vertices.

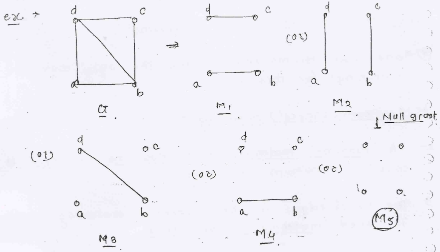
Matchings And Coverings :-

Matching :-

Let 'G' = (V, E) be a graph. A subgraph 'M' of G is called a 'matching of G', if each vertex of G is incident with at most one edge in M.

i.e. in a matching 'M', $\deg(v) \leq 1$, $\forall v \in G$.

ex :-



Note :-

1) In a matching, if $\deg(v) = 1$, then the vertex 'v' is said to "matched."

2) If $\deg(v) = 0$, then 'V' is not matched.

3) In a matching, no two edges are adjacent.

Maximal matching →

⊛ A matching of a graph (M) is said to be maximal if no other edges of G can be added to M.

From the previous matchings,
M_1, M_2 and M_3 are maximal matchings of G.

(and) M_4 and M_5 are not maximal.

⊛ → and also M_1, M_2 and M_3 are the only maximal matchings for this graph.

Maximum (Largest) matching →

A maximal matching with max no. of edges is called "maximum matching".

The no. of edges in the maximum matching of G is called "matching number" of G.

Ex. → From prev., M_1 and M_2 are "maximum matchings"

∴ Matching no. of the graph = 2.

Perfect matching →

A matching M of graph G is said to be perfect matching if every vertex of G is matched in M.

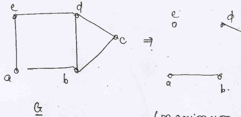
i.e. in a perfect matching, deg (v) = 1 ; ∀ v∈G.

For the prev. ex., M_1 and M_2 both are "Perfect matchings" of G.

Note →

1) Every perfect matching of a graph is also a "maximum matching" of graph.

2) A maximum matching of a graph need not be perfect.



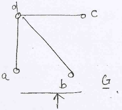
(maximum matching but not a perfect matching.)

3) If a graph G has a perfect matching, then no of vertices in the graph is even.

(A graph with odd no. of vertices cannot have perfect matching).

The converse of 3), need not be true.

ex

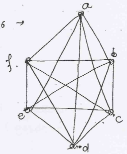


(This graph has even no of vertices, but no perfect matching possible.).

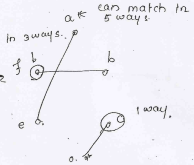
4) Complete graph 'K_n' has a perfect matching iff n is even. (no. of vertices are even).

Number of perfect matchings in a complete graph with $2n$ vertices i.e K_{2n} is
$(n = 1, 2, 3, \cdots)$

let $K_6 \rightarrow$



$\Rightarrow$



$\therefore$ No. of perfect matchings in $K_6 = 5 \times 3 \times 1 = \underline{15}$.

Now, for no. of perfect matchings in K_{2n} is

$(2n-1).(2n-3).(2n-5) \ldots 5.3.1$

$$= \frac{2n!}{2n.(2n-2).(2n-4)\ldots 6.4.2}$$

$$= \boxed{\frac{2n!}{2^n.\ n!}}$$

Q.1. How many perfect matchings in K_8 ?

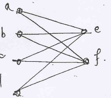
$2n = 8$

$n = 4$

→ $\frac{2n!}{2^n.\ n!} = \underline{105}$.

Note →

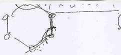
The complete bipartite graph has a perfect matching iff $m = n$.



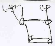
Here, perfect matching is not possible for $K_{4,2}$.

How many perfect matching possible in $K_{n,n}$

$= \boxed{n!}$



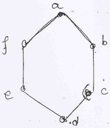
(105)



n/2.
5 × 3
2 ×

Q. 2. $n \geq 4$ and n is even. in a cycle graph C_n

→. In a cycle graph, only 2 perfect matchings are possible.



