DAY ELEVEN

Oscillations

Learning & Revision for the Day

- Periodic Motion
- Simple Harmonic Motion
- Oscillations of a Spring
- Force and Energy in SHM
- Composition of Two SHMs
- Simple Pendulum
- Free, Damped, Forced and Resonant Vibrations

Periodic Motion

A motion which repeats itself over a regular interval of time is called a periodic motion. A periodic motion in which a body moves back and forth repeatedly about a fixed point (called mean position) is called **oscillatory** or **vibratory motion.**

- **Period** The regular interval of time after which periodic motion repeats itself is called period of the motion.
- **Frequency** The number of times of motion repeated in one second is called frequency of the periodic motion. Every oscillatory motion is periodic but every periodic motion is not an oscillatory motion.
- **Displacement as a Function of Time** In a periodic motion each displacement value is repeated after a regular interval of time, displacement can be represented as a function of time.

$$y = f(t)$$

- **Periodic Function** A function which repeats its value after a fix interval of time is called a periodic function.

$$y(t) = y(t + T)$$

where, T is the period of the function.

Trigonometric functions $\sin\theta$ and $\cos\theta$ are simplest periodic functions having period of 2π.

Simple Harmonic Motion

Simple Harmonic Motion (SHM) is that type of oscillatory motion in which the particle moves to and fro or back and forth about a fixed point under a restoring force, whose magnitude is directly proportional to its displacement

i.e. $$F \propto x \text{ or } F = -kx$$

where, k is a positive constant called the **force constant** or **spring factor** and x is displacement.

Differential equations of SHM, for linear SHM, $\dfrac{d^2y}{dt^2} + \omega^2 y = 0$,

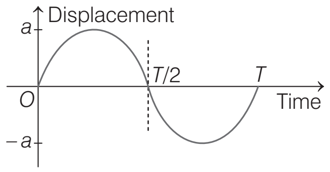
for angular SHM, $\dfrac{d^2\theta}{dt^2} + \omega^2 \theta = 0$

Terms Related to SHM

The few important terms related to simple harmonic motion are given as

- **Displacement** The displacement of a particle executing SHM is, in general, expressed as $y = A\sin(\omega t - \phi)$.

  where, A is the amplitude of SHM, ω is the angular frequency $\left(\text{where } \omega = \frac{2\pi}{T} = 2\pi\nu\right)$ and ϕ is the initial phase of SHM. However, displacement may also be expressed as $x = A\cos(\omega t - \phi)$.

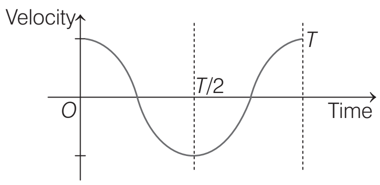


- **Amplitude** The maximum displacement on either side of mean position is called amplitude of SHM.
- **Velocity** The velocity of a particle executing SHM at an instant is defined as the time rate of change of its displacement at that instant.

  Velocity, $\frac{dy}{dt} = v = \omega\sqrt{A^2 - y^2}$

  At the mean position $(y = 0)$, during its motion $v = A\omega = v_{max}$ and at the extreme positions $(y = \pm A)$, $v = 0$.

  $\therefore$ Velocity amplitude, $v_{max} = A\omega$



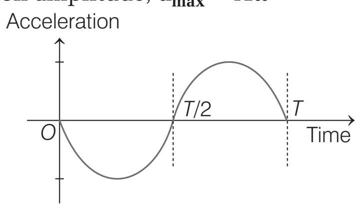
- **Acceleration** The acceleration of a particle executing SHM at an instant is defined as the time rate of change of velocity at that instant.

  Acceleration, $\frac{d^2y}{dt^2} = a = -\omega^2 y$

  The acceleration is also a variable.

  At the mean position $(y = 0)$, acceleration $a = 0$ and at the extreme position $(y = \pm A)$, the acceleration is $a_{max} = -A\omega^2$.

  $\therefore$ Acceleration amplitude, $a_{max} = A\omega^2$



- **Phase** Phase is that physical quantity which tells about the position and direction of motion of any particle at any moment. It is denoted by ϕ.
- **Phase Difference** If two particles perform S.H.M and their equations are

  $$y_1 = a\sin(\omega t + \phi_1) \text{ and } y_2 = a\sin(\omega t + \phi_2)$$

  phase difference $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$
- **Time Period** The time taken by a particle to complete one oscillation is called time period. It is denoted by T.

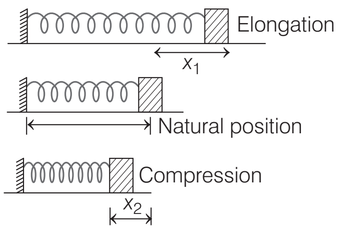
  $\therefore$ Time period of SHM,

  $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{|y|}{|a|}} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$
- **Frequency and Angular Frequency** It is defined as the number of oscillations executed by body per second. SI unit of frequency is hertz.

  Angular frequency of a body executing periodic motion is equal to product of frequency of the body with factor 2π. Angular frequency, $\omega = 2\pi n$.

Oscillations of a Spring

If the mass is once pulled, so as to stretch the spring and is then released, then a restoring force acts on it which continuously tries to restore its mean position.



Restoring force $F = -kl$,

where k is force constant and l is the change in length of the spring.

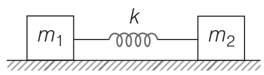
Here, $x_1 = x_2 = l$

- The spring pendulum oscillates simple harmonically having time period and frequency given by

  $$T = 2\pi\sqrt{\frac{m}{k}}$$

  and $$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
- If the spring is not light but has a mass m_s, then

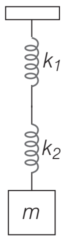
  $$T = 2\pi\sqrt{\frac{m + 1/3\, m_s}{k}}$$
- If two masses m_1 and m_2, connected by a spring, are made to oscillate on a horizontal surface, then its period will be $T = 2\pi\sqrt{\frac{\mu}{k}}$



  where, $\mu = \frac{m_1 m_2}{m_1 + m_2}$ = reduced mass of the system.

Series Combination of Springs

If two springs of spring constants k_1 and k_2 are joined in series (horizontally or vertically), then their equivalent spring constant k_s is given by

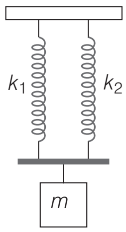


$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow k_s = \frac{k_1 k_2}{k_1 + k_2}$$

$$\therefore \qquad T = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

Parallel Combination of Springs

If two springs of spring constants k_1 and k_2 are joined in parallel as shown in figure, then their equivalent spring constant $k_p = k_1 + k_2$ hence,



$$T = 2\pi\sqrt{\frac{m}{k_p}} = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$$

Force and Energy in SHM

- **Force** For an object executing SHM, a force always acts on it, which tries to bring it in mean position, i.e. it is always directed towards mean position.

  The equation of motion, $\mathbf{F} = m\mathbf{a}$,

  $$\therefore \qquad F = -m\omega^2 x \qquad [\because a = -\omega^2 x]$$

  $$= -kx \qquad \left[\because \omega = \sqrt{\frac{k}{m}}\right]$$

  Here, negative sign shows that direction of force is always opposite to the direction of displacement.

- **Energy** If a particle of mass m is executing SHM, then at a displacement x from the mean position, the particle possesses potential and kinetic energy.

  At any displacement x,

  Potential energy, $U = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} k\, x^2$

  Kinetic energy, $K = \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} k\,(A^2 - x^2)$

  Total energy, $E = U + K = \frac{1}{2} m\omega^2 A^2 = 2\pi^2 m\nu^2 A^2$

  If there is no friction, the total mechanical energy, $E = K + U$, of the system always remains constant even though K and U change.

