CGPET 2021

Solved Paper

Question 1

A pendulum suspended from the roof of a railway carriage travelling at a speed v m / s round a curve a metre makes n oscillations per second. If the railway carriage is at rest the same pendulum makes n_1 oscillation per second when the carriage is stationary, the value of v^2 is

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Options:

A. $ag\sqrt{\left(\frac{n^4}{n_1^4}-1\right)}$

B. $ag\sqrt{(n^4-n_1^4)}$

C. $ag\sqrt{\left(1-\frac{n^4}{n_1^4}\right)^4}$

D. $\frac{a}{g}\sqrt{\left(1-\frac{n^4}{n_1^4}\right)}$

Answer: A

Solution:

Solution:

(a) We know that, centripital acceleration,

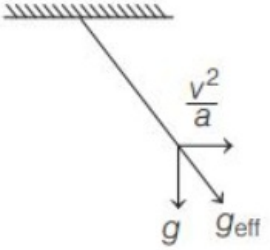
$g_c = \frac{v^2}{a}$

For circular path, effective acceleration of pendulum,

$g_{eff}^2 = g^2 + g_c^2$

$\Rightarrow g_{eff} = \sqrt{g^2 + g_c^2}$

$\Rightarrow g_{eff} = \sqrt{g^2 + (\frac{v^2}{a})^2}$



Now, time period, $T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

When the railway carraiage is travelling, then frequency of oscillation of simple pendulum is given as

$\frac{1}{n} = 2\pi\sqrt{\frac{l}{g_{eff}}}$ . . . . . (i) ($\because T = \frac{1}{n}$)

and when v = 0 (stationary state), then

$\frac{1}{n_1} = 2\pi\sqrt{\frac{l}{g}}$ .....(ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{\frac{1}{n}}{\frac{1}{n_1}} = \frac{2\pi\sqrt{\frac{l}{g_{eff}}}}{2\pi\sqrt{\frac{l}{g}}} \Rightarrow \frac{n_1}{n} = \sqrt{\frac{g}{g_{eff}}}$$

$$\Rightarrow \frac{n_1^2}{n^2} = \frac{g}{g_{eff}} \Rightarrow \frac{n_1^2}{n^2} = \frac{g}{\sqrt{g^2 + (\frac{v^2}{a})^2}}$$

Squaring both side of the above equation, we get

$$(\frac{n_1}{n})^4 = \frac{g^4}{g^2 + (\frac{v^2}{a})^2}$$

$$\Rightarrow g^2 + (\frac{v^2}{a})^2 = (\frac{n}{n_1})^4 g^2$$

$$\Rightarrow (v^2)^2 = a^2 g^2 [(\frac{n}{n_1})^4 - 1]$$

$$\Rightarrow v^2 = ag\sqrt{\left(\frac{n^4}{n_1^4} - 1\right)}$$

Question 2

In Young's double slit experiment, the ratio of amplitude of light coming from two slits is 2:3. If I_0 be the maximum intensity, then the resultant intensity I when they interfere at path difference $\frac{\lambda}{3}$ will be (λ is the wavelength of light used)

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Options:

A. $\frac{7}{25}I$

B. $\frac{9}{25}I$

C. $\frac{5}{7}I_0$

D. $\frac{3}{25}I$

Answer: A

Solution:

Solution:

Ratio of amplitude, $a_1 : a_2 = 2:3$

$$\Rightarrow \frac{a_1}{a_2} = \frac{2}{3}$$

$\therefore$ Ratio of intensities, $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{4}{9}$

Let $I_1 = 4x$ and $I_2 = 9x$

Maximum intensity is given as

$I_0 = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{4x} + \sqrt{9x})^2$

$= 25x$ .....(i)

When path difference $= \frac{\lambda}{3}$, then

phase difference $= \frac{2\pi}{\lambda} \times$ path difference

$= \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3} = 120°$

$\therefore I = I_1 + I_2 + 2\sqrt{I_1 I_2 \cos I\, 120°}$

$= 4x + 9x + 2\sqrt{4x \times 9x \cos I\, 120°}$

$= 13x - 6x$
$\Rightarrow I = 7x$.....(ii)
Hence, $\frac{I}{I_0} = \frac{7x}{25x} \Rightarrow I = \frac{7}{25} . I_0$

Question 3

A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass $= 5.98 \times 10^{24}$ kg) have to be compressed to be a black hole?

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Options:

A. 10^{-6}m

B. 10^{-2}m

C. 10^{2}m

D. 10^{-9}m

Answer: B

Solution:

Solution:
Given, mass of earth, $m = 5.98 \times 10^{24}$ kg
speed of light, $c = 3 \times 10^{8} m/s$
Gravitational constant,
$G = 6.67 \times 10^{-11} N - m^2/kg^2$
For earth to be black hole, the escape velocity should be at least equal to speed of light.
$\therefore$ Escape velocity = speed of light
$\sqrt{2\frac{Gm}{R}} = c$
$\Rightarrow R = \frac{2Gm}{c^2}$
$= \frac{2 \times 6.67 \times 10^{-11} \times 5.9 \times 10^{24}}{(3 \times 10^{8})^2}$
$= \frac{2 \times 6.67 \times 598 \times 10^{-11} \times 10^{24}}{10^{4} \times 9 \times 10^{16}}$
$= \frac{1334 \times 598}{9} \times 10^{-31} \times 10^{24}$
$= 10^{-2}$m

Question 4

What is the moment of inertia of a rod of mass M, length l about an axis perpendicular to it through one end?

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Options:

A. $\frac{Ml^2}{12}$

B. $\frac{Ml^2}{5}$

C. $\frac{Ml^2}{6}$

D. $\frac{Ml^2}{3}$

Answer: D

Solution:

Solution:
Moment of inertia of a rod of mass M and length l about an axis perpendicular to it passing through its centre of mass,
$I_{CM} = \frac{Ml^2}{12}$
According to parallel axes theorem, moment of inertia of rod perpendicular to it and passing through one end,
$I = I_{CM} + M(\frac{l}{2})^2$
$= \frac{Ml^2}{12} + \frac{Ml^2}{4} = \frac{Ml^2}{3}$

Question 5

Two radioactive substances A and B have decay constants 5λ and λ, respectively. At $t = 0$, they have the same number of nuclei, the ratio of number of nuclei of A to those of B will be $(\frac{1}{e})^2$ after a time

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Options:

A. 4λ

B. 2λ

C. $\frac{1}{4}\lambda$

D. $\frac{1}{2}\lambda$

E. None of above

Answer: E

Solution:

Solution:
Given, initial number of nuclei $= N_0$
After time t, remaining number of nuclei,
$N = N_0 e^{-\lambda t} (\text{where}, \lambda = \text{decay constant})$
Given, at time, t = 0
For radioactive substance A,
$N_A = N_0 e^{-5\lambda t}$
Similarly, for B,
$N_B = N_0 e^{-\lambda t}$

$\therefore$ Ratio $\frac{N_A}{N_B} = \frac{N_0 e^{5-\lambda t}}{N_0 e^{-\lambda t}} = \frac{e^{-5\lambda t}}{e^{-\lambda t}}$

$\Rightarrow \frac{N_A}{N_B} = e^{-4\lambda t}$ .....(i)

It is given that after time t,

the ratio, $\frac{N_A}{N_B} = (\frac{1}{2})^2$.....(ii)

From Eqs. (i) and (ii), we get

$\Rightarrow (\frac{I}{e})^2 = e^{-4\lambda t} \Rightarrow e^{-2} = e^{-\lambda}$

Comparing the powers, we get $-2 = -4\lambda t \Rightarrow t = \frac{I}{2\lambda}$

Hence, the ratio of number of nuclei of A to those of B will be $(\frac{I}{e})^2$ after a time of $\frac{I}{2\lambda}$

$\therefore$ No option in the given question is correct

Question 6

The half-life period of a radioactive substance is 5 min. The amount of substance decayed in 20 min will be

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Options:

A. 6.25 %

B. 25 %

C. 75 %

D. 93.75 %

Answer: D

Solution:

Solution:

Given, half - life period of a radioactive substance,

$T_{\frac{1}{2}} = 5$ min

Total time, T = 20 min

Number of half - life,

$n = \frac{\text{Total time}(T)}{\text{Half} - \text{life}\left(T_{\frac{1}{2}}\right)} = \frac{20}{5} = 4$

After n half - life, the amount of substance undecayed,

$N = N_0(\frac{1}{2})^n$

Fraction undecayed $= \frac{N}{N_0} = (\frac{1}{2})^4$

$\therefore$ Fraction decayed $= 1 - \frac{N}{N_0}$

$= 1 - (\frac{1}{2})^4 = \frac{15}{16}$

Therefore, percentage decayed amount

$= (\frac{15}{16} \times 100)\ \% = 93.75\ \%$

Question 7

A capacitor of 10μF charged upto 250V is connected in parallel with another capacitor of 5μF charged upto 100V. The common potential is

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Options:

A. 500V

B. 400V

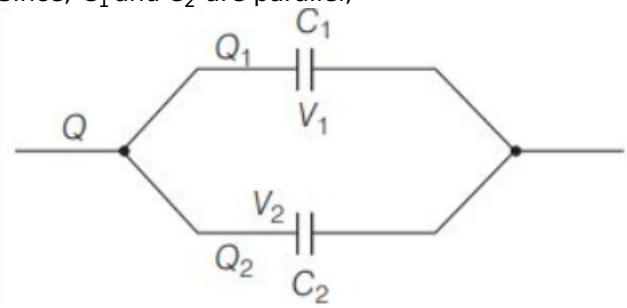
C. 300V

D. 200V

Answer: D

Solution:

Solution:
Given, capacitance of first capacitor,
$C_1 = 10\mu F$
Potential difference across first capacitor,
$V_1 = 250V$
Capacitance of second capacitor, $C_2 = 5\mu F$
Potential difference across second capacitor,
$V_2 = 100V$
Since, C_1 and C_2 are parallel,



$\therefore C_{eff} = C_1 + C_2$
and total charge, $Q = Q_1 + Q_2$
Here, $Q_1 = V_1C_1$ and $Q_2 = V_2C_2$
Now, common potential,

$$V = \frac{Q}{C_{eff}} = \frac{Q_1 + Q_2}{C_1 + C_2}$$

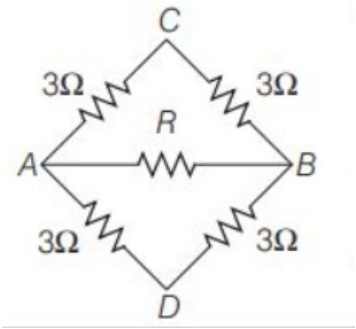
$$V = \frac{V_1C_1 + V_2C_2}{C_1 + C_2}$$

$$= \frac{250 \times 10 \times 10^{-6} + 100 \times 5 \times 10^{-6}}{10 \times 10^{-6} + 5 \times 10^{-6}}$$

$$= \frac{2500 + 500}{15} = \frac{3000}{15} = 200V$$

Question 8

The effective resistance between the points A and D is 3Ω. The value of R in above circuit is



Options:

A. 6Ω

B. 3Ω

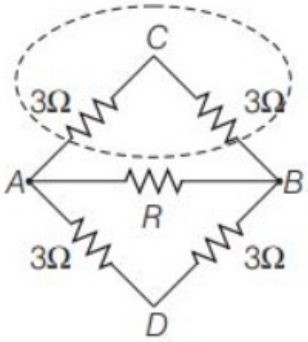
C. 2Ω

D. 1Ω

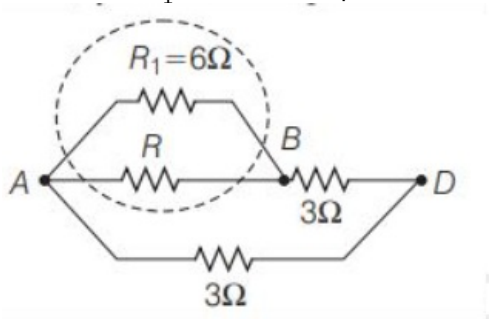
Answer: A

Solution:

Solution:
Given, effective resistance between points A and D,
$R_{AD} = 3\Omega$
The circuit is shown as



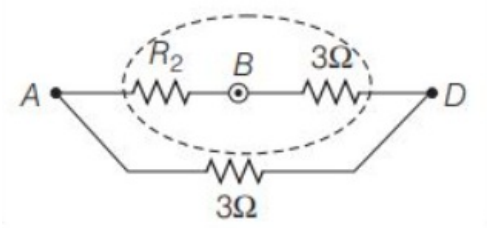
Resistance of sides AC and CB are in series
$\therefore R_1 = 3 + 3 = 6\Omega$.....(i)
Resistance R_1 and R are in paralel



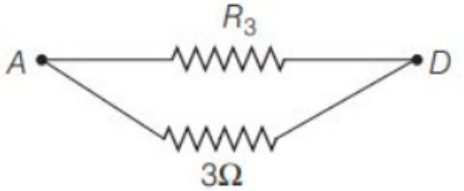
$$\therefore \frac{1}{R_2} = \frac{1}{R} + \frac{1}{R_1}$$

$$\Rightarrow R_2 = \frac{R \cdot R_1}{R + R_1} \text{ .....(ii)}$$

Resistance of sides AB and BD are series,



$R_3 = R_2 + 3$ ....(iii)
Now, resistance R_3 and 3Ω are parallel



$\therefore R_4 = \frac{R_3 \times 3}{R_3 + 3}$.....(iv)
Put the value from Eqs. (i), (ii) and (iii) in Eqs. (iv), we get
$R_4 = \frac{(R_2 + 3)3}{(R_2 + 3) + 3} = \frac{3R_2 + 9}{R_2 + 6}$
$= \frac{3\left(\frac{RR_1}{R + R_1}\right) + 9}{\left(\frac{RR_1}{R + R_1}\right) + 6}$
$\Rightarrow 3 = \frac{(\frac{6R}{R} + 6) + 9}{(\frac{6R}{R} + 6) + 6} (\text{Given}, R_4 = R_{AD} = 3\Omega)$
$\Rightarrow R = 6\Omega$

Question 9

A diatomic gas initially at $18^\circ C$ is compressed adiabatically to one eighth of its original volume. The temperature after compression will be

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Options:

A. $18^\circ C$

B. $395.4^\circ C$

C. $144^\circ C$

D. $887.4^\circ C$

Answer: B

Solution:

Solution:
Given, $T_1 = (18 + 273)K = 291K$ and $T_2 = ?$
Initial volume, $V_1 = V$ (let)
Final volume, $V_2 = \frac{V}{8}$
For diatomic gas, $\gamma = \frac{7}{5}$
Since, the process is adiabatic.
$\therefore TV^{\gamma - 1} =$ constant
$\Rightarrow T_1V_1^{\gamma - 1} = T_2V_2^{\gamma - 1}$

$\Rightarrow T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} . T_1$

$\Rightarrow T_2 = \left(\frac{V}{\frac{V}{8}}\right)^{(\frac{7}{5} - 1)} (291)$

$\Rightarrow T_2 = (8)^{0.4} . (291)$

$\therefore T_2 = 668.5K$

In Celsius scale, $668.5 = 66.85 - 273$

$\Rightarrow T_2 = 395.5°C \simeq 395.4°C$

Question 10

One mole of an ideal monochromatic gas expands till its temperature doubles under the process $V^2T =$ constant. If the initial temperature is 400K, the work done by the gas is

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Options:

A. $400R$

B. $200R$

C. $-200R$

D. indeterminate

Answer: C

Solution:

Solution:

Given, number of mole, $n = 1$

Initial temperature, $T_i = 400K$

Since, final temperature, $T_f = 2T_i$

$\therefore T_f = 800K$

Difference in temperature,

$\Delta T = T_f - T_i = 800K - 400K = 400K$

Now, internal energy, $\Delta U = nC_v . \Delta T$

$\Rightarrow \Delta U = 1 . (\frac{3}{2}R) . 400$

[For monoatomic gas, $C_v = \frac{3}{2}R$]

$\therefore \Delta U = 600R$.....(i)

According to the problem, process is given as

$V^2T = K$ .....(ii)

From the equation of ideal gas for 1 mole,

$pV = RT \Rightarrow T = \frac{pV}{R}$

$\Rightarrow V^2 . (\frac{pV}{R}) = K[\text{From Eq . (ii)}]$

$\Rightarrow V^3p = K$.....(iii)

Standard equation of adiatomic process is

$pV^x = K$

On comparing from Eq. (iii), we get

$x = 3$

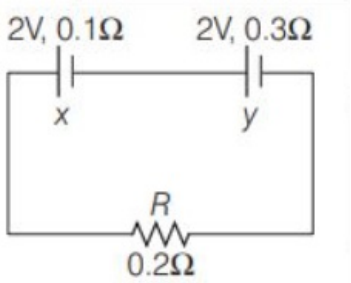
Since, the expression for molar heat capacity,

$C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$

$\Rightarrow C = \frac{R}{(\frac{5}{2} - 1)} + \frac{R}{1 - 3}$

(∵ For monoatomic gas, $\gamma = \frac{5}{2}$)
Solving above equation, we get $C = R$ and heat required,
$\Delta Q = n \,.\, C \,.\, \Delta T = 1 \times R \times 400 = 400R$ .....(iv)
Thus, work done, $\Delta W = \Delta Q - \Delta U$
From Eqs. (i) and (iv), we get
$\Delta W = 400R - 600R = -200R$

Question 11



In above circuit, potential difference between *A* and *B* is

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Options:

A. 0

B. 5V

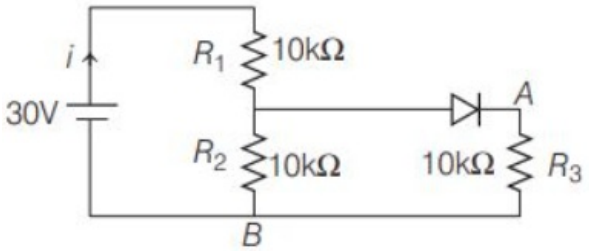
C. 10V

D. 15V

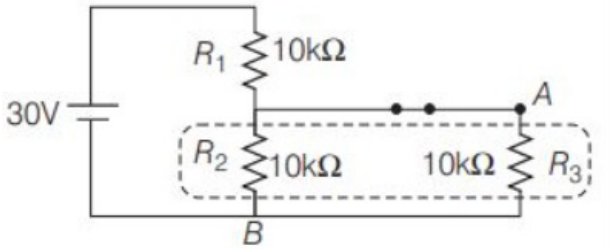
Answer: C

Solution:

Solution:
In this circuit, the diode is connected in forward bias.



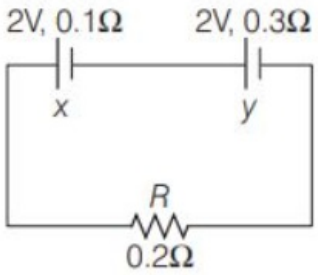
Hence, given circuit is redrawn as



Since, R_2 and R_3 are in parallel, hence their equivalent resistance,
$R' = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{10 \times 10}{10 + 10}$
$\therefore R' = 5k\Omega$
Now, $i = \frac{V}{R_{net}} = \frac{30}{10 + 5} = 2A$
Current through resistance R_3, $I_3 = i \times R_{AB}$

$= i \times \frac{R_2}{R_2 + R_3} = 2 \times \frac{10}{10 + 10} = 1A$
∴ Potential difference between points A and B,
$V_{AB} = I_3 R_3 = 1 \times 10 = 10V$

Question 12



The internal resistance of two cells shown are 0.1Ω and 0.3Ω. If $R = 0.2\Omega$, then the potential difference across the cell

Options:

A. y will be zero

B. x will be zero

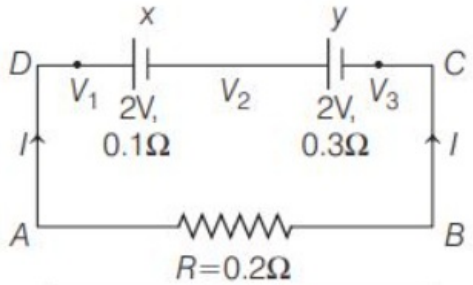
C. x and y will be $2V$

D. x will be $> 2V$ and y will be $< 2V$

Answer: A

Solution:

Solution:
From the given circuit diagram, the internal resistance of both the cells and resistance R connected in series.Consider current I is flowing through the circuit. We use Kirchhoff's voltage law for the closed path (ABCDA),



$-0.2I + 2 - 0.3I + 2 - 0.1I = 0 \Rightarrow 0.6I + 4 = 0$
$\therefore I = \frac{20}{3}A$
Potential difference across cell x,
$V_1 - 2 + 0.1 \times \frac{20}{3} = V_2 \Rightarrow V_1 - V_2 = \frac{4}{3}$
Potential difference acros cell y,
$V_2 - 2 + 0.3 \times \frac{20}{3} = V_3 \Rightarrow V_2 - V_3 = 0$
Hence, the potential difference across cell y is zero.

Question 13

Four rods with different radii r and length l are used to connect two reservoirs of heat at different temperatures. Which one will conduct most heat?

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Options:

A. $r = 1\,\text{cm}, l = 1\text{m}$

B. $r = 2\,\text{cm}, l = 2\text{m}$

C. $r = 1\,\text{cm}, l = \frac{1}{2}\,\text{cm}$

D. $r = 2\,\text{cm}, l = \frac{1}{2}\text{m}$

Answer: C

Solution:

Solution:
Conduction of heat in rod is given by relation
$\frac{Q = KA\,.\,\Delta\theta\,.\,t}{l}$
where, K = coefficient of thermal conductivity of heat,
A = cross - sectional area,
$\Delta\theta$ = temperature difference,
t = time takn by heat flow
and l = length of rod.
If the radius of rod be r, then
$A = \pi r^2 \Rightarrow Q = \frac{K\,.\,\pi r^{2.}\,\Delta\theta\,.\,t}{l}$
Rate of flow of heat through conductor,
$\frac{Q}{t} = \frac{K\pi r^2 \Delta\theta}{l}$
Here, K, π and $\Delta\theta$ are constants.
$\therefore Q \propto \frac{r^2}{l}$
For more value of Q,
(i) r should be maximum.
(ii) l should be minimum.
The value of $(\frac{r^2}{r})$ for each observation,
(a) $r = 1\,\text{cm} = 10^{-2}\text{m}, l = 1\text{m}$
$Q_1 \propto \frac{r^2}{l} = \frac{(10^{-2})^2}{1} = 10^{-4}$
(b) $r = 2\,\text{cm} = 2 \times 10^{-2}\text{m}, l = 2\text{m}$
$Q_2 \propto \frac{r^2}{l} = \frac{(2 \times 10^{-2})^2}{2} = 2 \times 10^{-4}$
(c) $r = 1\,\text{cm} = 10^{-2}\text{m}, l = \frac{1}{2}\,\text{cm} = \frac{1}{2} \times 10^{-2}\text{m}$
$Q_3 \propto \frac{r^2}{l} = \frac{(10^{-2})^2}{(\frac{1}{2} \times 10^{-2})} = 2 \times 10^{-2}$
(d) $r = 2\,\text{cm} = 2 \times 10^{-2}\text{m}, l = \frac{1}{2}\text{m}$
$Q_4 \propto \frac{r^2}{l} = \frac{(2 \times 10^{-2})^2}{\frac{1}{2}} = 8 \times 10^{-4}$
The value of $\frac{r^2}{l}$ is maximum for the dimensions given in option (c), hence it will conduct maximum heat.

Question 14

A wall has two layers A and B, each made of different materials. The thickness of both the layers is the same. The thermal conductivity of A, $K_A = 3K_B$. The temperature difference across the wall is $20^\circ C$. In thermal equilibrium $\left|\begin{matrix}\theta_1 \\ A\end{matrix}\right\|\left\|\begin{matrix}\theta_1 \\ B\end{matrix}\right|\theta_2$

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Options:

A. the temperature difference across $A = 15^\circ C$

B. rate of heat transfer across A is more than across B

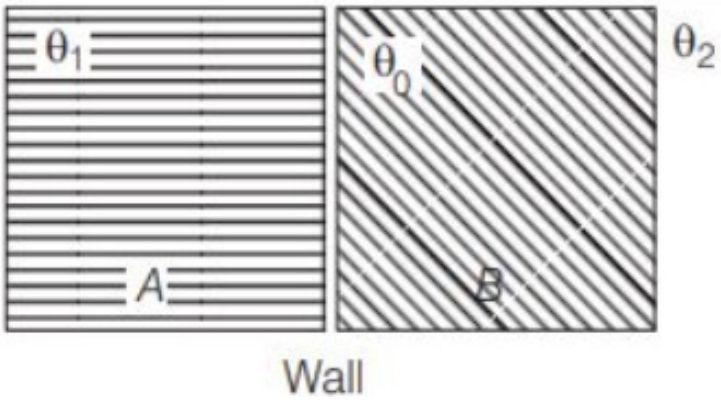
C. rate of heat transfer across both is same

D. temperature difference across B is $15^\circ C$

Answer: C

Solution:

Solution:
Given, $K_A = 3K_B$ and $(\theta_1 - \theta_2) = 20°C$
In the series combination, the rate of flow of heat is same.
i.e. $\frac{Q_A}{t} = \frac{Q_B}{t}$
$\Rightarrow \frac{K_A . A(\theta - \theta_0)}{l} = \frac{K_B . A(\theta_0 - \theta_2)}{l}$



$\Rightarrow 3K_B . (\theta_1 - \theta_0) = K_B . (\theta_0 - \theta_2)$
$\Rightarrow 3\theta_1 - 3\theta_0 = \theta_0 - \theta_2$
Adding θ_1 both sides,
$\Rightarrow 4\theta_1 - 4\theta_0 = \theta_1 - \theta_2$
Since, $(\theta_1 - \theta_2) = 20°C$ (Given)
$\therefore 4(\theta_1 - \theta_0) = 20 \Rightarrow (\theta_1 - \theta_0) = 5°C$
Hence, option (c) is correct.