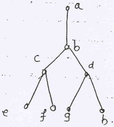
10/13

* Note →

A tree can have at most one perfect matching.

i.e. no. of perfect matchings in a tree ≤ 1. } (***).

Q.3. No. of perfect matchings in the tree shown below is



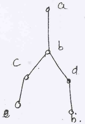
→ In a perfect matching, degree of every vertex of the graph is 1.

Note: Matching Number, - No. of edges in the maximum matching.

Therefore, we have to delete two edges at vertex c. By deleting any two edges of vertex c, perfect matching is not possible.

$\therefore$ No. of perfect matchings in the tree $= \boxed{0}$.

Q.4. No. of perfect matchings in the tree below?

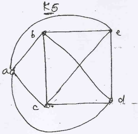


No. of perfect matchings $= \boxed{1}$.

5. Matching number of the complete graph K_n = ?

$\rightarrow$ Matching number of $K_n = \frac{n}{2}$ (if n is even).

If n is odd.



Here perfect matching is not possible. So, we can match only $n-1$ vertices.

$\therefore$ matching number $= \frac{(n-1)}{2}$ (if n is odd).

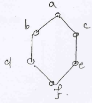
$\frac{n-1}{2}+1 \quad \frac{n-1+2}{2} \quad \boxed{\frac{n-1}{2}}$

$\therefore$ matching number of $K_n = \left\lfloor \frac{n}{2} \right\rfloor$

Q.6. Matching number of C_n ($n \geq 3$) = ?

if n is even. $\Rightarrow$

matching number = $\frac{n}{2}$.



if n is odd $\Rightarrow$

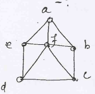
matching number = $\frac{n-1}{2}$.

$\therefore$ Matching no of $C_n = \lfloor n/2 \rfloor$.

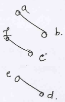
Q.7. Matching number of W_n ($n \geq 4$) = ?

$\rightarrow$ if n is even.

matching no. = $\frac{n}{2}$.



$\Rightarrow$

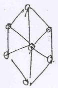


max. matching.

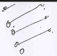
if n is odd.

matching no. = $\frac{n-1}{2}$.

W_7



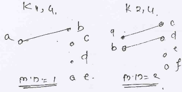
$\therefore$ matching no. of $W_n = \lfloor n/2 \rfloor$.

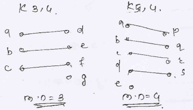


Q-8. Matching no of complete bipartite graph $K_{m,n}$ is ?

→ ∴ Matching no of

$K_{m,n}$ = $\underline{\min(m,n)}$

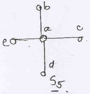




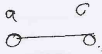
(n≥2)

Q. 9. Matching number of a star graph with 'n' vertices is ?

→ every star graph S_n is a bipartite graph of the form $K_{1,n-1}$.

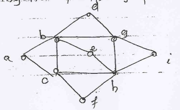


⇒



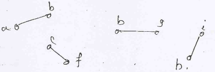
∴ matching no. of S_n = $\boxed{1}$.

Q.10. Matching no. of the graph shown below is ?



→ no of vertices in graph = 9.

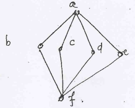
∴ Max no of vertices we can match = 8.



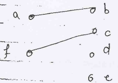
∴ max no of edges = 4.

∴ matching number of graph = 4.

Q.11. what is matching number of the graph shown below. ?

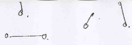


→ It is a complete bipartite graph $K_{2,4}$.

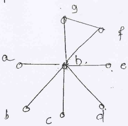


← matching

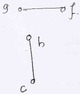
∴ matching no. = $\boxed{2}$.



2. Matching no. of the graph shown below is ?

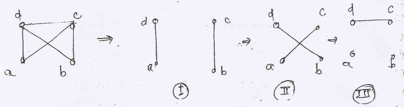


$\Downarrow$ max matching



$\therefore$ matching no. $= \underline{2}$.

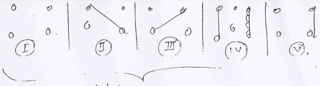
13. No of maximaml matchings in the graph shown below ?



perfect and maximum matchings.

maximal matching

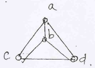
$\therefore$ total no. of maximal matchings $= \boxed{3}$.



matchings.

