CHAPTER 18

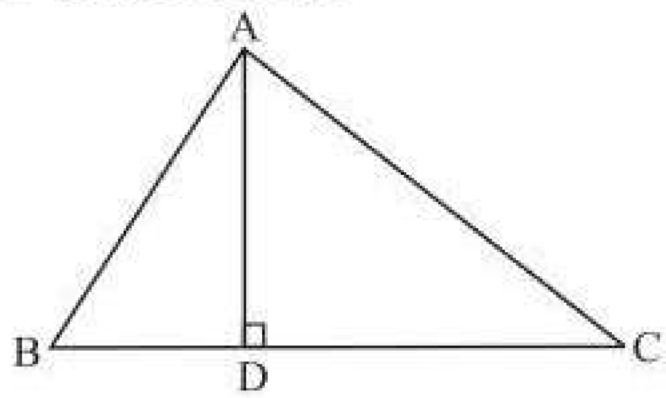
AREA AND PERIMETER

DEFINITION

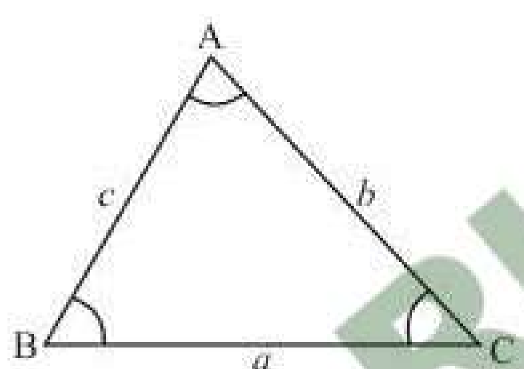
Perimeter: The perimeter of a plane geometrical figure is the total length of sides (or boundary) enclosing the figure. Units of measuring perimeter can be cm, m, km, etc.

Area: The area of any figure is the amount of surface enclosed within its bounding lines. Area is always expressed in square units.

AREA OF A TRIANGLE



- Area = $\frac{1}{2}$ × base × corresponding altitude



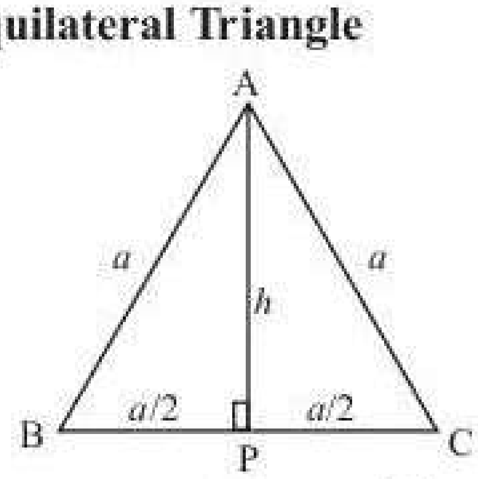
Semi-perimeter of ΔABC's $s = \frac{a+b+c}{2}$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ (Heron's formula)

- Area of $\Delta ABC = \frac{1}{2}$ × (Product of two sides) × (Sine of the included angle)

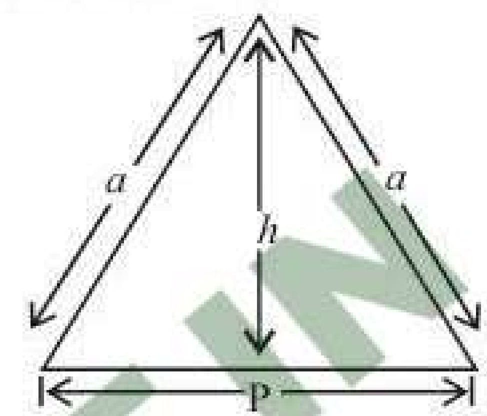
$$= \frac{1}{2} ac \sin B \text{ or } \frac{1}{2} ab \sin C \text{ or } \frac{1}{2} bc \sin A$$

- **Area of an Equilateral Triangle**



Area of an equilateral $\Delta = \frac{1}{2} \times a \times \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{4} a^2$, where a is the length of its one side

- **Area of Isoceles Triangle**



Where a is side of isoceles triangle b is the base of the isoceles triangle h is the height of the isoceles triangle. Area of an isoceles triangle = 2a + b

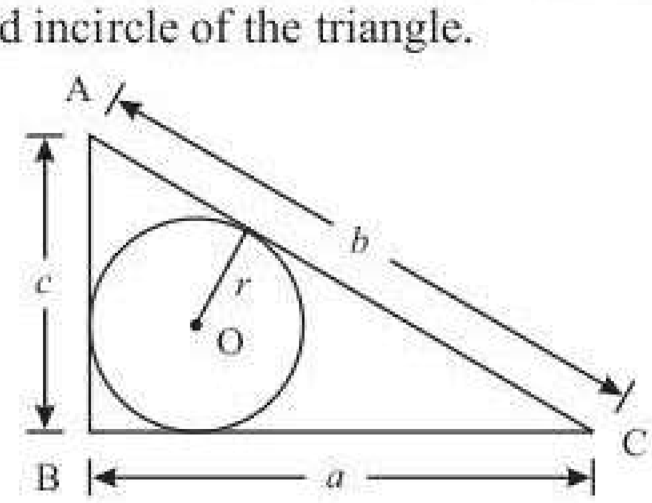
Attitude of an isoceles triangle $= \sqrt{a^2 - \frac{b^2}{4}}$

Remember

- ✧ among all the triangles that can be formed with a given perimeter, the equilateral triangle will have the maximum area.
- ✧ For a given area of triangle, the perimeter of equilateral triangle is minimum.

- **Area of Incircle and Circumcircle of a Triangle**

(i) If a circle touches all the three sides of a triangle, then it is called incircle of the triangle.



Area of incircle of a triangle = $r.s$, where r is the radius of the incircle and s is the half of the perimeter of the triangle.

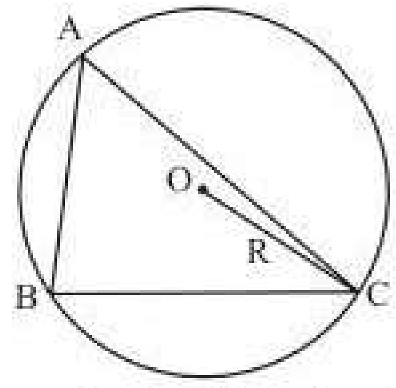
If a, b, c are the length of the sides of ΔABC, then $s = \frac{a+b+c}{2}$

For an equilateral triangle,

$$r = \frac{\text{Length of a side of the triangle}}{2\sqrt{3}} = \frac{h}{3},$$

where h is the height of the triangle.

(ii) If a circle passess through the vertices of a triangle, then the circle is called circumcircle of the triangle.



Area of Circumcircle of a triangle $= \dfrac{abc}{4R}$, where R is the radius of the circumcircle and a, b, c are the length of sides of the triangle.

For an equilateral triangle,

$$R = \frac{\text{Length of a side of the triangle}}{\sqrt{3}} = \frac{2h}{3},$$

where h is the height or altitude of the equilateral triangle. Hence for an equilateral triangle, $R = 2r$.

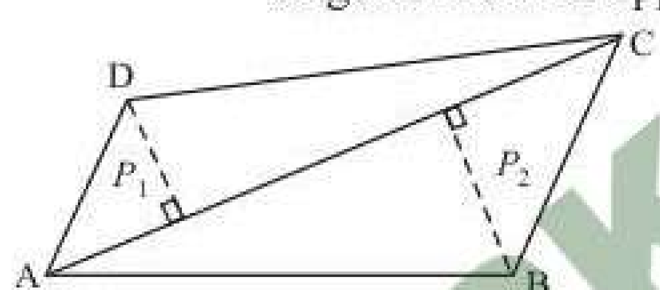
Remember

✧ Note that an equilateral triangle inscribed in a circle will have the maximum area compared to other triangles inscribed in the same circle.

AREA OF A QUADRILATERAL

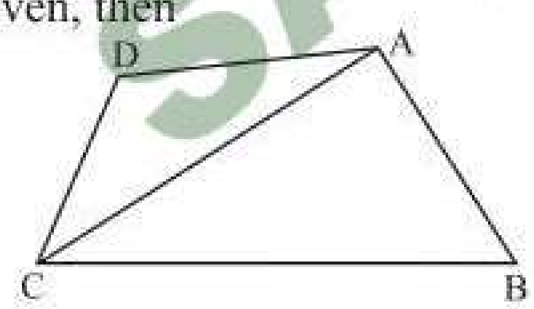
- **Area of quadrilateral *ABCD***

$= \dfrac{1}{2} \times$ (Length of the longest diagonal) $\times$ (Sum of length of perpendicular to the longest diagonal from its opposite vertices)



$= \dfrac{1}{2} \times d \times (p_1 + p_2)$, where $d = AC$ (i.e. longest diagonal)

- If length of four sides and one of its diagonals of quadrilateral *ABCD* are given, then



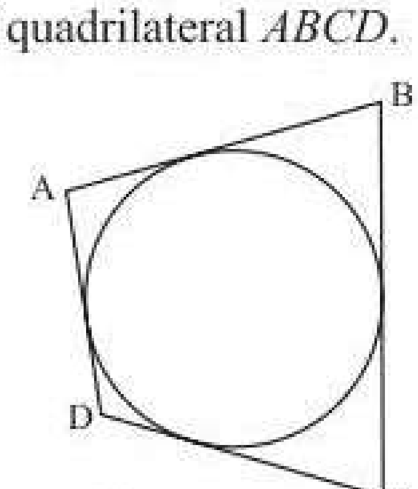
Area of the quadrilateral *ABCD*

= Area of ΔABC + Area of ΔADC

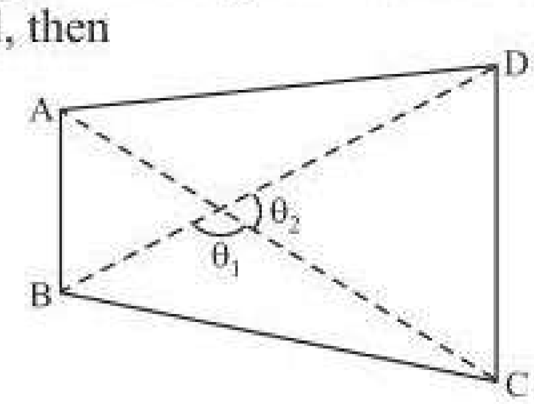
- **Area of circumscribed quadrilateral**

$$= \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where $s = \dfrac{a+b+c+d}{2}$ and a, b, c, d are length of sides of quadrilateral *ABCD*.



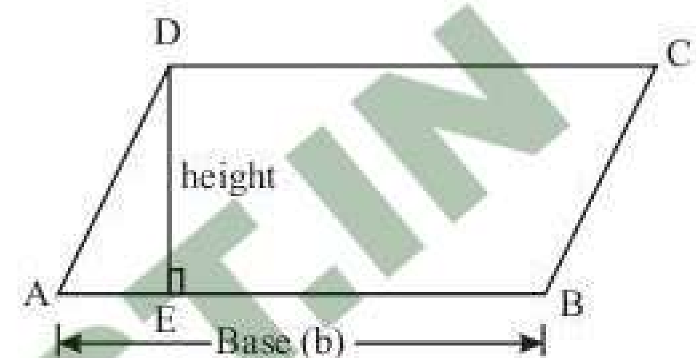
- If θ_1 and θ_2 are the angles between the diagonals of a quadrilateral, then



Area of the quadrilateral $= \dfrac{1}{2}\, d_1 d_2 \sin\theta_1$ or $\dfrac{1}{2}\, d_1 d_2 \sin\theta_2$

Here d_1 and d_2 are the length of the diagonals of the quadrilateral.

AREA OF A PARALLELOGRAM



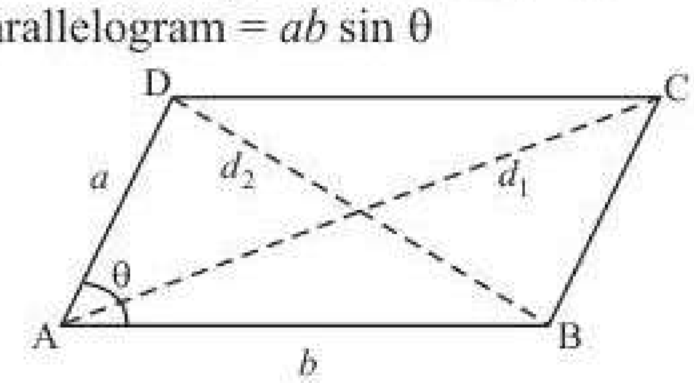
Area of parallelogram = Base × Corresponding height

$A = b \times h$

Perimeter of a parallelogram $= 2(a + b)$, where a and b are length of adjacent sides.

If θ be the angle between any two adjacent sides of a parallelogram whose length are a and b, then

Area of parallelogram $= ab \sin\theta$

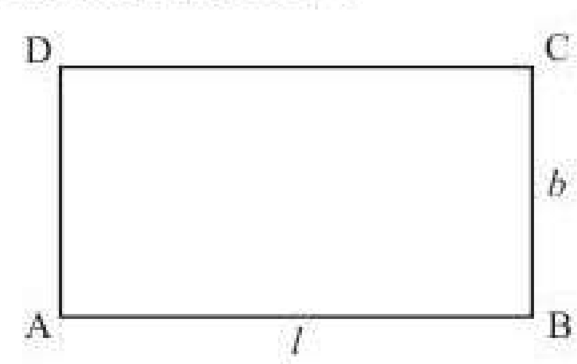


Remember

✧ In a parallelogram sum of squares of two diagonals = 2 (sum of squares of two adjacent sides)

i.e., $d_1^2 + d_2^2 = 2(a^2 + b^2)$

AREA OF A RECTANGLE

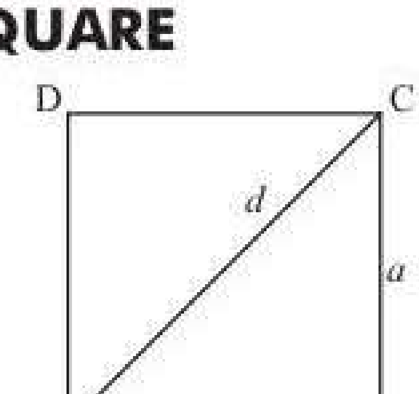


Area of a rectangle = Length × Breadth $= l \times b$

[If any one side and diagonal is given]

Perimeter of a rectangle $= 2(l + b)$

AREA OF A SQUARE



- **Area of square** = side × side = $a \times a = a^2$
- **Length of diagonal** $(d) = a\sqrt{2}$ (by Pythagoras theoram)

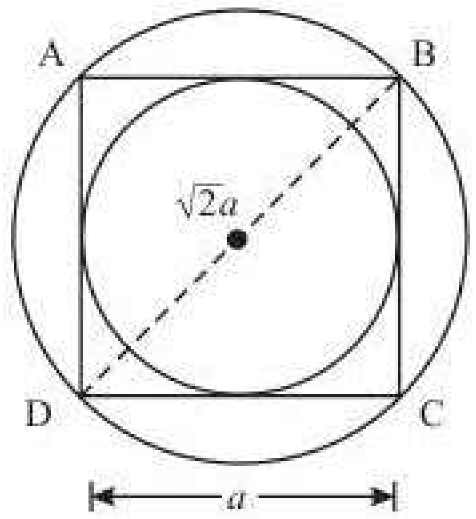
Hence, **area of the square** = $\left(\frac{d}{\sqrt{2}}\right)^2 = \frac{d^2}{2}$

Perimeter of square = 4 × side = 4 × a

For a given perimeter of a rectangle, a square has maximum area.

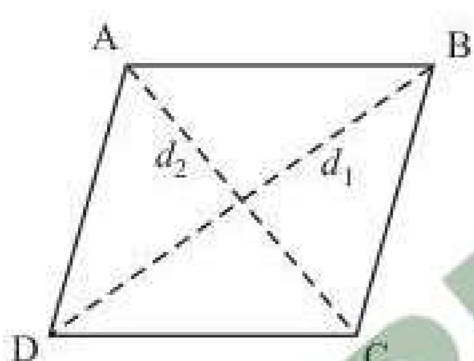
Remember

✧ The side of a square is the diameter of the inscribed circle and diagonal of the square is the diameter of the circumscribing circle.



Hence inradius = $\frac{a}{2}$ and circumradius = $\frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

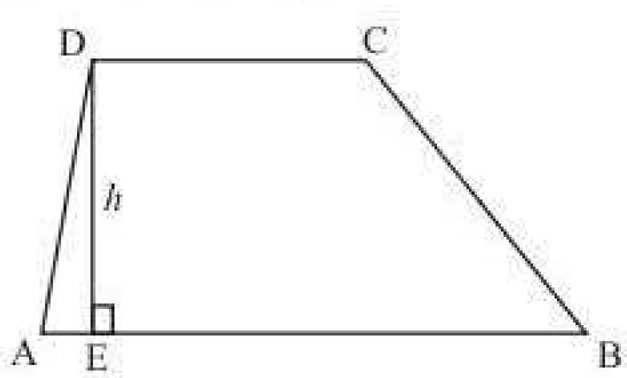
AREA OF A RHOMBUS



Area of a rhombus = $\frac{1}{2}$ × product of diagonals

$= \frac{1}{2} \times d_1 \times d_2$

AREA OF A TRAPEZIUM



- Distance between parallel sides of a trapezium is called height of trapezium.
- In fig. *ABCD* is a trapezium, whose sides *AB* and *CD* are parallel,

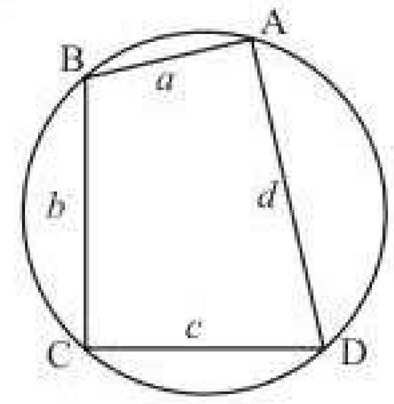
$DE = h$ = Height of the trapezium
= Distance between ∥ sides.

- Area of trapezium = $\frac{1}{2}$ (sum of ∥ sides) × height

$= \frac{1}{2} \times (AB + CD) \times DE$

AREA OF A CYCLIC QUADRILATERAL

- For a given quadrilateral *ABCD* inscribed in a circle with sides measuring a, b, c, and d;



$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where $s = \frac{a+b+c+d}{2}$

Example 1: A rectangular parking space is marked out by painting three of its sides. If the length of the unpainted side is 9 feet, and the sum of the lengths of the painted sides is 37 feet, then what is the area of the parking space in square feet?

Solution: Clearly, we have : $l = 9$ and $l + 2b = 37$ or $b = 14$.

∴ Area = $(l \times b) = (9 \times 14)$ sq. ft. = 126 sq. ft.

Example 2: A square carpet with an area 169 m² must have 2 metres cut-off one of its edges in order to be a perfect fit for a rectangular room. What is the area of rectangular room?

Solution: Side of square carpet $\sqrt{\text{Area}} = \sqrt{169} = 13$ m

After cutting of one side,

Measure of one side = 13 – 2 = 11 m

and other side = 13 m (remain same)

∴ Area of rectangular room = 13 × 11 = 143 m²

Example 3: The ratio between the length and the breadth of a rectangular park is 3 : 2. If a man cycling along the boundary of the park at the speed of 12 km/hr completes one round in 8 minutes, then the area of the park (in sq. m) is:

Solution: Perimeter = Distance covered in 8 min.

$$= \left(\frac{12000}{60} \times 8\right) \text{m} = 1600 \text{ m.}$$

Let length = $3x$ metres and breadth = $2x$ metres.

Then, $2(3x + 2x) = 1600$ or $x = 160$.

∴ Length = 480 m and Breadth = 320 m.

∴ Area = (480×320) m² = 153600 m².

Example 4: The length and breadth of a playground are 36m and 21 m respectively. Poles are required to be fixed all along the boundary at a distance 3m apart. The number of poles required will be

Solution: Given, playground is rectangular.

Length = 36 m, Breadth = 21 m

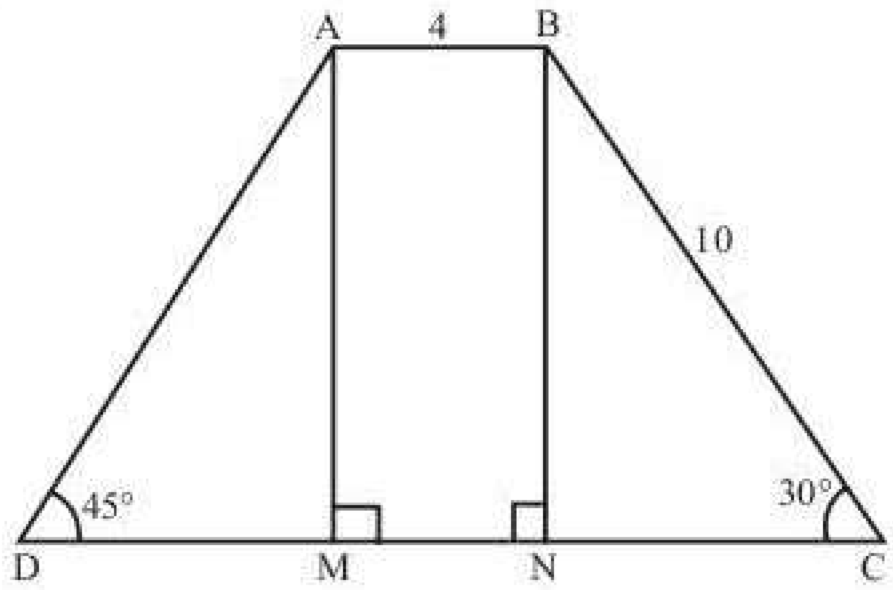
Now, perimeter of playground = 2(21 + 36) = 114

Now, poles are fixed along the boundary at a distance 3 m.