Question 15

If the frequency of light in a photoelectric experiment is doubled, the stopping potential will

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Options:

A. be doubled

B. be halved

C. become more than double

D. become less than double

Answer: C

Solution:

Solution:
According to Einstein's photoelectric equation,
$E = W_0 + K_{max}$ .....(i)
where, W_0 = work - function,
K_{max} = mximum kinetic energy of emitted electron
and E = energy of incident photon.
If v is the frequency of incident radiations, then
$E = hv$
$hv = W_0 + K_{max}$ [From eq. (i)]
$\Rightarrow K_{max} = hv - w_0$
According to problem

For Frequency (v)	For Frequency (2v)
$K_1 = hv - W_0$	$K_2 = h(2v) - W_0$ $= 2hv - W_0$

$K_2 - 2K_1 = 2hv - W_0 - hv + W_0$
$K_2 - 2K_1 = hv = E$
$\therefore K_2 > 2K_1$

Question 16

The energy of an electron in nth orbit is given by $E_n = \frac{-13.6}{n^2}\,\text{eV}$. The energy required to take an electron from ground state to the second excited state

©

Options:

A. $13.6\,\text{eV}$

B. $12.09\,\text{eV}$

C. $1.51\,\text{eV}$

D. $0.85\,\text{eV}$

Answer: B

Solution:

Solution:
Since the energy required to excite the electron energy levels n_1 to n_2 revolving round the nucleus is given by
$E_{n2} - E_{n1} = \Delta E = 13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right](\text{in eV})$
The energy required to excite the electron from ground state $(n_1 = 1)$ to second excited state $(n = 3)$ is given as
$\Rightarrow \Delta E = 13.6\left[\frac{1}{(1)^2} - \frac{1}{(3)^2}\right]$
$\Rightarrow \Delta E = 12.09\,\text{eV}$

Question 17

The electric potential at a point (x, y) in the xy-plane is given by $V = -kxy$. The electric field intensity at a distance r from the origin varies as

©

Options:

A. $2r^2$

B. $2r$

C. r^2

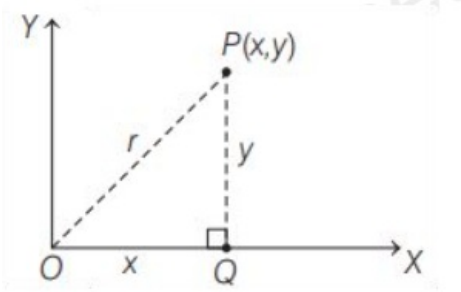
D. r

Answer: D

Solution:

Solution:
If ΔV is the potential difference and Δx is the effective displacement between two points, then the electric field is related with the potential as

$E = \frac{-\Delta V}{\Delta x}$ or $E = -\frac{dV}{dx}$
i.e. Negative rate of change of potential is equal to the electric fields.
$E_x = -\frac{dV}{dx} = \frac{-d}{dx}(-kxy) = +ky$ .....(i)
and along Y - axis,
$E_y = -\frac{dV}{dy} = \frac{-d}{dy}(-kxy) = +kx$ .....(ii)
∴ Resultant electric field,



Put the values from eq. (i) and (ii), we get
$E = \sqrt{(ky)^2 + (kx)^2}$
$\Rightarrow E = k\sqrt{x^2 + y^2}$ .....(iii)
From eq. (iii), we get
$E = k\sqrt{r^2} = kr$
∴ $E \propto r$

Question 18

Three charges $-q, Q$ and $-q$ are placed at equal distances on a straight line. If the potential energy of the system of three charges is zero, then the ratio of $Q:q$ is

©

Options:

A. 2:1

B. 1:2

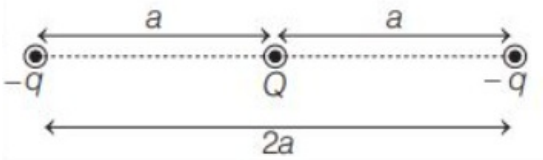
C. 1:4

D. 4:1

Answer: C

Solution:

Solution:
Consider charges $-q, Q$ and $-q$ are placed at equal distance a as shown in figure



For two charges q_1 and q_2, electrostatic potential energy is given by
$U = \frac{kq_1q_2}{r}$ (r = distance between two charges)
So, the potential energy of the given system of charges,
$U = -\frac{kqQ}{a} - \frac{kQq}{a} + \frac{kq \cdot q}{2a}$
According to the problem, U = 0
∴ $-2qQ - 2Qq + q^2 = 0 \Rightarrow -2Q - 2Q + q = 0$

$\therefore \frac{Q}{q} = \frac{1}{4} \Rightarrow Q : q = 1:4$

Question 19

A coefficient of static friction for steel on ice is 0.1. The coefficient of the sliding friction, therefore can be

©

Options:

A. 0.1

B. 0.11

C. 0.08

D. 1.1

Answer: C

Solution:

Solution:
Since, the static friction always less than that of kinetic friction.
Therefore, coefficient of sliding friction ($\mu_{sliding}$) is always less than coefficient of static friction(μ_{static}).
Hence, $\mu_{sliding} < \mu_{static}$
So, the correct option is (c)

Question 20

The time dependence of a physical quantity p is given by $p = p_0 \exp(-at^2)$, where a is a constant and t is the time. The constant a

©

Options:

A. is dimensionless

B. has dimensions $[T^{-2}]$

C. has dimensions $[T^2]$

D. has dimensions of [p]

Answer: B

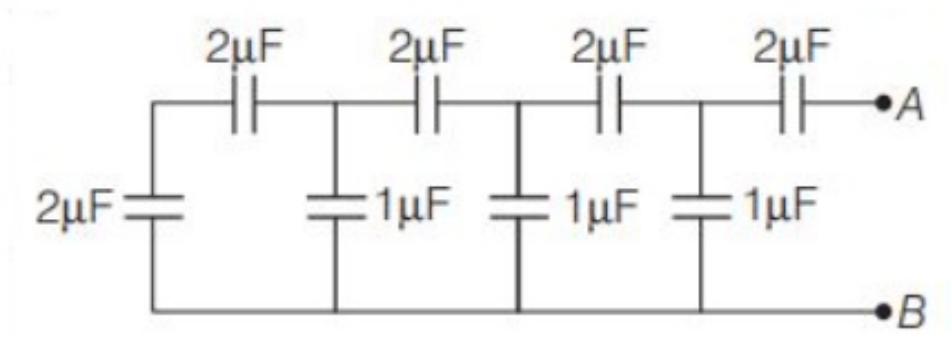
Solution:

Solution:
Given, $p = p_0 \exp(-at^2)$
Since, exponential power is dimensionless,

$\therefore [at]^2 = [M^0L^0T^0]$
$\Rightarrow [a] = \frac{1}{[t^2]} = \frac{1}{[T^2]} \Rightarrow [a] = [T^{-2}]$

Question 21

The equivalent capacitance of the combination as shown in figure between A and B is



Options:

A. $2\mu F$

B. $1\mu F$

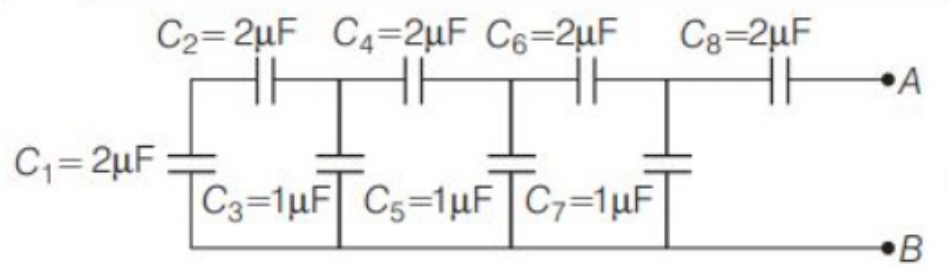
C. $3\mu F$

D. $6\mu F$

Answer: B

Solution:

Solution:
The circuit diagram is given as

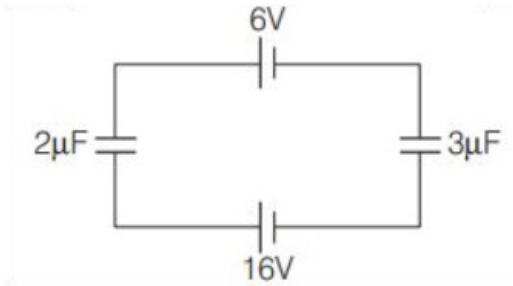


From above circuit diagram,
C_1 and C_2 are in series
$\therefore$ Equivalent capacitance of C_1 and C_2 is given as

$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{2 \times 2}{2 + 2} = 1\mu F$
Now, C_{12} and C_3 are in parallel, hence equivalent capacitance of C_{12} and C_3,
$C_{123} = C_{12} + C_3 = 1 + 1 = 2\mu F$
Now, C_{123} and C_4 are in series, hence their equivalent capacitance,
$C_{1234} = \frac{C_{123} \times C_4}{C_{123} + C_4} = \frac{2 \times 2}{2 + 2} = 1\mu F$
Again, C_{1234} and C_5 are in parallel, hence their equivalent capacitance,
$C_{12345} = C_{1234} + C_5 = 1 + 1 = 2\mu F$
Now, C_{12345} and C_6 are in series, hence their equivalent capacitance,
$C_{123456} = \frac{C_{12345} \times C_6}{C_{12345} + C_6} = \frac{2 \times 2}{2 + 2} = 1\mu F$
Now, C_{123456} and C_7 are in parallel, hence their equivalent capacitance,
$C_{1234567} = C_{123456} + C_7 = 1 + 1 = 2\mu F$
Now, $C_{1234567}$ and C_8 are in series, hence their equivalent capacitance,
$C_{12345678} = \frac{C_{1234567} \times C_8}{C_{1234567} + C_8} = \frac{2 \times 2}{2 + 2} = 1\mu F$
$\therefore C_{AB} = C_{12345678} = 1\mu F$

Question 22

The potential difference across $2\mu F$ capacitor in the circuit shown in



Options:

A. 12V

B. 4V

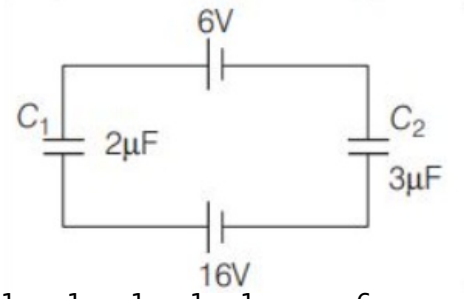
C. 6V

D. 18V

Answer: C

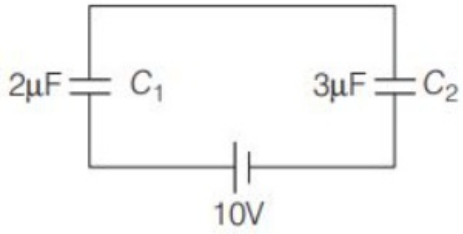
Solution:

Solution:
Since, C_1 and C_2 are connected in series, therefore equivalent capacitance is



$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2} + \frac{1}{3} \Rightarrow C_s = \frac{6}{5}\mu F$
Effective voltage, $V = 16 - 6 = 10V$

New equivalent circuit is



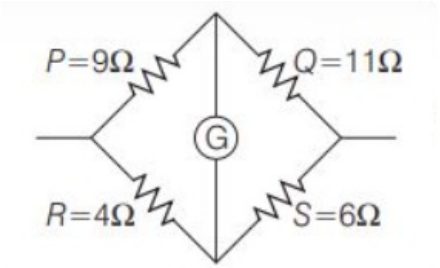
Charges on each capacitor,

$Q = C_s \times V = \frac{6}{5} \times 10 \Rightarrow Q = 12\mu C$

Thus, potential difference across $2\mu F$ capacitor is

$V_{C1}^{'} = \frac{Q}{C_1} = \frac{12}{2} \Rightarrow V_{C1}^{'} = 6V$

Question 23



How much resistance must be put in parallel to the resistance S to balance the above bridge?

Options:

A. 24Ω

B. $\frac{44}{9}\Omega$

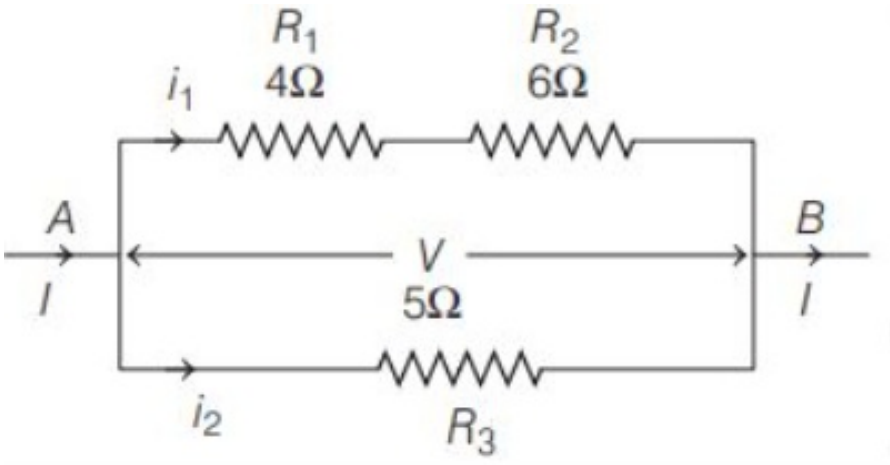
C. $\frac{132}{5}\Omega$

D. 182Ω

Answer: C

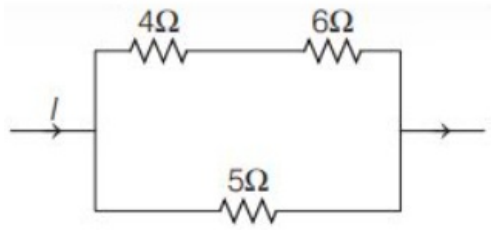
Solution:

Solution:
Given, resistance of arms as follows



$P = 9\Omega, Q = 11\Omega, R = 4\Omega, S = 6\Omega$
Consider resistance (r) is connected to parallel to the resistance (S).
So, that, the bridge is balance.
If r' be the equivalent resistance of r and S, then
$\Rightarrow \frac{1}{r} = \frac{1}{S} + \frac{1}{r} \Rightarrow (\frac{S.r}{S+r})$
Bridge is balanced, if $\frac{P}{Q} = \frac{R}{S}$
$\Rightarrow \frac{P}{q} = \frac{R}{(S.\frac{r}{S} + r)} = \frac{R(S+r)}{Sr}$
$\Rightarrow \frac{9}{11} = \frac{4(6+r)}{6r}$
Solving, $r = \frac{132}{5}\Omega$

Question 24



In the above circuit, the heat produced in 5Ω resistance is 10 cal / s. The heat produced in 4Ω resistance is

Options:

A. 1 cal / s

B. 2 cal / s

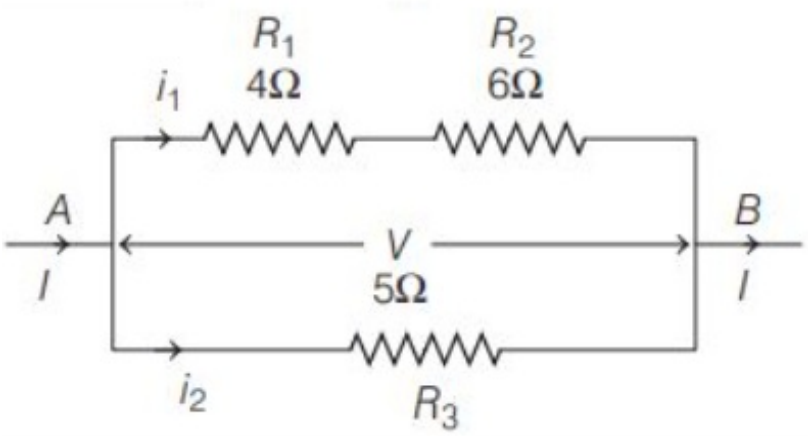
C. 3 cal / s

D. 4 cal / s

Answer: B

Solution:

Solution:
The given circuit diagram is shown as



Heat produced in 5Ω resistance,
$Q_s = 10 cal/s$
$\therefore Q_4 = ?$
Since, potential difference between points A and B is same.
i.e. $V = I_1(R_1 + R_2)$ and $V = I_2(R_3)$
$\Rightarrow I_1(R_1 + R_2) = I_2 R_3$
$\Rightarrow \frac{I_1}{I_2} = \frac{R_3}{R_1 + R_2} = \frac{5}{4+6}$
$\Rightarrow \frac{I_1}{I_2} = \frac{5}{10} = \frac{1}{2}$
$\because \frac{Q_5}{Q_4} = \frac{I_2^2 R_3}{I_1^2 R_1} = \frac{(\frac{2}{1})^{2.5}}{4} \Rightarrow \frac{10}{Q_4} = 5$
$\Rightarrow Q_4 = 2 cal/s$

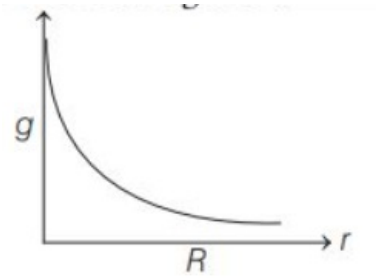
Question 25

The dependence of acceleration due to gravity g on the distance r from the centre of the earth assumed to be a sphere of radius R of uniform density is shown in figure below. The correct figure is

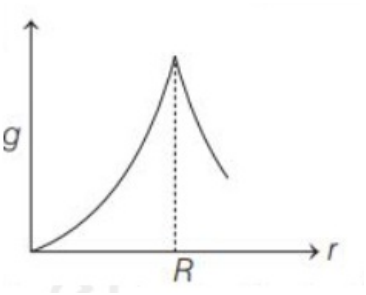
©

Options:

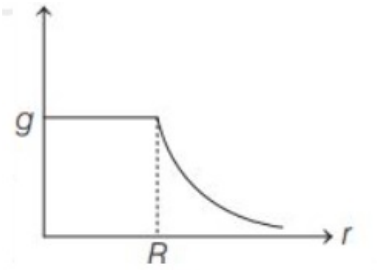
A.



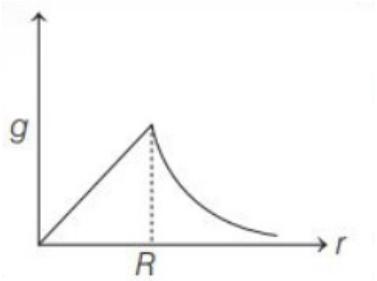
B.



C.



D.



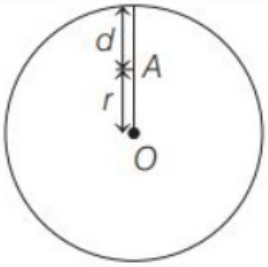
Answer: D

Solution:

Solution:

Gravitational acceleration at a depth d below the surface of earth or at a distance $r $g' = g(1 - \frac{d}{R})$ .....(i)

But $d + r = R \Rightarrow d = R - r$ .....(ii)



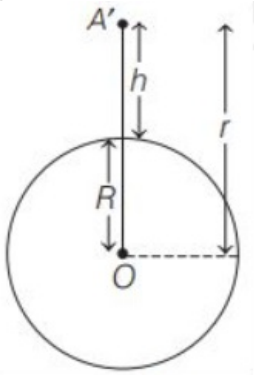
From eqs.(i) and (ii), we have

$g' = g(1 - \frac{R-r}{R}) = g(1 - \frac{R}{R} + \frac{r}{R}) = g(1 - 1 + \frac{r}{R})$

$\Rightarrow g' = \frac{gr}{R}$

$\therefore g' \propto r$ ....(iii)

Again, gravitational acceleration at a height h from the surface of earth or at distance $r > R$ from the centre of earth is given as



$$g'' = \frac{g}{(1+\frac{h}{R})^2}$$
$$= \frac{g}{(1+\frac{r-R}{R})^2} [\because r = h + R \Rightarrow h = r - R]$$
$$= \frac{g}{(\frac{r}{R})^2} = \frac{gR^2}{r^2}$$
$$\therefore g'' \propto \frac{1}{r^2} \text{ ....(iv)}$$

Hence, from Eqs. (iii) and (iv), we conclude that the correct variation of g with distance r is shown in option (d).

Question 26

The gravitational potential at the centre of four particles placed at the vertices of a square of side l

©

Options:

A. $-4\sqrt{2}\,\frac{Gm^2}{l}$

B. $-5.41\,\frac{Gm}{l}$

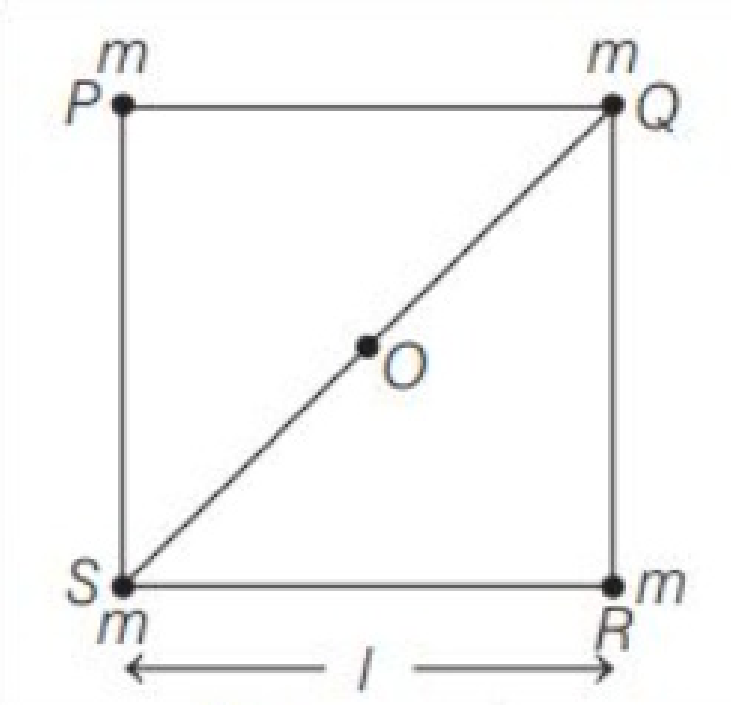
C. $-4\sqrt{2}\,\frac{Gm}{l}$

D. $5.41\,\frac{Gm^2}{l}$

Answer: C

Solution:

Solution:
Consider a square of side l and four particles each of mass $m\ kg$ placed at the vertices P, Q, R and S.



In ΔSRQ,
$SQ = \sqrt{(SR)^2 + (RQ)^2} \Rightarrow SQ = l\sqrt{2}$
Since, $SO = RO = QO = PO = \frac{l\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$
Now, potential at the centre due to one mass
$= -\frac{Gm}{\frac{l}{\sqrt{2}}} = -\sqrt{2}\frac{Gm}{l}$
Therefore, potential at the centre due to all four masses (each of m),
$V_{net} = 4 \times (-\sqrt{2}\frac{Gm}{l}) \Rightarrow V_{net} - 4\sqrt{2}\frac{Gm}{l}$

Question 27

A steel wire of length l has a magnetic moment M. It is then bent into a semicircular arc. The new magnetic moment is

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Options:

A. $\frac{2M}{\pi}$

B. $\frac{M}{\pi}$

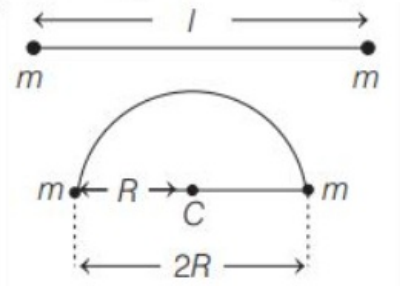
C. $\frac{M}{l}$

D. $M \times 1$

Answer: A

Solution:

Solution:
Given, length of steel wire = l



Let each pole strength is m, then magnetic moment of wire,
$M = m \, . \, l$ .....(i)
On bending a wire, its' pole strength does not change.
Let R is the radius of semicircular arc.
Then, new magnetic moment,
$M' = \frac{m}{(2R)}$ ....(ii)
But $l = \pi R$
$\therefore R = \frac{1}{\pi}$
From eq. (ii) we get
$M' = m.2\frac{l}{\pi}$
$M' = \frac{2M}{\pi}$ [From eq.(i)]

Question 28

The vertical component of earth's magnetic field at a place is $\sqrt{3}$ times the horizontal component. The value of angle of dip at this place is

©

Options:

A. 60°

B. 45°

C. 30°

D. 29°

Answer: A

Solution:

Solution:
Given, for the earth's magnetic field, vertically component (B_V) = $\sqrt{3}\times$ horizontal component (B_H)
Since, angle of dip (θ) is given as
$tan\theta = \frac{b_V}{B_H} = \frac{\sqrt{3}.B_H}{B_H} = \sqrt{3}$
$\Rightarrow \theta = tan^{-1}(\sqrt{3}) \Rightarrow \theta = 60°$

Question 29

The magnetic flux linked with a coil at any instant t given by $\varphi = 5t^3 - 100t$. The emf induced in the coil at $t = 2s$ is

©

Options:

A. $-40V$

B. 40V

C. 140V

D. 300V

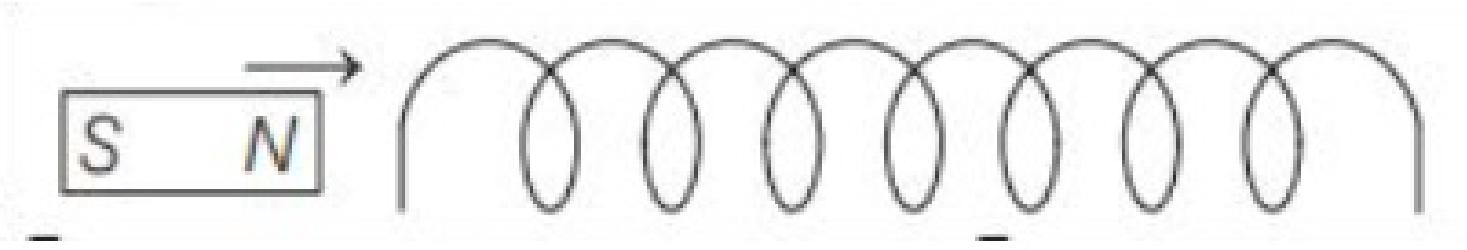
Answer: B

Solution:

Solution:
Given, magnetic flux linked with coil at any instant t,
$\phi = 5t^3 - 100t$
According to Faraday's law of electromagnetic induction, induced emf,
$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(5t^3 - 100t)$
$= -(15t^2 - 100)$
$= -(15 \times 2^2 - 100)[At\ t = 2s]$
$= -(60 - 100) = 40V$

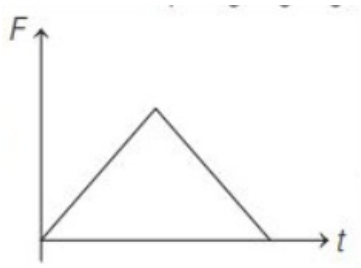
Question 30

The variation of induced emf (F) with time (t) in a coil if an short bar magnet is moved along its axis with a constant velocity is best represented as

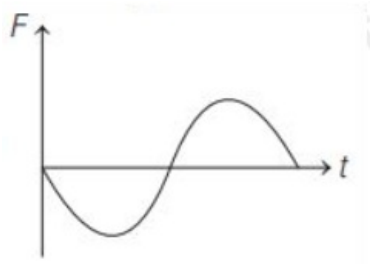


Options:

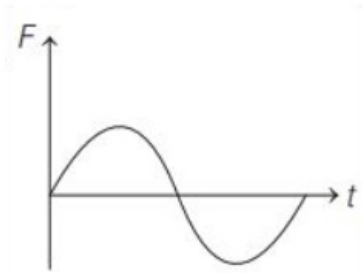
A.