∴ total no. of matchings in graph = $5+3 = \boxed{8}$.

Q.14. No. of maximal matchings in the graph given below is ?



$\underline{K_4}$.

→ no. of maximal matchings = $(n-1)(n-3)$.

$= 3 \times 1 = \boxed{3}$.

Total no. of matchings = $3+6+1 = \boxed{10}$.

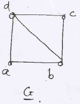
Coverings →

Line covering / Edge covering →

Let $G = (V, E)$ be a graph. A subset C of E is called a "line covering of G" if every vertex of G is incident with at ~~most~~ least one edge in C.
i.e. in a line covering, degree of every vertex is ≥ 1 for every $V \in G$.

i.e. $\deg_C(v) \geq 1 \quad ; \forall v \in G$.

ex →



$C_1 = \{\{a,b\}, \{c,d\}\}$. (minimal) ✓ (minimum) ✓

$C_2 = \{\{a,d\}, \{b,c\}\}$. (minimal) ✓ (minimum)

$C_3 = \{\{a,b\}, \{b,c\}, \{b,d\}\}$ (minimal) ✓ (not minimum) ×

$C_4 = \{\{a,b\}, \{b,c\}, \{c,d\}\}$ × (not minimal)

Note →

Line covering of a graph G does not exist iff G has an isolated vertex.

Line covering of a graph with n vertices has at least $\boxed{\lceil n/2 \rceil}$ edges.

Minimal line covering

A line covering 'c' of a graph G is said to be minimal if no edge can be deleted from 'C'.

For the graph given in above example, C_1, C_2, C_3 are minimal line coverings whereas, C_4 is not a minimal line covering.

Minimum line covering (Smallest minimal line covering)

A minimal line covering of G with minimum no. of edges is called "minimum line covering of G".

No. of edges in minimum line covering of G is called "line covering number of G".

- It is denoted by "α_1".

- For the graph given in above example, C_1 and C_2 are minimal line coverings of G and $\alpha_1 = 2$.

properties

- Every line covering contains a minimal line covering.

- but every line covering contains a minimum line covering is not true.

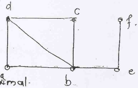
- The line covering C_3 does not contain any minimum line covering.

* No minimal line covering contains a cycle.

* If a line covering contains no paths of length 3 or more, then c is a minimal line covering because all the components of c are star graphs and from a stargraph, no edge can be deleted.

Independent Line Set →

Let $G=(V,E)$ be a graph. A subset L of E is called an independent line set of G if no two edges in L are adjacent.



$L_1 = \{\{a,b\}\}$ not maximal

$L_2 = \{\{b,d\},\{e,f\}\}$ ✓ maximal

$L_3 = \{\{a,b\},\{c,d\},\{e,f\}\}$ ✓ maximal

L_4 not possible.

$L_3 = \{\{a,d\},\{b,c\},\{e,f\}\}$ × not minimal.

Maximal Independent Line set → of a graph G.

A independependf line set is maximal if no other edge of G can be added to it.

For the graph given in the above excample, L_2 and L_3 are maximal indep line sets.

maximum indep. line set →

A maximal indep. line set with max. no. of edges is called "maximum independent line set" of

No. of edges in a maximum independent line set of G is called "line independence number" of G.

Denote it by β_1. = matching number of graph.

For the graph above, L_3 is a maximum independent line set of G and $\beta_1 = 3$.

Note → (line cov no.) (line independence no.)

For any graph G, $\alpha_1 + \beta_1 =$ no of vertices in G.
(with no isolated vertex)

Q.1. Line covering number of $K_n = \alpha_1 = ?$

→ line independence no. $= \beta_1 = \lfloor \frac{n}{2} \rfloor$ (matching no).

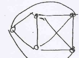
and $\alpha_1 + \beta_1 = n$. —①.

$\therefore \boxed{\alpha_1 = \lceil n/2 \rceil}$

K_4.

cov no. $= n/2$. (n is even).