Composition of Two SHMs

If a particle is acted upon two separate forces each of which can produce a simple harmonic motion. The resultant motion of the particle would be a combination of two SHMs.

For which $\qquad \mathbf{F}_1 + \mathbf{F}_2 = m\dfrac{d^2\mathbf{r}}{dt}$

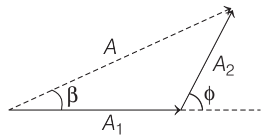
and $\mathbf{r}_1 + \mathbf{r}_2 = \mathbf{r}$ = resultant position of the particle

where, m = mass of the particle.

$\mathbf{r}_1, \mathbf{r}_2$ = positions of the particle under two forces.

There are two cases

- When two SHM are in same direction the resultant is given by

  $$x = x_1 + x_2 = A\sin(\omega t + \beta)$$

  where, $x_1 = A_1 \sin\omega t$,

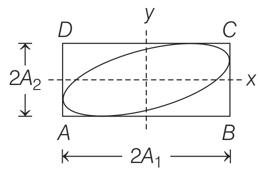
  $$x_2 = A_2 \sin(\omega t + \phi)$$

  $$A = \sqrt{A_1^2 + 2A_1A_2\cos\phi + A_2^2}$$

  and $\quad \tan\beta = \dfrac{A_2\sin\phi}{A_1 + A_2\cos\phi}$

  For any value of ϕ other than 0 and π resultant amplitude is between $|A_1 - A_2|$ and $A_1 + A_2$.

- When two SHM are mutually perpendicular to each other.

  The resultant SHM is given by

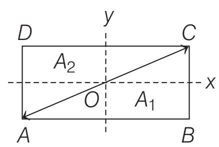
  $$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy\cos\phi}{A_1A_2} = \sin^2\phi \quad \text{(ellipse)}$$

  where, $x = A_1\sin\omega t$ and $y = A_2\sin(\omega t + \phi)$

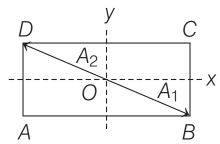
  Here, x is always between $-A_1$ to $+A_1$ and y is always between $-A_2$ to $+A_2$.

NOTE **Special Cases in Composition of Two SHMs**

- When $\phi = 0$, $y = \dfrac{A_2}{A_1}x$

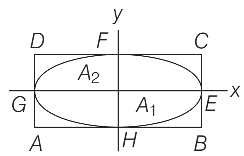


- When $\phi = \pi$, $y = -\dfrac{A_2}{A_1}x$



- When $\phi = \pi/2$. If $A_1 = A_2 = A$, then $\dfrac{x^2}{A_1^2} + \dfrac{y^2}{A_2^2} = 1$

  $x^2 + y^2 = A^2$ (circle)



Simple Pendulum

A simple pendulum, in practice, consists of a heavy but small sized metallic bob suspended by a light, inextensible and flexible string. The motion of a simple pendulum is simple harmonic for very small angular displacement (θ) whose time period and frequency are given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{and} \quad \nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

where, l is the effective length of the string and g is acceleration due to gravity.

- If a pendulum of length l at temperature $\theta°C$ has a time period T, then on increasing the temperature by $\Delta\theta°C$ its time period changes to ΔT,

  where, $$\frac{\Delta T}{T} = \frac{1}{2}\alpha\,\Delta\theta$$

  where, α is the temperature coefficient of expansion of the string.
- A second's pendulum is a pendulum whose time period is 2s. At a place where $g = 9.8\ \text{ms}^{-2}$, the length of a second's pendulum is 0.9929 m (or 1 m approx).
- If the bob of a pendulum (having density ρ) is made to oscillate in a non-viscous fluid of density σ, then it can be shown that the new period is

  $$T = 2\pi\sqrt{\frac{l}{g\left(1 - \frac{\sigma}{\rho}\right)}}$$
- If a pendulum is in a lift or in some other carriage moving vertically with an acceleration a, then the effective value of the acceleration due to gravity becomes $(g \pm a)$ and hence,

  $$T = 2\pi\sqrt{\frac{l}{(g \pm a)}}$$

  Here, positive sign is taken for an upward accelerated motion and negative sign for a downward accelerated motion.
- If a pendulum is made to oscillate in a freely falling lift or an orbiting satellite then the effective value of g is zero and hence, the time period of the pendulum will be infinity and therefore pendulum will not oscillate at all.
- If the pendulum bob of mass m has a charge q and is oscillating in an electrical field E, then

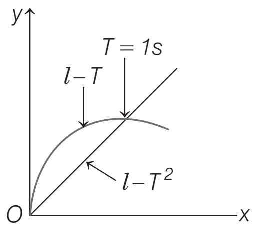
  $$T = 2\pi\sqrt{\frac{l}{\left(g \pm \frac{qE}{m}\right)}}$$

  The positive sign is to be used if the electrical force is acting vertically downwards and negative sign if the electrical force is acting vertically upwards.
- If pendulum of charge q is oscillating in an electric field E acting horizontally, then

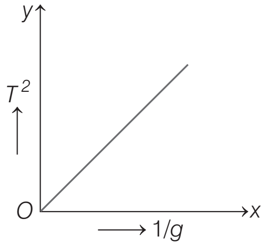
  $$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{q^2E^2}{m^2}}}}$$
- If the length of a simple pendulum is increased to such an extent that $l \to \infty$, then its time period is

  $$T = 2\pi\sqrt{\frac{R}{g}} = 84.6\ \text{min}$$

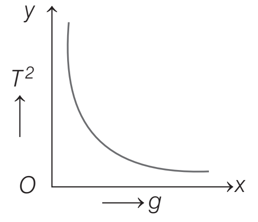
  where, R = radius of the earth.
- The graphs l-T and l-T^2 intersect at $T = 1$ s.



- The graph between T^2 and $1/g$ is a straight line.



- The graph between T^2 and g is a rectangular hyperbola.



Free, Damped, Forced and Resonant Vibrations

Some of the vibrations are described below.

Free Vibrations

If a body, capable of oscillating, is slightly displaced from its position of equilibrium and then released, it starts oscillating with a frequency of its own.

Such oscillations are called free vibrations.The frequency with which a body oscillates is called the **natural frequency** and is given by

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Here, a body continues to oscillate with a constant amplitude and a fixed frequency.

Damped Vibrations

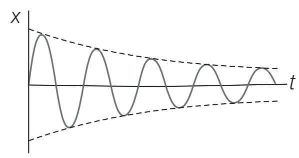
The oscillations in which the amplitude decreases gradually with the passage of time are called damped vibrations. Damping force, $F_d = -bv$

where, v is the velocity of the oscillator and b is a damping constant. The displacement of the oscillator is given by

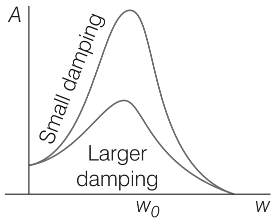
$$x(t) = Ae^{-bt/2m}\sin(\omega' t + \phi)$$

where, $\omega' =$ the angular frequency $= \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

The mechanical energy E of the oscillator is given by $E(t) = \frac{1}{2}kA^2e^{-bt/m}$



(a)



(b)

Forced Vibrations

The vibrations in which a body oscillates under the effect of an external periodic force, whose frequency is different from the natural frequency of the oscillating body, are called forced vibrations.

In forced vibrations the oscillating body vibrates with the frequency of the external force and amplitude of oscillations is generally small.

Resonant Vibrations

It is a special case of forced vibrations in which the frequency of external force is exactly same as the natural frequency of the oscillator.