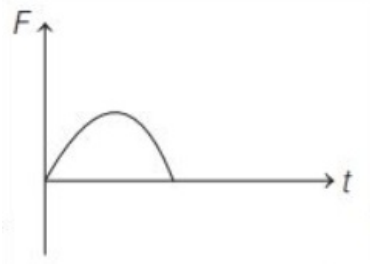
B.



C.



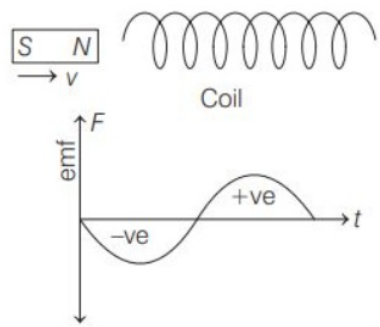
D.



Answer: B

Solution:

Solution:



Initially, the magnet moves towards the coil, the magnetic flux (ϕ_B) increases and due to this, the negative induced emf [$e = \frac{-d\phi}{dt}$] is also increases.

When the magnet moves inside the coil, the increase in ϕ_B goes up to maximum value. This shows that the $emfe$ induced reaches at its maximum negative value. After this, ϕ_B starts decreasing. Thus, e becomes positive and its magnitude starts increasing, i.e. there is a change in polarity of mathrm $emf(F)$ as the magnet passes through other side of coil.
Hence, correct variation of induced emf with time t is shown in the graph given in option (b).

Question 31

The stress at which extension of a material takes place more quickly as compared to the increase in load, is called

©

Options:

A. elastic point

B. plastic point

C. breaking point

D. None of the above

Answer: D

Solution:

Solution:
The stress at which extension of a material takes place more quickly as compared to the increase in load (i.e. small increase in stretching force) is known as yield point. A small increase in stress beyond the yielding point will give rise to a large elongation. Hence, option (d) is correct.

Question 32

The bulk modulus of a spherical object is B. If it is subjected to uniform pressure p, then the fractional decrease in radius is

©

Options:

A. $\frac{B}{3p}$

B. $\frac{3p}{B}$

C. $\frac{p}{3B}$

D. $\frac{P}{B}$

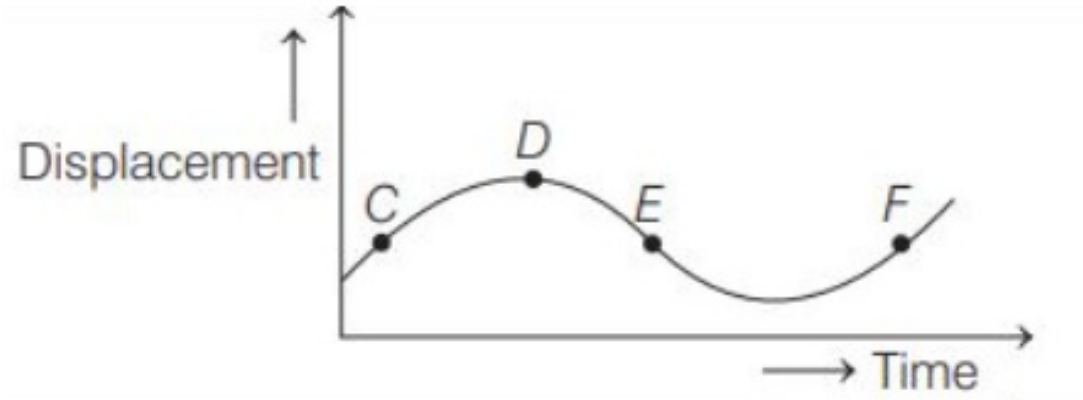
Answer: C

Solution:

Solution:
Given, bulk modulus of a spherical object $= B$Let a solid sphere of radius R.
Then, its volume,
$V = \frac{4}{3}\pi R^3 \Rightarrow \frac{\Delta V}{V} = (\frac{3\Delta R}{R})[\because z = x^n \therefore \frac{\Delta z}{z} = n\frac{\Delta x}{x}]$
$\therefore \frac{\Delta R}{R} = \frac{1}{3}\frac{\Delta V}{V}$ .....(i)
Now, bulk modules,
$B = \frac{p}{\frac{\Delta V}{V}}$
$\therefore \frac{\Delta V}{V} = \frac{p}{B}$ .....(ii)
From eqs. (i) and (ii), we get
$\frac{\Delta R}{R} = \frac{1}{3}\frac{p}{B}$

Question 33

The displacement-time graph of a moving particle is shown below. The instantaneous velocity of the particle is negative at the point



Options:

A. E

B. F

C. C

D. D

Answer: A

Solution:

Solution:
(a) According to given graph, point D represents the maximum displacement from origin and the slope of $(x - t)$ graph is zero and hence, the velocity is zero at point D.
While at point E, the slope of graph is negative, therefore the velocity at point E is negative.Hence, the correct option is (a).

Question 34

The heart of a man pumps 5L of blood through the arteries per min at a pressure of 150 mm of mercury column. If the density of the mercury be 13.6×10^3 kg / m^3 and $g = 10$ m / s^2, then the power of heart (in watt) is

©

Options:

A. 1.50

B. 1.70

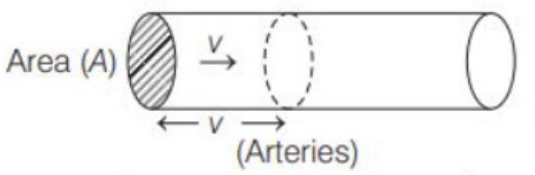
C. 2.35

D. 3.0

Answer: B

Solution:

Solution:
Given, heart pumps blood in volume per minutes,
$AV = 5L / min$
$AV = \frac{5 \times 10^{-3}}{60} m^3 / s$



Here as shown in figure, area of arteries is A and velocity of blood is v. Thus, the volume flows per second inside arteries is given by $(A . v)$.
Height of mercury, column, $h_{Hg} = 150 mm$
$= 150 \times 10^{-3} m$
Density of mercury, $\rho_{Hg} = 13.6 \times 10^3 kg / m^3$
and $g = 10 m / s^2$
Power (in watt) = ?
In this case, we find instantaneous power, P
$= F . v$ .....(i)
Since, F = pressure × area
and pressure $= h_{Hg} . \rho_{Hg} . g$
Put these values in eq. (i), we get
$P = h_{Hg} . \rho_{Hg} . g . (A . v)$ .....(ii)
$= 150 \times 10^{-3} \times 10^3 \times 10 \times \frac{5 \times 10^{-3}}{60}$
$= \frac{(15) \times (136) \times (5)}{6} \times 10^{-3}$
$= 1700 \times 10^{-3} = 1.70 W$

Question 35

A light wave travels from glass to water. The refractive index for glass and water are 3 / 2 and 4 / 3, respectively. The value of the critical angle will be

©

Options:

A. $sin^{-1}(\frac{8}{9})$

B. $sin^{-1}(\frac{9}{8})$

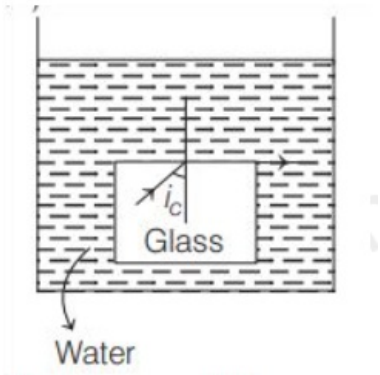
C. $sin^{-1}(\frac{1}{2})$

D. $sin^{-1}(\frac{3}{4})$

Answer: A

Solution:

Solution:
Given, refractive index for glass, ${}_a\mu_g = \frac{3}{2}$
and refractive index for water, ${}_a\mu_w = \frac{4}{3}$
Since, light ray tavels from glass (denser) to water (rarer).



So, the refractive index of glass with respect to Water is

${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} \Rightarrow {}_w\mu_g = \frac{\frac{3}{2}}{\frac{4}{3}} = \frac{9}{8}$.....(i)

If critical angle be i_c, then
$sini_c = \frac{1}{\mu_g} \Rightarrow sini_c = \frac{1}{\frac{9}{8}}$
$\therefore i_c = sin^{-1}(\frac{8}{9})$

Question 36

A convex lens in air produces a real image having the same size as object. When the object and the convex lens are immersed in a liquid, the real image formed is enlarged two times the object size. The refractive index of the liquid is

Options:

A. $\frac{11}{12}$

B. $\frac{12}{11}$

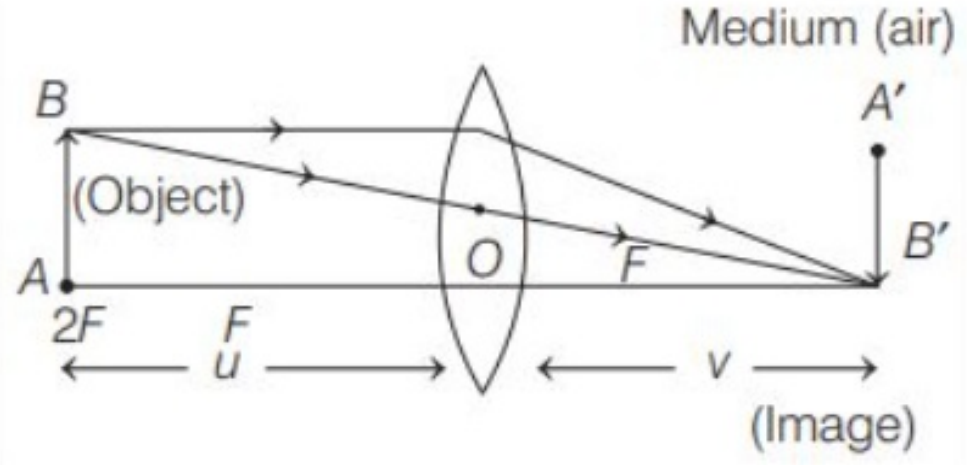
C. $\frac{13}{11}$

D. $\frac{3}{2}$

Answer: B

Solution:

Solution:
According to given situation, formation of image from convex lens is shown below



($\because AB = A'B'$)
Here, $u = -2f = -d(let)$
$\therefore f = \frac{d}{2}$
By lens Maker's formula,
$\frac{1}{f} = ({}_a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
$\Rightarrow \frac{1}{f} = ({}_a\mu_g - 1)(\frac{1}{R} - \frac{1}{(-R)})$
$\Rightarrow \frac{1}{f} = ({}_a\mu_g - 1) . \frac{2}{R}$
$\Rightarrow \frac{1}{\frac{d}{2}} = (1.5 - 1) . \frac{2}{R} \Rightarrow d = 2R$
When object and convex lens are immersed in liquid, then the image formed two times enlarged than object.
$m = -\frac{v}{u} \Rightarrow 1 = -\frac{v}{u}$
$\therefore$ v=-2u
But object remains at distance, $u = -d$.
Consider, then the focal length of lens in liquid becomes f_1, then
$\frac{1}{f_1} = \frac{1}{v} - \frac{1}{u} \Rightarrow f_1 = \frac{1}{-2u} - \frac{1}{u}$
$\Rightarrow f_1 = \frac{2d}{3}(\because u = -d)$
Let refractive index of liquid is μ_l ,
$\frac{1}{f_1} = (\frac{{}_a\mu_g}{{}_a\mu_l} - 1) . \frac{2}{R}$
$\frac{1}{\frac{2d}{3}} = (\frac{1.5}{\mu_l} - 1) . \frac{2}{\frac{d}{2}}$
After solving, $\mu_l = \frac{12}{11}$

Question 37

The potential energy of a particle executing SHM is 2.5J, when its displacement is half of amplitude. The total energy of the particle is

©

Options:

A. 2.5J

B. 5.0J

C. 7.5J

D. 10.0J

Answer: A

Solution:

Solution:
Given, potential energy, $U = 2.5J$
If mass of particle be m
and amplitude be a, then
$U = \frac{1}{2}m\omega^2a^2 \Rightarrow 2.5 = \frac{1}{2}m\omega^2a^2$
$\therefore m\omega^2a^2 = 5$ .....(i)
According tom the problem, the position of particle from mean position is given as
$x = \frac{a}{2}$
Thus, the total energy,
T = (potential energy) + (kinetic energy)
$= \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2(a^2 - x^2)$
$= \frac{1}{2}m\omega^2(\frac{a}{2})^2 + \frac{1}{2}m\omega^2[a^2 - (\frac{a}{2})^2]$
$= \frac{1}{2}m\omega^2a^2 = \frac{1}{2} \times 5$[From eq.(i)]
$= 2.5J$

Question 38

A horizontal stretched string fixed at two ends, is vibrating in its 5th harmonic frequency according to the equation $y(x,t) = (0.01m)[sin\,(62.8m^{-1})x]$ $[\cos(628s^{-1})\,t]$ Assuming, $\pi = 3.14$, the correct statement is

©

Options:

A. the number of nodes is 5.

B. the length of the string is $0.5m$.

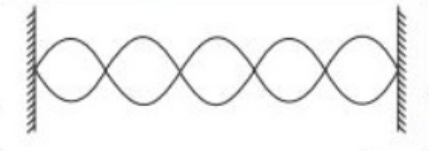
C. the fundamental frequency is $100\,Hz$.

D. the fifth harmonic frequency is $100\,Hz$.

Answer: D

Solution:

Solution:
Given, equation of wave,
$y(x,t) = (0.01)[sin(62.8m^{-1}).x][cos(628s^{-1}).t]$
and $\pi = 3.14$
Compare with standard equation given below,
we get
$y(x,t) = (2a)sin(kx), cos\omega t$
$\omega = 628s^{-1}$,
$k = 62.8m^{-1}$
Since, $k = \frac{2\pi}{\lambda}$
$\Rightarrow \lambda = \frac{2\pi}{k} = \frac{2 \times 3.14}{628}$
$\therefore \lambda = \frac{1}{10}m$
(A) According to the problem, string is vibrating in its fifth harmonic frequency, so there are 5 complete loops as shown in the figure.Thus, total number of nodes = 6



(B) The length of the string for 5 harmonics,
$l = \frac{5\lambda}{2} = \frac{5}{2} \times \frac{1}{10}$
$\therefore$ l=0.25 m
(C) The fundamental frequency, $n_f = \frac{v}{2l}$
$\Rightarrow n_f = \frac{\omega}{k}.(2l)(\because v = \frac{\omega}{k})$
$= \frac{628}{628}.\frac{1}{2 \times 0.25} = 20Hz$
(D) The fifth harmonic frequency, $n_5 = 5(\frac{v}{2l})$
$= 5(20) = 100Hz$

Question 39

In a current carrying long solenoid, the field produced does not depend upon

©

Options:

A. number of turns per unit length

B. current flowing

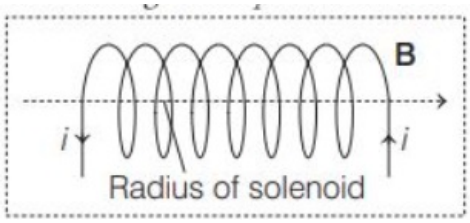
C. radius of the solenoid

D. All of the above

Answer: C

Solution:

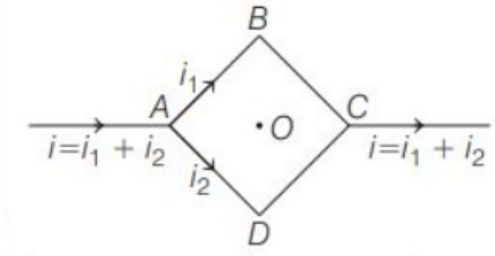
Solution:
The magnetic field due to current carrying long solenoid is given by



$B = \mu_0 n i$
where, n = number of turns per unit length
and i = current.
It is clear that B independent of radius of the solenoid.

Question 40

Above circuit shows a square loop $ABCD$ with edge length a. The resistance of the wire ABC is r and that of ADC is 2r. The value of magnetic field at the centre O of the loop assuming uniform wire is



Options:

A. $\frac{\sqrt{2}\mu_0 i}{3\pi a} \odot$

B. $\frac{\sqrt{2}\mu_0 i}{3\pi a} \otimes$

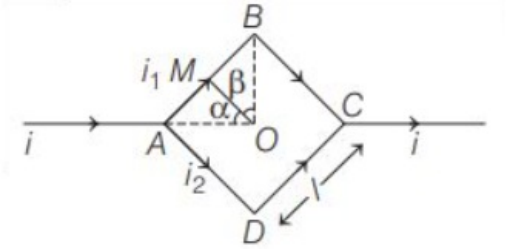
C. $\frac{\sqrt{2}\mu_0 i}{\pi a} \odot$

D. $\frac{\sqrt{2}\mu_0 i}{\pi a} \otimes$

Answer: B

Solution:

Solution:
Given, $i = i_1 + i_2$ .....(i)
∵ The length of part (ABC) is equal to length of part (ABD).



Here, resistance of part (ABC) is half that of part (ABD), i.e.
$i_1 = 2i_2$ .....(ii)
From eqs. (i) and (ii), we get
$i_1 = \frac{2i}{3}, i_2 = \frac{i}{3}$
Magnetic field due to AB and BC wire should be equal,

i.e. $B_{AB} = B_{BC}$
Magnetic field due to straight wire,
$B = \frac{\mu_0 i}{4\pi r}(sin\alpha + sin\beta)$
For wire AB,
From figure, $\alpha = \beta = 45°$, and $MO = \frac{l}{2}$
$\therefore B_AB = \frac{\mu_0}{4\pi} . \frac{i_1}{\frac{l}{2}}(sin45° + sin45°)$
$\Rightarrow B_{AB} = \frac{\mu_0}{4\pi} . \frac{2\sqrt{2}.I_1}{l} = B_{BC}$
Also, the magnetic field due to AD and DC wire should be equal, i.e. $B_{AD} = B_{DC}$
For wire AD,
$B_{AD} = \frac{\mu_0}{4\pi} . \frac{2\sqrt{2}.i_2}{l} = B_{DC}$
Net magnetic field at centre O is
$B_{net} = B_{AB} + B_{BC} - B_{AD} - B_{DC}$
Since, $(B_{AB} + B_{BC}) > (B_{AD} + B_{DC})$
$B_{net} = \frac{\mu_0}{4\pi} . \frac{2\sqrt{2} \times (\frac{2}{3}i) \times 2}{l} - \frac{\mu_0}{4\pi} . \frac{2\sqrt{2}.(\frac{i}{3}) \times 2}{l}$
$\Rightarrow B_{net} = \frac{\sqrt{2}\mu_0 i}{3\pi a}$
The direction of B_{net} will be inward to the plane of paper.

Question 41

A certain number of spherical drops of a liquid of radius r coalesce to form a single big drop of radius R and volume V. If T is the surface tension of the liquid, then

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Options:

A. energy $= 4VT(\frac{1}{r} - \frac{1}{R})$ is released

B. energy $= 3VT(\frac{1}{r} + \frac{1}{R})$ is absorbed

C. energy $= 3VT(\frac{1}{r} - \frac{1}{R})$ released

D. energy is neither released nor absorbed

Answer: C

Solution:

Solution:
According to problem, n small drop of radius r coalesce to form a big liquid drop of radius R. Since, the volume remain same.
i.e. $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$
$\therefore R = r \times n^{\frac{1}{3}}$.....(i)
During the formation of big drop, surface area decreases and hence, the energy released.
Now, change in surface area,
$\Delta A = n \times 4\pi r^2 - 4\pi R^2$
Therefore, energy released,
$\Delta E = T \times \Delta A = T \times [n \times 4\pi r^2 - 4\pi R^2]$
$= 4\pi T R^2 \left[\frac{n r^2}{R^2 - 1}\right]$
Using eq. (i), we get

$\Rightarrow \Delta E = 4\pi TR^2\left(n^{\frac{1}{3}} - 1\right)$
$= 4\pi TR^2(\frac{R}{r} - 1) = 4\pi TR^3[\frac{1}{r} - \frac{1}{R}]$
$= (\frac{3}{3}.4\pi R^3)\,T[\frac{1}{r} - \frac{1}{R}] = 3VT[\frac{1}{r} - \frac{1}{R}]$

Question 42

For a certain gas, the ratio of specific heats is given to be $\gamma = 1.5$. For this gas

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Options:

A. $C_p = \frac{3R}{J}$

B. $C_V = \frac{3R}{J}$

C. $C_p = \frac{5R}{J}$

D. $C_V = \frac{5R}{J}$

Answer: A

Solution:

Solution:
Given, ratio of specific heats,
$\frac{C_p}{C_V} = 1.5$ .....(i)
From the Mayer's equation,
$C_p - C_V = \frac{R}{J}$.....(ii)
Put the value of (C_V) from eq.(i) in eq.(ii),
we get
$C_p - \frac{C_p}{1.5} = \frac{R}{J} \Rightarrow C_p(1 - \frac{2}{3}) = \frac{R}{J}$
$\therefore C_p = \frac{3R}{J}$

Question 43

The focal lengths of the objective and the eyepiece of a telescope are 50 cm and 5 cm, respectively. Least distance of distinct vision is 25 cm. If the telescope is focussed for distinct vision on a scale placed at a distance of 200 cm away from objective, then the separation between the objective and the eyepiece is

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Options:

A. $100\,\text{cm}$

B. $75\,\text{cm}$

C. $70.8\,\text{cm}$

D. $60.8\,\text{cm}$

Answer: C

Solution:

Solution:
According to the problem, focal length of objective, $f_o = 50cm$
Object distance, $u_o = -200cm$
Image distance, $v_o = ?$
Now, from lens formula,
$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} \Rightarrow \frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o}$
$\Rightarrow \frac{1}{v_o} = \frac{1}{50} + \frac{1}{(-200)}$
$\therefore v_o = \frac{200}{3}cm$.....(i)
Also, focal length of eyepiece,
$f_e = 5cm, v_e = -D = -25cm$
Again for eyepiece, using lens formula,
$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$
$\Rightarrow \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$
Put the given values, we get
$\frac{1}{u_e} = \frac{1}{-25} - \frac{1}{5}$
$\therefore u_e = -\frac{25}{6}cm$ .....(ii)
Since, the separation between objective and eyepiece is
$L = |v_o| + |u_e|$
From eqs. (i) and (ii), we get
$L = \frac{200}{3} + \frac{25}{6}$
$\Rightarrow L = 70.83 \simeq 70.8cm$

Question 44

The angular resolution of a $10\,\text{cm}$ diameter telescope at a wavelength of $5000\mathring{A}$ is of the order of

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Options:

A. $10^{-6}\,\text{rad}$

B. $10^{6}\,\text{rad}$

C. $10^{-4}\,\text{rad}$

D. $10^{4}\,\text{rad}$

Answer: A

Solution:

Solution:
Given, diameter (aperture of objective),
$a = 10cm = (10 \times 10^{-2})\, m$
Wavelength, $\lambda = 5000\mathring{A} = 5000 \times 10^{-10} m$
The angular resolution of telescope,
$d\theta = \frac{1.22\lambda}{a}$
$= \frac{1.2 \times 500 \times 10^{-10}}{10 \times 10^{-2}}$
$= 6.10 \times 10^{-6} rad$
Hence, option (a) is correct.

Question 45

If an electron is going in the direction of magnetic field B with the velocity v, then the force on electron is

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Options:

A. zero

B. e(v · B)

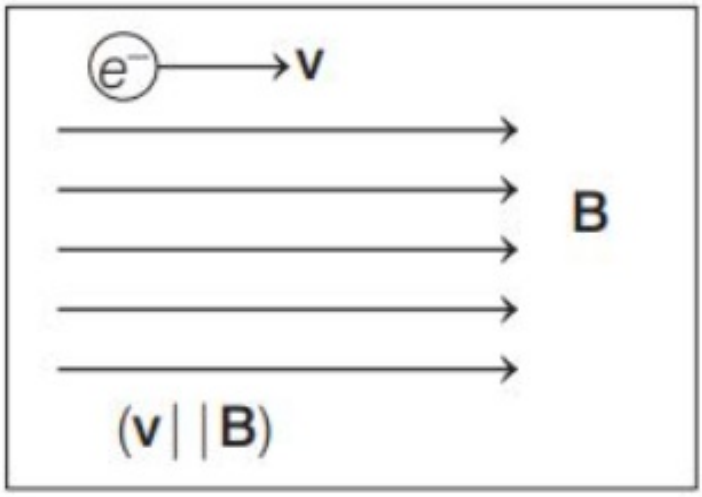
C. e(v × B)

D. None of these

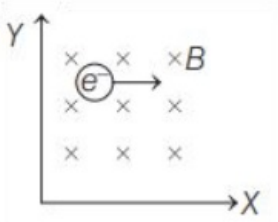
Answer: A

Solution:

Solution:
As given in problem, an electron is moving in the direction of magnetic field (B).
Hence, $\theta = 0°$
The expression of force is
$F = qvBsin\theta$ .....(i)
Since, v parallel to $B, i.e\theta - 0°$
Therefore, $sin\theta - sun0° = 0$
From eq. (i), we get
$F = qvB(0) \Rightarrow F = 0$



Question 46



In above figure electron enters into magnetic field B. It deflects in the direction

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Options:

A. + ve x-direction

B. -ve x-direction

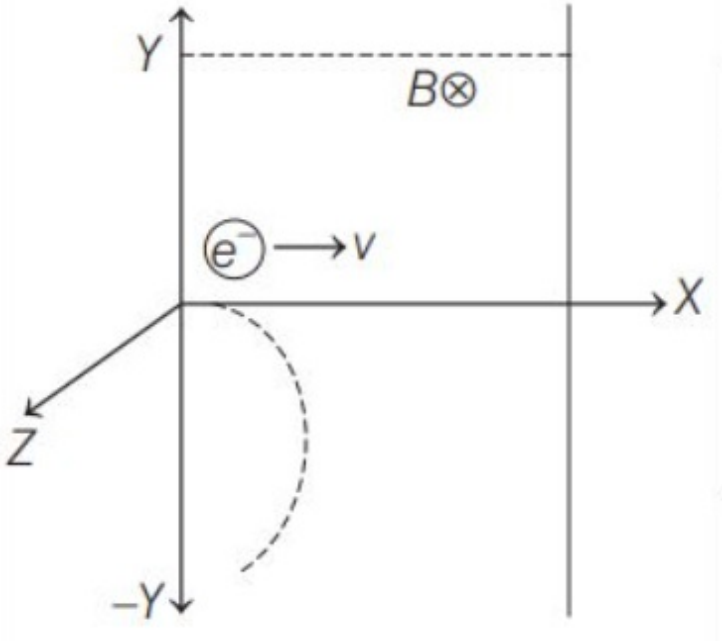
C. +ve y-direction

D. -ve y-direction

Answer: D

Solution:

Solution:
The force on electron (e^-) must be acting in the $-y$ - direction initially (according to Fleming's left hand rule).



Since, B is along $-z$ - direction,
v is along $+x$ - direction.
∴ Force on electron is in $-y$ - direction.

