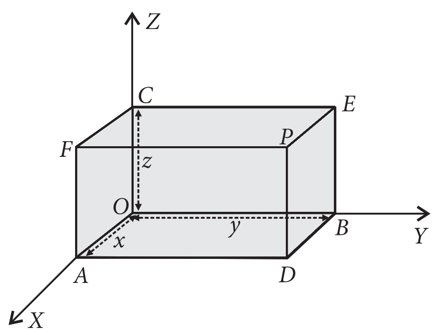
Introduction to Three Dimensional Geometry

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. In the given figure, if P is (2, 4, 5), then find the coordinates of F.



(a) (2, 4, 5) (b) (4, 0, 5)
(c) (2, 0, 5) (d) (4, 2, 5)

2. Find the octant in which the points (–3, 1, 2) and (–3, 1, –2) lie respectively.
(a) second, fourth (b) sixth, second
(c) fifth, sixth (d) second, sixth

3. Let L, M, N be the feet of the perpendiculars drawn from a point $P(7, 9, 4)$ on the x, y and z-axes respectively. Find the coordinates of L, M and N respectively.
(a) (7, 0, 0), (0, 9, 0), (0, 0, 4)
(b) (7, 0, 0), (0, 0, 9), (0, 4, 0)
(c) (0, 7, 0), (0, 0, 9), (4, 0, 0)
(d) (0, 0, 7), (0, 9, 0), (4, 0, 0)

4. Let A, B, C be the feet of the perpendicular segments drawn from a point $P(3, 4, 5)$ on the xy, yz and zx-planes, respectively. What are the coordinates of A, B and C ?
(a) (3, 4, 0), (0, 4, 4), (3, 0, 5)
(b) (3, 0, 4), (4, 5, 0), (3, 5, 0)
(c) (3, 5, 0), (0, 5, 4), (0, 3, 4)
(d) (3, 4, 0), (0, 4, 5), (3, 0, 5)

5. What is the perpendicular distance of the point $P(6, 7, 8)$ from xy-plane?
(a) 8 units (b) 7 units
(c) 6 units (d) 5 units

6. Let A, B, C be the feet of the perpendicular segments drawn from a point $P(3, 4, 5)$ on the xy, yz and zx - planes, respectively. The distance of the points A, B, C from the point P (in units) respectively are
(a) 5, 2, 4 (b) 3, 4, 5
(c) 5, 3, 4 (d) 3, 5, 4

7. Find the distance between the points $P(1, -3, 4)$ and $Q(-4, 1, 2)$.
(a) $\sqrt{5}$ units (b) $5\sqrt{3}$ units
(c) $3\sqrt{5}$ units (d) $2\sqrt{2}$ units

8. Find the equation of set of points P such that $PA^2 + PB^2 = 2k^2$, where A and B are the points (3, 4, 5) and (–1, 3, –7), respectively.
(a) $x^2 + y^2 + z^2 - 4x - 14y + 4z = 2k^2 - 109$
(b) $2x^2 + 2y^2 - 2z^2 - 4x - 14y - 4z = 2k^2 + 109$
(c) $2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = 2k^2 - 109$
(d) None of these

9. Find the equation of the set of the points P such that its distances from the points $A(3, 4, -5)$ and $B(-2, 1, 4)$ are equal.
(a) $10x + 6y - 18z - 29 = 0$
(b) $10x + 18y - 6z - 29 = 0$
(c) $5x + 3y - 9z - 29 = 0$
(d) $10x + 6y - 18z - 45 = 0$

10. Find the coordinates of a point which is equidistant from the four points $O(0, 0, 0)$, $A(l, 0, 0)$, $B(0, m, 0)$ and $C(0, 0, n)$.
(a) (l, m, n) (b) $\left(\frac{l}{2}, \frac{m}{2}, \frac{n}{2}\right)$
(c) $\left(\frac{l}{2}, m, \frac{n}{2}\right)$ (d) $(2l, 2m, 2n)$

11. Find the coordinates of the point which divides the line segment joining the points (1, –2, 3) and (3, 4, –5) in the ratio 2 : 3 externally.
(a) (–3, –14, 19) (b) (3, 14, 19)
(c) (–3, –14, –19) (d) (3, –14, –19)

12. Find the ratio in which the line segment joining the points (4, 8, 10) and (6, 10, –8) is divided by the yz-plane.

(a) 2 : 3 internally (b) 2 : 3 externally
(c) 5 : 7 internally (d) 5 : 7 externally

13. The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A and B are (3, –5, 7) and (–1, 7, –6), respectively, then find the coordinates of the point C.

(a) (2, 1, 1) (b) (1, 2, 1)
(c) (1, 1, 2) (d) (1, 2, 2)

14. Find the ratio in which the line segment joining the points (2, 4, 5) and (3, 5, –4) is divided by the xz-plane.

(a) 4 : 5 externally (b) 2 : 3 externally
(c) 1 : 3 externally (d) 4 : 5 internally

15. Find the coordinate of the point P which is five-sixth of the way from A(–2, 0, 6) to B(10, –6, –12).

(a) (–8, 5, –9) (b) (–8, –5, 9)
(c) (8, –5, –9) (d) (8, 5, 9)

16. M is the foot of the perpendicular drawn from the point A(6, 7, 8) on the yz-plane. What are the coordinates of point M?

(a) (6, 0, 0) (b) (6, 7, 0)
(c) (6, 0, 8) (d) (0, 7, 8)

17. L is the foot of the perpendicular drawn from a point (3, 5, 6) on x-axis. The coordinates of L are

(a) (3, 0, 0) (b) (0, 6, 0)
(c) (0, 0, 5) (d) (0, 5, 6)

18. What is the locus of a point for which $x = 0$, $z = 0$?

(a) equation of x-axis (b) equation of y-axis
(c) equation of z-axis (d) None of these

19. Equation of YOZ plane is

(a) $x = 0$ (b) $y = 0$
(c) $z = 0$ (d) None of these

20. The equations of x-axis are

(a) $x = 0, y = 0$ (b) $x = 0, z = 0$
(c) $y = 0, z = 0$ (d) $x = 0$

21. Find the point on x-axis which is equidistant from the point A(3, 2, 2) and B(5, 5, 4).

(a) (16, 0, 0) (b) $\left(\frac{5}{4}, 0, 0\right)$
(c) (9, 0, 0) (d) $\left(\frac{49}{4}, 0, 0\right)$

22. Find the point on y-axis which is at a distance of $\sqrt{10}$ units from the point (1, 2, 3).

(a) (0, 4, 0) (b) (0, 3, 0)
(c) (0, 2, 0) (d) (0, –1, 0)

23. Determine the point in yz-plane which is equidistant from three points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).

(a) (0, 1, 3) (b) (1, 0, 3)
(c) (0, 2, 3) (d) (0, 3, 1)

24. The distance of the point A(2, 3, 2) from the x-axis is

(a) 5 units (b) $\sqrt{13}$ units
(c) $2\sqrt{5}$ units (d) $5\sqrt{2}$ units

25. The perpendicular distance of the point (8, 15, 6) from y-axis is

(a) 5 units (b) 6 units
(c) 8 units (d) 10 units

26. The distance of the point $P(a, b, c)$ from the x-axis is

(a) $\sqrt{b^2+c^2}$ (b) $\sqrt{a^2+c^2}$
(c) $\sqrt{a^2+b^2}$ (d) None of these

27. The two vertices of a triangle are (4, 2, 1) and (5, 1, 4). If the centroid is (5, 2, 3), then the third vertex is

(a) (3, 4, 5) (b) (6, 2, 3)
(c) (6, 3, 2) (d) (6, 3, 4)

28. Ratio in which the xy-plane divides the join of (1, 2, 3) and (4, 2, 1) is

(a) 3 : 1 internally (b) 3 : 1 externally
(c) 2 : 1 internally (d) 2 : 1 externally

29. If P(3, 2, – 4), Q(5, 4, – 6) and R(9, 8, – 10) are collinear, then R divides PQ in the ratio

(a) 3 : 2 internally (b) 3 : 2 externally
(c) 2 : 1 internally (d) 2 : 1 externally

30. Mid-point of the line joining the points (– 1, 2, 3) and (2, – 1, 3) is

(a) (1, 1, 6) (b) $\left(\frac{1}{2}, \frac{1}{2}, 3\right)$
(c) (3, – 3, 0) (d) $\left(\frac{1}{3}, \frac{1}{3}, 2\right)$

31. The ratio in which the join of (1, – 2, 3) and (4, 2, – 1) is divided by the xy-plane is

(a) 1 : 3 externally (b) 3 : 1 internally
(c) 1 : 3 externally (d) None of these

32. The point which divides the line joining the points (1, 3, 4) and (4, 3, 1) internally in the ratio 2 : 1, is

(a) (2, – 3, 3) (b) (2, 3, 3)

(c) $\left(\frac{5}{2}, 3, \frac{5}{2}\right)$ (d) (3, 3, 2)

33. Find the coordinates of the point which is three-fifth of the way from (3, 4, 5) to (–2, –1, 0).

(a) (1, 0, 2) (b) (2, 0, 1)

(c) (0, 2, 1) (d) (0, 1, 2)

34. The distance of the point $A(2, 3, 4)$ from the y-axis is

(a) 5 units (b) $\sqrt{13}$ units

(c) $5\sqrt{2}$ units (d) $2\sqrt{5}$ units

35. The perpendicular distance of the point (6, 5, 8) from z-axis is

(a) $\sqrt{15}$ units (b) $\sqrt{61}$ units

(c) 8 units (d) 9 units

36. If the distance between the points (a, 0, 1) and (0, 1, 2) is $\sqrt{27}$, then the value of a is

(a) 5 (b) ±5

(c) –5 (d) None of these

37. The points (1, 2, 3), (– 1, – 1, – 1) and (3, 5, 7) are the vertices of

(a) an equilateral triangle

(b) an isosceles triangle

(c) a right triangle

(d) None of these

38. Find the coordinates of the points which trisect the line segment AB where $A(2, 1, -3)$ and $B(5, -8, 3)$.

(a) (4, –5, 1), (3, –2, –1)

(b) (–4, 5, 1), (3, –2, –1)

(c) (–5, 4, 1), (3, 2, 1)

(d) (4, 5, –1), (3, 2, 1)

39. A point R with x-coordinate 4 lies on the line segment joining the points $P(2, -3, 4)$ and $Q(8, 0, 10)$. Find the coordinates of the point R.