As a result, the oscillating body begins to vibrate with a large amplitude leading to the phenomenon of resonance to occur. Resonant vibrations play a very important role in music and in tuning of station/channel in a radio/TV etc.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 The displacement of a particle is represented by the equation $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$. The motion of the particle is

(a) simple harmonic with period $2\pi/\omega$
(b) simple harmonic with period π/ω
(c) periodic but not simple harmonic
(d) non-periodic

2 The displacement of a particle is represented by the equation $y = \sin^3 \omega t$. The motion is

(a) non-periodic
(b) periodic but not simple harmonic
(c) simple harmonic with period $2\pi/\omega$
(d) simple harmonic with period π/ω

3 Motion of an oscillating liquid column in a U-tube is

(a) periodic but not simple harmonic
(b) non-periodic
(c) simple harmonic and time period is independent of the density of the liquid
(d) simple harmonic and time period is directly proportional to the density of the liquid

4 The relation between acceleration and displacement of four particles are given below. Which one of the particle is exempting simple harmonic motion?

(a) $a_x = +2x$ (b) $a_x = +2x^2$
(c) $a_x = -2x^2$ (d) $a_x = -2x$

5 A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005\cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are

(a) $\alpha = 25.00\pi, \beta = \pi$ (b) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$
(c) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$ (d) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$

6 The maximum velocity of a particle executing simple harmonic motion with an amplitude 7 mm, is $4.4\ \text{ms}^{-1}$. The period of oscillation is

(a) 0.01 s (b) 10 s (c) 0.1 s (d) 100 s

7 A point mass oscillates along the x-axis according to the law $x = x_0\cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$, then

(a) $A = x_0, \delta = -\frac{\pi}{4}$ (b) $A = x_0\omega^2, \delta = \frac{\pi}{4}$
(c) $A = x_0\omega^2, \delta = -\frac{\pi}{4}$ (d) $A = x_0\omega^2, \delta = \frac{3\pi}{4}$

8 A body is executing SHM when its displacement from the mean position are 4 cm and 5 cm and it has velocity 10 cms^{-1} and 8 cms^{-1}, respectively. Its periodic time t

(a) $\frac{2\pi}{2}$ s (b) π s (c) $\frac{3\pi}{2}$ s (d) 2π s

9 A block rests on a horizontal table, which is executing SHM in the horizontal direction with an amplitude a. If the coefficient of friction is μ, then the block just starts to slip when the frequency of oscillation is

(a) $\frac{1}{2\pi}\sqrt{\frac{\mu g}{a}}$ (b) $2\pi\sqrt{\frac{a}{\mu g}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{a}{\mu g}}$ (d) $\sqrt{\frac{a}{\mu g}}$

10 A coin is placed on a horizontal platform, which undergoes horizontal SHM about a mean position O. The coin placed on the platform does not slip, when angular frequency of the SHM is ω. The coefficient of friction between the coin and platform is μ. The amplitude of oscillation is gradually increased. The coin will be begin to slip on the platform for the first time

(a) at the mean position
(b) at the extreme position of the oscillation
(c) for an amplitude of $\mu g/\omega^2$
(d) for an amplitude of $g/\mu\omega^2$

11 Two particles A and B are oscillating about a point O along a common line such that equation of A is given as $x_1 = a \cos \omega t$ and equation of B is given as $x_2 = b \sin\left(\omega t + \frac{\pi}{2}\right)$.

Then, the motion of A w.r.t. B is

(a) a simple harmonic motion with amplitude $(a - b)$
(b) a simple harmonic motion with amplitude $(a + b)$
(c) a simple harmonic motion with amplitude $\sqrt{a^2 + b^2}$
(d) not a simple harmonic motion but oscillatory motion

12 Two particles execute simple harmonic motion on same straight line with same mean position, same time period 6 s and same amplitude 5 cm. Both the particles start SHM from their mean position (in same direction) with a time gap of 1 s. Find the maximum separation between the two particles during their motion.

(a) 2 cm (b) 3 cm (c) 4 cm (d) 5 cm

13 A particle is acted simultaneously by mutually perpendicular simple harmonic motion $x = a \cos \omega t$ and $y = a \sin \omega t$. The trajectory of motion of the particle will be

(a) an ellipse (b) a parabola
(c) a circle (d) a straight line

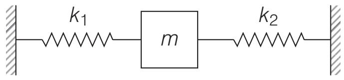
14 A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12} per second. What is the force constant of the bonds connecting one atom with the other? (Take, molecular weight of silver = 108 and Avogadro number = 6.02×10^{23} g mol^{-1}) **→ JEE Main 2018**

(a) 6.4 N/m (b) 7.1 N/m (c) 2.2 N/m (d) 5.5 N/m

15 If a spring of stiffness k is cut into two parts A and B of length $l_A : l_B = 2 : 3$, then the stiffness of spring A is given by **→ AIEEE 2011**

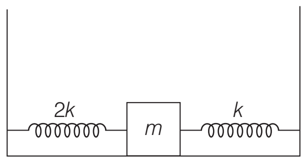
(a) $\frac{5}{2}k$ (b) $\frac{3k}{5}$ (c) $\frac{2k}{5}$ (d) k

16 Two springs of force constants k_1 and k_2, are connected to a mass m as shown. The frequency of oscillation of the mass is ν. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



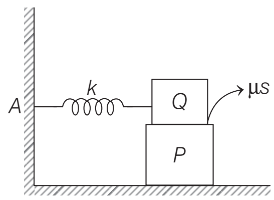
(a) $\frac{\nu}{2}$ (b) $\frac{\nu}{4}$ (c) 4ν (d) 2ν

17 Two springs of force constant k and $2k$ are connected to a mass as shown below. The frequency of oscillation of the mass is



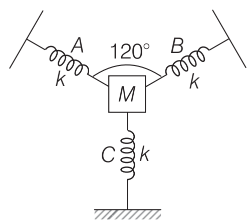
(a) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{m}{k}}$

18 A block P of mass m is placed on a horizontal frictionless plane. A second block Q of the same mass m is placed on it and is connected to a spring of spring constant k, the two blocks are pulled by a distance A. Block Q oscillates without slipping. What is the maximum value of frictional force between the two blocks?



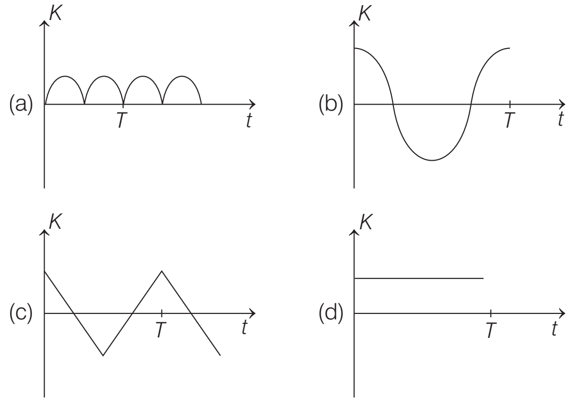
(a) $kA/2$ (b) kA (c) $\mu - mg$ (d) Zero

19 A particle of mass M is attached to three springs A, B and C having equal force constant k. If the particle is pushed a little towards any one of the springs and then left on its own, find the time period of its oscillation.