∴ Required no. of poles = $\frac{114}{3} = 38$.

Example 5: **Find the area of the trapezium *ABCD*.**

Solution:



AB and *DC* are the parallel sides

Height = *AM* = *BN*

AB = *MN* = 4

ΔBNC and ΔAMD are right angled triangles

In $\Delta BNC \Rightarrow \sin 30 = \dfrac{BN}{10} \Rightarrow BN = 5$

Using Pythagoras theorem $NC = \sqrt{10^2 - 5^2} = 5\sqrt{3}$

In ΔADM; $AM = 5$; $\tan 45 = \dfrac{AM}{DM} \Rightarrow 1 = \dfrac{5}{DM}$

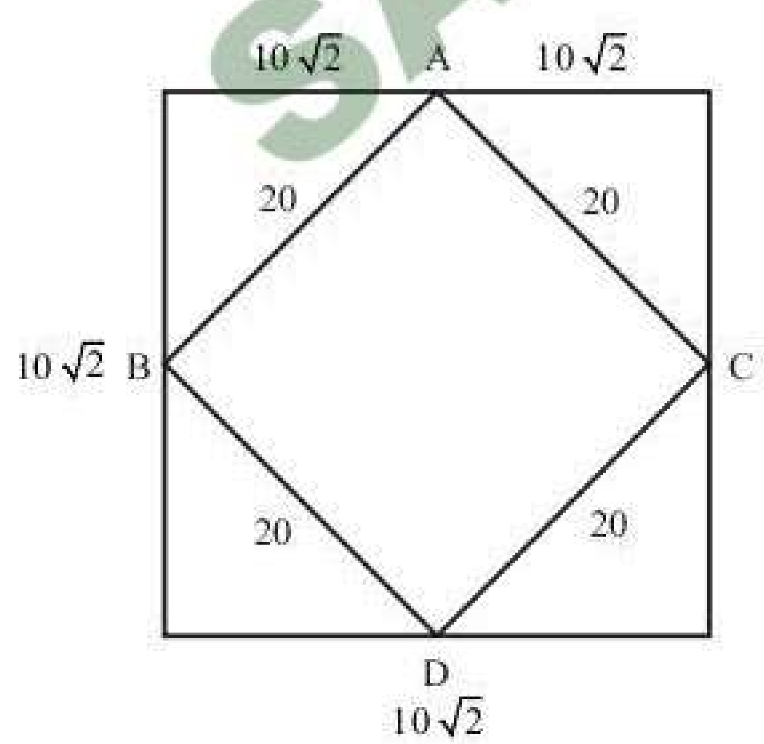
$\Rightarrow \quad DM = 5$

Area of trapezium $\Rightarrow \dfrac{1}{2}$ (Sum of parallel sides) × height

$$\Rightarrow \quad \frac{1}{2}(4 + 4 + 5\sqrt{3} + 5) \times 5 = \frac{5(13 + 5\sqrt{3})}{2}$$

Example 6: **Two goats tethered to diagonally opposite vertices of a field formed by joining the mid-points of the adjacent sides of another square field of side $20\sqrt{2}$. What is the total grazing area of the two goats?**

Solution:



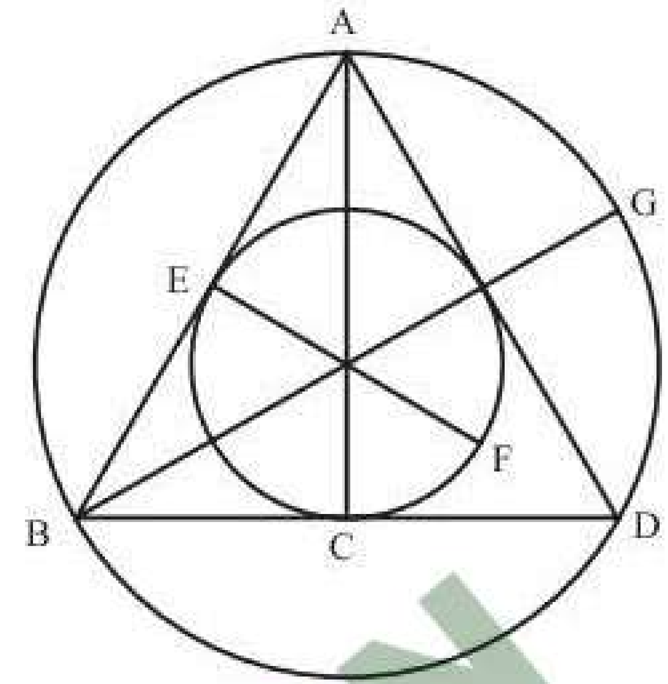
The length of rope of goat = $10\sqrt{2}$ m

Then the two goats will graze an area = Area of a semicircle with radius $10\sqrt{2}$ m.

So total area grazed = $\dfrac{\pi r^2}{2} \Rightarrow 100\ \pi\text{m}^2$

Example 7: **Find the ratio of the diameter of the circles inscribed and circumscribing an equilateral triangle to its height**

Solution:



Let arc side of equilateral triangle = a

Then height = $\dfrac{a\sqrt{3}}{2}$

Area = $\dfrac{\sqrt{3}}{4}a^2$; $S = \dfrac{a + a + a}{2} = \dfrac{3a}{2}$

Diameter of inner circle = $\dfrac{2 \times \text{Area}}{\text{Perimeter of triangle}}$

$$= \frac{\sqrt{3}}{4}a^2 \times \frac{2 \times 2}{3a} = \frac{a}{\sqrt{3}}$$

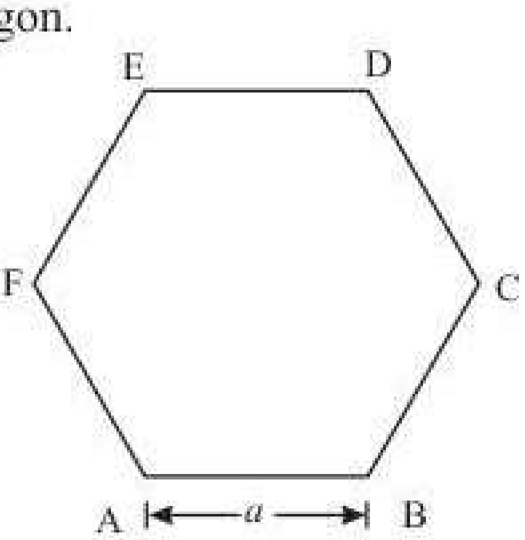
Diameter of outer circle = $\dfrac{a^3}{2 \times \text{Area}} = \dfrac{a^3}{2} \times \dfrac{4}{\sqrt{3}a^2}$

$$\Rightarrow \frac{2a}{\sqrt{3}}$$

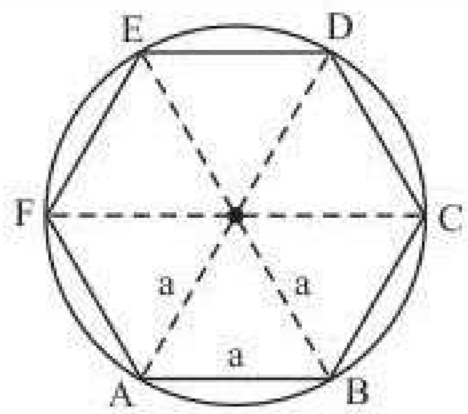
Ratio = $\dfrac{a}{\sqrt{3}} : \dfrac{2a}{\sqrt{3}} : \dfrac{a\sqrt{3}}{2} \Rightarrow$ Ratio = 2 : 4 : 3

AREA OF A REGULAR HEXAGON

- Area = $\dfrac{3\sqrt{3}}{2}a^2$, where 'a' is the length of each side of the regular hexagon.

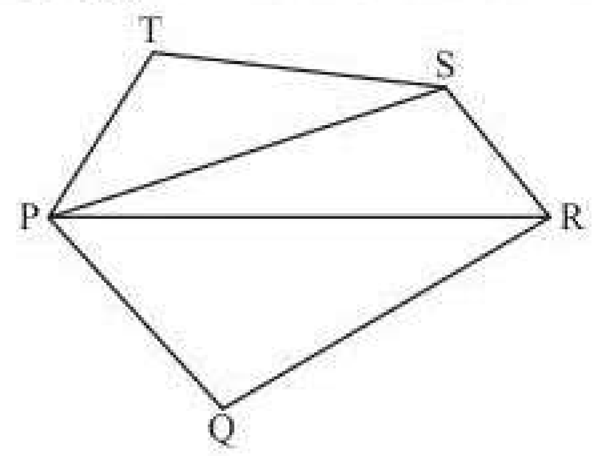


- Diagonals of a hexagon divide it into six equilateral triangle. Hence, **radius of the circumcircle of the hexagon** = Length of a side of the hexagon = a



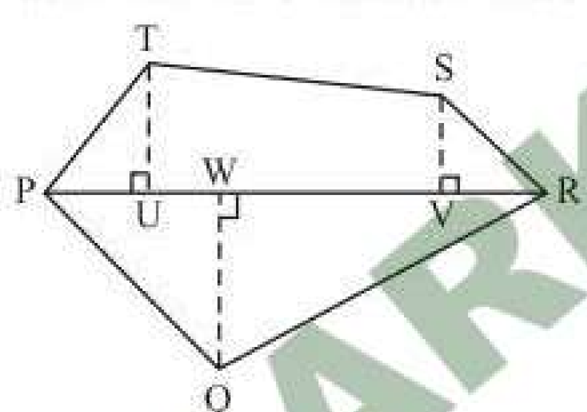
AREA OF IRREGULAR PLANE FIGURES

- By drawing all the diagonals from anyone vertex, The polygon divided into several triangles. Hence area of the polygon = Sum of area of all the triangles



For example,
Area of pentagon $PQRST$
= Area of $\Delta\ PQR$ + Area of $\Delta\ PRS$ + Area of $\Delta\ PST$.

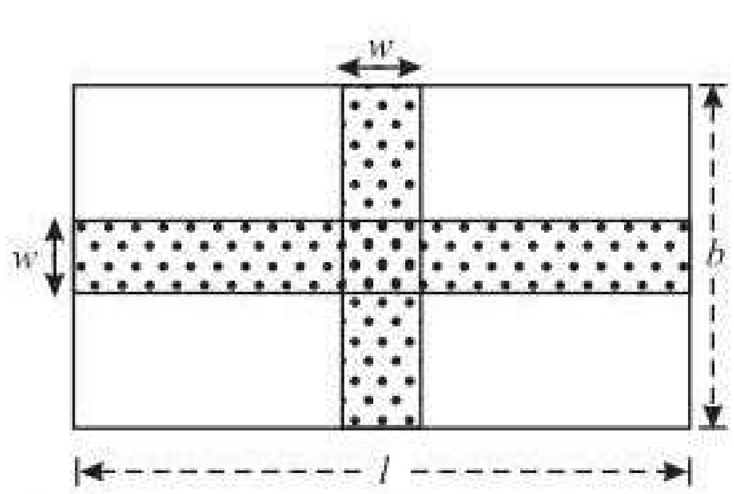
- By drawing longest diagonal and perpendicular from all vertices on two sides of the longest diagonal to the longest diagonals, the polygon is divided into several right triangles and trapeziums. By finding the sum of all the triangles and trapeziums, so formed we get the area of the polygon.



For example,
Area of pentagon $PQRST$ = Area of $\Delta\ PTU$ + Area of trapezium $(TUVS)$ + Area of $\Delta\ SVR$ + Area of $\Delta\ RQW$ + Area of $\Delta\ QWP$.

PATHS

- **Pathways Running Across the Middle of a Rectangle**

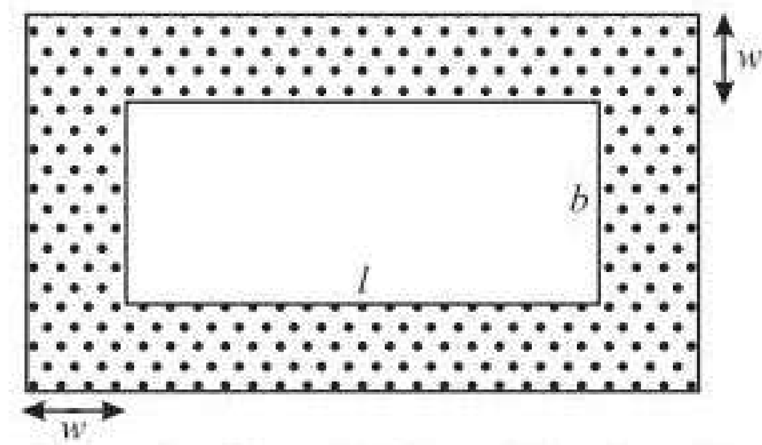


Area of the path $= l.w + b.w - w.w$
$= (l + b - w).w$

Perimeter of the path
$= 2l + 2b - 4w$
$= 2(l + b - 2w)$

Here w is the width of the path.

- **Pathways Outside a Rectangle**

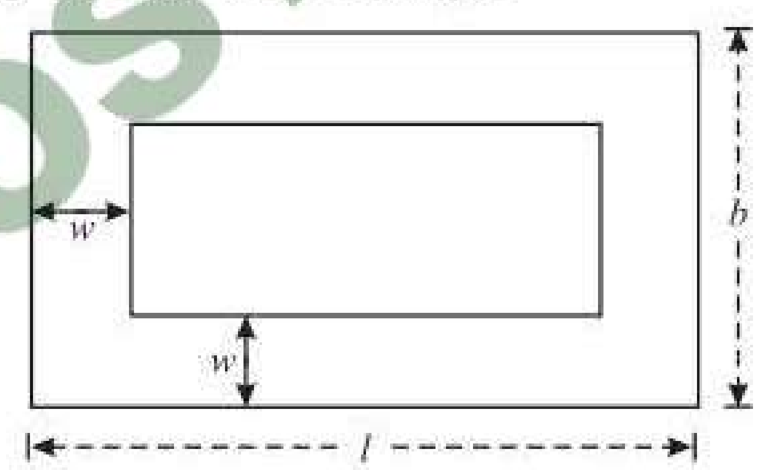


Area of path $= 2(lw) + 2(b.w) + 4(w.w)$
$= (l + b + 2w)2w$

Perimeter of path
= (Internal perimeter) + (External perimeter)
$= 2(l + b) + 2(l + b + 4w)$
$= 4(l + b + 2w)$

Here w is the width of the path.

- **Pathway Inside a Rectangle**



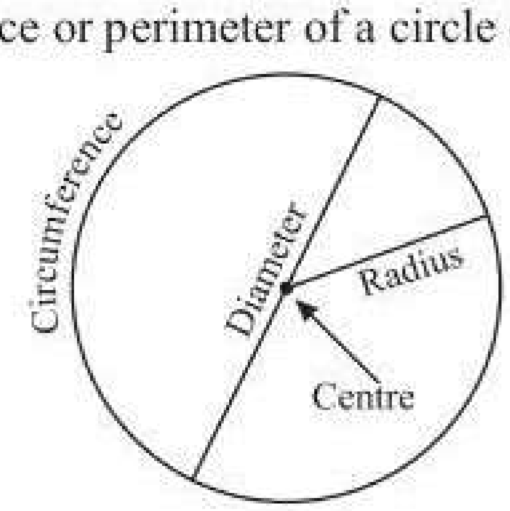
Area of path $= 2(l.w) + 2(b.w) - 4(w.w)$
$= (l + b - 2w).2w$

Perimeter of path
= Length of outer path + Length of inner path
$= 2(l + b) + 2(l + b - 4w)$
$= 4(l + b - 2w)$

AREA RELATED TO A CIRCLE

CIRCLE

- Circumference or perimeter of a circle of radius r is

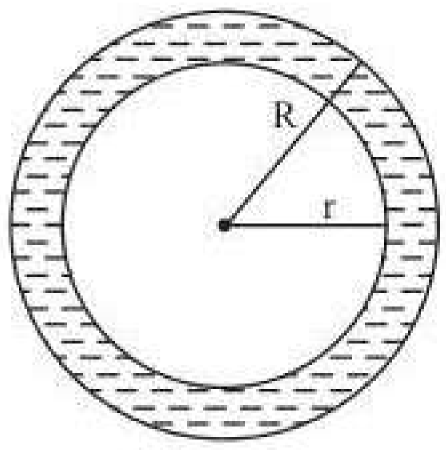


$c = 2\pi r = \pi d \quad (2r = d = \text{diameter})$

Area of the circle $= \pi r^2 = \dfrac{\pi d^2}{4} = \dfrac{c^2}{4\pi} = \dfrac{1}{2} \times c \times r$

Circular Ring

- Region enclosed between two concentric circles of different radii in a plane is called a ring.



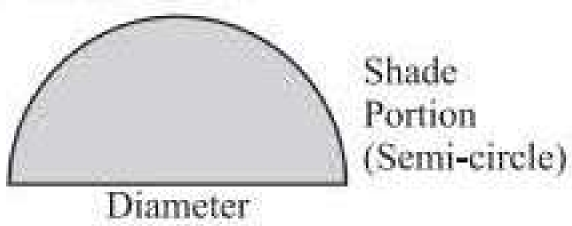
- Area of the ring $= \pi R^2 - \pi r^2 = \pi\ (R^2 - r^2)$
- Circumference of the ring

$= $ (External circumference) + (Internal circumference)

$= 2\pi R + 2\pi r = 2\pi(R + r)$

Semi-circle

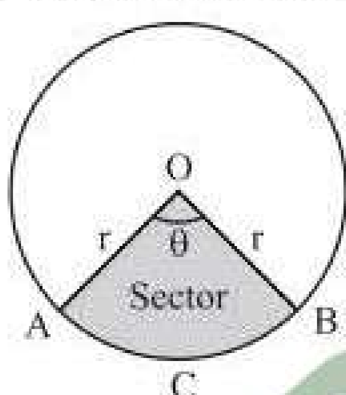
A semi-circle is a figure enclosed by a diameter and one half of the circumference of the circle.



- **Area of the semi-circle** $= \dfrac{\pi r^2}{2}$
- **Circumference of the semi-circle** $= \pi r + 2r = r(\pi + 2)$

Sector of a Circle

Sector of a circle is the portion of a circle enclosed by two radii and an arc of the circle. *OACB* is a sector of the circle.



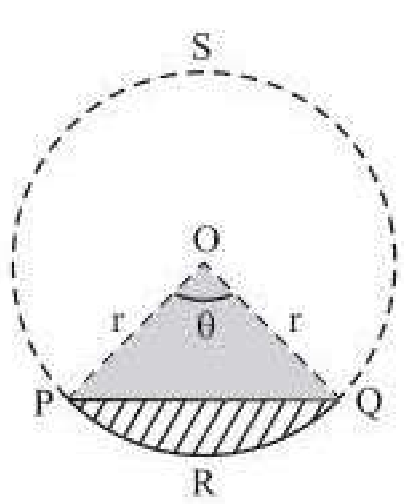
- Length of arc *ACB* (which make angle θ at the centre)

$$=(2\pi r) \times \frac{\theta}{360} = \frac{\pi r\theta}{180}$$

- Perimeter of the sector $OACB = 2r + \dfrac{\pi r\theta}{180}$
- Area of sector $OACB = (\pi r^2) \times \dfrac{\theta}{360}$

Segment of a Circle

A segment of a circle is a region enclosed by a chord and an arc of the circle.



Any chord of a circle which is not a diameter divides the circle into two segments, one of which is the major segment and other is minor segment.

- Perimeter of the segment *PRQP*

$$= \text{Length of the arc } PRQ + \text{Length of } PQ$$

$$= \frac{\pi r\theta}{180} + 2r\sin\frac{\theta}{2}$$

- Area of (minor) segment *PQR*

$$= \text{Area of sector } OPRQO - \text{Area of } \Delta OPQ$$

- Area of (major) segment *PSQ*

$$= \text{Area of circle} - \text{Area of segment } PQR$$

Example 8: A circular grass lawn of 35 metres in radius has a path 7 metres wide running around it on the outside. Find the area of path.

Solution: Radius of a circular grass lawn (without path) = 35 m

$\therefore$ Area $= \pi r^2 = \pi\,(35)^2$

Radius of a circular grass lawn (with path)

$= 35 + 7 = 42$ m

$\therefore$ Area $= \pi r^2 = \pi\,(42)^2$

$\therefore$ Area of path $= \pi(42)^2 - \pi(35)^2$

$= \pi(42^2 - 35^2)$

$= \pi(42 + 35)\ (42 - 35)$

$= \pi \times 77 \times 7 = \dfrac{22}{7} \times 77 \times 7 = 1694\ \text{m}^2$

Example 9: A wire can be bent in the form of a circle of radius 56 cm. If it is bent in the form of a square, then its area will be:

Solution: Length of wire $= 2\pi \times R = \left(2 \times \dfrac{22}{7} \times 56\right)$ cm

$= 352$ cm.

Side of the square $= \dfrac{352}{4}$ cm $= 88$ cm.

Area of the square $= (88 \times 88)\ \text{cm}^2 = 7744\ \text{cm}^2$.

Example 10: There are two concentric circular tracks of radii 100 m and 102 m, respectively. A runs on the inner track and goes once round on the inner track in 1 min 30 sec, while B runs on the outer track in 1 min 32 sec. Who runs faster?

Solution: Radius of the inner track = 100 m

and time = 1 min 30 sec ≡ 90 sec.

Also, Radius of the outer track = 102 m

and time = 1 min 32 sec ≡ 92 sec.

Now, speed of *A* who runs on the inner track

$$= \frac{2\pi\,(100)}{90} = \frac{20\pi}{9} = 6.98$$

And speed of *B* who runs on the outer track

$$= \frac{2\pi\,(102)}{90} = \frac{34\pi}{15} = 6.96$$

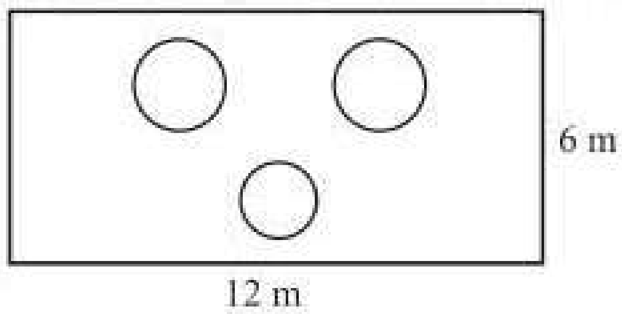
Since, speed of *A* > speed of *B*

$\therefore$ A runs faster than *B*.