(a) (1, 2, 9) (b) (4, 2, 6)

(c) (8, 2, 5) (d) (4, –2, 6)

40. If the origin is the centroid of the triangle with vertices $A(3a, 4, -5)$, $B(-2, 4b, 6)$ and $C(6, 10, c)$, then find the values of a, b, c.

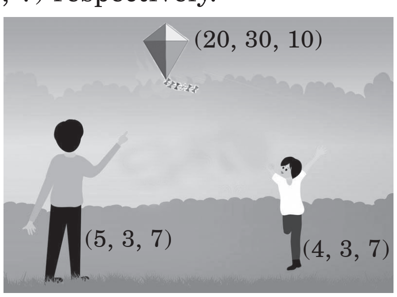
(a) $-\frac{4}{3}, -\frac{7}{2}, -1$ (b) $\frac{4}{3}, \frac{7}{2}, 1$

(c) $-\frac{4}{3}, -\frac{7}{2}, 1$ (d) None of these

Case Based MCQs

Case I : Read the following passage and answer the questions from 41 to 45.

Raj and his father were walking in a large park. They saw a kite flying in the sky. The position of kite, Raj and Raj's father are at (20, 30, 10), (4, 3, 7) and (5, 3, 7) respectively.



41. The distance between Raj and kite is

(a) 41.32 units (b) 31.52 units

(c) 38.32 units (d) 40.39 units

42. The distance between Raj's father and kite is

(a) 31.30 units (b) 38.43 units

(c) 31.03 units (d) 29.00 units

43. The co-ordinates of Raj lie in

(a) IV quadrant (b) III quadrant

(c) II quadrant (d) I quadrant

44. If co-ordinates of kite, Raj and Raj's father form a triangle, then find the centroid of it.

(a) (9.67, 12, 8) (b) (9.6, 8, 12)

(c) (12, 8, 10) (d) (7, 9, 7.2)

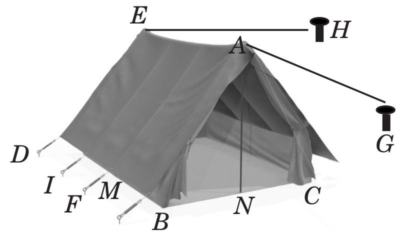
45. The co-ordinates of points in the XY-plane are of the form

(a) $(0, 0, z)$ (b) $(x, y, 0)$

(c) $(x, 0, y)$ (d) $(0, x, y)$

Case II : Read the following passage and answer the questions from 46 to 50.

Deepak and his friends went for camping for 2 or 3 days. There they set up a tent which is triangular in shape. The vertices of the tent are $A(4, 5, 9)$, $B(3, 2, 5)$, $C(5, 2, 5)$, $D(-3, 2, -5)$ and $E(-4, 5, -9)$ respectively.



The vertex A is tied up by the rope at the ends F and G and the vertex E is tied up at the ends I and H.

46. The octant in which D lies is

(a) II (b) III (c) V (d) VI

47. If M denotes the position of their bags inside the tent and it is just in middle of the vertices B and D, then the coordinates of M are

(a) (0, 2, 0) (b) (3, 2, 0)
(c) (0, 2, 5) (d) (3, 2, 5)

48. The length AE is

(a) $\sqrt{97}$ units (b) $2\sqrt{97}$ units
(c) $3\sqrt{97}$ units (d) $4\sqrt{97}$ units

49. If the length of the rope by which E is tied up with H is $5\sqrt{2}$ units, then the equation denotes the set of point of H is

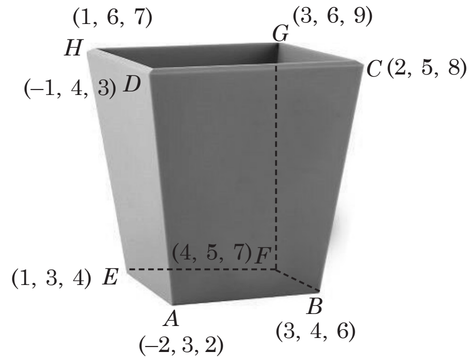
(a) $x^2 + y^2 + z^2 - 8x - 10y - 18z + 72 = 0$
(b) $x^2 + y^2 + z^2 - 8x - 10y - 18z - 72 = 0$
(c) $x^2 + y^2 + z^2 + 8x - 10y + 18z + 72 = 0$
(d) $x^2 + y^2 + z^2 + 8x + 10y + 18z + 72 = 0$

50. The length BC is

(a) 4 units (b) 2 units
(c) 16 units (d) 8 units

Case III : Read the following passage and answer the questions from 51 to 55.

Ravi makes a plan to gift his friend a hand made pen-stand of the trapezoidal shape given in the figure :



The vertices of the pen-stand are $A(-2, 3, 2)$, $B(3, 4, 6)$, $C(2, 5, 8)$, $D(-1, 4, 3)$, $E(1, 3, 4)$ $F(4, 5, 7)$, $G(3, 6, 9)$ and $H(1, 6, 7)$.

51. The points A, B, E and F are the vertices of

(a) Parallelogram (b) Rhombus
(c) Isoseles trapezium (d) None of these.

52. If I and J are the mid-point of AB and EF respectively, then the length of IJ is

(a) $\sqrt{2}$ units (b) $\sqrt{3}$ units
(c) $\sqrt{26}/2$ units (d) $\sqrt{7}$ units

53. Find the coordinates of the point which divides the line segment EH in 2 : 1.

(a) (1, 3, 6) (b) (1, 5, 6)
(c) (1, 4, 6) (d) (1, 2, 6)

54. The length of foot of perpendicular drawn from C on y-axis is

(a) (2, 0, 0) (b) (0, 5, 0)
(c) (0, 0, 8) (d) (2, 5, 0)

55. The ratio in which the line segment joining the vertices E and F is divided by yz-plane externally is

(a) 1 : 3 (b) 1 : 2
(c) 1 : 4 (d) 4 : 1

Assertion & Reasoning Based MCQs

Directions (Q.-56 to 60) : In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :

(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.
(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.
(c) Assertion is correct statement but Reason is wrong statement.
(d) Assertion is wrong statement but Reason is correct statement.

56. Assertion : The points $A(1, -1, 3)$, $B(2, -4, 5)$ and $C(5, -13, 11)$ are collinear.

Reason : If $AB + BC = AC$, then A, B, C are collinear.

57. Assertion : Coordinates of centroid of a triangle formed by the vertices $A(3, 2, 0)$, $B(5, 3, 2)$ and $C(0, 2, 4)$ is $\left(\frac{8}{3}, \frac{8}{3}, \frac{8}{3}\right)$.

Reason : Coordinates of centroid of a triangle with vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right).$$

58. Assertion : The foot of perpendicular drawn from the point $A(1, 2, 8)$ on the xy-plane is $(1, 2, 0)$.

Reason : Equation of xy-plane is $y = 0$.

59. Assertion : The distance between the points $(1+\sqrt{11}, 0, 0)$ and $(1, -2, 3)$ is $2\sqrt{6}$ units.

Reason : Distance between any two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is,

$$|AB| = \sqrt{(x_2+x_1)^2+(y_2+y_1)^2+(z_2+z_1)^2}$$

60. Assertion : The points $A(3, -1, 2)$, $B(1, 2, -4)$, $C(-1, 1, 2)$ and $D(1, -2, 8)$ are the vertices of a parallelogram.

Reason : Coordinates of mid-point of a line joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. Determine the octant of $(4, -3, -1)$.

2. If a point lies in yz-plane, then what is its x-coordinate?

3. In which octant does $(-1, -6, 5)$ lies?

4. If a point lies on the y-axis, then what are its x-coordinate and z-coordinate?

5. A point is on the x-axis. Write its y-coordinate and z-coordinate.

6. Find the distance of the point $P(4, -3, 5)$ from XY plane.

7. Write the distance of the point $A(3, 4, 5)$ from z-axis.

8. Find the distance of $(1, -2, 7)$ from the y-axis.

9. Find the distance of $(1, 2, 5)$ from x-axis.

10. What is the distance of the point $(3, 4, 5)$ from the YZ plane?

Short Answer Type Questions (SA-I)

11. Find the locus of the point which is equidistant from the points $A(0, 2, 3)$ and $B(2, -2, 1)$.

12. Verify that $(3, -2, 4)$, $(1, 0, -2)$, $(-1, 2, -8)$ are collinear.

13. Find the point on y-axis which is equidistant from the point $A(3, 2, 2)$ and $B(5, 5, 4)$.

14. Show that the points $A(0, 7, 10)$, $B(-1, 6, 6)$ and $C(-4, 9, 6)$ form an isosceles right-angled triangle.

15. Find the coordinates of the point which divides the join of $P(2, -1, 4)$ and $Q(4, 3, 2)$ in the ratio 2 : 3 internally.

16. A point R with x-coordinate 4 lies on the line segment joining the points $P(2, -6, 4)$ and $Q(8, 6, 10)$. Find the coordinates of the point R.

17. Let P and Q be any two points. Find the coordinates of the point R which divides PQ externally in the ratio 2 : 1 and verify that Q is the mid-point of PR.

