

ICSE Paper 2010

MATHEMATICS

SECTION A [40 Marks]

(Answer all questions from this Section.)

Question 1.

- (a) Solve the following inequation and represent the solution set on the number line.

$$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in R \quad [3]$$

- (b) Tarun bought an article for ₹ 8,000 and spent ₹ 1,000 for transportation. He marked the article at ₹ 11,700 and sold it to a customer. If the customer had to pay 10% sales tax, find

- (i) The customer's price.
(ii) Tarun's profit percent.

[3]

- (c) Mr. Gupta opened a recurring deposit account in a bank. He deposited ₹ 2,500 per month for two years. At the time of maturity he got ₹ 67,500. Find :

- (i) the total interest earned by Mr. Gupta.
(ii) the rate of interest per annum.

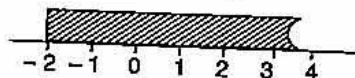
[4]

Solution :

- (a) Given : $-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in R$

$$\begin{aligned} -3 < -\frac{1}{2} - \frac{2x}{3} & \quad \text{and} \quad -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6} \\ -3 + \frac{1}{2} < -\frac{2x}{3} & \quad \text{and} \quad -\frac{2x}{3} \leq \frac{5}{6} + \frac{1}{2} \\ -\frac{5}{2} < -\frac{2x}{3} & \quad \text{and} \quad -\frac{2x}{3} \leq \frac{4}{3} \\ \frac{5}{2} > \frac{2x}{3} & \quad \text{and} \quad -x \leq 2 \\ x < \frac{15}{4} & \quad \text{and} \quad x \geq -2 \end{aligned}$$

$$\text{Solution set} = \left\{ x : \frac{15}{4} > x \geq -2 \right\}$$



- (b) Given : C.P. = ₹ 8,000 + ₹ 1,000 = ₹ 9,000, M.P. = ₹ 11,700, S.T. = 10%.

(i)

$$\text{Amount to be paid} = \text{M.P.} + \text{S.T. \% of M.P.}$$

$$\begin{aligned} &= 11,700 + \frac{10}{100} \times 11,700 \\ &= ₹ 12,870 \end{aligned}$$

Ans.

(ii)

$$\begin{aligned}\text{Profit} &= \text{M.P.} - \text{C.P.} = 11,700 - 9,000 \\ &= ₹ 2,700.\end{aligned}$$

$$\begin{aligned}\text{Profit percent} &= \frac{\text{Profit}}{\text{C.P.}} \times 100 \\ &= \frac{2,700}{9,000} \times 100 \\ &= 30\%.\end{aligned}$$

Ans.

(c) Total amount deposited = ₹ $(2,500 \times 24) = ₹ 60,000$

Equivalent principal for one month = ₹ $2,500 \times \frac{24(24+1)}{2} = ₹ (62,500 \times 12)$

(i) Total interest = $67,500 - 60,000$
= ₹ 7,500

(ii) Interest on ₹ $(62,500 \times 12)$ for 1 month

$$= ₹ \left(62,500 \times 12 \times \frac{R}{100} \times \frac{1}{12} \right) \quad \left[\because I = \frac{P \times R \times T}{100} \right]$$

$$7,500 = 625 R.$$

 \Rightarrow

$$R = 12\%.$$

Ans.

Question 2.

(a) Given $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ and $D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$.

Find $AB + 2C - 4D$.

[3]

(b) Nikita invests ₹ 6,000 for two years at a certain rate of interest compounded annually. At the end of the first year it amounts to ₹ 6,720. Calculate :

(i) the rate of interest.

(ii) the amount at the end of the second year.

[3]

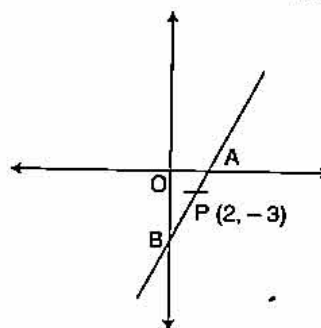
(c) A and B are two points on the x-axis and y-axis respectively. P (2, -3) is the mid point of AB. Find the

(i) Coordinates of A and B.

(ii) Slope of line AB.

(iii) Equation of line AB.

[4]


Solution :

(a) Given : $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$, $D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$$\begin{aligned}AB + 2C - 4D &= \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} - 4 \begin{bmatrix} 2 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 16 \\ -2 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0\end{aligned}$$

Ans.

- (b) (i) Given : Principal = ₹ 6,000, Time = 2 year, After one year amount = ₹ 6,720.

For 1st year : $P + I = ₹ 6,720$

$$6,000 + \frac{P \times R \times 1}{100} = 6,720$$

$$\frac{6,000 \times R}{100} = 720$$

\Rightarrow

$$R = 12\%$$

Ans.

$$(ii) \therefore A = P \left(1 + \frac{r}{100} \right)^n$$

$$\begin{aligned} \text{Amount at the end of 2nd year} &= 6,000 \left(1 + \frac{12}{100} \right)^2 \\ &= 6,000 \left(1 + \frac{3}{25} \right)^2 \\ &= 6,000 \left(\frac{28}{25} \times \frac{28}{25} \right) = \frac{37,632}{5} \\ &= ₹ 7,526.40. \end{aligned}$$

Ans.

