1. Geometrical Constructions

- **Construction of perpendicular bisector of a line segment**

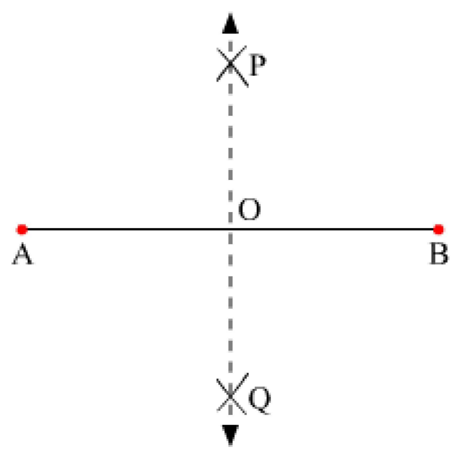
Perpendicular Bisector: A line that bisects a line segment at 90° is called the perpendicular bisector of the line segment.
Example:

Construct a perpendicular bisector of the line segment AB of length 8.2 cm.

Solution:

(1) Draw a line segment AB = 8.2 cm using a ruler.

(2) Draw two arcs taking A and B as centres and radius more than 4.1 cm on both sides of AB. Let the arcs intersect at points P and Q. Join PQ.



PQ is the required perpendicular bisector of line segment AB.

Note: We can verify the validity of construction of perpendicular bisector of a line segment using congruence.

- **Construction Of Bisector Of An Angle**

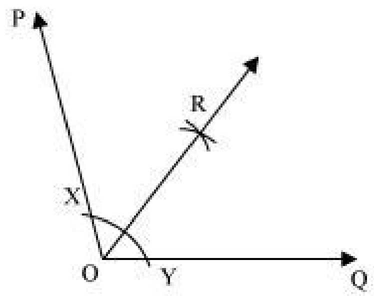
Bisector of an angle: A ray that divides an angle into two equal parts is called the bisector of the angle.

Example:
Construct 55° by bisecting an angle of measure 110°.
Solution:

(i) With the help of a protractor, draw ∠POQ = 110°.

(ii) Draw an arc of any radius taking O as centre. Let this arc intersect the arms OP and OQ at points X and Y respectively.

(iii) Taking X and Y as centres and radius more than half of XY, draw arcs to intersect each other, say at R. Join ray OR.



Now, OR is the bisector of ∠POQ i.e., ∠POR = ∠ROQ = 55°

Note: We can verify the validity of construction of angle bisector using congruence.

- **Construction of incircle of given triangle:**

  Example:

  Construct incircle of a right ΔPQR, right angled at Q, such that QR = 4 cm and PR = 6 cm.

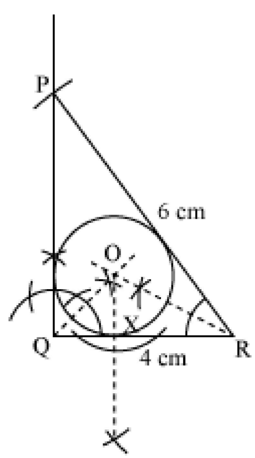
  Solution:

  Step 1: Draw a ΔPQR right-angled at Q with QR = 4 cm and PR = 6 cm.

  Step 2: Draw bisectors of ∠Q and ∠R. Let these bisectors meet at the point O.

  Step 3: From O, draw OX perpendicular to the side QR.

  Step 4: With O as centre and radius equal to OX, draw a circle.



The circle so drawn touches all the sides of ΔPQR and is the required incircle of ΔPQR.

- **Construction of circumcircle of given triangle:**

Example:

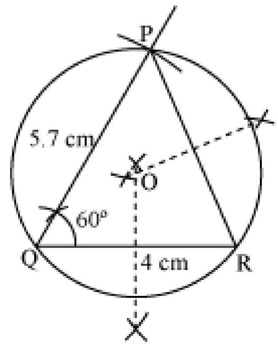
Construct the circumcircle of ΔPQR such that ∠Q = 60°, QR = 4 cm, and QP = 5.7 cm.

Solution:

Step 1: Draw a triangle PQR with ∠Q = 60°, QR = 4 cm, and QP = 5.7 cm

Step 2: Draw perpendicular bisector of any two sides, say QR and PR. Let these perpendicular bisectors meet at point O.

Step 3: With O as centre and radius equal to OP, draw a circle.



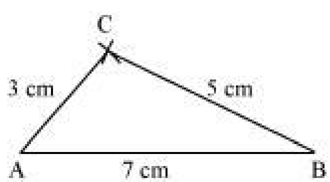
The circle so drawn passes through the points P, Q, and R, and is the required circumcircle of ΔPQR.

- A triangle can be constructed if all its sides are known.

Example:
Construct a triangle whose sides are 3 cm, 5cm and 7 cm.

