Gauss's law, flux of net electric field ( $E \vec{}$ ) through a closed surface S equals the net charge enclosed by the surface ( $q_{in}$ ) divided by  $\epsilon_0$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Using gauss law and eqns(i) and (ii)

$$2E \cdot \Delta S = \sigma \cdot \frac{\Delta S}{\epsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{2\epsilon_0}$$

Now

Magnitude of Electric field due to plate 1 is given by  $E_1 = \frac{\sigma}{2\epsilon_0}$

Magnitude of Electric field due to plate 2 is given by  $E_2 = \frac{\sigma}{2\epsilon_0}$

(a)at the left of the plates

Electric field due to plate 1 is in left direction(-ve) whereas electric field due to plate 2 is in right direction(+ve)

$\therefore$  net electric field

$$E = E_2 - E_1 = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

**Therefore net electric field due to both plates at the left of plates is zero**

(b)in between the plates,

The electric field due to plate 1 is in right direction(+ve) and electric field due to plate 2 is also in right direction(+ve)

$\therefore$  net electric field

$$E = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

**Therefore net electric field due to both plates in between the plates is given by  $\sigma/\epsilon_0$**

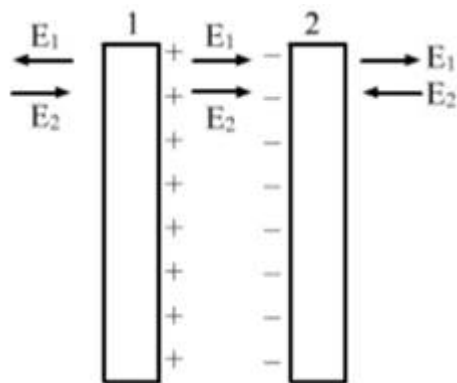
(c)at the right of the plates

Electric field due to plate 1 is in right direction (+ve) whereas electric field due to plate 2 is in left direction (-ve)

∴ net electric field

$$E = E_1 - E_2 = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

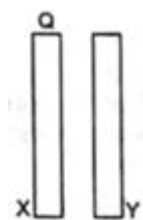
**Therefore net electric field due to both plates in the right of both plates is zero**



### 23. Question

Two conducting plates X and Y, each having large surface area A (on one side), are placed parallel to each other as shown in figure. The plate X is given a charge Q whereas the other is neutral. Find:

- the surface charge density at the inner surface of the plate X,
- the electric field at a point to the left of the plates
- the electric field at a point in between the plates and
- the electric field at a point to the right of the plates



### Answer

Given:

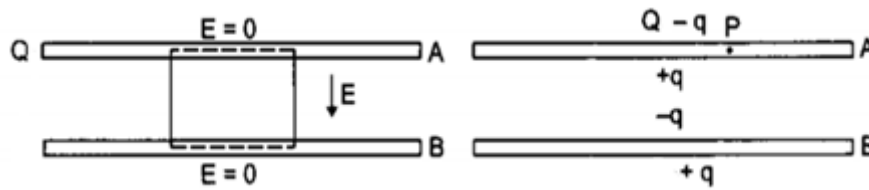
Charge on plate X=Q

Charge on plate Y=zero

Consider the gaussian surface as shown in fig.

Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The other parts of closed surface which are outside are

parallel to electric field and hence flux on these parts is zero. The total flux of electric field through this closed surface is zero. So, from Gauss's law total charge inside the closed surface be zero. The charge on inner surface of X should be equal and opposite to charge on inner surface of Y.



To find the value of q consider electric field at point P

We know that electric field due to a thin plate of charge Q is given by

$$E = \frac{Q}{2A\epsilon_0}$$

Where A=area of plate

Electric field at P

Due to charge  $Q-q = \frac{Q-q}{2A\epsilon_0}$  (towards right)

Due to charge  $q = \frac{q}{2A\epsilon_0}$  (towards left)

Due to charge  $-q = \frac{q}{2A\epsilon_0}$  (towards right)

Due to charge  $q = \frac{q}{2A\epsilon_0}$  (towards left)

The net electric field at P due to all four charged surfaces is

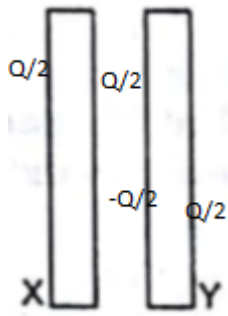
$$E = \frac{Q - 2q}{2A\epsilon_0}$$

Which is zero as P is inside conductor

$$\therefore \frac{Q-2q}{2A\epsilon_0} = 0$$

$$\therefore q = +\frac{Q}{2}$$

The final charge distribution is as shown in fig



(a) surface charge density at inner surface of plate X is given by

$$\frac{\text{charge at inner surface}}{\text{area of plate}}$$

$$\Rightarrow \frac{Q}{2A}$$

**Therefore surface charge density at inner surface of plate X is given by  $Q/2A$**

**Consider Right direction as negative and left as positive**

(b) at left of the plates

electric field due to,

$$\text{outer surface of plate X} = \frac{Q}{4A\epsilon_0} \text{ (towards left)}$$

$$\text{inner surface of plate X} = \frac{Q}{4A\epsilon_0} \text{ (towards left)}$$

$$\text{inner surface of plate Y} = \frac{+Q}{4A\epsilon_0} \text{ (towards right)}$$

$$\text{outer surface of plate Y} = \frac{Q}{4A\epsilon_0} \text{ (towards left)}$$

net electric field at left of plates is given by

$$E = \frac{Q}{4A\epsilon_0} + \frac{Q}{4A\epsilon_0} - \frac{Q}{4A\epsilon_0} + \frac{Q}{4A\epsilon_0} = \frac{Q}{2A\epsilon_0}$$

