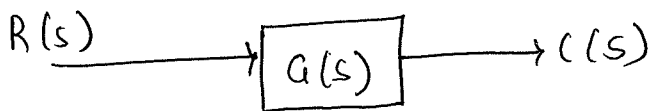
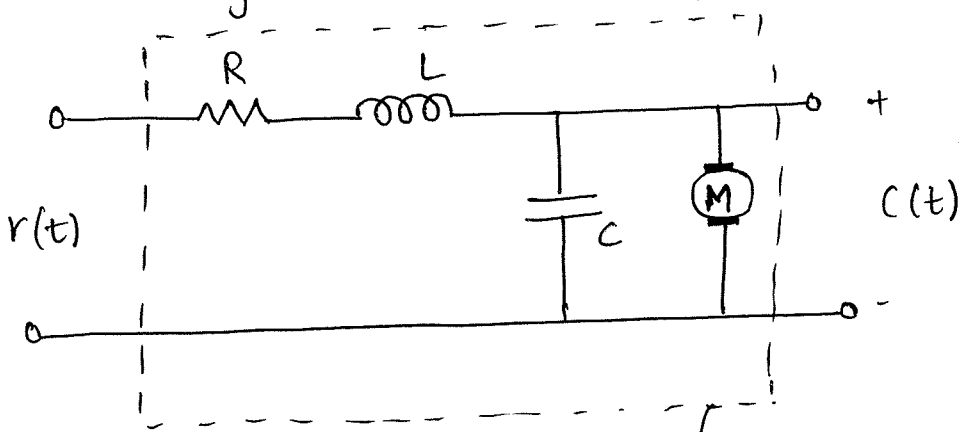


Ch- Block diagram and Signal Flow Graph (SFG)

What is purpose?

- Purpose of block diagram is to find the overall transfer function of the system between input and output.

- Block diagram is short hand pictorial representation of the system between input and output.



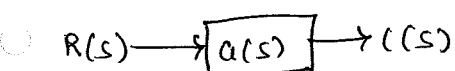
short hand pictorial representation of system betⁿ i/p & o/p.

- The system can be represented in two ways

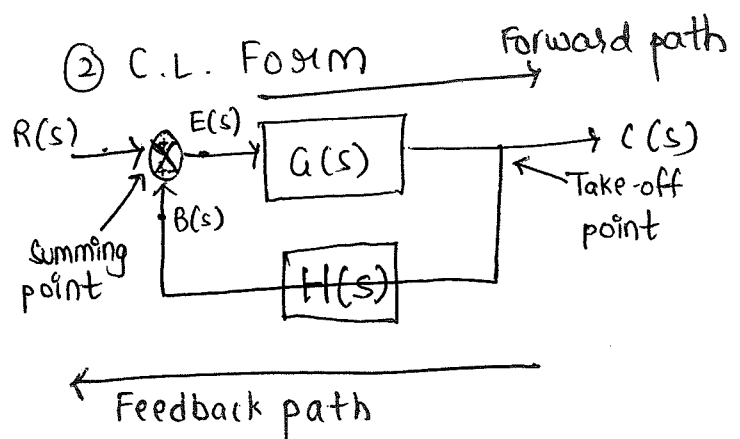
(1) Open loop form

(2) Close loop form

① O.L. Form



$$OL \text{ Gain} = OLTF = \frac{C(s)}{R(s)} = G(s)$$

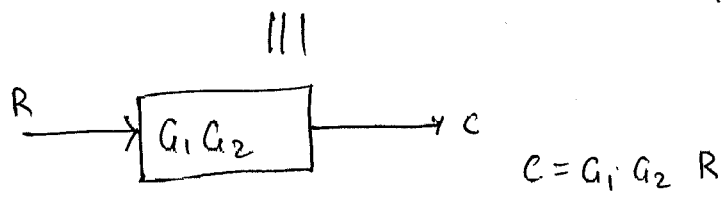
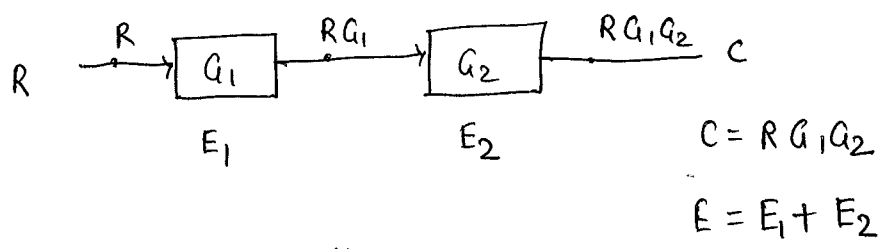


$$CLTF = CL \text{ gain} = \frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

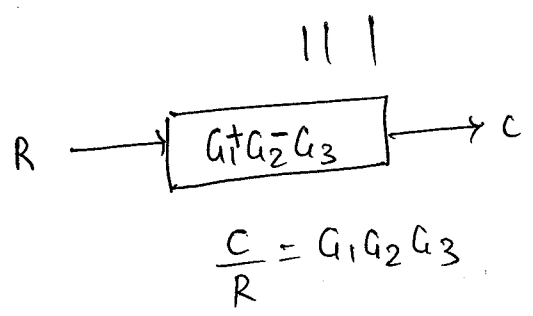
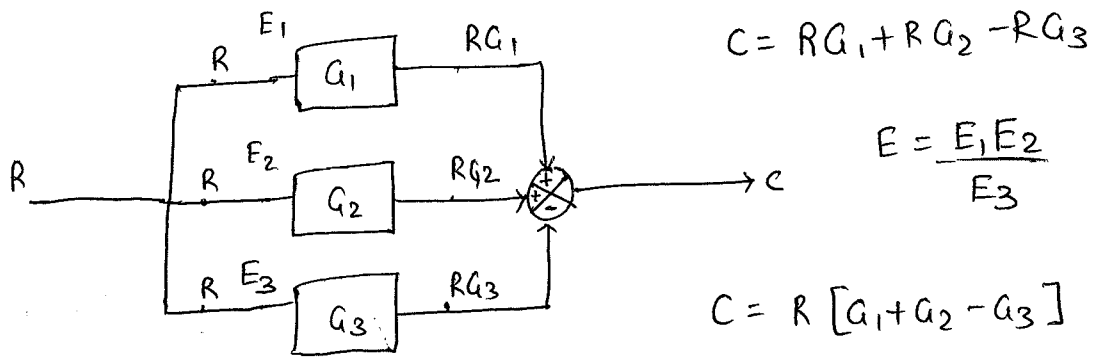
-ve feedback +ve feedback

* Block diagram reduction techniques

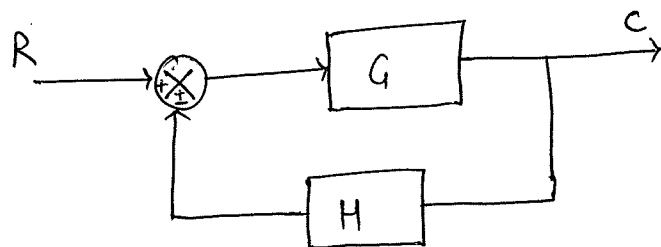
① Blocks are connected in series



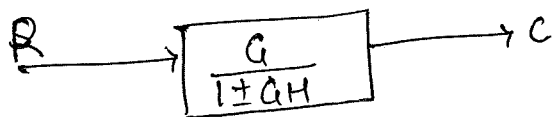
② Blocks are connected in parallel



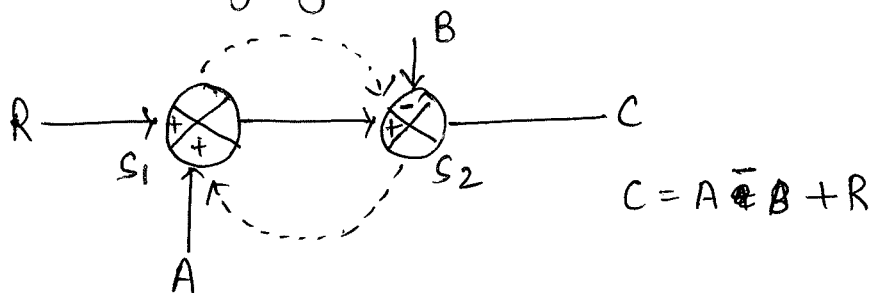
③ Closed loop



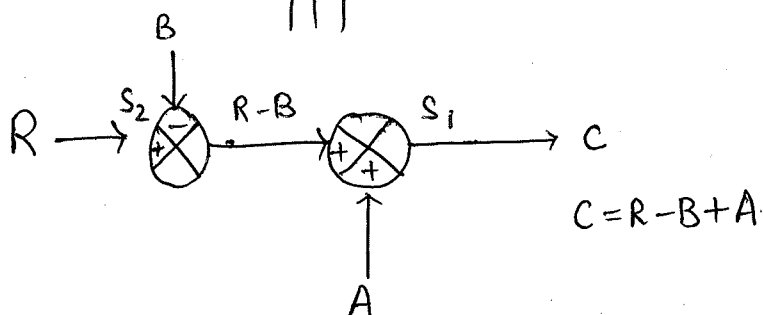
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④ Interchanging of summing point

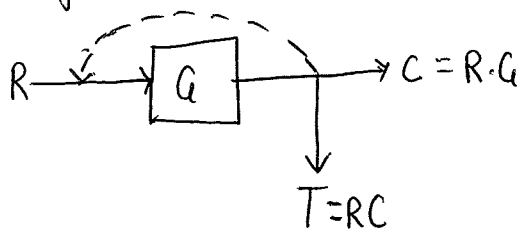


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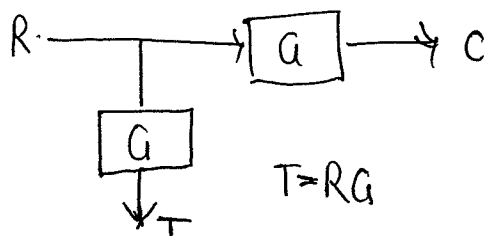
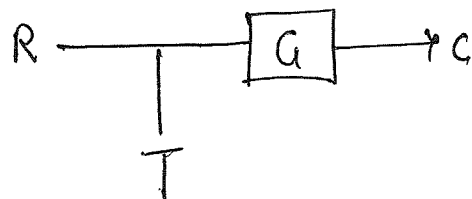
⑤ Shifting of Take-off point

(a) Shifting of take-off point before gain block



|||

Before modification

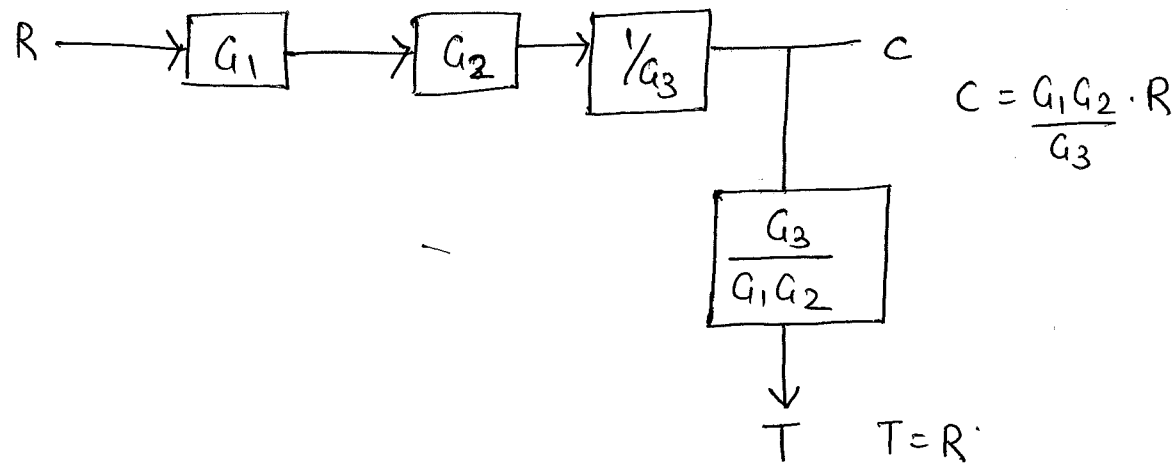
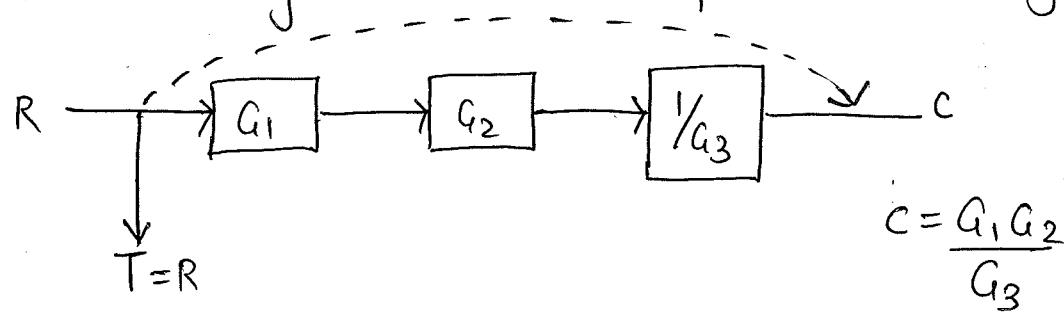


$$C = RG$$

$$T = RG$$

After modification

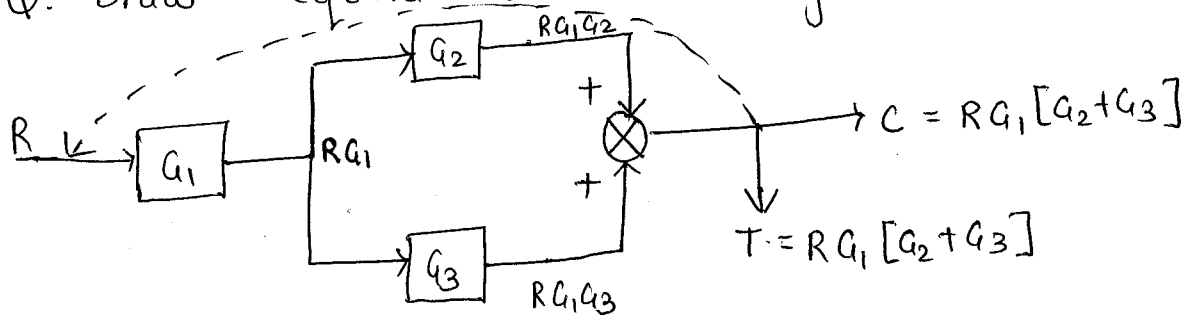
(b) Shifting of take-off point after gain block



NOTE:-

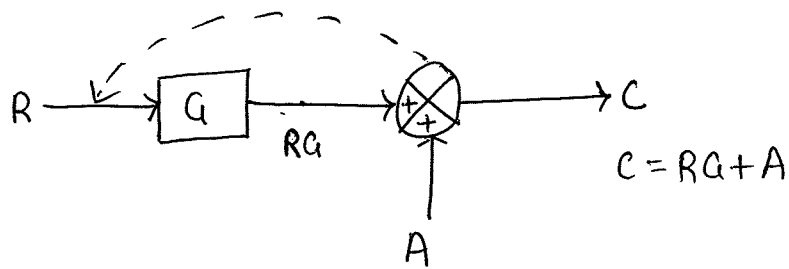
- While shifting of take-off point observes left hand side block changes. After shifting if there are extra gain blocks then divide (Gain) and if we are losing some gain block then multiply.

Q:- Draw equivalent block diagram?

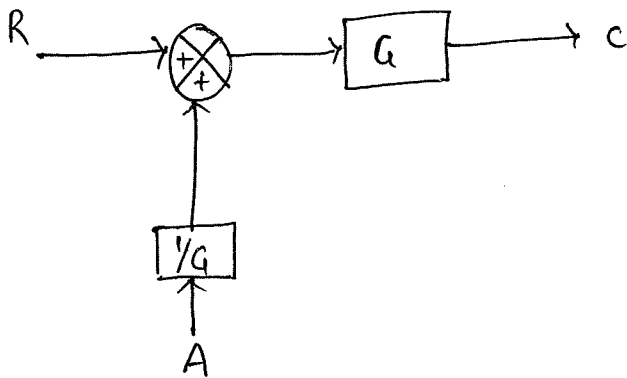


⑥ Shifting of summing point

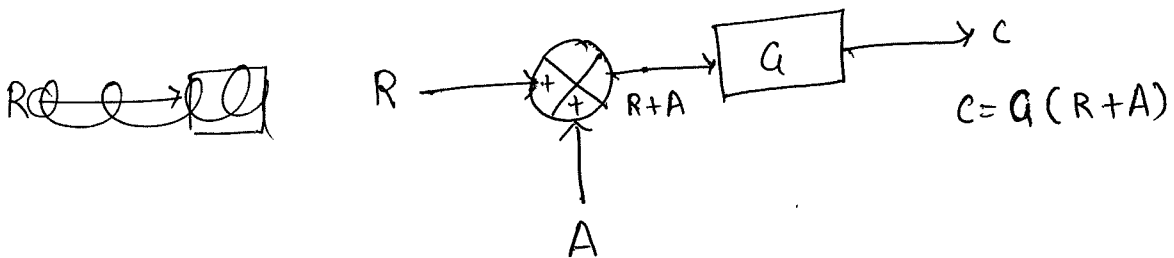
(a) Shifting of summing point before gain block



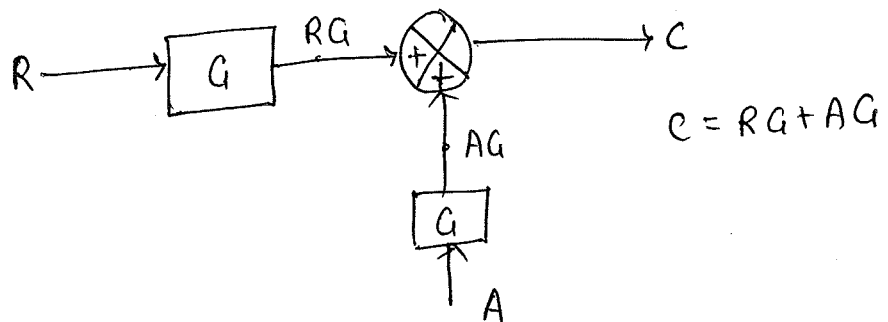
|||



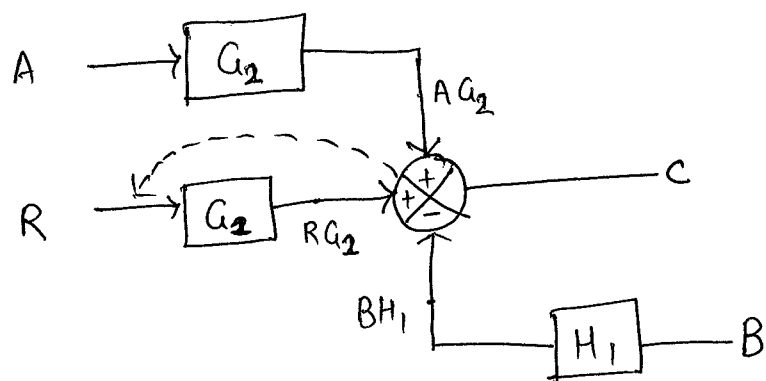
(b) Shifting of summing point after gain block



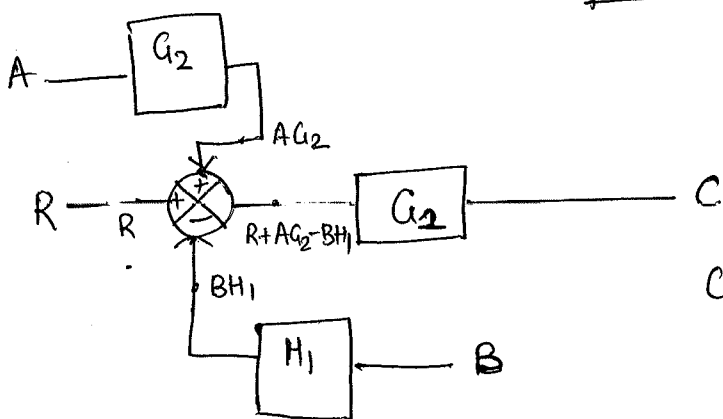
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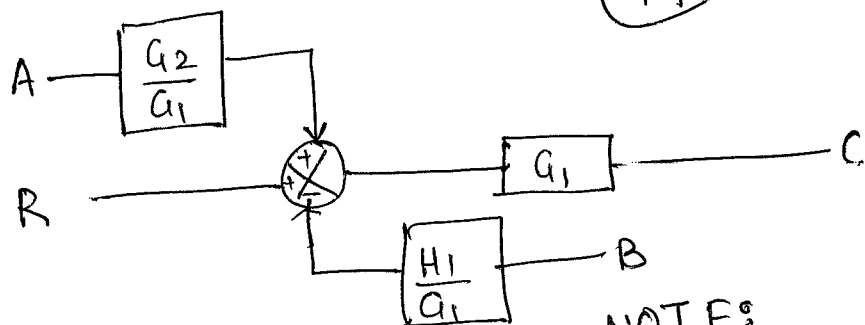
Q:- Draw eqⁿ block diagram:-



$$C = AG_2 + RG_2 - BH_1$$



$$C = RG_1 + AG_1G_2 - BH_1G_1$$



$$C = RG_1 + AG_2 - BH_1$$

NOTE:

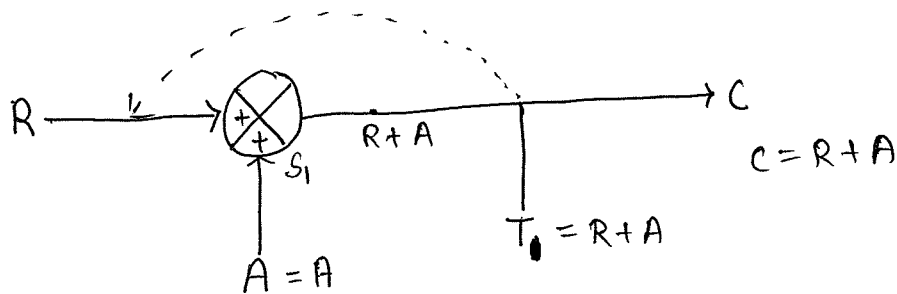
-while shifting operation changes are occurred only in additional forward path & feedback path connected to that summing point only.

-Before shifting & after shifting forward path gain should be remain same we don't want to loss and we don't want any extra gain so if it is extra gain after shifting then divide and if we miss any gain then we should multiply.