K_5



cov. no. $= \frac{n-1}{2} + 1 = \frac{n+1}{2}$ (n is odd)

$\therefore \boxed{\alpha_1 = \text{line covering no.} = \lceil n/2 \rceil}$

Note →
1) Line covering no. α of C_n $(n \geq 3)$

$= \lceil n/2 \rceil$

2) Line covering no of wheel graph W_n $(n \geq 4)$

$= \lceil n/2 \rceil$

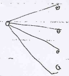
3) Line covering no of complete bipartite graph $K_{m,n}$

$= \max(m,n)$. $\left\{ \because \alpha_1 + \beta_1 = (m+n) \leftarrow \text{total no. of nodes} \right.$

$\uparrow$ $\min(m,n) \leftarrow$ matching no.

$\max(m,n)$

ex. for $K_{1,4}$.



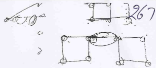
4 edges are reqd. in minimum line covering.

4) Line covering no. of a star graph with 'n' vertices $(n \geq 2)$ is

$\boxed{(n-1)}$

* Every star graph S_n can be represented as a complete bipartite graph $K_{1,n-1}$.

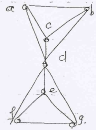
So, the matching no. (β_1) of $K_{1,n-1}$ is $\beta_1 = 1$.



$\therefore \ \alpha_1 + \beta_1 = n.$

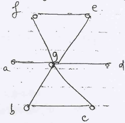
$\boxed{\alpha_1 = n-1}$ (for a star graph).

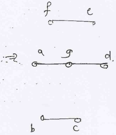
Q.2. Line covering of the graph shown below is ?



$\rightarrow \ |V| = n = 7. \quad \alpha_1 \geq \lceil n/2 \rceil = \boxed{4}$

3. Line covering of the graph show below is ?

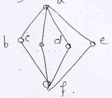




$\therefore \ \boxed{\alpha_1 = 4}$

Q.4. Line covering of a the " – " ?

→

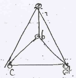


$K_{2,4}$

$\therefore \alpha_1 = 6 - 2 = \boxed{4}$

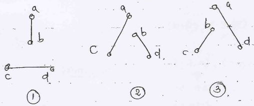
(no. of vertices) (matching no.)

5. " —" for the a graph shown below



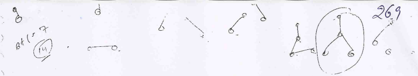
i) No. of min line coverings in graph.

→



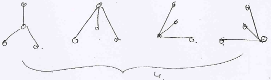
$\therefore$ no. of minimum line coverings = $\boxed{3}$

ii) No. of minimal line coverings



→ 3 minimum line coverings

+.



4.

∴ total minimal line coverings = 3+4 = 7

iii) which of the foll. is a minimal line covering of G ?

a) {(b,d), (a,b), (c,d)}.
(can be deleted)

b) {(a,c), (b,c), (b,d)}.
can be deleted.

✓ c) {(a,d), (d,c), (b,d)}. ✓ minimal.

d) {(a,d), (b,c), (a,c)} not minimal
can be deleted.

iv) Number of total line coverings in G.

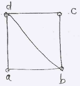
= 3+4+ 12 + C(6,4) + C(6,5) + 1.
(3 min. × 4 ways).

= $\boxed{41}$

Vertex - Coverings →

Let $G = (V, E)$ be a graph. A subset K of V is called as "vertex - covering" of G if every edge of G is covered by / incident with a vertex in K.

ex. →



G.

$K_1 = \{b, d\}$ minimal ✓

$K_2 = \{a, b, c\}$. minimal ✓

$K_3 = \{b, c, d\}$. not minimal ✗

$K_4 = \{a, d\}$. ✗ not a vertex covering.

⊛ Minimal vertex covering →

A vertex covering 'K' of a graph G is said to be minimal if no more vertex can be deleted from 'K'.

For the graph given above, K_1 and K_2 are minimal vertex coverings of G.

⊛ Minimum vertex covering → (smallest minimal vertex covering).

* A minimal vertex covering of G with min. no. of vertices is called "minimum vertex covering".