Question 47

The forbidden energy gap in conductor, semiconductors and insulators are EG_1, EG_2 and EG_3, respectively. The reation among them is

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Options:

A. $EG_1 = EG_2 = EG_3$

B. $EG_1 > EG_2 > EG_3$

C. $EG_1 < EG_2 < EG_3$

D. $EG_1 < EG_2 > EG_3$

Answer: C

Solution:

Solution:

Substance	Forbidden energy gap
Conductor	EG_1(Zero or very small)
Semiconductor	$EG_2\left(Ge = 0.2eV, Si = 0.7eV\right)$
Insulator	EG_3 (Very large)

It is clear from above observations,
$EG_1 < EG_2, EG_3$

Question 48

The dominant mechanisms for motion of charge carriers in forward and reverse biased silicon $p-n$ junctions are

Options:

A. drift in forward biased, diffusion in reverse bias

B. diffusion in forward biased, drift in reverse bias

C. diffusion in both forward and reverse bias

D. drift in both forward and reverse bias

Answer: B

Solution:

Solution:
In forward bias, depletion layer decreases and the diffusion takes place which results the electric current from p-side to n-side.
So, the diffusion current increases while drift current remains constant in forward bias. In reverse bias, the width of depletion layer increases, so the diffusion becomes more difficult, so net current (in very small magnitude) is due to the drift.

Therefore, dominant mechanisms for motion of charge carriers, diffusion in forward bias and drift in reverse bias.

Question 49

A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both light coincides. Further it is observed that the third bright fringe of known light coincides with the fourth bright of the unknown light. From this data, the wavelength of the unknown light is

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Options:

A. 442.5 nm

B. 398.4 nm

C. 532.8 nm

D. 672.3 nm

Answer: A

Solution:

Solution:
Given, mixture of light consist of two wavelengths
(i) Known wavelength, $\lambda_1 = 590nm$
(ii) Unknown wavelength, $\lambda_2 = ?$
Since, central maxima of both light coincides.
Also, the third bright fringe of light of wavelength λ_1, coincides with the fourth bright fringe of light of wavelength λ_2.
Therefore, $n_1\lambda_1 = n_2\lambda_2$
Here, $n_1(\text{known}) = 3(\text{unknown}), n_2(\text{unknown}) = 4$
Put the values, $3 \times 590 = 4 \times \lambda_2$
$\Rightarrow \lambda_2 = 4425nm$

Question 50

A light ray travels through two media A and B having, refractive indices of $\frac{4}{3}$ and $\frac{3}{2}$. respectively. If the thickness of the medium A is 4 cm and that of B is 6 cm, then the optical path length in the combined media will be

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Options:

A. $\frac{7}{3}$

B. $\frac{12}{3}$

C. $\frac{37}{3}$

D. $\frac{43}{3}$

Answer: D

Solution:

Solution:
Given,

Physical Quantities	Medium (A)	Medium (B)
Refractive index	$\mu_1 = \frac{4}{3}$	$\mu_2 = \frac{3}{2}$
Thickness	$t_1 = 4cm$	$t_2 = 6cm$

Optical path It is the distance travelled by light in vacuum in the same time in which, it travels a given path in medium.
∴ Optical path $= \mu_1 t_1 + \mu_2 t_2$
$= (\frac{4}{3} \times 4) + (\frac{3}{2} \times 6) = \frac{43}{3}$

Question 51

For the reaction, $N_2 + 3H_2 \rightleftharpoons 2NH_3 +$ Heat

Options:

A. $K_p = K_C$

B. $K_p = K_C(RT)^{-1}$

C. $K_p = K_C(RT)^{-2}$

D. $K_p = K_C RT$

Answer: C

Solution:

Solution:
Relation between K_p and K_C is as folows
$K_p K_C(RT)^{\Delta n}$
where, $K_p =$ equilibrium constant when concentrations at equilibrium are expressed in atmospheric pressure.
$K_C =$ equilibrium constant when concentrations at equilibrium are expressed in molarity.
$R =$ gas constant
$T =$ temperature
$\Delta n =$ the changed in number of moles of gas molecules.
[$\Delta n =$ number of gaseous moles in product- number of gaseous moles in reactant]
For reaction,
$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) + Heat$
$K_p = K_C(RT)^{2-(1+3)}$
$K_p = K_C(RT)^{-2}$
Hence, correct answer is (c).

Question 52

The pH of a solution is increased from 3 to 6 , its H^+ ion concentration will be

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Options:

A. reduced to half

B. doubled

C. reduced by 1000 times

D. increased by 1000 times

Answer: C

Solution:

Solution:
(c) If pH of solution increased from 3 to 6, H^+ ion concentration will be reduced to 1000 times.
$pH = -\log[H^+]$
$[H^+]$ when $pH = 3$,
$3 = -\log[H^+]$
antilog $(-3) = [H^+]$ .....(i)
$10^{-3} = [H^+]$.....(i)
Also, $[H^+]$ when $pH = 6$,
$6 = -\log[H^+]$
antilog $(-6) = [H^+]$
$10^{-6} = [H^+]$.....(ii).....(ii)

From (i) and (ii), it can be seen if pH is increased from 3 to 6, H^+ concentration will be reduced by 1000 times.

Question 53

Hess's law deals with

©

Options:

A. change in heat of reaction

B. rate of reaction

C. equilibrium constant

D. influence of pressure on volume of a gas

Answer: A

Solution:

Solution:
Hess's law deals with the change in heat, in a given chemical reaction.
Hess's law can be written as

$$\Delta H^\circ = \sum \Delta H_n$$

where, $\Delta H^\circ =$ heat obsorbed / evolved and

$\sum \Delta H_n =$ sum of heat absorbed or evolved in individual n steps of reaction.

Question 54

A reaction is not feasible, if

©

Options:

A. ΔH is positive and ΔS is also positive

B. ΔH is positive and ΔS is negative

C. ΔH is negative and ΔS is also negative

D. ΔH is negative and ΔS is positive

Answer: B

Solution:

Solution:
A reaction is not feasible or non-spontaneous, if enthalpy (ΔH) is positive and entropy change (ΔS) is negative because ΔG (Gibb's free energy change) will be positive.
The changes in Gibb's free energy is given as
$\Delta G = \Delta H - T\Delta S = (+ve) - T(-ve)$
$\Delta G = +ve$
Hence, ΔG will be positive for non - spontaneous reaction.

Question 55

Polymerisation reaction is initiated by strong bases or C_4HgLi or Grignard reagent known as

©

Options:

A. free radical polymerisation

B. step growth addition polymerisation

C. cationic addition polymerisation

D. anionic addition polymerisation

Answer: D

Solution:

Solution:
Anionic addition polymerisation is initiated by strong base or C_4HgLi or Grignard reagent as it involves initiation with anions. In this type of polymerisation monomers react with strong electronegative groups (strong bases, C_4HgLi or Grignard's reagent).

Question 56

Synthetic rubber that can be prepared by polymerising ethylene chloride and sodium polysulphide is known as

©

Options:

A. buna-S

B. thiokol

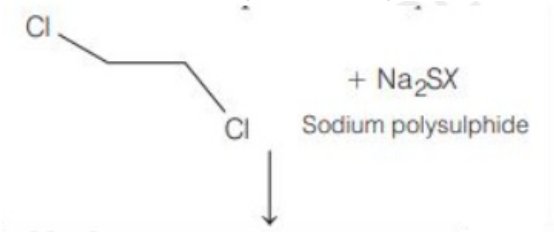
C. buna-N

D. neoprene

Answer: B

Solution:

Solution:
Thiokol is a liquid elastomer formed by polymerisation of ethylene chloride and sodium polysulphide.
It has low moisture permeability.



$HS(CH_2CH_2OCH_2OCH_2CH_2SS)_n$
$CH_2CH_2OCH_2OCH_2CH_2SH$

Question 57

The correct order of second ionisation potential of C, N, O and F is

©

Options:

A. F > O > N > C

B. O > F > N > C

C. C > N > O > F

D. O > N > F > C

Answer: B

Solution:

Solution:
The correct order of second ionisation potential of C, N, O, F is
$O > F > N > C$
The electronic configuration of N, O, F, and C are as follows
Nitrogen $(N) \longrightarrow [He]2s^2, 2p^3$
Oxygen $(O) \longrightarrow [He]2s^2, 2p^4$
Fluorine $(F) \longrightarrow [He]2s^2, 2p^5$
Carbon $(C) \longrightarrow [He]2s^2, 2p^2$
After removing one electron the configuration will be :
Nitrogen $(N^+) \longrightarrow [He]2s^2, 2p^3$
Oxygen $(O^+) \longrightarrow [He]2s^2, 2p^3$
Fluorine $(F^+) \longrightarrow [He]2s^2, 2p^4$
Carbon $(C^+) \longrightarrow [He]2s^2, 2p^1$
In nitrogen after removing one more electron, only one electron will be left in outermost p-subshell.
In oxygen, after removing one more electron the atom will become unstable as it will loose its half-filled stable configuration and will be left with only 2 electrons in valence p-subshell. So, have very high IP (Ionisation potential). In fluorine after removal of one more electron, it will occupy stable half-filled configuration and at last in carbon, after removal of one electron it will have ideal gas configuration. Hence, electron will be easily removed. So, it will have lowest second ionisation potential.

Question 58

Match list-I with list-II and select the correct answer using the codes given below

List-I	List-II
A. Fullerene	I. Lanthanoid
B. Promethium	II. Actinoid
C. Water	III. Allotrope
D. Lawrencium	IV. Lewis base

Options:

A. A - III, B - I, C - IV, D - II

B. A - III, B - II, C - IV, D - I

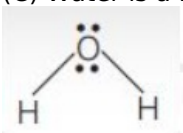
C. A - II, B - I, C - IV, D - III

D. A - II, B - III, C - I, D - IV

Answer: A

Solution:

Solution:
(A) Fullerene is the allotrope of carbon. Its molecular formula is mathrm C_{60} and has a shape like soccer ball. It contain twenty six-membered rings and twelve five-membered rings.
$\therefore$ Correct match of (A) is III.
(B) Promethium is a lanthanide with atomic symbol Pm and atomic number 61 .
$\therefore$ Correct match of (B) is I.
(C) Water is a Lewis base as it contain two lone pair of electrons.



$\therefore$ Correct match of (C) is IV.
(D) Lawrencium is a actinoid with symbol Lr and atomic number 103 .
$\therefore$ Correct match of (D) is II.
Hence, correct option is (a).

Question 59

Which one does not exhibit paramagnetism?

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Options:

A. NO_2

B. NO

C. ClO_2^-

D. ClO_2

Answer: C

Solution:

Solution:
ClO_2^- is diamagnetic in nature. It will not possess paramagnetism.Paramagnetism is due to presence of atleast one unpaired electron in molecule. Molecule with odd number of electrons will have atleast one unpaired electron.
Total number of electrons in
$NO_2: 7 + 8 \times 2 = 23$ electrons
$NO: 7 + 8 = 15$ electrons
$ClO_2^-: 17 + 8 \times 2 + 1 = 34$ electrons
$ClO_2: 17 + 8 \times 2 = 33$ electrons
$\therefore NO_2, NO$ and ClO_2, has odd number of electrons and therefore, paramagnetic in nature while ClO_2^- has even number of electrons and therefore exhibit diamagnetism.

Question 60

The non-metal which is not affected by NaOH.

©

Options:

A. Si

B. S

C. P

D. C

Answer: D

Solution:

Solution:
Carbon is not affected by $NaOH$. When carbon dioxide react with sodium hydroxide ($NaOH$) then oxidation state of carbon does not get affected.
$^{+4}CO_2 + NaOH \longrightarrow Na_2{}^{+4}CO_3$

Question 61

According to Bohr's theory, the energy required for the transition of H-atom from n = 6 to n = 8 state is

©

Options:

A. equal to energy required for the transition from $n = 5$ to $n = 7$ state

B. equal to energy required for the transition from $n = 7$ to $n = 9$ state

C. less than in option (a)

D. None of the above

Answer: C

Solution:

Solution:
According to Bohr's theory, the energy required for transition of H-atom from $n = 6$ to $n = 8$ is given as

$$\Delta\varepsilon_6 \to 8 = 13.6eV\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = 13.6eV\left[\frac{1}{6^2} - \frac{1}{8^2}\right]$$

$$= 13.6eV \times 0.012 = 0.1652eV \text{ .....(i)}$$

(a) Energy for transition of H-atom electron from $n = 5$ to $n = 7$.

$$\Delta\varepsilon_5 \to 7 = 13.6eV\left[\frac{1}{5^2} - \frac{1}{7^2}\right]$$

$$= 13.6eV \times 0.196$$

$$\Delta\varepsilon_5 \to 7 = 0.266$$

Energy for transition from $n = 5$ to $n = 7$ is not equal to transition from $n = 6$ to $n = 8$.
(b) Energy for transition from $n = 7$ to $n = 9$,

$$\Delta\varepsilon 7 \to 9 = 13.6eV\left[\frac{1}{7^2} - \frac{1}{9^2}\right]$$

$$= 13.6eV \times 0.0081 = 0.1096eV$$

∴ Energy for transition from $n = 7$ to $n = 9$ is not equal to transition from $n = 6$ to $n = 8$.
(c) The transition from $n = 6$ to $n = 8$ has energy less than transition from $n = 5$ to $n = 7$.
∴ Option (c) answer is correct.

Question 62

The correct set of quantum numbers for 4d-electrons is

©

Options:

A. $4, 3, 2, +\frac{1}{2}$

B. 4, 2,1, 0

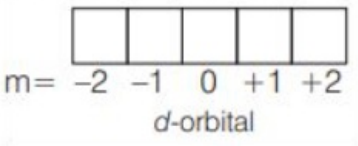
C. $4, 3, -2, +\frac{1}{2}$

D. $4, 2, 1, -\frac{1}{2}$

Answer: D

Solution:

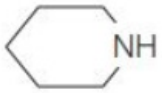
Solution:
For a $4d$ orbital,
n (principal quantum number) = 4
l (azimuthal quantum number)
= 2 since for d-orbital, $l = 2$
m (magnetic quantum number) ranges from $-l$ to l, i.e. from -2 to $+2$



s (spin quantum number) can be $+\frac{1}{2}$ or $-\frac{1}{2}$.
So, set of quantum number for $4d$-electrons are 4 ,2,1 and $\frac{-1}{2}$.

Question 63

In piperidine



the hybrid state assumed by N is

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Options:

A. sp

B. sp^2

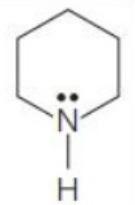
C. sp^3

D. dsp^2

Answer: C

Solution:

Solution:
In piperidine,



N has three σ-bonds and 1 lone pair.
Formula for finding hybridisation is as follows :
Number of lone pair + number of σ-bond + coordinate bond
Hybridisation of N in piperidine = 1 + 3 + 0 = 4
∴ Hybridisation = sp^3

Question 64

A binary liquid solution is prepared by mixing *n*-heptane and ethanol. Which one of the following statement is correct regarding the behaviour of the solution?

Options:

A. The solution is non-ideal, showing + ve deviation from Raoult's law.

B. The solution is non-ideal, showing - ve deviation from Raoult's law.

C. *n*-heptane shows +ve deviation while ethanol shows - ve deviation from Raoult's law.

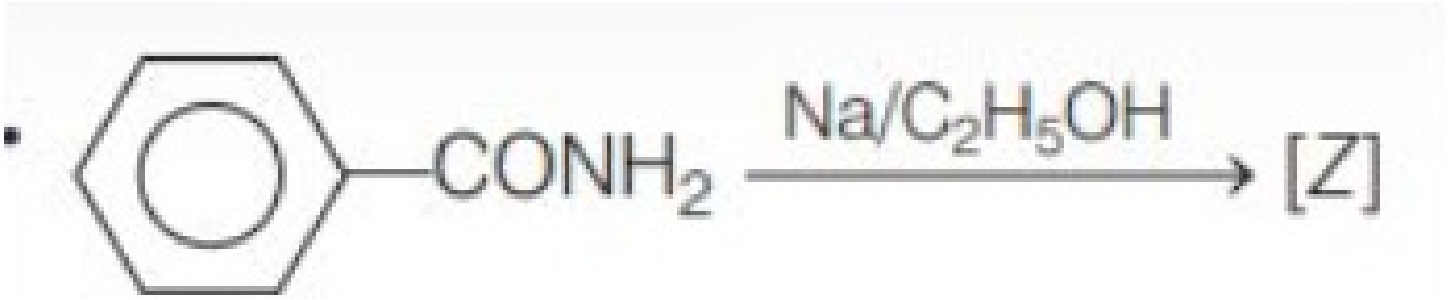
D. The solution formed is an ideal solution.

Answer: A

Solution:

Solution:
The solution prepared by mixing *n*-heptane and ethanol are non-ideal as *n*-heptane is non-polar and ethanol is polar. The stronger force of attraction between *n*-heptane- *n*-heptane and ethanol-ethanol are replaced by weaker force of attraction between *n*-heptane-ethanol in solution. Therefore, there will be increase in escaping tendencies of liquid molecules into vapour-phase which will increase vapour pressure more than expected value from Raoult's law. So, a positive deviation is observed.

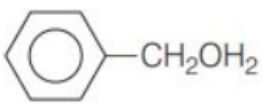
Question 65



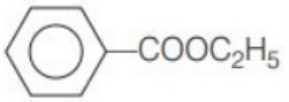
The product [Z] is

Options:

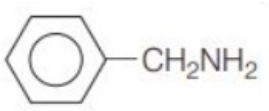
A.



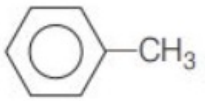
B.



C.



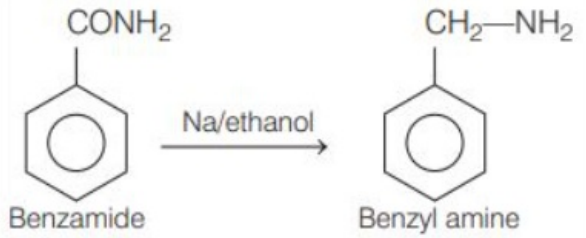
D.



Answer: C

Solution:

Solution:
Benzamide on reduction with sodium/ethanol produce benzyl amine as major product.



Question 66

The class of medicinal products used to treat stress is

©

Options:

A. analgesics

B. antiseptics

C. antihistamines

D. tranquilizers

Answer: D

Solution:

Solution:
Tranquilizers are used in treatment of stress. These affect the central nervous system and induce sleep in patients. They are also called psychotherapeutic drugs.

Question 67

In AgBr crystal, the ion size lies in the order $Ag^+ < < Br^-$. The AgBr crystal should have the following characteristics

Options:

A. defectless (perfect) crystal

B. Schottky defect only

C. Frenkel defect only

D. Both (b) and (c)

Answer: D

Solution:

Solution:
AgBr show both Frenkel and Schottky defect. Due to small size of Ag^+, ions misses from their lattice point and can occupy interstitial sites. Hence, due to missing of ions from their lattice points, Schottky defect arises and Frenkel defect arises when the missing ions occupy interstitial sites.

Question 68

Which of the following statement is not true about the hexagonal close packing?

Options:

A. The coordination number is 12.

B. It has 74% packing efficiency.

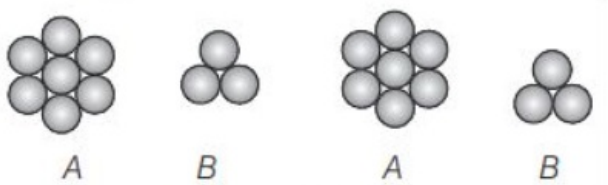
C. Tetrahedral voids of the second layer are covered by the sphere of the third layer.

D. In this arrangement spheres of the fourth layer are exactly aligned with those of the first layer.

Answer: D

Solution:

Solution:
Hexagonal close packing (hcp) can be arranged by two layers A and B one over another as shown in below figure.



$\therefore$ First layer and fourth layer are not exactly aligned.
Coordination number of an atom in hcp is 12 and packing fraction is 74%. The tetrahedral voids of second layer are covered by sphere of third layer.
Hence, statement (d) is incorrect.

Question 69

The half-life of two radioactive nuclides A and B are 1 and 2 min respectively. Equal weights of A and B are taken separately and allowed to disintegrate for 4 min. What will be the ratio of weight of A and B disintegrated?

©

Options:

A. 1:1

B. 1:4

C. 1:2

D. 1:3

Answer: B

Solution:

Solution:
Let weight of A and B be xg.
Half-life of A is 1 min that means it take $1 min$ for compound A to be half of its initial amount.
$\therefore$ Weight of A after 1 min $= \frac{x}{2}g$
After 2 min, it will be $= \frac{x}{4}g$
and similarly after 4 min, weight of A will be $\frac{x}{16}g$
Also, half-life of B is 2 min that means, it take 2 min for compound B to be half of its initial amount.
$\therefore$ Weight of B after 2 min $= \frac{x}{2}g$
$\therefore$ Weight of B after 4 min $= \frac{x}{4}g$
$\therefore$ Ratio of compound A and B disintegrated will be
$= \frac{\text{Weight of compound A}}{\text{weight of compound B}} = \frac{x}{16} \div \frac{x}{4} = \frac{x}{16} \times \frac{4}{x} = \frac{1}{4}$
$\therefore$ Ratio will be 1:4.

Question 70

A piece of wood when buried in Earth had 1% $^{14}C\left(t_{\frac{1}{2}} = 5760. years\right)$. Now as charcoal, it has only 0.25% ^{14}C . How long has the piece of wood been buried?

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Options:

A. 9133 years

B. 11520 years

C. 5760 years

D. 17280 years

Answer: B

Solution:

Solution:
(b) The half-life $t_{\frac{1}{2}}$ of C^{14} is 5760 years.
Initial amount of C^{14} is 1 %
Final amount of mathrm C^{14} is 0.25 %
$k = \frac{2303}{t} \log \frac{a}{a-x}$ .....(i)
where, $k =$ rate constant, $a =$ initial amount of reactant,
$a - x =$ amount left/final amount of reactant leftand $t =$ time taken.
Also, $k = \frac{0.6932}{t_{\frac{1}{2}}}$ .....(ii)
From (i) and (ii),
$\frac{0.6932}{t_{\frac{1}{2}}} = \frac{2.303}{t} \log \frac{a}{a-x}$
$\frac{0.6932}{5760} = \frac{2303}{t} \log \frac{1}{0.25}$
$\Rightarrow t = \frac{2303}{\frac{0.6932}{5760}} \log(4) = 11520 yr$

Question 71

One of the most widely used drugs in medicine iodex is

©

Options:

A. methyl salicylate

B. ethyl salicylate

C. acetyl salicylic acid

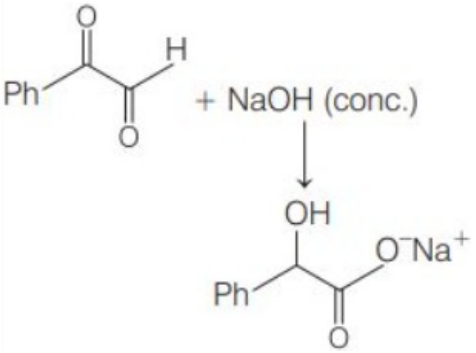
D. None of the above

Answer: A

Solution:

Solution:
Methyl salicylate is used as medicine in iodex for treatment of headaches and for sprains, aches and bruises.

Question 72



The reaction is known as

Options:

A. Cannizzaro's reaction

B. crossed Cannizzaro's reaction

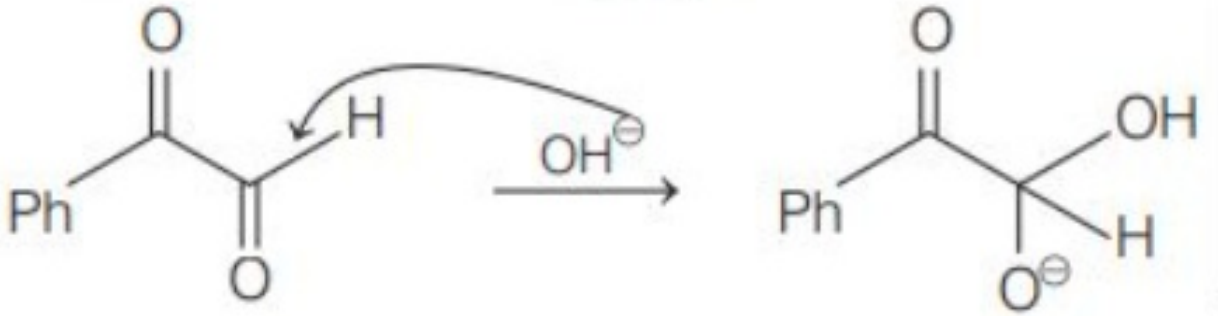
C. internal crossed Cannizzaro's reaction

D. Aldol condensation

Answer: C

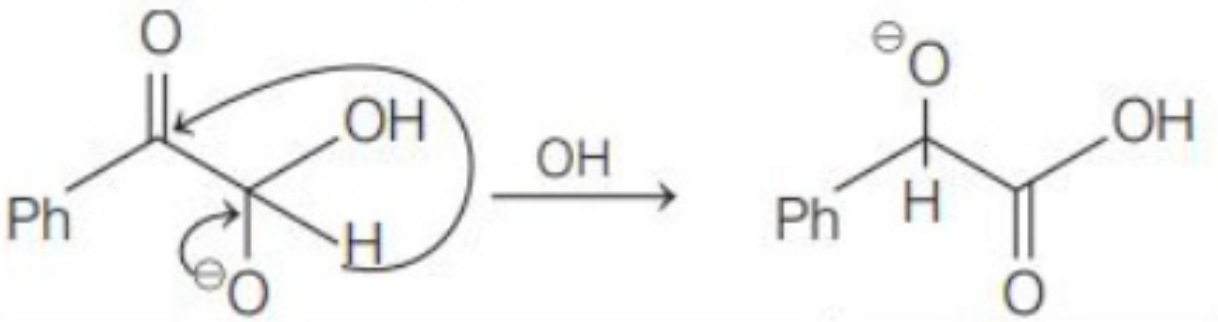
Solution:

Solution:
When 2-oxo-2-phenylacetaldehyde is treated with aqueous $NaOH$, it undergoes intramolecular cannizzaro reaction in which one carboxyl group is reduced to alcohol and other carboxyl group is reduced to sodium salt of carboxylic acid.
Mechanism
Firstly, base $NaOH$ attacks on carboxyl carbon of aldehyde (as aldehyde is more electrophilic)

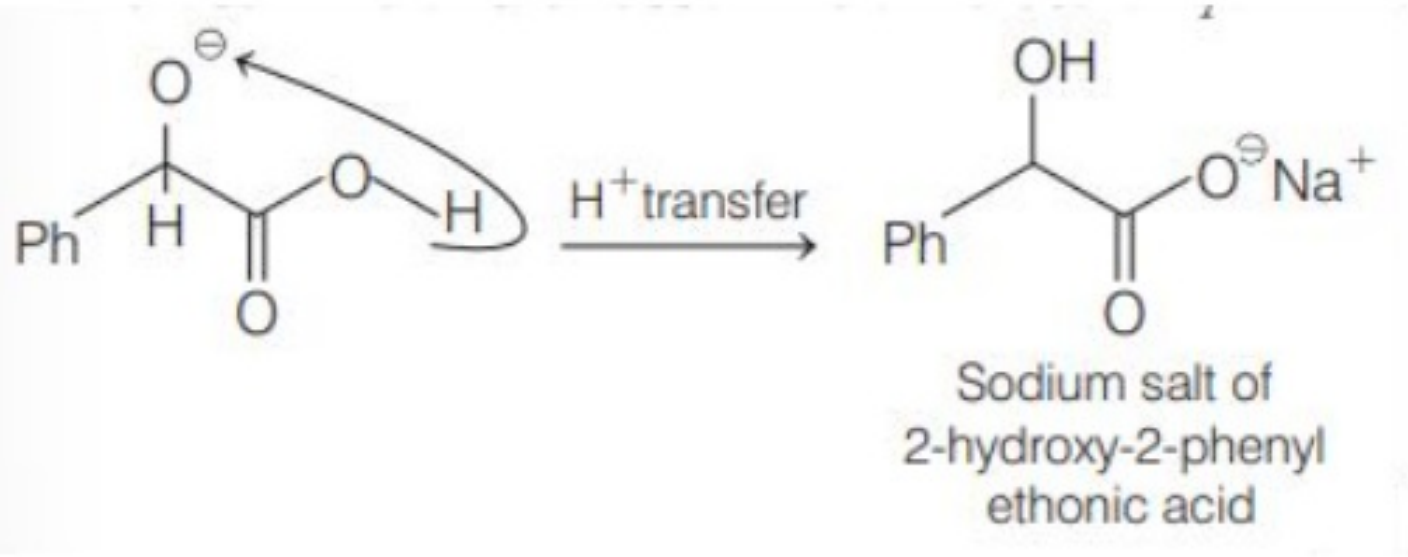


2-oxo-2-phenylacetaldehyde

Second step involves migration of H^+ion on ketone carbon.