- (c) Given : A ($x_1, 0$), B ($0, y_1$)

- (i) Mid point coordinates

$$\frac{x_1 + 0}{2} = 2 \Rightarrow x_1 = 4$$

$$\frac{0 + y_1}{2} = -3 \Rightarrow y_1 = -6$$

Coordinates of A (4, 0) and B (0, -6) Ans.

- (ii) Slope of line AB

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-6 - 0}{0 - 4} = \frac{-6}{-4} = \frac{3}{2} \end{aligned}$$

Ans.

- (iii) Equation of line

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{3}{2}(x - 4)$$

\Rightarrow

$$2y = 3x - 12$$

\Rightarrow

$$3x - 2y - 12 = 0$$

Ans.

Question 3.

- (a) Cards marked with numbers 1, 2, 3, 4 ... 20 are well shuffled and a card is drawn at random. What is the probability that the number of the cards is

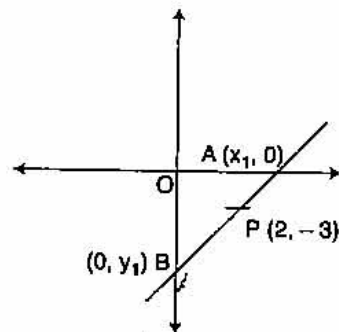
- a prime number
- divisible by 3
- a perfect square?

[3]

- (b) Without using trigonometric tables evaluate :

$$\frac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\operatorname{cosec}^2 10^\circ - \tan^2 80^\circ}$$

[3]



(c) (Use graph paper for this question)

$A(0, 3)$, $B(3, -2)$ and $O(0, 0)$ are the vertices of triangle ABO .

(i) Plot the triangle on a graph sheet taking $2 \text{ cm} = 1 \text{ unit}$ on both the axes.

(ii) Plot D the reflection of B in the Y axis, and write its co-ordinates.

(iii) Give the geometrical name of the figure $ABOD$.

(iv) Write the equation of the line of symmetry of the figure $ABOD$.

[4]

Solution :

(a) Given : Cards marked with numbers 1, 2, 20.

$$n(S) = 20$$

(i) Prime Numbers = 2, 3, 5, 7, 11, 13, 17, 19

$$n(E) = 8$$

$$P(\text{Prime number}) = P(A) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

Ans.

(ii) No. divided by 3 = 3, 6, 9, 12, 15, 18

$$n(E) = 6$$

$$P(\text{no. divided by 3}) = P(A) = \frac{n(E)}{n(S)} = \frac{6}{20} = \frac{3}{10}$$

Ans.

(iii) No. perfect square = 1, 4, 9, 16

$$n(E) = 4$$

$$P(\text{Perfect square}) = P(A) = \frac{n(E)}{n(S)} = \frac{4}{20} = \frac{1}{5}$$

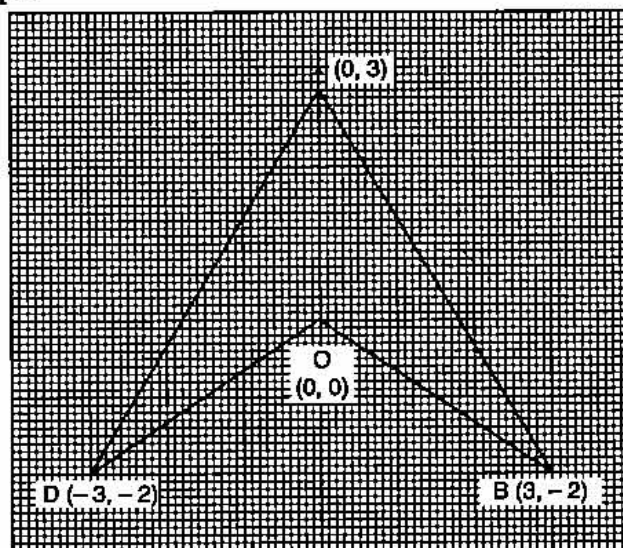
Ans.

(b) Given :

$$\begin{aligned} & \frac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\operatorname{cosec}^2 10^\circ - \tan^2 80^\circ} \\ &= \frac{\sin (90 - 55)^\circ \cos 55^\circ + \cos (90 - 55)^\circ \sin 55^\circ}{\operatorname{cosec}^2 10^\circ - \tan^2 (90 - 10)^\circ} \\ &= \frac{\cos 55^\circ \cos 55^\circ + \sin 55^\circ \sin 55^\circ}{(1 + \cot^2 10^\circ) - \cot^2 10^\circ} \\ &= \frac{\cos^2 55^\circ + \sin^2 55^\circ}{1 + \cot^2 10^\circ - \cot^2 10^\circ} = \frac{1}{1} = 1 \end{aligned}$$

Ans.