Example 11: **A rectangular plate is of 6 m breadth and 12 m length. Two apertures of 2 m diameter each and one apertures of 1 m diameter have been made with the help of a gas cutter. What is the area of the remaining portion of the plate?**

Solution: Given, Length = 12 m and Breadth = 6 m

$\therefore$ Area of rectangular plate $= 12 \times 6 = 72\ m^2$



Since, two apertures of 2 m diameter each have been made from this plate.

$\therefore$ Area of these two apertures $= \pi(1)^2 + \pi(1)^2 = \pi + \pi = 2\pi$

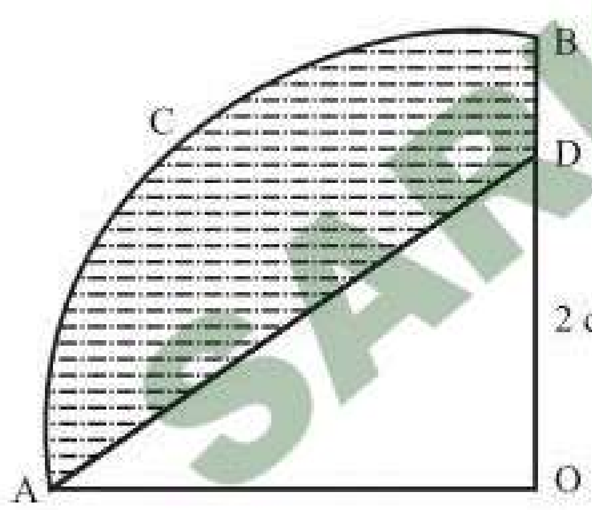
Area of 1 aperture of 1m diameter $= \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}$

$\therefore$ Total area of aperture $= 2\pi + \frac{\pi}{4} = \frac{9\pi}{4} = \frac{9}{4} \times \frac{22}{7} = \frac{99}{14}$

$\therefore$ Area of the remaining portion of the plate

$= 72 - \frac{99}{14}$ sq. m $= \frac{909}{14}$ sq. m ≈ 64.5 sq.m

Example 12: **In the adjoining figure, *AOBCA* represents a quadrant of a circle of radius 3.5 cm with centre *O*. Calculate the area of the shaded portion.**



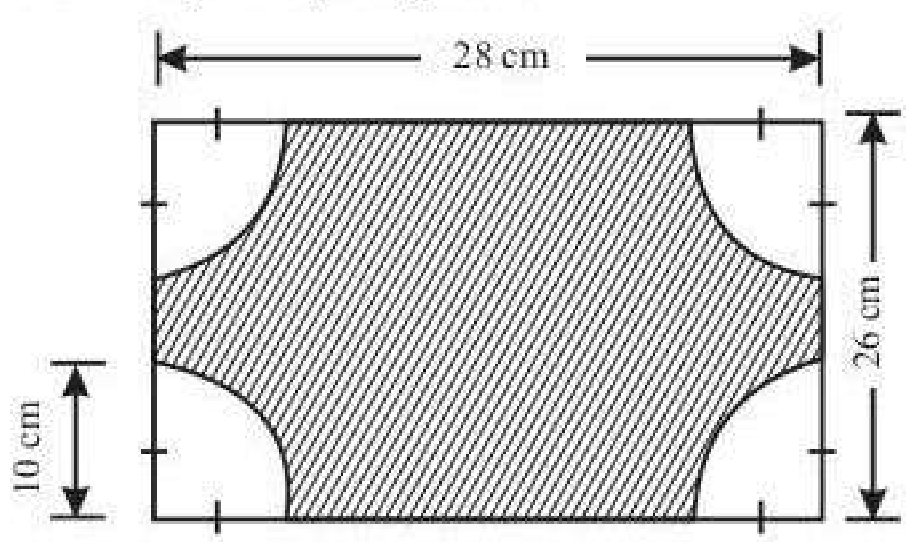
Solution:

Area of shaded portion = Area of quadrant – Area of triangle

$\Rightarrow \frac{\pi r^2}{4} - \frac{1}{2} \times 3.5 \times 2 = \frac{3.14 \times (3.5)^2}{4} - 3.5$

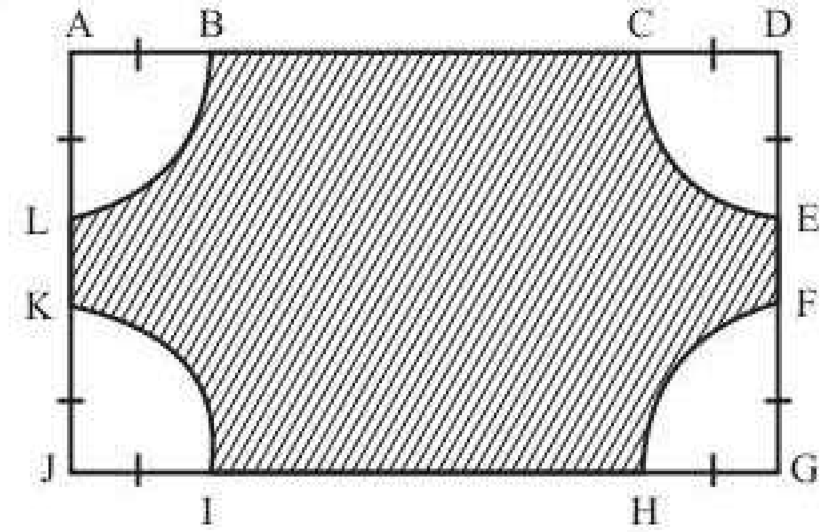
$\Rightarrow 6.125\ cm^2$

Example 13: **Find the perimeter and area of the shaded portion of the adjoining diagram:**



(a) 90.8 cm, 414 cm² **(b) 181.6 cm, 423.7 cm²**
(c) 90.8 cm, 827.4 cm² **(d) 181.6 cm, 827.4 cm²**

Solution:



KJ = radius of semicircles = 10 cm

4 quadrants of equal radius = 1 circle of that radius

Area of shaded portion $\Rightarrow$ Area of rectangle – Area of circle

$(28 \times 26) - (3.14 \times 100) \Rightarrow 414\ cm^2$

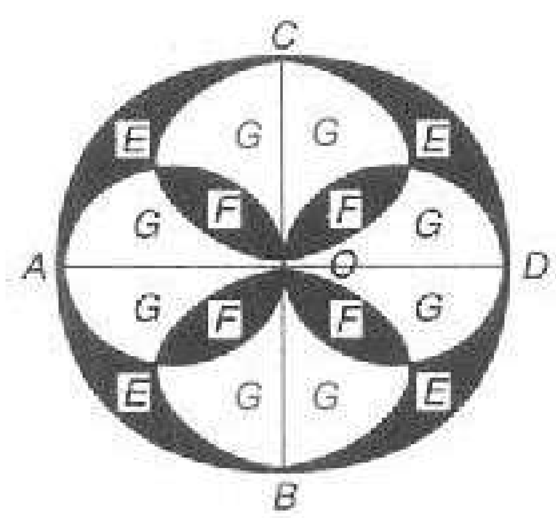
$BC = 28 - (10 + 10) = 8$ and $EF = 26 - (10 + 10) = 6$

Perimeter of shaded portion = 28 cm + $2\pi r$

Answer $\Rightarrow 414\ cm^2$ = Area and

Perimeter = 90.8

Example 14: ***ABDC* is a circle and circles are drawn with *AO*, *CO*, *DO* and *OB* as diameters. Areas *E* and *F* are shaded *E*/*F* is equal to**



Solution:

$AO = CO = DO = OB$ = radius of bigger circle = r (let)

Then area of $(G + F) = \frac{\pi r^2}{8}$

Area of $8(G + F) = \pi r^2$. Also area of $2G + F + E = \pi r^2$

i.e. $2G + F + F = 2G + F + E \Rightarrow F = E$

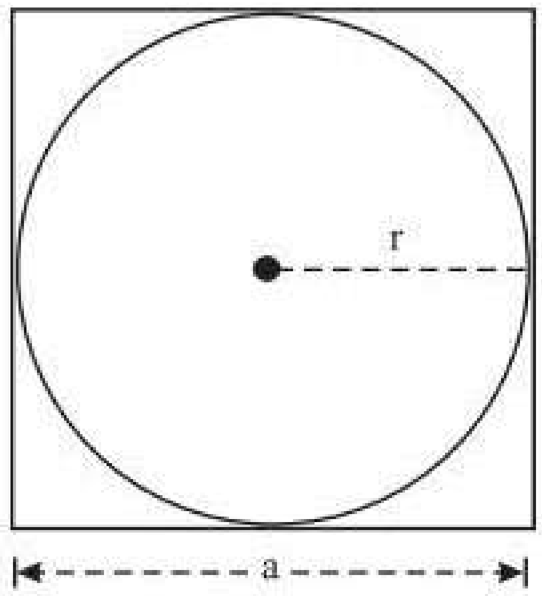
So the ratio of areas E and F = 1 : 1

CIRCLE PACKING IN A SQUARE

Let 'a' be the length of a side of the square and 'r' be the radius of the circle.

Case- (i): One circle

$$2r = a \Rightarrow r = \frac{a}{2}$$



Case- (ii): Two circles

In the Isosceles right angled $\Delta\, BCD$,

$$BD = \sqrt{2}r$$

In the Isosceles right angled Δ DFG,

$$DF = \sqrt{2}a$$

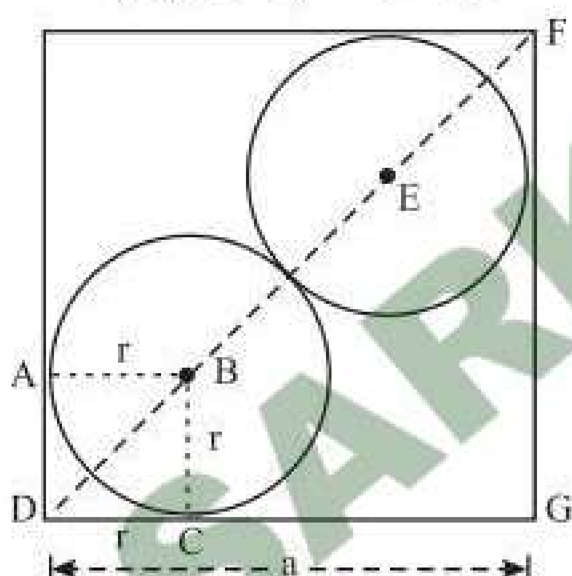
Now $\quad DF = DB + BE + EF$

$$= \sqrt{2}r + 2r + \sqrt{2}r$$

$$= 2r + 2\sqrt{2}r = 2(\sqrt{2}+1)r$$

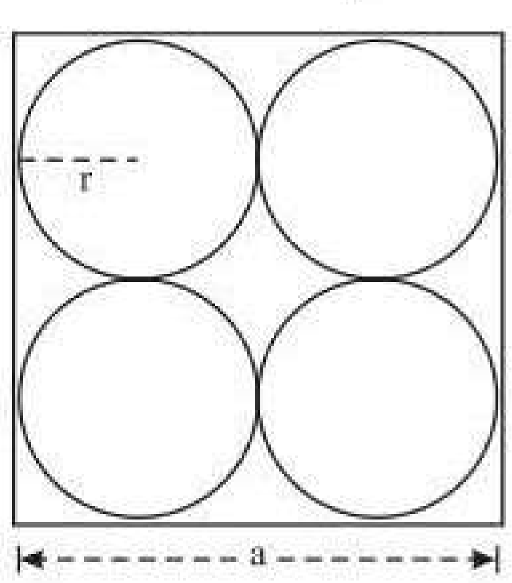
$$\therefore\ 2(\sqrt{2}+1)r = \sqrt{2}a$$

$$\Rightarrow \quad r = \frac{a}{\sqrt{2}(\sqrt{2}+1)} = \frac{a}{2+\sqrt{2}}$$



Case- (iii): Four circles

$$4r = a \Rightarrow r = \frac{a}{4}$$



Case- (iv): Five circles

In Isosceles right angled triangle ABC,

$$AC = \sqrt{2}r$$

$$AE = AC + CE = \sqrt{2}r + 2r = (2+\sqrt{2})r$$

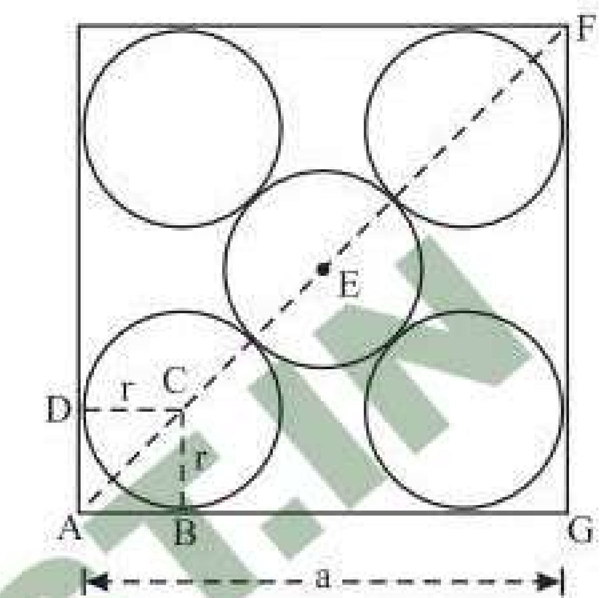
$$AF = 2AE = 2(2+\sqrt{2})r$$

In Isosceles right angle triangle AGF,

$$AF = \sqrt{2}a$$

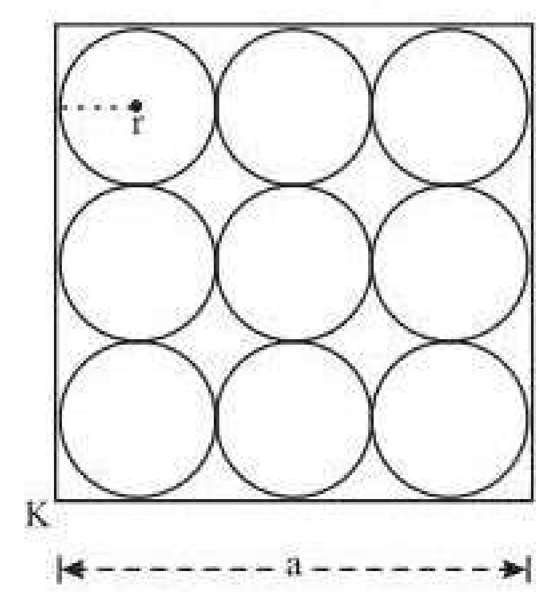
$$\therefore\quad 2(2+\sqrt{2})r = \sqrt{2}a$$

$$\Rightarrow \quad r = \frac{a}{2(\sqrt{2}+1)}$$



Case-(vi): Nine circles

$$6r = a \Rightarrow r = \frac{a}{6}$$

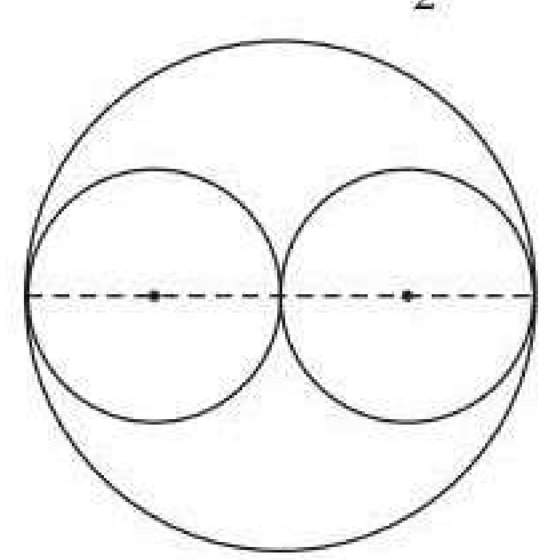


CIRCLES PACKING IN A CIRCLE

Let R be the radius of larger circle and r be the radius of smaller circle.

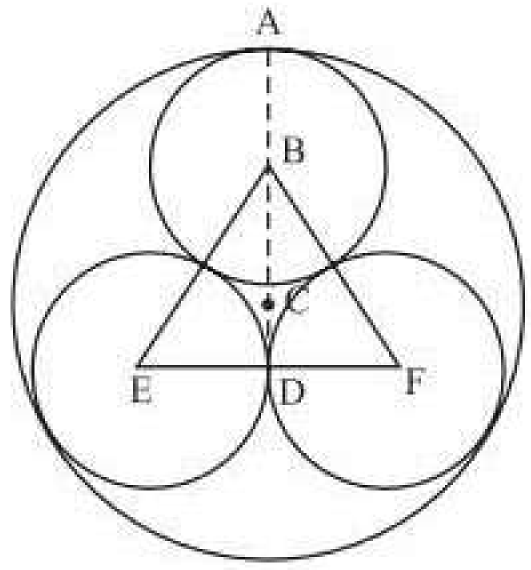
Case-(i): Two circles

$$R = 2r \Rightarrow r = \frac{R}{2}$$



Case-(ii): Three circles

C is the centroid of equilateral ΔBEF



$\therefore \quad BC : CD = 2 : 1$

$\therefore \quad BC = \frac{2}{3} BD$ ...(1)

In right angled ΔBDE,

$$BD = \sqrt{BE^2 - DE^2}$$

$$BD = \sqrt{4r^2 - r^2} = \sqrt{3}r \quad ...(2)$$

From (1) and (2),

$$BC = \frac{2}{3} \times \sqrt{3}r = \frac{2}{\sqrt{3}}r$$

Now $\quad AC = AB + BC$

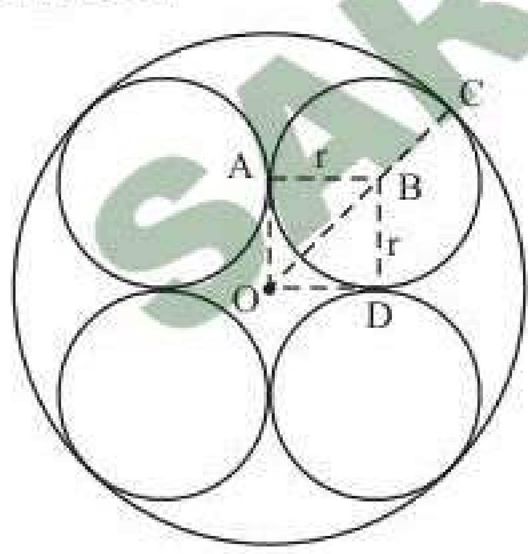
$$= r + \frac{2}{\sqrt{3}}r = \left(\frac{\sqrt{3}+2}{\sqrt{3}}\right)r$$

Also $\quad AC = R$

$$\therefore \quad \left(\frac{\sqrt{3}+2}{\sqrt{3}}\right)r = R \Rightarrow r = \frac{\sqrt{3}R}{\sqrt{3}+2}$$

$$\Rightarrow \quad r = \left(2\sqrt{3} - 3\right)R$$

Case- (iii): Four circles



In right ΔBDO,

$$OB = \sqrt{2}r$$

$$OC = OB + BC = \sqrt{2}r + r$$

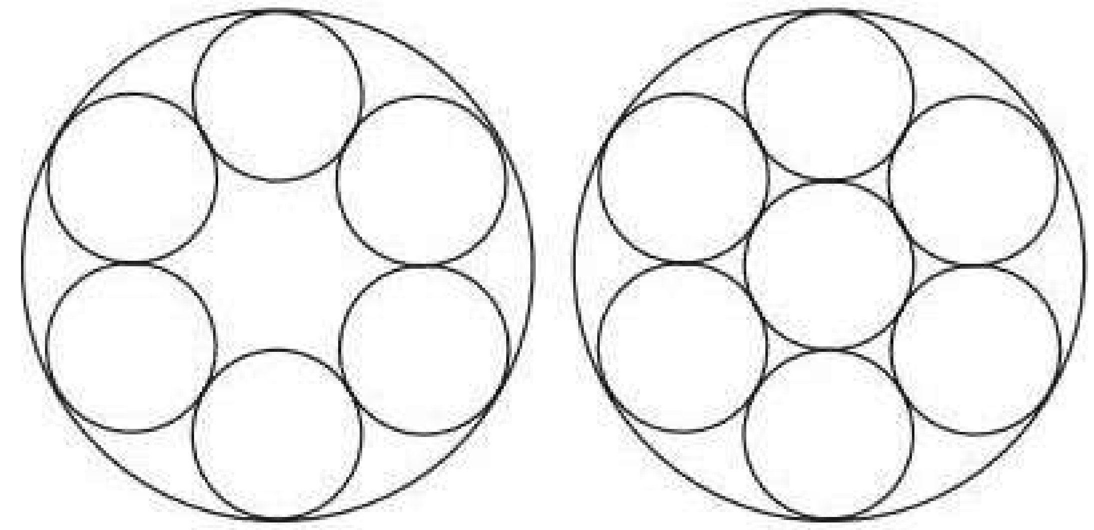
$$OC = \left(\sqrt{2}+1\right)r$$

Also $\quad OC = R$

$$\therefore \quad \left(\sqrt{2}+1\right)r = R$$

$$\Rightarrow \quad r = \frac{R}{\left(\sqrt{2}+1\right)} = \left(\sqrt{2}-1\right)R$$

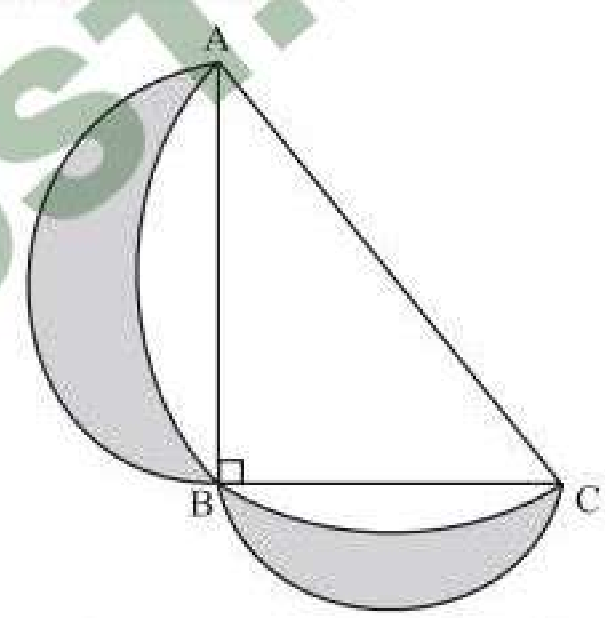
Case- (iv): Six/Seven circles



$$6r = 2R \Rightarrow r = \frac{1}{3}R$$

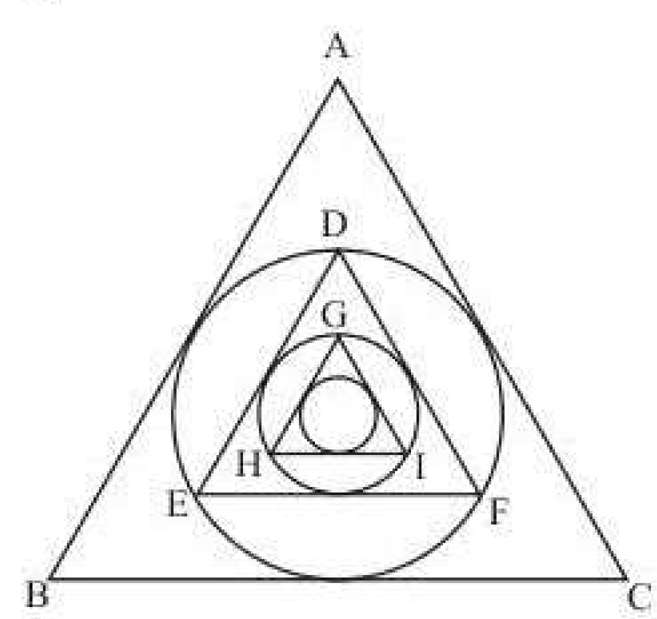
SOME OTHER IMPORTANT CONCEPTS

- In the figure ABC is a triangle right angled at B. Three semi-circles are drawn taking the three sides AB, BC and CA as diameter. The region enclosed by the three semi-circles is shaded.