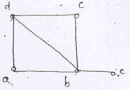
* No. of vertices in minimum vertex covering is called "vertex - covering number" of a graph, G.

It is denoted by 'α_2' (say).

For the graph given in above example, K_1 is a minimum vertex covering of G and $\boxed{\alpha_2 = 2}$

⑧ Independent vertex set →

Let $G = (V, E)$ be a graph. A subset 'S' of 'V' is called an "independent vertex set" of G if no two vertices in S are adjacent.



G

$S_1 = \{b\}$ maximal. ✓

$S_2 = \{d, e\}$. maximal ✓

$S_3 = \{a, c, e\}$ maximal ✓ maximum

$S_4 = \{c, e\}$. not maximal ✗ (a can be added).

⑨ Maximal Independent Vertex Set →

Let $G = (V, E)$ be a graph. An independent vertex set of G is said to be maximal if no other vertex of G can be added to it (S).

For the above ex., S_1, S_2 and S_3 are maximal indep. vertex sets of G.

(**) Maximum Independent Vertex Set (this set need not be a vertex covering of G).
(Largest maximal indep. vertex set)

A maximal independent vertex set of G with max. no. of vertices is called as "maximum indep. vertex set" of G.

* No. of vertices in maximum indep. vertex set of a graph G is called "vertex independence no. of G".

Let it denoted by 'β_2' (say).

For the graph given in above ex., S_3 is maximum independent vertex set of G and $\beta_2 = 3$.

Q.1. Vertex covering no. for the complete graph K_n = ?

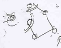
→ In a complete graph, each vertex is adjacent to rem. (n-1) vertices.

∴ a maximum indep. set of K_n contains only 1 vertex.

Note→ For any graph G, $G=(V,E)$.

i) $\alpha_2 + \beta_2 = |V|$.

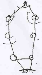
ii) if S is an indep. vertex set of G, then $|V-S|$ is a vertex covering of G.



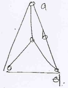
$\alpha_2 + \beta_2 = |V|$ $\alpha_1 = 2$

$\beta_2 = 2$

$|V| = 4$



→ ∴ $\beta_2 = \boxed{1}$



∴ $\alpha_2 = 4 - \beta_2$
$= 4 - 1$
$= \boxed{3}$

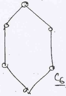
∴

For K_n, $\beta_2 = 1$.
$\alpha_2 = n-1$.

Q.2. For the cycle graph C_n, $(n \geq 3)$, what is α_2, β_2.

→ ∴ For C_n, (n even)

$\alpha_2 = \frac{n}{2}$

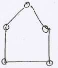


$\alpha_2 = 3. = \frac{6}{2}$

C_6.

(n odd).

∴ $\alpha_2 = \frac{(n-1)}{2} + 1$.



C_5

$5/2 = \boxed{3}$
$= \frac{4}{2} + 1$

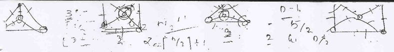
∴ For a cyclic graph C_n,

$\boxed{\alpha_2 = \lceil n/2 \rceil}$

∴ $\beta_2 = n - \lceil n/2 \rceil = \lfloor n/2 \rfloor$.

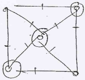
∴ $\boxed{\beta_2 = \lfloor n/2 \rfloor}$

For C_n
$\alpha_1 = \alpha_2$
$\beta_1 = \beta_2$



3. For the wheel graph W_n, $(n \geq 4)$. Find α_2 and β_2?

→ $$\alpha_2 = \begin{cases} \frac{n+2}{2}, & (n \text{ is even}) \\ \frac{n+1}{2}, & (n \text{ is odd}) \end{cases}$$

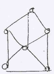


W_5

$\therefore \alpha_2 = 3.$

$$\cancel{\boxed{\alpha_2 = \lfloor n/2 \rfloor + 1}}$$

(for W_n.)



$\alpha_2 = 4$ W_6.

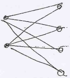
$$\boxed{\alpha_2 = \left\lceil \frac{n+1}{2} \right\rceil}$$

$$\boxed{\beta_2 = \left\lfloor \frac{n-1}{2} \right\rfloor}$$

For a complete bipartite graph $K_{m,n}$, find α_2 and β_2?

