

APPLICATION OF DERIVATIVES



BASIC CONCEPTS

1. Tangents and Normals:

Equation of tangent to the curve $y = f(x)$ at the point (x_1, y_1) is given by

$$(y - y_1) = \left(\frac{dy}{dx} \right)_{(x_1, y_1)} (x - x_1)$$

Equation of normal to the curve $y = f(x)$ at $P(x_1, y_1)$ is

$$(y - y_1) = \frac{-1}{\left[\frac{dy}{dx} \right]_{(x_1, y_1)}} (x - x_1)$$

2. Increasing Function: A function $f(x)$ is said to be an increasing function in (a, b) if

$$x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2) \quad \forall x_1, x_2 \in (a, b)$$

3. Decreasing Function: A function $f(x)$ is said to be decreasing in the interval (a, b) if

$$x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2) \quad \forall x_1, x_2 \in (a, b)$$

4. Maximum and Minimum Value of a Function

(or Absolute Maximum or Minimum Value)

A function f is said to attain maximum value at a point $a \in D_f$, if $f(a) \geq f(x) \quad \forall x \in D_f$ then $f(a)$ is called absolute maximum value of f .

A function f attains minimum value at $x = b \in D_f$ if $f(b) \leq f(x) \quad \forall x \in D_f$ then $f(b)$ is called absolute minimum value of f .

5. Local Maxima and Local Minima (or Relative Extrema)

Local Maxima: A function $f(x)$ is said to attain a local maxima at $x = a$, if there exists a neighbourhood $(a - \delta, a + \delta)$ of ' a ' such that $f(x) < f(a) \quad \forall x \in (a - \delta, a + \delta), x \neq a$, then $f(a)$ is the local maximum value of $f(x)$ at $x = a$.

Local Minima: A function $f(x)$ is said to attain a local minima at $x = a$, if there exists a neighbourhood $(a - \delta, a + \delta)$ of ' a ' such that $f(x) > f(a) \quad \forall x \in (a - \delta, a + \delta), x \neq a$, then $f(a)$ is called the local minimum value at $x = a$.

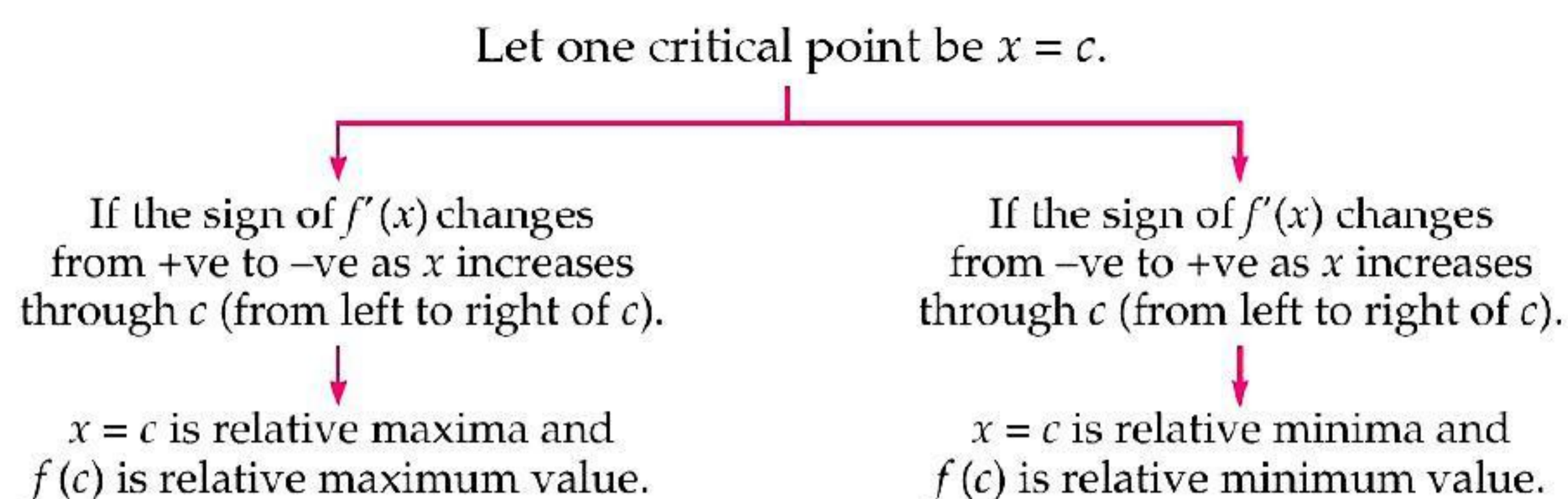
6. Test for Identifying Relative (Local) Maxima or Minima

(i) First Derivative Test

Step I: Find $f'(x)$

Step II: The equation $f'(x) = 0$ is solved to get critical points $x = c_1, c_2, \dots, c_n$.

Step III: The sign of $f'(x)$ is studied in the neighbourhood of each critical points.