**Therefore net electric field at a point to left to the plates is given by  $Q/2A\epsilon_0$  towards left**

(c) between the plates

electric field due to,

$$\text{outer surface of plate X} = \frac{Q}{4A\epsilon_0} \text{ (towards right)}$$

$$\text{inner surface of plate X} = \frac{Q}{4A\epsilon_0} \text{ (towards right)}$$

$$\text{inner surface of plate Y} = \frac{Q}{4A\epsilon_0} \text{ (towards right)}$$

$$\text{outer surface of plate Y} = \frac{Q}{4A\epsilon_0} \text{ (towards left)}$$

net electric field at a point between the plates is given by

$$E = -\frac{Q}{4A\epsilon_0} - \frac{Q}{4A\epsilon_0} - \frac{Q}{4A\epsilon_0} + \frac{Q}{4A\epsilon_0} = -\frac{Q}{2A\epsilon_0}$$

**Therefore net electric field at a point between the plates is given by  $Q/2A\epsilon_0$  towards right**

(d) to the right of the plates

electric field due to,

outer surface of plate X =  $\frac{Q}{4A\epsilon_0}$  (towards right)

inner surface of plate X =  $Q/4A\epsilon_0$  (towards right)

inner surface of plate Y =  $Q/4A\epsilon_0$  (towards left)

outer surface of plate Y =  $Q/4A\epsilon_0$  (towards right)

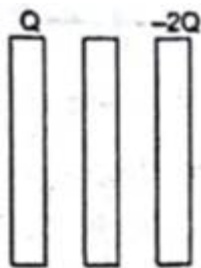
net electric field at a point between the plates is given by

$$E = -\frac{Q}{4A\epsilon_0} - \frac{Q}{4A\epsilon_0} + \frac{Q}{4A\epsilon_0} - \frac{Q}{4A\epsilon_0} = -\frac{Q}{2A\epsilon_0}$$

**Therefore net electric field at a point to the right of the plates is given by  $Q/2A\epsilon_0$  towards right**

#### 24. Question

Three identical metal plates with large surface areas are kept parallel to each other as shown in figure. The leftmost plate is given a charge  $Q$ , the rightmost a charge  $-2Q$  and the middle one remains neutral. Find the charge appearing on the outer surface of the rightmost plate.



#### Answer

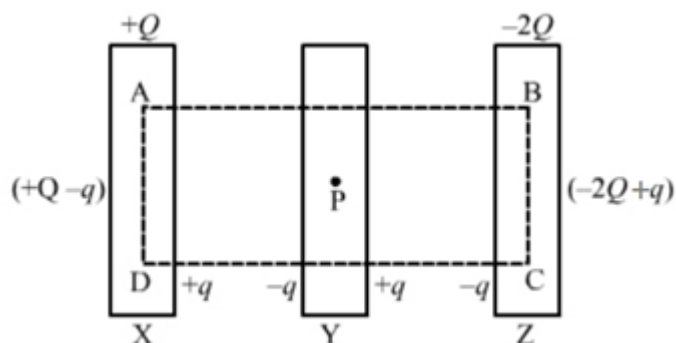
Given:

Charge on the left plate =  $Q$

Charge on the rightmost plate =  $-2Q$

Consider the gaussian surface as shown in fig.

Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The other parts of closed surface which are outside are parallel to electric field and hence flux on these parts is zero. The total flux of electric field through this closed surface is zero. So, from Gauss's law total charge inside the closed surface be zero



To find the value of  $q$  consider electric field at point P

We know that electric field due to a thin plate of charge  $Q$  is given by

$$E = \frac{Q}{2A\epsilon_0}$$

Where  $A$ =area of plate

Now electric field at point P due to,

Outer surface of plate X =  $\frac{Q-q}{2A\epsilon_0}$  (towards right)

Inner surface of plate X =  $\frac{q}{2A\epsilon_0}$  (towards right)

Left surface of plate Y =  $\frac{q}{2A\epsilon_0}$

(towards left)

Right surface of plate Y =  $\frac{q}{2A\epsilon_0}$

(towards left)

Inner surface of plate Z =  $\frac{q}{2A\epsilon_0}$  (towards right)

Outer surface of plate Z =  $\frac{2Q-q}{2A\epsilon_0}$  (towards right)

**Consider field towards right as positive and towards left as negative**

Net field at point P is given by

$$\frac{Q-q}{2A\epsilon_0} + \frac{q}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} + \frac{q}{2A\epsilon_0} + \frac{2Q-q}{2A\epsilon_0} = \frac{3Q-2q}{2A\epsilon_0}$$

We know that field inside a point inside conductor should be zero

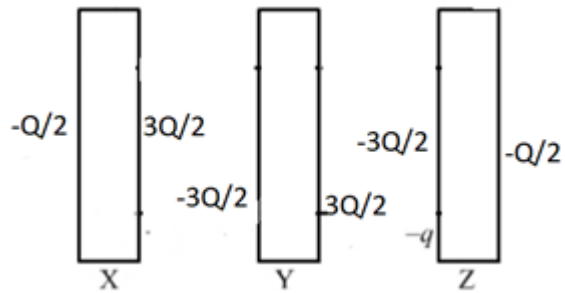


∴ net field at p=0

$$\frac{3Q - 2q}{2A\epsilon_0} = 0$$

$$q = \frac{3Q}{2}$$

The final charge distribution is shown in fig.



Charge on the outer surface of rightmost plate is given by

$$-2Q + q = -2Q + \frac{3Q}{2} = -\frac{Q}{2}$$

**Therefore, the charge on the outer surface of the rightmost plate is given by  $-Q/2$**