18. Three vertices of a parallelogram $PQRS$ are $P(3, -1, 2)$, $Q(1, 2, -4)$ and $R(-1, 1, 2)$. Find the coordinates of the fourth vertex.

19. Find the coordinates of the point which divides the join of $P(2, -1, 4)$ and $Q(4, 3, 2)$ in the ratio 2 : 3 externally.

20. Find the third vertex of triangle whose centroid is origin and two vertices are $(2, 4, 6)$ and $(0, -2, -5)$.

Short Answer Type Questions (SA-II)

21. Find the point in yz-plane which is equidistant from the points $A(3, 2, -1)$, $B(1, -1, 0)$ and $C(2, 1, 2)$.

22. Find the coordinates of the points which trisect the line segment joining the points $P(4, 2, -6)$ and $Q(10, -16, 6)$.

23. Show that the triangle ABC with vertices $A(0, 4, 1)$, $B(2, 3, -1)$ and $C(4, 5, 0)$ is right angled.

24. Show that the three points $A(2, 3, 4)$, $B(-1, 2, -3)$ and $C(-4, 1, -10)$ are collinear and find the ratio in which C divides AB.

25. Find the coordinates of a point on the y-axis which are at a distance of $5\sqrt{2}$ from the point $P(3, -2, 5)$.

26. Determine the values of a and b so that the points $(a, b, 3)$, $(2, 0, -1)$ and $(1, -1, -3)$ are collinear.

27. Find the lengths of the medians of the triangle with vertices $A(0, 0, 8)$, $B(0, 6, 0)$ and $C(6, 0, 0)$.

28. Prove that the points $(0, -1, -7)$, $(2, 1, -9)$ and $(6, 5, -13)$ are collinear. Find the ratio in which the first point divides the join of the other two.

29. Find the ratio in which YZ-plane divides the line segment joining points $(-2, 4, 7)$ and $(3, -5, 8)$. Also, find the coordinates of the point of intersection.

30. If the origin is the centroid of the triangle PQR with vertices $P(2a, 2, 6)$, $Q(-4, 3b, -10)$ and $R(8, 14, 2c)$, then find the values of a, b and c.

31. Find the points of trisection of the segment joining the points $A(1, 0, -6)$ and $B(-5, 9, 6)$.

32. Prove that the coordinates of the centroid of the triangle whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

33. Using section formula, show that the points $A(2, -3, 4)$, $B(-1, 2, 1)$ and $C\left(0, \frac{1}{3}, 2\right)$ are collinear.

34. Find the equation of the set of points which are equidistant from the points $(1, 2, 3)$ and $(3, 2, -1)$.

35. Let $A(2, 2, -3)$, $B(5, 6, 9)$, $C(2, 7, 9)$ be the vertices of a triangle. The internal bisector of the angle A meets BC at the point D. Find the coordinates of D.

Long Answer Type Questions (LA)

36. Determine the point in XY plane which is equidistant from three points $A(2, 0, 3)$, $B(0, 3, 2)$ and $C(0, 0, 1)$.

37. Find the point in the xy-plane which is equidistant from the points $(5, 0, 6)$, $(0, -3, 2)$ and $(4, 5, 0)$.

38. Find the equation of the set of points P, the sum of whose distances from $A(4, 0, 0)$ and $B(-4, 0, 0)$ is equal to 10.

39. The mid-points of the sides of a triangle are $(1, 5, -1)$, $(0, 4, -2)$ and $(2, 3, 4)$. Find its vertices.

40. Show that the points $A(1, 2, 3)$, $B(-1, -2, -1)$, $C(2, 3, 2)$ and $D(4, 7, 6)$ are the vertices of a parallelogram $ABCD$, but it is not a rectangle.

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. **(c) :** For the point F, the distance measured along OY is zero. Therefore, the coordinates of F are $(2, 0, 5)$.

2. **(d) :** The point $(-3, 1, 2)$ lies in second octant and the point $(-3, 1, -2)$ lies in sixth octant.

3. **(a) :** Since L is the foot of perpendicular from P on the x-axis, so its y and z-coordinates are zero. So, the coordinates of L is $(7, 0, 0)$. Similarly, the coordinates of M and N are $(0, 9, 0)$ and $(0, 0, 4)$, respectively.

4. **(d) :** Since A is the foot of perpendicular from P on xy-plane, so its z-coordinate is zero. Hence, coordinates of A is $(3, 4, 0)$. Similarly, the coordinates of B and C are $(0, 4, 5)$ and $(3, 0, 5)$ respectively.

5. (a) : Let L be the foot of perpendicular drawn from the point $P(6, 7, 8)$ on the xy-plane.
Then coordinates of $L \equiv (6, 7, 0)$
$\therefore$ Required distance PL

$$= \sqrt{(6-6)^2 + (7-7)^2 + (8-0)^2} = 8 \text{ units.}$$

6. (c) : We have, coordinates of $A \equiv (3, 4, 0)$, coordinates of $B \equiv (0, 4, 5)$, coordinates of $C \equiv (3, 0, 5)$
Now, $P \equiv (3, 4, 5)$

$$\therefore \quad PA = \sqrt{(3-3)^2 + (4-4)^2 + (5-0)^2} = 5 \text{ units}$$

$$PB = \sqrt{(3-0)^2 + (4-4)^2 + (5-5)^2} = 3 \text{ units}$$

$$PC = \sqrt{(3-3)^2 + (4-0)^2 + (5-5)^2} = 4 \text{ units}$$

7. (c) : The distance between the points $P(1, -3, 4)$ and $Q(-4, 1, 2)$ is

$$PQ = \sqrt{(-4-1)^2 + (1+3)^2 + (2-4)^2}$$

$$= \sqrt{25+16+4} = \sqrt{45} = 3\sqrt{5} \text{ units}$$

8. (c) : Let the coordinates of point P be (x, y, z).
Here $PA^2 = (x - 3)^2 + (y - 4)^2 + (z - 5)^2$
$PB^2 = (x + 1)^2 + (y - 3)^2 + (z + 7)^2$
By the given condition $PA^2 + PB^2 = 2k^2$

$$\Rightarrow \quad (x - 3)^2 + (y - 4)^2 + (z - 5)^2 + (x + 1)^2 + (y - 3)^2 + (z + 7)^2 = 2k^2$$

i.e., $2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = 2k^2 - 109$.

9. (a) : Let $P(x, y, z)$ be any point such that $PA = PB$

$$\text{Now } \sqrt{(x-3)^2 + (y-4)^2 + (z+5)^2} = \sqrt{(x+2)^2 + (y-1)^2 + (z-4)^2}$$

$$\Rightarrow \quad (x - 3)^2 + (y - 4)^2 + (z + 5)^2 = (x + 2)^2 + (y - 1)^2 + (z - 4)^2$$

$$\Rightarrow \quad 10x + 6y - 18z - 29 = 0.$$

10. (b) : Let $P(x, y, z)$ be the required point.
Then $OP = PA = PB = PC$.
Now $OP = PA \Rightarrow OP^2 = PA^2$

$$\Rightarrow \quad x^2 + y^2 + z^2 = (x - l)^2 + (y - 0)^2 + (z - 0)^2 \Rightarrow x = \frac{l}{2}$$

Similarly, $OP = PB \Rightarrow y = \dfrac{m}{2}$ and $OP = PC$

$$\Rightarrow \quad z = \frac{n}{2}$$

Hence, the coordinates of the required point are $\left(\dfrac{l}{2}, \dfrac{m}{2}, \dfrac{n}{2}\right)$.

11. (a) : Let $P(x, y, z)$ be the point which divides the line joining the points $A(1, -2, 3)$ and $B(3, 4, -5)$ externally in the ratio $2 : 3$. Then

$$x = \frac{2(3)-(3)(1)}{2-3} = -3,\ y = \frac{2(4)-(3)(-2)}{2-3} = -14,$$

$$z = \frac{2(-5)-(3)(3)}{2-3} = 19$$

Therefore, the required point is $(-3, -14, 19)$.

12. (b) : Let yz-plane divides the line segment joining $A(4, 8, 10)$ and $B(6, 10, -8)$ at $P(x, y, z)$ in the ratio $k : 1$. Then the coordinates of P are

$$\left(\frac{4+6k}{k+1}, \frac{8+10k}{k+1}, \frac{10-8k}{k+1}\right)$$

Since P lies on the yz-plane, so, its x-coordinate is zero

i.e., $\dfrac{4+6k}{k+1} = 0 \quad \Rightarrow \quad k = -\dfrac{2}{3}$

Therefore, yz-plane divides AB externally in the ratio $2 : 3$.

13. (c) : Let the coordinates of C be (x, y, z). Coordinates of the centroid is $(1, 1, 1)$. Then

$$\frac{x+3-1}{3} = 1,\ \frac{y-5+7}{3} = 1,\ \frac{z+7-6}{3} = 1$$

$\Rightarrow \quad x = 1, y = 1, z = 2.$
Hence, coordinates of C are $(1, 1, 2)$.

14. (a) : Let the join of $P(2, 4, 5)$ and $Q(3, 5, -4)$ be divided by xz-plane in the ratio $k : 1$ at the point $R(x, y, z)$.

Therefore $x = \dfrac{3k+2}{k+1}$, $y = \dfrac{5k+4}{k+1}$, $z = \dfrac{-4k+5}{k+1}$

Since the point $R(x, y, z)$ lies on xz-plane, its y-coordinate is zero, *i.e.*, $\dfrac{5k+4}{k+1} = 0 \Rightarrow k = -\dfrac{4}{5}$

Hence, the xz-plane divide the line joining given points in $4 : 5$ externally.