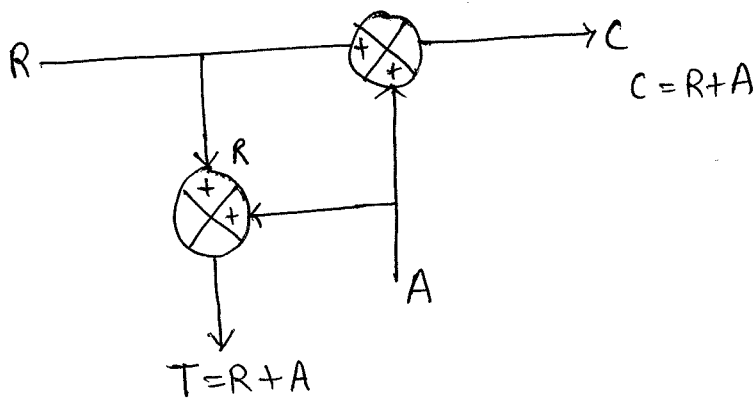
- For take-off point observe left hand side block change
- For summing point observe right hand side block change

⑦ Adjusting the take-off and summing point

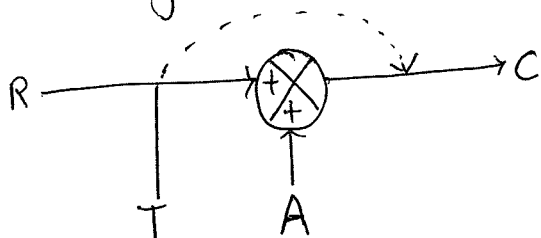
(a) Shifting of take-off point before summing point



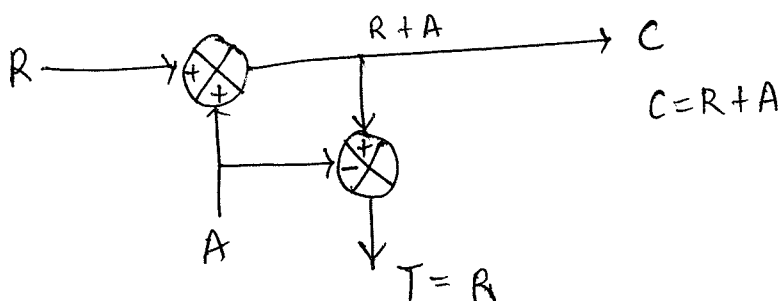
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(b) Shifting of take off point after summing point



|||



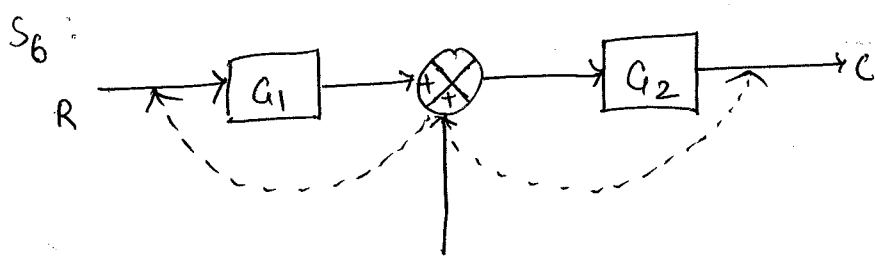
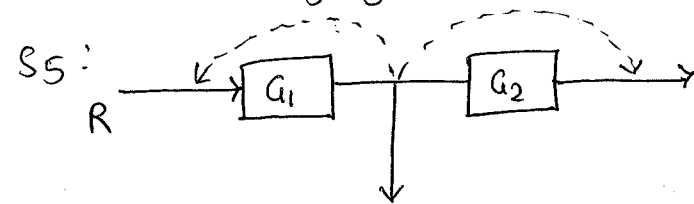
* Steps:-

S_1 : series

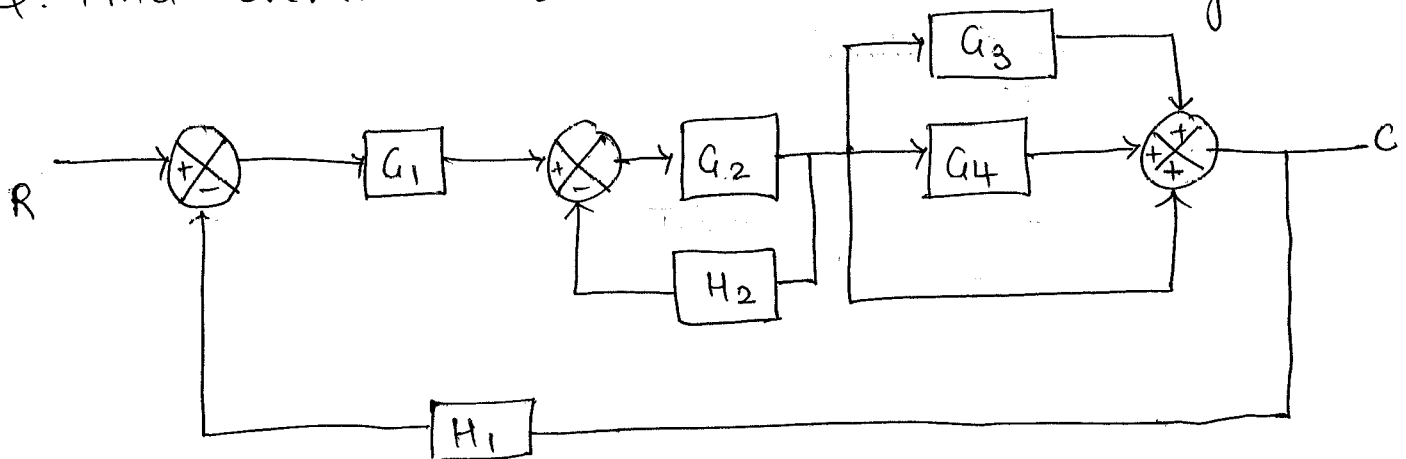
S_2 : parallel

S_3 : closed loop

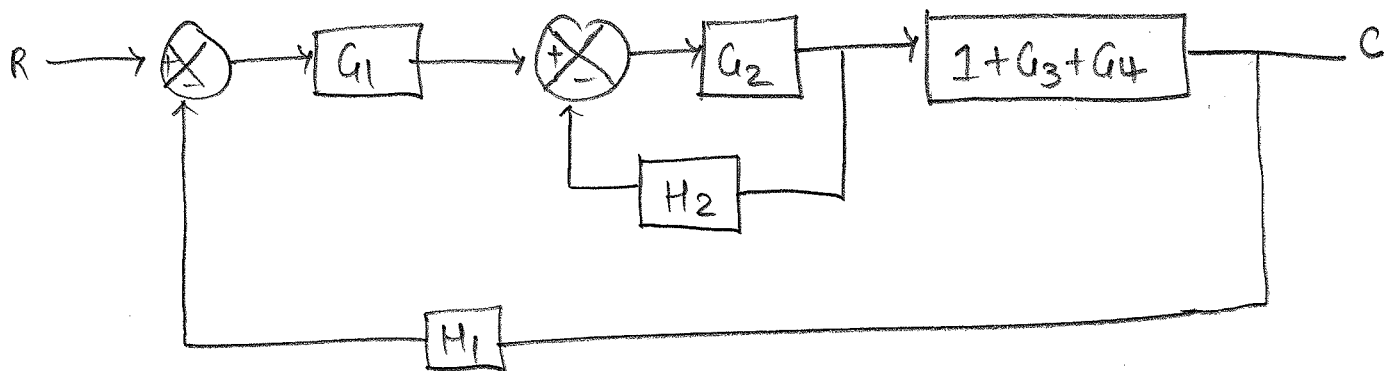
S_4 : interchanging of SP



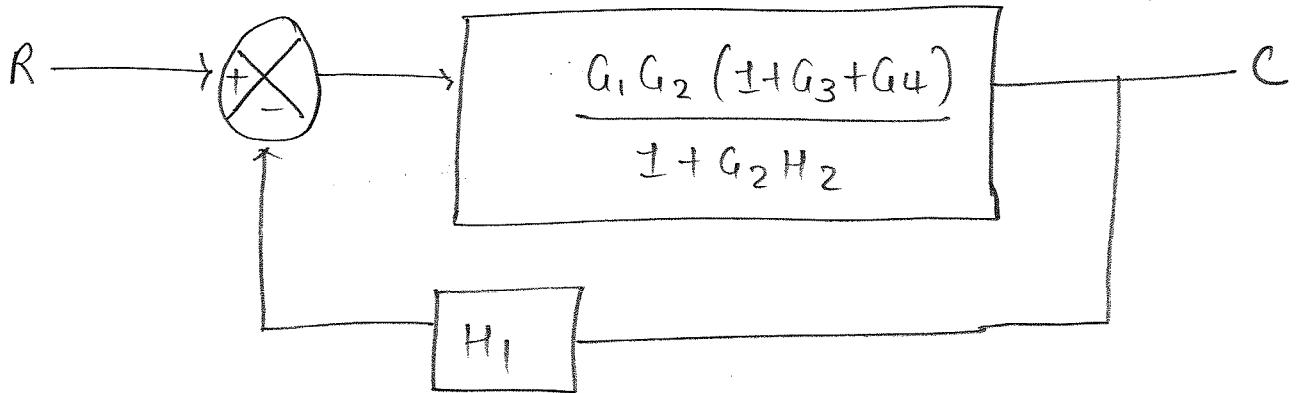
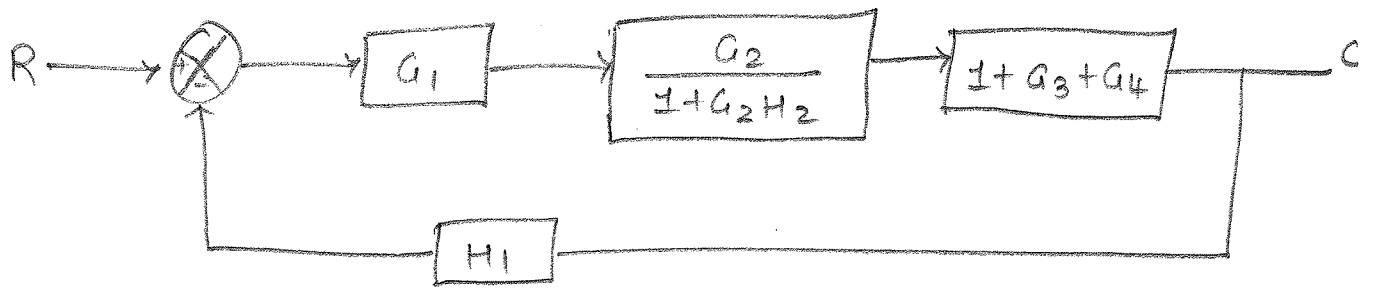
Q:- Find overall transfer function of block diagram:-



G_3, G_4 and 1 are in parallel

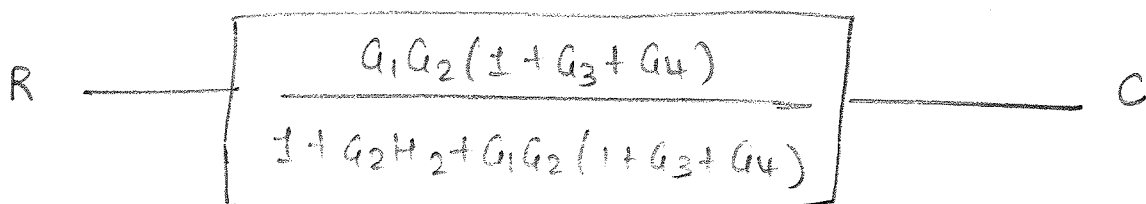


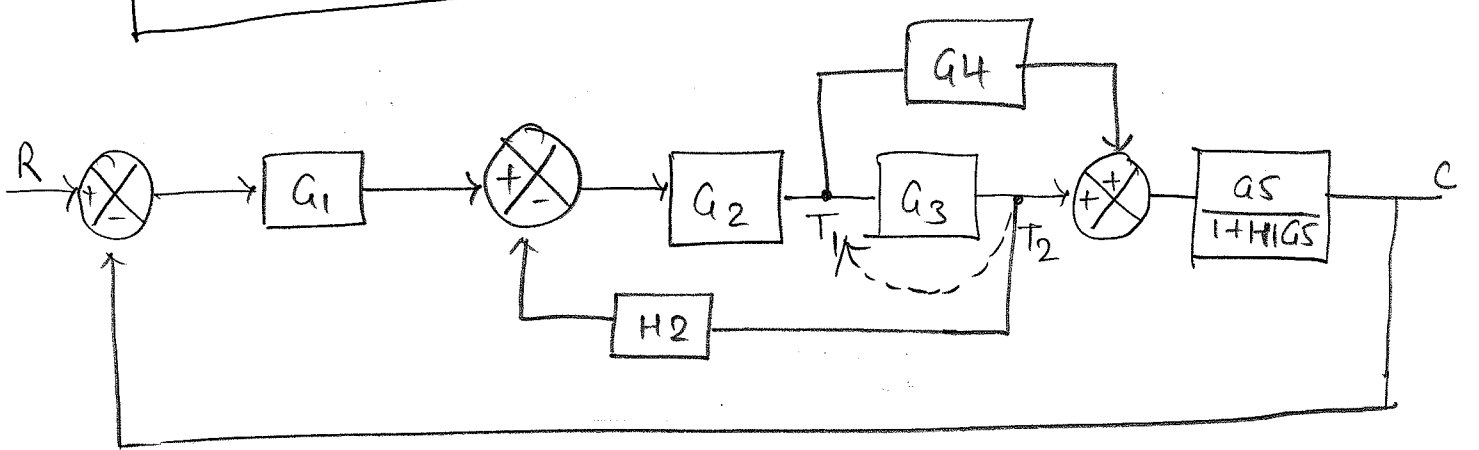
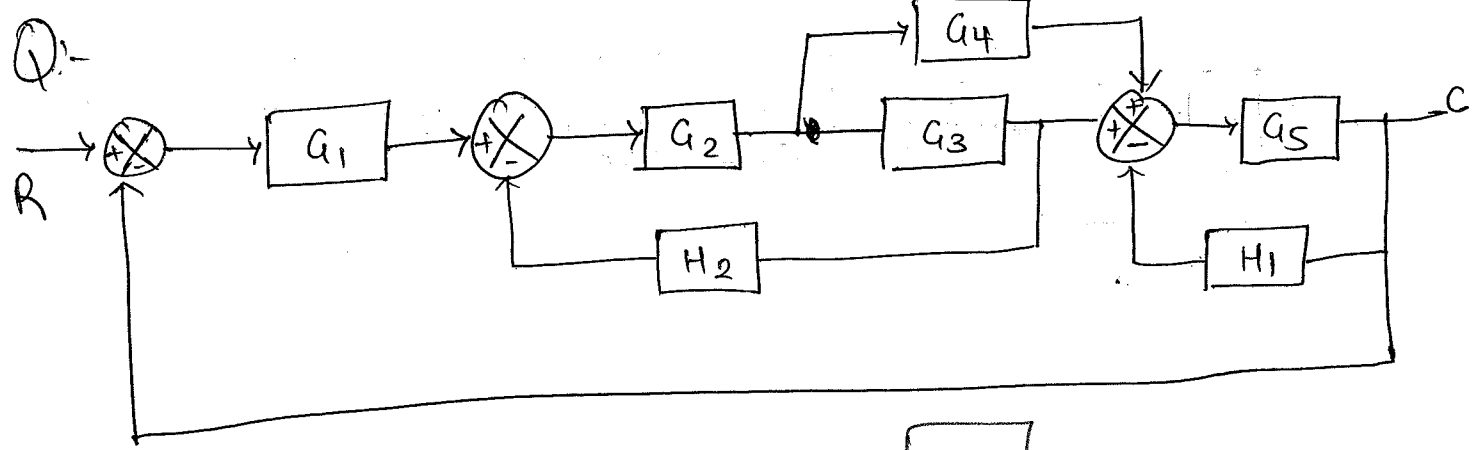
G_2 and H_2 are in feedback



$$\frac{C}{R} = \frac{G_1 G_2 (1 + G_3 + G_4)}{1 + G_2 H_2} \cdot \frac{1}{1 + \frac{G_1 G_2 (1 + G_3 + G_4) \cdot H_1}{1 + G_2 H_2}}$$

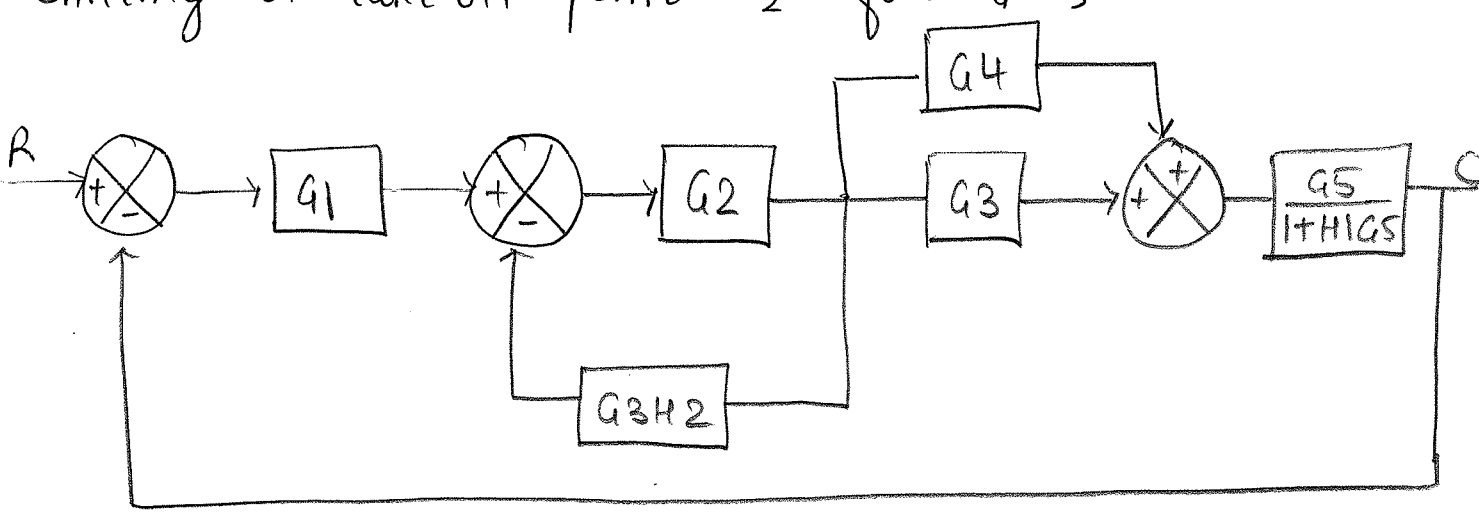
$$\frac{C}{R} = \frac{G_1 G_2 (1 + G_3 + G_4)}{1 + G_2 H_2 + G_1 G_2 (1 + G_3 + G_4) H_1}$$



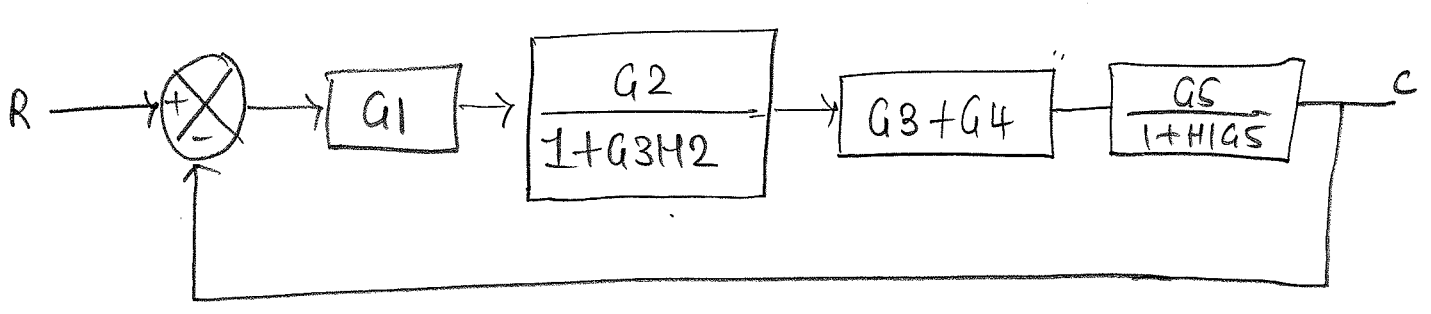


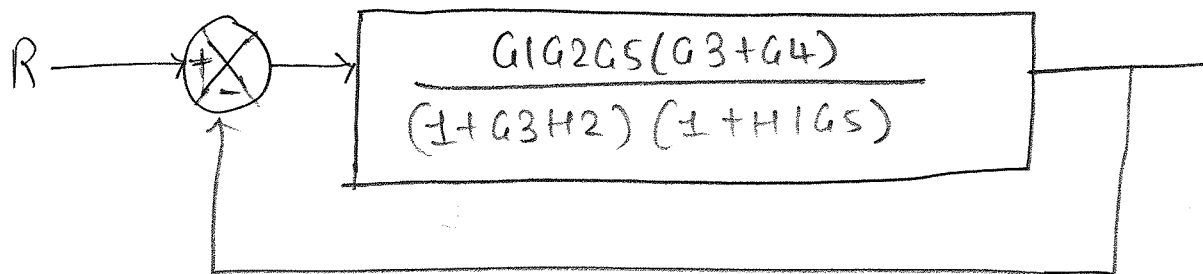
Method:-I

Shifting of take-off point T_2 before G_3

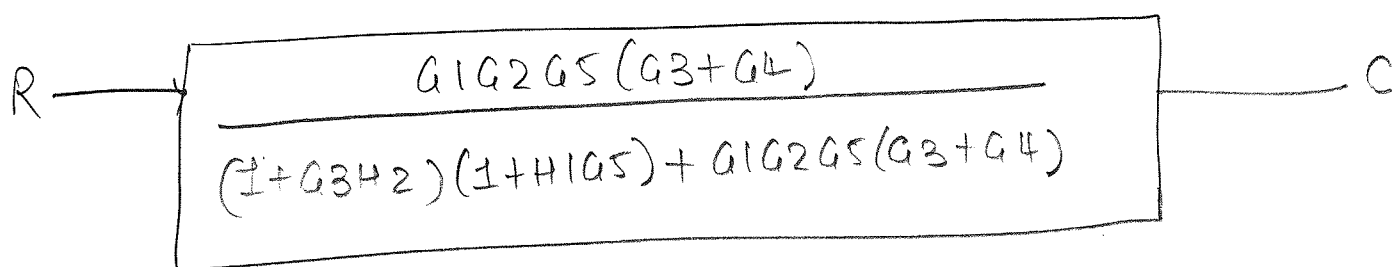


Now, G_3 & G_4 are in parallel, G_2 & G_3H_2 are in feedback form



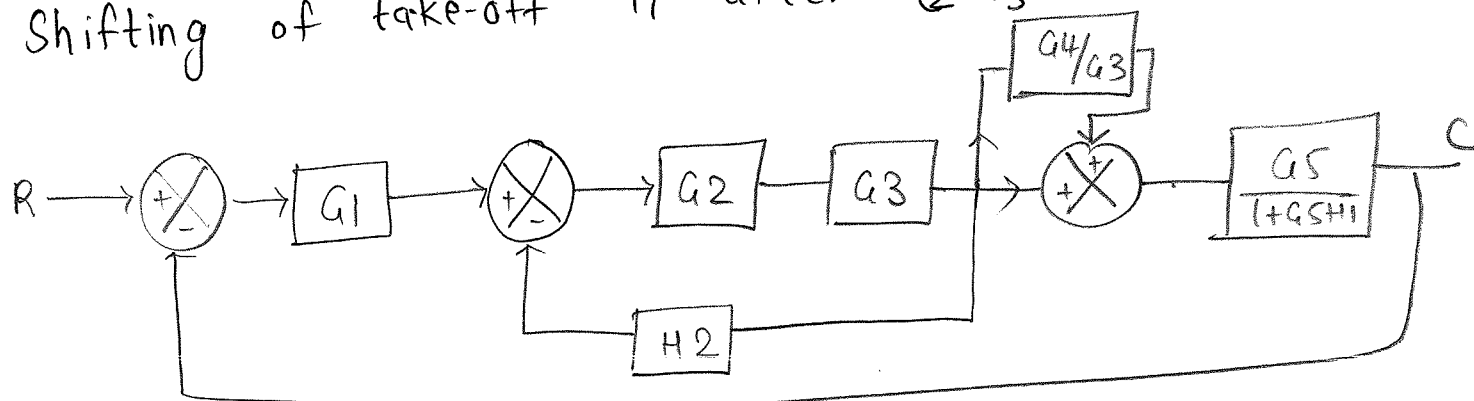


$$\frac{C}{R} = \frac{G_1 G_2 G_5 (G_3 + G_4)}{(1 + G_3 H_2) (1 + H_1 G_5) + G_1 G_2 G_5 (G_3 + G_4)}$$