Solution:

1. Draw a line segment AB of length 7 cm. With A as centre and radius equal to 3 cm, draw an arc.
2. With B as centre and radius 5 cm, draw another arc cutting the earlier drawn arc at C.
3. Join AC and BC to get ΔABC.

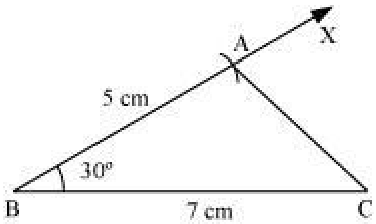


- A triangle can be constructed if the length of two sides and angle between them are given.

Example:
Construct ΔABC where BC = 7 cm, AB = 5 cm and ∠ABC = 30°

Solution:

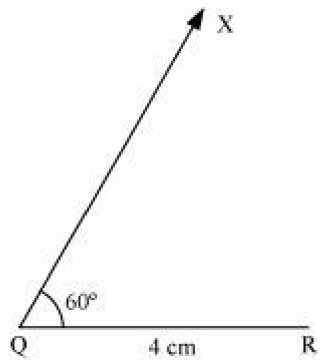
1. Draw a line segment BC of length 7 cm and at B draw a ray BX, making an angle of 30° with BC.
2. With B as centre and radius equal to 5 cm, draw an arc cutting BX at A.
3. Join AC to get the required ΔABC.



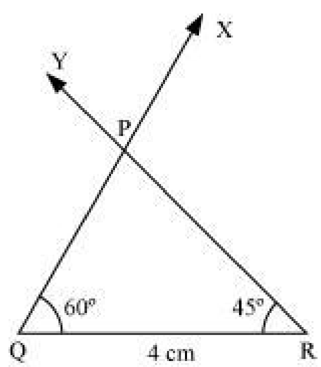
Example: Construct ΔPQR, where ∠PQR = 60°, ∠PRQ = 45° and QR = 4 cm.

Solution:

1. Draw a line segment QR of length 4 cm and draw a ray QX, making an angle of 60° with QR



2. Now, draw ray RY, making an angle of 45° with QR and intersecting QX at P. The resulting ΔPQR is the required triangle.

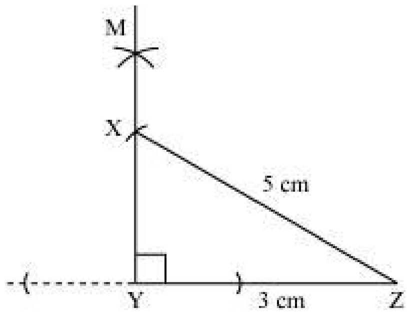


- A right-angled triangle can be constructed if the length of one of its sides or arms and the length of its hypotenuse are known.

  Example:
  Construct ΔXYZ, right-angled at Y, with XZ = 5 cm and YZ = 3 cm.
  Solution:

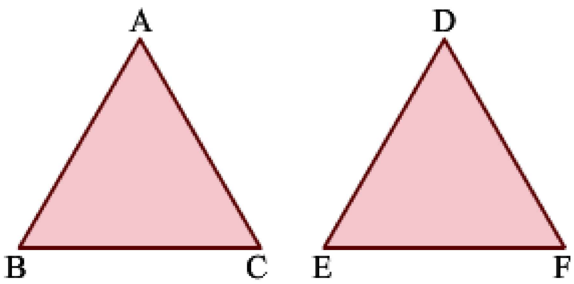
1. Draw a line segment YZ of length 3 cm. At Y, draw MY⊥YZ.

2. With Z as centre and radius equal to 5 cm, draw an arc intersecting MY at X. Join XZ to get the required ΔXYZ.



1. Congruence of triangles:

In the given triangles, ΔABC and ΔDEF are of the same shape and same size so they are congruent.



(i) Two s**calene triangles** are congruent for only one correspondence.

For example, in ΔPQR and ΔXYZ, if ∠P ≅ ∠X, ∠Q ≅ ∠Y, ∠R ≅ ∠Z and side PQ ≅ side XY, side QR ≅ side YZ, side PR ≅ side XZ then ΔPQR ≅ ΔXYZ.

(ii) Two **isosceles triangles** are congruent for two correspondences.

For example, for ΔABC and ΔXYZ with AB = AC and XY = XZ, the possible correspondences are ABC ↔ XYZ and ABC ↔ XZY. Thus, ΔABC ≅ ΔXYZ or ΔABC ≅ ΔXZY

(iii) Two **equilateral triangles** are congruent by all the possible correspondences.

2. Congruence of quadrilaterals:

Two quadrilaterals are congruent if they are of same shape and same size.

For example, in □ABCD and □PQRS, if AB ≅ PQ, BC ≅ QR, CD ≅ RS, DA ≅ SP and ∠A ≅ ∠P, ∠B ≅ ∠Q,∠C ≅ ∠R, ∠D ≅ ∠S then □ABCD ≅ □PQRS.

3. Congruence of circles:

Circles having equal radii are congruent.