15. (c) : Let $P(x, y, z)$ be the required point, *i.e.*, P divides AB in the ratio $5 : 1$. Then

$$P(x, y, z) = \left(\frac{5\times10+1\times(-2)}{5+1}, \frac{5\times(-6)+1\times0}{5+1}, \frac{5\times(-12)+1\times6}{5+1}\right)$$
$$= (8, -5, -9).$$

16. (d) : Since M is the foot of perpendicular from A on the yz-plane, so its x-coordinate is zero. Hence, coordinates of M are $(0, 7, 8)$.

17. (a) : Since L is the foot of perpendicular from point $(3, 5, 6)$ on x-axis. So, its y and z-coordinates are zero. Hence, the coordinates of L are $(3, 0, 0)$.

18. (b) : Locus of the point for which $x = 0$, $z = 0$ is y-axis, since on y-axis both $x = 0$ and $z = 0$.

19. (a) : On yz-plane, $x = 0$.

20. (c) : Equations of x-axis are $y = 0$ and $z = 0$.

21. (d) : The point on the x-axis is of the form $P(x, 0, 0)$. Since the points A and B are equidistant from P. Therefore $PA^2 = PB^2$

$$\Rightarrow \quad (x - 3)^2 + (0 - 2)^2 + (0 - 2)^2 = (x - 5)^2 + (0 - 5)^2 + (0 - 4)^2$$

$$\Rightarrow \quad 4x = 25 + 25 + 16 - 17 \Rightarrow x = \frac{49}{4}$$

Thus, the point P on the x-axis is $\left(\dfrac{49}{4}, 0, 0\right)$ which is equidistant from A and B.

22. (c) : Let the point P be on y-axis. Therefore, it is of the form $P(0, y, 0)$.

The point (1, 2, 3) is at a distance $\sqrt{10}$ units from $(0, y, 0)$.

$\Rightarrow \quad \sqrt{(1-0)^2+(2-y)^2+(3-0)^2}=\sqrt{10}$

$\Rightarrow \quad y^2 - 4y + 4 = 0 \Rightarrow (y - 2)^2 = 0 \Rightarrow y = 2$

Hence the required point is (0, 2, 0).

23. (a) : Since x-coordinate of every point in yz-plane is zero. Let $P(0, y, z)$ be a point in the yz-plane such that $PA = PB = PC$.

Now $PA^2 = PB^2$

$\Rightarrow (0-2)^2 + (y-0)^2 + (z-3)^2 = (0-0)^2 + (y-3)^2 + (z-2)^2$

i.e., $z - 3y = 0$ ...(i)

and $PB^2 = PC^2 \Rightarrow y^2 + 9 - 6y + z^2 + 4 - 4z = y^2 + z^2 + 1 - 2z$

i.e., $3y + z = 6$ ...(ii)

Solving (i) and (ii), we get $y = 1$, $z = 3$

Hence, the coordinates of the point P are (0, 1, 3).

24. (b) : We have to take the distance of point $A(2, 3, 2)$ from x-axis.

So, we have to take the distance of y and z coordinate of the point from origin $=\sqrt{3^2+2^2}=\sqrt{13}$ units.

25. (d) : Perpendicular distance of the point (8, 15, 6) from y-axis is $\sqrt{8^2+0^2+6^2}=\sqrt{64+36}=\sqrt{100}$

$= 10$ units.

26. (a) : The coordinates of the foot of the perpendicular from P on x-axis are $(a, 0, 0)$. Therefore, the required distance is $\sqrt{(a-a)^2+(b-0)^2+(c-0)^2}=\sqrt{b^2+c^2}$.

27. (d) : Let the third vertex be (α, β, γ).

$\therefore \quad 5=\dfrac{4+5+\alpha}{3} \Rightarrow \alpha=6, 2=\dfrac{2+1+\beta}{3} \Rightarrow \beta=3,$

$3=\dfrac{1+4+\gamma}{3} \Rightarrow \gamma=4$

$\therefore$ The third vertex is (6, 3, 4).

28. (b) : Suppose xy-plane divides the join of (1, 2, 3) and (4, 2, 1) in the ratio $\lambda : 1$. Then the coordinates of the point of division are $\left(\dfrac{4\lambda+1}{\lambda+1}, \dfrac{2\lambda+2}{\lambda+1}, \dfrac{\lambda+3}{\lambda+1}\right)$. This point lies on xy-plane. So, its z-coordinate = 0

$\Rightarrow \quad \dfrac{\lambda+3}{\lambda+1} = 0 \Rightarrow \lambda = -3$

Hence, xy-plane divides the join of (1, 2, 3) and (4, 2, 1) externally in the ratio 3 : 1.

29. (b) : Let $R(9, 8, -10)$ divides the join of $P(3, 2, -4)$ and $Q(5, 4, -6)$ in the ratio $k : 1$.

Then, $9 = \dfrac{5k+3}{k+1} \Rightarrow 9k + 9 = 5k + 3 \Rightarrow 4k = -6 \Rightarrow k = \dfrac{-3}{2}$

$\therefore$ Required ratio is 3 : 2 externally.

30. (b) : Mid-point is $\left(\dfrac{-1+2}{2}, \dfrac{2-1}{2}, \dfrac{3+3}{2}\right) = \left(\dfrac{1}{2}, \dfrac{1}{2}, 3\right)$.

31. (b) : Let $A(1, -2, 3)$ and $B(4, 2, -1)$.

Let xy-plane meet the line AB at the point C such that C divides AB in the ratio $k : 1$, then

$C \equiv \left(\dfrac{4k+1}{k+1}, \dfrac{2k-2}{k+1}, \dfrac{-k+3}{k+1}\right)$.

Since C lies in the xy plane. $\therefore z = 0$

$\Rightarrow \quad \dfrac{-k+3}{k+1}=0 \Rightarrow k=3$

$\therefore$ Required ratio is 3 : 1 internally.

32. (d) : Let the point (x, y, z) divides the join of (1, 3, 4) and (4, 3, 1) in the ratio 2 : 1. Then,

$x=\dfrac{1\times1+4\times2}{1+2}=\dfrac{9}{3}=3, \; y=\dfrac{1\times3+2\times3}{1+2}=\dfrac{9}{3}=3$

$z=\dfrac{1\times4+2\times1}{1+2}=\dfrac{6}{3}=2$

$\therefore$ Required point is (3, 3, 2).

33. (d) : Let $A \equiv (3, 4, 5)$, $B \equiv (-2, -1, 0)$ and $P(x, y, z)$ be the required point. As P is three-fifth of the way from A to B. So, P divides AB in the ratio 3 : 2.

$\therefore \quad P \equiv \left(\dfrac{3\times(-2)+2\times3}{3+2}, \dfrac{3\times(-1)+2\times4}{3+2}, \dfrac{3\times0+2\times5}{3+2}\right)$

$\Rightarrow \quad P \equiv (0, 1, 2)$.

34. (d) : We have to take the distance of point $A(2, 3, 4)$ from y-axis.

So, we have to take the distance of x and z coordinate of the point from origin $=\sqrt{2^2+4^2}=\sqrt{20}=2\sqrt{5}$ units.

35. (b) : Perpendicular distance of the point (6, 5, 8) from z-axis $=\sqrt{6^2+5^2+0^2}=\sqrt{36+25}=\sqrt{61}$ units.

36. (b) : Distance between the points $(a, 0, 1)$ and $(0, 1, 2)$ $=\sqrt{27}$

$\Rightarrow \quad \sqrt{(a-0)^2+(0-1)^2+(1-2)^2}=\sqrt{27}$

Squaring both sides, we get

$a^2 + 1 + 1 = 27 \Rightarrow a^2 = 25 \Rightarrow a = \pm 5$

37. (d) : Let the given points be $A(1, 2, 3)$, $B(-1, -1, -1)$ and $C(3, 5, 7)$.

$|AB|=\sqrt{2^2+3^2+4^2}=\sqrt{29},$

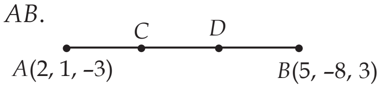
$|BC|=\sqrt{4^2+6^2+8^2}=\sqrt{116}=2\sqrt{29}$

and $|CA|=\sqrt{2^2+3^2+4^2}=\sqrt{29}$.

So, $|BC|=|AB|+|CA|$

$\Rightarrow$ The points A, B, C are collinear.

38. (a) : Let C and D be the two points which trisect the join of AB.



$\because$ C divides the join of AB in the ratio 1 : 2.

$\therefore$ Coordinates of C is

$$\left(\frac{1\times5+2\times2}{1+2}, \frac{1\times(-8)+2\times1}{1+2}, \frac{1\times3+2\times(-3)}{1+2}\right)$$

$\equiv (3, -2, -1)$.

Also, D divides the join of AB in the ratio 2 : 1

$\therefore$ Coordinates of D is

$$\left(\frac{2\times5+1\times2}{1+2}, \frac{2\times(-8)+1\times1}{1+2}, \frac{2\times3+1\times(-3)}{1+2}\right) \equiv (4, -5, 1).$$

39. (d) : Let $R(4, y, z)$ be any point which divides the join of $P(2, -3, 4)$ and $Q(8, 0, 10)$ in the ratio $k : 1$ internally.

$\therefore$ Coordinates of R is $\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$

But x-coordinate of R is 4.

So, $\frac{8k+2}{k+1}=4 \Rightarrow 8k + 2 = 4k + 4 \Rightarrow k=\frac{1}{2}$

Now, $y=\frac{-3}{\frac{1}{2}+1}=\frac{-3}{\frac{3}{2}}=-2$ and $z=\frac{\frac{10\times1}{2}+4}{\frac{1}{2}+1}=\frac{9}{\frac{3}{2}}=6$

Thus, coordinates of R is $(4, -2, 6)$.