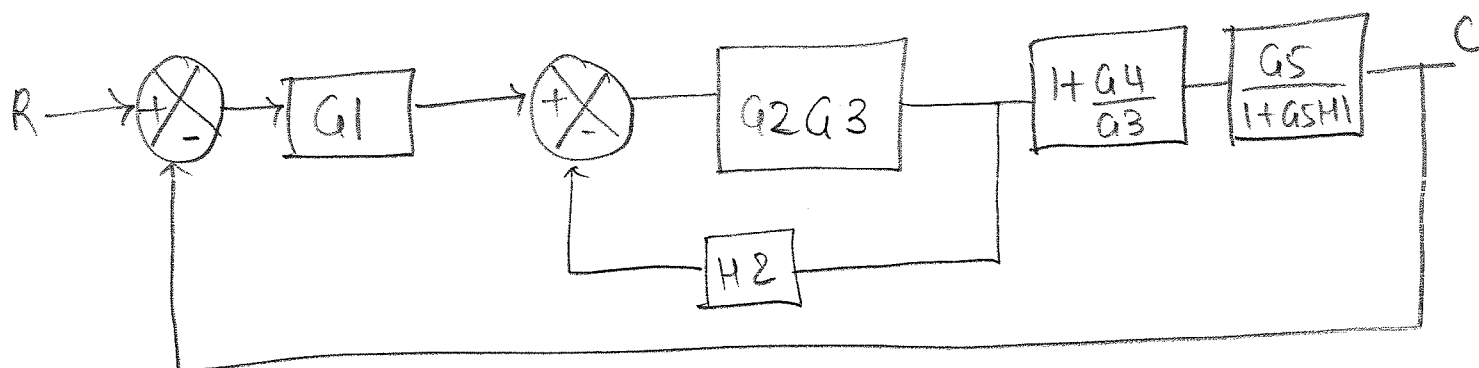


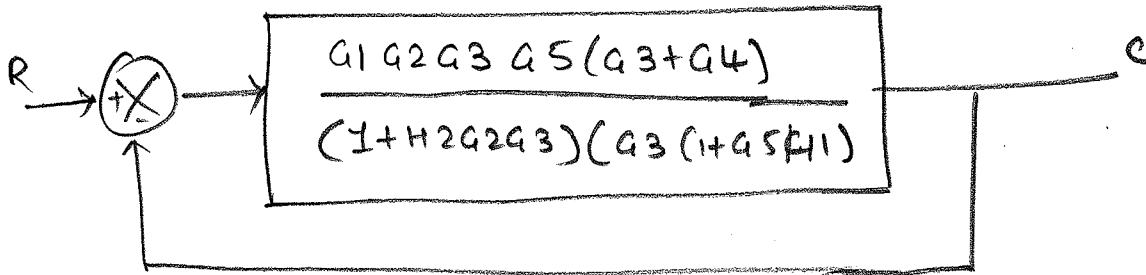
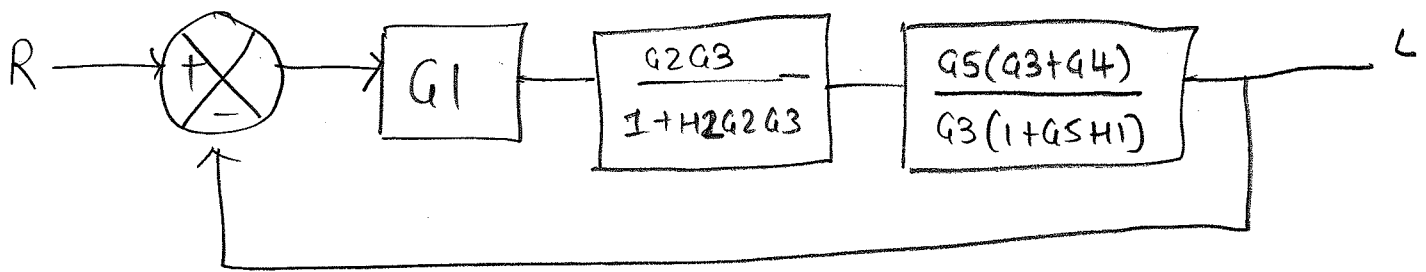
Method:- II

Shifting of take-off T_1 after G_3



Now, G_2 & G_3 are in series and whole with H_2 are in feedback, Also $\frac{G_4}{G_3}$ & $\pm$ are in parallel



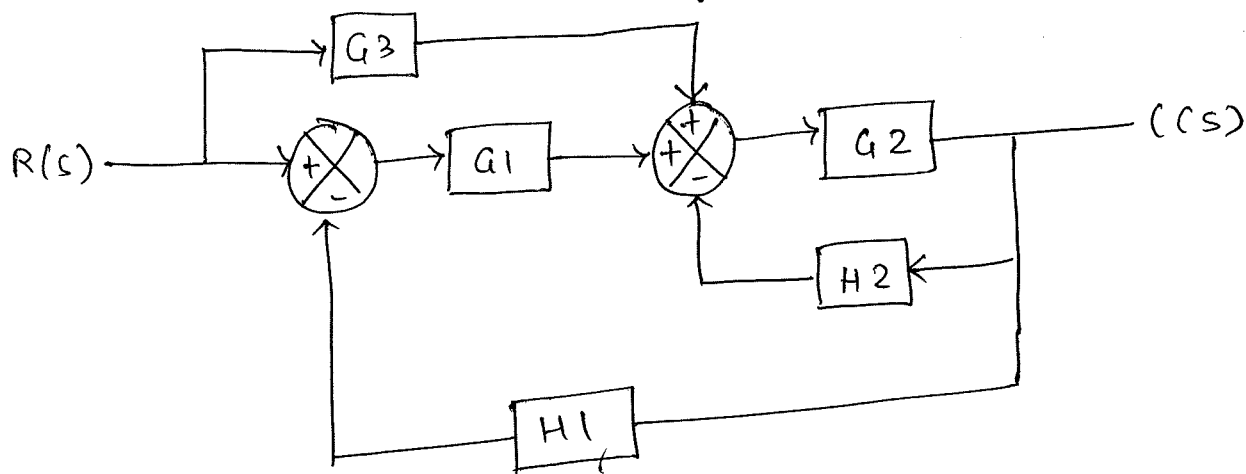


$$= \frac{G_1 G_2 G_3 G_5 (G_3 + G_4)}{(G_3 + G_5 G_3 H_1) (1 + H_2 G_2 G_3)}$$

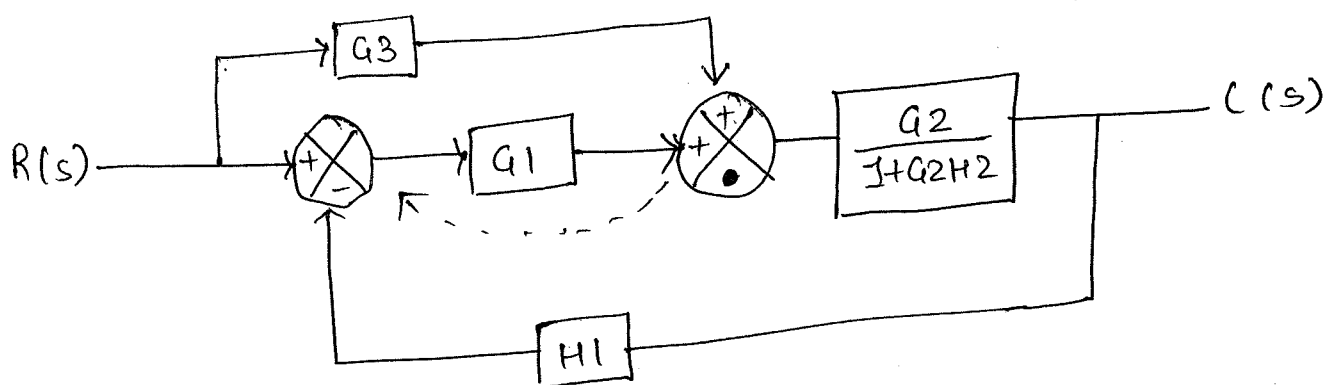
$$1 + \frac{G_1 G_2 G_3 G_5 (G_3 + G_4)}{(1 + H_2 G_2 G_3) (G_3 + G_3 G_5 H_1)}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_5 (G_3 + G_4)}{G_3 + G_3 G_5 H_1 + H_2 G_2 G_3^2 + H_2 G_2 G_3^2 G_5 H_1 + G_1 G_2 G_3 G_5 (G_3 + G_4)}$$

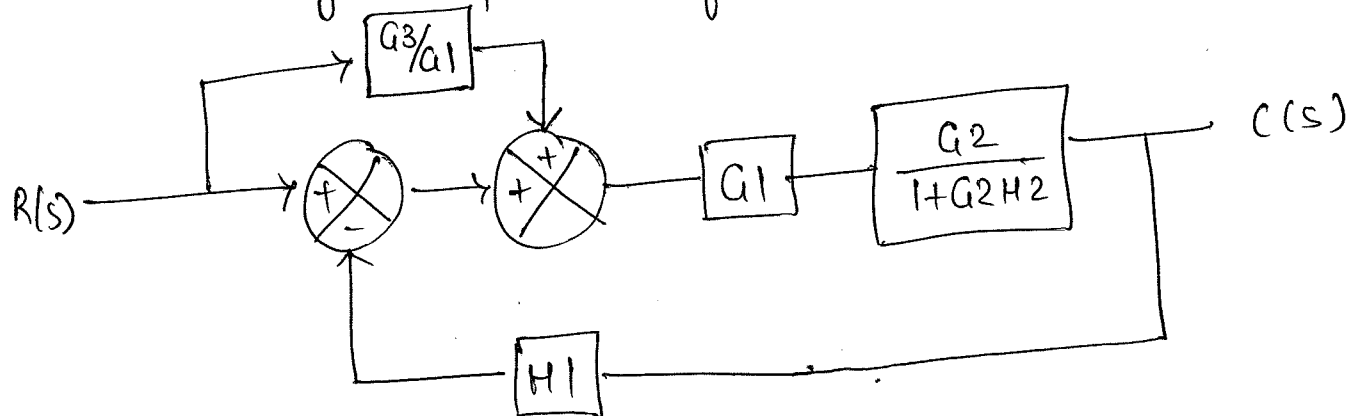
Q:- Find overall transfer function of block diagram



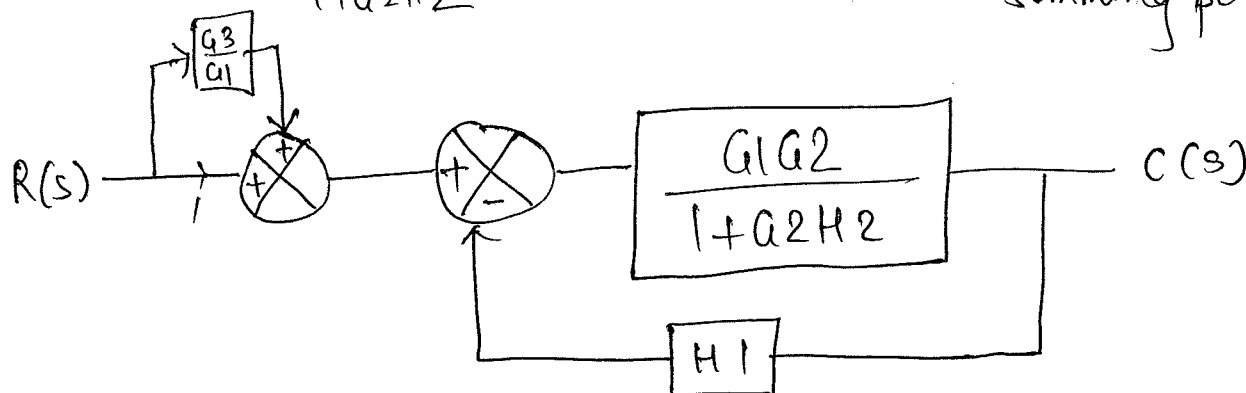
G_2 and H_2 are in feedback form

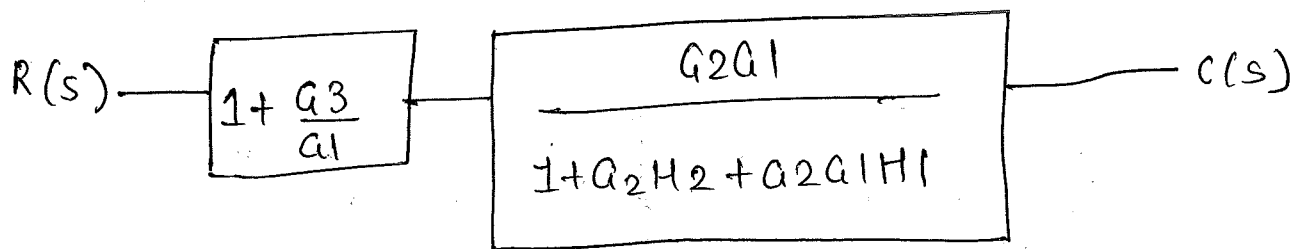


Now, shifting ^{summing} point before G_1

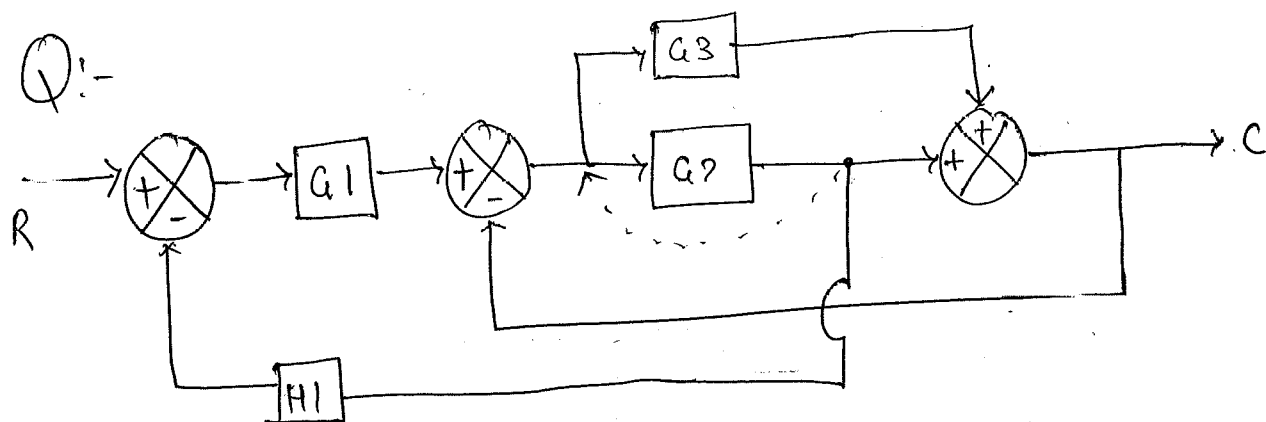
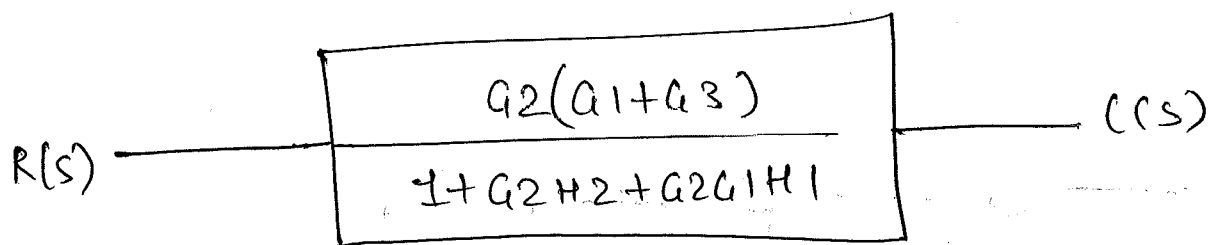


G_1 and $\frac{G_2}{1+G_2H_2}$ are in series & interchanging of summing point

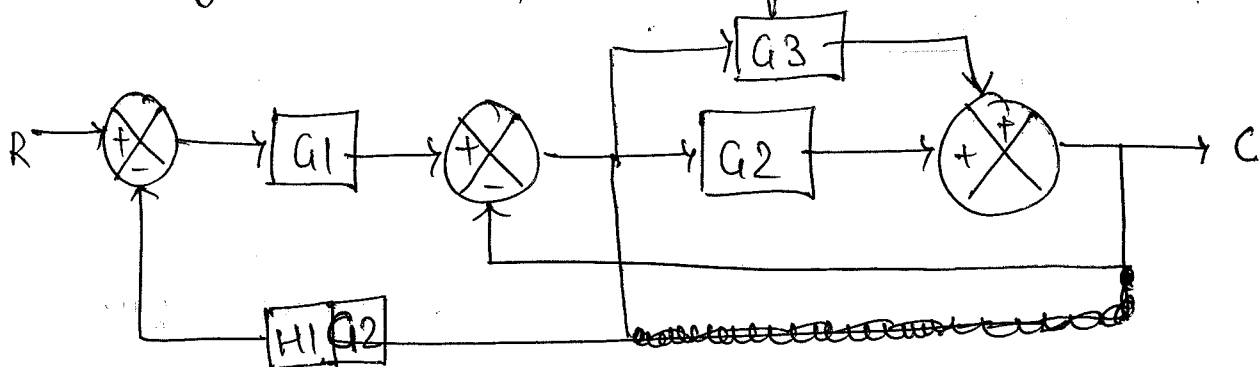




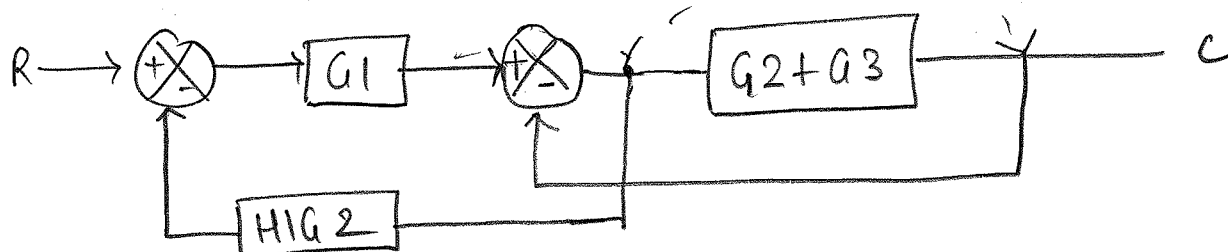
$$\frac{G1 + G3}{G1} \cdot \frac{G2G1}{1 + G2H2 + G2G1H1}$$

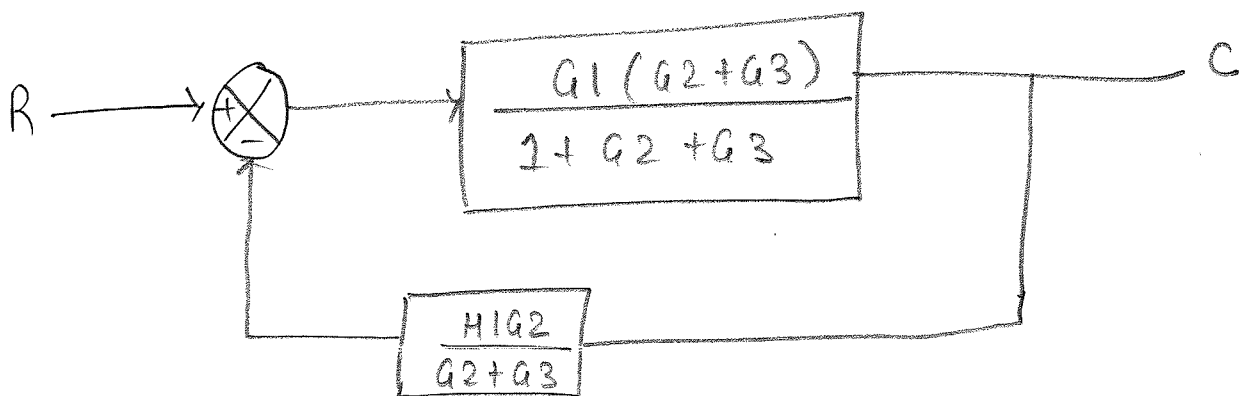
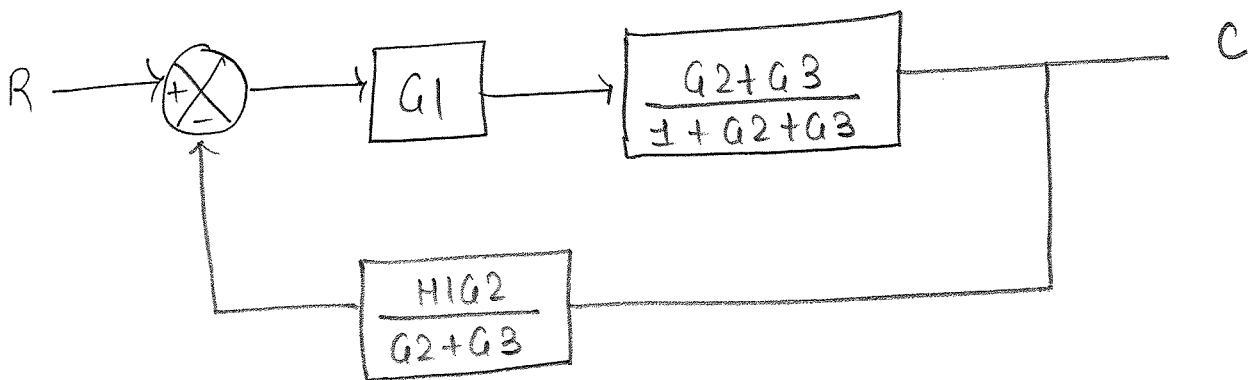
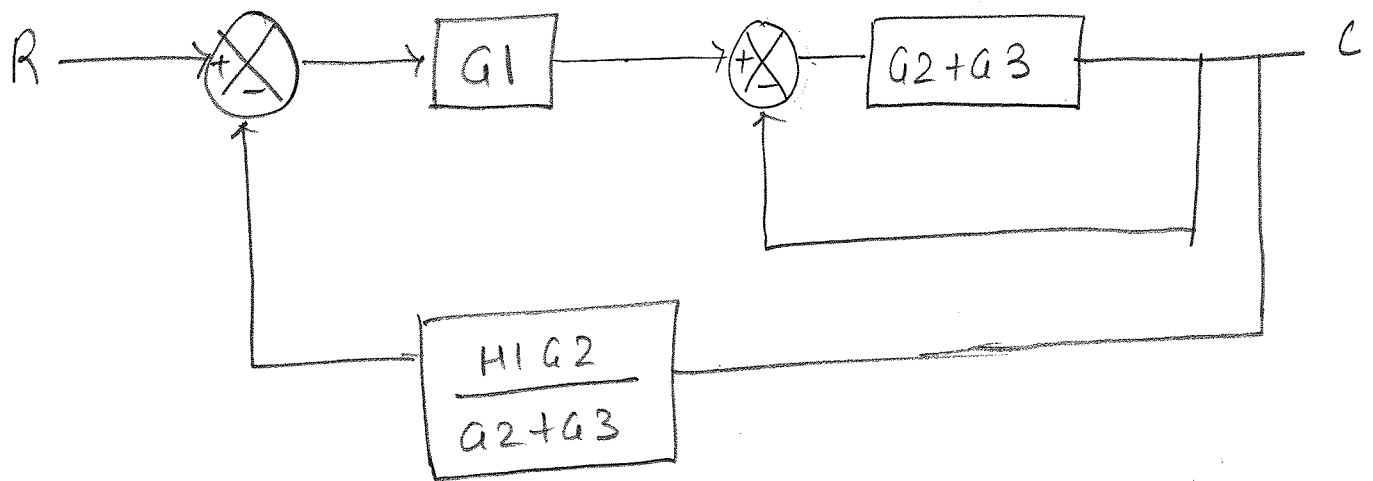