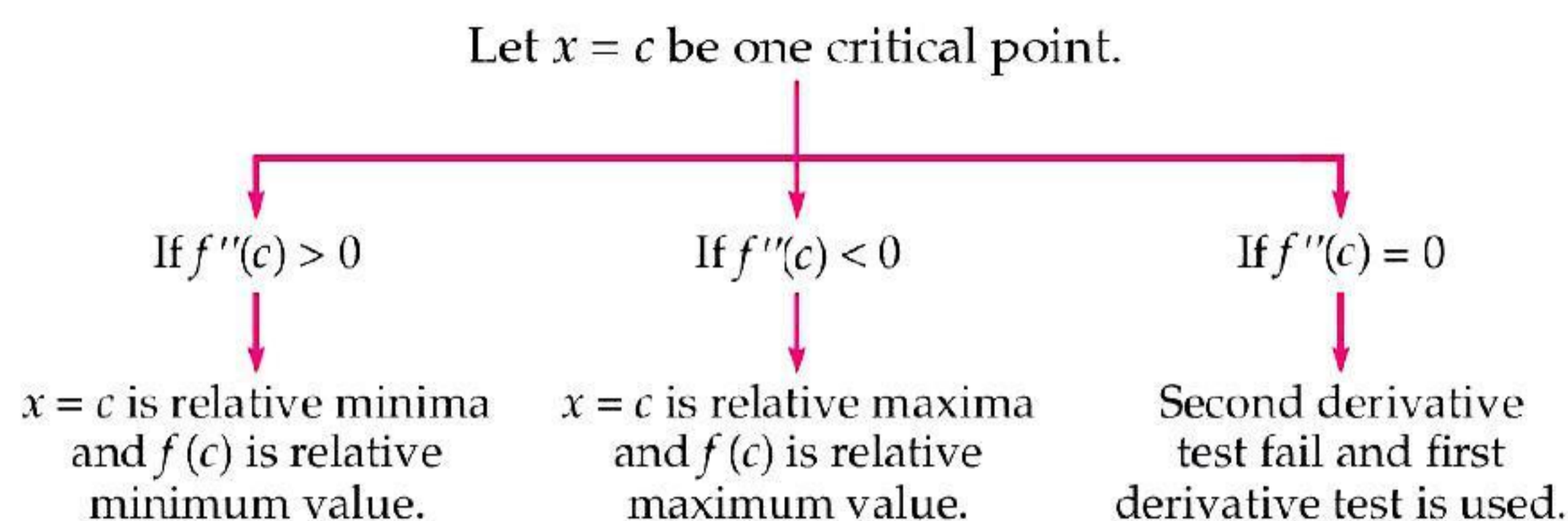


(ii) Second order derivative test

Step I: Find $f'(x) = 0$.

Step II: The equation $f'(x) = 0$ is solved to get critical points $x = c_1, c_2, \dots, c_n$.

Step III: $f''(x)$ is obtained and the sign of $f''(x)$ is studied for all critical points $x = c_1, c_2, \dots, c_n$.



7. Critical point: A point $x = c$ is called critical point of the function $f(x)$, if $f(c)$ exists and either $f'(c) = 0$ or $f'(c) = \infty$ (does not exist).

8. Point of Inflexion: If $f(x)$ is a function and $x = c$ is critical point, then $x = c$ is called point of inflexion if

- (i) $f'(c) = 0$ (ii) $f''(c) = 0$ (iii) $f'''(c) \neq 0$

MULTIPLE CHOICE QUESTIONS

Choose and write the correct option in the following questions.

1. The interval in which the function f given by $f(x) = x^2 e^{-x}$ is strictly increasing, is [CBSE 2020 (65/2/1)]

(a) $(-\infty, \infty)$	(b) $(-\infty, 0)$	(c) $(2, \infty)$	(d) $(0, 2)$
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2. The abscissa of the point on the curve $3y = 6x - 5x^3$, the normal at which passes through origin is

(a) 1	(b) $\frac{1}{3}$	(c) 2	(d) $\frac{1}{2}$
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3. $f(x) = x^x$ has a stationary point at

(a) $x = e$	(b) $x = \frac{1}{e}$	(c) $x = 1$	(d) $x = \sqrt{e}$
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4. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 = 2$ [NCERT Exemplar]

(a) touch each other	(b) cut at right angle
(c) cut at an angle $\frac{\pi}{3}$	(d) cut at an angle $\frac{\pi}{4}$
5. The slope of normal to the curve $y = 2x^2 + 3 \sin x$ at $x = 0$ is

(a) 3	(b) $\frac{1}{3}$	(c) -3	(d) $-\frac{1}{3}$
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6. The equation of the normal to the curve $y = \sin x$ at $(0, 0)$ is

