CBSE Sample Paper-02
SUMMATIVE ASSESSMENT –II
MATHEMATICS
Class – X

Time allowed: 3 hours Maximum Marks: 90

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 31 questions divided into four sections – A, B, C and D.
c) Section A contains 4 questions of 1 mark each which are multiple choice questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
d) Use of calculator is not permitted.

Section A

1. The probability that two different friend have different birthdays (ignoring a leap year) is:

(a) $\frac{364}{365}$ (b) $\frac{1}{365}$ (c) $\frac{1}{73}$ (d) $\frac{3}{73}$

2. The centroid of the triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is:

(a) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ (b) $(x_1+x_2+x_3, y_1+y_2+y_3)$

(c) $\left(\frac{x_1+x_2+x_3}{6}, \frac{y_1+y_2+y_3}{6}\right)$ (d) $\left(\frac{x_1+x_2+x_3}{4}, \frac{y_1+y_2+y_3}{4}\right)$

3. In an AP, if $a = 5,\ d = 2.5,\ a_n = 10,$ then the value of n is:

(a) 1 (b) 2 (c) 3 (d) 4

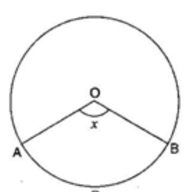
4. If in a Δ ABC, $\angle$C = 90° and $\angle$B = 45°, then state which of the following is true:

(a) Base = Perpendicular (b) Base = Hypotenuse
(c) Perpendicular = Hypotenuse (d) Base + Perpendicular = Hypotenuse

Section B

5. In the figure, O is the centre of the circle. The area of sector OAPB is $\frac{5}{18}$ of the area of the circle.

Find x.

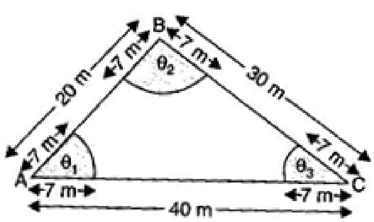


6. How many shots each having radius 3 cm can be made from a cubical lead solid of dimensions 49 cm x 36 cm x 22 cm?

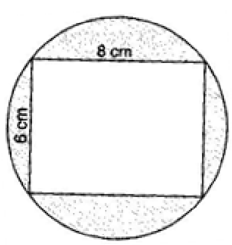
7. Three cubes of a metal whose edges are in the ratio 3 : 4 : 5 are melted and converted into a single cube of diagonal $24\sqrt{3}$ cm. Find the edges of the three cubes.
8. If 1 is a zero of the polynomial $p(x)=ax^2-3(a-1)x-1,$ then find the value of a.
9. If the first term of an AP is –4 and the common difference is 2, then find the sum of first 10 terms.
10. If a, b and c are the sides of a right angled triangle where c is the hypotenuse, then prove that the radius r of the circle which touches the sides of the triangle is given by $r=\dfrac{a+b-c}{2}$.

Section C

11. Find the ratio in which the point $(-3, k)$ divides the line segment joining the points $(-5,-4)$ and $(-2,3)$. Hence, find the value of k.
12. Show that the points A (1, 2), B (5, 4), C (3, 8) and D$(-1,6)$ are the vertices of a square.
13. Three horses are tethered at 3 corners of a triangular plot having sides 20 m, 30 m, 40 m with ropes of 7 m length each. Find the area of the plot which can be grazed by the horses. $\left(\text{Use } \pi=\dfrac{22}{7}\right)$



14. A rectangle 8 cm x 6 cm is inscribed in a circle as shown in figure. Find the area of the shaded region. $(\text{Use } \pi = 3.14)$

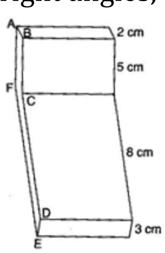


15. A solid iron rectangular block od dimensions 4.4 m x 2.6 m x 1 m is cast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.
16. Solve the quadratic equation: $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$
17. The sum of n terms of an AP is $3n^2+5n$. Find the AP and hence find its 16^{th} term.