Taking take-off point before G2 block

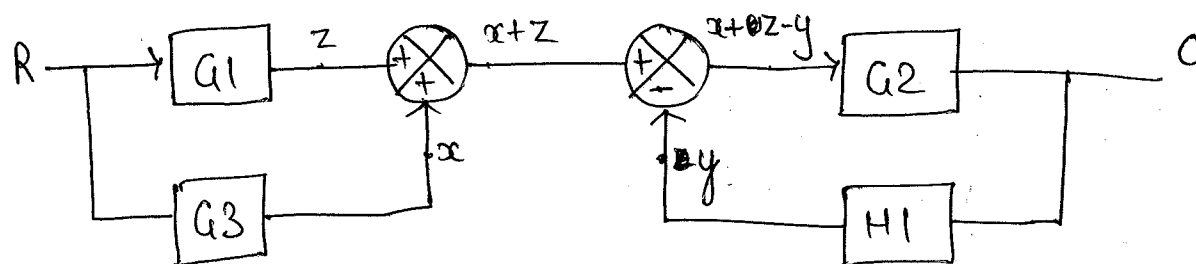
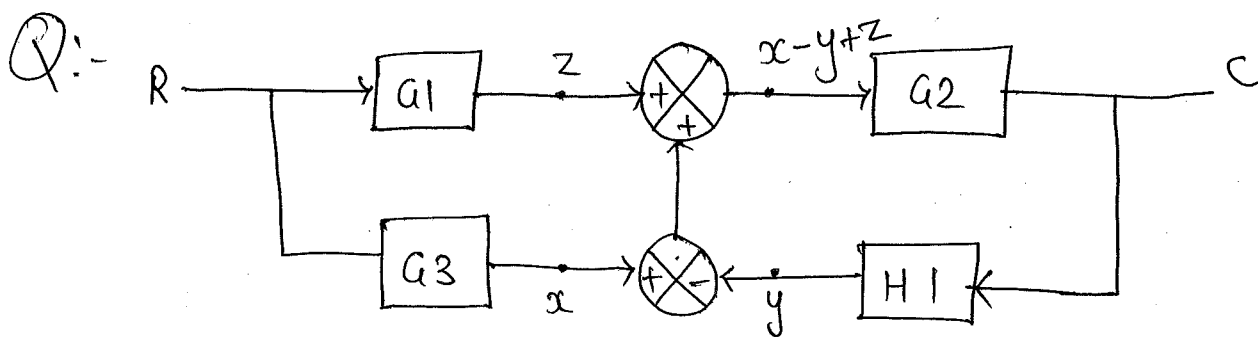


G2 & G3 are in parallel

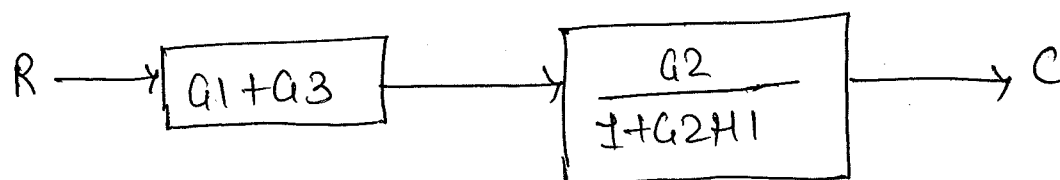




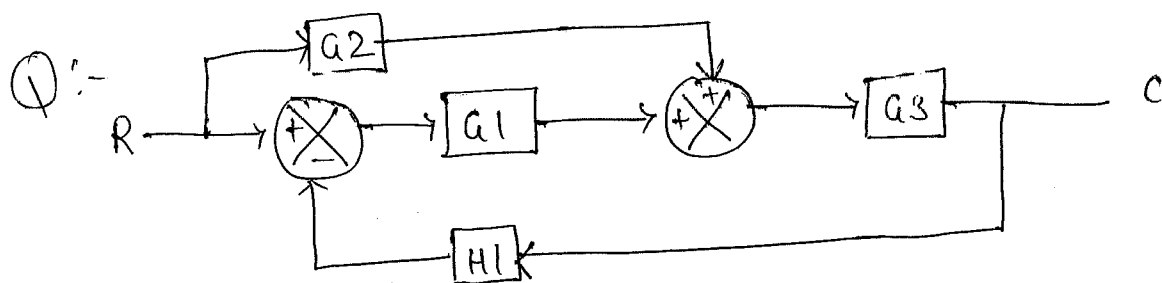
$$\frac{C}{R} = \frac{\frac{G_1(G_2+G_3)}{1+G_2+G_3}}{1 + \left(\frac{H_1 G_2}{G_2+G_3} \right) \left(\frac{G_1(G_2+G_3)}{(1+G_2+G_3)} \right)} = \frac{G_1(G_2+G_3)}{1+G_2+G_3+G_1 G_2 H_1}$$



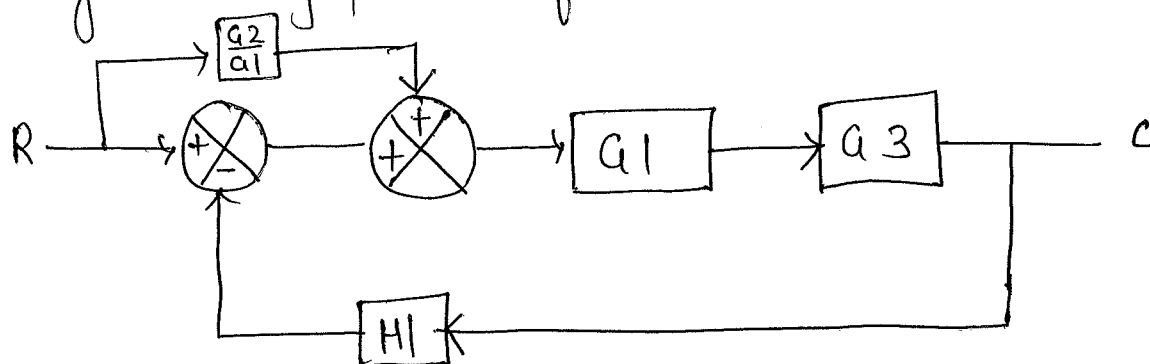
$G1$ & $G3$ are in parallel, $G2$ & $G1$ are in feedback form



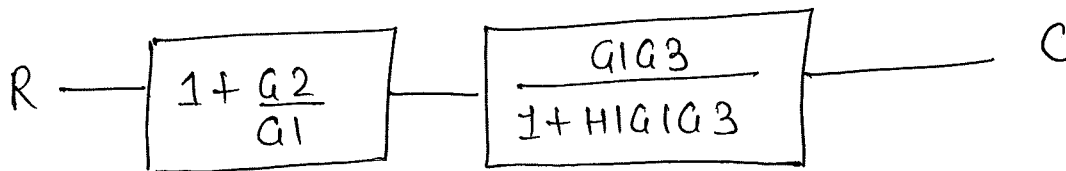
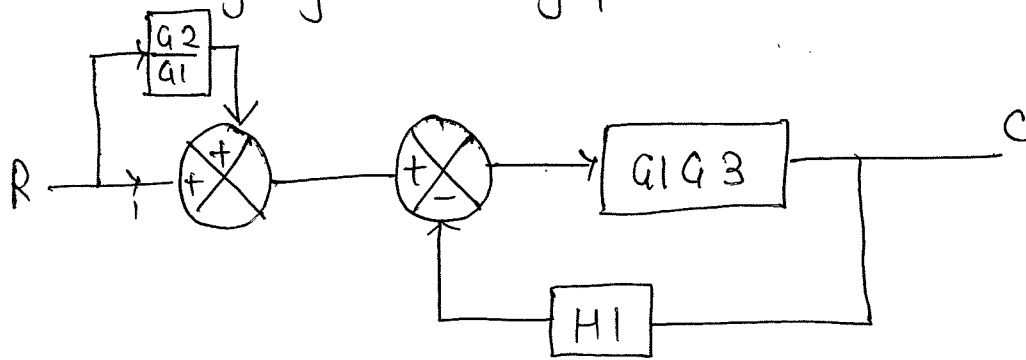
$$\frac{C}{R} = \frac{(G1+G3)G2}{1+G2H1}$$



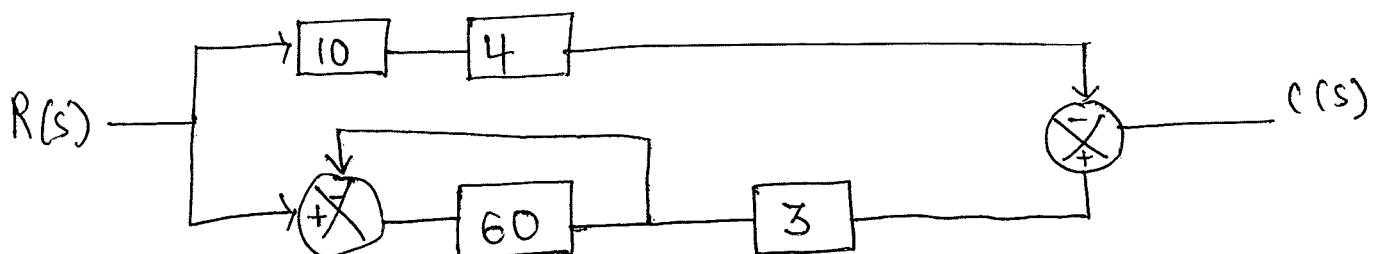
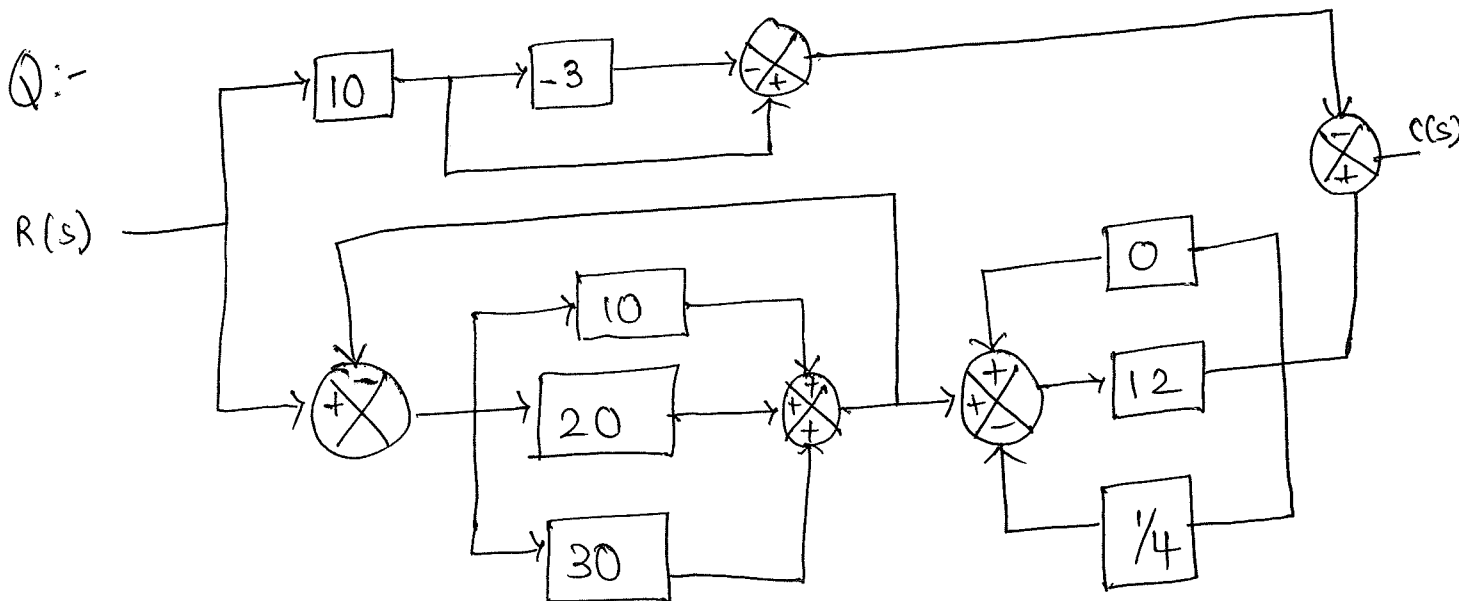
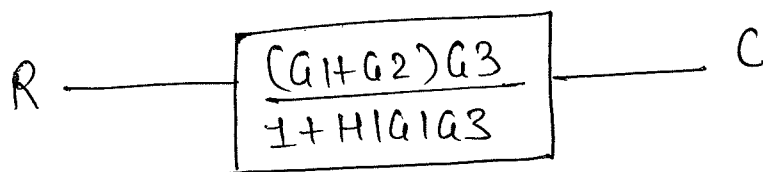
Taking summing point before $G1$

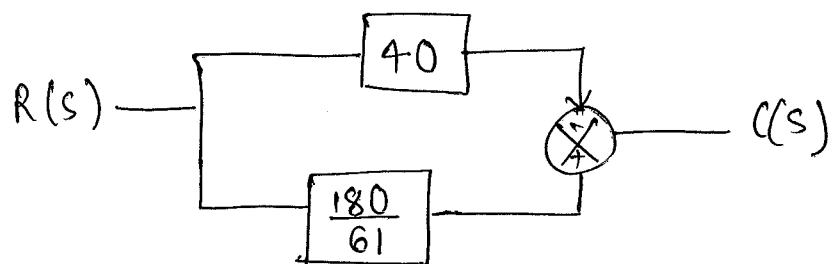
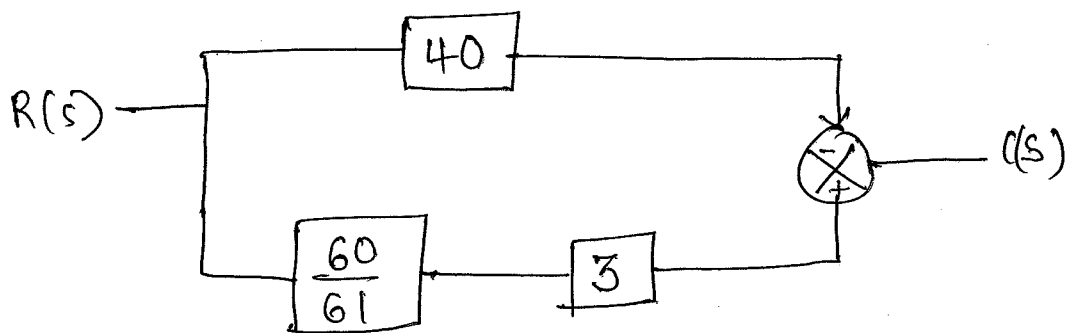


Interchanging summing point & G_1 & G_3 are in series



$$\frac{C}{R} = \frac{(G_1 + G_2) G_3}{1 + H_1 G_1 G_3}$$





$$\frac{C(s)}{R(s)} = \frac{180}{61} - 40 = \frac{180 - 2440}{61} = \frac{-2260}{61}$$

Q:- construct

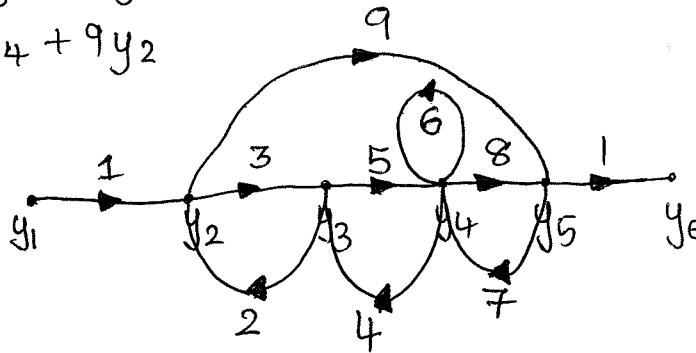
$$y_2 = y_1 + 2y_3$$

$$y_3 = 3y_2 + 4y_4$$

$$y_4 = 5y_3 + 6y_4 + 7y_5$$

$$y_5 = 8y_4 + 9y_2$$

Sol:-



Input node:- The node which has only output outgoing branches.

Output node:- The node which has only incoming branch.

Chain Node/Link node:- The node which has both incoming and outgoing branches is called chain node

In above SFG,

y_1 = input node

y_2, y_3, y_4, y_5 = chain node

y_6 dummy node

Forward path:- It is the path to input to output

Loop:- It is the path which terminates where it has started.

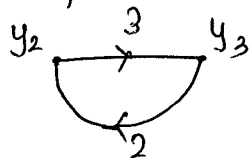
(1) No. of forward path:

$$P_1 = 1 \cdot 3 \cdot 5 \cdot 8 \cdot 1 = 120$$

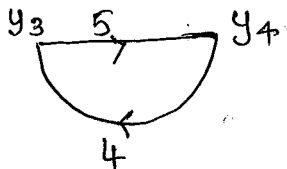
$$P_2 = 1 \cdot 9 \cdot 1 = 9$$

(2) No. of individual loops:

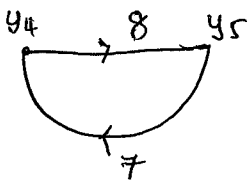
1) $L_1 = 3 \cdot 2$ Node (y_2, y_3)



2) $L_2 = 5 \cdot 4$ (y_3, y_4)



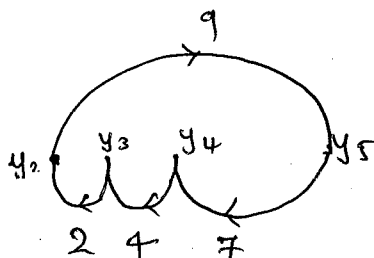
3) $L_3 = 8 \cdot 7$ (y_4, y_5)



4) $L_4 = 6$ (y_4)



5) $L_5 = 9 \cdot 7 \cdot 4 \cdot 2$ (y_2, y_3, y_4, y_5)



(3) No. of two non-touching loops:-

If there is no common node between two individual ~~and~~ loop is called non-touching loops.

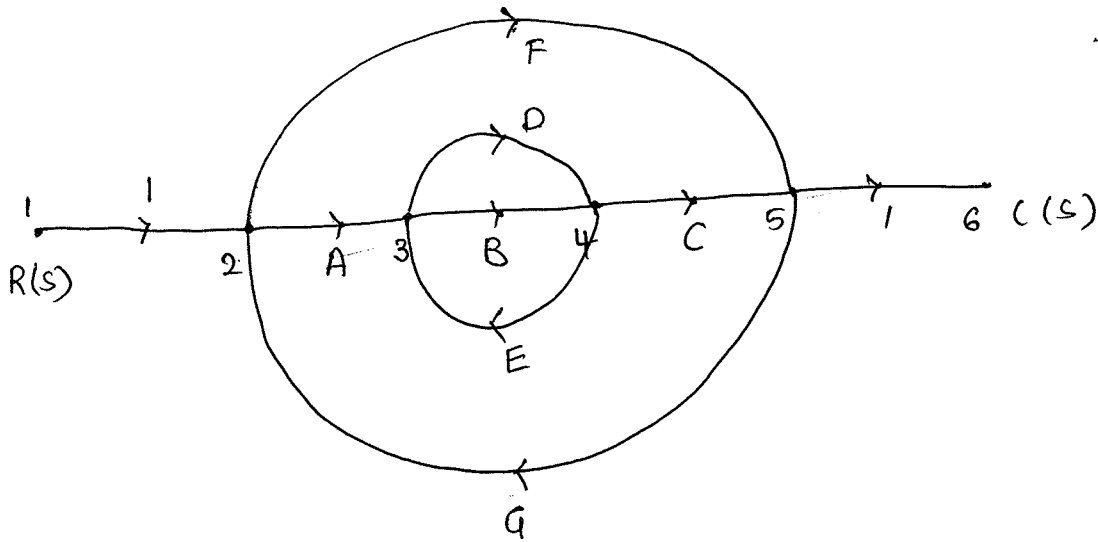
1) $L_1 L_3$

2) $L_1 L_4$

(4) No. of three non-touching loops:

There is no three non-touching loops.

Q:- For the given SFG find no. of forward path, individual loops, two non-touching loops & three non-touching loops



No. of forward path = $1 \cdot A \cdot B \cdot C \cdot 1 = P_1$

$= 1 \cdot F \cdot 1 = P_2$

$= 1 \cdot A \cdot D \cdot C \cdot 1 = P_3$

Individual loops:- $L_1 = F \cdot G$

$L_2 = D \cdot E$

$L_3 = B \cdot E$

$L_4 = A \cdot B \cdot C \cdot G$

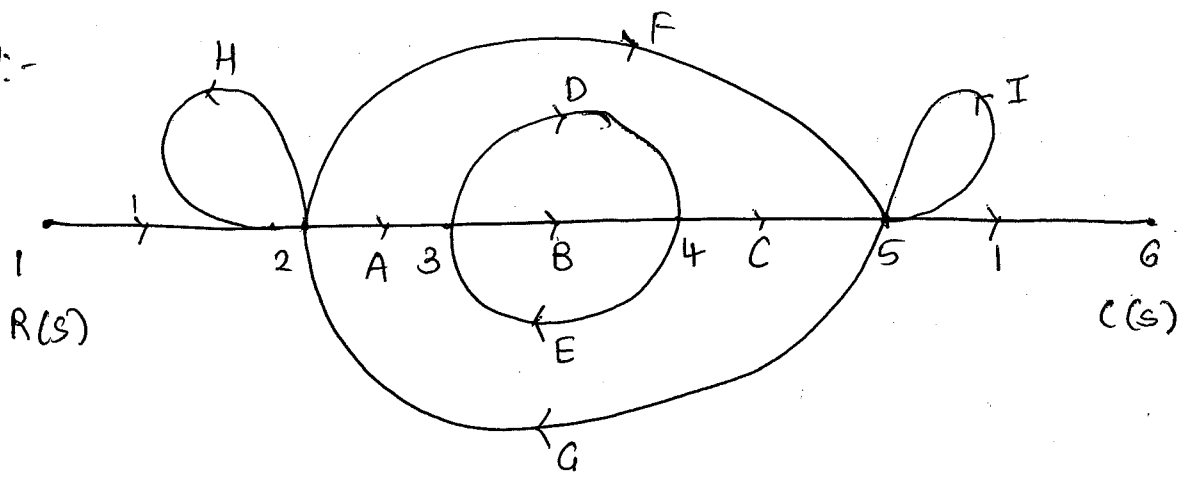
$L_5 = A \cdot D \cdot C \cdot G$

Two non-touching loops: $L_1 \cdot L_2 = F \cdot G \cdot D \cdot E$

$L_1 \cdot L_3 = F \cdot G \cdot B \cdot E$

Three non-touching loops: zero.