(c) (i) See graph.



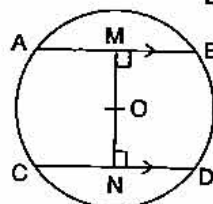
- (ii) Coordinate of D = (-3, -2)
 (iii) Geometrical name of ABOD is arrow.
 (iv) Equation of the line of symmetry is

$$x = 0$$

Question 4.

- (a) When divided by $x - 3$ the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of 'p'. [3]

- (b) In the figure given alongside AB and CD are two parallel chords and O is the centre. If the radius of the circle is 15 cm, find the distance MN between the two chords of length 24 cm and 18 cm respectively. [3]



- (c) The distribution given below shows the marks obtained by 25 students in an aptitude test. Find the mean, median and mode of the distribution. [4]

Marks obtained	5	6	7	8	9	10
No. of students	3	9	6	4	2	1

Solution :

- (a) Given :

$$f(x) = x^3 - px^2 + x + 6$$

$$g(x) = 2x^3 - x^2 - (p + 3)x - 6$$

when $f(x)$ is divided by $(x - 3)$ remainder $f(3)$ and $f(x)$ is divided by $(x - 3)$ remainder $g(3)$.

$$f(3) = g(3)$$

$$(3)^3 - (3)^2 p + 3 + 6 = 2(3)^3 - (3)^2 - (p + 3)3 - 6$$

$$27 - 9p + 3 + 6 = 54 - 9 - (p + 3)3 - 6$$

$$\Rightarrow 36 - 9p = 30 - 3p$$

$$\Rightarrow 9p - 3p = 36 - 30$$

$$\Rightarrow 6p = 6$$

$$\Rightarrow p = 1$$

Ans.

- (b) Given : OA = OC = 15 cm, AB = 24 cm, CD = 18 cm.

Now

In $\triangle OAM$,

$$AM = 12, CN = 9$$

$$OA^2 = OM^2 + AM^2$$

$$OM^2 = OA^2 - AM^2$$

$$= 15^2 - 12^2$$

$$= 225 - 144$$

$$= 81$$

$$OM = 9$$

$$OC^2 = ON^2 + CN^2$$

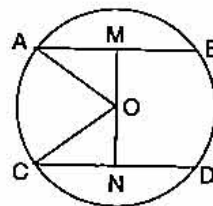
$$ON^2 = OC^2 - CN^2 = 15^2 - 9^2$$

$$= 225 - 81 = 144$$

$$ON = 12$$

Similarly, in $\triangle OCN$,

$$MN = OM + ON = 9 + 12 = 21 \text{ cm.}$$

Ans.

(c)

x_i	f_i	$x_i f_i$	cf
5	3	15	3
6	9	54	12
7	6	42	18
8	4	32	22
9	2	18	24
10	1	10	25
	$\Sigma f = 25$	$\Sigma x_i f_i = 171$	

$$\text{Mean} = \frac{\Sigma x_i f_i}{N} = \frac{171}{25} = 6.84 \quad \text{Ans.}$$

$\therefore n = 25$ (odd)

$$\text{Median} = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ term} = 13^{\text{th}} \text{ term} = 7 \quad \text{Ans.}$$

$$\text{Mode} = 6 \text{ (maximum freq.)} \quad \text{Ans.}$$

SECTION B [40 Marks]

Answer any **four** Questions in this Section.

Question 5.

- (a) Without solving the following quadratic equation, find the value of 'p' for which the roots are equal.

$$px^2 - 4x + 3 = 0 \quad [3]$$

- (b) Rohit borrows ₹ 86,000 from Arun for two years at 5% per annum simple interest. He immediately lends out this money to Akshay at 5% compound interest compounded annually for the same period. Calculate Rohit's profit in the transaction at the end of the two years. [3]

- (c) Mrs. Kapoor opened a Saving Bank Account in State Bank of India on 9th January 2008. Her pass book entries for the year 2008 are given below :

Date	Particulars	Withdrawals (in ₹)	Deposits (in ₹)	Balance (in ₹)
Jan 9, 2008	By Cash	—	10,000	10,000
Feb 12, 2008	By Cash	—	15,500	25,500
April 6, 2008	To Cheque	3,500	—	22,000
April 30, 2008	To Self	2,000	—	20,000
July 16, 2008	By Cheque	—	6,500	26,500
Aug. 4, 2008	To Self	5,500	—	21,000
Aug. 20, 2008	To Cheque	1,200	—	19,800
Dec. 12, 2008	By Cash	—	1,700	21,500

Mrs. Kapoor closed the account on 31st December, 2008. If the bank pays interest at 4% per annum, find the interest Mrs. Kapoor receives on closing the account. Give your answer correct to the nearest rupee. [4]

Solution :

(a) Roots are equal \Rightarrow $b^2 - 4ac = 0$
 $b^2 = 4ac$

Given : $a = p, b = 4, c = 3$.