18. A point P is 13 cm from the centre of the circle. The length of the tangent drawn from P to the circle is 12 cm. Find the radius of the circle.
19. The angles of depression of the top and the bottom of a building 50 meters high as observed from the top of a tower are 30° and 60° respectively. Find the height of the tower and the horizontal distance between the building and the tower. $\left(\text{Take } \sqrt{3} = 1.73\right)$
20. Cards marked with the numbers 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from this box. Find the probability that the number on the card is:
    (i) an even number.
    (ii) a number less than 14.
    (iii) a number which is a perfect square.

Section D

21. Draw a circle of radius 3 cm. From a point 5 cm away from the centre of the circle, draw two tangents to the circle. Find the lengths of the tangents.
22. The angle of elevation of the top of a tower as observed from a point on the ground is $'\alpha'$ and moving $'a'$ meters towards the tower, the angle of elevation is $'\beta'$. Prove that the height of the tower is $\dfrac{a \tan\alpha \tan\beta}{\tan\beta - \tan\alpha}$.
23. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is:
    (i) a king or a jack (ii) a non-ace
    (iii) a red card (iv) neither a king nor a queen.
24. Find the coordinates of the points which divide the line segment joining the points $(-4, 0)$ and (0, 6) in three equal parts.
25. In figure, the shape of a solid copper piece (made of two pieces) with dimensions is shown. The face of ABCDEFA is the uniform cross-section. Assume that the angles at A, B, C, D, E and F are right angles, calculate the volume of the piece.



26. A milk container is in the form of a frustum of cone of height 18 cm with radius of its upper and lower ends as 8 cm and 32 cm respectively. Find the amount of milk which can completely fill the container and its cost at the rate of Rs.20 per litre. $\left(\text{Use } \pi = 3.14\right)$
27. Solve for $x: abx^2 + \left(b^2 - ac\right)x - bc = 0$
28. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, then find the sides of two squares.

29. Ram asks the labour to dig a well up to a depth of 10 m. Labour charges `150 for first meter and `50 for each subsequent meters. As labour was uneducated, he claims `550 for the whole work.

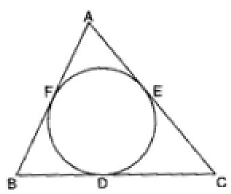
Read the above passage and answer the following questions:

(i) What should be the actual amount to be paid to the labour?

(ii) What value of Ram is depicted in the question, if he pays `600 to the labour?

30. The incircle of Δ ABC touches the sides BC, CA and AB at D, E and F respectively. Show that:

AF + BD + CD = AE + BF + CE = $\frac{1}{2}$ (Perimeter of Δ ABC)



31. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Using the above result, prove the following:

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre of a point Q so that OQ = 13 cm. Find the length of PQ.

CBSE Sample Paper-02
SUMMATIVE ASSESSMENT –II
MATHEMATICS
Class – X

(Solutions)

SECTION-A

1. (a)
2. (a)
3. (b)
4. (a)
5. Area of sector = $\frac{\theta}{360°}\pi r^2$

According to question,

$$\frac{\theta}{360°}\pi r^2 = \frac{5}{18}\pi r^2$$

$\Rightarrow \quad x = 100°$

6. Let n shots be made. Then,

According to question,

Volume of n shots = Volume of cuboid

$\Rightarrow \quad n.\frac{4}{3} = \pi r^3 = 49\times 36\times 22$

$\Rightarrow \quad n.\frac{4}{3}\times\frac{22}{7}.(3)^3 = 49\times 36\times 22 \quad \Rightarrow \quad n = \frac{49\times 36\times 22\times 7\times 3}{4\times 22\times 27} \quad \Rightarrow n = 363.4$

7. Let the edges of three cubes (in cm) be $3x, 4x$ and $5x$ respectively. Then,

Volume of the cubes after melting = $(3x)^3 + (4x)^3 + (5x)^3$ = $216x^3$ cm^3

Let the edge of the new cube be a cm. Then,

$a^3 = 216x^3 \quad \Rightarrow \quad a = 6x$

$\therefore \quad$ Diagonal = $a\sqrt{3} = 6\sqrt{3}x$

And $\quad 6\sqrt{3}x = 24\sqrt{3} \quad \Rightarrow \quad x = 4$

Hence, the edges of the three cubes are 12 cm, 16 cm and 20 cm.