Area of the shaded region = Area of the right angled triangle.

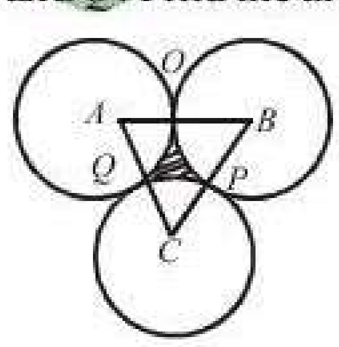
- In the figure given below all triangles are equilateral triangles and circles are inscribed in these triangles. If the side of triangle $ABC = a$, then the side of triangle $DEF = \frac{a}{2}$ and the side of triangle $GHI = \frac{a}{4}$



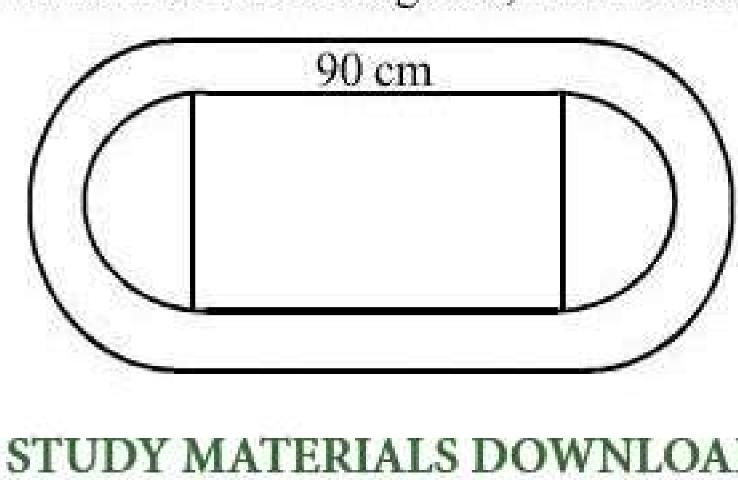
Thus length of a side of an inner triangle is half the length of immediate outer triangle. Similarly the radius of an inner circle is half the radius of immediate outer circle.

EXERCISE

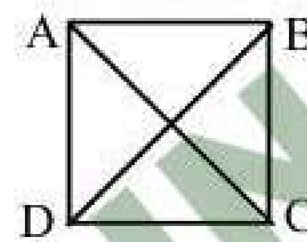
1. If the ratio of areas of two squares is 9 : 1, the ratio of their perimeter is :
   (a) 9 : 1 (b) 3 : 4
   (c) 3 : 1 (d) 1 : 3
2. A circle and a rectangle have the same perimeter. The sides of the rectangle are 18 cm and 26 cm. What is the area of the circle ?
   (a) 88 cm^2 (b) 154 cm^2
   (c) 1250 cm^2 (d) 616 cm^2
3. A square carpet with an area 169 m^2 must have 2 metres cut-off one of its edges in order to be a perfect fit for a rectangualar room. What is the area of rectangular room?
   (a) 180 m^2 (b) 164 m^2
   (c) 152 m^2 (d) 143 m^2
4. The length of a rectangular field is double its width. Inside the field there is a square-shaped pond 8 m long. If the area of the pond is 1/8 of the area of the field, what is the length of the field?
   (a) 32 m (b) 16m
   (c) 64 m (d) 20m
5. A garden is 24 m long and 14 m wide. There is a path 1 m wide outside the garden along its sides. If the path is to be constructed with square marble tiles 20 cm × 20 cm, the number of tiles required to cover the path is
   (a) 1800 (b) 200
   (c) 2000 (d) 2150
6. The cost of the paint is ₹ 36.50 per kg. If 1 kg of paint covers 16 square feet, how much will it cost to paint outside of a cube having 8 feet each side?
   (a) ₹692 (b) ₹768
   (c) ₹876 (d) ₹972
7. From a circular sheet of paper with a radius of 20 cm, four circles of radius 5 cm each are cut out. What is the ratio of the areas of uncut to the cut portion?
   (a) 1 : 3 (b) 4 : 1
   (c) 3 : 1 (d) 4 : 3
8. Three circles with centres *A*, *B* and *C* and with unit radii touch each other at *O*, *P* and *Q*. Find the area of the shaded region .



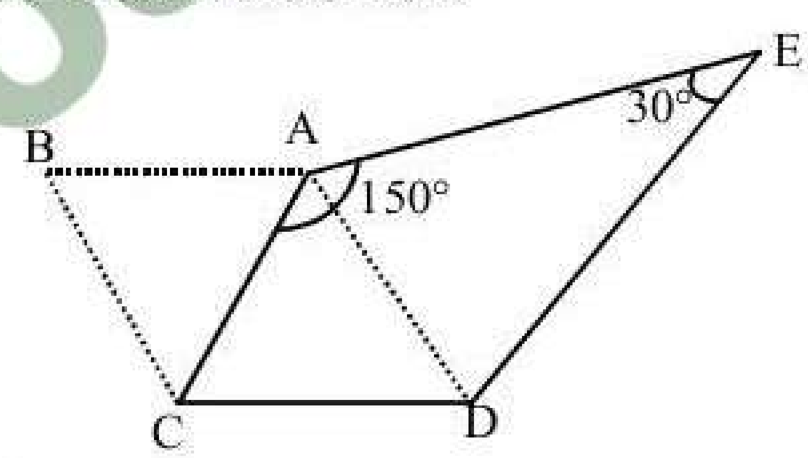
   (a) 0.16 sq. units (b) 1.21 sq. units
   (c) 0.03 sq. units (d) 0.32 units
9. The inside perimeter of a practice running track with semi-circular ends and straight parallel sides is 312 m. The length of the straight portion of the track is 90 m. If the track has a uniform width of 2 m throughout, find its area.



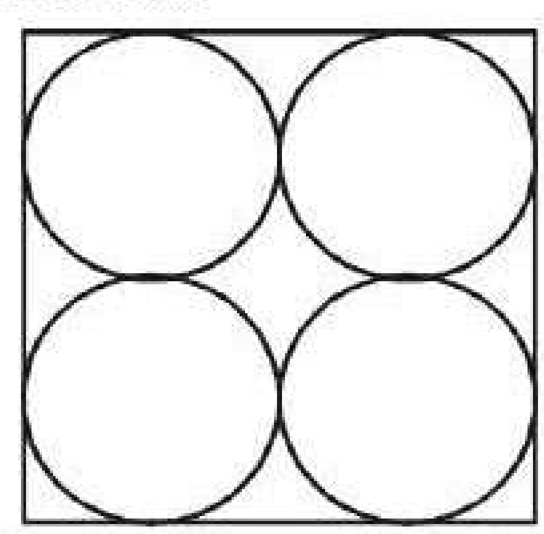
   (a) 5166 m^2 (b) 5802.57 m^2
   (c) 636.57 m^2 (d) 1273.14 m^2
10. The perimeter of a sector of a circle of radius 5.7 m is 27.2 m. Find the area of the sector.
   (a) 90.06 cm^2 (b) 135.09 cm^2
   (c) 45 cm^2 (d) None of these
11. *ABCD* is a square of area 4, which is divided into four non overlapping triangles as shown in the fig. Then the sum of the perimeters of the triangles is



   (a) $8(2+\sqrt{2})$ (b) $8(1+\sqrt{2})$
   (c) $4(1+\sqrt{2})$ (d) $4(2+\sqrt{2})$
12. In ΔACD, $AD = AC$ and $\angle C = 2\angle E$. The distance between parallel lines AB and CD is h.

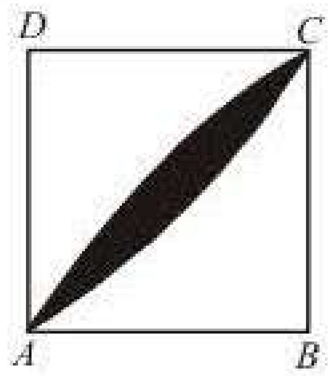


   Then
   I. Area of parallelogram *ABCD*
   II. Area of ΔADE
   (a) I > II (b) I < II
   (c) I = II (d) Nothing can be said
13. Four identical coins are placed in a square. For each coin, the ratio of area to circumference is same as the ratio of circumference to area.



   Then, find the area of the square that is not covered by the coins
   (a) $16(\pi - 1)$ (b) $16(8 - \pi)$
   (c) $16(4 - \pi)$ (d) $16\left(4 - \frac{\pi}{2}\right)$
14. Find the area of an isosceles triangle whose equal sides are 8 cm each and the third side is 10 cm ?
   (a) 10 cm^2 (b) 48 cm^2
   (c) $5\sqrt{39}$ cm^2 (d) $10\sqrt{10}$ cm^2

15. In the figure given below, $ABCD$ is a square of side 4 cm. Two quadrants of a circle with B and D as centres are draw. The radius of each of the quadrants is 4cm. What is the area of the shaded portion?



(a) 4.56 sq. cm (b) 9.12 sq. cm
(c) 13.68 sq. cm (d) 7.76 sq.cm

16. What is the area of a regular hexagon inscribed in a circle of radius r ?

(a) $2\sqrt{3}\,r^2$ sq. units (b) $\frac{3\sqrt{3}}{2}r^2$ sq. units
(c) $\frac{2}{\sqrt{3}}r^2$ sq. units (d) $\frac{\sqrt{3}}{2}r^2$ sq. units

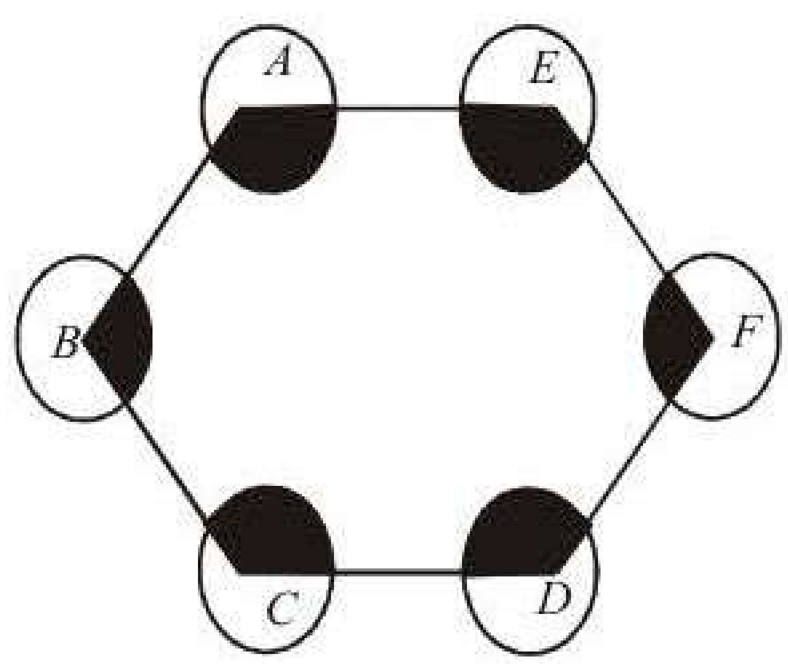
17. The central park of the city is 40 metres long and 30 metres wide. The mayor wants to construct two roads of equal width in the park such that the roads intersect each other at right angles and the diagonals of the park are also the diagonals of the small square formed at the intersection of the two roads. Further, the mayor wants that the area of the two roads to be equal to the remaining area of the park. What should be the width of the roads?

(a) 10 metres (b) 12.5 metres
(c) 14 metres (d) 15 metres

18. The sides of a triangle are 21, 20 and 13 cm. Find the area of the larger triangle into which the given triangle is divided by the perpendicular upon the longest side from the opposite vertex.

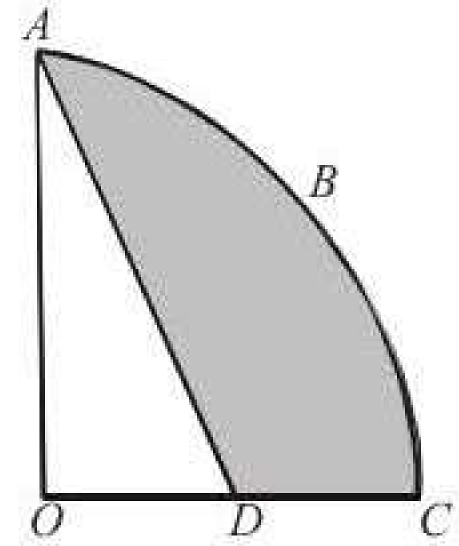
(a) 72 cm^2 (b) 96 cm^2
(c) 168 cm^2 (d) 144 cm^2

19. Find the sum of the areas of the shaded sectors given that $ABCDEF$ is any hexagon and all the circles are of same radius r with different vertices of the hexagon as their centres as shown in the figure.



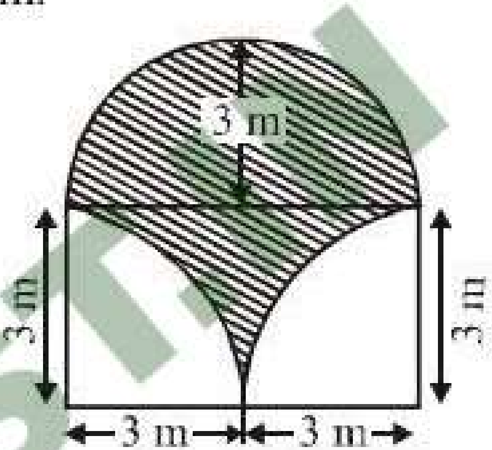
(a) πr^2 (b) $2\pi r^2$
(c) $5\pi r^2/4$ (d) $3\pi r^2/2$

20. In the figure given below, $ABCO$ represents a quadrant of a circle of radius 10.5 cm with centre O. Calculate the area of shaded portion, if $OD = DC$.



(a) 59 cm^2 (b) 69 cm^2
(c) 79 cm^2 (d) 49 cm^2

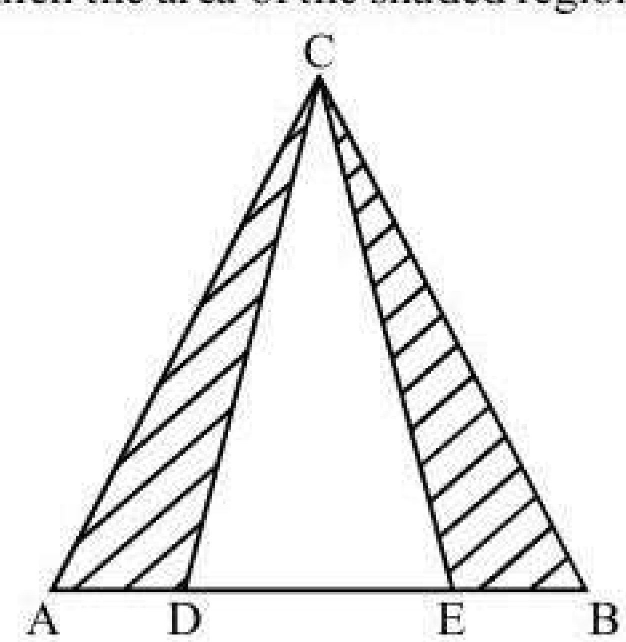
21. In the adjoining figure is a park in which shaded area is to be covered by grass. If the rate of covering with grass is ₹ 0.70 per sq. m.



Find the expenditure of covering its field with grass ($\pi = 22/7$)

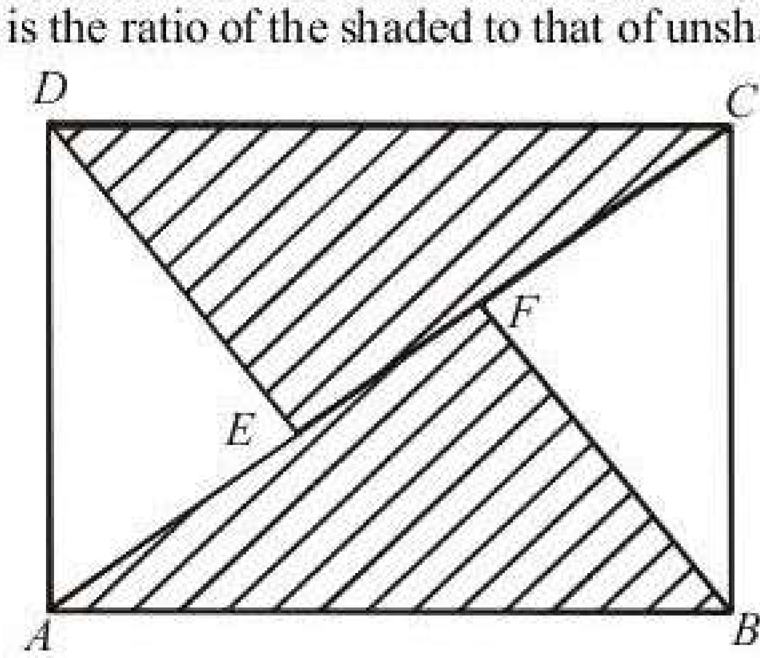
(a) ₹ 12.60 (b) ₹ 6.30
(c) ₹ 9.30 (d) ₹ 10.30

22. In the equilateral triangle ABC, $AD = DE = BE$, D and E lies on the AB. If each side of the triangle (i.e., AB, BC and AC) be 6 cm, then the area of the shaded region is:



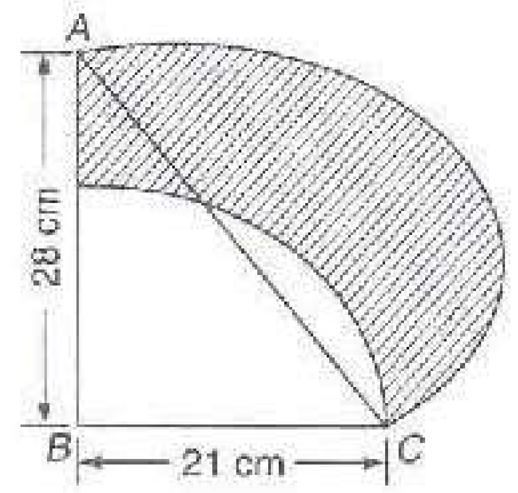
(a) 9 cm^2 (b) $6\sqrt{3}$ cm^2
(c) $5\sqrt{3}$ cm^2 (d) None of these

23. $ABCD$ is a rectangle of dimensions 6 cm × 8 cm. DE and BF are the perpendiculars drawn on the diagonal of the rectangle. What is the ratio of the shaded to that of unshaded region?