Q:-



(1) No. of forward path:

$$P_1 = 1 \cdot A \cdot B \cdot C \cdot 1 = ABC$$

$$P_2 = 1 \cdot F \cdot 1 = F$$

$$P_3 = 1 \cdot ADC \cdot 1 = ADC$$

(2) No. of loops:

$$L_1 = BE \quad L_6 = H$$

$$L_2 = DE \quad L_7 = I$$

$$L_3 = ABCG$$

$$L_4 = FG$$

$$L_5 = ADG$$

(3) Two non-touching loops

$$L_1 L_4 = BE \cdot FG$$

$$L_6 L_7 = HI$$

$$L_2 L_4 = DEFG$$

$$L_6 L_1 = HBE$$

$$L_6 L_2 = HDE$$

$$L_7 L_1 = IBE$$

$$L_7 L_2 = IDE$$

Three non-touching loops:

$$L_1 = H I B E$$

$$L_2 = H I D E$$

Q:- Represent

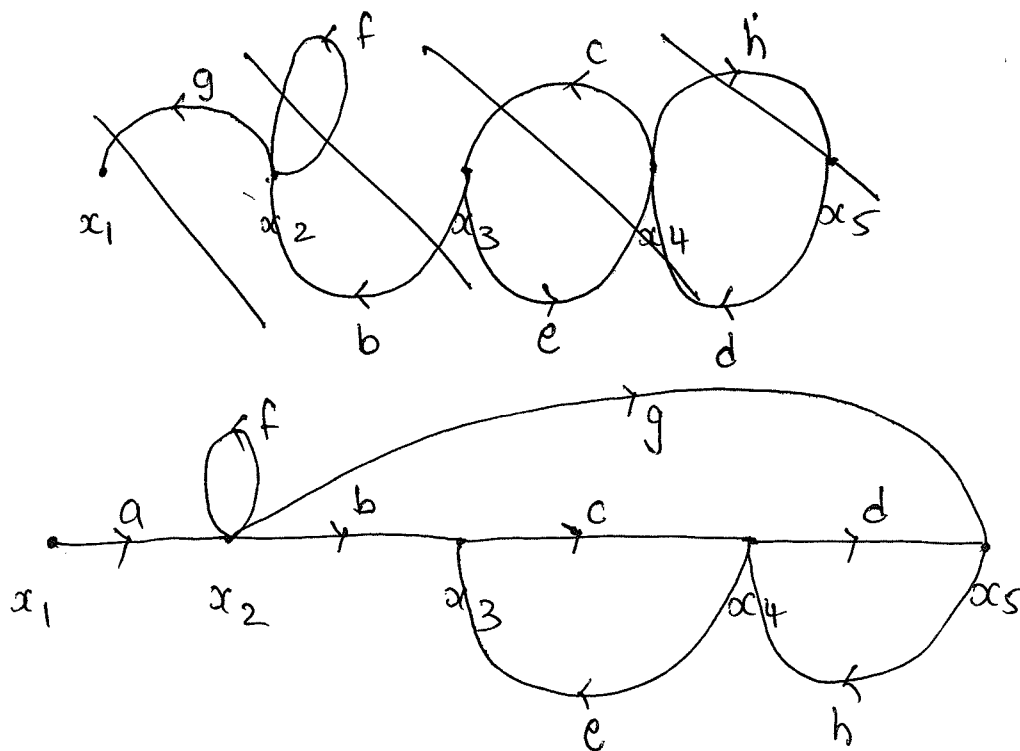
by SFG and find forward path, two non-touching loops and three non-touching loops.

$$x_2 = a x_1 + f x_2$$

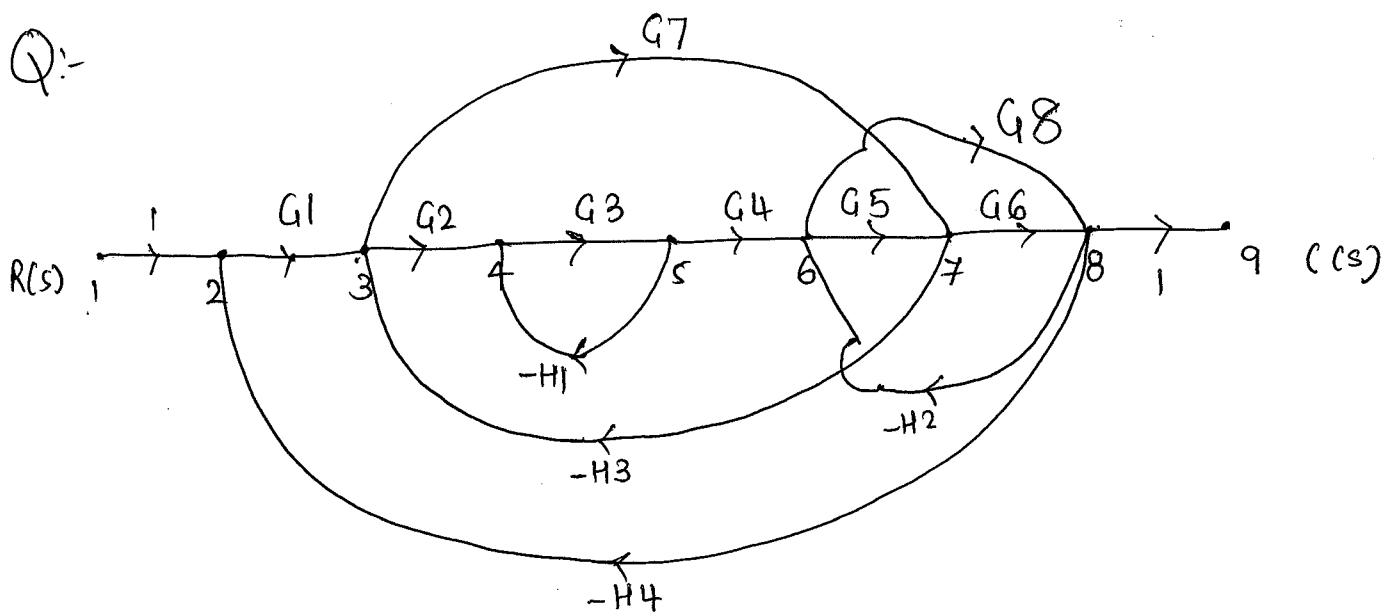
$$x_3 = b x_2 + e x_4$$

$$x_4 = c x_3 + h x_5$$

$$x_5 = d x_4 + g x_2$$



Q:-



NOTE:

If there is one feedback path between two nodes then no. of individual loops is equal to no. of forward path between that two nodes.

Sol:- Forward path:-

$$P_1 = G_1 G_2 G_3 G_4 G_5 G_6$$

$$P_2 = G_1 G_7 G_6$$

$$P_3 = G_1 G_2 G_3 G_4 G_8$$

No. of single loops:-

$$L_1 = -H_3 G_7$$

$$L_7 = G_1 G_2 G_3 G_4 G_8 (-H_4)$$

$$L_2 = -G_1 G_3 G_4 G_5 H_3$$

$$L_8 = G_1 G_7 G_6 (-H_4)$$

$$L_3 = -H_1 G_3$$

$$L_4 = -H_2 G_6 G_5$$

$$L_5 = -H_2 G_8$$

$$L_6 = -H_4 G_1 G_2 G_3 G_4 G_5 G_6$$

No. of two non-touching loops:

$$L_3 L_1 = G_3 G_7 H_1 H_3$$

$$L_1 L_5 = G_7 H_3 G_8 H_2$$

$$L_3 L_5 = G_3 H_1 G_8 H_2$$

$$L_1 L_4 = H_3 G_7 G_5 G_6 H_2$$

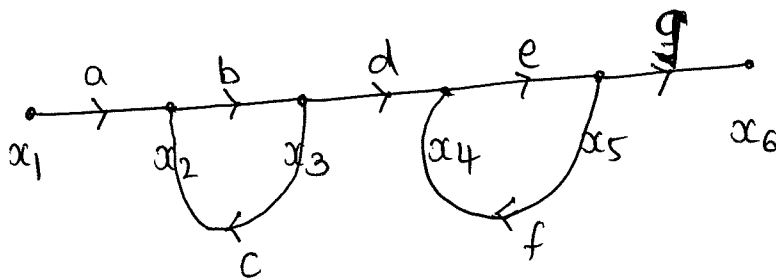
$$L_3 L_4 = G_3 H_1 G_5 G_6 H_2$$

~~$$L_3 L_5 = H_3 G_7 G_5 G_6 H_2$$~~

No. of three non-touching

$$L_3 L_1 L_5 = H_1 G_3 G_7 H_3 G_5 G_6 H_2$$

Q:- The SFG shown in figure has ^{one} ~~two~~ forward path, ^{two isolated loop} determine overall transmittance or overall gain relating x_1 and x_6



$$x_2 = ax_1 + cx_2$$

$$x_3 = bx_2$$

$$x_4 = dx_3 + fx_5$$

$$x_5 = ex_4$$

$$x_6 = gx_5$$

We have to find $\frac{x_6}{x_1} = P$

Now, $x_3 = b(ax_1 + cx_3)$

$$x_3(1 - cb) = bax_1$$

$$\rightarrow x_3 = \frac{ab}{1 - cb} \cdot x_1$$

$$\rightarrow x_4 = \frac{abd}{1 - cb} x_1 + fx_5$$

$$\rightarrow x_4 = \frac{abd}{1 - cb} x_1 + fe x_4$$

$$x_4(1 - fe) = \frac{abd}{1 - cb} x_1$$

$$x_4 = \frac{abd}{(1 - cb)(1 - fe)} \cdot x_1$$

$$x_6 = gx_4$$

$$x_6 = \frac{abdeg}{(1 - cb)(1 - fe)} \cdot x_1$$

$$\frac{x_6}{x_1} = \frac{abdeg}{1 - cb - fe + cbfe}$$

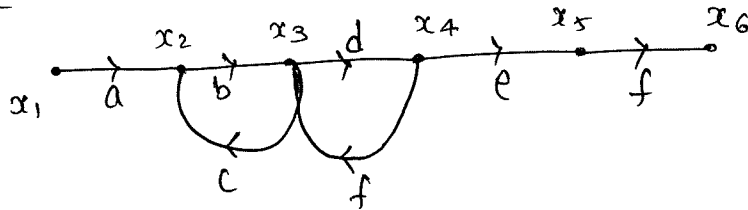
$$\frac{x_6}{x_1} = \frac{abdeg \xrightarrow{\text{Forward path}}}{1 - \underbrace{(cb + fe)}_{\text{loops}} + \underbrace{cbfe}_{\text{two non touching loop}}}$$

$$\frac{x_6}{x_1} = \frac{P}{\Delta}$$

$$\Delta = 1 - (cb + fe) + cbfe = 1 - (L_1 + L_2) + L_1L_2$$

$$\Delta = 1 - [\text{sum of individual loop}] + [\text{sum of gain product of two non-touching}] - [\text{sum of gain product of 3 non-touching loop}] + \dots$$

Q:-



$$\frac{x_6}{x_1} = ?$$

$$x_2 = ax_1 + cx_3$$

$$x_3 = bx_2 + fx_4$$

$$x_4 = dx_3$$

$$x_5 = x_4 e$$

$$x_6 = fx_5$$

$$\rightarrow x_4 = d[bx_2 + fx_4]$$

$$x_4(1-df) = dbx_2 \quad \therefore x_4 = \frac{dbx_2}{1-df}$$

$$x_4(1-df) = db[ax_1 + cx_3]$$

$$= db[ax_1 + c[bx_2 + fx_4]]$$

$$x_5 = \frac{dbx_2 \cdot e}{1-df}$$

$$x_6 = \frac{fedbx_2}{1-df}$$

$$x_4(1-df) = db[ax_1 + cbx_2]$$

$$x_6 = \frac{fedb}{1-df} [ax_1 + \frac{cbx_2}{d}]$$

$$x_6 = \frac{fedb}{1-df} \cdot (ax_1 + cx_3)$$

$$= \frac{fedb}{1-df} \left[ax_1 + \frac{cbx_2}{d(1-df)} \right]$$

$$x_2 = ax_1 + cx_3$$

$$x_3 = bx_2 + fx_4$$

$$x_4 = dx_3$$

$$x_5 = ex_4$$

$$x_6 = gx_5$$

$$x_3 = b(ax_1 + cx_3) + fdx_3$$

$$x_3 = bax_1 + bcx_3 + fdx_3$$

$$x_3(1 - bc - fd) = bax_1$$

$$\frac{x_3}{x_1} = \frac{ab}{1 - bc - fd}$$

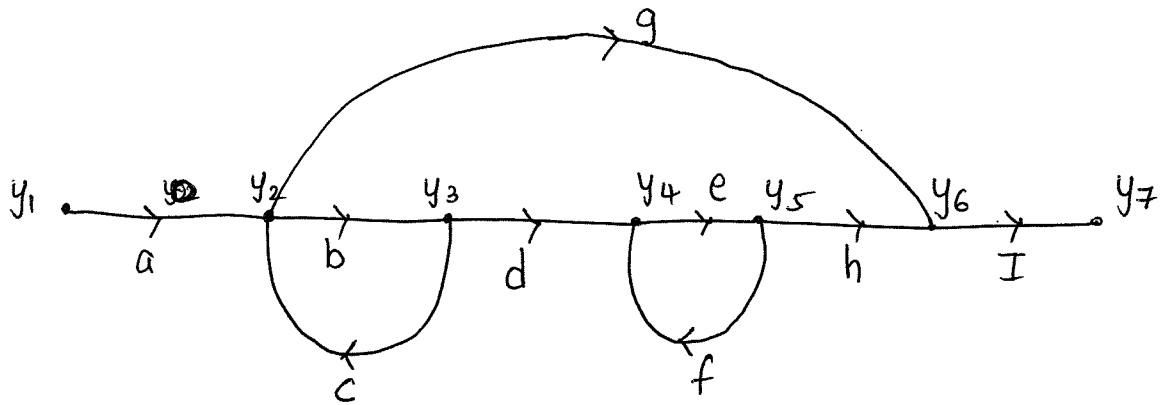
$$x_4 = \frac{abd x_1}{1 - bc - fd}$$

$$x_5 = \frac{abde x_1}{1 - bc - fd}$$

$$x_6 = \frac{abdeg x_1}{1 - bc - fd}$$

$$\frac{x_6}{x_1} = \frac{abdeg}{1 - bc - fd}$$

Q:-



$$\frac{y_7}{y_1} = ?$$

$$y_2 = ay_1 + cy_3$$

$$y_3 = by_2$$

$$y_4 = dy_3 + fy_5$$

$$y_5 = ey_4$$

$$y_6 = hy_5 + gy_2$$

$$y_7 = y_6 \cdot i$$

$$\Rightarrow y_7 = [hy_5 + gy_2] i \quad \text{--- (1)}$$

$$y_3 = b[ay_1 + cy_3]$$

$$y_3(1 - bc) = aby_1$$

$$y_3 = \frac{aby_1}{1 - bc}$$

$$y_4 = \frac{abd}{1 - bc} y_1 + fy_5$$

$$y_4 = \frac{abd}{1-bc} \cdot y_1 + fey_4$$

$$y_4[1-fe] = \frac{abd}{1-bc} \cdot y_1$$

$$y_4 = \frac{abd}{(1-bc)(1-fe)} \cdot y_1$$

$$y_5 = \frac{abde}{(1-bc)(1-fe)} \cdot y_1$$

Now,

$$y_2 = ay_1 + \frac{cab}{(1-cb)} y_1$$

$$y_2 = \left(a + \frac{abc}{1-cb} \right) y_1$$

$$= \left(\frac{a(1-cb) + abc}{1-cb} \right) y_1$$

$$= \left(\frac{a - abc + abc}{1-bc} \right) y_1$$

$$y_2 = \frac{a}{1-bc} y_1$$

$$y_7 = \left[\frac{abdehi}{(1-bc)(1-fe)} + \frac{agi}{(1-bc)} \right] y_1$$

$$\frac{y_7}{y_1} = \frac{abdehi(1) + agi(1-fe)}{1 - (bc + fe) + bcfe}$$

$$\frac{y_2}{y_1} = \frac{P_1 \overrightarrow{abdehi}(1)^{\Delta_1} + P_2 \overrightarrow{agi}(1-fe)^{\Delta_2}}{1 - \underset{\substack{\uparrow \\ L_1 + L_2}}{(cb+fe)} + \underset{\substack{\uparrow \\ L_1 L_2}}{cbfe}}$$

* Mason's Gain Formula

$$\text{Overall T.F.} = \frac{\sum_{k=1}^n P_k \cdot \Delta_k}{\Delta} = \frac{P_1 \Delta_1 + P_2 \Delta_2 + \dots}{\Delta}$$

P_k = no. of forward path gain

Δ_k = It is obtained from Δ which can be obtained by removing all loops which are connected to that forward path

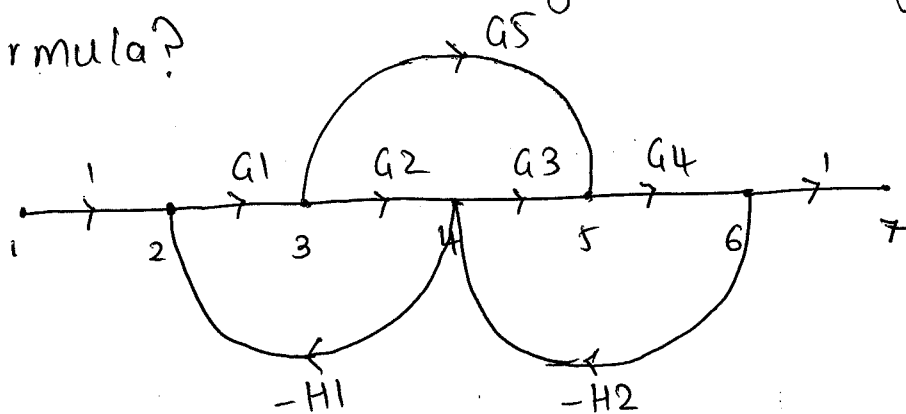
Means here for above examples:-

$$\Delta_1 = 1 - (0+0) + 0 = 1$$

$$\Delta_2 = 1 - (fe+0) + 0 = 1-fe$$

$$\Delta = 1 - [\sum \text{loop}] + [\sum \text{two non-touching loop}] - [\sum \text{3 non-touching loops}]$$

Q:- Find overall T.F. using SFG's using Mason's Gain Formula?



(1) No. of forward path

$$P_1 = G_1 G_2 G_3 G_4, \Delta_1 = 1$$

$$P_2 = G_1 G_5 G_4, \Delta_2 = 1$$

(2) No. of loops

(3) Two non-touching loops
Zero

$$L_1 = -G_1 G_2 H_1$$

$$L_2 = -G_3 G_4 H_2$$

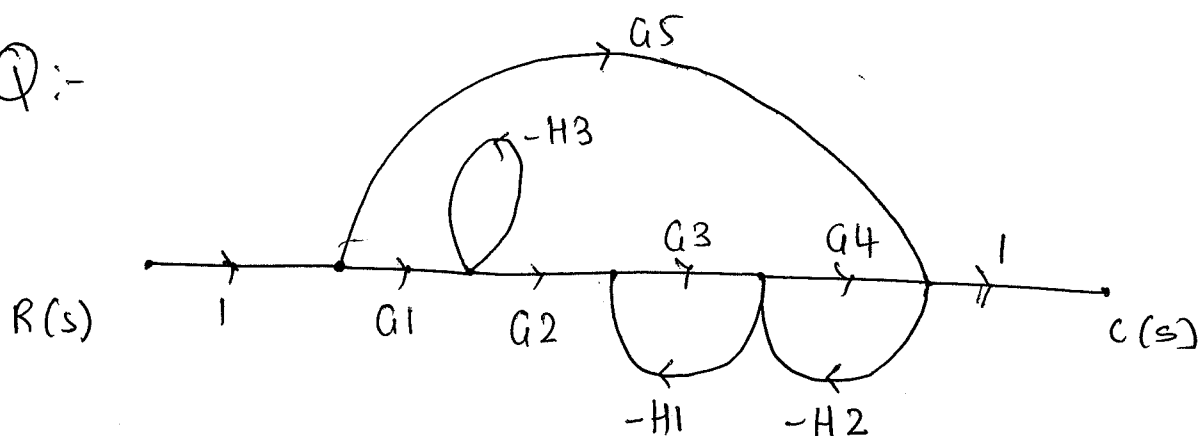
$$L_3 = G_1 G_5 G_4 H_1 H_2$$

$$\Delta = 1 + G_1 G_2 H_1 + G_3 G_4 H_2 - G_1 G_5 G_4 H_1 H_2$$

$$\text{T.F.} = \frac{G_1 G_2 G_3 G_4 (1) + G_1 G_5 G_4 (1)}{\Delta}$$

$$\text{T.F.} = \frac{G_1 G_2 G_3 G_4 + G_1 G_5 G_4}{1 + G_1 G_2 H_1 + G_3 G_4 H_2 - G_1 G_5 G_4 H_1 H_2}$$

Q:-



(1) No. of forward path

$$P_1 = G_1 G_2 G_3 G_4, \Delta_1 = 1$$

$$P_2 = G_5, \Delta_2 = 1 + H_1 G_3 + H_3 + G_3 H_1 H_3$$

(2) No. of single loops

$$L_1 = -H_3$$

$$L_2 = -H_1 G_3$$

$$L_3 = -H_2 G_4$$

(3) Two non-touching loops

$$L_1 L_2 = H_1 H_3 G_3$$

$$L_1 L_3 = H_2 H_3 G_4$$

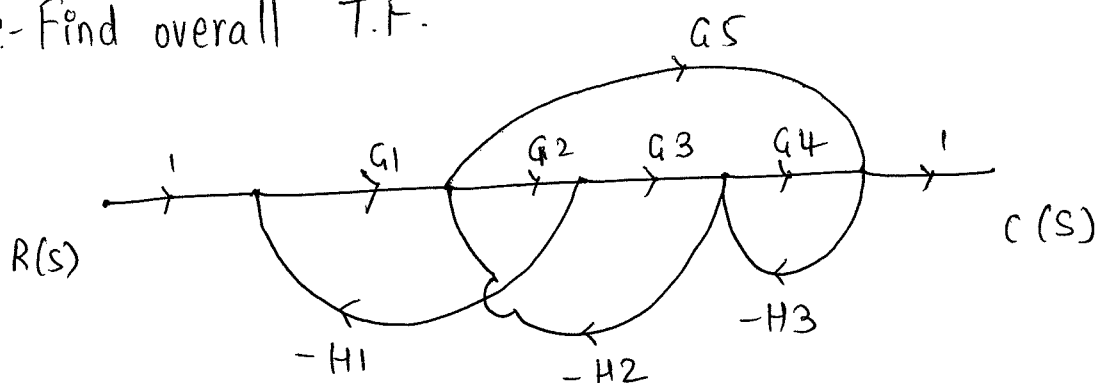
$$\Delta = 1 + H_3 + H_1 G_3 + H_2 G_4 + H_1 H_3 G_3 + H_2 G_3 G_4$$

From Mason's gain formula,

$$T.F. = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\frac{C(s)}{R(s)} = T.F. = \frac{G_1 G_2 G_3 G_4 + G_5 (1 + H_1 G_3 + H_3 + G_3 H_1 H_3)}{1 + H_3 + H_1 G_3 + H_2 G_4 + H_1 H_3 G_3 + H_2 G_3 G_4}$$

Q:- Find overall T.F.



$$P_1 = G_1 G_2 G_3 G_4, \Delta_1 = 1$$

$$P_2 = G_1 G_5, \Delta_2 = 1$$

$$\Delta = 1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_4 H_3 + G_1 G_2 H_1 G_4 H_3 + G_5 H_2 H_3$$

single loops:

$$L_1 = -G_1 G_2 H_1$$

$$L_2 = +G_2 G_3 H_2$$

$$L_3 = -G_4 H_3$$

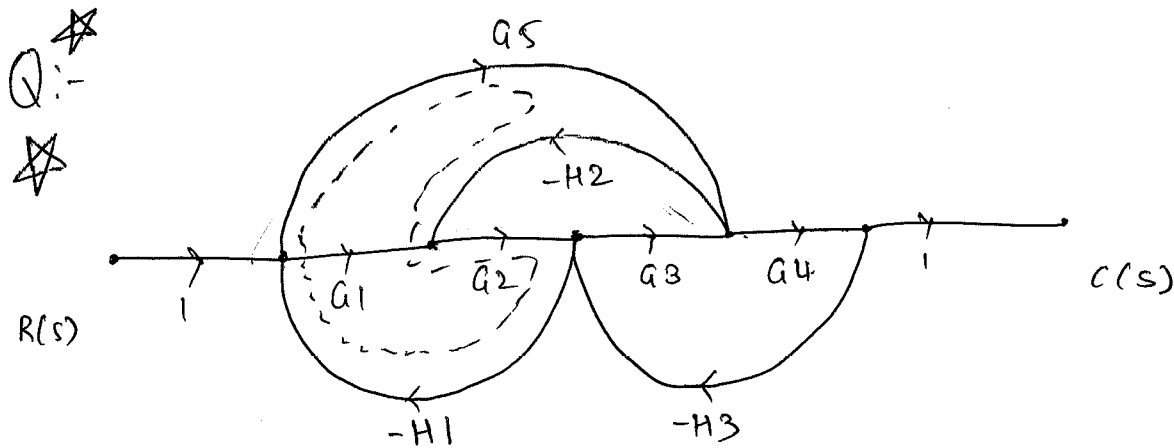
$$L_4 = G_5 H_2 H_3$$

two non-touching loops

$$L_1 L_3 = G_1 G_2 H_1 G_4 H_3$$

$$T.F. = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 G_3 G_4 + G_1 G_5}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_4 H_3 + G_5 H_2 H_3 + G_1 H_1 G_2 G_4 H_3}$$