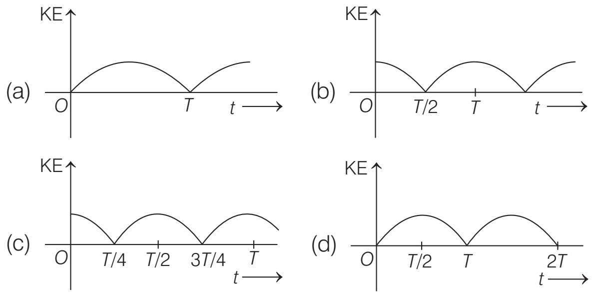


(a) $2\pi\sqrt{(M/k)}$ (b) $2\pi\sqrt{(2M/k)}$
(c) $2\pi\sqrt{(M/2k)}$ (d) $2\pi\sqrt{(M/3k)}$

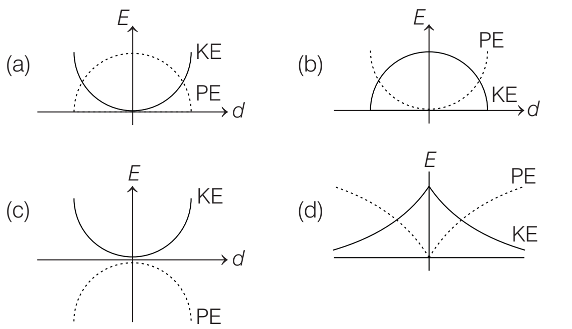
20 A body performs SHM. Its kinetic energy K varies with time T as indicated in the graph



21 A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look, like → **JEE Main 2017 (Offline)**



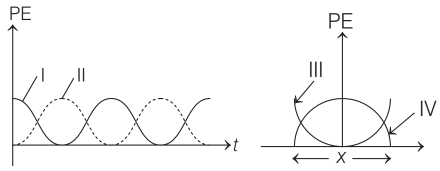
22 For a simple pendulum, a graph is plotted between its Kinetic Energy (KE) and Potential Energy (PE) against its displacement d. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale) → **JEE Main 2015**



23 The total energy of a particle, executing simple harmonic motion is

(a) $\propto x$
(b) $\propto x^2$
(c) independent of x
(d) $\propto x^{1/2}$
where, x is the displacement from the mean position.

24 For a particle executing SHM, the displacement x is given by $x = A \cos \omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x.



(a) I and III (b) II and IV
(c) II and III (d) I and IV

25 A simple pendulum performs simple harmonic motion about $x = 0$ with an amplitude a, and time period T. The speed of the pendulum at $x = a/2$ will be

(a) $\frac{\pi a \sqrt{3}}{T}$ (b) $\frac{\pi a \sqrt{3}}{2T}$ (c) $\frac{3\pi^2 a}{T}$ (d) $\frac{\pi a}{T}$

26 The value of g decrease by 0.1% on a mountain as compared to sea level. If a simple pendulum is used to record the time, then the length must be

(a) increased by 0.1% (b) decreased by 0.1%
(c) increased by 0.2% (d) decreased by 0.2%

27 Two pendulums have time periods T and $\frac{5T}{4}$. They start SHM at the same time from the mean position. What will be the phase difference between them after the bigger pendulum completes one oscillation?

(a) 45° (b) 90° (c) 60° (d) 30°

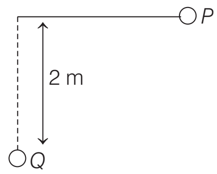
28 A simple pendulum of length l is suspended from the roof of a train which is moving in a horizontal direction with an acceleration a. Then, the time period T is given by

(a) $2\pi\sqrt{l/g}$ (b) $2\pi\sqrt{l/(a^2 + g^2)^{1/2}}$
(c) $2\pi\sqrt{l/(a+g)}$ (d) $2\pi\sqrt{l/(g-a)}$

29 Two simple pendulums of length 1 m and 4 m respectively are both given small displacement in the same direction. The shorter pendulum has completed number of oscillations equal to → **JEE Main (Online) 2013**

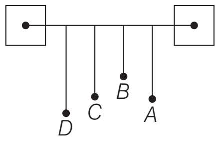
(a) 2 (b) 7
(c) 5 (d) 3

30 A pendulum of length 2m lift at P. When it reaches Q, it losses 10% of its total energy due to air resistance. The velocity of Q is



(a) 2 m/s (b) 1 m/s
(c) 6 m/s (d) 8 m/s

31 Four pendulums A, B, C and D are hung from the same elastic support as shown alongside. A and C are of the same length while B is smaller than A and D is larger than A. A is given a displacement then in steady state



(a) D will vibrate with maximum amplitude
(b) C will vibrate with maximum amplitude
(c) B will vibrate with maximum amplitude
(d) All the four will oscillate with equal amplitude

32 Bob of a simple pendulum of length l is made of iron. The pendulum is oscillating over a horizontal coil carrying direct current. If the time period of the pendulum is T, then → **JEE Main (Online) 2013**

(a) $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is smaller than in air alone
(b) $T = 2\pi\sqrt{\frac{l}{g}}$ and damping is larger than in air alone
(c) $T > 2\sqrt{\frac{l}{g}}$ and damping is smaller than in air alone
(d) $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is larger than in air alone

33 The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5s. In another 10 s it will decreases to α times its original magnitude, where α equals → **JEE Main 2013**

(a) 0.7 (b) 0.81
(c) 0.729 (d) 0.6

Direction (Q. Nos. 34-38) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

34 If two springs S_1 and S_2 of force constants k_1 and k_2, respectively are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement I If stretched by the same amount, work done on S_1, will be more than that on S_2.

Statement II $k_1 < k_2$

35 **Statement I** A particle performing SHM at certain instant is having velocity v. It again acquires a velocity v for the first time after a time interval of T second, then the time period of oscillation is T second.

Statement II A particle performing SHM can have the same velocity at two instants in one cycle.

36 **Statement I** A particle performing SHM while crossing the mean position is having a minimum potential energy, this minimum potential energy could be non-zero.

Statement II In the equilibrium position, the net force experienced by the particle is zero, hence potential energy would be zero at the mean position.

37 **Statement I** A circular metal hoop is suspended on the edge by a hook. The hoop can oscillate from one side to the other in the plane of the hoop, or it can oscillate back and forth in a direction perpendicular to the plane of the hoop.

The time period of oscillation would be more when oscillations are carried out in the plane of the hoop.

Statement II Time period of physical pendulum is more if the moment of inertia of the rigid body about the corresponding axis, passing through the pivoted point is more.

38 **Statement I** The time period of a pendulum, in a satellite orbiting around the earth, is infinity.

Statement II Time period of a pendulum is inversely proportional to the square root of acceleration due to gravity.

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A 15 g ball is shot from a spring gun whose spring has a force constant of 600 Nm^{-1}. The spring is compressed by 5 cm. The greatest possible horizontal range of the ball for this compression is ($g = 10 ms^{-2}$).

(a) 10.0 m (b) 6.0 m (c) 12.0 m (d) 8.0 m

2 Two simple harmonic motions are represented by the equations $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$.

The phase difference of the velocity of particle 1, with respect to the velocity of particle 2 is (at $t = 0$)

(a) $\frac{-\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{-\pi}{3}$ (d) $\frac{\pi}{6}$

3 A piece of wood has dimension $a \times b \times c$. It is floating in a liquid of density ρ such that side a is vertical. It is now pushed down gently and released. The time period is

(a) $2\pi\sqrt{\rho a/g}$ (b) $2\pi\sqrt{abc/g}$
(c) $2\pi\sqrt{g/\rho a}$ (d) $2\pi\sqrt{bc/\rho g}$

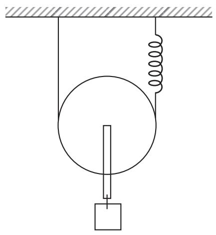
4 The length of a spring is α when a force of 4N is applied on it. The length of a spring is β when a force of 5N is applied on it. Then find the length of the spring when a force of 9N is applied on the spring.

(a) $5\beta - 4\alpha$ (b) $\beta - \alpha$
(c) $5\alpha - 4\beta$ (d) $9(\beta - \alpha)$

5 A simple pendulum of length l has a bob of mass m with a charge q on it. A vertical sheet of charge having surface charge density σ passes through the point of suspension. At equilibrium, the string makes an angle θ with the vertical. If the tension in the string is T then,

(a) $\tan\theta = \frac{\sigma q}{2\varepsilon_0 mg}$ (b) $\tan\theta = \frac{\sigma q}{\varepsilon_0 mg}$
(c) $T > 2\pi\sqrt{\frac{l}{g}}$ (d) $T = 2\pi\sqrt{\frac{l}{g}}$

6 A mass m is suspended from a massless pulley which itself is suspended with the help of a massless extensible spring as shown alongside.