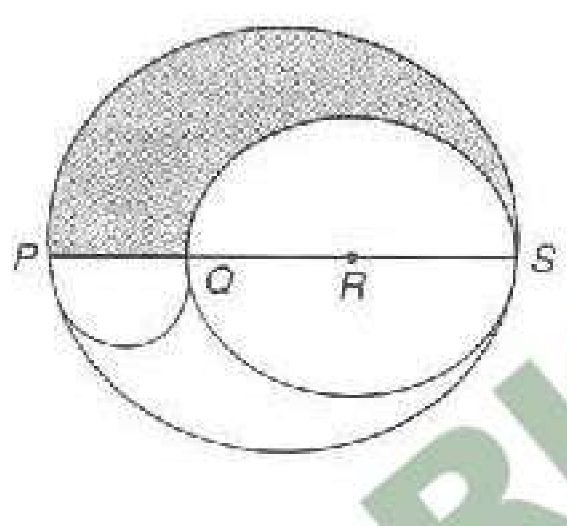
(a) 7 : 3 (b) 16 : 9
(c) $4 : 3\sqrt{2}$ (d) Data insufficient

24. In the figure, ABC is a right angled triangle with $B = 90°$, $BC = 21$ cm and $AB = 28$ cm. With AC as diameter of a semicircle and with BC as radius, a quarter circle is drawn. Find the area of the shaded portion correct to two decimal places



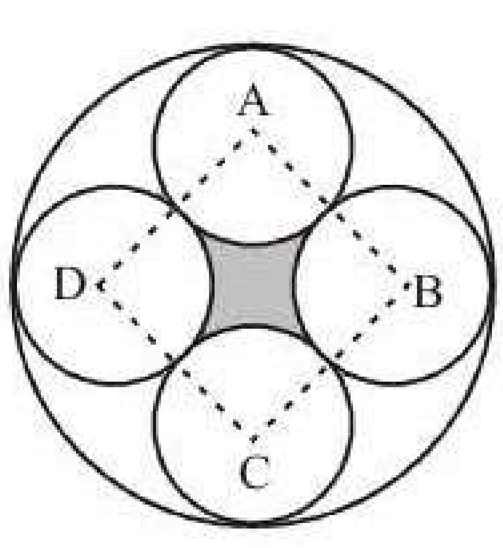
(a) 428.75 cm^2 (b) 857.50 cm^2
(b) 214.37 cm^2 (d) 371.56 cm^2

25. PQRS is the diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn with PQ and QS as diameters as shown in the figure alongside. Find the ratio of the area of the shaded region to that of the unshaded region.



(a) 1 : 2 (b) 25 : 121
(c) 5 : 18 (d) 5 : 13

26. In the figure below, the radius of the bigger circle is $\left(\sqrt{2}+1\right)$ cm and the radius of all the smaller circles are equal. Each of the smaller circles touches two of the other three smaller circles and the larger circle as shown. Find the area (in cm^2) of the shaded portion.



(a) $2 - \frac{\pi}{2}$ (b) 1
(c) $\pi/4$ (d) Cannot to determined

27. In an isosceles right angled triangle, the perimeter is 20 metre. Find its area.

(a) $100\left(3-2\sqrt{2}\right)$ m^2 (b) $150\left(5-\sqrt{3}\right)$m^2
(c) 500 m^2 (d) None of these

28. The diameter of a garden roller is 1.4 m and it is 2 m long. How much area will it cover in 5 revolutions ? $\left(\text{use } \pi = \frac{22}{7}\right)$

(a) 40 m^2 (b) 44 m^2
(c) 48 m^2 (d) 36 m^2

29. From a square piece of a paper having each side equal to 10 cm, the largest possible circle is being cut out. The ratio of the area of the circle to the area of the original square is nearly :

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$
(c) $\frac{5}{6}$ (d) $\frac{6}{7}$

30. A picture 30" × 20" has a frame 2½" wide. The area of the picture is approximately how many times the area of the frame?

(a) 4 (b) $2\frac{1}{2}$
(c) 2 (d) 5

31. A rectangular paper, when folded into two congruent parts had a perimeter of 34 cm for each part folded along one set of sides and the same is 38 cm when folded along the other set of sides. What is the area of the paper?

(a) 140 cm^2 (b) 240 cm^2
(c) 560 cm^2 (d) None of these

32. The length and breadth of the floor of the room are 20 feet and 10 feet respectively. Square tiles of 2 feet length of different colours are to be laid on the floor. Black tiles are laid in the first row on all sides. If white tiles are laid in the one-third of the remaining and blue tiles in the rest, how many blue tiles will be there?

(a) 16 (b) 24
(c) 32 (d) 48

33. Four equal circles are described about the four corners of a square so that each touches two of the others. If a side of the square is 14 cm, then the area enclosed between the circumferences of the circles is :

(a) 24 cm^2 (b) 42 cm^2
(c) 154 cm^2 (d) 196 cm^2

34. The ratio of height of a room to its semi-perimeter is 2 : 5. It costs ₹ 260 to paper the walls of the room with paper 50 cm wide at ₹ 2 per metre alllowing an area of 15 sq. m for doors and windows. The height of the room is:

(a) 2.6 m (b) 3.9 m
(c) 4 m (d) 4.2 m

35. If the diagonal of a square is doubled, then its area will be

(a) three times (b) four times
(c) same (d) none of these

36. A wire is bent into the form of a circle, whose area is 154 cm^2. If the same wire is bent into the form of an equilateral triangle, the approximate area of the equilateral triangle is
(a) 93.14 cm^2 (b) 90.14 cm^2
(c) 83.14 cm^2 (d) 39.14 cm^2

37. The lengths of two sides of a right angled triangle which contain the right angle are a and b, respectively. Three squares are drawn on the three sides of the triangle on the outer side. What is the total area of the triangle and the three squares?
(a) $2(a^2+b^2)+ab$ (b) $2(a^2+b^2)+2.5ab$
(c) $2(a^2+b^2)+0.5ab$ (d) $2.5(a^2+b^2)$

38. In the ΔABC, the base BC is trisected at D and E. The line through D, parallel to AB, meets AC at F and the line through E parallel to AC meets AB at G. If EG and DF intersect at H, then what is the ratio of the sum of the area of parallelogram AGHF and the area of the ΔDHE to the area of the ΔABC?
(a) 1/2 (b) 1/3
(c) 1/4 (d) 1/6

39. A square, a circle and an equilateral triangle have same perimeter.
Consider the following statements
I. The area of square is greater than the area of the triangle.
II. The area of circle is less than the area of triangle.
Which of the above statement is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

40. The area of a rectangle, whose one side is a and other side is $2a^2$. What is the area of a square having one of the diagonals of the rectangle as side?
(a) $2a^2$ (b) $3a^2$
(c) $4a^2$ (d) $5a^2$

41. Consider the following statements
I. Area of a segment of a circle is less than area of its corresponding sector.
II. Distance travelled by a circular wheel of diameter $2d$ cm in one revolution is greater than $6d$ cm.
Which of the above statements is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

42. If the circumference of a circle is equal to the perimeter of square, then which one of the following is correct?
(a) Area of circle = Area of square
(b) Area of circle ≥ Area of square
(c) Area of circle > Area of square
(d) Area of circle < Area of square

43. If an isosceles right angled triangle has area 1 sq unit, then what is its perimeter?
(a) 3 units (b) $2\sqrt{2}+1$ units
(c) $(\sqrt{2}+1)$ units (d) $2(\sqrt{2}+1)$ units

44. If the area of a regular hexagon is $96\sqrt{3}$ sq cm, then its perimeter is
(a) 36 cm (b) 48 cm
(c) 54 cm (d) 64 cm

45. A rectangular field is 22 m long and 10 m wide. Two hemispherical pitholes of radius 2 m are dug from two places and the mud is spread over the remaining part of the field. The rise in the level of the field is.
(a) $\frac{8}{93}$m (b) $\frac{13}{93}$m
(c) $\frac{16}{93}$m (d) $\frac{23}{93}$m

46. The area of an isosceles ΔABC with AB = AC and altitude AD = 3 cm is 12 sq cm. What is its perimeter?
(a) 18 cm (b) 16 m
(c) 14 cm (d) 12 cm

47. A hospital room is to accommodate 56 patients. It should be done in such a way that every patient gets 2.2 m^2 of floor and 8.8 m^3 of space. If the length of the room is 14 m, then breadth and the height of the room are respectively
(a) 8.8 m, 4 m (b) 8.4 m, 4.2 m
(c) 8 m, 4 m (d) 7.8 m, 4.2 m

48. What is the area between a square of side 10 cm and two inverted semi-circular, cross-sections each of radius 5 cm inscribed in the square?
(a) 17.5 cm^2 (b) 18.5 cm^2
(c) 20.5 cm^2 (d) 21.5 cm^2

49. If the diagonals of a rhombus are 4.8 cm and 1.4 cm, then what is the perimeter of the rhombus?
(a) 5 cm (b) 10 cm
(c) 12 cm (d) 20 cm

50. How many circular plates of diameter d be taken out of a square plate of side $2d$ with minimum loss of material?
(a) 8 (b) 6
(c) 4 (d) 2

51. What is the total area of three equilateral triangles inscribed in a semi-circle of radius 2 cm?
(a) 12 cm^2 (b) $\frac{3\sqrt{3}}{4}$ cm^2
(c) $\frac{9\sqrt{3}}{4}$ cm^2 (d) $3\sqrt{3}$ cm^2

52. The area of sector of a circle of radius 36 cm is 72π cm^2. The length of the corresponding arc of the sector is
(a) π cm (b) 2π cm
(c) 3π cm (d) 4π cm

53. A square is inscribed in a circle of diameter $2a$ and another square is circumscribing circle. The difference between the areas of outer and inner squares is
(a) a^2 (b) $2a^2$
(c) $3a^2$ (d) $4a^2$

54. ABC is a triangle right angled at A. AB = 6 cm and AC = 8 cm. Semi-circles drawn (outside the triangle) on AB, AC and BC as diameters which enclose areas x, z square units, respectively. What is $x+y-z$ equal to?
(a) 48 cm^2 (b) 32 cm^2
(c) 0 (d) None of these

55. What is the area of the larger segment of a circle formed by a chord of length 5 cm subtending an angle of 90° at the centre?

(a) $\frac{25}{4}\left(\frac{\pi}{2}+1\right)cm^2$ (b) $\frac{25}{4}\left(\frac{\pi}{2}-1\right)cm^2$

(c) $\frac{25}{4}\left(\frac{3\pi}{2}+1\right)cm^2$ (d) None of these

56. A rectangle of maximum area of drawn inside a circle of diameter 5 cm. What is the maximum area of such a rectangle?

(a) 25 cm^2 (b) 12.5 cm^2

(c) 12 cm^2 (d) None of these

57. If AB and CD are two diameters of a circle of radius *r* and they are mutually perpendicular, then what is the ratio of the area of the circle to the area of the ΔACD?

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{4}$ (d) 2π

58. The sides of a triangular field are 41 m, 40 m and 9 m. The number of rose beds that can be prepared in the field if each rose bed, on an average, needs 900 square cm space, is **(CDS)**

(a) 2000 (b) 1800

(c) 900 (d) 800

59. The ratio of the outer and inner perimeters of a circular path is 23 : 22. If the path is 5 m wide, the diameter of the inner circle is **(CDS)**

(a) 55 m (b) 110m

(c) 220m (d) 230m

60. Four equal-sized maximum circular plates are cut off from a square paper sheet of area 784 square cm. The circumference of each plate is **(CDS)**

(a) 11 cm (b) 22 cm

(c) 33 cm (d) 44 cm

61. A truck moves along a circular path and describes 100 m when it has traced out 36° at the centre. The radius of the circle is equal to **(CDS)**

(a) $\frac{100}{x}m$ (b) $\frac{250}{x}m$

(c) $\frac{500}{x}m$ (d) $\frac{600}{x}m$

62. A rhombus is formed by joining midpoints of the sides of a rectangle in the suitable order. If the area of the rhombus is 2 square units, then the area of the rectangle is **(CDS)**

(a) $2\sqrt{2}$ square units (b) 4 square units

(c) $4\sqrt{2}$ square units (d) 8 square units

63. The base of an isosceles triangle is 300 unit and each fot is equal sides is 170 units. Then the area of the triangle is **(CDS)**

(a) 9600 square units (b) 10000 square units

(c) 12000 square units (d) None of the above

64. Four equal discs are placed such that each one touches two others. If the area of empty space enclosed by them is 150/847 square centimetre, then the radius of each disc is equal to **(CDS)**

(a) 7/6 cm (b) 5/6 cm

(c) 1/2 cm (d) 5/11 cm

65. A circular path is made from two concentric circular rings in such a way that the smaller ring when allowed to roll over the circumference of the bigger ring, it takes three full revolutions. If the area of the pathway is equal to *n* times the area of the smaller ring, then *n* is equal to **(CDS)**

(a) 4 (b) 6

(c) 8 (d) 10

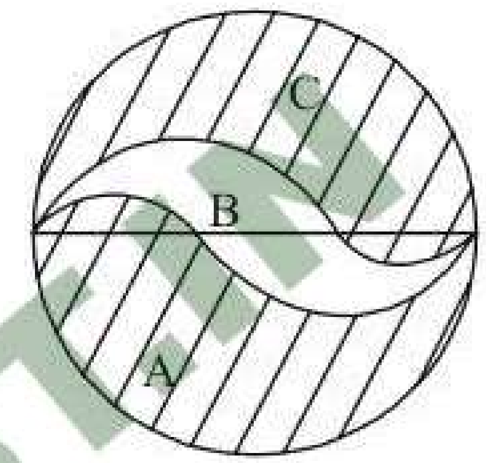
66. *ABC* is a triangle in which *D* is the midpoint of *BC* and *E* is the midpoint of *AD*. Which of the following statements is/are correct? **(CDS)**

1. The area of triangle *ABC* is equal to four times the area of triangle *BED*.
2. The area of triangle *ADC* is twice the area of triangle *BED*.

Select the correct answer using the code given below.

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

67. 

A circle of 3 m radius is divided into three areas by semicircles of radii 1m and 2m as shown in the figure above. The ratio of the three areas *A*, *B* and *C* will be **(CDS)**

(a) 2 : 3 : 2 (b) 1 : 1 : 1

(c) 4 : 3 : 4 (d) 1 : 2 : 1

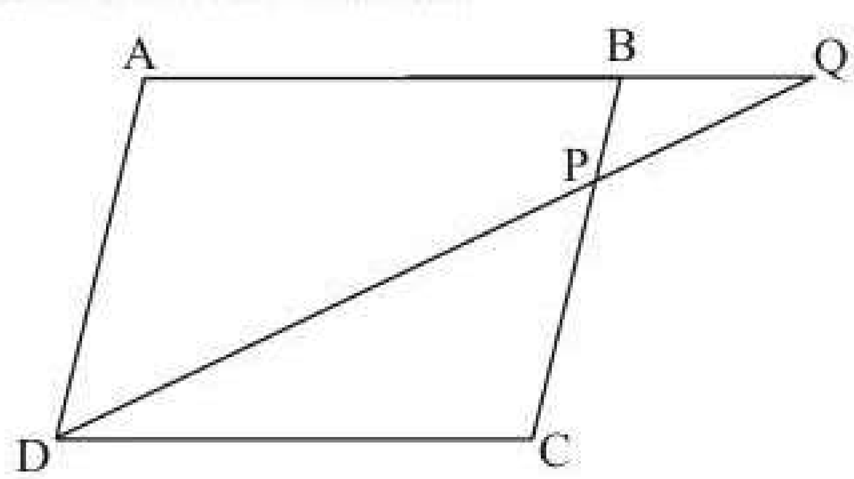
68. There are 437 fruit plants in an orchard planted in rows. The distance between any two adjacent rows is 2 m and the distance between any two adjacent plants is 2m. Each row has the same number of plants. There is 1 m clearance on all sides of the orchard. What is the cost of fencing the area at the rate of ₹100 per metre **(CDS)**

(a) ₹15,600

(b) ₹16,800

(c) ₹18,200

(d) More information is required

69. 

In the above figure, ABCD is a parallelogram. P is a point on BC such that PB : PC = 1 : 2. DP and AB when both produced meet at Q. If area of triangle BPQ is 20 square unit, the area of triangle DCP is **(CDS)**

(a) 20 square unit (b) 30 square unit

(c) 40 square unit (d) None of the above

70. A circle of radius r is inscribed in a regular polygon with n sides (the circle touches all sides of the polygon). If the perimeter of the polygon is p, then the area of the polygon is **(CDS)**

(a) (p + n) r (b) (2p – n) r

(c) $\frac{pr}{2}$ (d) None of the above

HINTS & SOLUTIONS

1. (c) Let the area of two squares be $9x$ and x respectively.
So, sides of both squares will be
$\sqrt{9x}$ and $\sqrt{x}$ respectively. [since, side = $\sqrt{\text{area}}$]
Now, perimeters of both squares will be
$4\times\sqrt{9x}$ and $4\sqrt{x}$ respectively.
[since, perimeter = 4 × side]
Thus, ratio of their perimeters $= \dfrac{4\sqrt{9x}}{4\sqrt{x}} = 3:1$

2. (d) Perimeter of the circle $= 2\pi r = 2(18+26)$
$\Rightarrow \; 2\times\dfrac{22}{7}\times r = 88 \;\Rightarrow r = 14$
$\therefore$ Area of the circle
$= \pi r^2 = \dfrac{22}{7}\times 14\times 14 = 616 \text{ cm}^2$.

3. (d) Side of square carpet $= \sqrt{\text{Area}} = \sqrt{169} = 13$ m
After cutting of one side,
Measure of one side = 13 – 2 = 11 m
and other side = 13 m (remain same)
$\therefore$ Area of rectangular room = 13 × 11 = 143 m^2

4. (a) Let width of the field = b m
$\therefore$ length = 2 b m
Now, area of rectangular field = $2b \times b = 2b^2$
Area of square shaped pond = 8 × 8 = 64
According to the question,
$64 = \dfrac{1}{8}(2b^2) \Rightarrow b^2 = 64\times 4 \Rightarrow b = 16\text{m}$
$\therefore$ length of the field = 16 × 2 = 32 m

5. (c) Given, length of garden = 24 m and
breadth of garden = 14 m
$\therefore$ Area of the garden = 24 × 14 m^2 = 336 m^2.
Since, there is 1 m wide path outside the garden
$\therefore$ Area of Garden (including path)
= (24 + 2) × (14 + 2) = 26 × 16 m^2 = 416 m^2.
Now, Area of Path = Area of garden (inculding path) – Area of Garden
= 416 – 336 = 80 m^2.
Now , Area of Marbles = 20 × 20 = 400 cm^2
$\therefore$ Marbles required $= \dfrac{\text{Area of Path}}{\text{Area of Marbles}}$
$= \dfrac{80,0000}{400} = 2000$

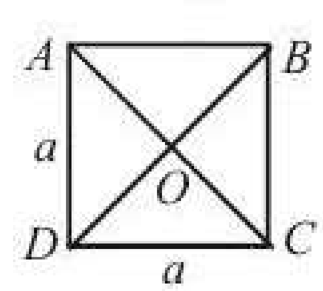
6. (c) Surface area of the cube = (6×8^2) sq. ft. = 384 sq. ft.
Quantity of paint required $= \left(\dfrac{384}{16}\right)$kg = 24 kg.
$\therefore$ Cost of painting = ₹ (36.50 × 24) = ₹ 876.

7. (c) $\dfrac{\text{Area of uncut portion}}{\text{Area of cut portion}} = \dfrac{(\pi\times 20\times 20)-(100\pi)}{(4\times\pi\times 5\times 5)}$
$= \dfrac{300\pi}{100\pi} = \dfrac{3}{1}$

8. (a) Area of shaded region = Area of equilateral $\Delta ABC - 3$ (Area of sector AQO)
$= \dfrac{\sqrt{3}}{4}\times(2)^2 - 3\times\dfrac{60}{360}\times\dfrac{22}{7}\times(1)^2$
$= \sqrt{3} - \dfrac{11}{7} = 1.73 - 1.57 = 0.16$ sq. units.

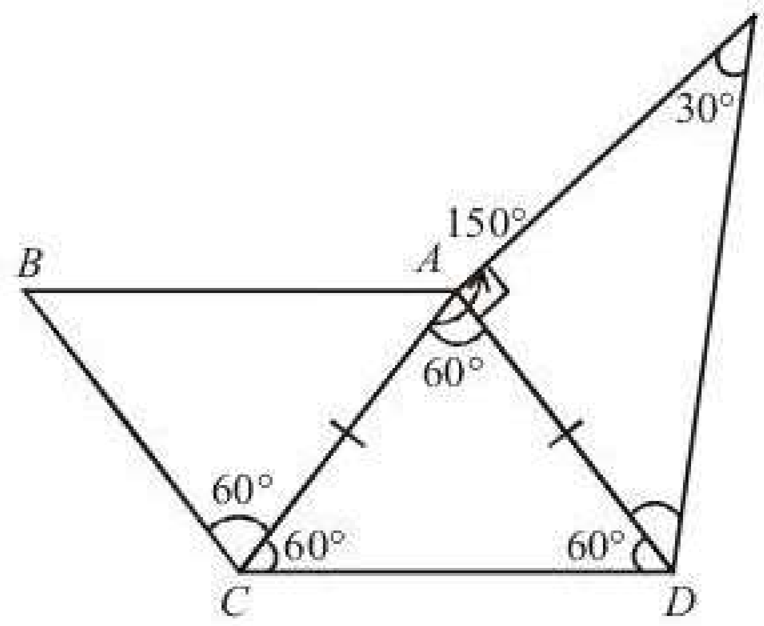
9. (c) 2 semicircles = 1 circle with equal radius
So $2\pi r = 132 \Rightarrow 2r = \dfrac{132}{3.14} = 42$ m diameter
Area of track = Area within external border – Area within internal border.
$\Rightarrow \pi(23^2 - 21^2) + 90\times 46 - 90\times 42$
$\Rightarrow 88\pi + 360 \Rightarrow 636.3\text{ m}^2$

10. (d) Let the angle subtended by the sector at the centre be = θ
Then, $5.7 + 5.7 + (2\pi)\times 5.7\times\dfrac{8}{360} = 27.2$
$11.4 + \dfrac{11.4\times 3.14\times\theta}{360} = 27.2$
$\Rightarrow \dfrac{\theta}{360} = 0.44$
Area of the sector
$= \pi r^2\dfrac{\theta}{360} \Rightarrow (22/7)\times(5.7)^2\times 0.44$
= 44.92 approx.