No. of forward path

$$P_1 = G_1 G_2 G_3 G_4, \Delta_1 = 1$$

$$P_2 = G_5 G_4, \Delta_2 = 1$$

No. of individual loops

$$L_1 = -G_1 G_2 H_1$$

$$L_5 = G_5 H_2 G_2 H_1$$

$$L_2 = -G_3 G_4 H_3$$

$$L_3 = \cancel{G_1 G_2 G_3 G_4 H_1 H_3}^{G_5} G_4 H_1 H_3$$

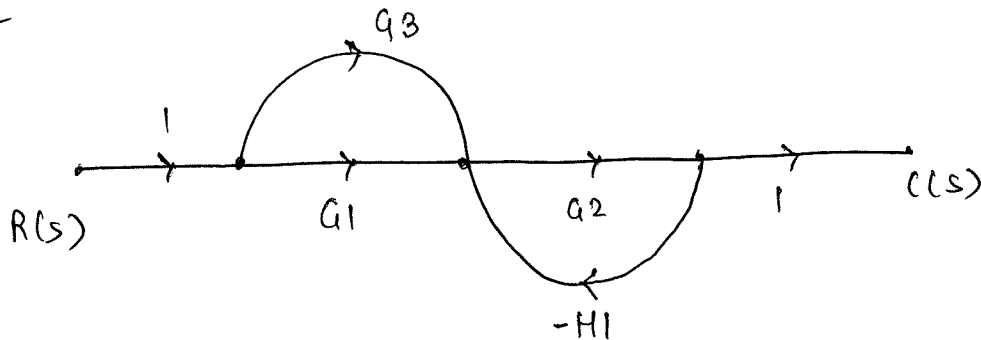
$$L_4 = -G_2 G_3 H_2$$

Two non-touching loops:-

Zero

$$T.F. = \frac{G_1 G_2 G_3 G_4 + G_5 G_4}{1 + G_1 G_2 H_1 + G_3 G_4 H_3 - G_5 G_4 H_1 H_3 + G_2 G_3 H_2 - G_5 H_2 G_2 H_1}$$

①:-



Forward path:-

$$P_1 = G_1 G_2, \Delta_1 = 1$$

$$P_2 = G_2 G_3, \Delta_2 = 1$$

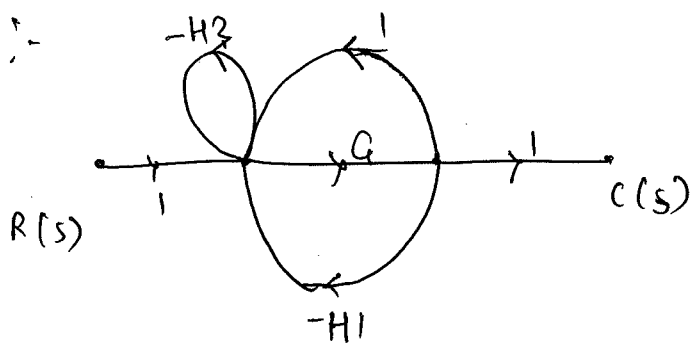
single loop:-

$$L_1 = -G_2 H_1$$

$$\Delta = 1 + G_2 H_1$$

$$T.F. = \frac{G_1 G_2 + G_2 G_3}{1 + G_2 H_1}$$

Q:-



Forward path

$$P1 = G, \Delta1 = 1$$

single loop:-

two non-touching loop:-
zero

$$L1 = -H2$$

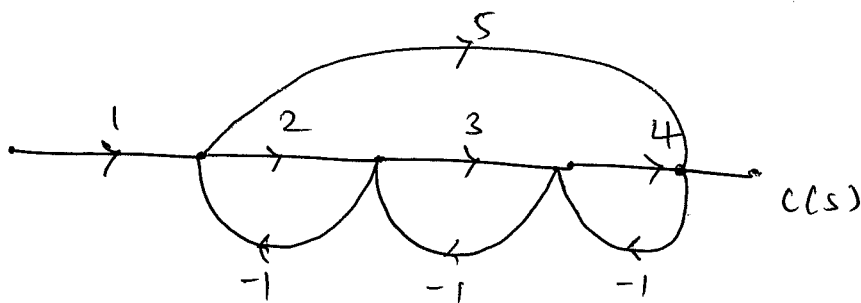
$$L2 = G$$

$$L3 = -GH1$$

$$\Delta = 1 + H2 - G + GH1$$

$$T.F. = \frac{G}{1 + H2 + GH1 - G}$$

Q:-



Forward path

single loop

two non-touching loops

$$P1 = 24$$

$$L1 = -2$$

$$L1 L3 = 8$$

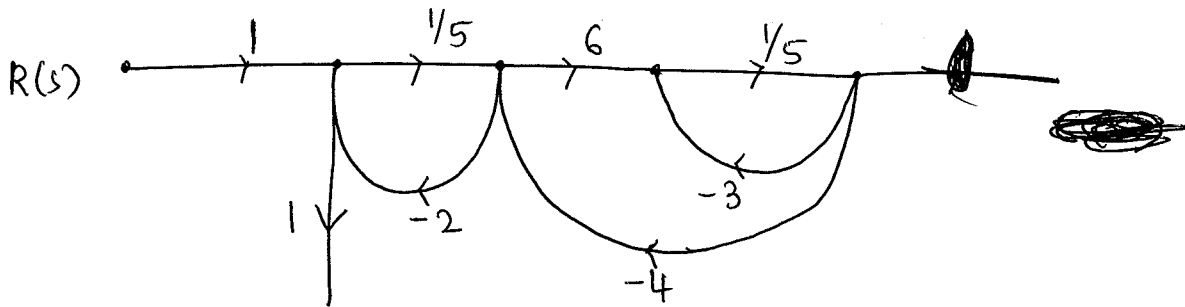
$$P2 = 5$$

$$L2 = -3$$

$$L3 = -4$$

$$L4 = -5$$

Q:-



* Forward gain $C(s)$

$$P_1 = 1$$

* Single loop

$$L_1 = -\frac{2}{5}$$

$$L_2 = -\frac{3}{5}$$

$$L_3 = 6 \cdot \frac{1}{5} (-4) = -\frac{24}{5}$$

* Two non-touching loop

$$L_1 L_2 = \frac{6}{25}$$

$$\Delta = 1 + \frac{2}{5} + \frac{3}{5} + \frac{24}{5} + \frac{6}{25} = \frac{25 + 10 + 15 + 120 + 6}{25}$$

$$\Delta = \frac{176}{25}$$

$$\Delta_1 = 1 + \frac{3}{5} + \frac{24}{5} = \frac{5 + 27}{5} = \frac{32}{5}$$

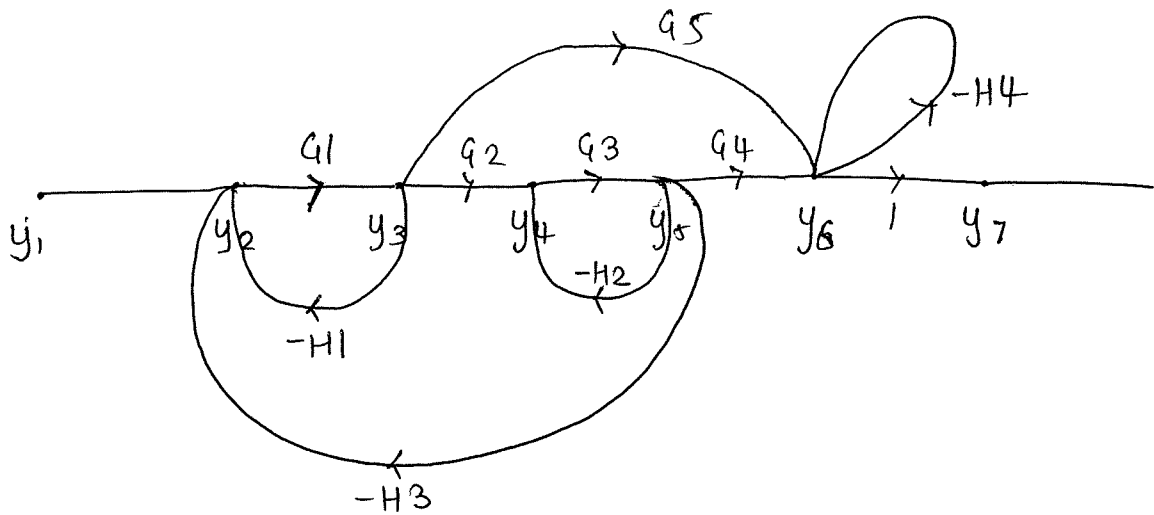
$$T.F. = \frac{P_1 \Delta_1}{\Delta} = \frac{1 \cdot \frac{32}{5}}{\frac{176}{25}} = \frac{160}{176}$$

IMP ★ -

① :- $\frac{y_7}{y_1}, \frac{y_5}{y_1}, \frac{y_2}{y_1}, \frac{y_7}{y_2}, \frac{y_5}{y_3}, \frac{y_5}{y_4}$ and so on ratio

★

of any two node for SFG.



Forward Path

$$P_1 = G_1 G_2 G_3 G_4$$

$$P_2 = G_1 G_5$$

loop

$$L_1 = -G_1 H_1$$

$$L_2 = -G_3 H_2$$

$$L_3 = -H_4$$

$$L_4 = -G_1 G_2 G_3 H_3$$

Two non-touching loop

$$L_1 L_2 = G_1 G_3 H_1 H_2$$

$$L_1 L_3 = G_1 H_1 H_4$$

$$L_2 L_3 = G_3 H_2 H_4$$

$$L_4 L_3 = G_1 G_2 G_3 H_3 H_4$$

Three non-touching loop

$$L_1 L_2 L_3 = -G_1 G_3 H_1 H_2 H_4$$

$$\Delta = 1 + G_1 H_1 + G_3 H_2 + H_4 + G_1 G_2 G_3 H_3 + G_1 G_3 H_1 H_2 + G_1 H_1 H_4 + G_3 H_2 H_4 + G_1 G_2 G_3 H_3 H_4 + G_1 G_3 H_1 H_2 H_4$$

$$\frac{y_6}{y_1} = \frac{G_1 G_2 G_3 G_4 (1) + G_1 G_5 (1 + G_3 H_2)}{\Delta}$$

$$\frac{y_7}{y_1} = \frac{G_1 G_2 G_3 G_4 (1) + G_1 G_5 (1 + G_3 H_2)}{\Delta}$$

$$\frac{y_5}{y_1} = \frac{G_1 G_2 G_3 (1 + H_4)}{\Delta}$$

$$\frac{y_2}{y_1} = \frac{1 \cdot (1 + G_3 H_2 + H_4 + G_3 H_4 H_2)}{\Delta}$$

NOTE:-

$$\frac{y_7}{y_2} = \frac{y_7/y_1}{y_2/y_1}$$

$$\frac{y_5}{y_3} = \frac{G_2 G_3 (1)}{\Delta} \Rightarrow \text{wrong}$$

$$\frac{y_5}{y_3} = \frac{y_5/y_1}{y_3/y_1} = \frac{G_1 G_2 G_3 (1 + H_4)}{G_1} = G_2 G_3 (1 + H_4)$$

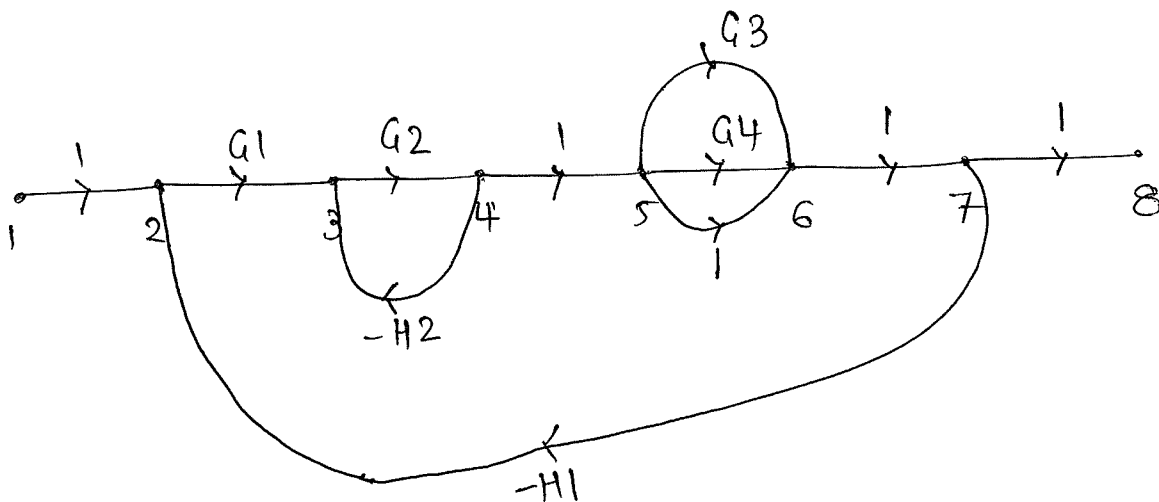
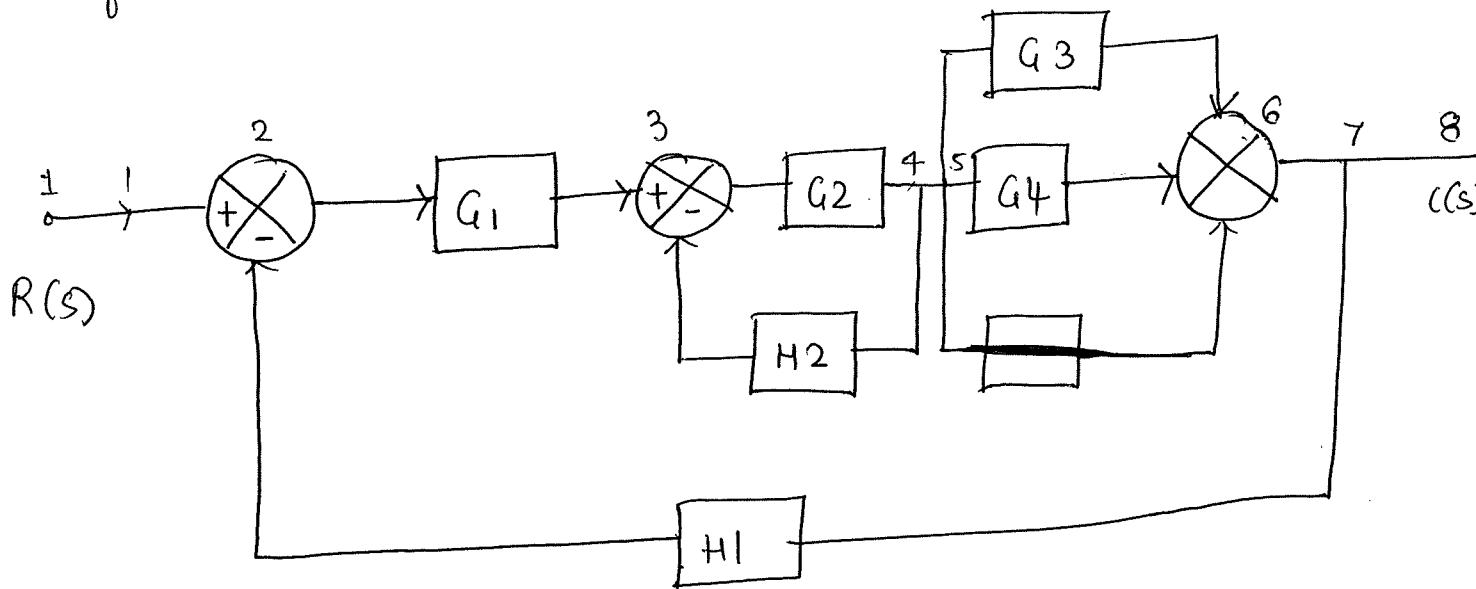
$$\frac{y_3}{y_1} = \frac{G_1 (1)}{\Delta}$$

$$\frac{y_5}{y_4} = \frac{y_5/y_1}{y_4/y_1} = \frac{G_1 G_2 G_3 (1 + H_4)}{G_1 G_2 (1)} = G_3 (1 + H_4)$$

$$\frac{y_4}{y_1} = \frac{G_1 G_2 (1)}{\Delta}$$

* Construction of SFG from block diagram

Q:- From block diagram construct SFG and find overall transfer function from Meson's gain formula.



$$P1 = G1G2G4$$

$$P2 = G1G2G3$$

$$P3 = G1G2$$

single loop:

$$L1 = -G2H2$$

$$L2 = -G1G2G4H1$$

$$L3 = -G1G2G3H1$$

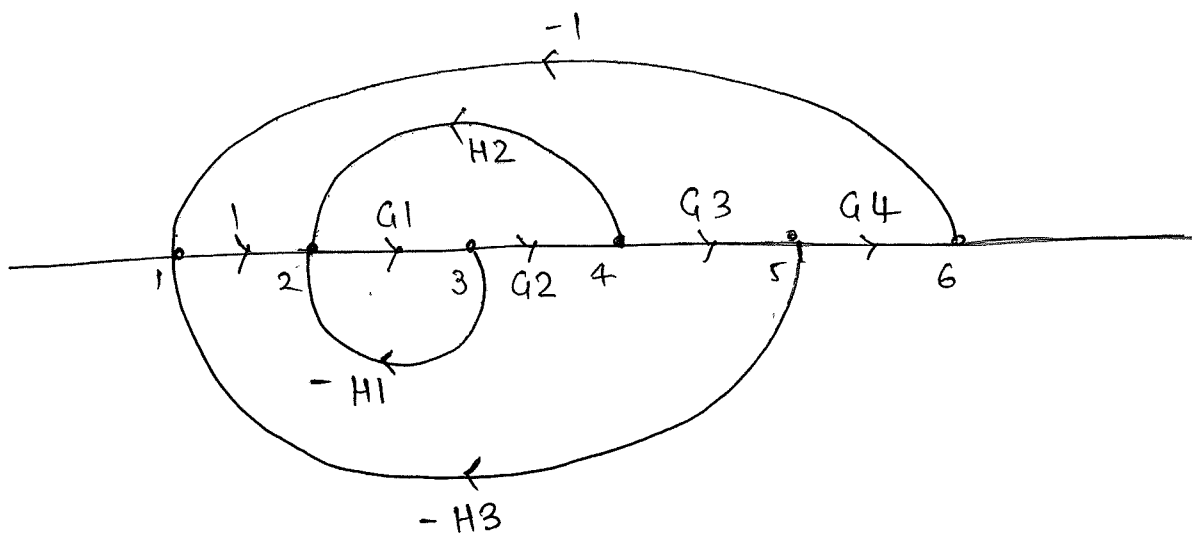
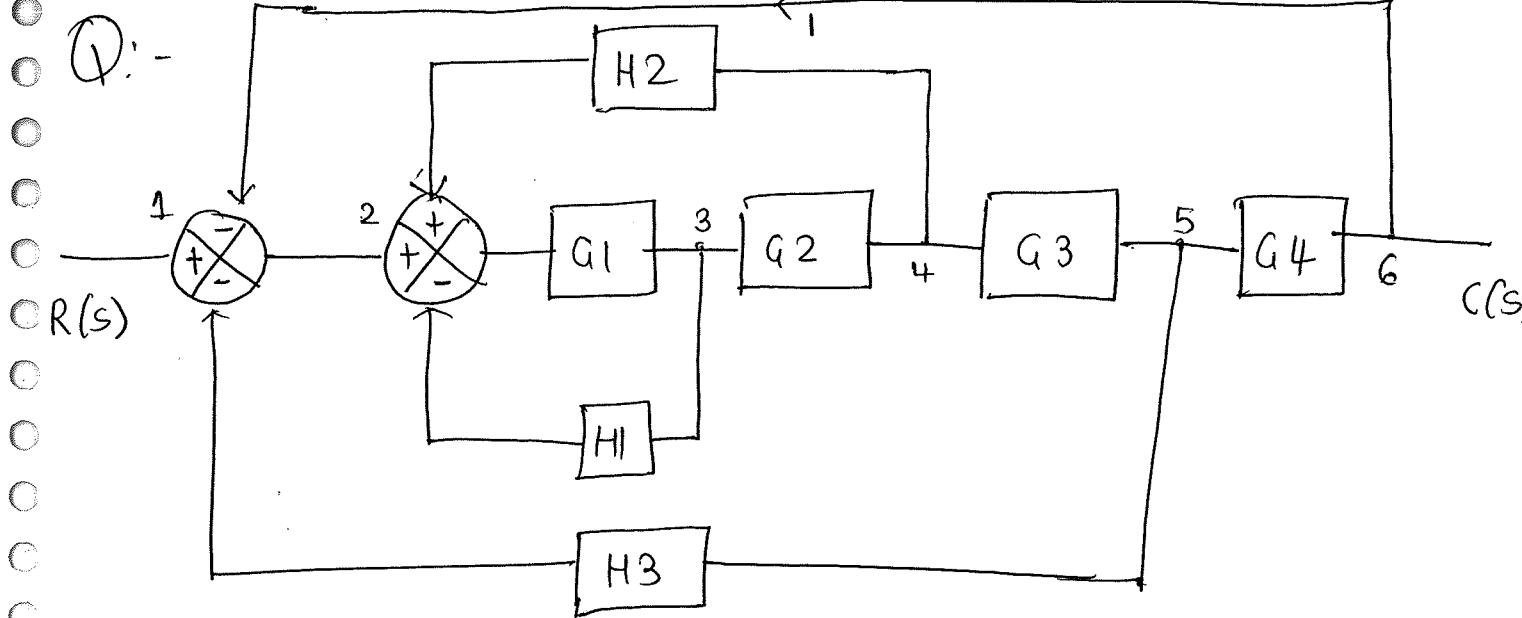
$$L4 = -G1G2H1$$

Two - non ~~touch~~ touching loop:-

Zero.