8. $p(1) = 0$

$\Rightarrow \quad a(1)^3 - 3(a-1)(1) - 1 = 0 \quad \Rightarrow \quad a - 3a + 2 = 0$

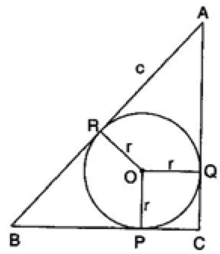
$\Rightarrow \quad -2a + 2 = 0 \quad \Rightarrow \quad a = 1$

9. $a = -4,\ d = 2,\ n = 10$

$\because \quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\therefore \qquad S_{10} = \frac{10}{2}\left(2\times(-4)+(10-1)2\right)$ = 5 x 10 = 50

10. AB = AR + BR



$\Rightarrow \qquad c$ = AQ + BP

$\Rightarrow \qquad c$ = (AC – CQ) + (BC – PC)

$\Rightarrow \qquad c = (b-r)+(a-r)$

$\Rightarrow \qquad c = a+b-2r$

$\Rightarrow \qquad r = \frac{a+b-c}{2}$

11. Let the ratio be $\lambda:1$. Then, according to question,

$\frac{(-2\lambda)+(-5)}{\lambda+1} = -3 \qquad \Rightarrow \qquad \frac{-2\lambda-5}{\lambda+1} = -3$

$\Rightarrow \qquad -2\lambda-5 = -3\lambda-3 \qquad \Rightarrow \qquad -2\lambda+3\lambda = -3+5$

$\Rightarrow \qquad \lambda = 2$

Also, $\frac{3\lambda-4}{\lambda+1} = k \qquad \Rightarrow \qquad \frac{3\times2-4}{2+1} = k$

$\Rightarrow \qquad k = \frac{6-4}{3} \qquad \Rightarrow \qquad k = \frac{2}{3} = 2:3$

12. Given, A (1, 2), B (5, 4), C (3, 8) and D $(-1,6)$

AB = $\sqrt{(5-1)^2+(4-2)^2} = \sqrt{4^2+2^2} = \sqrt{20} = 2\sqrt{5}$ units

BC = $\sqrt{(3-5)^2+(8-4)^2} = \sqrt{(-2)^2+(4)^2} = \sqrt{20} = 2\sqrt{5}$

CD = $\sqrt{(-1-3)^2+(6-8)^2} = \sqrt{(-4)^2+(-2)^2} = \sqrt{20} = 2\sqrt{5}$

DA = $\sqrt{(-1-1)^2+(6-2)^2} = \sqrt{(-2)^2+(4)^2} = \sqrt{20} = 2\sqrt{5}$

And diagonals

AC = $\sqrt{(3-1)^2+(8-2)^2} = \sqrt{(2)^2+(6)^2} = \sqrt{40} = 2\sqrt{10}$

BD = $\sqrt{(-1-5)^2+(6-4)^2} = \sqrt{(-6)^2+(2)^2} = \sqrt{40} = 2\sqrt{10}$

Hence, all four sides and two diagonals are equal.

Therefore ABCD is a square.

13. Required area,

$$= \pi r^2 \frac{\theta_1}{360°} + \pi r^2 \frac{\theta_2}{360°} + \pi r^2 \frac{\theta_3}{360°}$$

$$= \frac{\pi r^2}{360°}(\theta_1 + \theta_2 + \theta_3)$$

$$= \frac{\pi r^2}{360°}(180°) \qquad [\because \text{ Sum of all the angles of a triangle is } 180°]$$

$$= \frac{22}{7} \times 7 \times 7 \times \frac{180°}{360°}$$

= 77 m^2

14. Diagonal of the rectangle = $\sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10$ cm

$\therefore$ Radius of the circle = $\frac{10}{2}$ = 5 cm

$\therefore$ Area of the circle = $\pi.5^2$ = 3.14 x 5 x 5 = 78.5 cm^2

$\therefore$ Area of rectangle = 8 x 6 = 48 cm^2

$\therefore$ Required area = 78.5 – 48 = 30.5 cm^2

15. Let the length of the pipe be x cm.

According to question,

Volume of hollow cylinder = Volume of rectangular block

$\Rightarrow$ $\pi\left(r_1^2 - r_2^2\right)h - l \times b \times h$

$\Rightarrow$ $\pi\left[(30+5)^2 - (30)^2\right]x =$ 4.4 x 2.6 x 1 x 100 x 100 x 100