11. (b)


$ABCD$ is square $a^2 = 4 \Rightarrow a = 2$
$ac = BD = 2\sqrt{2}$
perimeters of four triangles
$= AB + BC + CD + DA + 2(AC + BD)$
$= 8 + 2(2\sqrt{2} + 2\sqrt{2}) = 8(1+\sqrt{2})$

12. (c)



$\angle A = \angle C = 60$ (alternative angles)

$\angle C = \angle D = 60°$

(since $AC = AD$ and $\angle A = 60°$)

ΔACD is equilateral

so its area $= \frac{x^2\sqrt{3}}{4}$ (where x is side)

Area of parallelogram $ABCD = 2 \times \frac{x^2\sqrt{3}}{4} = \frac{x^2\sqrt{3}}{2}$

Area of $\Delta ADE = \frac{1}{2} \times AD \times AE$

$= \frac{1}{2} \times x \times x \tan 60° = \frac{x^2\sqrt{3}}{2}$

therefore we see,

Area of parallelogram $ABCD$ = Area of ΔADE

13. (c) Let r be the radius of each circle.

Then by given condition,

$$\frac{\pi R^2}{2\pi R} = \frac{2\pi R}{\pi R^2} \Rightarrow R^2 = 4 \Rightarrow R = 2$$

$\therefore$ The length of the side of the square = 8

Now the area covered by 4 coins $= 4 \times \pi(2)^2 = 16\pi$

and area of the square = 64

$\therefore$ The area which is not covered by the coins

$= 64 - 16\pi = 16(4 - \pi)$

14. (c) Area of isosceles triangle $= \frac{b}{4}\left(\sqrt{4a^2 - b^2}\right)$

where b is the base and a is any of the equal sides.

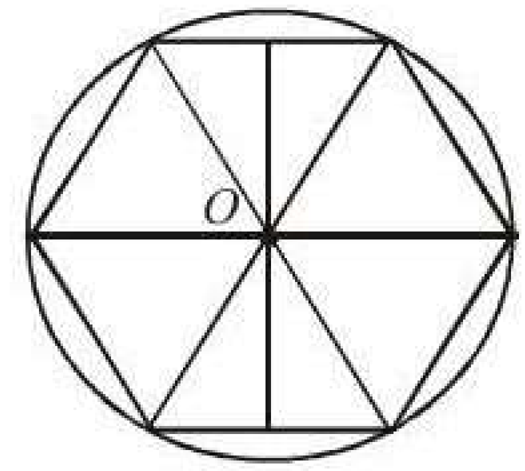
Area of the required triangle $= \frac{10}{4}\left(\sqrt{4(8)^2 - (10)^2}\right)$

$= \frac{10}{4}\sqrt{156} = 5\sqrt{39}$ cm^2

15. (b) Area of the shaded portion = (Area of quadrant ABC + Area of quadrant ACD – Area of square $ABCD$).

$$= \left(\frac{\pi}{2} \times 4^2 - 4^2 = \left(\frac{\pi}{2} - 1\right)4^2\right) = (\pi - 2)\,4 = 4.56 \text{ sq. cm}.$$

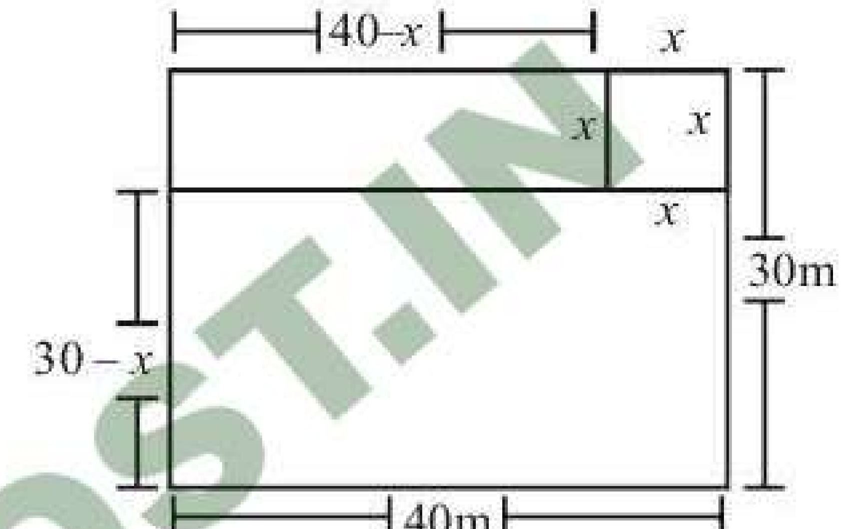
16. (b) We can divide the regular hexagon into 6 equilateral triangles. Since the hexagon is in a circle the radius r is the side of the equilateral triangle.



$\therefore$ Area of the hexagon

$= 6 \times \frac{\sqrt{3}}{4}r^2 = \frac{3\sqrt{3}}{2}r^2$ sq. units.

17. (a)



Hence, $(x)(40 - x) + (x)(30 - x) + x^2$

$= 1200 - [(x)(40 - x) + (x)(30 - x) + x^2]$

$2[(x)(40 - x) + (x)(30 - x) + x^2] = 1200$

$40x - x^2 + 30x - x^2 + x^2 = 600$

$-x^2 + 70x - 600 = 0$

$x^2 - 70x + 600 = 0$

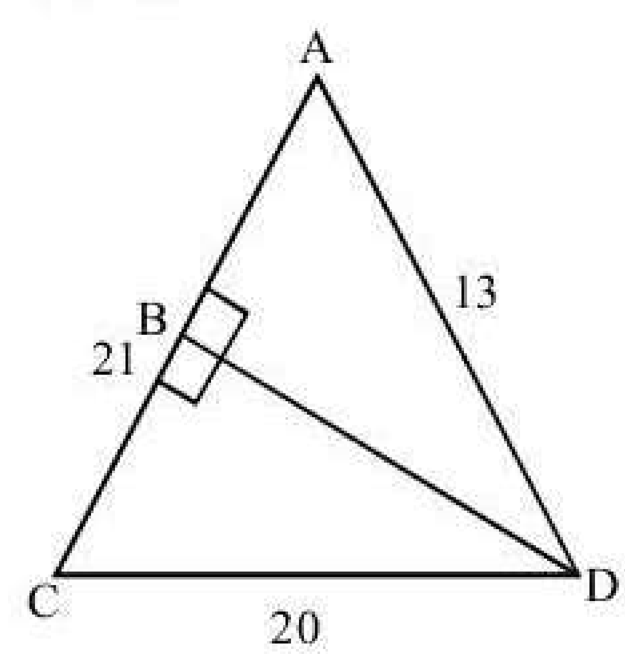
$(x - 60)(x - 10) = 0$

$x = 10$ or 60.

As x must be less than 30.

$\therefore x = 10$

18. (b)



Let the original triangle be ACD

Longest side $= AC = 21$ cm

In the right angled ΔABD, by Pythagorean triplets, we get $AB = 5$ cm and $BD = 12$ cm

Then, $BC = 21 - 5 = 16$

By Pythagoras theorem,

$BD^2 = CD^2 - BC^2 \Rightarrow BD = 12$ cm

Area of the larger $\Delta BDC = \frac{1}{2} \times 16 \times 12 = 96$ cm^2

19. (b) Sum of interior angles of a hexagon = 720°
6 sectors with same radius $r = 2$ full circles of same radius.
So area of shaded region $\Rightarrow 2\pi r^2$

20. (a) Since $ABCDOA$ is a quadrant of circle of radius 10.5 cm $OA = OC = r = 10.5$ cm and

$$OD = DC = \frac{10.5}{2} = 5.25 \text{ cm}$$

Area of shaded portion = (Area of the quadrant) – (Area of ΔAOD)

$$\text{Area} = \frac{\theta}{360}\pi r^2 - \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{90}{360} \times \frac{22}{7} \times (10.5)^2 - \frac{1}{2} \times 5.25 \times 10.5$$

$= 86.625 - 27.5625 = 59.06 \text{ cm}^2$.

21. (a) From the fig. the shaded area
= (Area of the rectangle – 2 × quarter of circle) + area of rectangle

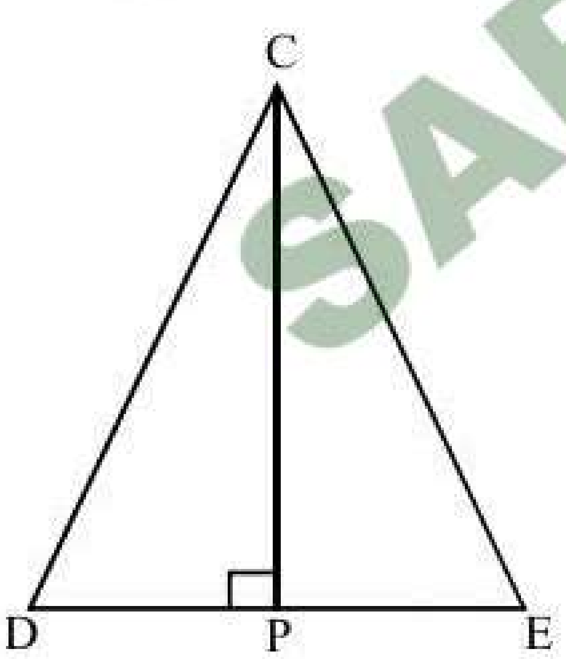
$$= \left[\left(3 \times 6 - 2 \times \frac{\pi}{4} \times 3^2\right) + \frac{1}{2}\pi \times 3^2\right] \text{ sq. m}$$

$$= \left[18 - \frac{9\pi}{2} + \frac{9\pi}{2}\right] = 18 \text{ sq. m}$$

$\therefore$ Cost of covering with grass = ₹ $\frac{18 \times 70}{100}$

$$= ₹\, \frac{630 \times 2}{100} = ₹\, 12.60$$

22. (b)



Area of equilateral triangle

$$ABC = \frac{\sqrt{3}}{4} \times (6)^2 = 9\sqrt{3} \text{ cm}^2$$

$$\text{Area of } \Delta CDE = \frac{1}{2} \times DE \times CP$$

$$= \frac{1}{2} \times 2 \times \frac{\sqrt{3}}{2} \times 6$$

$= 3\sqrt{3}$ cm²

$\therefore$ Area of shaded region $= 9\sqrt{3} - 3\sqrt{3} = 6\sqrt{3}$ cm²

23. (b) $$\frac{\text{Area of } \Delta DAE}{\text{Area of } \Delta DEC} = \frac{\frac{1}{2} \times DE \times AE}{\frac{1}{2} \times DE \times CE}$$

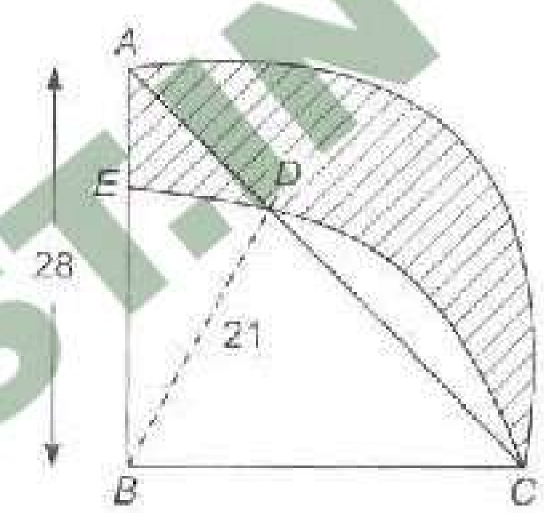
$$= \frac{AE}{CE} = \frac{(AD)^2}{(DC)^2} = \left(\frac{6}{8}\right)^2 = \frac{9}{16}$$

Similarly, in ΔABC,

$$\frac{\text{Area of } \Delta BCF}{\text{Area of } \Delta BFA} = \frac{9}{16}$$

$\therefore$ The area of shaded to unshaded region $= \frac{16}{9}$

24. (a)



Area of shaded portion = Area of ADC – Area of sector DC + Area of ΔADB – sector BED

$\Rightarrow$ Area of $ADC = \pi \times (17.5)^2 \times \frac{1}{2} = 481$ cm²

$$\frac{\angle DBC}{\angle ABC} = \frac{21}{28} \Rightarrow \angle DBC = 67.5 \text{ and } \angle DBA = 22.5$$

$$\Rightarrow \text{Area of sector } DC = \left(\pi \times 21^2 \times \frac{67.5}{360}\right) - \left(\frac{1}{2} \times 21^2 \times \sin 67.5\right) = 56 \text{ cm}^2$$

$$\text{Area of ADE} = \left(\frac{1}{2} \times 28 \times 21\right) - \left(204 + \frac{1}{2} \times 21^2 \times \sin 22.5\right) = 5.6 \text{cm}^2$$

Thus area of shaded portion = 480 – 56 + 5.6 = 429 cm²

25. (d) $PQ = QR = RS = \frac{12}{3} = 4$ cm

Area of unshaded region $\Rightarrow \frac{\pi 6^2}{2} + \frac{\pi 4^2}{2}$

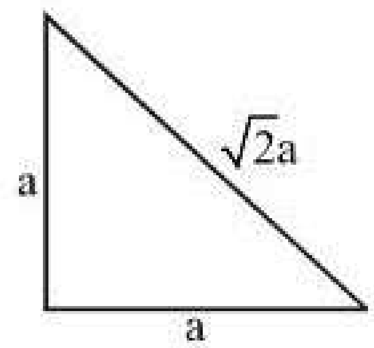
$\Rightarrow 18\pi + 8\pi \Rightarrow 26\pi$

Area of shaded region $\Rightarrow \frac{\pi 6^2}{2} - \frac{\pi 4^2}{2}$

$\Rightarrow 18\pi - 8\pi = 10\pi$

$\text{Ratio} = \frac{10\pi}{26\pi} \Rightarrow \frac{5}{13} \Rightarrow 5:13$

26. (d) If the radius of smaller circle is 1 unit, then the radius of the bigger circle is $\sqrt{2}+1$ units.
So, the answer in this case would be the area of square ABCD– 4 quadrants of the smaller circle.
$= 4 - \pi$

27. (a)


Perimeter of triangle $= a + a + \sqrt{2}\,a = 20$m

$$a(2+\sqrt{2}) = 20$$

$$a = \frac{20}{2+\sqrt{2}} \times \frac{(2-\sqrt{2})}{(2-\sqrt{2})} = 10\left(2-\sqrt{2}\right)\text{m}$$

Area of triangle $= \frac{1}{2} \times a \times a$

$$= \frac{1}{2} \times 10\left(2-\sqrt{2}\right) \times 10\left(2-\sqrt{2}\right)$$

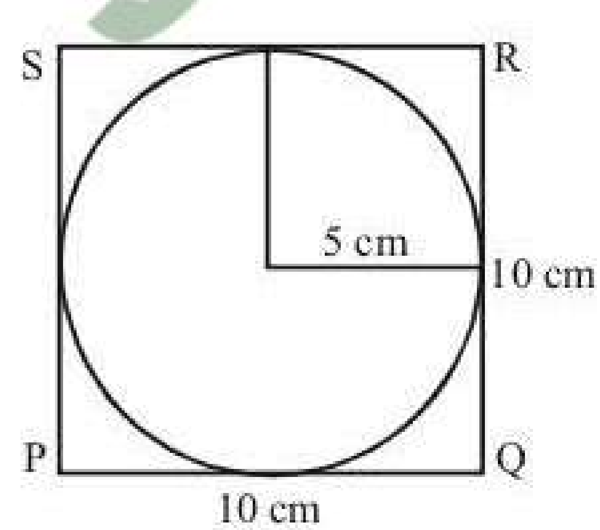
$= 50\,(4 + 2 - 4\sqrt{2}\,)$

$= 100\,(3 - 2\sqrt{2}\,)\text{m}^2$

28. (b) Required area covered in 5 revolutions

$$= 5 \times 2\pi rh = 5 \times 2 \times \frac{22}{7} \times 0.7 \times 2 = 44\text{ m}^2$$

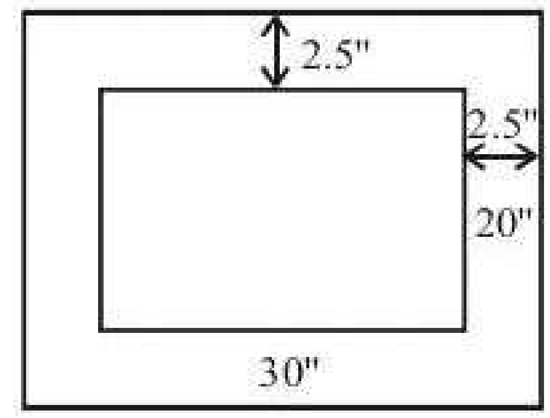
29. (a) Area of the square $= (10)^2 = 100$ cm^2
The largest possible circle would be as shown in the figure below :



Area of the circle $= \frac{22}{7} \times (5)^2 = \frac{22 \times 25}{7}$

Required ratio $= \frac{22 \times 25}{7 \times 100} = \frac{22}{28} = \frac{11}{14}$

$$= 0.785 \approx 0.8 = \frac{4}{5}$$

30. (a)


Length of frame $= 30 + 2.5 \times 2 = 35$ inch
Breadth of frame $= 20 + 2.5 \times 2 = 25$ inch
Now, area of picture $= 30 \times 20 = 600$ sq. inch
Area of frame $= (35 \times 2.5) + (25 \times 2.5) = 150$

$$x = \frac{600}{150} = 4\text{times}$$

31. (a) When folded along breadth, we have :

$$2\left(\frac{l}{2} + \text{b}\right) = 34 \text{ or } l + 2\text{b} = 34 \qquad \text{...(i)}$$

When folded along length, we have :

$$2\left(l + \frac{\text{b}}{2}\right) = 38 \text{ or } 2l + \text{b} = 38 \qquad \text{....(ii)}$$

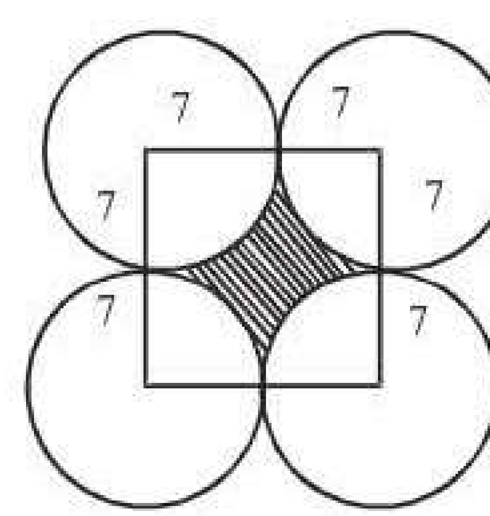
Solving (i) and (ii), we get :
$l = 14$ and b = 10.
$\therefore$ Area of the paper $= (14 \times 10)$ cm^2 = 140 cm^2.

32. (a) Area left after laying black tiles
$= [(20 - 4) \times (10 - 4)]$ sq. ft. = 96 sq. ft.

Area under white tiles $= \left(\frac{1}{3} \times 96\right)$ sq. ft = 32 sq. ft.

Area under blue tiles = (96 – 32) sq. ft = 64 sq. ft.

Number of blue tiles $= \frac{64}{(2 \times 2)} = 16.$

33. (b)


The shaded area gives the required region.
Area of the shaded region = Area of the square – area of four quadrants of the circles

$$= (14)^2 - 4 \times \frac{1}{4}\pi(7)^2$$

$$= 196 - \frac{22}{7} \times 49 = 196 - 154 = 42\text{ cm}^2$$

34. (c) Let h = 2x metres and $(l + \text{b}) = 5$x metres.

Length of the paper $= \frac{\text{Total cost}}{\text{Rate per m}} = \frac{260}{2}$ m = 130 m.

Area of the paper $= \left(130 \times \frac{50}{100}\right) m^2 = 65 m^2$.

Total area of 4 walls $= (65 + 15) m^2 = 80 m^2$.

$\therefore 2(l + b) \times h = 80 \Rightarrow 2 \times 5x \times 2x = 80$

$\Rightarrow x^2 = 4 \Rightarrow x = 2$.

$\therefore$ Height of the room = 4 m.

35. (b) Diagonal of a square (d) = $\sqrt{2} \times$ side of square (a).

$$d = \sqrt{2}a \Rightarrow a = \frac{d}{\sqrt{2}}$$

Area of square $\Rightarrow a^2 = \frac{d^2}{2}$

Now, diagonal gests doubled

$$a = \frac{(2d)}{\sqrt{2}}$$

Area of square $= \left(\frac{2d}{\sqrt{2}}\right)^2 = 4\left(\frac{d^2}{2}\right)$

$\frac{d^2}{2}$ is area of square

Therefore, Area will be four temes.

36. (b) Let r be the radius of circle.

$\pi r^2 = 154 \text{ cm}^2$

$r^2 = \frac{154}{22} \times 7 = 49$

r = 7 cm

length of wire = circumference of circle

$= 2 \times \frac{22}{7} \times 7 = 44$ cm

Now, Perimeter of equilateral triangle = 44 cm

side $= \frac{44}{3}$ cm

Area of equilateral triangle $= \frac{\sqrt{3}}{4} \times \left(\frac{44}{3}\right)^2$

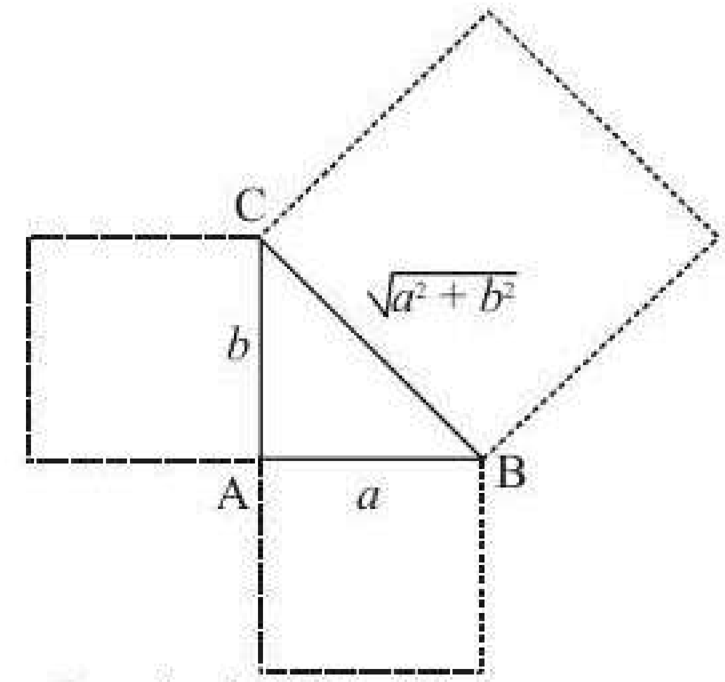
$= \frac{484\sqrt{3}}{9} = 91.42 \text{ cm}^2$

Area of equilateral triangle is nearly equal to 90.14 cm^2

Hence, option (b) is correct.