What will be the time period of oscillation of the mass? The force constant of the spring is k.

(a) $\pi\sqrt{m/k}$
(b) $2\pi\sqrt{m/k}$
(c) $4\pi\sqrt{m/k}$
(d) $2\pi\sqrt{m/2k}$

7 If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then which of the following does not change with time?

(a) $a^2T^2 + 4\pi^2v^2$ (b) $\frac{aT}{x}$
(c) $aT + 2\pi v$ (d) $\frac{aT}{v}$

8 A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = kt^2, (k = 1 ms^{-2})$, where y is the vertical displacement. The time period now becomes T_2. The ratio of $\frac{T_1^2}{T_2^2}$ is (take, $g = 10 ms^{-2}$)

(a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) 1 (d) $\frac{4}{5}$

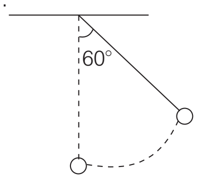
9 A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes T_M. If the Young's modulus of the material of the wire is Y, then $1/Y$ is equal to (g = gravitational acceleration) **→ JEE Main 2015**

(a) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$ (b) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$
(c) $\left[1 - \left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (d) $\left[1 - \left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

10 The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till the water is coming out, the time period of oscillation would

(a) first increase and then decrease to the original value
(b) first decrease and then increase to the original value
(c) remain unchanged
(d) increase towards a saturation value

11 A pendulum of length $l = 1$ m is released from $\theta_0 = 60°$. The rate of change of speed of the bob at $\theta = 30°$ is (take, $g = 10 m/s^2$).



(a) $5\sqrt{3}$ m/s^2 (b) 5 m/s^2
(c) 10 m/s^2 (d) 2.5 m/s^2

12 A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, Then,

(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$
(c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

13 A particle performs simple harmonic motion with amplitude A. Its speed is tripled at the instant that it is at a distance $\frac{2}{3}A$ from equilibrium position. The new amplitude of the motion is **→ JEE Main 2016 (Offline)**

(a) $\frac{A}{3}\sqrt{41}$ (b) $3A$
(c) $A\sqrt{3}$ (d) $\frac{7}{3}A$

14 A wooden cube (density of wood d) of side l floats in a liquid of density ρ with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period T. Then, T is equal to

(a) $2\pi\sqrt{\frac{l\rho}{(\rho - d)g}}$ (b) $2\pi\sqrt{\frac{ld}{\rho g}}$
(c) $2\pi\sqrt{\frac{l\rho}{dg}}$ (d) $2\pi\sqrt{\frac{ld}{(\rho - d)g}}$

15 Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

16 A particle moves with simple harmonic motion in a straight line. In first τ sec, after starting from rest it travels a distance a and in next τ sec, it travels $2a$, in same direction, then **→ JEE Main 2014**

(a) amplitude of motion is $3a$
(b) time period of oscillations is 8τ
(c) amplitude of motion is $4a$
(d) time period of oscillations is 6τ

17 An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely, isolated from its surrounding, the piston executes a simple harmonic motion with frequency **→ JEE Main 2013**

(a) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (b) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$
(c) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

18 If a simple pendulum has significant amplitude (up to a factor of $1/e$ of original) only in the period between $t = 0$ s to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity with b as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds

(a) $\frac{0.693}{b}$ (b) b
(c) $\frac{1}{b}$ (d) $\frac{2}{b}$

ANSWERS

SESSION 1

1 (b)	**2** (b)	**3** (c)	**4** (d)	**5** (a)	**6** (a)	**7** (d)	**8** (b)	**9** (a)	**10** (c)
11 (a)	**12** (d)	**13** (c)	**14** (b)	**15** (a)	**16** (d)	**17** (c)	**18** (a)	**19** (c)	**20** (a)
21 (c)	**22** (b)	**23** (c)	**24** (a)	**25** (a)	**26** (b)	**27** (b)	**28** (b)	**29** (a)	**30** (c)
31 (b)	**32** (d)	**33** (c)	**34** (d)	**35** (d)	**36** (c)	**37** (a)	**38** (a)		

SESSION 2

1 (a)	**2** (a)	**3** (a)	**4** (a)	**5** (a)	**6** (a)	**7** (b)	**8** (a)	**9** (a)	**10** (a)
11 (b)	**12** (b)	**13** (d)	**14** (b)	**15** (a)	**16** (d)	**17** (c)	**18** (d)		

Hints and Explanations

SESSION 1

1 Given, $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$ ...(i)

Velocity, $v = \frac{dy}{dt}$

$= 3 \times 2\,\omega \sin\left(\frac{\pi}{4} - 2\omega t\right)$

Acceleration, $a = \frac{dv}{dt}$

$= -4\omega^2 \times 3\cos\left(\frac{\pi}{4} - 2\,\omega\, t\right)$

$= -4\,\omega^2 y$

As $a \propto y$ and negative sign shows that, it is directed towards equilibrium (or mean position), hence particle will execute SHM.
Comparing Eq. (i) with equation

$y = r\cos(\phi - \omega' t)$

We have, $\omega' = 2\omega$

or $\frac{2\pi}{T'} = 2\omega$ or $T' = \frac{\pi}{\omega}$

2 Given equation of motion is

$y = \sin^3 \omega t$

$= (3\sin\omega t - \sin 3\omega t)/4$

$[\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta]$

$\Rightarrow \frac{dy}{dt} = \left[\frac{d}{dt}(3\sin\omega t) - \frac{d}{dt}(\sin 3\omega t)\right]/4$

$\Rightarrow 4\frac{dy}{dt} = 3\omega\cos\omega t - [3\omega\cos 3\omega t]$

$\Rightarrow 4 \times \frac{d^2 y}{dt^2} = -3\omega^2 \sin\omega t + 9\omega^2 \sin 3\omega t$

$\Rightarrow \frac{d^2 y}{dt^2} = -\left(\frac{3\omega^2 \sin\omega t - 9\omega^2 \sin 3\omega t}{4}\right)$

$\Rightarrow \frac{d^2 y}{dt^2}$ is not proportional to y.

Hence, motion is not SHM.

As the expression is involving sine function, hence it will be periodic.

3 The motion of an oscillating liquid column in a U-tube is simple harmonic and the time period is independent of the density of the liquid.

$T = 2\,\pi\sqrt{\frac{h}{g}}$

where, h = height of liquid in each column.

4 For motion to be SHM acceleration of the particle must be proportional to negative of displacement.

i.e. $a \propto -(y \text{ or } x)$

We should be clear that y has to be linear.

5 Given, $y = 0.005\cos(\alpha x - \beta t)$

Comparing the equation with the standard form,

$y = A\cos\left(\frac{x}{\lambda} - \frac{t}{T}\right)2\,\pi$

we have, $2\pi/\lambda = \alpha$

and $2\pi/T = \beta$

$\Rightarrow$ $\alpha = 2\,\pi/0.08 = 25.00\,\pi$

and $\beta = \pi$

6 Maximum velocity $v = A\omega$,

(where, A is the amplitude and ω is the angular frequency of oscillation).