$\Rightarrow$ $3.14[1225 - 900]x =$ 11.44 x 1000000

$\Rightarrow$ 1020.5. x = 11440000

$\Rightarrow$ x = 112 m

16. $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$ $\Rightarrow$ $\sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow$ $\sqrt{3}x\left(x - \sqrt{3}\sqrt{2}\right) + \sqrt{2}\left(x - \sqrt{2}\sqrt{3}\right) = 0$

$\Rightarrow$ $\left(x - \sqrt{6}\right)\left(\sqrt{3}x + \sqrt{2}\right) = 0$

$\Rightarrow$ $x = \sqrt{6}, \frac{-\sqrt{2}}{\sqrt{3}}$

17. $S_n = 3n^2 + 5n$

$S_1 = 3(1)^2 + 5(1) = 8$

$S_2 = 3(2)^2 + 5(2) = 22$

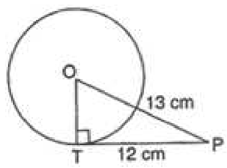
$\therefore$ $a_1 = 8,$ $a_2 = S_2 - S_1$ = 22 – 8 = 14

$\therefore$ $d = a_2 - a_1$ = 14 – 8 = 6

$\therefore$ AP is 8, 14, 20, 26, ……..

And $a_{16} = a + 15d$ = 8 + 15 x 6 = 98

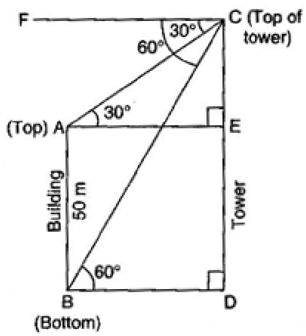
18. $OT^2 = OP^2 - PT^2$ [By Pythagoras theorem]



$\Rightarrow$ $OT^2 = 13^2 - 12^2 = 169 - 144 = 25$

$\Rightarrow$ OT = 5 cm

19. In right Δ CEA,



$\tan 30^\circ = \frac{CE}{AE}$

$\Rightarrow$ $\frac{1}{\sqrt{3}} = \frac{CE}{AE}$ ……….(i)

In right Δ CDB,

$\tan 60^\circ = \frac{CD}{BD}$

$\Rightarrow$ $\sqrt{3} = \frac{CE + 50}{BD}$ ……….(ii)

Dividing eq. (i) by eq. (ii), we get,

$\frac{1}{3} = \frac{CE}{CE + 50}$ [$\because$ BD = AE]

$\Rightarrow$ CE = 25 m

$\therefore$ CD = CE + ED = 25 + 50 = 75 m

From eq. (i),

$\frac{1}{\sqrt{3}} = \frac{CE}{AE}$ $\Rightarrow$ AE = $25\sqrt{3}$ = 25 x 1.73 = 43.25 m

20. There are 100 cards in the box, out of which one card can be drawn in 100 ways.

$\therefore$ Total number of possible outcome = 100

(i) From number 2 to 101, there are 50 even numbers, namely [2, 4, 6, …..100]. Out of these 50 even numbered cards, one card can be chosen in 50 ways.-

Hence, P (getting an even numbered card) = $\frac{50}{100} = \frac{1}{2}$

(ii) There are 12 cards bearing numbers less than 14, i.e., namely [2, 3, 4, ……13].

Hence required probability = $\frac{12}{100} = \frac{3}{25}$

(iii) The perfect squares numbers from 2 to 101 are 4, 9, 16, 25, 36, 49, 64, 81, 100 i.e. squares of 2, 3, 4, 5……10 respectively.

Therefore there are 9 cards marked with the numbers which are perfect squares.

Hence, required probability = $\frac{9}{100}$

21. **Steps of construction**:

(a) Draw a circle with O as centre and radius equal to 3 cm.