37. (c) In ΔABC, Use Pythagorias theorem,

$BC = \sqrt{AB^2 + AC^2} = \sqrt{a^2 + b^2}$

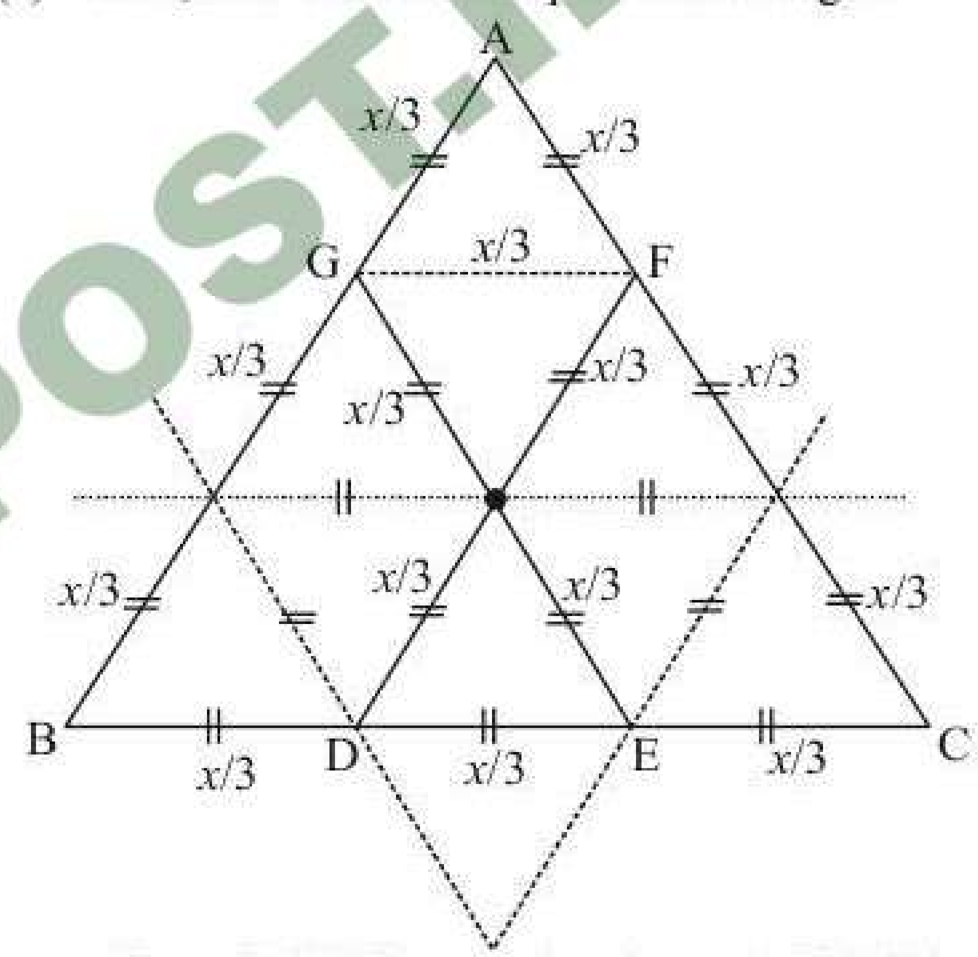


$\therefore$ Required total area

$= a^2 + b^2 + \left(\sqrt{a^2 + b^2}\right)^2 + \frac{1}{2}ab$

$= 2(a^2 + b^2) + 0.5ab$

38. (b) Here, ΔABC forms an equilateral triangle.



where, AGFH form a rhombus and ΔHDE is also an equilateral triangle.

$\therefore$ Area of rhombus AGHF

= (Area of ΔAGF + Area of ΔGFH)

$= \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2 + \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2 = 2 \times \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2$

Now, area of $\Delta HDE = \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2$

and area of $\Delta ABC = \frac{\sqrt{3}}{4}x^2$

By given condition,

$$\frac{\text{Area of rhombus AGHF} + \text{Area of } \Delta \text{HDE}}{\text{Area of } \Delta \text{ABC}}$$

$$= \frac{2 \times \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2 + \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2}{\frac{\sqrt{3}}{4}x^2}$$

$$= \frac{3\times\frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2}{\frac{\sqrt{3}}{4}x^2} = \frac{3}{9} = \frac{1}{3}$$

39. (c) Let the radius of circle is r and the side of a square is a, then by given condition.

$$2\pi r = 4a \Rightarrow a = \frac{\pi r}{2}$$

$\therefore$ Area of square

$$= \left(\frac{\pi r}{2}\right)^2 = \frac{\pi^2 r^2}{4} = \frac{9.86r^2}{4} = 2.46r^2$$

and area of circle $= \pi r^2 = 3.14r^2$

and let the side of equilateral triangle is x.

Then, by given condition,

$$3x = 2\pi r \Rightarrow x = \frac{2\pi r}{3}$$

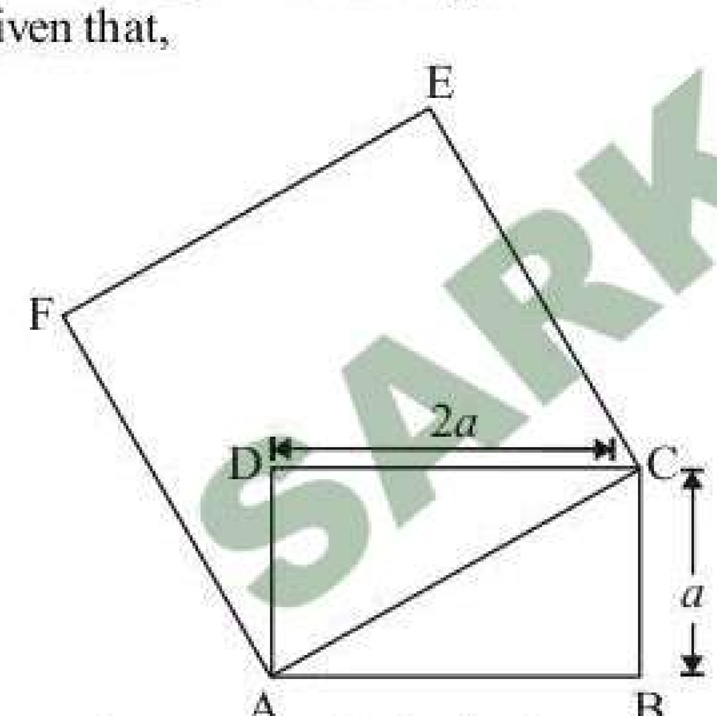
$\therefore$ Area of equilateral triangle

$$= \frac{\sqrt{3}}{4}x^2 = \frac{\sqrt{3}}{4}\times\frac{4\pi^2 r^2}{9}$$

$$= \frac{\pi^2}{3\sqrt{3}}r^2 = 1.89r^2$$

Hence, Area of circle > Area of square > Area of equilateral triangle

40. (d) Given that,



Area of rectangle $= 2a^2 = l \times b$

$\Rightarrow l \times b = 2a^2 = l \times a \Rightarrow l = 2a$

Now, in ΔACD,

$AC^2 = AD^2 + CD^2 = a^2 + 4a^2 = 5a^2$

$\therefore$ $AC = \sqrt{5}a$ unit

Hence, area of square

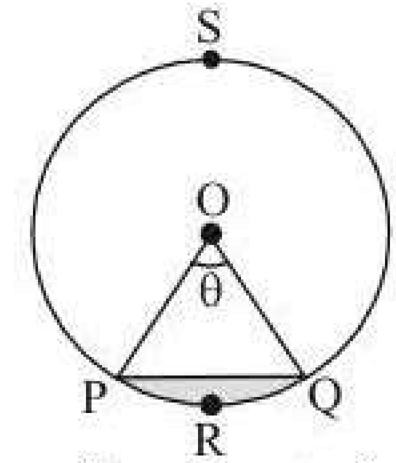
$= (a\sqrt{5})^2$

$= 5a^2$ sq units

41. (c) I. We know that, Area of segment (PRQP)

= Area of sector (OPRQO) – Area of ΔOPQ

$$= \frac{\pi r^2\theta}{360} - \frac{1}{2}r^2\sin\theta$$



So, the area of a segment of a circle is always less than area of its corresponding sector.

II. Distance travelled by a circular wheel of diameter $2d$ cm in one revolution $= 2\pi\frac{(2d)}{2} = 2\times 3.14\times d$

$= 6.28\,d$

which is greater than $6d$ cm.

Therefore, statement I and II both are correct.

42. (c) Let the radius of a circle is r and a be the length of the side of a square.

Given, circumference of a circle = Perimeter of a square

$\Rightarrow 2\pi r = 4a$

$\Rightarrow a = \frac{\pi}{2}r = 1.57r$

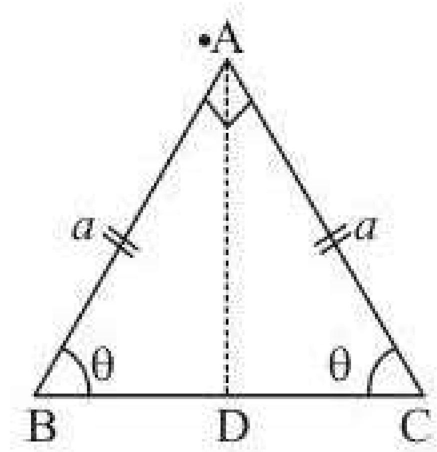
Now, area of the circle $(A_c) = \pi r^2 = 3.14r^2$

and area of the square $(A_s) = a^2 = 2.4649r^2$

$\therefore$ Area of circle > Area of square

43. (d) Let AB = AC = a

$\therefore$ $BC^2 = AB^2 + AC^2$ (by Pythagoras theorem)



In ΔABC, $= a^2 + a^2 = 2a^2 \Rightarrow BC = a\sqrt{2}$

$90^\circ + \theta + \theta = 180^\circ$

(since, sum of all interior angles of any triangle is 180°)

$\Rightarrow 2\theta = 90^\circ$

$\therefore \theta = 45^\circ$

Now, in ΔABD,

$$\sin 45^\circ = \frac{AD}{a} \Rightarrow AD = \frac{a}{\sqrt{2}}$$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\times AD\times BC$

44. (b) Area of regular polygon

$$= \frac{na^2}{4}\cot\frac{180^\circ}{n}$$

here polygon is hexagon so $n = 6$

Now, $\frac{6\times a^2}{4}\cot\frac{180^\circ}{6} = 96\sqrt{3}$

$\Rightarrow \dfrac{6a^2}{4} \times \cot 30^\circ = 96\sqrt{3}$

$\Rightarrow \dfrac{6a^2}{4} \times \sqrt{3} = 96\sqrt{3}$

$a^2 = 64$

$\boxed{\because a = 8}$

Perimeter of a regular hexagon

$= 6 \times \text{side} = 6 \times 8 = 48$ cm

$= \dfrac{1}{2} \times a\sqrt{2} \times \dfrac{a}{\sqrt{2}} = 1$ sq unit (given)

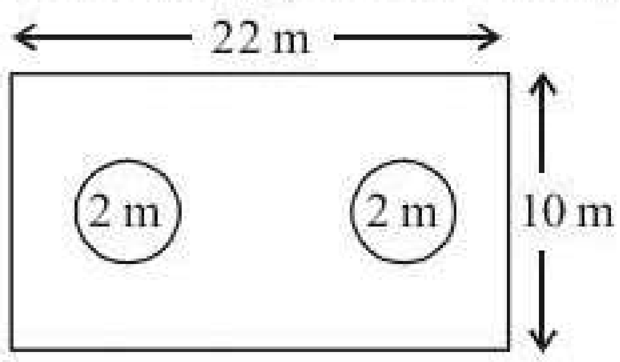
$\Rightarrow \dfrac{a^2}{2} = 1$

$\therefore a = \sqrt{2}$

$\therefore$ Perimeter of $\Delta ABC = 2a + a\sqrt{2} = 2\sqrt{2} + \sqrt{2} \times \sqrt{2}$

$= 2(1+\sqrt{2})$ units

45. (c) Volume of mud dug out in two hemispherical pitholes



$= 2 \times \dfrac{2}{3}\pi r^3 = 2 \times \dfrac{2}{3} \times \dfrac{22}{7} \times 2^3 = \dfrac{2 \times 2 \times 22 \times 8}{21} = \dfrac{704}{21}$ m^3

Area on which the mud is spread over

= Area of field – Area of pitholes

$= l \times b - 2 \times \pi r^2 = 22 \times 10 - 2 \times \dfrac{22}{7} \times 2^2$

$= 220 - \dfrac{176}{7} = \dfrac{1540 - 176}{7} = \dfrac{1364}{7}$ m^2

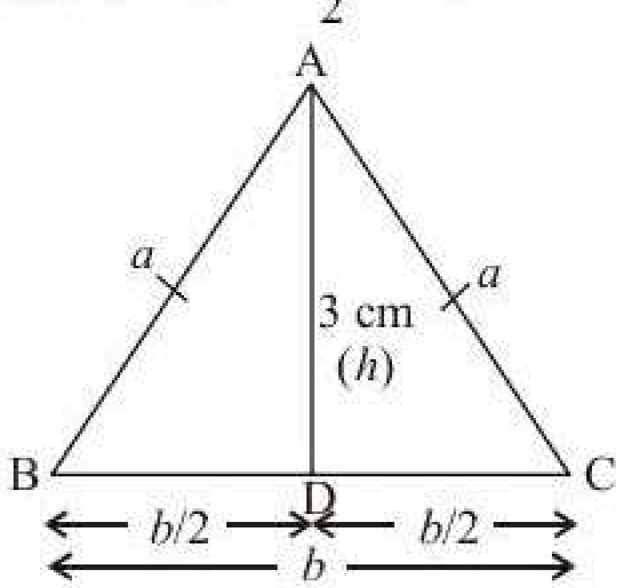
Now, let the rise in level by h m, then

Area of remaining field $\times h$ = Volume of mud dugged out

$\Rightarrow \dfrac{1364}{7} \times h = \dfrac{704}{21}$

$\therefore h = \dfrac{704 \times 7}{1364 \times 21} = \dfrac{16}{93}$ m

46. (a) Area of the $\Delta ABC = \dfrac{1}{2} \times b \times h$



$\Rightarrow 12 = \dfrac{1}{2} \times b \times 3$

$\therefore b = \dfrac{12 \times 2}{3} = 8$ cm

Here, $BD = CD = \dfrac{b}{2} = \dfrac{8}{2} = 4$ cm

In right angled ΔABD, by Pythagoras theorem,

$AB = \sqrt{BD^2 + AD^2}$

$\Rightarrow a = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$ cm

Now, perimeter of an isosceles triangle

$= 2a + b = 2 \times 5 + 8 = 10 + 8 = 18$ cm

47. (a) Let the breadth and height of room be b and h m, respectively.

Then, according to the question,

$\Rightarrow l \times b = n \times$ Area occupied by one patient

$\Rightarrow 14 \times b = 56 \times 22$

$\Rightarrow b = \dfrac{56 \times 2.2}{14} = 8.8$ m

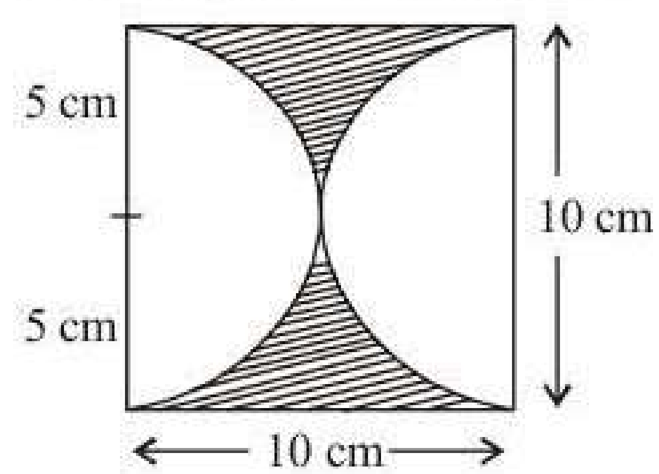
Now, total volume of the room is equal to total patients multiplied by volume occupied by each patient.

Then, $14 \times 8.8 \times h = 8.8 \times 56$

$\therefore h = \dfrac{8.8 \times 56}{14 \times 8.8} = 4$ m

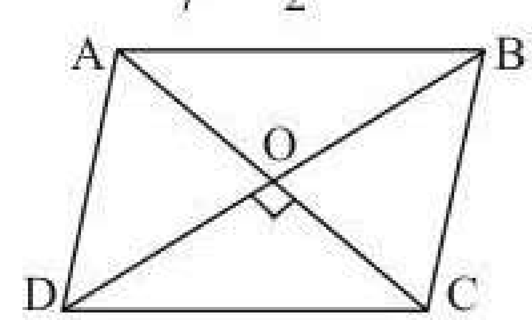
48. (a) Area between square and semi-circles

= Area of square – 2x Area of semi-circle



$= (10)^2 - 2\text{x}\dfrac{22}{7} \times \dfrac{(5)^2}{2} = 100 - 78.5 = 21.5$ cm^2

49. (b)



Here, $OD = \dfrac{BD}{2} = \dfrac{4.8}{2} = 2.4$; $OC = \dfrac{AC}{2} = \dfrac{1.4}{2} = 0.7$

Since, in rhombus diagonal bisect at 90°. Then, in ΔODC.

$OD^2 + OC^2 = CD^2$

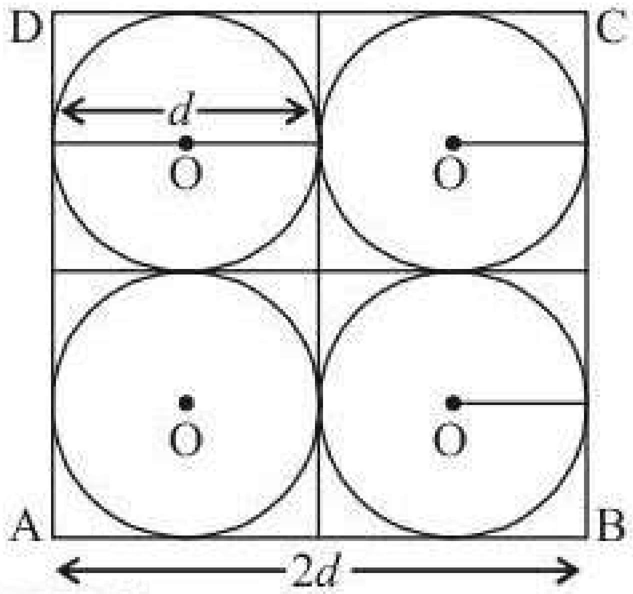
$\Rightarrow CD = \sqrt{OD^2 + OC^2}$

$= \sqrt{(2.4)^2 + (0.7)^2}$

$CD = \sqrt{6.25} \Rightarrow CD = 2.5 = \dfrac{5}{2}$

$\Rightarrow$ Perimeter of rhombus = 4(side) $= 4 \times \dfrac{5}{2} = 10$ cm

50. (c) From the figure it is clear that, 4 circular plates of diameter can be made of a Square plate of side 2d with minimum loss of material.



Let ABCD be square

Diameter of circle = d

Radius of circle = $\frac{d}{2}$

Here from figure it is clear that side of the square is equal to diameter of two circle

Side of square = $d + d = 2d$

= AB = BC = CD = DA

Therefore, perimeter of square = no. of circular plates × sum of diameter two circular plates

$\Rightarrow$ (2d + 2d + 2d + 2d) no. of circular plates × 2d

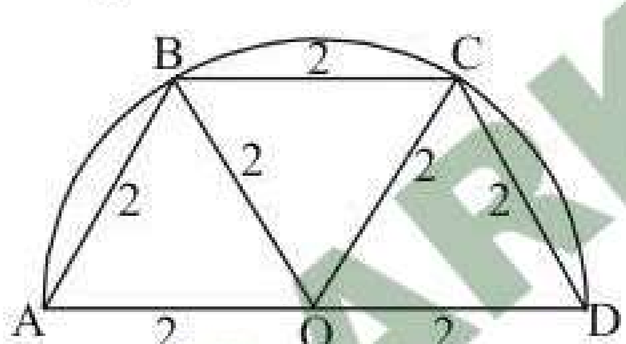
no. of circular plates = $\frac{8d}{2d} = 4$

51. (d) Here, $\Delta AOB, \Delta BOC$ and ΔCOD are equilateral triangles.

$\therefore$ Side = 2 cm

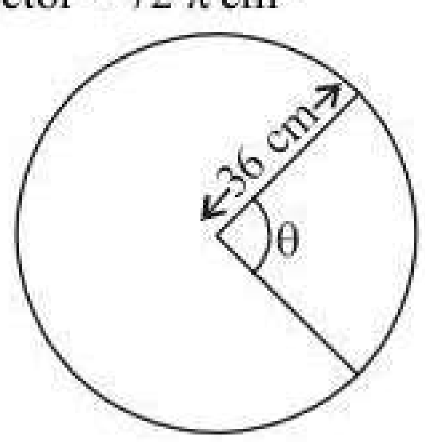
Total area of three equilateral triangles

$= 3\times\frac{\sqrt{3}}{4}\,(\text{Side})^2$



$= 3\times\frac{\sqrt{3}}{4}\times 4 = 3\sqrt{3}\ \text{cm}^2$

52. (d) Area of sector = $72\,\pi\ \text{cm}^2$

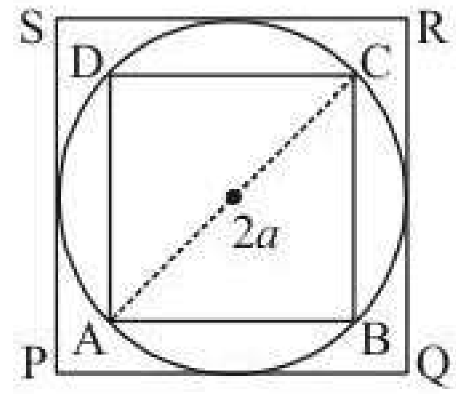


$\Rightarrow \frac{\pi r^2\theta}{360^o} = 72\,\pi$

$\therefore\ \theta = \frac{72\times 360}{36\times 36} = 20^o$

Length of arc = $\frac{\pi r\,\theta}{180^o} = \frac{\pi\times 36\times 20}{180} = 4\pi$ cm

53. (b)



For inscribed square.