\Rightarrow

$$16 = 4p.3$$

\Rightarrow

$$p = \frac{16}{12} = \frac{4}{3}$$

Ans.

(b) Given : $P = 86,000, R = 5\%, T = 2$ years.

$$S.I. = \frac{P \times R \times T}{100} = \frac{86,000 \times 5 \times 2}{100} = ₹ 8,600$$

$$C.I. = P \left[\left(1 + \frac{R}{100} \right)^T - 1 \right]$$

$$= 86,000 \left[\left(1 + \frac{5}{100} \right)^2 - 1 \right] = 86,000 \left[\left(\frac{21}{20} \right)^2 - 1 \right] = 86,000 \times \frac{41}{400} = ₹ 8,815$$

$$\text{Profit} = C.I. - S.I. = 8,815 - 8,600 = ₹ 215$$

Ans.

(c) Minimum balance for the month

January	-	10,000
February	-	10,000
March	-	25,500
April	-	20,000
May	-	20,000
June	-	20,000
July	-	20,000
August	-	19,800
September	-	19,800
October	-	19,800
November	-	19,800

$$\text{Principal} = ₹ 2,04,700, R = 4\%$$

$$S.I. = \frac{P \times R \times T}{100} = \frac{2,04,700 \times 4 \times 1}{100 \times 12}$$

$$= ₹ 682.33 = ₹ 682.$$

Ans.

Question 6.

(a) A manufacturer marks an article for ₹ 5,000. He sells it to a wholesaler at a discount of 25% on the market price and the wholesaler sells it to a retailer at a discount of 15% on the market price. The retailers sells it to a consumer at the market price and at each stage the VAT is 8%. Calculate the amount of VAT received by the Government from :

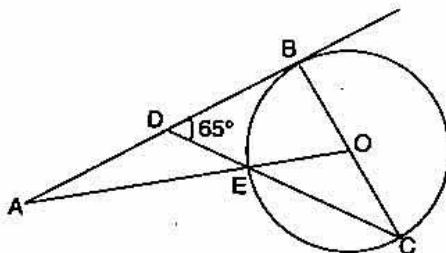
(i) the wholesaler

(ii) the retailer.

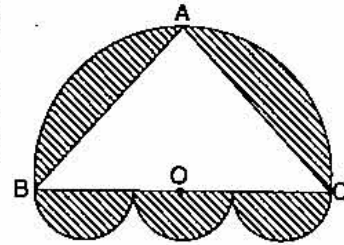
[3]

(b) In the following figure O is the centre of the circle and AB is a tangent to it at point B. $\angle BDC = 65^\circ$. Find $\angle BAO$.

[3]



- (c) A doorway is decorated as shown in the figure. There are four semi-circles. BC , the diameter of the larger semi circle is of length 84 cm. Centres of the three equal semi-circles lie on BC . ABC is an isosceles triangle with $AB = AC$. If $BO = OC$, find the area of the shaded region. (Take $\pi = \frac{22}{7}$) [4]



Solution :

(a) Given :

Cost of manufacturer = ₹ 5,000

S.P. of manufacturer = C.P. of wholesaler

$$= 5,000 - \frac{25}{100} \times 5,000$$

$$= 5,000 - 1,250$$

$$= ₹ 3,750$$

S.P. of wholesaler = C.P. of retailer

$$= 5,000 - \frac{15}{100} \times 5,000$$

$$= 5,000 - 750$$

$$= ₹ 4,250$$

S.P. of retailer = C.P. of consumer

$$= ₹ 5,000$$

$$(i) \quad \text{VAT by the wholesaler} = \frac{8}{100} \times 3,750$$

$$= ₹ 300$$

Ans.

$$(ii) \quad \text{VAT by retailer} = \frac{8}{100} \times (4,250 - 3,750)$$

$$= \frac{8}{100} \times 500$$

$$= ₹ 40.$$

Ans.

(b) AB is tangent $\Rightarrow \angle ABO = 90^\circ$

$$\Rightarrow \begin{aligned} \angle BDC &= 65^\circ \text{ (given)} \\ \angle BCD &= 90^\circ - 65^\circ = 25^\circ \\ \angle BOE &= 2 \times 25^\circ && \text{(angle at centre)} \\ &= 50^\circ \\ \angle BAO &= 90^\circ - \angle BOE \\ \angle BAO &= 90^\circ - 50^\circ \\ &= 40^\circ \end{aligned}$$

Ans.

(c) Let $AB = AC = x$ cm.