(b) Draw OP = 5 cm and bisect it. Let M be the mid-point of OP.

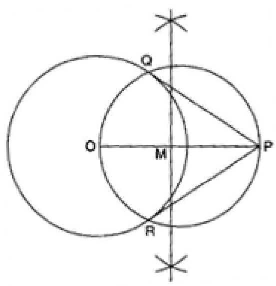
(c) Taking M as centre and OM as radius , draw a circle. Let it intersect the given circle at Q and R.

(d) Join PQ and PR.

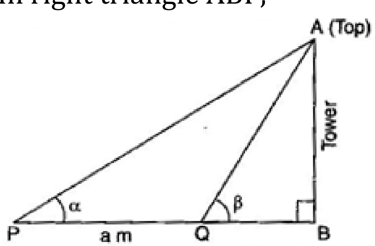
Then PQ and PR are the required two tangents.

On measurement,

PQ = PR = 4 cm



22. In right triangle ABP,



$$\tan \alpha = \frac{AB}{a + QB} \qquad \text{……….(i)}$$

In right triangle ABQ, $\tan \beta = \frac{AB}{QB}$

$$\Rightarrow \qquad QB = \frac{AB}{\tan \beta} \qquad \text{……….(ii)}$$

Putting the value of QB from eq. (ii) in eq. (i), we get,

$$\tan\alpha = \frac{AB}{a+\frac{AB}{\tan\beta}} = \frac{AB\ \tan\beta}{a\tan\beta + \ AB}$$

$\Rightarrow$ $a\tan\alpha\tan\beta + \ AB\ \tan\alpha = \ AB\ \tan\beta$

$\Rightarrow$ $AB(\tan\beta - \tan\alpha) = a\tan\alpha\tan\beta$

$\Rightarrow$ $AB = \dfrac{a\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$

23. Total number of cards in the deck = 52

$\therefore$ Number of all possible outcomes = 52

(i) Number of a king or a jack = 4 + 4 = 8

$\therefore$ Required probability = $\dfrac{8}{52} = \dfrac{2}{13}$

(ii) Number of a non-ace = 52 – 4 = 48

$\therefore$ Required probability = $\dfrac{42}{52} = \dfrac{12}{13}$

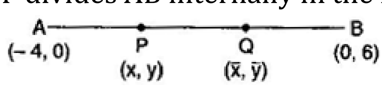
(iii) Number of a red card = 13 + 13= 26

$\therefore$ Required probability = $\dfrac{26}{52} = \dfrac{1}{2}$

(iv) Number of neither a king nor a queen = 52 – (4 + 4) = 4

$\therefore$ Required probability = $\dfrac{44}{52} = \dfrac{11}{13}$

24. P divides AB internally in the ratio 1 : 2.



$\therefore$ $x = \dfrac{1\times 0 + 2\times(-4)}{1+2} = \dfrac{-8}{3}$

And $y = \dfrac{1\times 6 + 2\times 0}{1+2} = 2$

$\therefore$ $P \to \left(\dfrac{-8}{3}, 2\right)$

Since Q is the mid-point of PB.

$\therefore$ $\bar{x} = \dfrac{\frac{-8}{3}+0}{2} = \dfrac{-4}{3}$

And $\bar{y} = \dfrac{2+6}{2} = 4$

$\therefore$ $Q \to \left(\dfrac{-4}{3}, 0\right)$

25. Volume of horizontal cuboid = lbh

= 22 x (8 + 2) x 3 = 22 x 10 x 3 = 660 cm^3

Volume of vertical cuboid = lbh

= 22 x 2 x 5 = 220 cm³

∴ Total volume of piece

= 660 cm³ + 220 cm³

= 1180 cm³

26. Given, $h = 18$ cm

$r_1 = 32$ cm

$r_2 = 8$ cm

According to the question,

Amount of milk = Volume of frustum

$\Rightarrow$ Volume of frustum = $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= \frac{1}{3}\times 3.14\times 18\left[(32)^2 + (8)^2 + 32\times 8\right]$

$= \frac{3.14\times 18}{3}(1024 + 64 + 256)$

$= \frac{3.14}{3}\times 18\times 1344 = 25320.96$ cm³

Cost of milk = $\frac{25320.96\times 20}{1000} = \frac{506419.2}{1000}$ = Rs. 506.42