Diameter of circle = Diagonal of square

using Pythagoras theorem,

$\Rightarrow AB^2 + BC^2 = AC^2$

$\Rightarrow 2AB^2 = (2a)^2$ [$\therefore$ AB = BC]

$2AB^2 = 4a^2$

$\therefore\ AB^2 = 2a^2$

$\because\ AB = \sqrt{2}a$

Area of square ABCD = $(\sqrt{2}a)^2 = 2a^2$

For circumscribed circle,

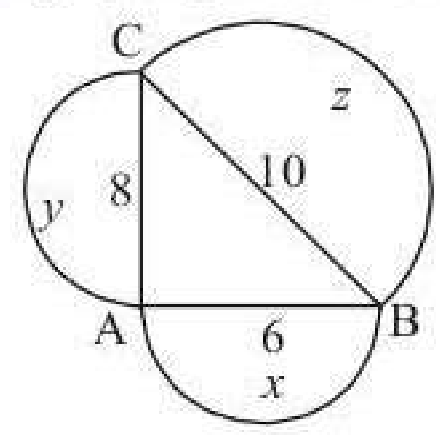
Diameter of circle = Side of square = $2a$

Area of square PQ RS = $(2a)^2 = 4a^2$

Difference between area of outer square and inner square

$= 4a^2 - 2a^2 = 2a^2$

54. (c) In ΔABC, by Pythagoras theorem,



$BC^2 = AB^2 + AC^2 = 36 + 64 = 100$ cm

$\therefore$ BC = 10 cm.

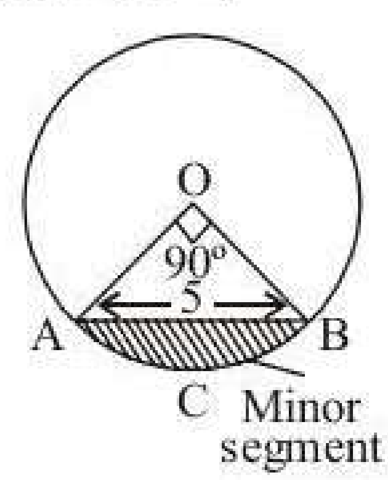
Now, area of semi-circle $x = \frac{\pi(3)^2}{2} = \frac{9\pi}{2}\ \text{cm}^2$

Area of semi-circle $y = \frac{16\pi}{2}\ \text{cm}^2$

Area of semi-circle $z = \frac{25\pi}{2}\ \text{cm}^2$

Now, value of $x + y - z = \left(\frac{9\pi}{2}+\frac{16\pi}{2}\right) - \frac{25\pi}{2} = 0$

55. (c) In ΔAOB, AO = OB = r



Using Pythagoras theorem,

$AB^2 = OA^2 + OB^2 \Rightarrow (5)^2 = r^2 + r^2$

$\therefore\ r^2 = \frac{25}{2}$ cm

Now, area of sector

$AOB = \frac{\theta}{360^\circ} \times \pi r^2 = \frac{90^\circ}{360^\circ} \times \pi \times \frac{25}{2} = \frac{25\pi}{8} \text{ cm}^2$

Now, area of minor segment

= Area of sector – Area of triangle

$= \frac{25\pi}{8} - \frac{r^2}{2} = \frac{25\pi}{8} - \frac{25}{4} = \left(\frac{25\pi - 50}{8}\right)$

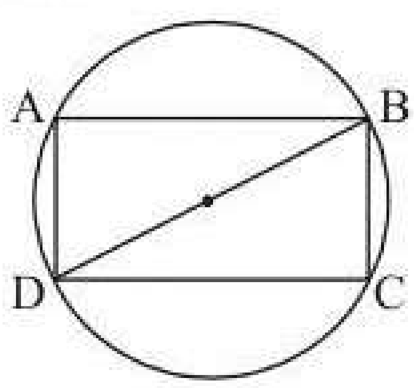
Area of major segment = Area of circle – Area of minor segment

$= \pi r^2 - \left(\frac{25\pi - 50}{8}\right) = \frac{25\pi}{2} - \frac{(25\pi - 50)}{8}$

$= \frac{100\pi - 25\pi + 50}{8} = \frac{75\pi + 50}{8}$

$= \frac{25}{8}(3\pi + 2) = \frac{25}{4}\left(\frac{3\pi}{2} + 1\right) \text{cm}^2$

56. (c) ABCD be the rectangle inscribed in the circle of diameter 5 cm.



∴ Diameter = Diagonal of rectangle

Now, let x and y be the lengths and breadths of rectangle, respectively.

Now in ΔABD,

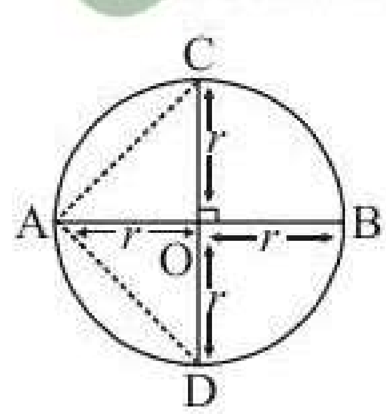
$AB^2 + AD^2 = (5)^2 \Rightarrow x^2 + y^2 = 25$

Since, they form Pythagoras triplet,

∴ $x = 4$ and $y = 3$

So, area of rectangle = 3 × 4 = 12 cm^2

57. (b) Required ratio $= \frac{\text{Area of circle}}{\text{Area of } \Delta ACD}$



$= \frac{\pi r^2}{\frac{1}{2} \times 2r \times r} = \pi$

58. (a) Area of triangular field

$= \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{a+b+c}{2} = \frac{41+40+9}{2} = 45 \text{ m}$

Area $= \sqrt{45 \times (45-41) \times (45-40) \times (45-9)}$

$= \sqrt{45 \times 4 \times 5 \times 36}$

$= 180 \text{ m}^2 = 1800000 \text{ cm}^2$

Number of rose bed $= \frac{1800000}{900} = 2000.$

59. (c) $\frac{r_2}{r_1} = \frac{23}{22}$

$\Rightarrow \frac{r_2 - r_1}{r_1} = \frac{23 - 22}{22}$

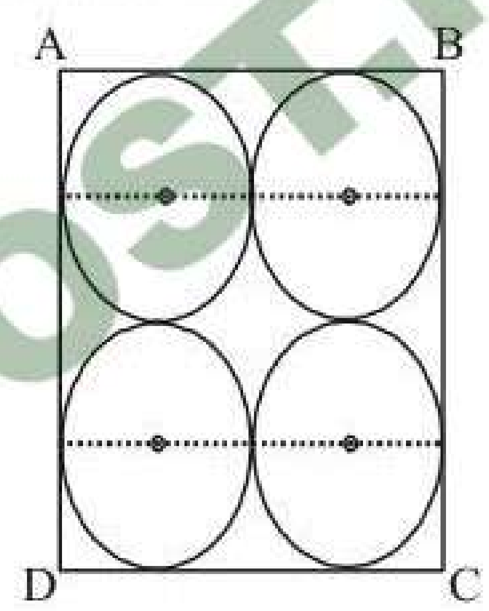
$\Rightarrow \frac{r_2 - r_1}{r_1} = \frac{1}{22}$

$\Rightarrow \frac{5}{r_1} = \frac{1}{22}$

$r_1 = 110 \text{ m}$

Diameter of inner circle = 110 × 2 m = 220 m

60. (d)



Let ABCD is square

Side of square $(a) = \sqrt{784} = 28$ cm

Diameter of single circle $= \frac{28}{2} = 14$ cm

Radius of single circle $= \frac{14}{2} = 7$ cm

Circumference of each plate $= 2\pi r$

$= 2 \times \frac{22}{7} \times 7 = 44 \text{ cm}$

61. (c) Let radius = r

Length of an arc = 100 m

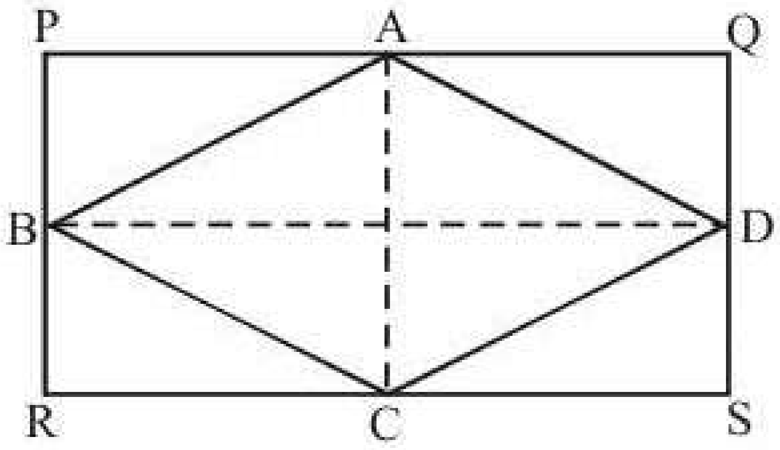
But length of arc $= 2\pi r\left(\frac{\theta}{360}\right)$

$\Rightarrow 100 = 2\pi \times r \times \frac{36}{360}$

$\Rightarrow r = \frac{100 \times 360}{2\pi \times 36} = \frac{500}{\pi} \text{m}$

∴ Option (c) is correct.

62. (b) Let PQRS be a rectangle and ABCD be a rhombus which is formed by joining the mid points of a rectangle.



Given Area of rhombus = 2 unit.

But Area $= \frac{1}{2} \times d_1 \times d_2 = 2$

$d_1 d_2 = 2 \times 2 = 4$ units

where d_1 = diagonal AC

d_2 = diagonal BD

But AC = Breadth of rectangle

BD = length of rectangle

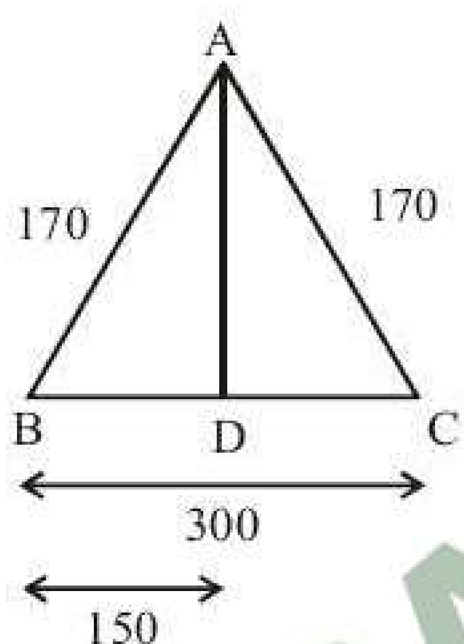
$\Rightarrow$ Area of rectangle = AC × BD

$= d_1 \times d_2$

= 4 units

$\therefore$ option (b) is correct.

63. (c) Let ABC be an isosceles triangle.



Area $= \frac{1}{2} \times AD \times BC$

$= \frac{1}{2} \times \sqrt{(170)^2 - (150)^2} \times 300$

$= \frac{1}{2} \times \sqrt{28900 - 22500} \times 300$

($\because \Delta ADC$ is a right angled triangle then by pythagoras theorem, we find AD)

$= 150 \times \sqrt{6400}$

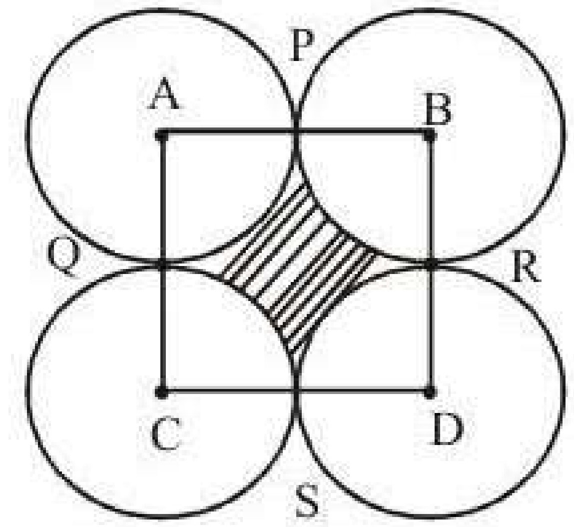
$= 150 \times 80$

= 12000 units.

$\therefore$ Option (c) is correct.

64. (d) Given Area of shaded region $= \frac{150}{847} \text{cm}^2$

We have to find radius of each disc.



Since these discs are equal then their radius are also equal.

$\Rightarrow$ AC = AB = BD = DC = Diameter of disc.

Let r be radius of disc.

Area of square ABCD $= (AB)^2 = (2r)^2 = 4r^2$

Area of shaded region = Area of square – 4 (Area of sector APQ)

Area of sector APQ $= \frac{\theta}{360} \times \pi r^2$

$[\because \angle A = \angle B = \angle C = \angle D = 90°]$

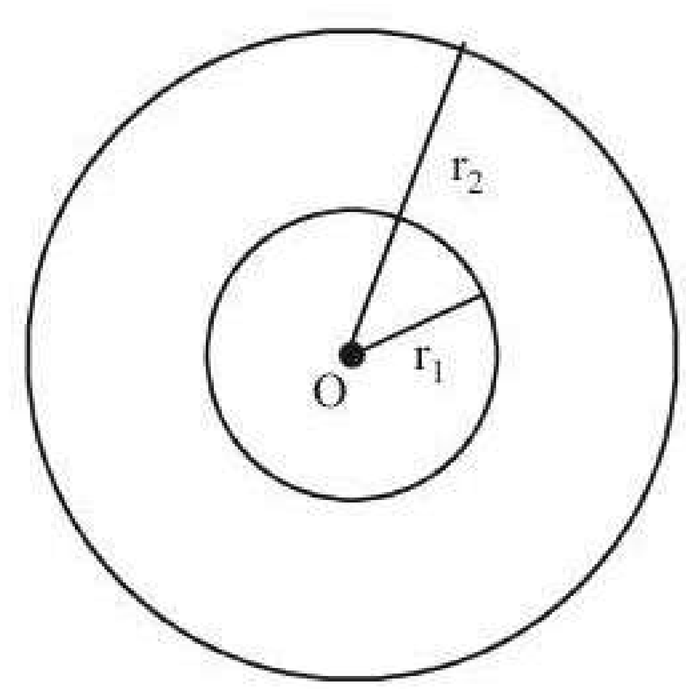
$\frac{150}{847} = 4r^2 - 4 \times \frac{90}{360} \times \frac{22}{7} \times (r)^2$

$\frac{150}{847} = 4r^2 - \frac{22}{7} r^2 \Rightarrow \frac{6r^2}{7} = \frac{150}{847}$

$r^2 = \frac{25}{121} \Rightarrow r = \frac{5}{11}$ cm

$\therefore$ Option (d) is correct.

65. (c) Let two cocentric circular rings with centre O and



Radius of large ring = r_2

Radius of smaller ring = r_1

Area of circular both

= Area of larger ring – Area of smaller ring ...(1)

Given circumference of larger ring = 3 × circumference of smaller ring

$\Rightarrow 2\pi r_2 = 3 \times 2\pi r_1$

$\Rightarrow r_2 = 3r_1$

Also given

Area of circular both = n (area of smaller ring) ...(2)

Comparing (1) & (2) we have

Area of larger ring – Area of smaller ring

= n(Area of smaller ring)

$\Rightarrow \pi r_2^2 - \pi l_1^2 = n\left(\pi r_1^2\right)$

$\Rightarrow \pi r_2^2 = (n+1)\pi r_1^2$

$\Rightarrow (n+1)r_1^2 = r_2^2$

$\Rightarrow (n+1)r_1^2 = (3r_1)^2$

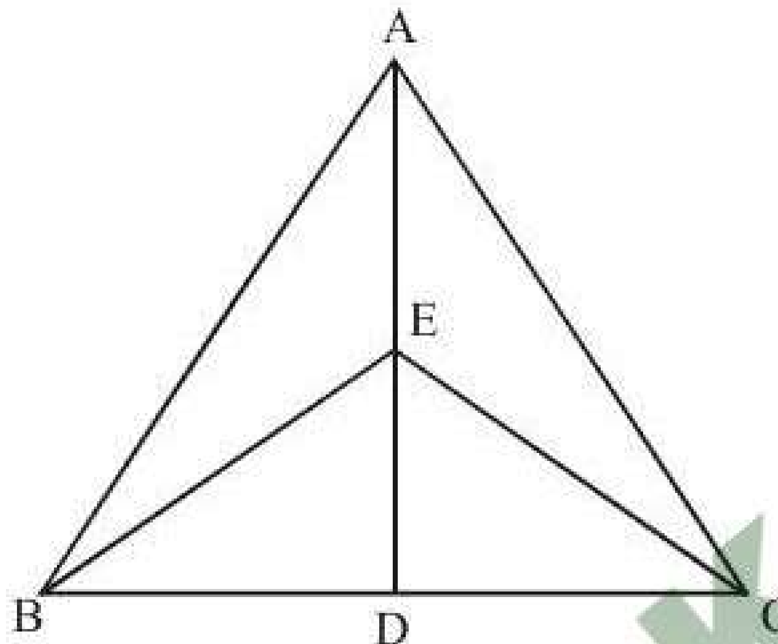
$(\because r_2 = 3l_1)$

$\Rightarrow n+1 = 9$

$\Rightarrow n = 8$

$\therefore$ Option (c) is correct.

66. (c) ABC is a triangle in which D and E are the mid points of BC and AD respectively.



$\Rightarrow AE = ED = \frac{AD}{2}$ and $BD = DC = \frac{BC}{2}$

Statement (1) $\frac{\text{Area of triangle ABC}}{\text{Area of triangle BED}}$

$= \frac{\frac{1}{2}\times BC\times AD}{\frac{1}{2}\times ED\times BD} = \frac{\frac{1}{2}\times BC\times AD}{\frac{1}{2}\times\frac{1}{2}AD\times\frac{1}{2}BC}$

$= \frac{1}{\frac{1}{4}} = 4$

Area of ΔABC = 4 Area of ΔBED

(1) is true.

Statement (2) $\frac{\text{Area of }\Delta ADC}{\text{Area of }\Delta BED} = \frac{\frac{1}{2}\times AD\times DC}{\frac{1}{2}\times BD\times ED}$

$= \frac{AD\times DC}{DC\times\frac{1}{2}AD}$

$\left(\because BD = DC \text{ and } ED = \frac{1}{2}AD\right)$

$= 2$

Area of ΔADC = 2 Area of ΔBED

2 is true.

$\therefore$ Option (c) is correct.

67. (d)

68. (b) 437 = 19 (23)

There are 19 rows with 23 trees in each row.

The distance between any two adjacent plants is 2m and the distance between any two adjacent rows is 2m. $\therefore$ The dimensions of the orchard are

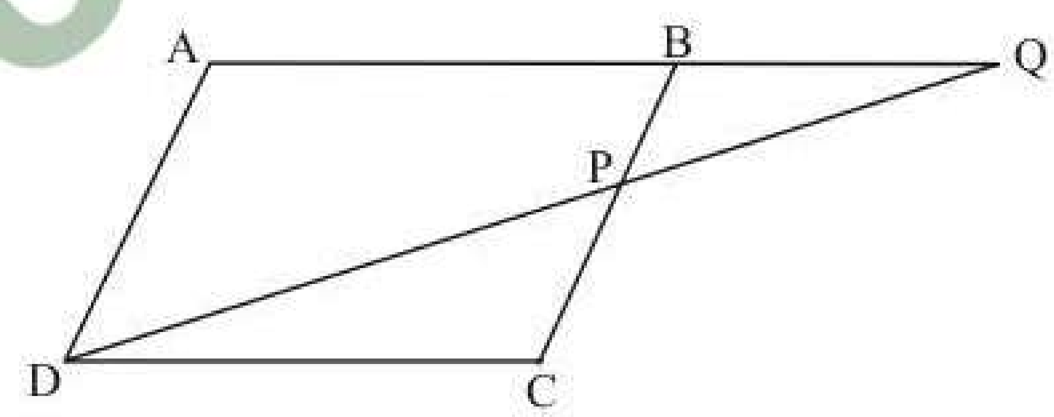
Length = [1 + 22 × 2 + 1] = 46m

Breadth = [1 + 18 × 2 + 1] = 38m

Perimeter = 2(46 + 38) = 168 m

Cost of fencing = 100 × 168 = ₹16800

69. (d)



ΔBPQ and ΔCPD are similar then,

$\Rightarrow \frac{\text{area of }\Delta BPQ}{\text{area of }\Delta CPD} = \left(\frac{BP}{PC}\right)^2$

$\Rightarrow \frac{20}{\text{area of }\Delta CPD} = \left(\frac{1}{2}\right)^2$

Area of Δ CPD = 4 × 20 = 80 square unit

So, option (d) is correct.

70. (c) The n-sided polygon can be dinded into 'n' triangle with O, the Centre of the circle as one veotex for each triange. The altitude of each triangle is r. Let the sides of the polygon be 'a_1', a_2 ____a_n. (Given $a_1 = a_2$ = _____a_n)

$\therefore$ The area of polygon is $\frac{nr}{2} = \frac{pr}{2}$

Area of polygon = $\frac{a_1 r}{2} + \frac{a_2 r}{2} + --- \frac{a_n r}{2} = \frac{pr}{2}$

So, option (c) is correct.