$$\Delta = 1 + G2H2 + G1G2G4H1 + G1G2G3H1 + G1G2H1$$

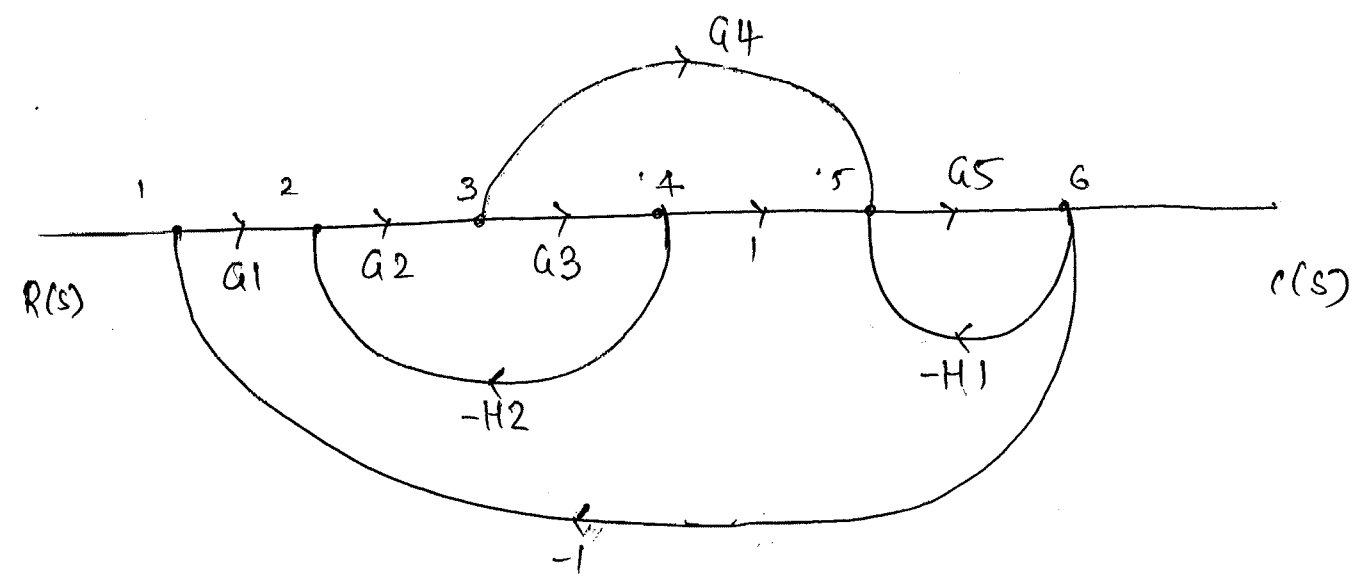
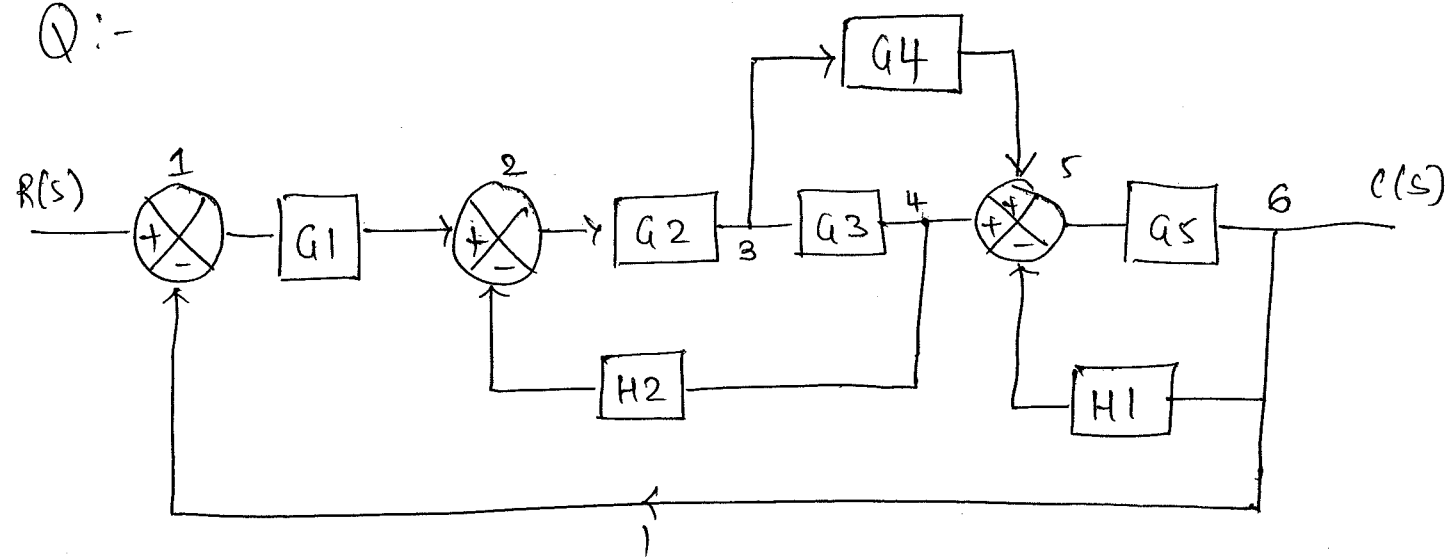
$$T.F. = \frac{G1G2G4 + G1G2G3 + G1G2}{1 + G2H2 + G1G2G4H1 + G1G2G3H1 + G1G2H1}$$



$$P1 = G1G2G3G4$$

$$T.F. = \frac{G1G2G3G4}{1 + G1H1 - G1G2H2 + G1G2G3H3 + G1G2G3G4}$$

Q:-



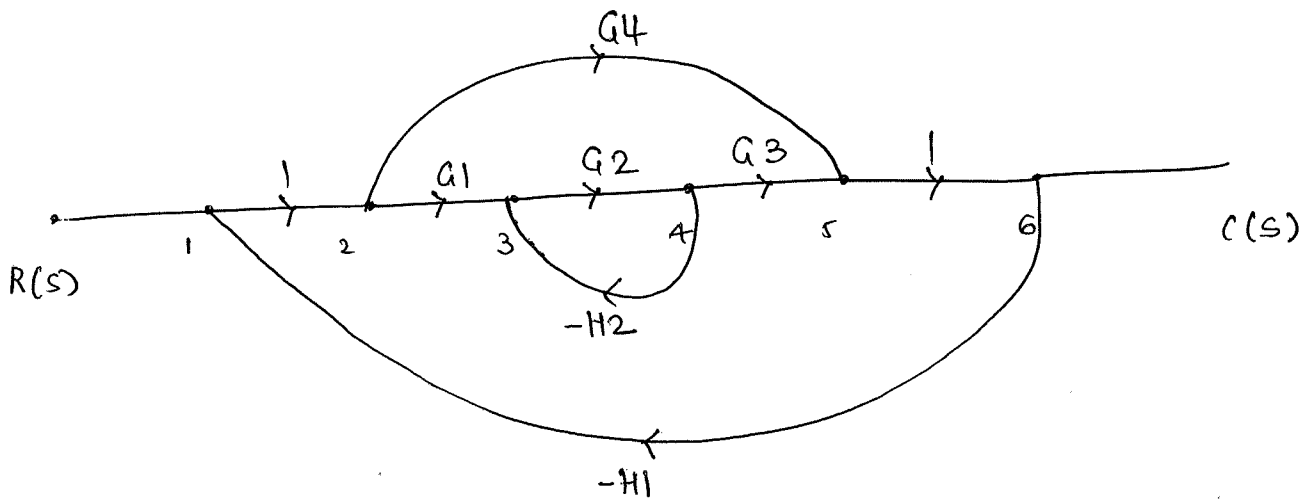
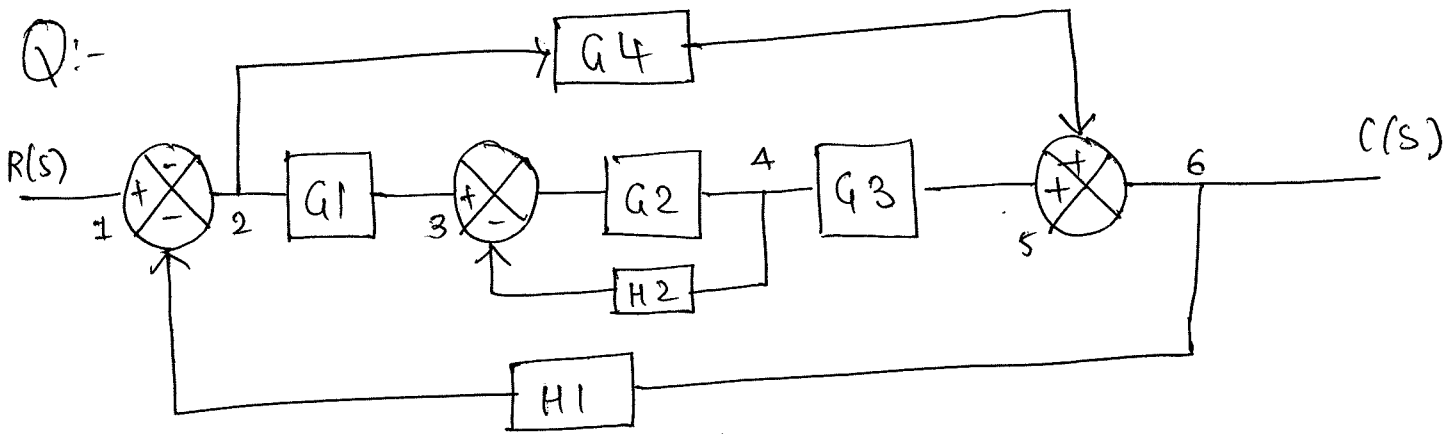
Forward path:-

$$P_1 = G_1 G_2 G_3 G_5$$

$$P_2 = G_1 G_2 G_4 G_5$$

$$T.F. = \frac{G_1 G_2 G_3 G_5 + G_1 G_2 G_4 G_5}{1 + G_2 G_3 H_2 + G_5 H_1 + G_1 G_2 G_3 G_5 + G_1 G_2 G_4 G_5}$$

Q:-



Forward path

$$P_1 = G_1 G_2 G_3$$

$$P_2 = G_4$$

single loop

$$L_1 = -G_2 H_2$$

$$L_2 = -G_4 H_1$$

$$L_3 = -G_1 G_2 G_3 H_1$$

two non-touching loop

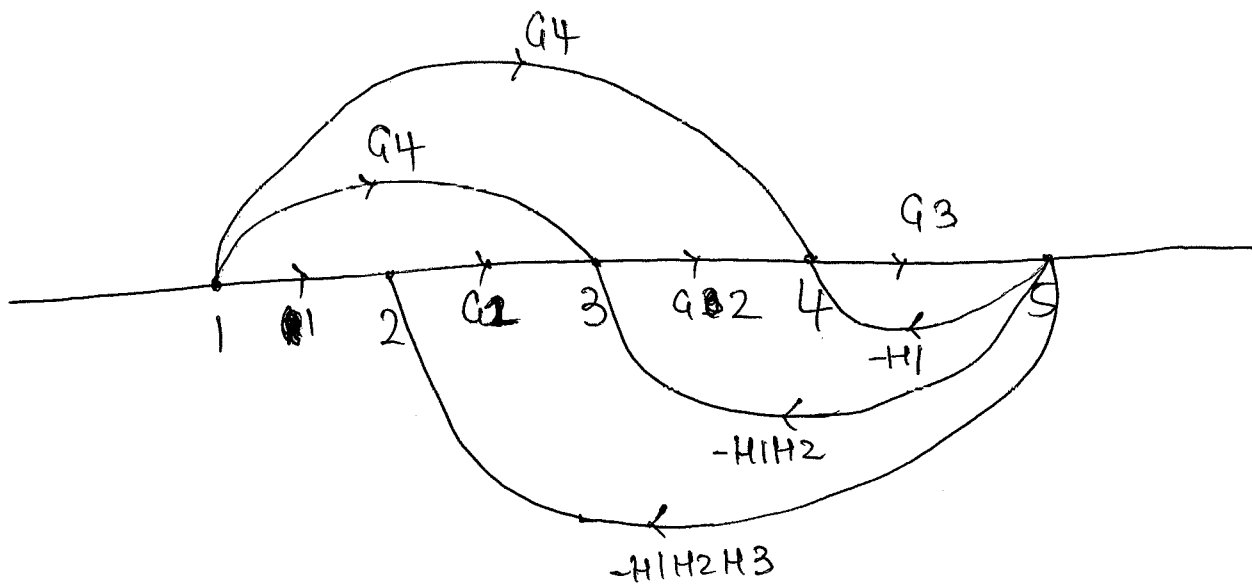
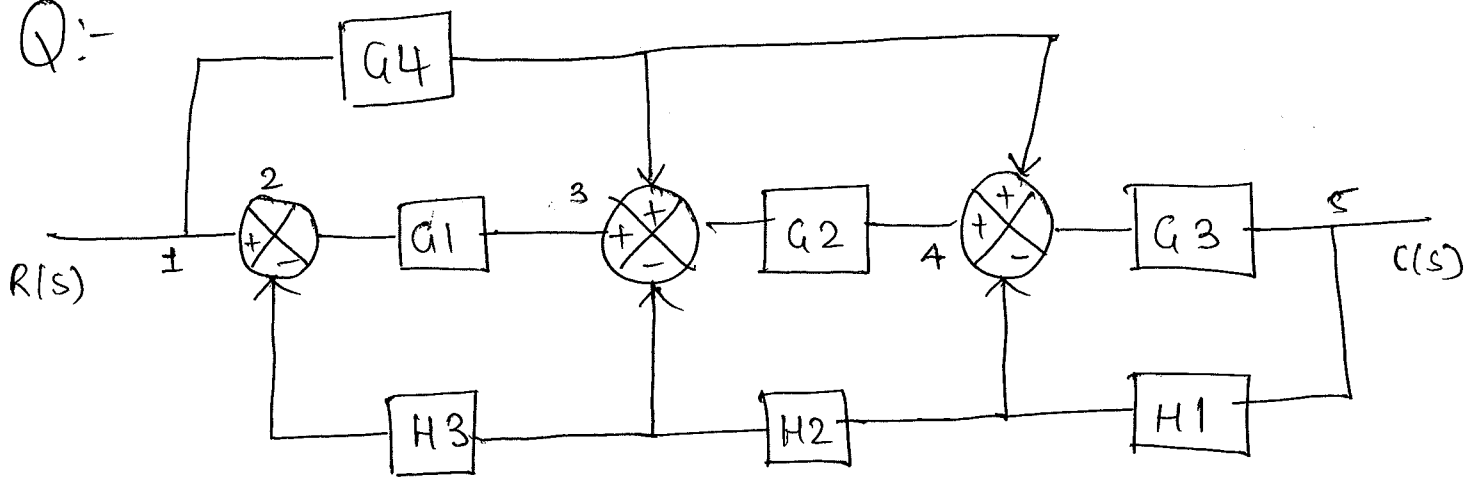
$$L_1 L_2 = G_2 G_4 H_1 H_2$$

$$\Delta = 1 + G_2 H_2 + G_4 H_1 + G_1 G_2 G_3 H_1$$

$$+ G_2 G_4 H_1 H_2$$

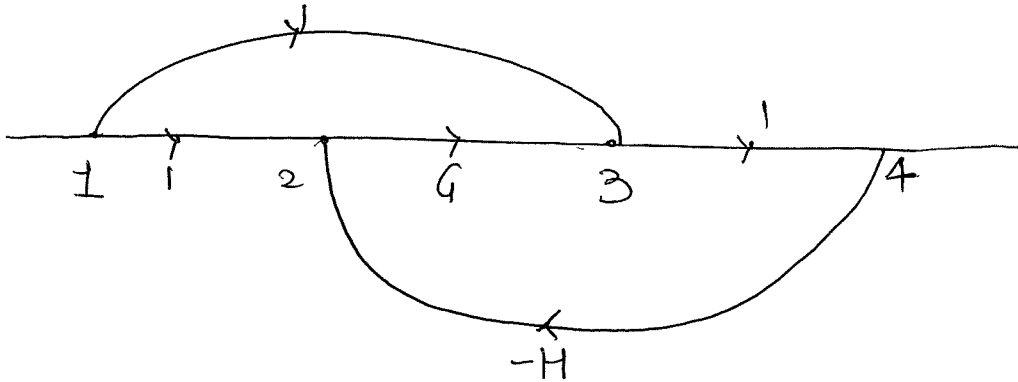
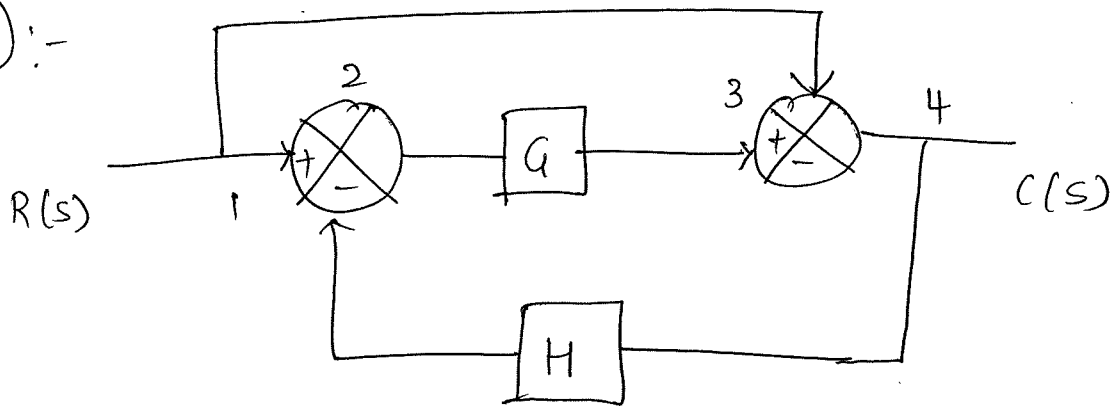
$$\text{T.F.} = \frac{G_1 G_2 G_3 + G_4 (1 + G_2 H_2)}{1 + G_2 H_2 + G_4 H_1 + G_1 G_2 G_3 H_1 + G_2 G_4 H_1 H_2}$$

Q:-



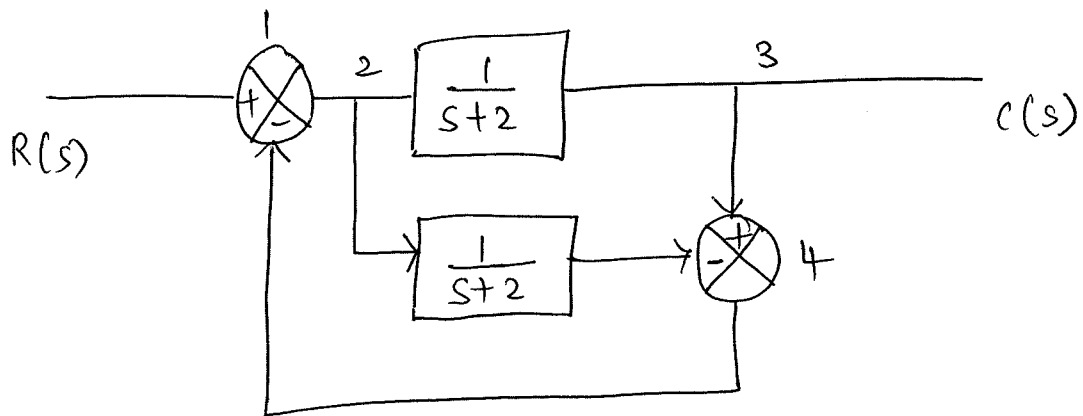
$$T.F. = \frac{G_1 G_2 G_3 + G_4 G_2 G_3 + G_4 G_3}{1 + H_1 G_3 - G_3 G_2 H_1 H_2 + G_1 G_2 G_3 H_1 H_2 H_3}$$

Q:-

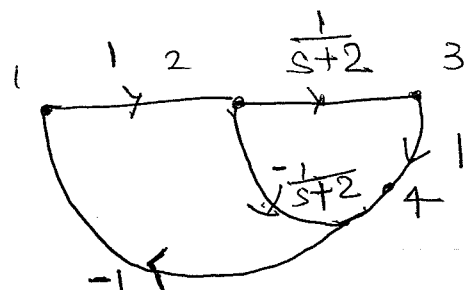


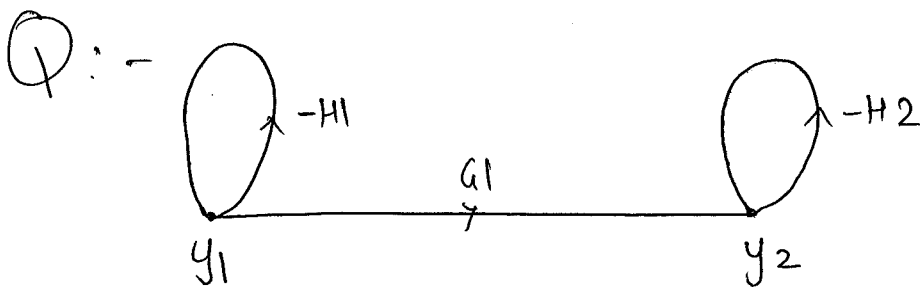
$$I.F. = \frac{1 + G}{1 + GH}$$

Q:-

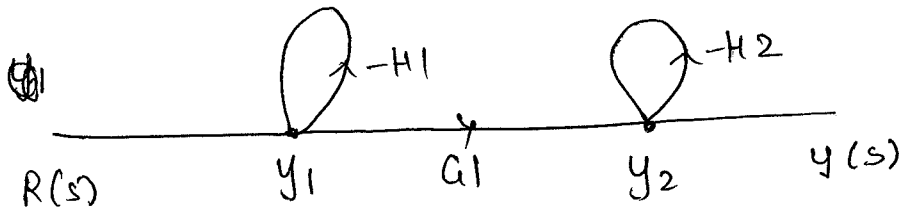


$$T.F. = \frac{\frac{1}{s+2}}{1 + \frac{1}{s+2} - \frac{1}{s+2}} = \frac{1}{s+2}$$





$$\frac{y_2}{y_1} = ?$$



$$\frac{y_1}{R(s)} = \frac{1(1+H_2)}{1+H_1+H_2+H_1H_2}$$

$$\frac{y_2}{R(s)} = \frac{G_1(1)}{1+H_1+H_2+H_1H_2}$$

$$\frac{y_2}{y_1} = \frac{y_2/R(s)}{y_1/R(s)} = \frac{G_1}{1+H_2}$$

$$\boxed{\frac{y_2}{y_1} = \frac{G_1}{1+H_2}}$$

Method: 2

$$y_2 = G_1 y_1 - H_2 y_2$$

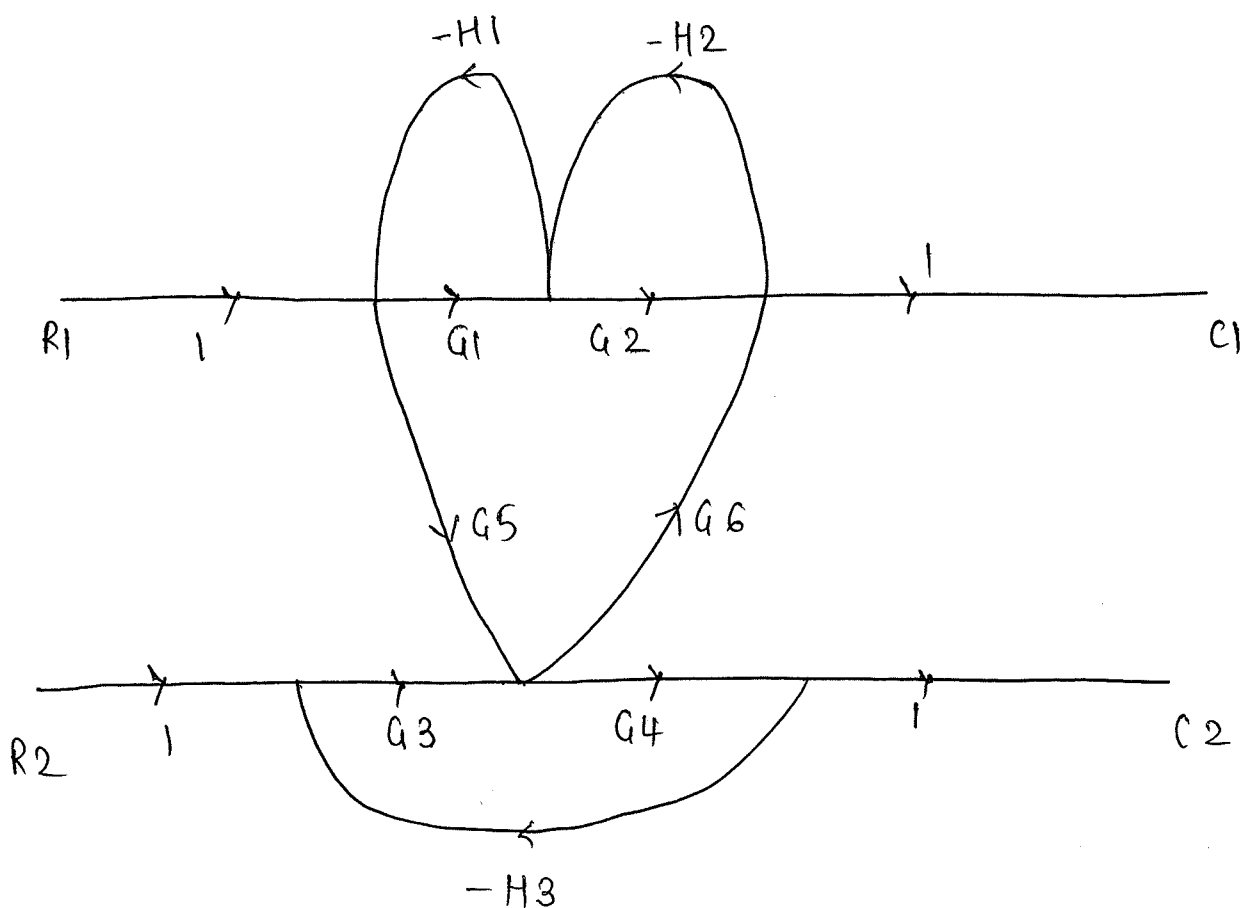
$$y_2(1+H_2) = G_1 y_1$$

$$y_2 = \frac{G_1}{1+H_2} y_1$$

$$\frac{y_2}{y_1} = \frac{G_1}{1+H_2} //$$

- Method of Meson's gain formula gives ratio with respect to input node only it cannot give ratio with respect to middle node.