40. (a) : Here $A(3a, 4, -5)$, $B(-2, 4b, 6)$ and $C(6, 10, c)$ are the three vertices of ΔABC, then coordinates of centroid are $\left[\frac{3a-2+6}{3}, \frac{4+4b+10}{3}, \frac{-5+6+c}{3}\right]$.

But it is given that coordinates of centroid are (0, 0, 0).

$\therefore$ $\frac{3a-2+6}{3}=0 \Rightarrow 3a = -4 \Rightarrow a=-\frac{4}{3}$,

$\frac{4+4b+10}{3}=0 \Rightarrow 4b = -14 \Rightarrow b=-\frac{7}{2}$,

$\frac{-5+6+c}{3}=0 \Rightarrow c = -1$

41. (b) : Required distance

$=\sqrt{(20-4)^2+(30-3)^2+(10-7)^2}$

$=\sqrt{16^2+27^2+3^2}$

$=\sqrt{256+729+9}=\sqrt{994}=31.52$ units

42. (c) : Required distance

$=\sqrt{(20-5)^2+(30-3)^2+(10-7)^2}$

$=\sqrt{15^2+27^2+3^2}$

$=\sqrt{225+729+9}=\sqrt{963}=31.03$ units

43. (d) : Because in (4, 3, 7) ; all are positive. Thus, the coordinate lie in the I quadrant.

44. (a) : Centroid $=\left(\frac{20+4+5}{3}, \frac{30+3+3}{3}, \frac{10+7+7}{3}\right)$

$= (9.67, 12, 8)$

45. (b) : For XY-plane, $z = 0$

$\Rightarrow$ The co-ordinates are of the form $(x, y, 0)$.

46. (d)

47. (a) : As, M is the middle point of $B(3, 2, 5)$ and $D(-3, 2, -5)$

$\therefore$ The coordinates of M are

$\left(\frac{3-3}{2}, \frac{2+2}{2}, \frac{5-5}{2}\right)=(0,2,0)$

48. (b) : The length AE is

$=\sqrt{(-4-4)^2+(5-5)^2+(-9-9)^2}$

$=\sqrt{64+0+324}$

$=\sqrt{388}=2\sqrt{97}$ units.

49. (c) : As, the distance of $H(x, y, z)$ from $E(-4, 5, -9)$ is $5\sqrt{2}$ units.

$\therefore$ $EH=5\sqrt{2}$

$\Rightarrow$ $\sqrt{(x+4)^2+(y-5)^2+(z+9)^2}=5\sqrt{2}$

Squaring both sides, we get

$(x + 4)^2 + (y - 5)^2 + (z + 9)^2 = 25 \times 2$

$x^2 + y^2 + z^2 + 8x - 10y + 18z + 122 = 50$

$\Rightarrow$ $x^2 + y^2 + z^2 + 8x - 10y + 18z + 72 = 0$

50. (a) : The length $BC = \sqrt{(5-3)^2+(2-2)^2+(5-5)^2}$

$=\sqrt{4+0+0}=4$ units

51. (d) : Now, $AB=\sqrt{(3+2)^2+(4-3)^2+(6-2)^2}=\sqrt{42}$

and $EF=\sqrt{(4-1)^2+(5-3)^2+(7-4)^2}=\sqrt{22}$

$AF=\sqrt{(4+2)^2+(5-3)^2+(7-2)^2}=\sqrt{65}$

$BE=\sqrt{(1-3)^2+(3-4)^2+(4-6)^2}=\sqrt{9}=3$

$\Rightarrow$ $AB \neq EF$

52. (c) : As, I is the mid-point of A and B

$\therefore$ Coordinates of $I=\left(\frac{3-2}{2}, \frac{4+3}{2}, \frac{6+2}{2}\right)=\left(\frac{1}{2}, \frac{7}{2}, 4\right)$

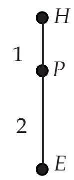
and J is midpoint of E and F

$\therefore$ Coordinates of $J=\left(\frac{4+1}{2}, \frac{5+3}{2}, \frac{7+4}{2}\right)=\left(\frac{5}{2}, 4, \frac{11}{2}\right)$

$\therefore$ $IJ=\sqrt{\left(\frac{5}{2}-\frac{1}{2}\right)^2+\left(4-\frac{7}{2}\right)^2+\left(\frac{11}{2}-4\right)^2}$

$=\frac{\sqrt{26}}{2}$ units.

53. **(b) :** Let P be the point which divides EH in 2 : 1.



$\therefore$ The coordinates of P are

$$\left(\frac{2+1}{2+1}, \frac{12+3}{2+1}, \frac{14+4}{2+1}\right) = (1, 5, 6)$$

54. **(b) :** As we know, on y-axis $x = 0$, $z = 0$

$\therefore$ The coordinates of foot of perpendicular drawn from C are (0, 5, 0)

55. **(c) :** Let YZ plane divides the line segment joining $E(1, 3, 4)$ and $F(4, 5, 7)$ in $h : 1$ at $P(x, y, z)$

$\therefore$ Coordinates of $P = \left(\frac{4h+1}{h+1}, \frac{5h+3}{h+1}, \frac{7h+4}{h+1}\right)$

Since P is in YZ plane

$\therefore$ Its x coordinates is zero.

$\therefore$ $4h + 1 = 0$

$$h = \frac{-1}{4}$$

$\therefore$ YZ-plane divides EF externally in the ratio 1 : 4

56. **(a) :** $|AB| = \sqrt{(1)^2+(-3)^2+(2)^2} = \sqrt{1+9+4} = \sqrt{14}$

$|BC| = \sqrt{(3)^2+(-9)^2+(6)^2} = \sqrt{9+81+36} = 3\sqrt{14}$

$|AC| = \sqrt{(4)^2+(-12)^2+(8)^2} = \sqrt{16+144+64} = 4\sqrt{14}$

$\because$ $AB + BC = 4\sqrt{14} = AC$

$\therefore$ Points A, B and C are collinear.

57. **(d) :** Coordinates of centroid of a triangle with vertices $A(3, 2, 0)$, $B(5, 3, 2)$ and $C(0, 2, 4)$ is

$$\left(\frac{3+5+0}{3}, \frac{2+3+2}{3}, \frac{0+2+4}{3}\right) = \left(\frac{8}{3}, \frac{7}{3}, 2\right)$$

$\therefore$ Assertion is wrong but Reason is correct.

58. **(c) :** We know that in xy-plane, z-coordinate is 0. So, coordinate of foot of perpendicular drawn from point $A(1, 2, 8)$ on xy-plane is (1, 2, 0).

Equation of xy-plane is $z = 0$

$\therefore$ Reason is wrong.

59. **(c) :** Let $A \equiv (1+\sqrt{11}, 0, 0)$ and $B \equiv (1, -2, 3)$

$\therefore$ $AB = \sqrt{(1-1-\sqrt{11})^2+(-2-0)^2+(3-0)^2}$

$= \sqrt{11+4+9} = \sqrt{24} = 2\sqrt{6}$ units

$\therefore$ Assertion is correct but Reason is wrong.

60. **(a) :** Mid-point of $AC = \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right)$

$= (1, 0, 2)$

Mid-point of $BD = \left(\frac{1+1}{2}, \frac{2-2}{2}, \frac{-4+8}{2}\right) = (1, 0, 2)$

$\because$ Mid-points of AC and BD coincides.

$\therefore$ $ABCD$ is a parallelogram.

SUBJECTIVE TYPE QUESTIONS

1. Since, x coordinate is positive while y and z, coordinates are negative.

$\therefore$ (4, –3, –1) lies in VIII octant.

2. If a point lies in yz-plane, then its x-coordinate is 0.

3. Since, x and y coordinates are negative while z coordinate is positive.

$\therefore$ (–1, –6, 5) lies in III octant.

4. If a point lies on y-axis, then its x-coordinate is 0 and z-coordinate is 0.

5. A point on the x-axis is of the form$(x, 0, 0)$.

$\therefore$ y-coordinate = 0 and z-coordinate = 0.

6. The coordinates of foot of perpendicular L from $P(4, -3, 5)$ on XY-plane is given by $L(4, -3, 0)$

$\therefore$ Required distance

$= \sqrt{(4-4)^2+(-3+3)^2+(0-5)^2} = \sqrt{25} = 5$ units.

7. The coordinates of foot of perpendicular R from $A(3, 4, 5)$ on z-axis is given by $R(0, 0, 5)$

$\therefore$ Required distance $= \sqrt{(0-3)^2+(0-4)^2+(5-5)^2}$

$= \sqrt{9+16+0} = \sqrt{25} = 5$ units.

8. The coordinates of foot of perpendicular from (1, –2, 7) on y-axis are (0, –2, 0)

$\therefore$ Required distance $= \sqrt{(0-1)^2+(-2+2)^2+(0-7)^2}$

$= \sqrt{1+0+49} = \sqrt{50} = 5\sqrt{2}$ units.

9. Let L be the foot of perpendicular drawn from $A(1, 2, 5)$ to the x-axis.

Then, coordinates of $L \equiv (1, 0, 0)$.

$\therefore$ Required distance AL

$= \sqrt{(1-1)^2+(2-0)^2+(5-0)^2} = \sqrt{4+25}$

$= \sqrt{29}$ units.

10. The coordinates of foot of perpendicular from the point (3, 4, 5) on YZ plane are (0, 4, 5).

$\therefore$ Required distance $= \sqrt{(0-3)^2+(4-4)^2+(5-5)^2}$

$= \sqrt{9+0+0} = 3$ units.