At last H^+transfer occur intramolecularly.



Question 73

The ion that cannot be precipitated by HCl **and** H_2S

Options:

A. Pd^{2+}

B. Zn^{2+}

C. Ag^{+}

D. None of these

Answer: D

Solution:

Solution:
Lead (Pb^{2+}), zinc (Zn^{2+}), silver (Ag^{+}) can be precipitated by HCl and H_2S.
Hence, answer will be none of the above.
(a) Lead cation react with hydrochloric acid to give white ppt. of lead chloride.

$$\underset{(White ppt.)\,Lead chloride}{Pb^{2+} + HCl \longrightarrow PbCl_2 \downarrow}$$

Lead cation reacts with hydrogen sulphide to form insoluble ppt. of lead sulphide.

$$\underset{Lead sulphide}{Pb^{2+} + H_2S \longrightarrow PbS \downarrow}$$

(b) Zinc cation react with HCl to give white ppt. of zinc chloride.

$$\underset{Zinc chloride}{Zn^{2+} + HCl \longrightarrow ZnCl_2 \downarrow}$$

Zinc cation reacts with H_2S to give yellowish ppt. of zinc sulphide.

$$\underset{Zinc sulphide}{Zn^{2+} + H_2S \longrightarrow ZnS \downarrow}$$

(c) Silver cation reacts with hydrochloric acid to give white ppt. of $AgCl$ (silver chloride).

$$\underset{(White ppt.)\,Silver chloride}{Ag^{+} + HCl \longrightarrow AgCl \downarrow}$$

Silver cation react with hydrogen sulphide to form black colour ppt. of Ag_2mS (silver sulphide).

$$\underset{Silver sulphide}{Ag^{+} + H_2S \longrightarrow Ag_2S \downarrow}$$

Question 74

The geometrical isomerism is shown by

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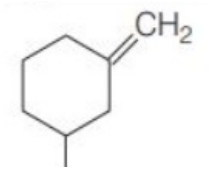
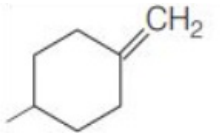
Options:

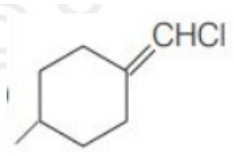
A.



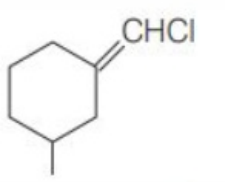
B.



C.



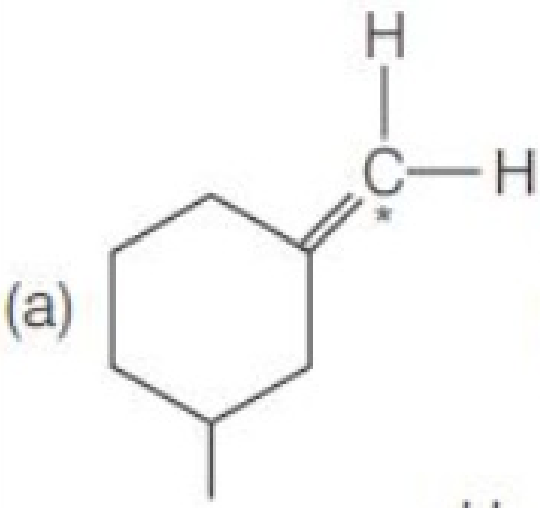
D.



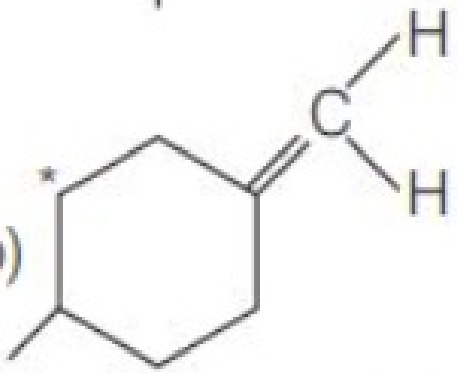
Answer: D

Solution:

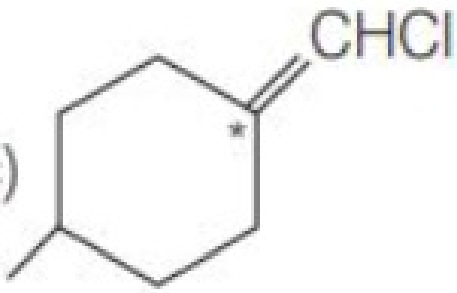
Solution:
Geometrical isomerism is exhibited by
(i) compounds having $C = C$
(ii) the two groups attached to same carbon atom should be different.
(a), (b) and (c) have same two groups attached to one carbon atom.



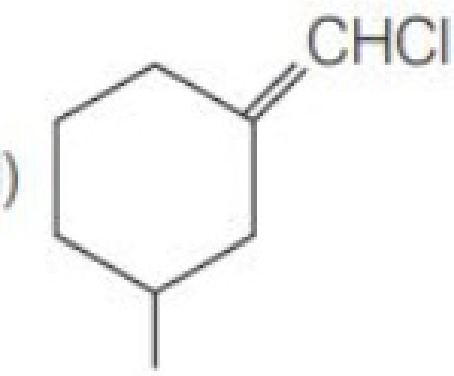
(a) with $\overset{*}{C}$ carbon both group are hydrogen atoms



(b) with $\overset{*}{C}$ carbon both group are hydrogen atoms



(c) with $\overset{*}{C}$ carbon both group are same



(d) Both the carbon of double bond have different group

Hence, they will not show geometrical isomerism.

Question 75

A dibasic acid containing C, H and O was found to contain C = 26.7% and H = 2.2%. The vapour density of diethyl ester was found to be 73 . What is the molecular formula of the acid?

©

Options:

A. CH_2O_2

B. $C_2H_2O_4$

C. $C_3H_3O_4$

D. $C_4H_4O_4$

Answer: B

Solution:

Solution:
First we need to find empirical formula.

Percentage of elements present	Elements	Convert % to grams	Divide mass by molar masses to find moles	Divide all moles with smallest number of moles
26.7%	C	$0.267g$	$\frac{0.267}{16} = 0.022$	$\frac{0.022}{0.022} = 1$
2.2%	H	$0.022g$	$\frac{0.022}{1} = 0.022$	$\frac{0.022}{0.022} = 1$
71.7%	O	$0.71g$	$\frac{0.71}{16} = 0.044$	$\frac{0.044}{0.022} = 2$

$\therefore C:H:O = 1:1:2$
Hence, empirical formula is CHO_2.
Molecular mass of diethyl ether is $(C_2H_5OC_2H_5)$ $2 \times 73 = 146u$
$\therefore$ Formula mass of ethyl group $= 29u$
In diethyl ether two ethyl group must have replaced the H-atom of dibasic acid.
$\therefore$ The molecular mass of acid = Molecular mass of diethyl ether $-(2 \times$ mass of ethyl group $) +$ mass of hydrogen
$= 149 - (2 \times 29) + 2 = 90u$
$\therefore$ Molecular formula of acid

$$= \frac{\text{Molecular mass of acid}}{\text{Empiricial mass formula}} \times CHO_2$$

$$= \frac{90}{(12 + 1 + 16 \times 2)} \times CHO_2$$

$$= \frac{90}{45} \times CHO_2 = 2 \times CHO_2$$

$\therefore$ Molecular formula of acid $= C_2H_2O_4$

Question 76

For the molecular formula C_5H_{10} possible structural isomers are

©

Options:

A. 6

B. 3

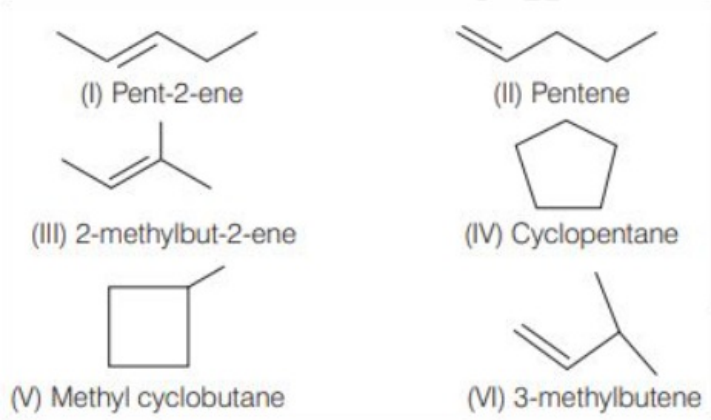
C. 4

D. 5

Answer: A

Solution:

Solution:
The number of possible structural isomers of C_5H_{10} are 6. Structural isomers are isomers having different atomic arrangement (structure) but have same molecular formula.



Question 77

The octane number of petrol generally available is

©

Options:

A. 20 to 40

B. 40 to 60

C. 80 to 100

D. 100 to 120

Answer: C

Solution:

Solution:
The octane number of petrol generally available is 80-100. Regular (the lowest octane fuel) has octane number 87 , midgrade (middle range octane fuel) has octane number 89-90 and premium (the highest octane fuel) has octane number 91-94.

Question 78

Nitrobenzene on electrolytic reduction in strongly acidic medium gives

©

Options:

A. aniline

B. p-aminophenol

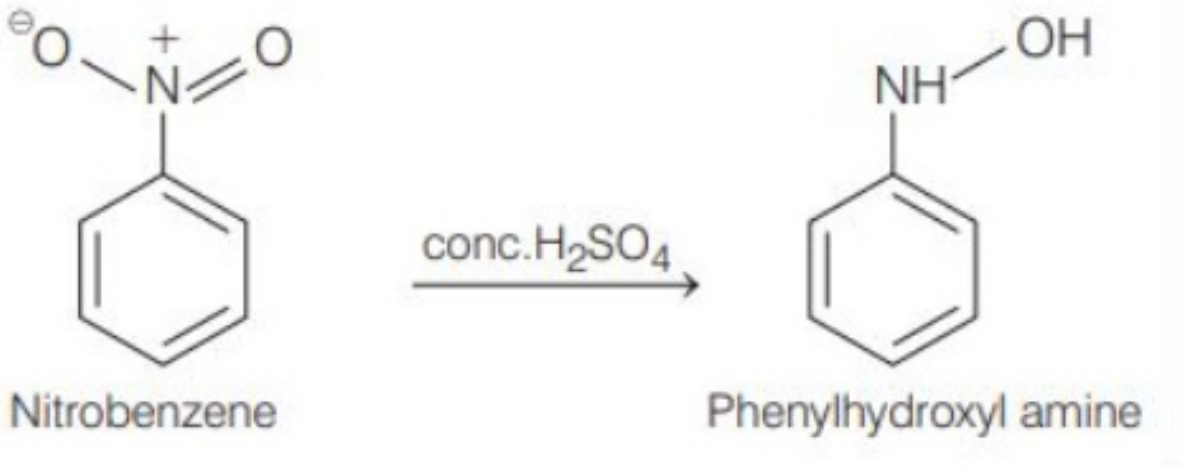
C. m-nitroaniline

D. nitrosobenzene

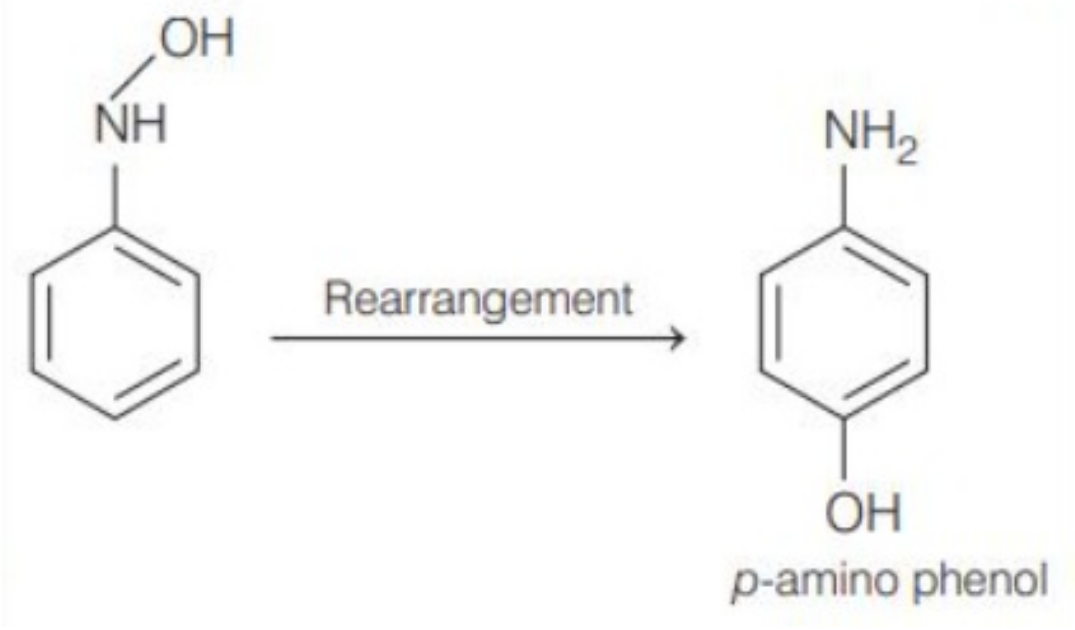
Answer: B

Solution:

Solution:
Nitrobenzene on electrolytic reduction in strong acidic medium will give *p*-aminophenol. This conversion takes place in two steps.
Step 1 Nitrobenzene reacts with conc. H_2SO_4 to produce phenylhydroxylamine.



Step 2 Phenylhydroxyl amine rearranges to form p-aminophenol.



Question 79

Catalyst increases rate of reaction because

©

Options:

A. it decreases ΔH

B. it increase ΔH

C. it decreases activation energy

D. it increase activation energy

Answer: C

Solution:

Solution:
A catalyst increases the rate of reaction by decreasing the activation energy. This is also known as threshold energy. It is the minimum amount of energy for a effective collision to occur.

Question 80

A first order reaction has half-life of 14.5 min. What percentage of the reactant will remain after 24 min ?

©

Options:

A. 68.2 %

B. 18.3 %

C. 31.8 %

D. 45.5 %

Answer: C

Solution:

Solution:
For first order reaction, rate constant is given as
$k = \frac{2.303}{t} \log \frac{a}{(a-x)}$ .....(i)
(a= amount of reactant used and $a - x =$ amount of reactant left after time t)
Given, $t_{\frac{1}{2}}$ (half-life) = 14.5 min
$\therefore t_{\frac{1}{2}} = \frac{0.6932}{k}$ (for first order reaction)

$14.5\& = \frac{0.6932}{k} \Rightarrow k = 0.0478\text{min}^{-1}$
From (i),
$0.0478 min^{-1} = \frac{2.303}{24 min} \log \frac{100\%}{a-x}$
$\frac{0.0478 \times 24}{2.303} = \log 100 - \log(a-x) [\log \frac{a}{b} = \log a - \log b]$
$0.4982 = 2 - \log(a-x)$
$1.5 = \log(a-x)$
$\text{antilog}(1.5) = (a-x)$
$31.8\% = (a-x)$
∴ 31.8% of reactant will remain after 24 min.

Question 81

The oxidation state of sulphur in Caro's and Marshall's acid are respectively

©

Options:

A. $+4, +6$

B. $+8, +6$

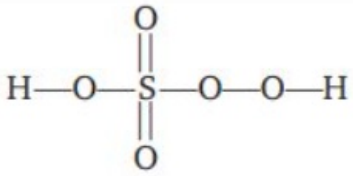
C. $+6, +6$

D. $+6, +4$

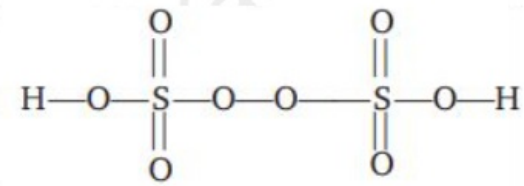
Answer: C

Solution:

Solution:
The formula for Caro's acid is H_2SO_5.Structure of Caro's acid



Oxidation number of sulphur in H_2SO_5 is : $x + 3$ (oxygen) $+2$ (oxygen in peroxide bond)+ 2(hydrogen) = 0
$x + 3(-2) + 2(-1) + 2(1) = 0$
$x - 6 - 2 + 2 = 0$
$x = +6$
The formula of Marshall's acid is $H_2S_2O_8$.



Oxidation number of sulphur in $H_2S_2O_8$ is : $2x + 6$ (oxygen) $+2$ (oxygen in peroxide bond)$+2$(hydrogen) $= 0$
$2x + 6(-2) + 2(-1) + 2(1) = 0$
$2x - 12 + (-2) + 2 = 0$
$2x = 12 \Rightarrow x = +6$
∴ Oxidation numbers of sulphur in Caro's acid and Marshall's acid are $+6$ and $+6$ respectively.

Question 82

When water is electrolysed, hydrogen and oxygen gases are produced. If 1.008g of H_2 is liberated at cathode. What mass of O_2 is formed at the anode?

©

Options:

A. 4g

B. 8g

C. 16g

D. 32g

Answer: B

Solution:

Solution:
When water is electrolysed, H_2 gas and O_2 gas are liberated in ratio 2:1.

$$2H_2O \xrightarrow{Electrolysis} 2H_2 + O_2$$

$$\therefore \frac{\text{Number of moles of } H_2}{2} = \frac{\text{Number of moles } O_2}{1}$$

$$\text{Number of moles} = \frac{\text{weight}}{\text{Molecular mass}}$$

$$\therefore \frac{\text{weight of } H_2}{\text{Molecular mass of } H_2} \times \frac{1}{2} = \text{weight of } \frac{O_2}{\text{Molecular mass of } O_2}$$

$$\frac{1.008}{4} = \frac{\text{weight of } O_2}{32}$$

$$\Rightarrow \text{Weight of } O_2 = \frac{1.008 \times 32}{4} = 8.016g$$

$\therefore$ Amount of O_2 liberated at anode is $8.016g \approx 8g$

Question 83

The pK_{a_1} and pK_{a_2} values of alanine are 2.3 and 9.7 respectively. The isoelectric point of alanine is

©

Options:

A. 3

B. 7

C. 8

D. 6

Answer: D

Solution:

Solution:
Isoelectric point can be calculated as :
Isoelectric point $= \frac{1}{2}[pK_{a_1} + pK_{a_2}]$
$\therefore$ Isoelectric point $= \frac{1}{2}[2.3 + 9.7] = \frac{1}{2} \times 12 = 6$

Question 84

Consider following statement about enzymes.
I. Enzymes lack in nucleophilic groups.
II. Enzymes are highly specific in catalysing reactions.
III. Enzymes catalyse chemical reactions by lowering of activation energy.
IV. Pepsin is a proteolytic enzyme.
Correct statements are

©

Options:

A. (I) only

B. (I) and (IV)

C. (I) and (III)

D. (II), (III) and (IV)

Answer: D

Solution:

Solution:
The only incorrect statement is (I). Enzymes possess nucleophiles. e.g. amino acids like lysine, histidine etc. serve as nucleophile and are active sites of enzymes.

Question 85

On strong heating lead nitrate gives

©

Options:

A. PbO_2, PbO, NO_2

B. PbO, NO_2, O_2

C. PbO, NO, O_2

D. PbO, NO, NO_2

Answer: B

Solution:

Solution:
On heating lead nitrate, it decomposes to form lead oxide (PbO), nitrogen dioxide (NO_2) and oxygen (O_2).
$2Pb(NO_2)_2 \xrightarrow{\Delta} 2PbO + 4NO_2 + O_2$

Question 86

A pale blue liquid is obtained by equimolecular mixture of two gases at $-30^\circ C$ is

©

Options:

A. N_2O

B. N_2O_3

C. N_2O_4

D. N_2O_5

Answer: B

Solution:

Solution:
A pale blue liquid of N_2O_3 is obtained by equimolecular mixture of two gases at $-30°C$. These two gases are NO and NO_2.
$NO + NO_2 \overset{-30°C}{\rightleftharpoons} \underset{(Pale blue)}{N_2O_3}$

Question 87

The pair of elements with almost similar atomic radii is

©

Options:

A. Ti, Zr

B. Mo, W

C. Ni, Pd

D. Cr, Mo

Answer: B

Solution:

Solution:
Mo and W has similar atomic radius due to lanthanoid contraction. This contraction occur due to poor shielding effect of $4f$-orbital due to which the electron in s-orbital are more attracted towards nucleus which will lead to decrease in size of atom.

Question 88

The number of moles of $KMnO_4$ that will be needed to react with one mole of sulphite in an acidic solution is

©

Options:

A. $\frac{2}{5}$

B. $\frac{3}{5}$

C. $\frac{4}{5}$

D. 1

Answer: A

Solution:

Solution:
The reaction with $KMnO_4$ and sulphite is as follows :
$MnO_4^- + 5SO_3^{2-} + 6H^+ \longrightarrow 2Mn^{2+} + 5SO_4^{2-} + 3H_2O$
So, one mole of sulphite (SO_3^{2-}) is needed to react with $\frac{2}{5}$ mole of MnO_4^- according to given reaction.
Note 2 moles of MnO_4^- react with 5 moles of sulphite in acidic medium.

Question 89

A dibromo derivative of an alkane reacts with sodium metal to form an alicyclic hydrocarbon. The derivative is

©

Options:

A. 2,2 dibromobutane

B. 1, 1 dibromopropane

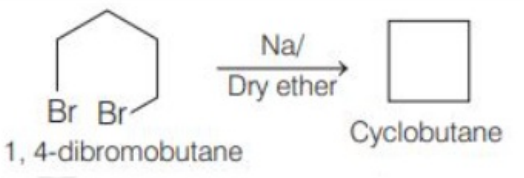
C. 1,4 dibromobutane

D. 1,2 dibromoethane

Answer: C

Solution:

Solution:
Dibromo derivative of alkane react with sodium metal to form alicyclic hydrocarbon. This reaction is known as intramolecular Wurtz reaction.
eg.



∴ Derivative is 1,4 - dibromobutane.

Question 90

$$X \xleftarrow[\text{catalyst}]{H_2 / \text{ Lindlar's}} H_3CC \equiv CCH_3$$

$$\xrightarrow[\text{liquid } NH_3]{Na /} Y.$$

X and Y respectively are

©

Options:

A. cis, trans-but-2-ene

B. Both trans-but-2-ene

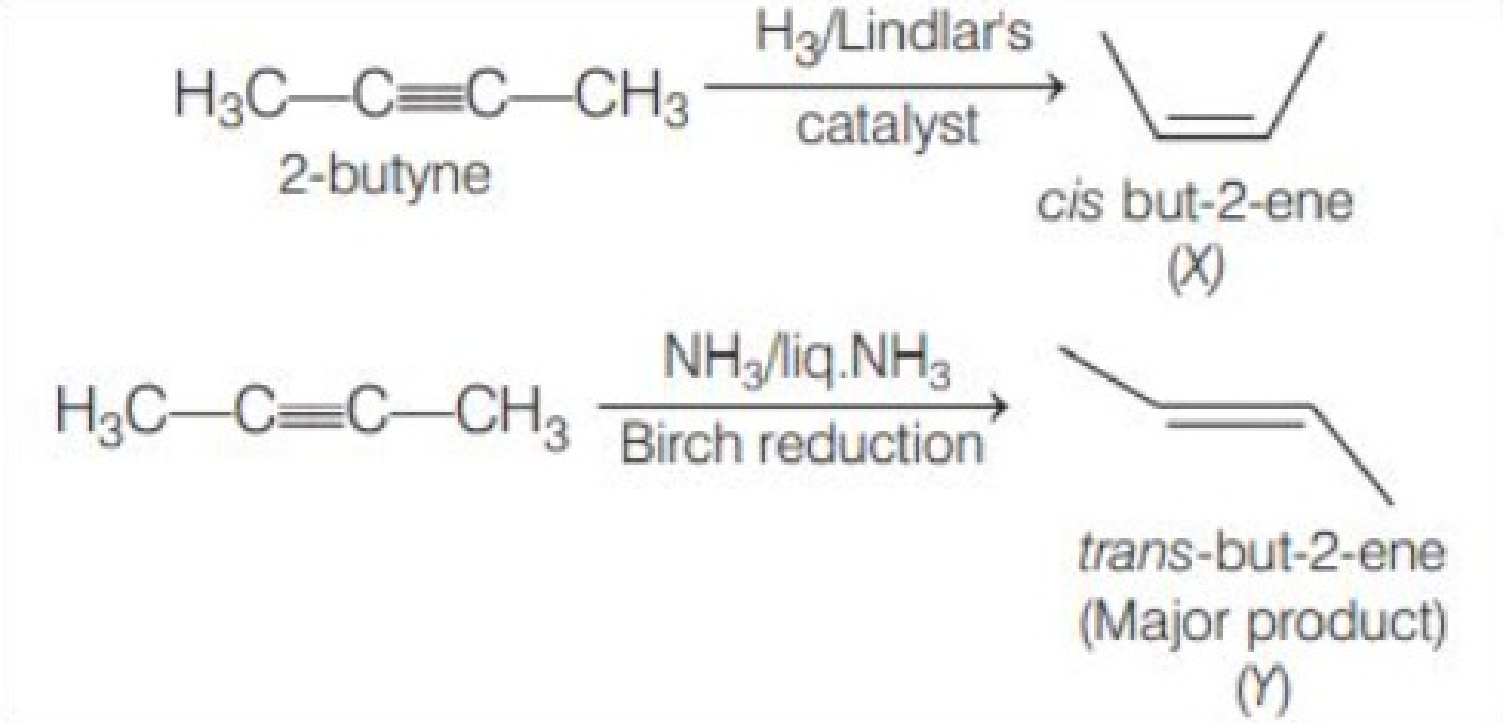
C. trans, cis-but-2-ene

D. Both cis-but-2-ene

Answer: A

Solution:

Solution:
2-butyne is treated with H_2 /Lindlar's catalyst to give compound X (cis -2-butene) and 2-butyne is treated with Na/ liq. NH_3 to produce trans-2-butene (Y) as major product.
Note Lindlar's catalyst is $H_2 / Pd / CaCO_3$ which is used for hydrogenation of alkyne to form cis-alkene.