As angle in semi circle is 90°

$$\text{i.e.,} \quad \angle A = 90^\circ$$

In right angled $\triangle ABC$, by Pythagoras theorem, we get

$$AB^2 + AC^2 = BC^2$$

$$x^2 + x^2 = 84^2$$

$$2x^2 = 84 \times 84$$

$$x^2 = 84 \times 42$$

Now

$$\text{Area of } \triangle ABC = \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2} \times 84 \times 42$$

$$= 1764 \text{ cm}^2.$$

$$\text{Diameter of semicircle } (2r) = 84 \text{ cm}$$

$$\text{Radius } (r) = \frac{1}{2} \times 84 = 42 \text{ cm}$$

$$\therefore \text{Area of semicircle} = \frac{1}{2} \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 42 \times 42$$

$$= 2772 \text{ cm}^2.$$

$$\text{Diameter of each (three equal) semicircles} = \frac{1}{3} \times 84 = 28 \text{ cm.}$$

$$\text{Radius of the 3 equal semicircles} = \frac{1}{2} \times 28 = 14 \text{ cm.}$$

$$\therefore \text{Area of three equal semi circles} = 3 \times \frac{1}{2} \pi r^2$$

$$= 3 \times \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$$

$$= 924 \text{ cm}^2.$$

$$\begin{aligned} \text{Area of shaded region} &= \text{Area of semicircles} + \text{Area of three equal circles} \\ &\quad - \text{Area of } \triangle ABC \\ &= 2772 + 924 - 1764 \\ &= 3696 - 1764 \\ &= 1932 \text{ cm}^2. \end{aligned}$$

Ans.

Question 7.

- (a) Use ruler and compasses only for this question :
- Construct $\triangle ABC$, where $AB = 3.5 \text{ cm}$, $BC = 6 \text{ cm}$ and $\angle ABC = 60^\circ$.
 - Construct the locus of points inside the triangle which are equidistant from BA and BC .
 - Construct the locus of points inside the triangle which are equidistant from B and C .
 - Mark the point P which is equidistant from AB , BC and also equidistant from B and C . Measure and record the length of PB . [3]
- (b) The equation of a line is $3x + 4y - 7 = 0$. Find
- the slope of the line.
 - the equation of a line perpendicular to the given line and passing through the intersection of the lines $x - y + 2 = 0$ and $3x + y - 10 = 0$. [3]
- (c) The mean of the following distribution is 52 and the frequency of class interval 30-40 is ' f '. Find ' f '.

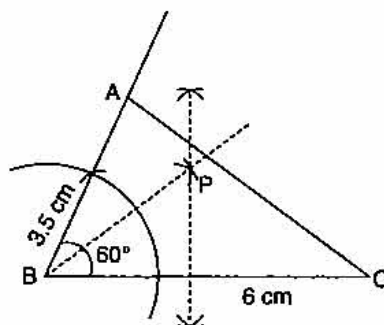
Class Interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	3	f	7	2	6	13

[4]

Solution :

(a) **Steps of Construction :**

- (i) Draw $BC = 6$ cm and make an angle at $B = 60^\circ$. Cut $BA = 3.5$ cm and meet A to C . This is the required ΔABC .
- (ii) Draw the bisector of ΔABC and perpendicular bisector of BC ; both intersecting at P .
- (iii) P is the required point. $PB = 3.5$ cm.



(b) **Given :** Equation of the line is

$$3x + 4y - 7 = 0$$

$$4y = -3x + 7$$

$$y = -\frac{3}{4}x + \frac{7}{4}$$

(i) Slope of the line (m_1) = $-\frac{3}{4}$ **Ans.**

(ii) Slope of the perpendicular line (m_2) = $\frac{-1}{m_1} = \frac{-1}{-3/4} = \frac{4}{3}$

Intersection of the lines $x - y + 2 = 0$...(i)

and $3x + y - 10 = 0$...(ii)

By Adding equation (i) and (ii) $4x = 8 \Rightarrow x = 2$

Put $x = 2$, in equation (i) we get

$$2 - y + 2 = 0 \Rightarrow y = 4$$

Equation of line $y - y_1 = m_2(x - x_1)$

$$y - 4 = \frac{4}{3}(x - 2)$$

$\Rightarrow 4x - 3y + 4 = 0$ **Ans.**

(c)

Interval	Frequency (f_i)	x_i	$d_i = x_i - A$	$f_i d_i$
10-20	5	15	-30	-150
20-30	3	25	-20	-60
30-40	f	35	-10	$-10f$
40-50	7	45 A	0	0
50-60	2	55	10	20
60-70	6	65	20	120
70-80	13	75	30	390
	$36 + f$			$\Sigma f_i d_i = 320 - 10f$

$$\text{Mean} = A + \frac{\sum f_i d_i}{N}$$

$$52 = 45 + \frac{320 - 10f}{36 + f}$$

 \Rightarrow

$$7 = \frac{320 - 10f}{36 + f}$$

 \Rightarrow

$$252 + 7f = 320 - 10f$$

 \Rightarrow

$$17f = 68$$

 \Rightarrow

$$f = 4$$

Ans.

Question 8.