$\therefore$ $4.4 = (7 \times 10^{-3}) \times 2\,\pi/T$

or $T = \frac{7 \times 10^{-3}}{4.4} \times \frac{2 \times 22}{7} = 0.01$ s

7 Given, $x = x_0 \cos\left(\omega\, t - \frac{\pi}{4}\right)$

Acceleration, $a = \frac{d^2 x}{dt^2}$

$= -\omega^2 x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$

$= \omega^2 x_0 \cos\left(\omega t + \frac{3\pi}{4}\right)$

So, $A = \omega^2 x_0$ and $\delta = \frac{3\pi}{4}$

8 Using $v^2 = \omega^2(a^2 - y^2)$, we have

$10^2 = \omega^2(a^2 - 4^2)$

and $8^2 = \omega^2(a^2 - 5^2)$

So, $10^2 - 8^2 = \omega^2(5^2 - 4^2) = (3\omega)^2$

$\Rightarrow$ $6 = 3\omega$ or $\omega = 2$

$\therefore$ $T = 2\,\pi/\omega$

$= 2\pi/2 = \pi$ s

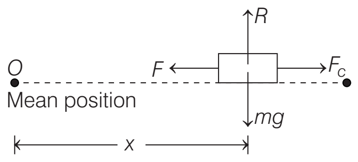
9 Force of friction $= \mu\, m\, g = m\omega^2 a$

$= m\;(2\,\pi\nu)^2 a$

$\Rightarrow$ $\nu = \frac{1}{2\pi}\sqrt{\frac{\mu\, g}{a}}$

10 Let O be the mean position and x be the distance of the coin from O. The coin will slip, if centrifugal force on the coin just becomes equal to the force of friction i.e.

$m\, x\omega^2 = \mu\, m\, g$



From the diagram,
$m\, A\omega^2 = \mu\, mg$ or $A = \mu g/\omega^2$

11 The displacement of A relative to B is

$x = x_1 - x_2$

$x = a\cos\omega t - b\sin\left(\omega t + \frac{\pi}{2}\right)$

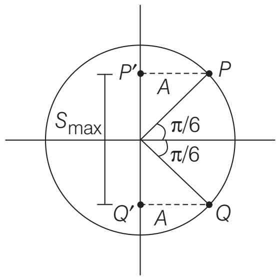
$= a\cos\,\omega t - b\cos\omega t$

$= (a - b)\cos\omega t$

Which is a simple harmonic motion with amplitude $(a - b)$.

12 Phase difference,

$\phi = \omega t = \frac{2\pi}{6} \times 1 = \frac{\pi}{3}$ rad



The maximum separation between the two particles is

$S_{\text{max}} = 2A\sin\frac{\pi}{6}$

or $S_{\text{max}} = 2 \times 5 \times \frac{1}{2} = 5$ cm

13 Given, $x = a\cos\omega t$ ...(i)

$Y = a\sin\omega t$ ...(ii)

Squaring and adding Eqs.(i) and (ii), we get

$x^2 + y^2 = a^2(\cos^2\omega t + \sin^2\omega t)$

$= a^2$

$[\because \cos^2\omega t + \sin^2\omega t = 1]$

This is the equation of a circle.

Clearly, the locus is a circle of constant radius a.

14 For a harmonic oscillator,

$T = 2\pi\sqrt{\frac{m}{k}}$

where, k = force constant and $T = \frac{1}{\nu}$

$\therefore\ k = 4\pi^2\nu^2 m$

$= 4 \times \left(\frac{22}{7}\right)^2 \times (10^{12})^2 \times \frac{108 \times 10^{-3}}{6.02 \times 10^{23}}$

$\Rightarrow\ k = 7.1$ N/m

15 For spring, $k \propto \frac{1}{l}$

$\therefore\ \frac{k_A}{k_B} = \frac{l_B}{l_A} \Rightarrow k_A = \frac{l_A + l_B}{l_A}k = \frac{5}{2}k$

16 We know that, $\nu = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$

When k_1 and k_2 are made four times their original value.

Then, and

$$\nu' = \frac{1}{2\pi}\cdot 2\sqrt{\frac{k_1 + k_2}{m}} = 2\nu$$

17 The effective spring constant is $K = k + 2k = 3k$.
The time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{m}{3k}} \text{ and } \nu = \frac{1}{T}$$

so, we get

$$\nu = \frac{1}{2\pi}\sqrt{\frac{3k}{m}}$$

18 Angular frequency of the system,

$$\omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}$$

Maximum acceleration of the system will be, $\omega^2 A$ or $\frac{kA}{2m}$. This acceleration of the lower block, is provided by friction.
Hence, $f_{\max} = ma_{\max} = m\,\omega^2 A$

$$= m\left(\frac{kA}{2m}\right) = \frac{kA}{2}$$

19 When the mass m is pushed in a downward direction through a distance x, the effective restoring force, in magnitude is

$F = kx + kx\cos 60^\circ + kx\cos 60^\circ$
$= 2kx$

$\therefore$ Spring factor, $k' = 2k$
and Inertia factor $= M$

So time period, $T = 2\pi\sqrt{\frac{M}{2k}}$

20 The frequency of kinetic energy is twice that of a particle executive SHM.

21 KE is maximum at mean position and minimum at extreme position $\left(\text{at } t = \frac{T}{4}\right)$.

22 During oscillation, motion of a simple pendulum KE is maximum at the mean position where PE is minimum. At extreme position, KE is minimum and PE is maximum. Thus, correct graph is depicted in option (b).

23 In a simple harmonic motion, when a particle is displaced to a position from its mean position, its kinetic energy is converted into potential energy. Hence, total energy of a particle remains constant or the total energy in simple harmonic motion does not depend on the displacement x.

24 Potential energy is minimum (in this case zero) at the mean position $(x = 0)$ and maximum at the extreme positions $(x = \pm A)$.

At time $t = 0$, $x = A$, the potential energy should be maximum. Therefore, graph I is correct. Further in graph III, potential energy is minimum at $x = 0$. Hence, this is also correct.

25 Since, $v = \omega\sqrt{a^2 - y^2}$,

At, x or $y = a/2$

$$\Rightarrow \quad v = \omega\sqrt{a^2 - \frac{a^2}{4}} = \omega\sqrt{\frac{3a^2}{4}}$$

$$= \frac{2\pi}{T} \times \frac{\sqrt{3}\,a}{2} = \frac{\pi\sqrt{3}\,a}{T}$$

26 As, $T = 2\pi\sqrt{l/g}$

Taking log and differentiating the expression, keeping T constant we have

$$\frac{dl}{l} = \frac{dg}{g} = -\frac{0.1}{100}$$

$$\therefore \quad (dl/l) \times 100 = -0.1/100 \times 100$$
$$= -0.1\%$$

27 When bigger pendulum of time period $(5T/4)$ completes one oscillation, the smaller pendulum will complete $(5/4)$ oscillation. It means, the smaller pendulum will be leading the bigger pendulum by a phase of $T/4 = \pi/2$ rad $= 90^\circ$.

28 Effective acceleration $= \sqrt{a^2 + g^2}$

$$\therefore \text{Time period, } T = 2\pi\sqrt{\frac{l}{(a^2 + g^2)^{1/2}}}$$

29 Let T_1 and T_2 be the time period of shorter length and larger length pendulums respectively. According to question,

$$nT_1 = (n-1)T_2$$

So, $n\,2\pi\sqrt{\frac{1}{8}} = (n-1)\,2\pi\sqrt{\frac{4}{8}}$

or $n = (n-1)2 = 2n - 2 \Rightarrow n = 2$

30 By applying conservation of energy between P and Q

$\frac{1}{2}mv^2 = 0.9(mgh)$

$\Rightarrow v^2 = 2 \times 0.9 \times 10 \times 2 = 36 \Rightarrow v = 6$ m/s

31 As A and C are of same length, so they will be in resonance, hence C will vibrate with the maximum amplitude.

32 $T < 2\pi\sqrt{\frac{l}{g}}$

As, current passed through in the coil which attracts the molecules of air closer to it, thus density of air increases which produces larger damping than that in air alone.