Question 91

Which one of the following is correct for adsorption?

©

Options:

A. $\Delta G > 0$

B. $\Delta S > 0$

C. $\Delta S < 0$

D. $\Delta H > 0$

Answer: C

Solution:

Solution:
Entropy change (ΔS) is randomness of gas molecule which decreases when adsorption of molecules occur due to restriction in their motion.therefore Delta S is negative or Delta $S < 0$.

Question 92

Which of the following is not an ore of iron?

©

Options:

A. Limonite

B. Cassiterite

C. Magnetite

D. Siderite

Answer: B

Solution:

Solution:
Cassiterite is not an ore of iron.Limonite is iron ore and is a mixture of hydrated iron (III) oxide-hydroxide. Cassiterite is an tin oxide mineral, SnO_2.
Magnetite is an iron ore, Fe_3O_4.
Siderite is an iron ore, $FeCO_3$.

Question 93

Which one of the following reaction is an example for calcination process?

©

Options:

A. $MgCO_3 \xrightarrow{\Delta} MgO + CO_2$

B. $2\,ZnS + 3O_2 \rightarrow 2\,ZnO + 2SO_2$

C. $2\,Zn + O_2 \rightarrow 2\,ZnO$

D. $2\,Ag + 2\,HCl + [\,O\,] \rightarrow 2\,AgCl + H_2O$

Answer: A

Solution:

Solution:
The process of conversion of carbonate ore or hydroxide into its oxide by heating in absence of oxygen is known as calcination.
$MgCO_3 \xrightarrow{\Delta} MgO + CO_2$ is an example of calcination process.

Question 94

Ellingham diagram represents

©

Options:

A. change of ΔH with temperature

B. change of ΔG with pressure

C. change of $(\Delta G - T\Delta S)$ with temperature

D. change of ΔG with temperature

Answer: D

Solution:

Solution:
The Ellingham diagram represents change in free energy ΔG with temperature. It is a plot of Delta G vs temperature which is used to choose reducing agents for metal extraction.

Question 95

Benzene vapour mixed with air when passed over V_2O_5 catalyst as $500^\circ C$ gives

©

Options:

A. oxalic acid

B. glyoxal

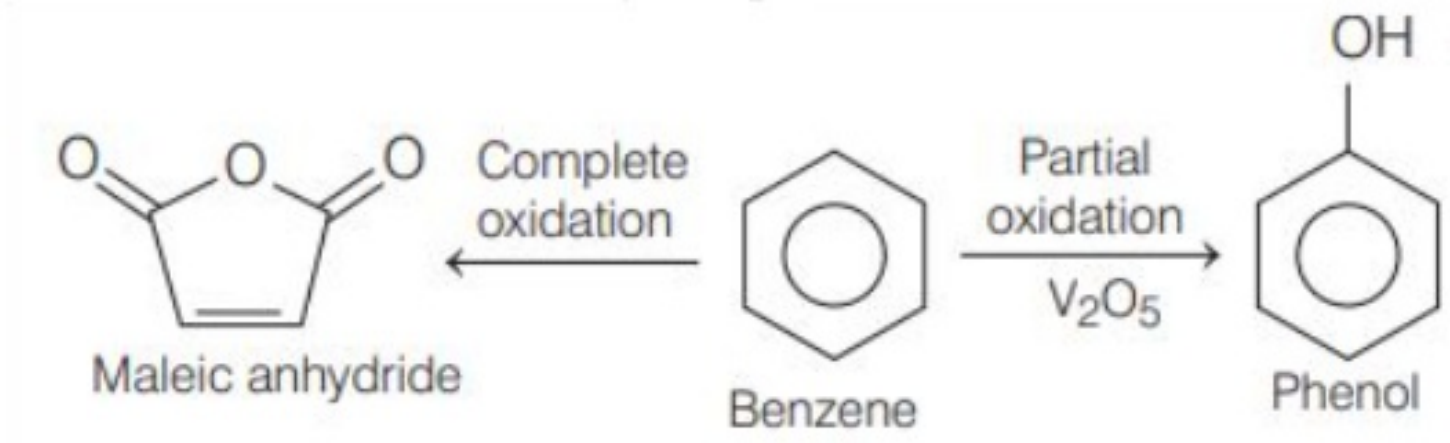
C. fumaric acid

D. maleic anhydride

Answer: D

Solution:

Solution:
Benzene when partially oxidised by V_2O_5 in presence of air, amounts to phenol and when it undergoes a process of complete oxidation by V_2O_5, it gives maleic anhydride.



Question 96

Ozonolysis of 2, 3-dimethyl-l-butene followed by reduction with zinc and

water gives

Options:

A. methanoic acid and 3-methyl-2- butanone

B. methanal and 3-methyl-2-butanone

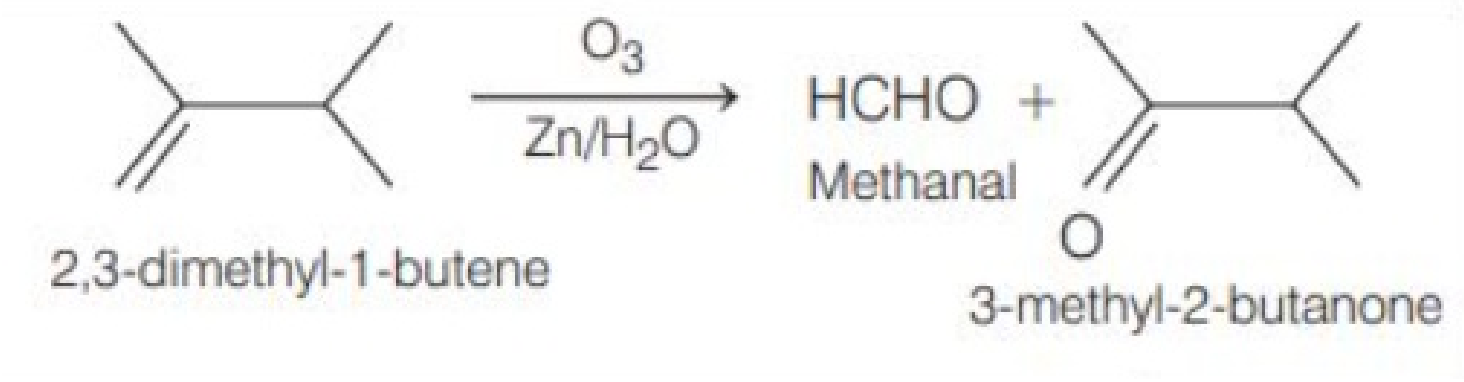
C. methanal and 2-methyl-3-butanone

D. methanoic acid and 2-methyl-3-butanone

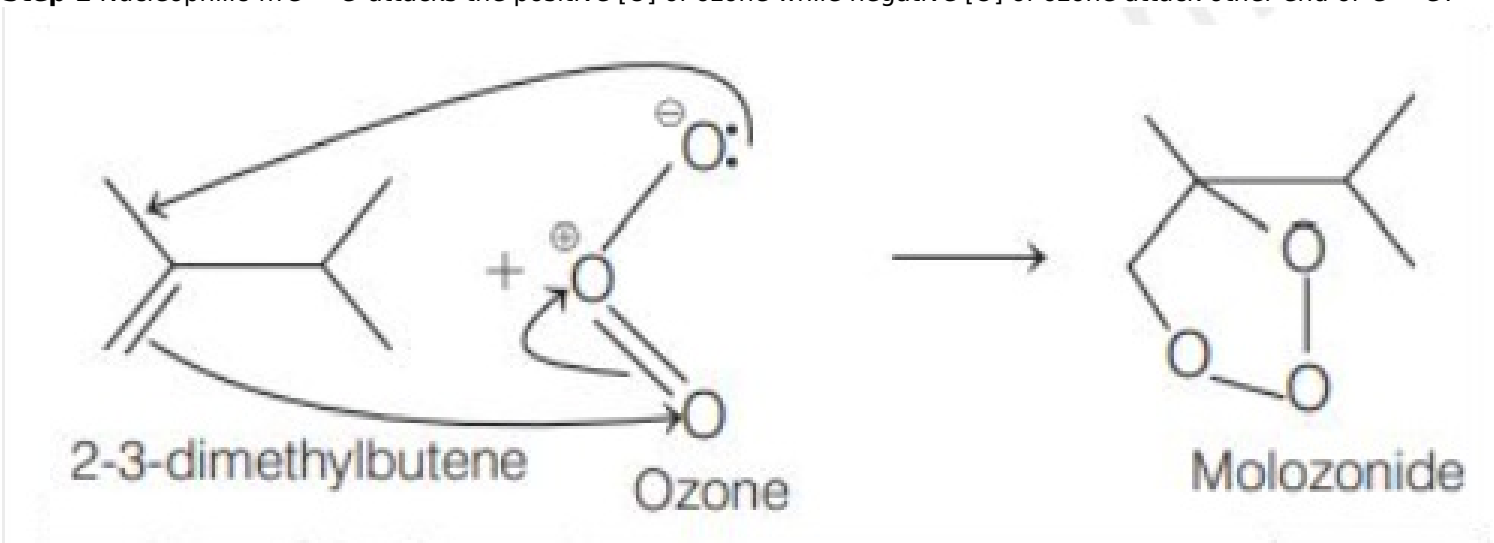
Answer: B

Solution:

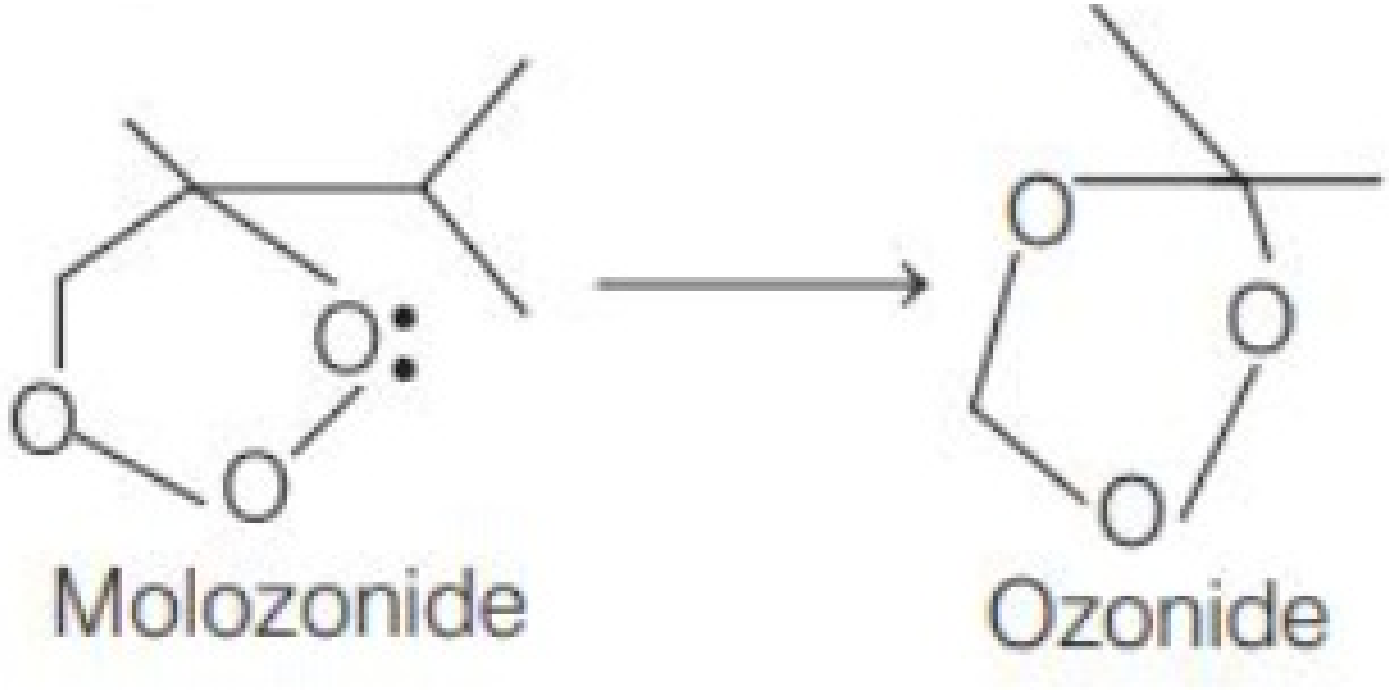
Solution:
Ozonolysis of 2, 3-dimethyl-1-butene gives methanal and 3-methyl-2-butanone as major product. Ozonolysis is a reaction in which unsaturated bonds of alkene are cleaved by ozone (O_3) in presence of Zn / H_2O to give carbonyl compounds.



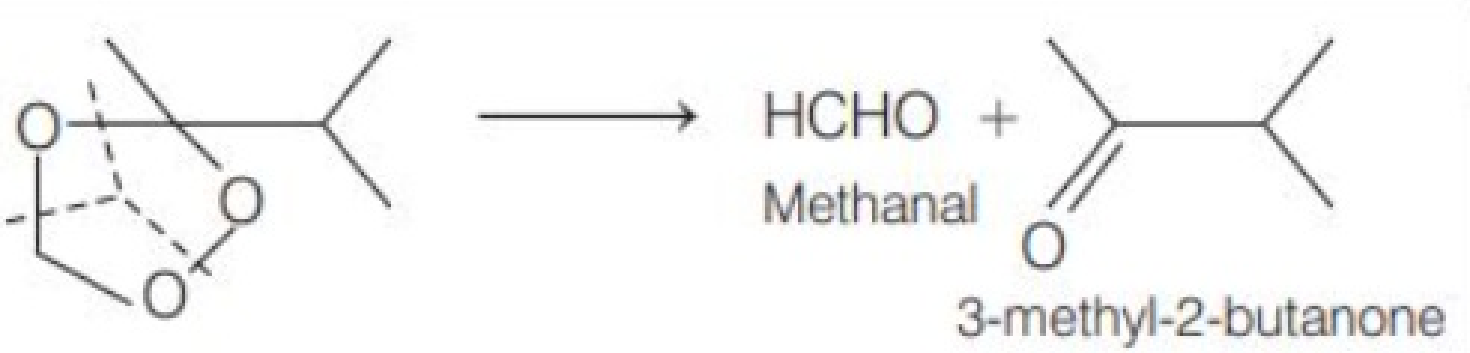
Mechanism
Step 1 Nucleophilic $mC = C$ attacks the positive [O] of ozone while negative [O] of ozone attack other end of $C = C$.



Step 2 The cyclic molozonide rearranges itself to ozonide.



Step 3 In last step, ozonide decompose to form carbonyl compounds in presence of Zn / H_2O.



Question 97

Standardisation of $Na_2S_2O_3$ using $K_2Cr_2O_7$ by iodometry, the equivalent $K_2Cr_2O_7$ is

©

Options:

A. molecular weight / 2

B. molecular weight / 6

C. molecular weight / 3

D. same as molecular weight

Answer: B

Solution:

Solution:
The reaction takes place when $Na_2S_2O_3$ is standardised using $K_2Cr_2O_7$ is as follows :
$4Cr_2O_7^{2-} + 3S_2O_3^{3-} + 26H^+ \longrightarrow 6SO_4^{2-} + 8Cr^{3+} + 13H_2O$

The oxidation state of chromium in $K_2Cr_2O_7$ changes from $+6$ to $+3$. The net change in oxidation number per formula unit is 6 .
$\therefore$ Value of n - factor = 6

$$\text{Equivalent weight, E } = \frac{\text{Molecular weight}}{\text{n} - \text{factor}}$$

$$= \frac{\text{Molecular weight}}{6}$$

Question 98

The IUPAC name for the complex $Na[BH(OCH_3)_3]$ is

©

Options:

A. sodium hydridotrimethoxoborate (III)

B. sodium hydridotrimethoxoyborate (II)

C. sodium hydridotrimethoxoboron

D. sodium hydridotrimethoxoborate (III)

Answer: A

Solution:

Solution:
IUPAC name of $Na[BH(OCH_3)_3 \text{right}]$ is sodium hydridotrimethoxoborate (III).
The ligand OCH_3 is named as methoxo.

Question 99

The oxidation number of iron in $Fe_4[Fe(CN)_6]_3$ are respectively

©

Options:

A. $+2, +3$

B. $+2, +2$

C. $+3, +3$

D. $+3, +2$

Answer: D

Solution:

Solution:
In $Fe_4[Fe(CN)_6]_3$, the counter ion and complex iron has different oxidation state.
$Fe_4[Fe(CN)_6]_3 \longrightarrow 4Fe^{3+} + 3[Fe(CN)_6]^{4-}$
$\therefore$ Counter iron will have $+3$ oxidation state.

∴ In complex ion,
$x + 6(CN) = -4$
$x + 6(-1) = -4 \Rightarrow x = +2$
∴ In complex, iron has $+2$ oxidation state.
∴ Oxidation number of iron in $Fe_4[Fe(CN)_6]_3$ are $+3$ and $+2$ respectively

---

Question 100

The numbers of bridging carbonyl group in $Fe_2(CO)_9$ has

©

Options:

A. one

B. two

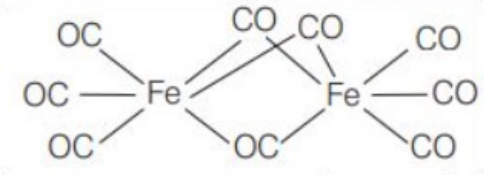
C. three

D. four

Answer: C

Solution:

Solution:
The number of bridging carbonyl group in $Fe(CO)_9$ is 3.



$Fe_2(CO)_9$ is an inorganic compound. IUPAC name of $Fe_2(CO)_9$ is diironnonacarbonyl.

Question 101

If between two numbers, two geometrical mean G_1 and G_2 and arithmetic mean A are placed, then the value of $\frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$ is

©

Options:

A. $\frac{A}{2}$

B. A

C. 2A

D. None of these

Answer: C

Solution:

Solution:
Let a and b are two numbers.
Given, G_1 and G_2 are two geometric mean between a and b.
$\therefore a, G_1, G_2, b$ are in GP.
$\because t_n = ar^{n-1}$ (nth term or GP)
$b = a.r^{4-1} \Rightarrow r = (\frac{b}{a})^{\frac{1}{3}}$
$G_1 = ar = a \times (\frac{b}{a})^{\frac{1}{3}} = a^{\frac{2}{3}}b^{\frac{1}{3}}$
$G_2 = ar^2 = a(\frac{b}{a})^{\frac{2}{3}} = a^{\frac{1}{3}}b^{\frac{2}{3}}$
$\therefore$ A be the arithmetic mean of a and b.
$\Rightarrow A = \frac{a+b}{2}$
Now, the value $\frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$
$= \frac{a^{\frac{4}{3}}.b^{\frac{2}{3}}}{a^{\frac{1}{3}}b^{\frac{2}{3}}} + \frac{a^{\frac{2}{3}}.b^{\frac{4}{3}}}{a^{\frac{2}{3}}b^{\frac{1}{3}}}$
$= a + b = 2A$

Question 102

The value of $4 + 2(1+2)\log 2 + \frac{2(1+2^2)}{2!}(\log 2)^2 + \frac{2(1+2^3)}{3!}(\log 2)^3 + ...$ is

©

Options:

A. 10

B. 12

C. $\log(3^2 \cdot 4^2)$

D. $\log(2^2 \cdot 3^2)$

Answer: B

Solution:

Solution:
$4 + 2(1+2)log2 + \frac{2(1+2^2)}{2!}(log2)^2$
$+\frac{2(1+2^3)}{3!}(log2)^3 + ...$
$\Rightarrow 2(1+1) + 2(1+2)log2 + \frac{2(1+2^2)}{2!}(log2)^2$
$+\frac{2(1+2^3)}{3!}(log2)^3 + ...$
$\Rightarrow 2\left[1 + log2 + \frac{(log2)^2}{2!} + \frac{(log2)^3}{3!}.....\right]$
$+2\left(1 + 2log2 + \frac{(2log2)^2}{2!} + \frac{(2log2)^3}{3!} + ...\right)$

$\because e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots.$
$\Rightarrow 2[e^{log 2}] + 2(e^{2log 2})$
$= 2(2^{log e}) + 2[(2^2)^{log_e}][\because a^{log_e b} = b^{log_e a}]$
$= 2.2 + 2.2^2[\because log_e e = 1]$
$= 4 + 8 = 12$

Question 103

The angle of a triangle are $\cot^{-1}2$ and $\cot^{-1}3$, then the third angle is

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Options:

A. $\frac{\pi}{4}$

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

Answer: B

Solution:

Solution:
$\because cot^{-1}2$ and $cot^{-1}3$ are two angles of a triangle.
$\therefore$ Third angle $= \pi - (cot^{-1}2 + cot^{-1}3)$
$= \pi - (tan^{-1}\frac{1}{2} + tan^{-1}\frac{1}{3})[\because cot^{-1}x = tan^{-1}\frac{1}{x}]$
$= \pi - tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}}\right)[\because tan^{-1}x + tan^{-1}y = tan^{-1}(\frac{x+y}{1-xy})]$
$= \pi - tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) = \pi - tan^{-1}1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

Question 104

In a $\triangle ABC, \angle B = \frac{\pi}{3}$ and $C = \frac{\pi}{4}$. Let D divides BC internally in the ratio 1:3, then $\frac{sin\angle BAD}{sin\angle CAD}$ equals

©

Options:

A. $\frac{1}{\sqrt{6}}$

B. $\frac{1}{3}$

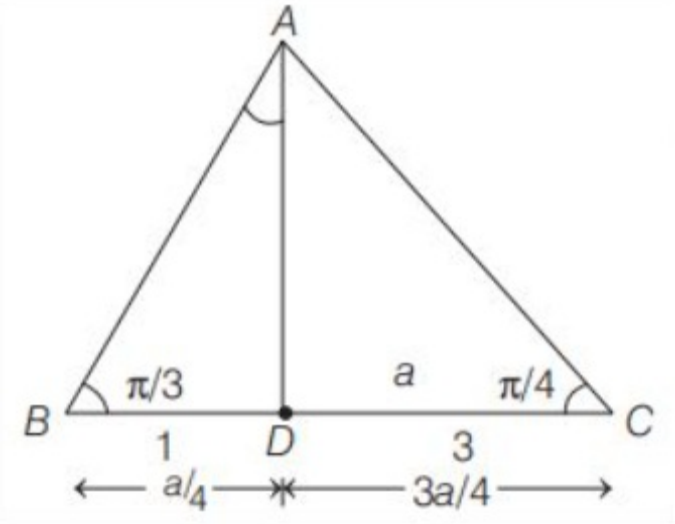
C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{\frac{2}{3}}$

Answer: A

Solution:

Solution:



Let $BC = a$,
$AB = c$
and $AC = b$
$\angle B = \frac{\pi}{3}$
$\angle C = \frac{\pi}{4}$
$\angle A = \pi - \frac{\pi}{3} - \frac{\pi}{4} = \frac{5\pi}{12}$
$\because B\frac{D}{C}D = \frac{1}{3}$ [given]
$\therefore BD = \frac{a}{4}$
and $CD = \frac{3a}{4}$
Using sine rule.
In ΔABD,

$\Rightarrow \frac{BD}{sin\angle BAD} = \frac{AD}{\frac{sin\pi}{3}}$

$\frac{\frac{a}{4}}{sin\angle BAD} = \frac{AD}{\frac{sin\pi}{3}}$ .....(i)

In ΔACD,

$\frac{CD}{sin\angle CAD} = \frac{AD}{\frac{sin\pi}{4}}$

$\Rightarrow \frac{\frac{3a}{4}}{sin\angle CAD} = \frac{AD}{\frac{sin\pi}{4}}$ .....(ii)

Divide eq. (i) by eq. (ii),

$\frac{\frac{\frac{a}{4}}{sin\angle BAD}}{\frac{\frac{3a}{4}}{sin\angle CAD}} = \frac{\frac{AD}{\frac{sin\pi}{3}}}{\frac{AD}{\frac{sin\pi}{4}}}$

$\Rightarrow \frac{sin\angle CAD}{3sin\angle BAD} = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow \frac{sin\angle BAD}{sin\angle CAD} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{\sqrt{2}}} = \frac{1}{\sqrt{6}}$

Question 105

The equation of a straight line passing through $(-3, 2)$ and cutting an intercept equal in magnitude but opposite in sign from the axes is given by

©

Options:

A. $x - y + 5 = 0$

B. $x + y - 5 = 0$

C. $x - y - 5 = 0$

D. $x + y + 5 = 0$

Answer: A

Solution:

Solution:

Let equation of libe is

$\frac{x}{a} + \frac{y}{b} = 1$

$\because$ According to equation, $b = -a$

$\frac{x}{a} + \frac{y}{-a} = 1$

Now it passes through $(-3, 2)$,

Then, $\frac{-3}{a} + \frac{2}{-a} = 1$

$\Rightarrow \frac{-3-2}{a} = 1$

$a = -5$

∴ Equation of line
$\frac{x}{-5}+\frac{y}{5}=1$
$\Rightarrow x-y=-5$
$\Rightarrow x-y+5=0$

Question 106

The angle between the tangents drawn from the point $(1,4)$ to the parabola $y^2=4x$ is

©

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

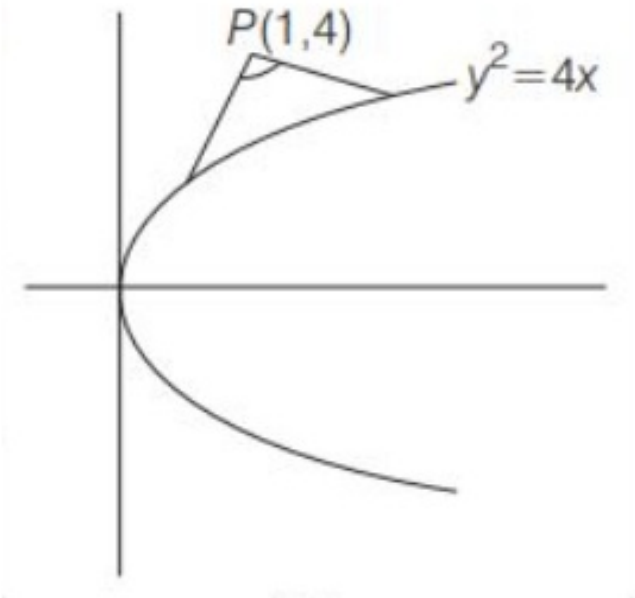
C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: C