→ $\boxed{\alpha_2 = \min.(m,n)}$

$K_{2,4}$.



$\alpha_2 + \beta_2 = m+n.$

$\therefore \boxed{\beta_2 = \max.(m,n).}$

$\frac{n+1}{2}$ $n-$ $\lfloor n/2 \rfloor + 1$ $\frac{n+3}{2}$ $\frac{n^2}{2}$ 2 $n = \lceil n/2 \rceil - 1$ $\beta_2 = \lceil n/2 \rceil - 1$

$n-$ $\frac{n}{2}+1$ $n+3$ $\alpha_0 + \beta_2 = n$. $n - \lfloor n/2 \rfloor - 1$

$\lceil n/2 \rceil -)$

275

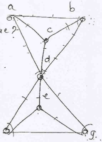
Q.5. For a star graph with n vertices, $(n \geq 2)$.

$\alpha_2 = ?$ $\beta_2 = ?$

$\rightarrow$ $S_n = K_{1,n-1}$.

$\therefore$ $\boxed{\alpha_2 = 1, \quad \beta_2 = n-1}$

Q.6. For the graph shown below, which of the foll. statements is/are not true?



S1) The set $\{b, f\}$ is a max. independent set.

S2) $\beta_2 = 2$ S3) $\alpha_2 = 5$.

S4) $\{a, b, c, e, g\}$ is a minimum vertex cover.

$\rightarrow$ S1) $\Rightarrow$ The indep. vertex set of 3 vertices is not possible because there is cycles of length 3. So, this is maximum.

So, S1 is true.

S2) From S1, S2 is true. $\therefore$ $\beta_2 = 2$. $\therefore$ S2 true

S3) $\alpha_2 + \beta_2 = 7$. $\therefore \alpha_2 = 5$. $\therefore$ S3 true.

S4) These vertices cannot cover all the edges.

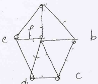
So, not true

30/10/2013

Q-7. For the graph shown below, which of the foll. are true

✓ S1) α_2 is 4.

✓ S2) $\beta_2 = 2$



✗ S3) $\{a, b, \}$ c, f. is a minimum vertex cover.

✓ S4) $\{a, c\}$ is maximum independent vertex set.

→ $\alpha_2 =$ (for a wheel graph)

$= \left\lceil \frac{n+1}{2} \right\rceil \quad (n = 6)$

$= \left\lceil \frac{7}{2} \right\rceil = \boxed{4}.$

$\therefore$ S1) is true.

$\therefore \alpha_2 + \beta_2 = n. \quad \therefore \boxed{\beta_2 = 2.}$

$\therefore$ S2) is true.

S3) → $\{a, b, c, f\}$ cannot cover the edge 'ed'. So,

S3) is not true.

S4). true- (Indep. vertex set with 3 vertices is not possible).

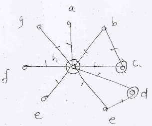
1.10. For graph given below,

→

$\boxed{\alpha_2 = 3}$

$\beta_2 = 9 - 3 = \underline{6}$

$\boxed{\beta_2 = 5}$



{d, c, h} ← minimum vertex covering.

{c, d, a, g, f, e} ← maximum indep. vertex set.

Complete Matchings →

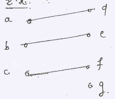
Let $G = (V, E)$ be a bipartite graph with vertex partition $V = \{V_1, V_2\}$

A matching from V_1 to V_2 is called a "complete matching" if every vertex in V_1 is matched

i.e. in a complete matching,

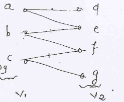
$$\deg(v) = 1 \; ; \; \forall v \in V_1$$

Ex. →



complete matching from V_1 to V_2.

⇐ (complete matching from V_1 to V_2)



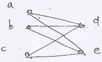
G.

Note →

1) Every complete matching in a bipartite graph is a maximum matching.