27. $abx^2 + \left(b^2 - ac\right)x - bc = 0$

$\Rightarrow \quad abx^2 + b^2x - acx - bc = 0$

$\Rightarrow \quad bx(ax+b) - c(ax+b) = 0$

$\Rightarrow \quad (ax+b)(bx-c) = 0$

$\Rightarrow \quad x = \frac{-b}{a}, \frac{c}{b}$

28. Let the side of the larger square be x m. Then its perimeter = $4x$ m

Perimeter of the larger square – Perimeter of the smaller square = 24 m

$\Rightarrow \quad 4x -$ Perimeter of the smaller square = 24

$\Rightarrow$ Perimeter of the smaller square = $(4x - 24)$ m

$\Rightarrow$ Side of the smaller square = $\frac{4x-24}{4} = (x-6)$ m

According to the question,

Area of the larger square + Area of the smaller square = 468 m²

$\Rightarrow \quad x^2 + (x-6)^2 = 468$

$\Rightarrow \quad x^2 + x^2 - 12x - 432 = 0$

$\Rightarrow \quad 2x^2 - 12x - 432 = 0$

$\Rightarrow \quad x^2 - 6x - 216 = 0$

$\Rightarrow \quad x^2 - 18x + 12x - 216 = 0$

$\Rightarrow \quad x(x-18) + 12(x-18) = 0$

$\Rightarrow \quad (x-18)(x+12)=0 \quad \Rightarrow \quad x=18,-12$

$x=-12$ is inadmissible as x is the length of a side which cannot be negative.

$\therefore \quad x=18$ and $x-6=12$

Hence, the sides of the two squares are 18 m and 12 m.

29. (i) Here, amount form an AP.

First term, a = Labour charge for first meter = `150

Since Labour charge increasing by `50 for each subsequent meters.

$\therefore \quad d=50$

Total depth = 10 m

$\therefore$ Labour charge for 10 m = $a+(n-1)d$

= 150 + (10 – 1) x 50 = 150 + 9 x 50

= 150 + 450 = 600

Hence `600 should be paid to the labours.

(ii) If Ram pays `600 to the labour, then it shows his honesty and sincerety.

30. $\because$ Tangent segments from an external point to a circle are equal in length.

$\therefore$ $\therefore$ AE = AF BF = BD CD = CE

$\Rightarrow$ AF + BD + CD = AE + BF + CE ……….(i)

Also, Perimeter of ΔABC

= AB + BC + CA

= (AF + BF) + (BD + CD) + (CE + AE)

= (AF + BD + CD) + (AE + BF + CE)

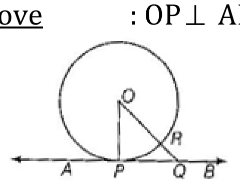
= 2(AF + BD + CD) [From eq. (i)]

= 2(AE + BF + CE)

$\therefore$ Perimeter of ΔABC = 2 (AE + BF + CE)

31. **First part**:<u>Given</u> : A circle with centre O and radius r and a tangent AB at a point P.

<u>To Prove</u> : OP $\perp$ AB



<u>Construction</u>: Take any point Q, other than P on the tangent AB. Join OQ. Suppose OQ meets the circle at R.

<u>Proof</u> :Clearly OP = OQ [Radii]

Now, OQ = OR + RQ

$\Rightarrow$ OQ> OR

$\Rightarrow$ OQ> OP [OP = OQ]

$\Rightarrow$ OP <OQ

Thus, OP is shorter than any segment joining O to any point of AB.

So, OP is perpendicular to AB.

Hence,

$OT = OT'$ .... (Radii of the same circle)

and $OP = OP$ ....(Common)

$\therefore \Delta OTP \cong \Delta OT'P$ ....(RHS congruency)

Hence, $OP \perp AB$

Second part: Using the above, we get,

$\angle OPQ = 90^\circ$

$\therefore PQ = \sqrt{OQ^2 - OP^2}$ [By Pythagoras theorem]

$\Rightarrow PQ = \sqrt{13^2 - 5^2} = 12$ cm