Solution:

Solution:
Parabola : $S \Rightarrow y^2-4x=0$
Point : $P(1,4)$
∵ Pair of tangents is
$SS_1=T^2$



where, $S = Y^2 - 4x$
$S_1 = (4)^2 - 4(1) = 16 - 4 = 12$
$T = y(4) - 4(\frac{x+1}{2}) = 4y - 2x - 2$
$\therefore$ Pair of tangent from $P(1,4)$ is
$(y^2 - 4x)(12) = (4y - 2x - 2)^2$
$\Rightarrow 12(y^2 - 4x) = 4(2y - x - 1)^2$
$\Rightarrow 3(y^2 - 4x) = 4y^2 + x^2 + 1 - 4xy + 2x - 4y$
$\Rightarrow 3y^2 - 12x = 4y^2 + x^2 + 1 - 4xy + 2x - 4y$
$\Rightarrow x^2 + y^2 - 4xy + 14x - 4y + 1 = 0 \ldots\ldots\ldots (i)$
$\therefore a = 1, b = 1, h = -2, g = 7, f = -2, c = 1$
Let θ be the angle between pair of tangents represented by eq. (i)
$\therefore tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \left|\frac{2\sqrt{4-1}}{1+1}\right|$
$tan\theta = \sqrt{3}$
$\theta = \frac{\pi}{3}$

Question 107

If $\sin(x+y) = \log(x+y)$, then $\frac{dy}{dx}$ is equal to

©

Options:

A. 2

B. -2

C. 1

D. -1

Answer: D

Solution:

Solution:
$sin(x+y) = log(x+y)$
$\Rightarrow x+y = e^{sin(x+y)}$
Differenciating w.r.t.x,
$(1+\frac{dy}{dx}) = e^{sin(x+y)} . cos(x+y)(1+\frac{dy}{dx})$
$\Rightarrow (1+\frac{dy}{dx})[1-e^{sin(x+y)} . cos(x+y)] = 0$
$1+\frac{dy}{dx} = 0 or 1-e^{sin(x+y)} cos(x+y) = 0$
$\therefore \frac{dy}{dx} = -1$

Question 108

If straight line $y = 4x - 5$ is tangent to the curve $y^2 = px^3 + q$ at $(2, 3)$, then

©

Options:

A. $p = 2, q = -7$

B. $p = -2, q = 7$

C. $p = -2, q = -7$

D. $p = 2, q = 7$

Answer: A

Solution:

Solution:
$\because$ y=4x-5 is tangent to the curve
$y^2 = px^3 + q \text{ at } (2,3)$
$\therefore$ Slope of tangent to the curve $y^2 = px^3 + q \text{ at } (2,3)$
$= \left.\frac{dy}{dx}\right|_{(2,3)}$

Differenciating $y^2 = px^3 + qw.r.t.x$,
$2y\frac{dy}{dx} = 3px^2 \Rightarrow \frac{dy}{dx} = \frac{3px^2}{2y}$
$\therefore \left.\frac{dy}{dx}\right|_{(2,3)} = \frac{3p(2)^2}{2(3)} \Rightarrow 2p$
Slope of line $y = 4x - 3$ is equal to 4
$\therefore \left.\frac{dy}{dx}\right|_{(2,3)} = 4 \Rightarrow 2p = 4 \Rightarrow p = 2$
$\because (2,3)$ lies on the curve $y^2 = px^3 + q$
$\Rightarrow (3)^2 = p(2)^3 + q$
$9 = 2^4 + q \Rightarrow q = 9 - 16 = -7$
$\therefore p = 2, q = -7$

Question 109

The coordinates of a point on the line $\frac{x}{2} = \frac{y+1}{-3} = z - 1$ at a distance $\sqrt{11}$ from the point $(1, -1, 1)$ are

©

Options:

A. $(2, -4, 2)$

B. $(1, -2, 4)$

C. $(\frac{1}{7}, \frac{-2}{7}, \frac{3}{7})$

D. $(-2, 4, -2)$

Answer: A

Solution:

Solution:
Line : $\frac{x}{2} = \frac{y+1}{-3} = z - 1 = \lambda$
Any point P on this line is $P(2\lambda, -3\lambda - 1, \lambda + 1)$
$\therefore$ P is $(2\lambda, 3\lambda - 1, \lambda + 1)$.
$\therefore$ According to question,
Distance between $P(2\lambda, -3\lambda - 1, \lambda + 1)$ and $(1, -1, 1)$ is $\sqrt{11}$,
$\therefore \sqrt{(2\lambda - 1)^2 + (-3\lambda - 1 + 1)^2 + (\lambda + 1 - 1)^2} = \sqrt{11}$
$(2\lambda - 1)^2 + (-3\lambda)^2 + (\lambda)^2 = 11$
$4\lambda^2 - 4\lambda + 1 + 9\lambda^2 + \lambda^2 = 11$
$\Rightarrow 14\lambda^2 - 4\lambda - 10 = 0 \Rightarrow 7\lambda^2 - 2\lambda - 5 = 0$
$\Rightarrow 7\lambda^2 - 7\lambda + 5\lambda - 5 = 0$
$\Rightarrow 7\lambda(\lambda - 1) + 5(\lambda - 1) = 0$
$\Rightarrow (\lambda - 1)(7\lambda + 5) = 0$
$\lambda = 1$ or $\lambda = \frac{-5}{7}$
when $\lambda = 1$, then P is $(2, -4, 2)$ and when
$\lambda = -\frac{5}{7}$, then P is $(-\frac{10}{7}, \frac{8}{7}, \frac{2}{7})$.

Question 110

If the lines $x - 2 = y - 3 = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = z - 5$ are coplanar,

then k have

Options:

A. any value

B. exactly one value

C. exactly two values

D. exactly three values

Answer: C

Solution:

Solution:

$L_1 = \frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$

$L_2 = \frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$

$\because$ L_1 and L_2 are coplanar.

$\therefore (a_2 - a_1) . (b_1 \times b_2) = 0$

$[(a_2 - a_1) b_1 b_2] = 0$

Now, $a_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $b_1 = \hat{i} + \hat{j} - \hat{k}$

$a_2 = \hat{i} + 4\hat{j} + 5\hat{k}$ and $b_2 = k\hat{i} + 2\hat{j} + \hat{k}$

Now, $[(a_2 - a_1) b_1 b_2] = 0$

$$\Rightarrow \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$-(1 + 2k) - 1(1 + k^2) + 1(2 - k) = 0$

$\Rightarrow -1 - 2k - 1 - k^2 + 2 - k = 0$

$\Rightarrow -k^2 - 3k = 0 \Rightarrow k^2 + 3k = 0$

$\Rightarrow k(k + 3) = 0 \Rightarrow k = -3, 0$

$\therefore$ k have exactly two values.

Question 111

The value of $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cot x}$ is

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{1}{2}$

D. 1

Answer: B

Solution:

Solution:

$I = \int_0^{\frac{\pi}{2}} \frac{dx}{1+\cot x}$

$I = \int_0^{\frac{\pi}{2}} \frac{sinxdx}{sinx+cosx} [cotx = \frac{cosx}{sinx}]$.....(i)

$\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

Now,

$I = \int_0^{\frac{\pi}{2}} \frac{sin(\frac{\pi}{2}-x)\,dx}{sin(\frac{\pi}{2}-x)} + cos(\frac{\pi}{2}-x)$

$I = \int_0^{\frac{\pi}{2}} \frac{cosxdx}{cosx+sinx}$ .....(ii)

On adding eqs. (i) and (ii), we get

$2I = \int_0^{\frac{\pi}{2}} \frac{(sinx+cosx)}{sinx+cosx} dx = \int_0^{\frac{\pi}{2}} 1dx$

$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}} \Rightarrow 2I = \frac{\pi}{2} - 0 \Rightarrow 2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

Question 112

The area between the curve $y^2 = 4ax$, X-axis and the ordinate $x = 0$ and $x = a$ is

©

Options:

A. $\frac{4}{3}a^2$

B. $\frac{8}{3}a^2$

C. $\frac{2}{3}a^2$

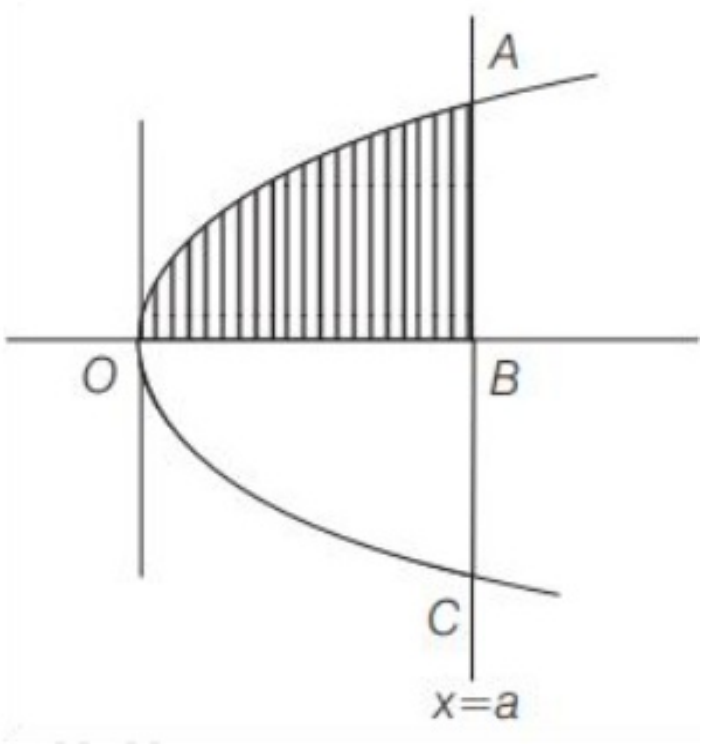
D. $\frac{5}{3}a^2$

Answer: A

Solution:

Solution:

$\because$ Area between the curve $y^2 = 4ax, X-axis$,
$x = 0$ and $x = a$



= Area of region $OABO$

$$= \int_a^b y\,dx = \int_a^b \sqrt{4ax}\,dx$$

$$= 2\sqrt{a}\int_a^b \sqrt{x}\,dx = 2\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$$

$$= 2\sqrt{a}\,.\frac{a^{\frac{3}{2}}}{\frac{3}{2}} = \frac{4a^2]}{3}$$

Question 113

The function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the interval $(4, 5)$ is

Options:

A. increasing

B. decreasing

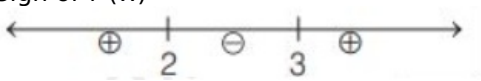
C. constant

D. nothing can be said

Answer: A

Solution:

Solution:
$f(x) = 2x^3 - 15x^2 + 36x - 48$
$f'(x) = 6x^2 - 30x + 36$
$= 6(x^2 - 5x + 6) = 6(x^2 - 3x - 2x + 6)$
$= 6(x - 3)(x - 2)$
Sign of $f'(x)$



$\therefore f(x)$ is increasing in $x \in (-\infty, 2) \cup (3, \infty)$
and $f(x)$ is decreasing in $x \in (2, 3)$.
$\therefore$ In the interval $(4, 5)$.
$f(x)$ is increasing

Question 114

Which of the following functions is inverse of itself?

Options:

A. $f(x) = \dfrac{1 - x}{1 + x}$

B. $f(x) = 5^{\log x}$

C. $f(x) = 2^{x(x-1)}$

D. None of these

Answer: A

Solution:

Solution:
We know that
If $f^{-1}(x)$ is inverse of $f(x)$, then
$fof^{-1}(x) = x$
(a) $f(x) = \dfrac{1 - x}{1 + x}$

$\therefore fof(x) = \frac{1-f(x)}{1+f(x)} = \frac{1-(\frac{1-x}{1+x})}{1+(\frac{1-x}{1+x})}$
$= \frac{1+x-1+x}{1+x+1-x} = x$
$\therefore f-1(x) = f(x)$
(b) $f(x) = 5^{logx}$
$\therefore fof(x) = 5^{logf(x)} = 5^{log(5^{logx})} = 5^{logxlog5} \neq x$
$\therefore f-1(x) \neq f(x)$
(c) $f(x) = 2^{x(x-1)}$
$\therefore fof(x) = 2^{f(x)(f(x)-1)}$
$= 2^{2^{x(x-1)}(2^{x(x-1)}-1)}$
$\neq x$
$\therefore f-1(x) \neq f(x)$

Question 115

Three dice are thrown together. The probability that all will show even number is

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Options:

A. $\frac{3}{216}$

B. $\frac{9}{216}$

C. $\frac{27}{216}$

D. None of these

Answer: C

Solution:

Solution:
Three dice are thrown together.
$\therefore$ Total number of outcomes
$= 6 \times 6 \times 6 = 216$
Now, all shows even numbers.
Then, total favourable outcomes count
$= 3 \times 3 \times 3 = 27$
[$\because$ total 3 even numbers are there, i.e. $2, 4, 6$].
$\therefore$ Required probability $= \frac{27}{216}$

Question 116

Correlation coefficient is

©

Options:

A. arithmetic mean of regression coefficient

B. harmonic mean of regression coefficient

C. geometric mean of regression coefficient

D. None of the above

Answer: C

Solution:

Solution:
∵ Correlation coefficient is the geometric mean of the regression coefficient.

Question 117

Let z_1 and z_2 are two complex numbers whose principal arguments are α and β, such that $\alpha + \beta > \pi$, then the principal argument of $(z_1 z_2)$ is

©

Options:

A. $\alpha + \beta + \pi$

B. $\alpha + \beta - \pi$

C. $\alpha + \beta - 2\pi$

D. $\alpha + \beta$

Answer: C

Solution:

Solution:
$\arg(z_1) = \alpha, \arg(z_2) = \beta$ and $\alpha + \beta > \pi$
∵ we know that,
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_1) + 1m\pi$
$= \alpha + \beta + 2m\pi, m \in I$
∵ Principal argument lies between $-\pi$ to π and $\alpha + \beta > \pi$
∴ Principal argument of $z_{1.}z_2 = \alpha + \beta - 2\pi$

Question 118

The common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{1985} + z^{100} + 1 = 0$ are

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Options:

A. $-1, \omega$

B. $-1, \omega^2$

C. ω, ω^2

D. $1, \omega, \omega^2$

Answer: C

Solution:

Solution:
$z^3 + 2z^2 + 2z + 1 = 0$
$\Rightarrow (z+1)(z^2 + z + 1) = 0$
$z + 1 = 0 or z^2 + z + 1 = 0$
when $z = -1, z^{1985} + z^{100} + 1 = (-1)^{1985} + (-1)^{100} + 1$
$= -1 + 1 + 1 = 1 \neq 0$
$\because z = -1$ do not satisfy the equation
$z^{1985} + z^{100} + 1 = 0$
Then, it is not the common roots.
$\therefore$ The common roots are the roots of $z^2 + z + 1 = 0$
We know that ω and ω^2 are the roots of
$z^2 + z + 1 = 0$
(where ω is cube root of unity)
When $z = \omega$,
$(\omega)^{1985} + (\omega)^{100} + 1 = (\omega^3)^{661}\omega^2 + (\omega^3)^{33}\omega + 1$
$= \omega^2 + \omega + 1 = 0$ (true)
When $z = \omega$,
$(\omega^2)^{1985} + (\omega^2)^{100} + 1 = (\omega)^{3970} + \omega^{200} + 1$
$= (\omega^2)^{1323}\omega + (\omega^3)^{66}\omega^2 + 1$
$= (1)^{1323}\omega + (1)^{66}\omega^2 + 1$
$= \omega + \omega^2 + 1 = 0$ (true)
$\therefore \omega, \omega^2$ are the common roots of $z^{1985} + z^{100} + 1 = 0$
and $z^3 + 2z^2 + 2z + 1 = 0$.

Question 119

The order and degree of the differential equation $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$ are

©

Options:

A. $\frac{1}{2}, 1$

B. 2,1

C. 1,1

D. 1,2

Answer: D

Solution:

Solution:
$\frac{\sqrt{dy}}{dx} - \frac{4dy}{dx} - 7x = 0$
$\frac{\sqrt{dy}}{dx} = \frac{4dy}{dx} + 7x$
Squaring on both sides,

$\frac{dy}{dx} = (\frac{4dy}{dx} + 7x)^2$
$\Rightarrow 16(\frac{dy}{dx})^2 + \frac{dy}{dx}(56x - 1) + 49x^2 = 0$
$\because$ Now equation is radical free.
$\therefore$ Order $= 1$, Degree $= 2$

Question 120

The solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$ is

©

Options:

A. $\log(1 + y) = x + \frac{x^2}{2} + c$

B. $(1 + y)^2 = x + \frac{x^2}{2} + x$

C. $\log(1 + y) = \log(1 + x) + c$

D. None of the above

Answer: A

Solution:

Solution:
$\frac{dy}{dx} = 1 + x + y + xy$
$\frac{dy}{dx} = (1 + x) + y(1 + x)$
$\frac{dy}{dx} = (1 + x)(1 + x) \Rightarrow \frac{dy}{1 + y} = (1 + x)\,dx$
Integrating both sides,
$\int \frac{dy}{1 + y} = \int (1 + x)\,dx$
$\Rightarrow log(1 + x) = x + \frac{x^2}{2} + c$

Question 121

A curve passes through the following points

x	1	2	3	4
y	1	4	9	16

using trapezoidal rule, find the area bounded by the curve, X-axis and lines $x = 1, x = 4$.

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Options:

A. 20.5 sq. units

B. 21.5 sq. units

C. 22.5 sq. units

D. 23.5 sq. units

Answer: B

Solution:

Solution:

x	1	2	3	4
y	1 y_1	4 y_2	9 y_3	16 y_4

Using trapezoidal rule
$a = 1, b = 4, n = 3$
$h =$ width of sub - interval $= 1$
$\int_1^4 f(x)\,dx = \frac{1}{2}[(y_1 + y_4) + 2(y_2 + y_3)]$
$= \frac{1}{2}[(1 + 16) + 2(4 + 9)]$
$= \frac{1}{2}(17 + 26)$
$= \frac{1}{2} \times 43 = \frac{43}{2} = 21.5$

Question 122

The iteration formula for Newton- Raphson method is

©

Options:

A. $X_{n+1} = X_n + \frac{f(X_n)}{f'(X_n)}$

B. $x_{n+1} = X_n + \frac{f(X_{n-1})}{f'(x_n)}$

C. $X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$

D. $X_{n+1} = X_n - \frac{f(X_{n-1})}{f'(X_n)}$

Answer: C

Solution:

Solution:
Newton Rapshon Method
Iteration formula $\Rightarrow X_{n+1} = X_n = \frac{f(X_n)}{f'(X_n)}$

Question 123

If $f(x)$ { $\begin{matrix} x & x \geq 0 \\ -x & x < 0 \end{matrix}$., then at $x = 0$

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Options:

A. $f(x)$ is not continuous

B. $f(x)$ is differentiable

C. $f(x)$ is continuous but not differentiable

D. None of the above

Answer: C

Solution:

Solution:
$f(x) = \{ \begin{matrix} x & x \geq 0 \\ -x & x < 0 \end{matrix}$
Contuinuity at $x = 0$
$f(0) = 0$
RHL $= f(o^+) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} + x$
$= \lim_{h \to 0} + (0 + h) = 0$
LHL $= f(0^-) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} - x$
$= \lim_{h \to 0} - (-h) = \lim_{h \to 0} h = 0$
$\because f(0) = f(0^+) = f(0^-) = 0$
$\because f(x)$ is continuous at $x = 0$
Differentiability at $x = 0$
LHD $= f'(0^-) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \to 0} \frac{-(-h) - 0}{-h}$
$\lim_{h \to 0} \frac{h}{-h} = \lim_{h \to 0} - 1 = 1$
RHD $= f'(0^+) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$
$= \lim_{h \to 0} \frac{h - 0}{h} = \lim_{h \to 0} = 1$
$\because f'(0^+) \neq f'(0^+)$
$\therefore f(x)$ is not differentiable at $x = 0$

Question 124

The equivalent function of $\log x^2$ is

Options:

A. $2 \log x$

B. $2 \log |x|$

C. $|\log x^2|$

D. $(\log x)^2$

Answer: B

Solution:

Solution:
$y = logx^2 = log|x|^2 = log|x|$

Question 125

Solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\varphi(\frac{y}{x})}{\varphi'(\frac{y}{x})}$ **is**

Options:

A. $\varphi(\frac{y}{x}) = kx$

B. $x\varphi(\frac{y}{x}) = k$

C. $\varphi(\frac{y}{x}) = ky$

D. $y\varphi(\frac{y}{x}) = k$

Answer: A

Solution:

Solution:
$\frac{dy}{dx} = \frac{y}{x} + \frac{\phi(\frac{y}{x})}{\phi'(\frac{y}{x})}$
Let $\frac{y}{x} = V$
$y = Vx$
$\frac{dy}{dx} = V + \frac{xdv}{dx}$
$\therefore$ Equation becomes
$V + \frac{xdv}{dx} = V + \frac{\phi(V)}{\phi'(V)}$
$\Rightarrow \frac{xdV}{dx} = \frac{\phi(V)}{\phi'(V)}$
$\Rightarrow \frac{\phi'(V)}{\phi(V)} dV = \frac{dx}{x}$

Integrating both sides,
$\int \frac{\phi'(V)}{\phi(V)} dV = \int \frac{dx}{x}$
$\Rightarrow \ln[\phi(V)] = \ln x + c$
$\Rightarrow \ln(\phi(V)) - \ln x = c$
$\Rightarrow \ln\left(\frac{\phi(\frac{y}{x})}{x}\right) = c$
$\Rightarrow \frac{\phi(\frac{y}{x})}{x} = e^e = k \Rightarrow \phi(\frac{y}{x}) = kx$

Question 126

Differential equation of those circles which pass through origin and their centres lie on Y-axis will be

©

Options:

A. $(x^2 - y^2)\frac{dy}{dx} + 2xy = 0$

B. $(x^2 - y^2)\frac{dy}{dx} - 2xy = 0$

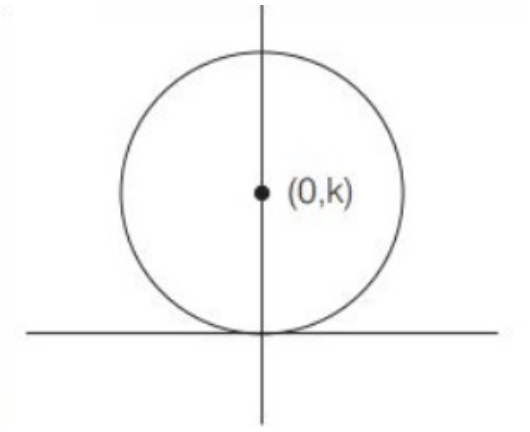
C. $(x^2 - y^2)\frac{dy}{dx} - xy = 0$

D. $(x^2 - y^2)\frac{dy}{dx} + xy = 0$

Answer: B

Solution:

Solution:
Family of circles which pass through origin and their centre lies on Y-axis has $C(0, k)$ and radius $= k$
Then, equation of circle will be
$(x - 0)^2 + (y - k)^2 = k^2$



$x^2 + y^2 + k^2 - 2ky = k^2$
$x^2 + y^2 - 2ky = 0$ .....(i)
Differentiating eq.(i) w.r.t.x,
$2x + 2y\frac{dy}{dx} - 2k\frac{dy}{dx} = 0$.....(ii)
From eq. (i) , $k = \frac{x^2 + y^2}{2y}$. Put it in eq.(ii),
$2x + \left[2y - 2\left(\frac{x^2 + y^2}{2y}\right)\right]\frac{dy}{dx} = 0$
$\Rightarrow [2y^2 - x^2 - y^2]\frac{dy}{dx} + 2xy = 0$
$\Rightarrow (y^2 - x^2)\frac{dy}{dx} + 2xy = 0$
$\Rightarrow (x^2 - y^2)\frac{dy}{dx} - 2xy = 0$

Question 127

If $3 \leq 3t - 18 \leq 18$, then which one of the following is correct?

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Options:

A. $15 \leq 2t + 1 \leq 20$

B. $8 \leq t \leq 12$

C. $8 \leq t+1<13$

D. $21 \leq 3t \leq 24$

Answer: C

Solution:

Solution:
$3<3t-18<18$
Add 18
$3+18 \leq 3t \leq 18+18$
$21 \leq 3t \leq 36$
Divide by 3
$7 \leq t \leq 12$ .....(i)
Add 1 in eq. (i).
$7+1 \leq t+1 \leq 12+1$
$8 \leq t+1 \leq 13$ .....(ii)
Multiply eq.(i) by (2)
$14 \leq 2t \leq 24$
Add 1, $15 \leq 2t+1 \leq 25$ .....(ii)
Multiply eq.(i) by 3
$21 \leq 3t \leq 36$ .....(iv)

Question 128

If $C_r = {}^nC_r$ and $(C_0+C_1)(C_1+C_2)\ldots(C_{n+1}+C_n) = K\frac{(n+1)^n}{n!}$, then the value of K is

©

Options:

A. $C_0C_1C_2\ldots C_n$

B. $C_1^2C_2^2\ldots C_n^2$

C. $C_1+C_2+\ldots+C_n$

D. $C_0C_1+C_1C_2+C_2C_3+\ldots+C_nC_{n+1}$

Answer: A

Solution:

Solution:
$C_r = {}^nC_r$
$(C_0+C_1).(C_1+C_2).(C_2+C_3).\ldots\ldots\ldots\ldots.(C_{n-1}+C_n)$
$= K\frac{(n+1)^2}{n!}$
$\because C_r+C_{r+1} = C_r\left(1+\frac{C_{r+1}}{C_r}\right) = C_r(1+\frac{n-r}{r+1})$
$\left[\because {}^nC_{r+1} = \frac{n-r}{r+1}\right]$
$= C_r(\frac{n+1}{r+1})$
$\sum_{r=0}^{n} C_r + C_{r+1} = \sum_{r=0}^{n} C_r(\frac{n+1}{r+1})$
$= (\frac{n+1}{1})C_0.(\frac{n+1}{2})C_1.\ldots\ldots\ldots.\frac{(n+1)}{n}C_{n-1}.\frac{n+1}{n+1}C_n$

$= (\frac{n+1}{1}) C_0 . (\frac{n+1}{2}) C_1 . \ldots\ldots\ldots \frac{(n+1)}{n} C_{n-1} . C_n$
$(n+1)^n . \frac{C_0 . C_1 . C_2 . \ldots\ldots C_n}{1.2.3 . \ldots\ldots n}$
$= \frac{(n+1)^n}{n!} . C_0 . C_1 . C_2 . \ldots\ldots C_n$
$\therefore K = C_0 . C_1 . C_2 . \ldots . C_n$

Question 129

If $|a| = 2, |b| = 3, a \cdot b = 0$ and $c = a \times \{a \times (a \times (a \times b))\}$, then $|c| =$?