(a) Use the Remainder Theorem to factorise the following expression :

$$2x^3 + x^2 - 13x + 6$$

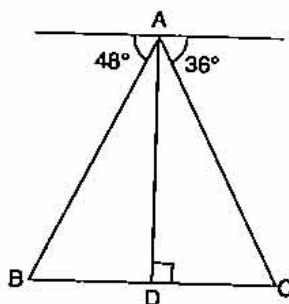
[3]

(b) If x, y, z are in continued proportion, prove that $\frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}$.

[3]

(c) From the top of a light house 100 m high the angles of depression of two ships on opposite sides of it are 48° and 36° respectively. Find the distance between the two ships to the nearest metre.

[4]

**Solution :**

(a) Given :

$$f(x) = 2x^3 + x^2 - 13x + 6$$

$$f(1) = 2 + 1 - 13 + 6 \neq 0$$

$$f(-1) = -2 + 1 + 13 + 6 \neq 0$$

$$f(2) = 16 + 4 - 26 + 6 = 0$$

So, $x - 2$ is one factor of $f(x)$ by remainder theorem.

$$\begin{array}{r} 2x^2 + 5x - 3 \\ x - 2 \overline{) 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 - 4x^2} \\ 5x^2 - 13x + 6 \\ \underline{5x^2 - 10x} \\ -3x + 6 \\ \underline{-3x + 6} \\ 0 \end{array}$$

\therefore The other factors of $f(x)$ are the factors of $2x^2 + 5x - 3$.

$$= 2x^2 + 6x - x - 3$$

$$= 2x(x + 3) - 1(x + 3)$$

$$= (2x - 1)(x + 3)$$

Hence, $2x^3 + x^2 - 13x + 6 = (2x - 1)(x + 3)(x - 2)$

Ans.

(b) If x, y, z are in continued proportion

$$\begin{aligned} \frac{x}{y} &= \frac{y}{z} = k \\ \Rightarrow y &= kz \\ \text{and } x &= xy = k^2z \\ \text{L.H.S.} &= \frac{(x+y)^2}{(y+z)^2} = \frac{(k^2z + kz)^2}{(kz + z)^2} \\ &= \frac{k^2z^2(k+1)^2}{z^2(k+1)^2} \\ &= k^2 \\ \text{R.H.S.} &= \frac{x}{z} = \frac{k^2z}{z} = k^2 \end{aligned}$$

Hence

$$\text{L.H.S.} = \text{R.H.S.}$$

Proved

(c) In $\triangle ABD$,

$$\tan 48^\circ = \frac{AD}{BD}$$

\Rightarrow

$$1.11 = \frac{100}{BD}$$

$$BD = \frac{100}{1.11} = 90.09 \text{ m}$$

In $\triangle ACD$,

$$\tan 36^\circ = \frac{AD}{DC}$$

\Rightarrow

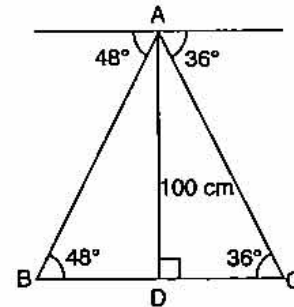
$$0.7265 = \frac{100}{DC}$$

\Rightarrow

$$DC = \frac{100}{0.7265} = 137.64 \text{ m}$$

\therefore

$$\begin{aligned} BC &= BD + DC \\ &= 90.09 + 137.64 \\ &= 227.73 \text{ m.} \end{aligned}$$



Ans.

Question 9.

(a) Evaluate :

$$\begin{bmatrix} 4 \sin 30^\circ & 2 \cos 60^\circ \\ \sin 90^\circ & 2 \cos 0^\circ \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \quad [3]$$

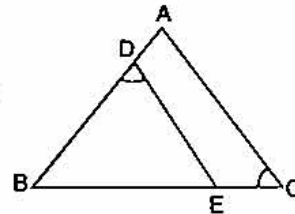
(b) In the figure ABC is a triangle with $\angle EDB = \angle ACB$.

Prove that $\triangle ABC \sim \triangle EBD$.

If $BE = 6 \text{ cm}$, $EC = 4 \text{ cm}$, $BD = 5 \text{ cm}$ and area of $\triangle BED = 9 \text{ cm}^2$. Calculate the :

- length of AB
- area of $\triangle ABC$.

[4]



(c) Vivek invests ₹ 4,500 in 8%, ₹ 10 shares at ₹ 15. He sells the shares when the price rises to ₹ 30, and invests the proceeds in 12% ₹ 100 shares at ₹ 125. Calculate :

- the sale proceeds.
- the number of ₹ 125 shares he buys.
- the change in his annual income from dividend.