11. Let $P(x, y, z)$ be any point which is equidistant from $A(0, 2, 3)$ and $B(2, -2, 1)$.

Then, $PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow$ $(x-0)^2+(y-2)^2+(z-3)^2 = (x-2)^2+(y+2)^2+(z-1)^2$

$\Rightarrow$ $4x - 8y - 4z + 4 = 0 \Rightarrow x - 2y - z + 1 = 0$

Hence, the required locus is $x - 2y - z + 1 = 0$

12. Let $A(3, -2, 4)$, $B(1, 0, -2)$ and $C(-1, 2, -8)$ be given points.

$\therefore$ By distance formula,

$$AB = \sqrt{(1-3)^2+(0+2)^2+(-2-4)^2}$$

$$= \sqrt{4+4+36} = \sqrt{44} = 2\sqrt{11} \text{ units}$$

$$BC = \sqrt{(-1-1)^2+(2-0)^2+(-8+2)^2}$$

$$= \sqrt{4+4+36} = \sqrt{44} = 2\sqrt{11} \text{ units}$$

$$AC = \sqrt{(-1-3)^2+(2+2)^2+(-8-4)^2}$$

$$= \sqrt{16+16+144} = \sqrt{176} = 4\sqrt{11} \text{ units}$$

$\therefore$ $AB + BC = 4\sqrt{11}$ units $= AC$

Thus, the given points are collinear.

13. The point on the y-axis is of the form $P(0, y, 0)$. Since the points $A(3, 2, 2)$ and $B(5, 5, 4)$ are equidistant from P.

$\therefore$ $PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow$ $(0-3)^2 + (y-2)^2 + (0-2)^2 = (0-5)^2 + (y-5)^2 + (0-4)^2$

$\Rightarrow$ $6x = 25 + 25 + 16 - 17$ *i.e.*, $y = \dfrac{49}{6}$

Thus, the point $P\left(0, \dfrac{49}{6}, 0\right)$, on the y-axis is equidistant from A and B.

14. Given, $A(0, 7, 10)$, $B(-1, 6, 6)$ and $C(-4, 9, 6)$

$\therefore$ Using distance formula, we get,

$$AB=\sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}=\sqrt{18}=3\sqrt{2}$$

$$BC=\sqrt{(-4+1)^2+(9-6)^2+(6-6)^2}=\sqrt{18}=3\sqrt{2}$$

$$AC=\sqrt{(-4-0)^2+(9-7)^2+(6-10)^2}=\sqrt{36}=6$$

Clearly, $AB = BC$ and $AB^2 + BC^2 = 18 + 18 = 36 = AC^2$.

Hence, triangle ABC is an isosceles right-angled triangle.

15. Let $R(x, y, z)$ be the required point.

If R divides PQ internally in the ratio 2 : 3, then

$$x=\frac{2\times4+3\times2}{2+3}, y=\frac{2\times3+3(-1)}{2+3}$$

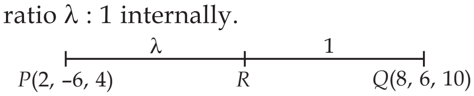
and $z=\dfrac{2\times2+3\times4}{2+3}$

$$\left[\because\ (x,y,z)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)\right]$$

$\Rightarrow$ $x=\dfrac{14}{5}, y=\dfrac{3}{5}$ and $z=\dfrac{16}{5}$

So, the coordinates of point R are $\left(\dfrac{14}{5},\dfrac{3}{5},\dfrac{16}{5}\right)$.

16. Suppose $R(4, y, z)$ be any point which divides PQ in the ratio $\lambda : 1$ internally.



Then, the coordinates of R are

$$\left(\frac{8\lambda+2}{\lambda+1},\frac{6\lambda-6}{\lambda+1},\frac{10\lambda+4}{\lambda+1}\right)$$

Since x-coordinate of R is 4.

$\therefore$ $\dfrac{8\lambda+2}{\lambda+1}=4 \Rightarrow 8\lambda+2=4\lambda+4 \Rightarrow 4\lambda=2 \Rightarrow \lambda=\dfrac{1}{2}$.

$\therefore$ $y=\dfrac{6\left(\frac{1}{2}\right)-6}{\frac{1}{2}+1}=-2$ and $z=\dfrac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}=6$

Hence, the coordinates of R are $(4, -2, 6)$.

17. Let the coordinates of points P and Q be (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively.

Then, the coordinates of the point R which divides PQ externally in the ratio 2 : 1 are

$$\left(\frac{2x_2-x_1}{2-1},\frac{2y_2-y_1}{2-1},\frac{2z_2-z_1}{2-1}\right)$$

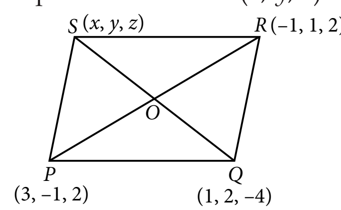
$=(2x_2 - x_1, 2y_2 - y_1, 2z_2 - z_1)$

The coordinates of the mid-point of PR are

$$\left(\frac{x_1+2x_2-x_1}{2},\frac{y_1+2y_2-y_1}{2},\frac{z_1+2z_2-z_1}{2}\right)=(x_2, y_2, z_2)$$

Clearly, these are the coordinates of point Q. Hence, Q is the mid-point of PR.

18. Let the required vertex be $S(x, y, z)$



In a parallelogram, diagonals bisect each other.

$\therefore$ Mid point of PR = Mid point of QS

$$\Rightarrow \left(\frac{3-1}{2},\frac{-1+1}{2},\frac{2+2}{2}\right)=\left(\frac{x+1}{2},\frac{y+2}{2},\frac{z-4}{2}\right)$$

$$\left[\because\ (x,y,z)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)\right]$$

$$\Rightarrow (1,0,2)=\left(\frac{x+1}{2},\frac{y+2}{2},\frac{z-4}{2}\right)$$

$$\Rightarrow \frac{x+1}{2}=1,\ \frac{y+2}{2}=0,\ \frac{z-4}{2}=2$$

$\Rightarrow$ $x = 1,\ y = -2,\ z = 8$

$\therefore$ The coordinates of fourth vertex is $(1, -2, 8)$.

19. Let $R(x, y, z)$ be any point which divides PQ externally in the ratio 2 : 3, then

$$x=\frac{2\times4-3\times2}{2-3}, y=\frac{2\times3-3\times(-1)}{2-3},$$

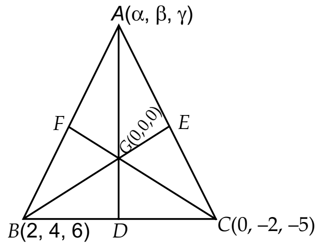
$$z=\frac{2\times2-3\times4}{2-3}$$

$$\left[\because\ (x,y,z)=\left(\frac{mx_2-nx_1}{m-n},\frac{my_2-ny_1}{m-n},\frac{mz_2-nz_1}{m-n}\right)\right]$$

$\Rightarrow$ $x = -2, y = -9, z = 8$

So, the coordinates of R are $(-2, -9, 8)$.

20. Let third vertex of ΔABC be $A(\alpha, \beta, \gamma)$



Since the coordinates of centroid G are (0, 0, 0)

$$\Rightarrow \quad 0=\frac{\alpha+2+0}{3} \Rightarrow \alpha=-2$$

$$\Rightarrow \quad 0=\frac{\beta+4-2}{3} \Rightarrow \beta=-2$$

$$\Rightarrow \quad 0=\frac{\gamma+6-5}{3} \Rightarrow \gamma=-1$$

So, the third vertex of the triangle is $A(-2, -2, -1)$.

21. Let P be a point in the yz plane. The point in the yz plane is of the form $P(0, y, z)$.

$\because$ P is equidistant from $A(3, 2, -1)$, $B(1, -1, 0)$ and $C(2, 1, 2)$

$\therefore$ $AP = BP = CP \Leftrightarrow AP^2 = BP^2$ and $BP^2 = CP^2$

$\Rightarrow$ $(0-3)^2 + (y-2)^2 + (z+1)^2 = (0-1)^2 + (y+1)^2 + (z-0)^2$

and $(0-1)^2 + (y+1)^2 + (z-0)^2 = (0-2)^2 + (y-1)^2 + (z-2)^2$

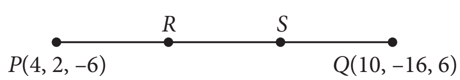
$\Rightarrow$ $3y - z - 6 = 0$ ...(i)

and $4y + 4z - 7 = 0$ ...(ii)

On solving (i) and (ii), we get $y=\frac{31}{16}$ and $z=\frac{-3}{16}$

Hence, the required point is $\left(0, \frac{31}{16}, \frac{-3}{16}\right)$.

22. **(b) :** Let R and S be two points which trisect the join of PQ.



$\because$ Point R divides the join of PQ in the ratio 1 : 2.

$\therefore$ Coordinates of R is

$$\left(\frac{1\times10+2\times4}{1+2}, \frac{1\times(-16)+2\times2}{1+2}, \frac{1\times6+2\times(-6)}{1+2}\right)$$

$= (6, -4, -2)$

Also point S divides the join of PQ in the ratio 2 : 1.

$\therefore$ Coordinates of S is

$$\left(\frac{2\times10+1\times4}{1+2}, \frac{2\times(-16)+1\times2}{1+2}, \frac{2\times6+1\times(-6)}{1+2}\right)$$

$= (8, -10, 2)$.

23. We have given $A(0, 4, 1)$, $B(2, 3, -1)$ and $C(4, 5, 0)$. By using distance formula, we get

$$AB=\sqrt{(0-2)^2+(4-3)^2+(1+1)^2}$$

$$=\sqrt{4+1+4}=\sqrt{9}=3 \text{ units}$$

$$BC=\sqrt{(2-4)^2+(3-5)^2+(-1-0)^2}$$

$$=\sqrt{4+4+1}=\sqrt{9}=3 \text{ units}$$

$$AC=\sqrt{(0-4)^2+(4-5)^2+(1-0)^2}$$

$$=\sqrt{16+1+1}=\sqrt{18} \text{ units}$$

Now, $AB^2 + BC^2 = 9 + 9 = 18 = AC^2$.