33 Amplitude of damped oscillator,

$A = A_0 e^{-\frac{bt}{2m}}$

After 5 s, $\quad 0.9A_0 = A_0 e^{-\frac{b(5)}{2m}}$

$$\Rightarrow \quad 0.9 = e^{-\frac{b(5)}{2m}} \quad \ldots\text{(i)}$$

After 10 more second,

$$A = A_0 e^{-\frac{b(15)}{2m}} = A_0\left(e^{-\frac{5b}{2m}}\right)^3 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$A = 0.729A_0$$

Hence, $\quad \alpha = 0.729$

34 As no relation between k_1 and k_2 is given in the question, that is why, nothing can be predicted about Statement I. But as in Statement II, $k_1 < k_2$

Then, for same force

$$W = F\cdot x = F\cdot\frac{F}{K} = \frac{F^2}{K}$$

$\Rightarrow \quad W \propto \frac{1}{k}$

i.e. $\quad W_1 > W_2$

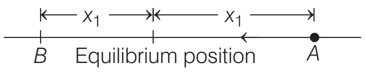
But for same displacement,

$W = F\cdot x = \frac{1}{2}kx\cdot x = \frac{1}{2}kx^2$

$\Rightarrow W \propto k$, i.e. $W_1 < W_2$

Thus, in the light of Statement II, Statement I is false.

35 Consider the situation as shown in the adjoint figure. Let us say at any instant t_1, the particle crosses A as shown, the particle again acquires the same velocity, when it crosses B let us say at instant t_2. According to statement I, $(t_2 - t_1)$ is the time period of SHM which is wrong.



36 At the mean position,

$$F = 0 = -\frac{dU}{dx} = 0$$

$\Rightarrow U =$ constant which can be zero or non-zero.

37 When the hoop oscillates in its plane, moment of inertia is

$$I_1 = mR^2 + mR^2 \text{ i.e. } I_1 = 2mR^2$$

While when the hoop oscillates in a direction perpendicular to the plane of the hoop, moment of inertia is

$$I_2 = \frac{mR^2}{2} + mR^2 = \frac{3mR^2}{2}$$

Time period of physical pendulum is,

$T = 2\pi\sqrt{\frac{I}{mgd}}$; d is same in both the cases.

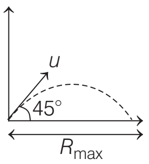
38 From the relation of the time period,

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \frac{1}{\sqrt{g}}$$

When the satellite is orbiting around the earth, the value of g inside it is zero. Hence, the time period of pendulum in a satellite will be infinity and it is also clear that time period of pendulum is inversely proportional to square root of acceleration due to gravity g.

SESSION 2

1 For getting horizontal range, there must be some inclination of spring with ground to project ball.



$$R_{max} = \frac{u^2}{g}$$

But KE acquired by ball
= PE of spring gun

$$\Rightarrow \quad \frac{1}{2}mu^2 = \frac{1}{2}kx^2 \Rightarrow u^2 = \frac{kx^2}{m}$$

$$\Rightarrow \quad R_{max} = \frac{kx^2}{mg} = \frac{600 \times (5 \times 10^{-2})^2}{15 \times 10^{-3} \times 10}$$

$$= 10 \text{ m}$$

2 Given, $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$

$$\Rightarrow \frac{dy_1}{dt} = v_1 = 0.1 \times 100\,\pi \cos\left(100\,\pi\, t + \frac{\pi}{3}\right)$$

or $v_1 = 10\,\pi \sin\left(100\pi t + \frac{\pi}{3} + \frac{\pi}{2}\right)$

or $v_1 = 10\pi \sin\left(100\pi t + \frac{5\pi}{6}\right)$

and $\quad y_2 = 0.1 \cos\,\pi t$

$\Rightarrow \quad \frac{dy_2}{dt} = v_2 = -0.1 \sin\,\pi t$

or $\quad v_2 = 0.1 \sin(\pi t + \pi)$

Hence, the phase difference

$$\Delta\phi = \phi_1 - \phi_2 = \left(100\pi t + \frac{5\pi}{6}\right) - (\pi t + \pi)$$

$$= \frac{5\pi}{6} - \pi = -\frac{\pi}{6} \qquad (\text{at } t = 0)$$

3 Force of buoyancy $= b \times c \times \rho_w \times g$

$= bc\,g \qquad (\because \rho_w = 1)$

and mass of piece of wood $= abc\,\rho$

So, acceleration

$= -bc\,g/abc\,\rho = -(g/a\rho)$

Hence, time period, $T = 2\pi\sqrt{\frac{\rho a}{g}}$

4

$$4 = k(\alpha - l)$$
$$5 = k(\beta - l)$$
$$9 = k(\gamma - l) \Rightarrow \frac{4}{5} = \frac{\alpha - l}{\beta - l}$$

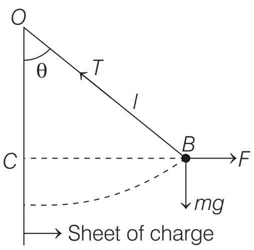
or $\quad 4\beta - 4l = 5\alpha - 5l$

$\quad l = 5\alpha - 4\beta$

Now, $9\alpha - 9l = 4\gamma - 4l$

$$4\gamma = 9\alpha - 5l = 9\alpha - 5(5\alpha - 4\beta)$$
$$= 9\alpha - 25\alpha + 20\beta$$
$$= 20\beta - 16\alpha$$
$$\gamma = 5\beta - 4\alpha$$

5 In the figure, we represent the electric intensity at B due to the sheet of charge,



$$E = \frac{1}{2}\frac{\sigma}{\varepsilon_0}$$

Force on bob due to the sheet of charge,

$$F = qE = \frac{1}{2}\frac{\sigma q}{\varepsilon_0}$$

As the bob is in equilibrium,

so $\frac{mg}{OC} = \frac{F}{CB} = \frac{T}{BO}$

or $\frac{CB}{OC} = \frac{F}{mg} = \frac{\sigma q}{2\varepsilon_0\, mg} = \tan\theta$

6 If mass m moves down a distance y, then the spring is pulled by $2y$ and the force with which the spring is pulled will be $F = R = mg/2$.

Hence, $mg/2 = k(2y)$

$\Rightarrow \quad y/g = m/4k$

$\Rightarrow \quad T = 2\pi\sqrt{y/g}$

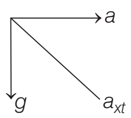
$= 2\pi\sqrt{m/4k} = \pi\sqrt{m/k}$

7 As, $\frac{aT}{x} = \frac{\omega^2 xT}{x}$

$$= \frac{4\pi^2}{T^2} \times T = \frac{4\pi^2}{T}$$

= constant.

8 Given, $y = kt^2 \Rightarrow a = \frac{d^2y}{dt^2} = 2k$



or $\quad a_y = 2 \text{ m/s}^2 \qquad (\text{as, } k = 1 \text{ m/s}^2)$

$\therefore \quad T_1 = 2\pi\sqrt{\frac{l}{g}}$

and $\quad T_2 = 2\pi\sqrt{\frac{l}{g + a_y}}$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{g + a_y}{g} = \frac{10 + 2}{10} = \frac{6}{5}$$

9 We know that time period,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

When additional mass M is added to its bob

$$T_M = 2\pi\sqrt{\frac{L + \Delta L}{g}},$$

where, ΔL is increase in length.