2) But the converse of the above statement need not be true. i.e. a maximum matching of a bipartite graph from V_1 to V_2 need not be a complete matching

ex. →

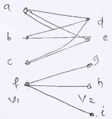


In the max. matching from V_1 to V_2 only two vertices can be matched. So, it is not complete matching.

3) In a bipartite graph, a complete matching from V_1 to V_2 exists only when no. of vertices in V_1 should be less than or equal to no. of vertices in V_2.

5) The converse of the above statement need not be true.

ex →



One vertex among a, b, c cannot be mapped. So, the complete matching from V_1 to V_2 does not exist even if $m \le n$.

a, b, c are collectively adjacent to 'd' and 'e'. So, only two of a, b, c can be mapped.

Half theorem →

Let G be a bipartite graph with vertex partition $V = \{V_1, V_2\}$.

A complete matching from V_1 to V_2 exists iff every subset of 'k' vertices in V_1 are collectively adjacent to at least 'k' vertices in V_2.

$(k = 1, 2, 3, \ldots, |V_1|)$.

Note :

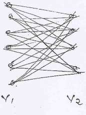
1) In a bipartite graph with vertex partition $\{V_1, V_2\}$, a complete matching from V_1 to V_2 exists if

$\delta(V_1) \geq \Delta(V_2)$.

i.e. if min. degree of all vertices in V_1 is greater than or equal to max. degree of all vertices in V_2.

Q. Which of the following bipartite graphs has a complete matching from V_1 to V_2?

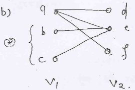
a) $K_{6,4}$



Here, no. of vertices in V_1 is greater than no. of vertices in V_2.

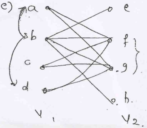
∴ Complete matching is not possible.

b)



Complete matching is not possible because in set V_1, the two vertices b and c are adjacent to same vertex e. So, only one among b and c can be mapped.

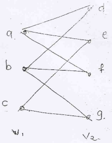
c)



In set V_1, we have 3 vertices a, c and d which are collectively adj to only f and g in V_2.

∴ Complete matching is not possible.

d)



$\delta(V_1) \geq \Delta(V_2)$

$\therefore$ complete matching is possible.

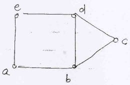
Spanning Trees →

Let G be a connected graph. A subgraph 'H' of 'G' is called a "spanning tree of G" if

i) H is a tree and H

ii) H contains all vertices of G

ex →



G.

Circuit Rank of graph G :-

Let 'G' be a connected graph with 'n' vertices and 'm' edges.

A spanning tree 'T' of 'G' contains (n-1) edges
∴ the no. of edges we have to delete from G in order to get a spanning tree is equal to (m-n+1), which is called "circuit-Rank" of the graph G.

∴ Circuit Rank of G = $|E| - |V| + 1$.

For the graph given in above ex,
circuit Rank of G = $7 - 5 + 1 = \boxed{3}$.

1. Let G be a connected graph with 6 vertices and degree of each vertex 3. Find circuit Rank of G.

→ By sum of degrees of vertices thm,

$$\sum_{i=1}^{6} \deg(v_i) = 2 \times \text{no. of edges}$$

$\therefore$ $6 \times 3 = 2 \times$ no. of edges.

$\therefore$ no. of edges $= |E| = 9$.

$\therefore$ Circuit-Rank of $G = |E| - |V| + 1$

$= 9 - 6 + 1 = \boxed{4}$

Q.2. Let G be a connected graph with 7 vertices, no cycles of odd length and max. no. of edges. Find circuit graph of G.

→ If a graph has no cycle of odd length, then it is a bipartite graph.

$K_{3,4}$ $\therefore$ no. of edges will be maximum $= 4 \times 3 = \boxed{12}$

or max. no of edges in bipartite graph with 'n' vertices

$$= \left\lfloor \frac{n^2}{4} \right\rfloor = \left\lfloor \frac{49}{4} \right\rfloor = \boxed{12}$$

$\therefore$ Circuit-Rank of $G = 12 - 7 + 1 = \boxed{6}$.

? n+2.

n12 &·n·

Q.3. Circuit rank of a complete graph K_n ?