Hence, ΔABC is a right angled triangle.

24. We have given, $A(2, 3, 4)$, $B(-1, 2, -3)$ and $C(-4, 1, -10)$.

Now, $AB=\sqrt{(2+1)^2+(3-2)^2+(4+3)^2}$

$$=\sqrt{9+1+49}=\sqrt{59} \text{ units}$$

$$BC=\sqrt{(-1+4)^2+(2-1)^2+(-3+10)^2}$$

$$=\sqrt{9+1+49}=\sqrt{59} \text{ units}$$

$$AC=\sqrt{(2+4)^2+(3-1)^2+(4+10)^2}$$

$$=\sqrt{36+4+196}=\sqrt{236}=2\sqrt{59} \text{ units}$$

Now, $AC = AB + BC$

So, the points A, B and C are collinear.

Also, $AC : BC = 2\sqrt{59} : \sqrt{59} = 2 : 1$

Hence, C divides AB in the ratio 2:1 externally.

25. Let $Q(0, y, 0)$ be any point on y-axis.

Then, $PQ=\sqrt{(0-3)^2+(y+2)^2+(0-5)^2}$

$$=\sqrt{9+y^2+4+4y+25}=\sqrt{y^2+4y+38}$$

Given, $PQ = 5\sqrt{2}$

$\Rightarrow \sqrt{y^2+4y+38}=5\sqrt{2}$.

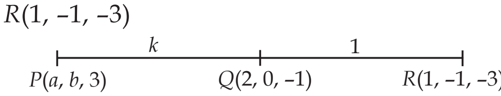
On squaring both sides, we get

$y^2 + 4y + 38 = 50 \quad \Rightarrow \quad y^2 + 4y - 12 = 0$

$\Rightarrow \quad (y-2)(y+6) = 0 \quad \Rightarrow \quad y = 2, -6$

Thus, coordinates of point Q are (0, 2, 0) and (0, –6, 0).

26. Suppose the given points are $P(a, b, 3)$, $Q(2, 0, -1)$ and $R(1, -1, -3)$



Let Q divides the line segment PR in the ratio $k : 1$.

Then coordinates of Q are $\left(\frac{k+a}{k+1}, \frac{-k+b}{k+1}, \frac{-3k+3}{k+1}\right)$

But coordinates of Q are (2, 0, –1)

$$\therefore \quad (2, 0, -1) \equiv \left(\frac{k+a}{k+1}, \frac{-k+b}{k+1}, \frac{-3k+3}{k+1}\right)$$

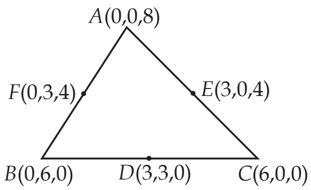
$$\Rightarrow \quad \frac{k+a}{k+1}=2, \frac{-k+b}{k+1}=0, \frac{-3k+3}{k+1}=-1$$

Now $\frac{-3k+3}{k+1}=-1 \Rightarrow -3k+3=-k-1 \Rightarrow k=2$

$$\therefore \quad \frac{k+a}{k+1}=2 \Rightarrow \frac{2+a}{3}=2 \Rightarrow a=6-2=4$$

and $\frac{-k+b}{k+1}=0 \Rightarrow \frac{-2+b}{3}=0 \Rightarrow b=2$.

27. Let D, E and F be the mid-points of sides BC, CA and AB respectively.



The coordinates of D, E and F are

$$D\left(\frac{0+6}{2},\frac{6+0}{2},\frac{0+0}{2}\right)=(3,3,0)$$

$$E\left(\frac{6+0}{2},\frac{0+0}{2},\frac{0+8}{2}\right)=(3,0,4)$$

and $F\left(\frac{0+0}{2},\frac{0+6}{2},\frac{8+0}{2}\right)=(0,3,4)$ respectively.

$$\therefore \quad AD=\sqrt{(0-3)^2+(0-3)^2+(8-0)^2}$$

$$=\sqrt{9+9+64}=\sqrt{82} \text{ units}$$

$$BE=\sqrt{(0-3)^2+(6-0)^2+(0-4)^2}$$

$$=\sqrt{9+36+16}=\sqrt{61} \text{ units}$$

and, $CF=\sqrt{(6-0)^2+(0-3)^2+(0-4)^2}$

$$=\sqrt{36+9+16}=\sqrt{61} \text{ units}$$

28. Let the given points be $A(0, -1, -7)$, $B(2, 1, -9)$ and $C(6, 5, -13)$

Now, $AB=\sqrt{(0-2)^2+(-1-1)^2+(-7+9)^2}$

$$=\sqrt{4+4+4}=2\sqrt{3} \text{ units}$$

$$BC=\sqrt{(2-6)^2+(1-5)^2+(-9+13)^2}$$

$$=\sqrt{16+16+16}=4\sqrt{3} \text{ units}$$

$$AC=\sqrt{(0-6)^2+(-1-5)^2+(-7+13)^2}$$

$$=\sqrt{36+36+36}=6\sqrt{3} \text{ units}$$

Now, $AC = AB + BC$

$\therefore$ The points A, B and C are collinear.

Now, $AB:AC=2\sqrt{3}:6\sqrt{3}=1:3$

Hence, point A divides B and C in the ratio 1 : 3 externally.

29. Let point $R(x, y, z)$ divides the join of $(-2, 4, 7)$ and $(3, -5, 8)$ in the ratio $k : 1$, then point of division is

$$R\left(\frac{3k-2}{k+1},\frac{-5k+4}{k+1},\frac{8k+7}{k+1}\right) \qquad \text{...(i)}$$

As the point R lies on YZ-plane, $\Rightarrow$ $x = 0$

$$\Rightarrow \quad \frac{3k-2}{k+1}=0 \Rightarrow k=\frac{2}{3}$$

$\therefore$ Required ratio is 2 : 3 internally.

Substituting the value of k in (i), we get point of division as $\left(0,\dfrac{\frac{-10}{3}+4}{\frac{2}{3}+1},\dfrac{\frac{16}{3}+7}{\frac{2}{3}+1}\right)$

i.e., $\left(0,\frac{2}{5},\frac{37}{5}\right)$.

30. Vertices of ΔPQR are $P(2a, 2, 6)$, $Q(-4, 3b, -10)$ and $R(8, 14, 2c)$.

Centroid of ΔPQR is :

$$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$$

$$=\left(\frac{2a-4+8}{3},\frac{2+3b+14}{3},\frac{6-10+2c}{3}\right)$$

$$=\left(\frac{2a+4}{3},\frac{3b+16}{3},\frac{2c-4}{3}\right)$$

Also, centroid is at origin *i.e.,* (0, 0, 0)

$$\therefore \quad \frac{2a+4}{3}=0 \quad \Rightarrow \quad a=-2$$

$$\frac{3b+16}{3}=0 \quad \Rightarrow \quad b=\frac{-16}{3}$$

$$\frac{2c-4}{3}=0 \quad \Rightarrow \quad c=2$$

Thus, $a = -2$, $b = \dfrac{-16}{3}$ and $c = 2$

31. Let P and Q be the points of trisection of the segment $[AB]$, then P divides $[AB]$ in the ratio 1 : 2 and Q divides $[AB]$ in the ratio 2 : 1.

$$\therefore \quad P \equiv \left(\frac{1\times(-5)+2\times 1}{1+2},\frac{1\times 9+2\times 0}{1+2},\frac{1\times 6+2\times(-6)}{1+2}\right),$$

i.e., P is $(-1, 3, -2)$

and $Q \equiv \left(\frac{2\times(-5)+1\times 1}{2+1},\frac{2\times 9+1\times 0}{2+1},\frac{2\times 6+1\times(-6)}{2+1}\right)$,

i.e., Q is $(-3, 6, 2)$.

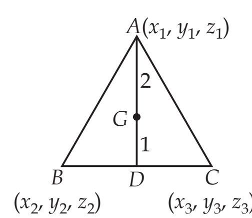
Hence, the required points of trisection are $P(-1, 3, -2)$ and $Q(-3, 6, 2)$.

32. Let G be the centroid of a triangle ABC. We know, centroid of a triangle divides each median in the ratio 2 : 1.

As, D is mid-point of BC.

$\therefore$ Coordinates of D are

$$\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2},\frac{z_2+z_3}{2}\right)$$



Since, G divides AD in the ratio 2 : 1.

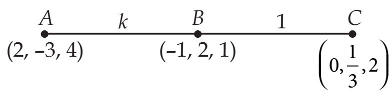
$\therefore$ Coordinates of G are

$$\left(\frac{1\cdot x_1+2\left(\frac{x_2+x_3}{2}\right)}{1+2}, \frac{1\cdot y_1+2\left(\frac{y_2+y_3}{2}\right)}{1+2},\right.$$

$$\left.\frac{1\cdot z_1+2\left(\frac{z_2+z_3}{2}\right)}{1+2}\right)$$

i.e. $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

33. Suppose the given points are collinear



Let the ratio in which B divides AC be $k : 1$.

Coordinates of B are $\left(\frac{2}{k+1}, \frac{\frac{1}{3}k-3}{k+1}, \frac{2k+4}{k+1}\right)$

But coordinates of B are (–1, 2, 1)

$\therefore$ $\frac{2}{k+1}=-1, \frac{\frac{1}{3}k-3}{k+1}=2, \frac{2k+4}{k+1}=1$

From each of these equations, we get $k = -3$

Since each of these equations give the same value of k.

$\therefore$ The given points are collinear.