Options:

A. 32

B. 48

C. 96

D. 24

Answer: B

Solution:

Solution:
$|a| = 2, |b| = 3, a . b = 0$
$c = a \times \{a \times (a \times (a \times b))\}$
$\because a \times (b \times c = (a . c) b - (a . b) . c$
$\therefore c = a \times \{a \times [(a . b) a - (a . b) b]\}$
$= a \times \{a \times (-|A|^{2 \cdot} b)\} [\because a . b = 0]$
$= a \times (a \times (-|a^2| . b))$
$= a \times (a \times (-4b))$
$= 4\{a \times (a \times (-b))\}$
$= 4\{a \times b \times a\} \{\because a \times b = -b \times a\}$
$= 4[(a . a) b - (a . b) a]$
$= 4(|a|^2 b - 0] = 4(4b) = 16b$
$\therefore |c| = 16|b| = 16 \times 3 = 48$

Question 130

Projection of $2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\hat{i} + 2\hat{j} + \hat{k}$ will be

Options:

A. $\frac{5}{3}\sqrt{6}$

B. $\sqrt{6}$

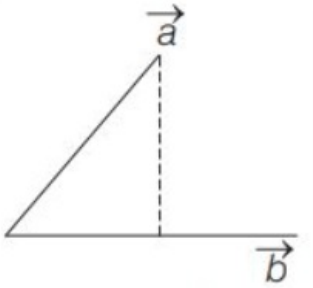
C. $\frac{\sqrt{3}}{2}$

D. cannot be projected

Answer: A

Solution:

Solution:
Let $a = 2\hat{i} + 3\hat{j} + 2\hat{k}$
$b = \hat{i} + 2\hat{j} + \hat{k}$
Projection of a on $b = |a| cos\theta$



$= |a| \cdot \frac{|a.b|}{|a||b|} = \frac{a.b}{|b|}$
$\because a.b = (2\hat{i} + 3\hat{j} + 2\hat{k}).(\hat{i} + 2\hat{j} + \hat{k})$
$= 2 + 6 + 2 = 10$
$|b| = \sqrt{1^2 + 2^2 + (1)^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$
$\therefore$ Projection $= \frac{10}{\sqrt{6}} \times \frac{10\sqrt{6}}{6} = \frac{5\sqrt{6}}{3}$

Question 131

The value of $\int x \log x \, dx$ is

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Options:

A. $\frac{x^2}{2} \log x - \frac{x^2}{2} + c$

B. $\frac{x^2}{2} \log x - \frac{x^2}{4} + c$

C. $\frac{x^2}{2} \log x + \frac{x^2}{2} + c$

D. None of these

Answer: B

Solution:

Solution:
$I = \int x \log x$
Using integration using by parts
$I = logx \int x dx - \int [\frac{d}{dx}(logx) . \int x dx] dx$
$= logx(\frac{x^2}{2}) - \int (\frac{1}{x} . \frac{x^2}{2}) dx + C$
$\frac{x^2}{2} logx - \int \frac{x}{2} dx + C = \frac{x^2}{2} logx - \frac{x^2}{4} + C$

Question 132

The value of $\int e^x[\tan x - \log(\cos x)]\,dx$

©

Options:

A. $e^x \log(sec x) + c$

B. $e^x \log(\text{cosec } x) + c$

C. $e^x \log(\cos x) + c$

D. $e^x \log(sin\ x) + c$

Answer: A

Solution:

Solution:
$I. \int e^x[\tan x - \log(\cos x)]\,dx$
$\because$ We know that,
$\int e^x(f(x) + f'(x)\,dx = e^x f(x) + c)$
$\because\ I = \int e^x(\tan x + \log \sec x)\,dx$
$\{\ \because \cos x = -\log \sec x \text{ and } \sec x = \frac{1}{\cos x}\}$
Let $f(x) = \log \sec x$
$f'(x) = \frac{1}{\sec} x\,.\sec x \tan x = \tan x$
$\therefore\ I = e^x \log \sec x + c$

Question 133

The equation of conic with focus at $(1, -1)$, directrix along $x - y + 1 = 0$ and eccentricity $\sqrt{2}$ is

Options:

A. $x^2 - y^2 = 1$

B. $xy = 1$

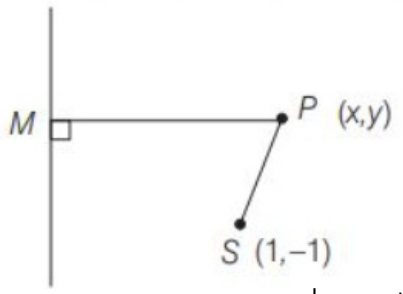
C. $2xy - 4x + 4y + 1 = 0$

D. $2xy + 4x - 4y - 1 = 0$

Answer: C

Solution:

Solution:
Focus $(1, -1)$; Eccentricity $= \sqrt{2}$
Directrix, $x - y + 1 = 0$
$\because$ Equation of conic
$SP = ePM$



$\sqrt{(x-1)^2+(y+1)^2} = \sqrt{2}.\left|\frac{x-y+1}{\sqrt{1^2+1^2}}\right|$

$\Rightarrow \sqrt{(x-1)^2+(y+1)^2} = \sqrt{2}\left|\frac{x-y+1}{\sqrt{2}}\right|$

Squaring both sides

$x^2-2x+1+y^2+2y+1$

$= x^2+y^2+1-2xy-2y+2x$

$\Rightarrow 2xy-4x+4y+1=0$

Question 134

The mirror image of parabola $y^2 = 4x$ relative to tangent to the parabola at the point $(1, 2)$ is

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Options:

A. $(x-1)^2 = 4(y+1)$

B. $(x+1)^2 = 4(y+1)$

C. $(x+1)^2 = 4(y-1)$

D. $(x-1)^2 = 4(y-1)$

Answer: C

Solution:

Solution:

Given,

Parabola $y^2 = 4x, 4a = 4 \Rightarrow a = 1$

Let point on the curve $P(at^2, 2at), i.e. P(t^2, 2t)$

Equation of tangent a point $P(1, 2)$ is

$(y-2) = m(x-1)$

$m = \left.\frac{dy}{dx}\right|_{(1,2)}$

$\Rightarrow y^2 = 4x \Rightarrow 2y\frac{dy}{dx} = 4$

$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$

$\Rightarrow \left.\frac{dy}{dx}\right|_{(1,2)} = \frac{2}{2} = 1$

$\therefore$ Equation of tangent is

$(y-2) = 1(x-1)$

$x-y+1=0$

Let the mirror image of point $P(t^2, 2t)\, be (h, k)$

$\frac{h-t^2}{1} = \frac{k-2t}{-1} = \frac{-2(t^2-2t+1)}{1^2+1^2}$

$\Rightarrow \frac{h-t^2}{1} = \frac{k-2t}{-1} = \frac{-2(t^2-2t+1)}{2}$

$\Rightarrow \underset{I}{\frac{h-t^2}{1}} = \underset{II}{\frac{k-2t}{-1}} = \underset{III}{\frac{-t^2+2t-1}{1}}$

Comparing I and III

$h = 2t - 1 \Rightarrow t = \left.\frac{h+1}{2}\right|$
Comparing II and III
$k = t^2 + 1 \Rightarrow k - 1 = t^2$
$\therefore$ Eliminating t
$k - 1 = (\frac{h+1}{2})^2$
$\Rightarrow (h+1)^2 = 4(k-1)$
Put $h = x, k = y$, we get $(x+1)^2 = 4(y-1)$

Question 135

The probabilities of solving a problem by A, B and C are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. It they work independently, then the probability that the problem will be solved is

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Options:

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{3}{4}$

D. $\frac{4}{5}$

Answer: C

Solution:

Solution:
P(A)=Probability of solving a problem by $A + \frac{1}{2}$
P(B) = Probability of solving a problem by $B = \frac{1}{3}$
P(C) = Probability of solving a problem by $C + \frac{1}{4}$
$\because$ A, B and C are independent events.
$\Rightarrow P(A \cap B \cap C) = P(A) \,.\, P(B) \,.\, P(C)$
$= \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{24}$
$\Rightarrow$ Probability that problem is solved
$= P(A \cup B \cup C)$
$1 - P(A \cup \bar{B} \cup C)$
$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$
$= 1 - P(\bar{A}) \,.\, P(\bar{B}) \,.\, P(\bar{C})$
$= 1 - (1 - P(A)(1 - P(B)(1 - P(C))$
$= 1 - (1 - \frac{1}{2})(1 - \frac{1}{3})(1 - \frac{1}{4})$
$= 1 - (\frac{1}{2})(\frac{2}{3})(\frac{3}{4})$
$= 1 - \frac{1}{4} = \frac{3}{4}$

Question 136

The probability that the number formed by taking all the digits $1, 2, 3, 4, 5$ is divisible by 4 is

Options:

A. $\frac{1}{5}$

B. $\frac{1}{4}$

C. $\frac{1}{3}$

D. None of these

Answer: A

Solution:

Solution:
Digits $1, 2, 3, 4, 5$
The total numbers formed by digits $1, 2, 3, 4, 5$ is
$= 5! = 120$
Now, event A = Number formed by all digits $1, 2, 3, 4, 5$ is divisible by 4

Last two digits can be $12, 24, 32, 52$
Number of favourable case $= 3! \times 4$
$\therefore P(A) = \frac{4 \times 3!}{5!} = \frac{4}{5 \times 4} = \frac{1}{5}$

Question 137

In an equilateral triangle, the ratio of the incircle, circumcircle and ex-circle are in the ratio

Options:

A. 1:2:3

B. 2:3:4

C. 1:3:2

D. 1:1:1

Answer: A

Solution:

Solution:
Let a be the side of equilateral triangles.

Area of an eqilateral triangle, $\Delta = \frac{\sqrt{3}}{4}a^2$

and $s = \frac{a+a+a}{2} = \frac{3a}{2}$

In radius $r = \frac{\Delta}{s} = \frac{\frac{\sqrt{3}a^2}{4}}{\frac{3a}{2}} = \frac{a}{2\sqrt{3}}$

Circumradius $R = \frac{abc}{4\Delta} = \frac{a^3}{4\frac{\sqrt{3}a^2}{4}} = \frac{a}{\sqrt{3}}$

Ex - radius $r_1 = \frac{\Delta}{s-a} = \frac{\frac{\sqrt{3}}{4}a^2}{\frac{3a}{2}-a} = \frac{\sqrt{3}}{2}a$

Hence, $r : R : r_1$

$\Rightarrow \frac{a}{2\sqrt{3}} : \frac{a}{\sqrt{3}} : \frac{\sqrt{3}}{2}a = 1:2:3$

Question 138

The period of the function $f(x) = \tan(5x+3)$ in radians is

©

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{10}$

C. $\frac{\pi}{5}$

D. $\frac{\pi}{6}$

Answer: C

Solution:

Solution:

$f(x) = tan(5x+3)$

$\because$ Period of tan x, is π.

$\therefore f(x) = \tan c$, period is π

$f(5x) = tan(5x)$; period is $\frac{\pi}{5}$

$f(5x+3) = tan(5x+3)$, period does not change,

i.e. $\frac{\pi}{5}$

$\therefore$ period of $f(x)$ is $\frac{\pi}{5}$

Question 139

If $f(a) < 0$ and $f(b) > 0$, then one root of the equation $f(x) = 0$ is(a) either a or b(b) less than a an greater, than b(c) lies between a and b (d) None of the above

©

Options:

A. either a or b

B. less than a

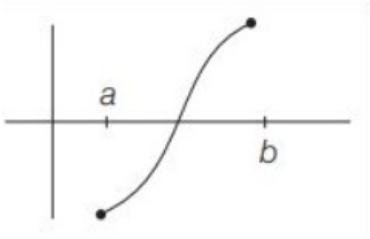
C. lies between a and b

D. None of the above

Answer: C

Solution:

Solution:
$f(a) < 0$ and $f(b) > 0$
$f(x) = 0$
$\because f(a) \cdot f(b) < 0$



$\therefore$ One root of equation
$f(x) = 0$
Lies between a and b.

Question 140

Which of the following is primary storage of computer?

©

Options:

A. RAM

B. ROM

C. Hard Disc

D. None of these

Answer: A

Solution:

Solution:
$\because$ Primary storage of computer is RAM.

Question 141

The equation of straight line passing through the points $(4, -5, -2)$ and $(-1, 5, 3)$ is

Options:

A. $x-4=\frac{y+5}{-2}=-z-1$

B. $x+1=\frac{y-5}{2}=3-z$

C. $\frac{x}{-1}=\frac{y}{5}=\frac{z}{-3}$

D. $\frac{x}{4}=\frac{y}{-5}=\frac{z}{-2}$

Answer: A

Solution:

Solution:
∵ Equation of line passing through two parts
$(x_1, y_1, z_1) and (x_2, y_2, z_2)$ is
$\frac{x-x_1}{x_2-x_1}=\frac{y_1-y_1}{y_2-y_2}=\frac{z-z_1}{z_2-z_1}$
or $\frac{x-x_2}{x_2-x_1}=\frac{y-y_2}{y_2-y_1}=\frac{z-z_2}{z_2-z_1}$
∴ Equation of line passing through $(4,-5,-2)$ and $(-1,5,3)$ is
$\frac{x-4}{-1-4}=\frac{y+5}{5+5}=\frac{z+2}{3+2}$
$\Rightarrow \frac{x-4}{-5}=\frac{y+5}{10}=\frac{z+2}{5}$
$\Rightarrow x-4=\frac{y+5}{-2}=\frac{z+1}{-1}$
or $\frac{x+1}{-1-4}=\frac{y-5}{5+5}=\frac{z_3}{3+2}$
$\Rightarrow \frac{x+1}{-5}=\frac{y-5}{10}=\frac{z-3}{-1}$
$\Rightarrow \frac{x+1}{1}=\frac{y-5}{-2}=\frac{z-3}{-1}$
Hence, the required equation is
$x-4=\frac{y+5}{-2}=-z-1$ or $x+1=\frac{y-5}{-2}=-z+3$

Question 142

If the direction ratio of two lines are given by $3/m-4/n+mn=0$ and $/+2m+3n=0$, then the angle between the lines is

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: A

Solution:

Solution:
$3lm - 4nl + mn = 0$ .....(i)
and $l + 2m + 3n = 0$ .....(ii)
$l = -2m - 3n$
Put $l = -2m - 3n$ in eq. (i),
$3(-2m - 3n)m - 4(-2m - 3n)n + mn = 0$
$\Rightarrow -6m^2 - 9mn + 8mn + 12n^2 + mn = 0$
$\Rightarrow -6m^2 + 12n^2 = 0$
$\Rightarrow 12n^2 = 6m^2 \Rightarrow 2n^2 = m^2$
$\Rightarrow m^2 - 2n^2 \Rightarrow m = \pm\sqrt{2}n$
$\because$ If $m = \sqrt{2}n \Rightarrow l = -2(2\sqrt{2} + 2)n$
$\Rightarrow m = -\sqrt{2}n \Rightarrow l = -(-2\sqrt{2} + 3)n \therefore$ Directions ratios are
$-(2\sqrt{2} + 3)n, \sqrt{2}n, n and -(-2\sqrt{2} + 3)n, -\sqrt{2}n, n)$
$\Rightarrow -(2\sqrt{2} + 3), \sqrt{2}, 1 and -(-2\sqrt{2} + 3), -\sqrt{2}, 1$
$\because$ We know that
$cos\theta = a_1a_2 + b_1b_2 + c_1c_2$
$= (3 + 2\sqrt{2})(3 - 2\sqrt{2}) + (\sqrt{2})(-\sqrt{2}) + (1)(1)$
$= 9 - 8 - 2 + 1 = 0$
$\therefore cos\theta = 0, \theta = \frac{\pi}{2}$

Question 143

The value of $\lim_{x \to 1}(1 - x)\tan(\frac{\pi x}{2})$ is equal to

©

Options:

A. $\frac{\pi}{2}$

B. π

C. $\frac{2}{\pi}$

D. 0

Answer: C

Solution:

Solution:
$\lim_{x \to 1}(1 - x)tan\pi x|/2$
$\lim_{x \to 1}\frac{1 - x}{cot\frac{\pi x}{2}}$
$\because \frac{1 - 1}{cot\frac{\pi}{2}} = \frac{0}{0}$ [Indeterminate form]
Using L'Hospital rule,
$\lim_{x \to 1}\frac{-1}{-cosec^2\frac{\pi x}{2} \cdot \frac{\pi}{2}} = \frac{-1}{-cosec^2\frac{\pi}{2} \cdot \frac{\pi}{2}} = \frac{2}{\pi}$

Question 144

If $f(x) = \begin{cases} \frac{x}{\frac{1}{x}} & x \neq 0 \\ e^x + 1 & \\ 0 & x = 0 \end{cases}$ **, then** **.**

©

Options:

A. $\lim_{x \to 0^+} f(x) = 1$

B. $\lim_{x \to 0^-} f(x) = 1$

C. $f(x)$ is continuous at $x = 0$

D. None of the above

Answer: C

Solution:

Solution:

$f(x) = \begin{cases} \frac{x}{\frac{1}{x}} & x \neq 0 \\ e^x + 1 & \\ 0 & x = 0 \end{cases}$.

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{x}{e^{\frac{1}{x}} + 1}$

$\lim_{h \to 0} \frac{h}{e^{\frac{1}{h}} + 1} = 0$

$\left[\because h \to 0, \frac{1}{h} \to \infty \Rightarrow e^{\frac{1}{h}} \to \infty, \frac{1}{e^{\frac{1}{h}} + 1} \to 0 \right]$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{x}{e^{\frac{1}{x}} + 1}$

$\lim_{n \to 0} = \frac{-h}{e^{\frac{-1}{h}} + 1} = \frac{0}{0} + 1 = 0 \left[\because h \to 0, e^{\frac{-1}{h}} \to e^{-\infty} \to 0 \right]$

$\because \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = f(0) = 0$

$f(x)$ is continuous at $x = 0$.

Question 145

Which is not an input device of a computer?

©

Options:

A. scanner

B. joystick

C. keyboard

D. None of these

Answer: D

Solution:

Solution:
∵ Input device of computer is scanner, joystick and keyboard.

Question 146

An operating system is a/an

Options:

A. System Software

B. Utility Software

C. Application Software

D. None of these

Answer: C

Solution:

Solution:
∵ An operating system is an Application Software.

Question 147

The unit vector parallel to the resultant vector of $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$ is

Options:

A. $\frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$

B. $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

C. $\frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}$

D. $\frac{-\hat{i} - \hat{j} + 8\hat{k}}{\sqrt{69}}$

Answer: A

Solution:

Solution:
Resultant vector of $a = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and
$b = \hat{i} + \hat{j} + 3\hat{k}$ is $a + b$
$a + b = 2\hat{i} + 4\hat{j} - 5\hat{k} + \hat{i} + 2\hat{j} + 3\hat{k} = 3\hat{i} + 6\hat{j} - 2\hat{k}$
$\therefore$ Unit vector of $a + b = \frac{a + b}{|a + b|}$
$= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{3^2 + 6^2 + (-2)^2}}$
$= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{49}} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$

Question 148

If a and b are two unit vectors such that a + 2b and 5a − 4b are perpendicular to each other, then angle between a and b is

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Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\cos^{-1}(\frac{1}{3})$

D. $\cos^{-1}(\frac{2}{7})$

Answer: B

Solution:

Solution:
$\because$ a and b are unit vectors
$|a| = |b| = 1$
$\because (a + 2b)$ and $5a - 4b$ are perpendicular.
$\therefore (a + 2b) . (5a - 4b) = 0$
$\Rightarrow 5(a . b) - 4a . b + 10a . b - 8(b . b) = 0$
$\Rightarrow 5|a|^2 + 6a . b - 8|b|^2 = 0$
$\Rightarrow 5 + 6a . b - 8 = 0$
$\Rightarrow 6a . b = 3$
$\Rightarrow a . b = \frac{3}{6} = \frac{1}{2}$
$\because a . b = |a||b| cos\theta$
$\theta =$ angle between a and b

$\Rightarrow |a||b\left|cos\theta = \frac{1}{2}\right.$
$1 \times 1 \times cos\theta = \frac{1}{2}$
$cos\theta = \frac{1}{2}$
$\theta = \frac{\pi}{3}$

Question 149

If A is a square matrix of order $n \times n$ and K is a scalar, then adj (KA) is equal to

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Options:

A. K adj A

B. K^n adj A

C. K^{n-1} adj A

D. K^{n+1} adj A

Answer: C

Solution:

Solution:
$\because A\,\text{adj}(A) = |A|I_n$.....(i)
$\Rightarrow KA\,\text{adj}(KA) = |KA|I_n$
$A.\,\text{adj}(KA) = \frac{K^n|A|}{K}I_n$
$A.\,\text{adj}(KA) = K^{n-1}|A|I_n$
$A\,\text{adj}(KA) = K^{n-1}.A\,\text{adj}(A)$
$\Rightarrow$ Pre multiply by A^{-1}
$A^{-1}\,\text{adj}(KA) = K^{n-1}(A^{-1}A)\,\text{adj}\,A$
$\text{adj}(KA) = K^{n-1}\,\text{adj}\,A[\because AA^{-1} = A^{-1}A = I, IA = A]$

Question 150

The number of all three digited even numbers such that, if 5 is one of the digits, then next digit is 7 is

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Options:

A. 360

B. 365

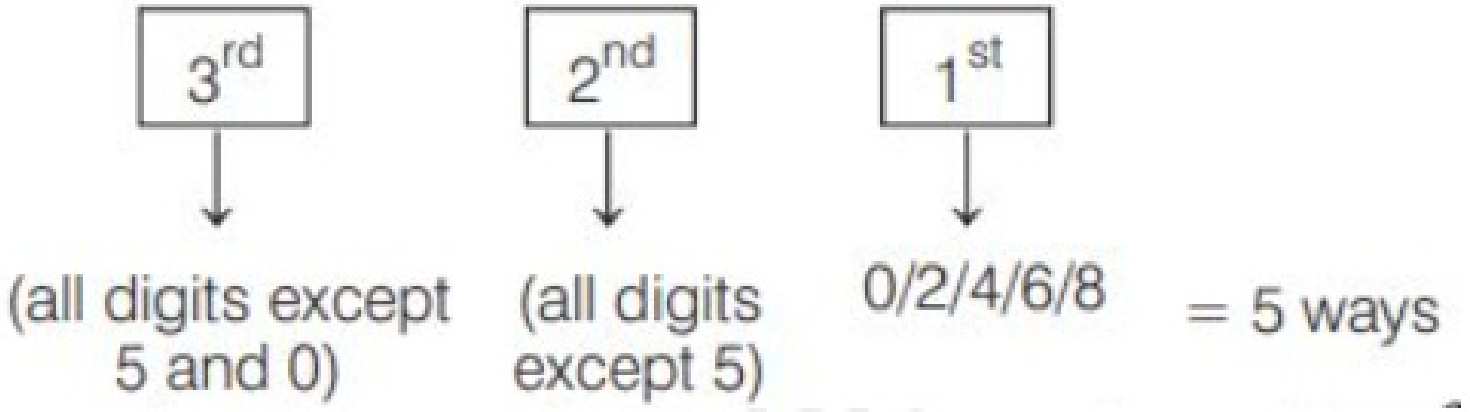
C. 370

D. 375

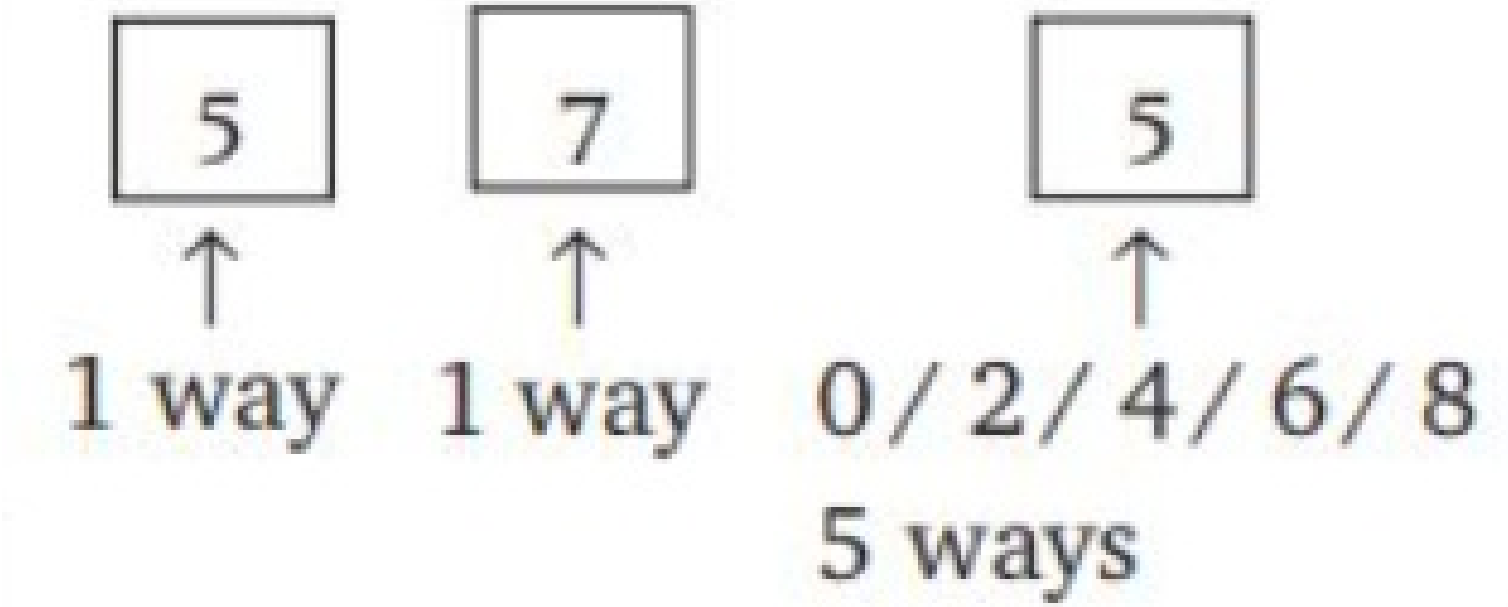
Answer: B

Solution:

Solution:
3-digit even numbers
Case I When 5 is not there.
Then, 3 digit even numbers



and second and third places can be fill by 9 ways and 8 ways respectively.
Total ways $= 8 \times 9 \times 5 = 360$
Case 2 When 5 is there
Then, 3-digit even number



Total ways $= 1 \times 1 \times 5 = 5$
Required number of ways $= 360 + 5 = 365$