Q:- Find $\frac{C_1}{R_1}$, $\frac{C_1}{R_2}$, $\frac{C_2}{R_1}$, $\frac{C_2}{R_2}$ to given multiple input multiple output system.



Sol:-

$$\Delta = 1 + G_1 H_1 + G_2 H_2 - G_5 G_6 H_1 H_2 + G_3 G_4 H_3 + G_3 G_4 G_1 H_3 H_1 + G_3 G_2 G_4 H_2 H_3$$

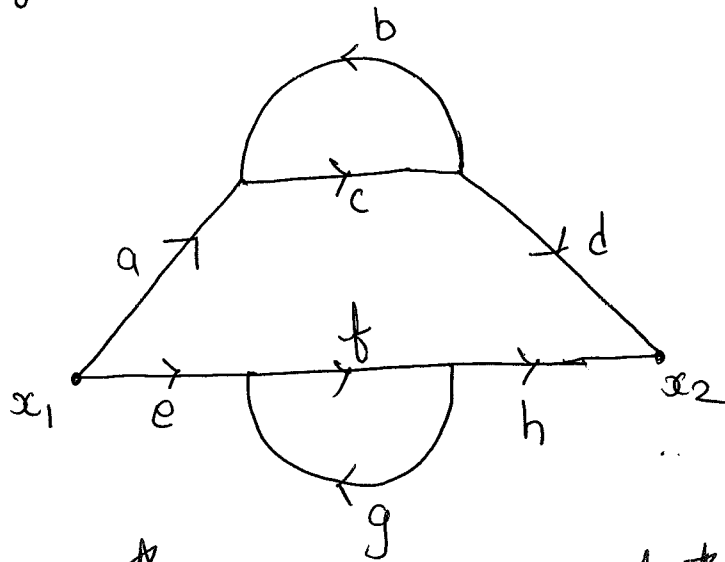
$$\frac{C_1}{R_1} = \frac{G_1 G_2 (1 + G_3 G_4 H_3) + G_5 G_6 (1)}{\Delta}$$

$$\frac{C1}{R2} = \frac{G3G6(1+G1H1)}{\Delta}$$

$$\frac{C2}{R2} = \frac{G3G4(1+G2H2+G1H1)}{\Delta}$$

$$\frac{C2}{R1} = \frac{G5G4(1+G2H2)}{\Delta}$$

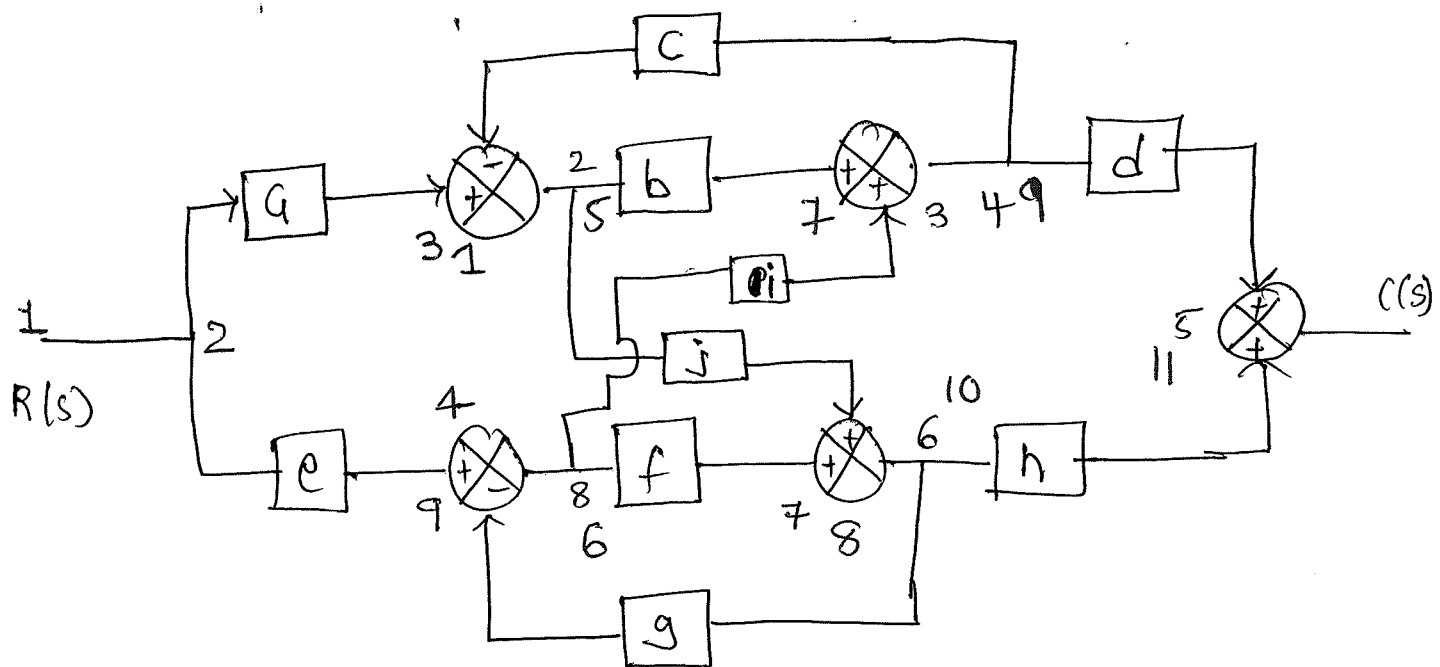
Q:- For the SFG determine overall transmittance relating x_2, x_1



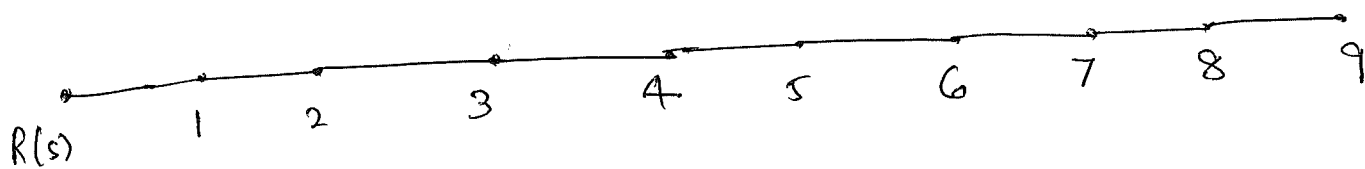
★ No need to add dummy node ★

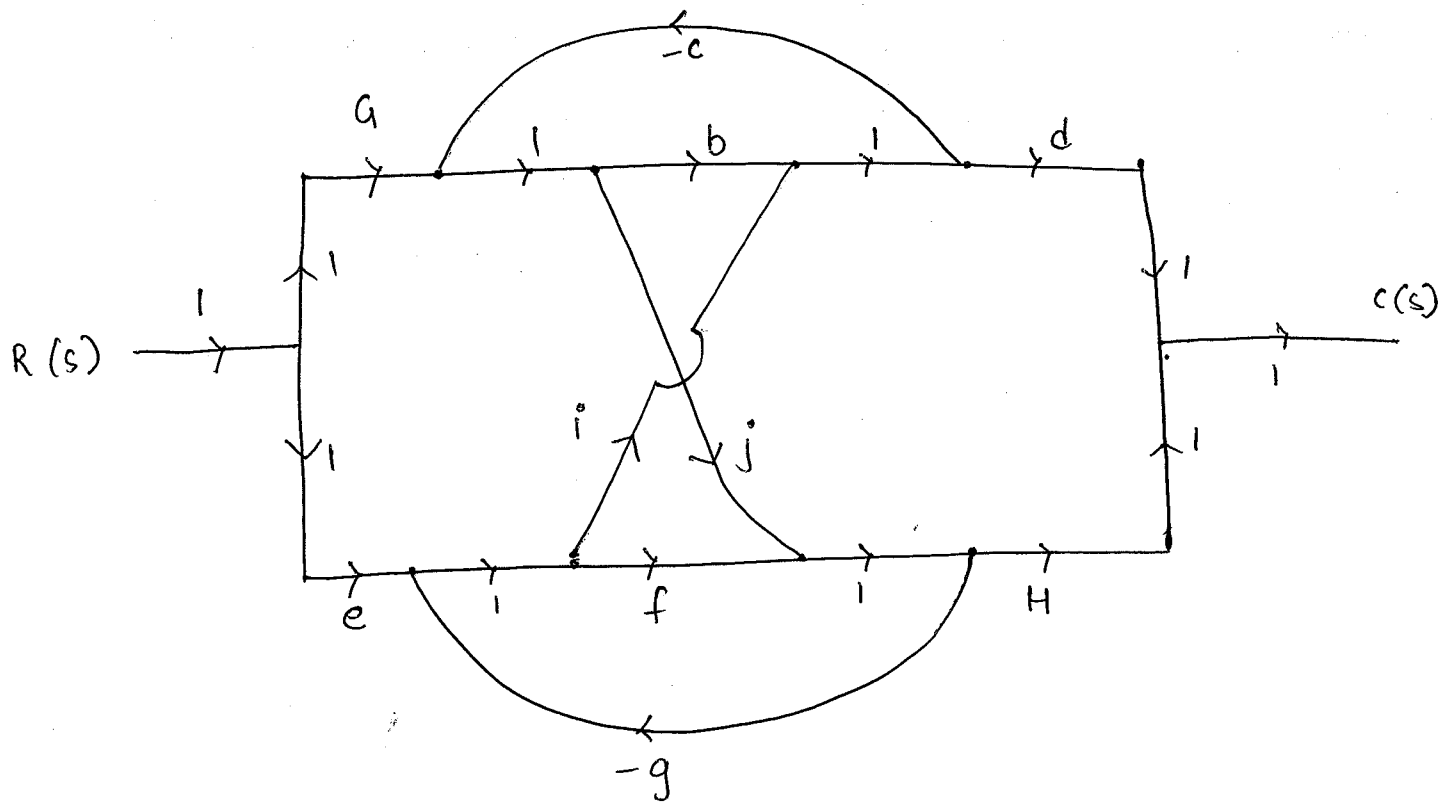
$$\frac{x_2}{x_1} = \frac{acd(1-fg) + efn(1-bc)}{1 - fg - bc + bcfg}$$

- Obtain SFG representation of the system whose block diagram is given below
- (1) Forward path
- (2) Individual loop
- (3) Non-touching loops
- (4) Overall transfer function



1 → wrong node
 ± → right node





$$P_1 = Gbd, P_2 = GjH, P_3 = efH, P_4 = eid$$

$$P_5 = -Gjgid, P_6 = -eicjH$$

single loops:

$$L_1 = fg$$

$$L_2 = -bc$$

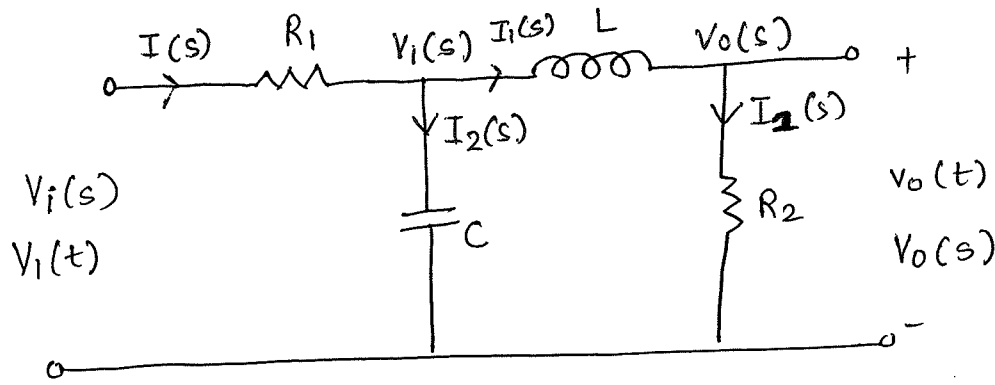
$$L_3 = jgic$$

Two non-touching loops

$$L_1 L_2 = fgbc$$

$$\text{T.F.} = \frac{Gbd(1+fg) + GjH + efH(1+bc) + eid - Gjgid - eicjH}{1 + fg + bc^{-jgic} + fgbc}$$

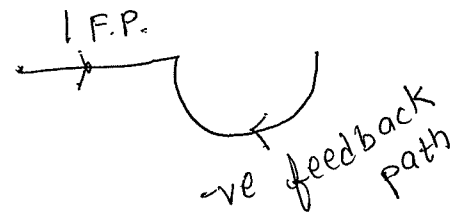
*Construction of SFG from electrical network:



$$\Rightarrow I(s) = \frac{V_i(s) - V_1(s)}{R_1} = \frac{1}{R_1} [V_i(s) - V_1(s)] \quad \text{--- (1)}$$

$$V_1(s) = \frac{1}{sC} \cdot I_2(s)$$

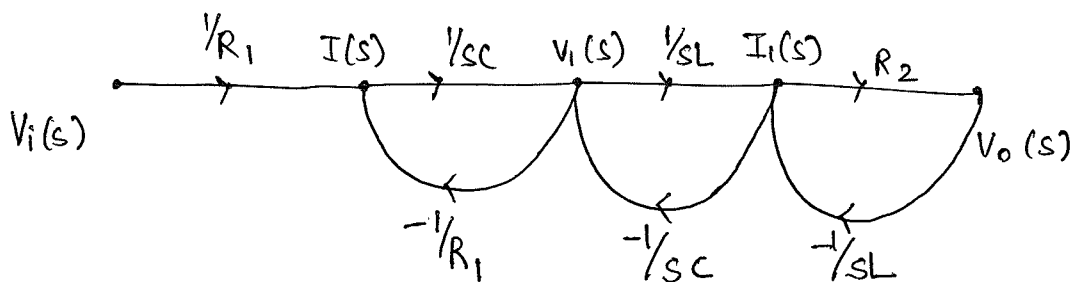
$$I_2(s) = I(s) - I_1(s)$$



$$\Rightarrow V_1(s) = \frac{1}{sC} [I(s) - I_1(s)] \quad \text{--- (2)}$$

$$\Rightarrow I_1(s) = \frac{V_1(s) - V_0(s)}{R} \quad \text{--- (3)}$$

$$\Rightarrow V_0(s) = R_2 I_1(s) \quad \text{--- (4)}$$



$$\frac{C(s)}{R(s)} = \frac{R_2}{s^2 L C R_1} \cdot \frac{1}{1 + \frac{1}{s C R_1} + \frac{1}{s^2 L C} + \frac{R_2}{s L} + \frac{R_2}{s^2 L C R_1}}$$

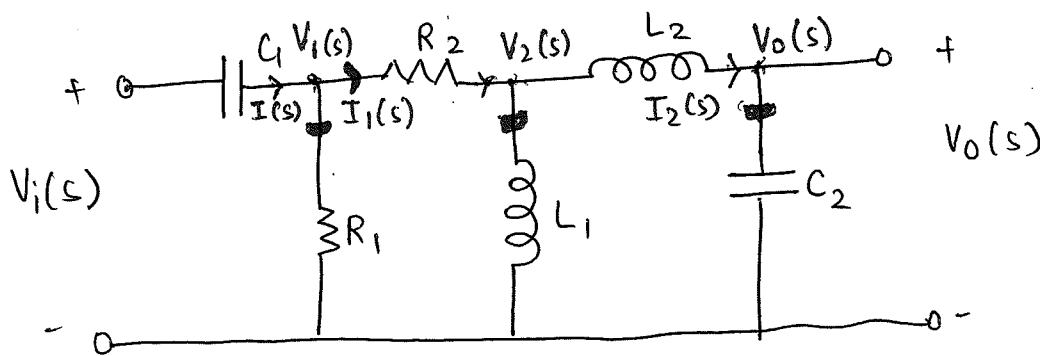
Also, from ~~above~~ result

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2 \cdot Z_4}{Z_1(Z_2 + Z_3 + Z_4) + Z_2(Z_3 + Z_4)}$$

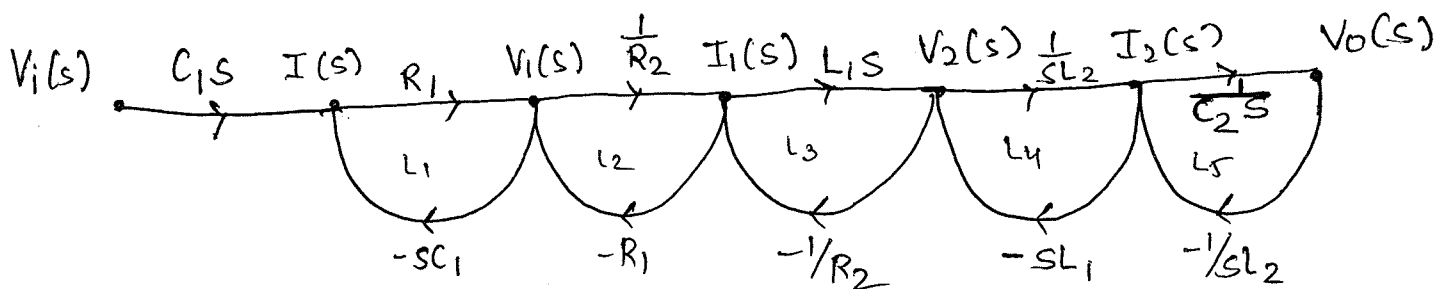
* Procedure for drawing SFG from electrical network.

- Mark voltage & current along the series path element.
- No. of nodes in SFG are variables along the series path
- Each and every element of electrical network gives one forward path and one negative feedback path except last element gives only one forward path
- Take inverse of impedance as a path gain for series element and take same impedance as a path gain for shunt element.

Q:- Find T.F. and draw SFG.



In parallel
R, L, C write
as it is.
In series -1
write ()
of it.



$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{R_1 L_1 C_1}{R_2 L_2 C_2}}{1 + s R_1 C_1 + \frac{R_1}{R_2} + \frac{s L_1}{R_2} + \frac{L_1}{L_2} + \frac{1}{s^2 L_2 C_2} + \frac{s^2 R_1 L_1 C_1}{R_2} + \frac{s R_1 C_1 L_1}{L_2} + \cancel{\frac{s R_1 C_1}{R_2}} + \frac{s R_1 C_1}{s L_2 C_2} + \frac{L_1}{s R_2 C_2 L_2} + \frac{R_1 L_1}{R_2 L_2} + \frac{R_1}{s^2 R_2 C_2 L_2} + \frac{R_1 C_1 L_1}{R_2 C_2 L_2}}$$

If $R=L=C$ then

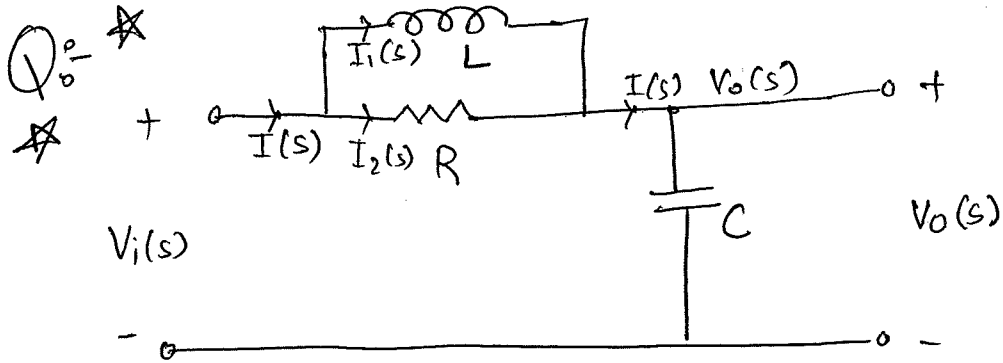
$$\begin{aligned} \frac{V_o(s)}{V_i(s)} &= \frac{1}{1 + s + 1 + s + 1 + \frac{1}{s^2} + s^2 + s + \frac{1}{s} + \frac{1}{s} + 1 + \frac{1}{s^2} + 1} \\ &= \frac{1}{\frac{2}{s^2} + s^2 + 3s + \frac{2}{s} + 5} \\ &= \frac{s^2}{2 + s^4 + 3s^3 + 2s + 5s^2} \end{aligned}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{s^2}{s^4 + 3s^3 + 5s^2 + 2s + 2}$$

Total order = 4

storage element = 4

DC gain = 0

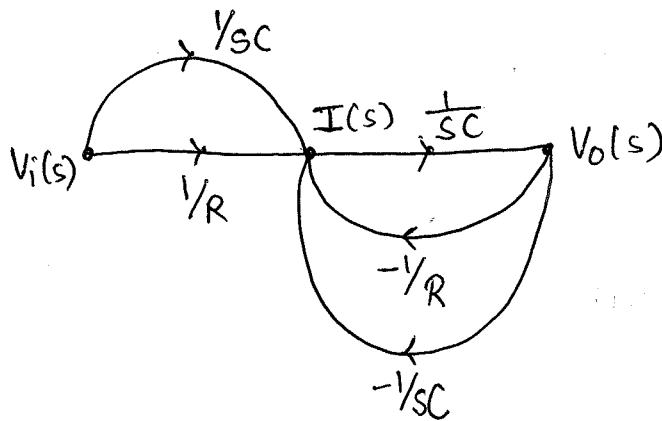


$$I(s) = I_1(s) + I_2(s)$$

$$I(s) = \frac{V_i(s) - V_o(s)}{sL} + \frac{V_i(s) - V_o(s)}{R}$$

$$I(s) = V_i(s) \left[\frac{1}{R} + \frac{1}{sL} \right] - V_o(s) \left[\frac{1}{R} + \frac{1}{sL} \right]$$

$$V_o(s) = \frac{1}{sC} I(s)$$



$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sCR} + \frac{1}{s^2C^2}}{1 + \frac{1}{sCR} + \frac{1}{s^2C^2}}$$