34. Let $A(x, y, z)$ be any point which is equidistant from points $B(1, 2, 3)$ and $C(3, 2, -1)$.

Then $AB=\sqrt{(x-1)^2+(y-2)^2+(z-3)^2}$

$AC=\sqrt{(x-3)^2+(y-2)^2+(z+1)^2}$

It is given that $AB = AC$

$\sqrt{(x-1)^2+(y-2)^2+(z-3)^2}$

$=\sqrt{(x-3)^2+(y-2)^2+(z+1)^2}$

$\Rightarrow$ $(x-1)^2+(y-2)^2+(z-3)^2$
$=(x-3)^2+(y-2)^2+(z+1)^2$

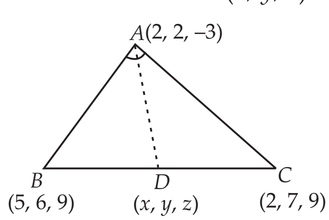
$\Rightarrow$ $x^2+1-2x+z^2+9-6z=x^2+9-6x+z^2$
$+1+2z$

$\Rightarrow$ $-2x-6z+10=-6x+2z+10$

$\Rightarrow$ $-2x-6z+6x-2z=0$

$\Rightarrow$ $4x-8z=0$ $\Rightarrow$ $x-2z=0$

35. Let the coordinates of D be (x, y, z)



$AB=\sqrt{9+16+144}=\sqrt{169}=13$ units

$AC=\sqrt{0+25+144}=\sqrt{169}=13$ units

Now, $\frac{BD}{DC}=\frac{AB}{AC}=\frac{13}{13}=\frac{1}{1}$

$\Rightarrow$ $BD = DC$

Since, D divides the line BC in two equal parts. So, D is the mid-point of BC.

$\therefore$ By using the mid-point formula,

$x=\frac{5+2}{2}=\frac{7}{2}$

$y=\frac{6+7}{2}=\frac{13}{2}$ and $z=\frac{9+9}{2}=9$

Hence, the coordinates of D are $\left(\frac{7}{2}, \frac{13}{2}, 9\right)$

36. Let P be the point equidistant from three given points. The point in XY plane is of the form $P(x, y, 0)$.

According to question, $AP = BP = CP$

$\Rightarrow$ $AP^2 = BP^2 = CP^2$

Now, $AP^2 = BP^2$

$\Rightarrow$ $(x-2)^2+(y-0)^2+(0-3)^2$
$=(x-0)^2+(y-3)^2+(0-2)^2$

$\Rightarrow$ $x^2+4-4x+y^2+9=x^2+y^2+9-6y+4$

$\Rightarrow$ $6y=4x$ $\Rightarrow$ $x=\frac{6}{4}y$ ...(i)

Again, $BP^2 = CP^2$

$\Rightarrow$ $(x-0)^2+(y-3)^2+(0-2)^2=(x-0)^2+(y-0)^2$
$+(0-1)^2$

$\Rightarrow$ $x^2+y^2+9-6y+4=x^2+y^2+1$

$\Rightarrow$ $6y=12 \Rightarrow y=2$

Put $y = 2$ in (i) we get, $x=\frac{6}{4}\times 2=3$

$\therefore$ Required point is (3, 2, 0)

37. Any point in xy-plane is of the form $(x, y, 0)$.

Let $P(x, y, 0)$ be equidistant from $A(5, 0, 6)$, $B(0, -3, 2)$ and $C(4, 5, 0)$.

$\therefore$ $PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow$ $(x-5)^2+(y-0)^2+(0-6)^2=(x-0)^2+(y+3)^2+(0-2)^2$

$\Rightarrow$ $x^2+25-10x+y^2+36=x^2+y^2+9+6y+4$

$\Rightarrow$ $5x+3y=24$ ...(i)

Also, $PB = PC$ $\Rightarrow$ $PB^2 = PC^2$

$\Rightarrow$ $(x-0)^2+(y+3)^2+(0-2)^2=(x-4)^2+(y-5)^2+(0-0)^2$

$\Rightarrow$ $x^2+y^2+9+6y+4=x^2+16-8x+y^2+25-10y$

$\Rightarrow$ $2x+4y=7$ ...(ii)

Solving (i) and (ii), we get, $x=\frac{75}{14}$ and $y=\frac{-13}{14}$.

$\therefore$ Coordinates of P are $\left(\frac{75}{14}, \frac{-13}{14}, 0\right)$

38. Let $P(x, y, z)$ be any point.

Then, $PA=\sqrt{(x-4)^2+(y-0)^2+(z-0)^2}$

$=\sqrt{x^2+16-8x+y^2+z^2}$

and $PB = \sqrt{(x+4)^2 + (y-0)^2 + (z-0)^2}$

$$= \sqrt{x^2 + 16 + 8x + y^2 + z^2}$$

It is given that $PA + PB = 10$

$\therefore \quad \sqrt{x^2 + 16 - 8x + y^2 + z^2} + \sqrt{x^2 + 16 + 8x + y^2 + z^2} = 10$

$\Rightarrow \quad \sqrt{x^2 + 16 - 8x + y^2 + z^2}$

$= 10 - \sqrt{x^2 + 16 + 8x + y^2 + z^2}$ ...(i)

Squaring (i) on both sides, we get

$x^2 + 16 - 8x + y^2 + z^2 = 100 + x^2 + 16 + 8x + y^2 + z^2$
$-20\sqrt{x^2 + 16 + 8x + y^2 + z^2}$

$\Rightarrow \quad 20\sqrt{x^2 + 16 + 8x + y^2 + z^2} = 16x + 100$

$\Rightarrow \quad 5\sqrt{x^2 + 16 + 8x + y^2 + z^2} = 4x + 25$ ...(ii)

Squaring (ii) on both sides, we get

$25(x^2 + 16 + 8x + y^2 + z^2) = 16x^2 + 625 + 200x$

$\Rightarrow \quad 25x^2 + 400 + 200x + 25y^2 + 25z^2 - 16x^2 - 625 - 200x = 0$

$\Rightarrow \quad 9x^2 + 25y^2 + 25z^2 - 225 = 0$

$\therefore$ Required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$

39. Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ be the vertices of the given triangle, and let $D(1,5,-1)$, $E(0,4,-2)$ and $F(2,3,4)$ be the mid-points of the sides BC, CA and AB respectively.

As, D is the mid-point of BC

$\therefore \quad \dfrac{x_2 + x_3}{2} = 1, \dfrac{y_2 + y_3}{2} = 5, \dfrac{z_2 + z_3}{2} = -1$

$\Rightarrow \quad x_2 + x_3 = 2,\ y_2 + y_3 = 10,\ z_2 + z_3 = -2$ ...(i)

As, E is the mid-point of CA

$\therefore \quad \dfrac{x_1 + x_3}{2} = 0, \dfrac{y_1 + y_3}{2} = 4, \dfrac{z_1 + z_3}{2} = -2$

$\Rightarrow \quad x_1 + x_3 = 0,\ y_1 + y_3 = 8,\ z_1 + z_3 = -4$ ...(ii)

As, F is the mid-point of AB

$\therefore \quad \dfrac{x_1 + x_2}{2} = 2, \dfrac{y_1 + y_2}{2} = 3, \dfrac{z_1 + z_2}{2} = 4$

$\Rightarrow \quad x_1 + x_2 = 4,\ y_1 + y_2 = 6,\ z_1 + z_2 = 8$ ...(iii)

Adding first three equations in (i), (ii) and (iii), we get,

$2(x_1 + x_2 + x_3) = 6$

$\Rightarrow \quad x_1 + x_2 + x_3 = 3$ ...(iv)

Solving first equations in (i), (ii) and (iii) with (iv), we get, $x_1 = 1$, $x_2 = 3$, $x_3 = -1$

Adding second equations in (i), (ii) and (iii), we get,

$2(y_1 + y_2 + y_3) = 10 + 8 + 6$

$\Rightarrow \quad y_1 + y_2 + y_3 = 12$ ...(v)

Solving second equations in (i), (ii) and (iii) with (v), we get, $y_1 = 2$, $y_2 = 4$, $y_3 = 6$

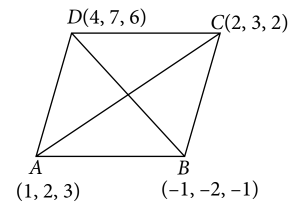
Adding last equations in (i), (ii) and (iii), we get,

$2(z_1 + z_2 + z_3) = -2 - 4 + 8 \Rightarrow z_1 + z_2 + z_3 = 1$ ...(vi)

Solving last equations in (i), (ii) and (iii) with (vi), we get, $z_1 = 3$, $z_2 = 5$, $z_3 = -7$

Thus, the vertices of the triangle are $A(1,2,3)$, $B(3,4,5)$ and $C(-1, 6, -7)$.

40. Given vertices are $A(1, 2, 3)$, $B(-1, -2, -1)$, $C(2, 3, 2)$ and $D(4, 7, 6)$.



Coordinates of mid-point of AC are

$$\left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right)$$

Coordinates of mid-point of BD are

$$\left(\frac{-1+4}{2}, \frac{-2+7}{2}, \frac{-1+6}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right)$$

As mid points of AC and BD are same

So, A, B, C and D are the vertices of a parallelogram.

Now, $AC = \sqrt{(2-1)^2 + (3-2)^2 + (2-3)^2}$

$$= \sqrt{1+1+1} = \sqrt{3}$$

$BD = \sqrt{(4+1)^2 + (7+2)^2 + (6+1)^2}$

$$= \sqrt{25+81+49} = \sqrt{155}$$

As, $AC \neq BD \quad \Rightarrow$ Diagonals are not equal.

$\therefore \quad ABCD$ is not a rectangle

Hence, A, B, C and D are vertices of a parallelogram but not rectangle.