[4]

Solution :

$$\begin{aligned}
 \text{(a) Given : } & \begin{bmatrix} 4 \sin 30^\circ & 2 \cos 60^\circ \\ \sin 90^\circ & 2 \cos 0^\circ \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} 4 \times \frac{1}{2} & 2 \times \frac{1}{2} \\ 1 & 2 \times 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}
 \end{aligned}$$

Ans.**(b)**

$$\angle EDB = \angle ACB \text{ (given)}$$

$$\angle DBE = \angle ABC$$

$$\angle DEB = \angle BAC$$

 \Rightarrow

$$\Delta ABC \sim \Delta EBD$$

(AA axiom)

Proved**(i) Given :** BE = 6 cm, EC = 4 cm, BD = 5 cm.

$$\frac{AB}{EB} = \frac{BC}{BD} = \frac{AC}{ED}$$

$$\frac{AB}{EB} = \frac{BC}{BD}$$

 \Rightarrow

$$\frac{AB}{6} = \frac{BE + EC}{5} = \frac{6 + 4}{5} = \frac{10}{5} = 2$$

 \Rightarrow

$$AB = 12 \text{ cm}$$

Ans.**(ii)**

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta EBD} = \frac{AB^2}{EB^2} = \frac{144}{36}$$

$$\frac{\text{Area of } \Delta ABC}{9} = \frac{(12)^2}{(6)^2}$$

 \Rightarrow

$$\text{Area of } \Delta ABC = \frac{144 \times 9}{36} = 36 \text{ m.}$$

Ans.**(c)**

$$\text{Number of shares bought} = \frac{4,500}{15}$$

$$= 300$$

$$\text{Total face value} = ₹ 300 \times 10$$

$$= ₹ 3,000$$

$$\text{Dividend} = \frac{8}{100} \times 3,000$$

$$= ₹ 240.$$

Amount received on selling 300 shares for ₹ = $300 \times 30 = ₹ 9,000$

$$\text{(i) Sale proceeds} = ₹ 9,000 - ₹ 4,500 = ₹ 4,500$$

Ans.

$$\text{(ii) Number of shares bought at ₹ 125} = \frac{9,000}{125}$$

$$= 72$$

Ans.

$$\begin{aligned} \text{(iii) Total face value of 72 shares} &= ₹ 72 \times 100 \\ &= ₹ 7,200 \end{aligned}$$

$$\begin{aligned} \text{Dividend} &= \frac{12}{100} \times 7,200 \\ &= ₹ 864. \end{aligned}$$

$$\begin{aligned} \text{Change in his annual income} &= 864 - 240 \\ &= ₹ 624. \end{aligned}$$

Ans.

Question 10.

- (a) A positive number is divided into two parts such that the sum of the squares of the two parts is 208. The square of the larger part is 8 times the smaller part. Taking x as the smaller part of the two parts, find the number. [4]
- (b) The monthly income of a group of 320 employees in a company is given below :

Monthly Income	No. of Employees
6000–7000	20
7000–8000	45
8000–9000	65
9000–10000	95
10000–11000	60
11000–12000	30
12000–13000	5

Draw an ogive of the given distribution on a graph sheet taking 2 cm = ₹ 1,000 on one axis and 2 cm = 50 employees on the other axis. From the graph determine :

- (i) the median wage.
 (ii) the number of employees whose income is below ₹ 8,500
 (iii) If the salary of a senior employee is above ₹ 11,500, find the number of senior employees in the company.
 (iv) the upper quartile. [6]

Solution :

- (a) Let x and y are the two parts.

$$x^2 + y^2 = 208 \quad \dots(1)$$

$$y^2 = 8x \quad \dots(2)$$

$$\Rightarrow x^2 + 8x - 208 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 1$, $b = 8$, $c = -208$

$$= \frac{-8 \pm \sqrt{(8)^2 - 4 \times 1 \times (-208)}}{2 \times 1}$$

$$= \frac{-8 \pm \sqrt{64 + 832}}{2}$$

$$= \frac{-8 \pm 29.93}{2} = \frac{-8 + 29.93}{2} \text{ or } \frac{-8 - 29.93}{2}$$