→ $= |E| - |V| + 1$

$= \frac{n \cdot (n-1)}{2} - n + 1$

$= \boxed{\frac{(n-1)\cdot(n-2)}{2}}$

Q.4. Circuit Rank of a wheel graph W_n →

→ $= |E| - |V| + 1$

$= 2(n-1) - (n-1)$

$= \boxed{(n-1)}$

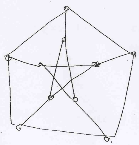
Q.5. Circuit rank of cycle graph C_n ?

$= \underline{1}$. (if we delete one edge, we get a tree).

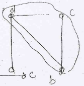
Q.6. Circuit Rank of Graph shown below.

Circuit Rank $= 15 - 10 + 1$

$= \boxed{6}$.

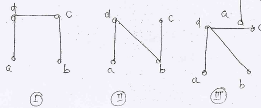


7. Number of spanning trees in the graph shown below



cycle of length 3.

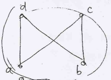
→ No. of spanning trees = 3.



I and III are isomorphic.

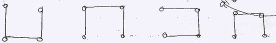
∴ for the graph, no. of nonisomorphic spanning trees = $\boxed{2}$.

3.8. No. of spanning trees in the graph shown below,



cycle graph of length 4

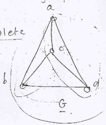
→ No. of spanning trees = $\boxed{4}$



But all are isomorphic.

∴ No. of nonisomorphic spanning trees = $\boxed{1}$.

Q.9. Number of spanning trees in the graph G is ?



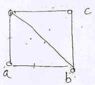
→ * No. of spanning trees in complete graph $K_n = \boxed{(n)^{n-2}}$

(Cayley's formula).

∴ No. of spanning trees in K_4.

$= 4^{(4-2)} = 4^2 = \boxed{16}$

Q.10. No of spanning trees in the graph shown below is



→ No. of spanning trees $= \boxed{8}$

For a complete graph K_n.

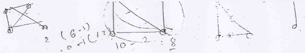
(**). To get the no of spanning trees.

Kirchoff's theorem →

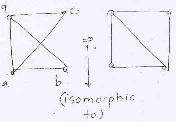
* Let 'A' be the adjacency matrix of the connected graph 'G'.

* Let 'M' be the matrix obtained from 'A' by replacing each '1' with '-1' and replacing each '0' in the principle diagonal of A with the degree of the corresponding vertex.

* The cofactor of any element of M = no. of spanning trees in G.



11. No. of spanning trees in the graph shown below is ?



$A =$

	a	b	c	d
a	0	1	1	1
b	1	0	0	1
c	1	0	0	1
d	1	1	1	0.

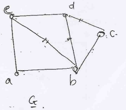
$$M = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$$

cofactor of 3 = $2\begin{bmatrix} 6-1 \end{bmatrix} - 0 + (-1)(+2)$

$= 10 - 2 = \boxed{8}$

1.12 No. of spanning trees in the graph shown below is ?

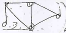
$\rightarrow$



$A =$

	a	b	c	d	e
a	0	1	0	0	1
b	~~0~~1	0	1	1	1
c	0	1	0	1	0
d	0	~~0~~1	1	0	1
e	1	1	0	1	0

6 + 6 = 10
3 × 4 = 12 − 4 =



5 + 9 + 4
18 (16)

$$M = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & \cancel{0}-1 & -1 & 3 & -1 \\ -1 & -1 & 0 & -1 & 3 \end{bmatrix}$$

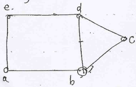
$$2 \left| 4\{10 \cancel{00} - 3\} + 1\{ \quad \} \right. \times 1$$

cofactor of M_{51} = cofactor (−1)

$$= (-1)^{5+1} \begin{vmatrix} -1 & 0 & 0 & -1 \\ 4 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 3 & -1 \end{vmatrix}$$

$= \boxed{21}$

9.13. No. of spanning trees in the graph shown below, is ?



$^{n}C_3 =$ ...
4 +
5 + 2 × 4
13.

Ans = 11