We know that Young modulus of the material

$$Y = \frac{Mg/A}{\Delta L/L} = \frac{MgL}{A\Delta L}$$

$$\Rightarrow \quad \Delta L = \frac{MgL}{AY}$$

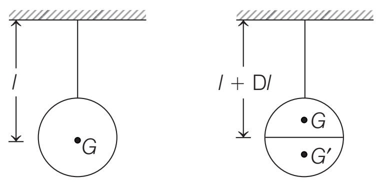
$$T_M = 2\pi\sqrt{\frac{L + \frac{MgL}{AY}}{g}}$$

$$\Rightarrow \quad \left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY}$$

or $\quad \frac{Mg}{AY} = \left(\frac{T_M}{T}\right)^2 - 1$

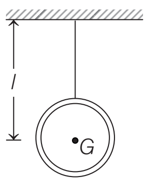
or $\quad \frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$

10



Spherical hollow ball filled with water $\qquad$ Spherical hollow ball half filled with water

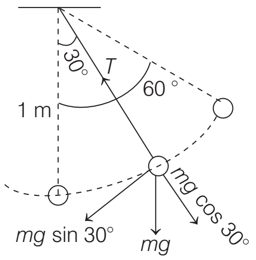
$$T = 2\pi\sqrt{\frac{l}{g}} \qquad T_1 = 2\pi\sqrt{\frac{l + \Delta l}{g}}$$



Spherical hollow ball

$$T_2 = 2\pi\sqrt{\frac{l}{g}} \text{ and } T_1 > T_2$$

Hence, time period first increases and then decreases to the original value.

11 

Rate of change of speed $\frac{dv}{dt}$

$= $ tangential acceleration

$= \frac{\text{tangential force}}{\text{mass}} = \frac{mg\sin 30°}{m}$

$= g\sin 30° = 10\left(\frac{1}{2}\right)\text{m/s}^2 = 5\,\text{m/s}^2$

12 Time period of the spring,

$$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$

Here, k be the force constant of spring. For the first spring,

$$t_1 = 2\pi\sqrt{\left(\frac{m}{k_1}\right)} \quad \text{...(i)}$$

For the second spring,

$$t_2 = 2\pi\sqrt{\left(\frac{m}{k_2}\right)} \quad \text{...(ii)}$$

The effective force constant in the series combination is

$$k = \frac{k_1k_2}{k_1 + k_2}$$

Time period of combination

$$T = 2\pi\sqrt{\left[\frac{m(k_1+k_2)}{k_1k_2}\right]}$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2 m(k_1+k_2)}{k_1k_2} \quad \text{...(iii)}$$

From Eqs. (i) and (ii), we get

$$t_1^2 + t_2^2 = 4\pi^2\left(\frac{m}{k_1} + \frac{m}{k_2}\right)$$

or $$t_1^2 + t_2^2 = 4\pi^2 m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

or $$t_1^2 + t_2^2 = \frac{4\pi^2 m(k_1+k_2)}{k_1k_2}$$

$$\Rightarrow \quad t_1^2 + t_2^2 = T^2 \quad \text{[from Eq. (iii)]}$$

13 The velocity of a particle executing SHM at any instant, is defined as the time rate of change of its displacement at that instant.

$$v = \omega\sqrt{A^2 - x^2}$$

where, ω is angular frequency, A is amplitude and x is displacement of a particle.

Suppose that the new amplitude of the motion be A'.

Initial velocity of a particle performs SHM,

$$v^2 = \omega^2\left[A^2 - \left(\frac{2A}{3}\right)^2\right] \quad \text{... (i)}$$

where, A is initial amplitude and ω is angular frequency.

Final velocity,

$$(3v)^2 = \omega^2\left[A'^2 - \left(\frac{2A}{3}\right)^2\right] \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

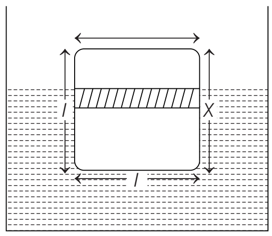
$$\frac{1}{9} = \frac{A^2 - \frac{4A^2}{9}}{A'^2 - \frac{4A^2}{9}} \Rightarrow A' = \frac{7A}{3}$$

14 Let at any instant, cube is at a depth x from the equilibrium position, then net force acting on the cube = upthrust on the portion of length x

$\therefore \; F = -\rho l^2 x g = -\rho l^2 g\, x \quad \text{...(i)}$

Negative sign shows that, force is opposite to x.

Hence, equation of SHM



$$F = -kx \quad \text{...(ii)}$$

Comparing Eqs.(i) and (ii), we get

$$k = \rho l^2 g$$

$$\therefore \; T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l^3 d}{\rho l^2 g}} = 2\pi\sqrt{\frac{ld}{\rho g}}$$

15 Let $x_1 = A\sin(\omega t + \phi_1)$

and $x_2 = A\sin(\omega t + \phi_2)$

$x_2 - x_1 = A$

$[\sin(\omega t + \phi_2) - \sin(\omega t + \phi_1)]$

$$= 2A\cos\left(\frac{2\omega t + \phi_1 + \phi_2}{2}\right)\sin\left(\frac{\phi_2 - \phi_1}{2}\right)$$

The resultant motion can be treated as a simple harmonic motion with amplitude $2A\sin\left(\frac{\phi_2 - \phi_1}{2}\right)$

Given, maximum distance between the particles $= X_0 + A$

$\therefore$ Amplitude of resultant SHM

$$= X_0 + A - X_0 = A$$

$$\therefore \; 2A\sin\left(\frac{\phi_2 - \phi_1}{2}\right) = A \Rightarrow$$

$$\phi_2 - \phi_1 = \pi/3$$

16 In SHM, a particle starts from rest, we have

i.e. $x = A\cos\omega t$, at $t = 0, x = A$

When $t = \tau$, then $x = A - a$ ...(i)

When $t = 2\tau$, then $x = A - 3a$ ...(ii)

On comparing Eqs. (i) and (ii), we get

$$A - a = A\cos\omega\tau$$

$$A - 3a = A\cos 2\omega\tau$$

As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$

$$\frac{A - 3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$$A^2 - 3aA = A^2 + 2a^2 - 4Aa$$

$$a^2 = 2aA, \quad A = 2a$$

Now, $A - a = A\cos\omega\tau \Rightarrow \cos\omega\tau = 1/2$

$$\Rightarrow \quad \frac{2\pi}{T}\tau = \frac{\pi}{3} \Rightarrow T = 6\tau$$

17 $\frac{Mg}{A} = P_0 \Rightarrow Mg = P_0A$

$$P_0V_0^\gamma = (P_0 + \Delta V_0)(V_0 - \Delta V_0)^\gamma$$

$$\Rightarrow \quad P_0 = (P_0 + \Delta P_0)\left(1 - \frac{\Delta V_0}{V_0}\right)^\gamma$$

$$= (P_0 + \Delta P_0)\left(1 - r\frac{\Delta V_0}{V_0}\right)$$

$$= \left(P_0 - VP_0\frac{\Delta V_0}{V_0} + \Delta P_0\right)$$

or $\Delta P_0 = VP_0\frac{\Delta V_0}{V_0}$

But $\Delta V = Ax$,

where, A = area at cross section of piston

$\therefore \; \Delta P_0 = \frac{\gamma P_0 A}{V_0}x$

Restoring force $F = -\Delta P_0 \times A = -\frac{\gamma P_0 A^2}{V_0}x$

Comparing it with, $F_{res} = -kx$

$$k = \frac{\gamma P_0 A^2}{V_0}$$

$$\therefore \; f = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A^2}{MV_0}}$$

18 For damped harmonic motion,

$$ma = -kx - mbv$$

or $$ma + mbv + kx = 0$$

Solution to above equation is

$$x = A_0 e^{-\frac{bt}{2}}\sin\omega t; \text{ with } \omega^2 = \frac{k}{m} - \frac{b^2}{4m}$$

where, amplitude drops exponentially with time.

$$A_\tau = A_0 e^{-\frac{b\tau}{2}}$$

Average time τ is that duration when amplitude drops by 63%, i.e. becomes A_0/e

Thus, $$A_\tau = \frac{A_0}{e}A_0 e^{-\frac{b\tau}{2}}$$

or $$\frac{b\tau}{2} = 1 \text{ or } \tau = \frac{2}{b}$$