$$= -18.96 \text{ or } 10.97$$

$$y^2 = 8x$$

$$= 8 \times 10.97$$

$$= 87.76$$

$$y = 9.37$$

$$\text{Number} = x + y$$

$$= 10.97 + 9.37$$

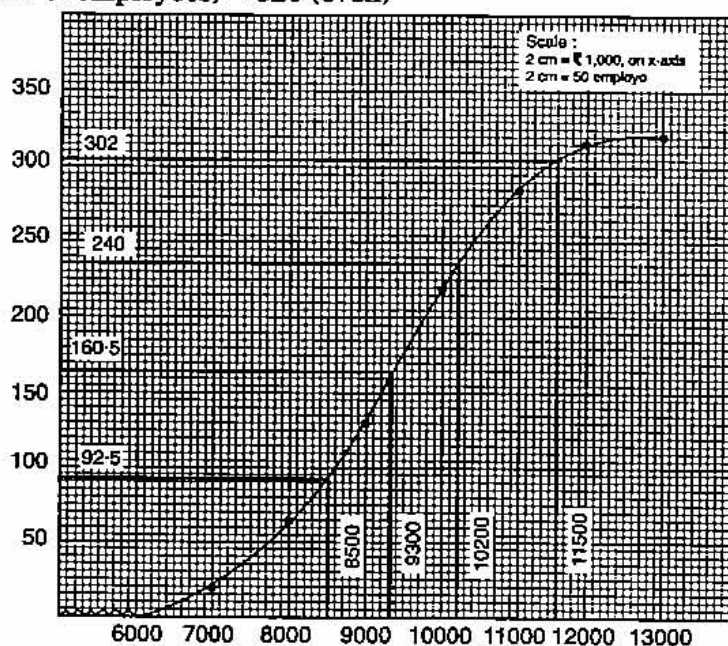
$$= 20.34$$

Ans.

(b)

Monthly Income	No. of Employees	C.F.
6000-7000	20	20
7000-8000	45	65
8000-9000	65	130
9000-10000	95	225
10000-11000	60	285
11000-12000	30	315
12000-13000	5	320
	320	

Here n (no. of employees) = 320 (even)



$$(i) \quad \text{Median} = \frac{1}{2} \left[\frac{n}{2} + \left(\frac{n}{2} + 1 \right) \right] = \frac{1}{2} [160 + 161] = 160.5$$

Required median = ₹ 9,300 (from graph)

Ans.

(ii) Number of employees whose income is below ₹ 8,500 = 92.5 approx. Ans.

(iii) Number of senior employees in the company = $320 - 302 = 18$. **Ans.**

(iv) Upper quartile = $\frac{3n}{4} = \frac{3 \times 320}{4} = 240$

Upper quartile = 10,200. **Ans.**

Question 11.

(a) Construct a regular hexagon of side 4 cm. Construct a circle circumscribing the hexagon. **[3]**

(b) A hemispherical bowl of diameter 7.2 cm is filled completely with chocolate sauce. This sauce is poured into an inverted cone of radius 4.8 cm. Find the height of the cone. **[3]**

(c) Given $x = \frac{\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2}}$

Use componendo and dividendo to prove that $b^2 = \frac{2a^2x}{x^2 + 1}$. **[4]**

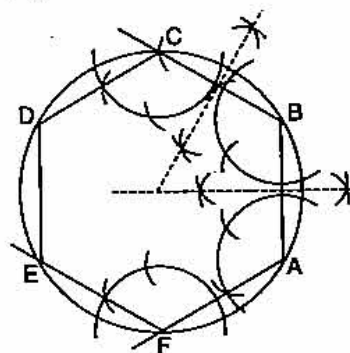
Solution :

(a) Steps of Construction :

(i) Using the given data, construct the regular hexagon ABCDEF with each side equal to 4 cm.

(ii) Draw the perpendicular bisectors of sides AB and AF which intersect each other at point O.

(iii) With O as centre and OA as radius draw a circle which will pass through all the vertices of the regular hexagon ABCDEF.



(b) Given : Diameter of hemispherical bowl = 7.2 cm

Radius of hemispherical bowl = 3.6 cm

$$\begin{aligned} \text{Volume of hemispherical bowl} &= \frac{2}{3} \times \pi r^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times 3.6 \times 3.6 \times 3.6 \\ &= 97.76 \text{ cm}^3. \end{aligned}$$

$$\begin{aligned} \text{Volume of cone} &= \frac{1}{3} \times \pi R^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times 4.8 \times 4.8 \times h \\ &= 24.14 \times h \text{ cm}^3 \end{aligned}$$

Volume of cone = Volume of hemispherical bowl

$$\Rightarrow 24.14 \times h = 97.76$$

$$\Rightarrow h = \frac{97.76}{24.14}$$

$$= 4.05 \text{ cm.}$$

Ans.

(c) Given : $x = \frac{\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2}}$

Componendo and dividendo

$$\begin{aligned} \frac{x+1}{x-1} &= \frac{(\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}) + (\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2})}{(\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}) - (\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2})} \\ &= \frac{2(\sqrt{a^2 + b^2})}{2\sqrt{a^2 - b^2}} \end{aligned}$$

$$\Rightarrow \frac{(x+1)^2}{(x-1)^2} = \frac{a^2 + b^2}{a^2 - b^2}$$

Again componendo and dividendo

$$\Rightarrow \frac{(x+1)^2 + (x-1)^2}{(x+1)^2 - (x-1)^2} = \frac{a^2 + b^2 + a^2 - b^2}{a^2 + b^2 - a^2 + b^2}$$

$$\frac{2x^2 + 2}{4x} = \frac{2a^2}{2b^2}$$

$$\Rightarrow \frac{x^2 + 1}{2x} = \frac{a^2}{b^2}$$

$$\Rightarrow b^2 = \frac{2a^2x}{x^2 + 1}$$

Proved

● ●