(a) $x = 0$	(b) $y = 0$	(c) $x + y = 0$	(d) $x - y = 0$
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7. The tangent to the curve $y = e^{2x}$ at the point $(0, 1)$ meets x -axis at
 (a) $(0, 1)$ (b) $(0, 2)$ (c) $\left(-\frac{1}{2}, 0\right)$ (d) $(2, 0)$
8. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point
 (a) $(1, 2)$ (b) $(2, 1)$ (c) $(1, -2)$ (d) $(-1, 2)$
9. The point on the curve $x^2 = 2y$ which is nearest to the point $(0, 5)$ is
 (a) $(2\sqrt{2}, 4)$ (b) $(2\sqrt{2}, 0)$ (c) $(0, 0)$ (d) $(2, 2)$
10. The slope of tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is
 (a) $\frac{22}{7}$ (b) $\frac{6}{7}$ (c) $\frac{7}{6}$ (d) $\frac{-6}{7}$
11. The greatest of the numbers $1, 2^{1/2}, 3^{1/3}, 4^{1/4}, 5^{1/5}, 6^{1/6}$ and $7^{1/7}$, is
 (a) $2^{1/2}$ (b) $3^{1/3}$ (c) $4^{1/4}$ (d) $7^{1/7}$
12. The minimum value of x^x ($x > 0$) is
 (a) 1 (b) $e^{-1/e}$ (c) $\left(\frac{1}{e}\right)^e$ (d) none of these
13. The values of a for which $y = x^2 + ax + 25$ touches the x -axis are
 (a) 0 (b) ± 10 (c) 4, -6 (d) ± 5
14. If $f(x) = \frac{1}{4x^2 + 2x + 1}$, then its maximum value is
 (a) 0 (b) $\frac{4}{3}$
 (c) ± 5 (d) Maximum value does not exist.
15. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in
 (a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$ (c) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
16. Let the $f: R \rightarrow R$ be defined by $f(x) = 2x + \cos x$, then f
 (a) has a maximum, at $x = 0$ (b) has a minimum, at $x = \pi$
 (c) is an increasing function (d) is a decreasing function
17. At $x = \frac{5\pi}{6}$, $f(x) = 2 \sin 3x + 3 \cos 3x$ is
 (a) 0 (b) minimum (c) maximum (d) none of these
18. At $(0, 0)$ the curve $y = x^{1/2}$ has a
 (a) a horizontal tangent parallel to x -axis (b) a vertical tangent parallel to y -axis
 (c) an oblique tangent (d) tangent does not exist.
19. If the tangent at (a, b) to the curve $x^3 + y^3 = c^3$ meets the curve again in (a_1, b_1) then $\frac{a_1}{a} + \frac{b_1}{b}$ is equal to
 (a) 1 (b) -1 (c) c (d) $\frac{c}{2}$
20. The point of intersection of the tangent drawn to the curve $x^2y = 1 - y$ at the points where it is met by the curve $xy = 1 - y$, is given by
 (a) $(0, -1)$ (b) $(1, 1)$ (c) $(0, 1)$ (d) $(-1, 0)$
21. The values of a for which the function $f(x) = \sin x - ax + b$ increases on R are
 (a) $(-\infty, \infty)$ (b) $[-1, 1]$ (c) $(-\infty, -1)$ (d) none of these
22. The function $f(x) = \frac{2x^2 - 1}{x^4}$, $x > 0$, decreases in the interval
 (a) $(-\infty, 0)$ (b) $[1, \infty)$ (c) $[-1, -1]$ (d) none of these

23. The least value of the function $f(x) = ax + \frac{b}{x}$ ($a > 0, b > 0, x > 0$) is
 (a) $\frac{a}{b}$ (b) $2\sqrt{ab}$ (c) 0 (d) none of these
24. The condition that the curve $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ may cut each other orthogonally is
 (a) $a_1 + a = b_1 + b$ (b) $a_1 - a = b_1 - b$
 (c) $\frac{1}{a_1} - \frac{1}{a} = \frac{1}{b_1} - \frac{1}{b}$ (d) $\frac{1}{a_1} + \frac{1}{a} = \frac{1}{b_1} + \frac{1}{b}$
25. If m be the slope of a tangent to the curve $e^y = 1 + x^2$ then
 (a) $|m| > 1$ (b) $m < 1$ (c) $|m| < 1$ (d) $|m| \leq 1$
26. If at each point of the curve $y = x^3 - ax^2 + x + 1$ the tangent is inclined at an acute angle with positive direction of the x -axis then
 (a) $a > 0$ (b) $a \leq \sqrt{3}$ (c) $-\sqrt{3} \leq a \leq \sqrt{3}$ (d) none of these
27. Slope of tangent to the curve $y = x^2 - x$ at the point where the line $y = 2$ cuts the curve in the first quadrant is
 (a) 2 (b) 3 (c) -3 (d) None of these
28. The slope of the tangent of the locus $y = \cos^{-1}(\cos x)$ at $x = -\frac{\pi}{4}$ is
 (a) 1 (b) 0 (c) 2 (d) -1
29. Let the parabolas $y = x^2 + ax + b$ and $y = x(c - x)$ touch each other at the point (1, 0). Then
 (a) $a = -3$ (b) $b = 1$ (c) $c = 2$ (d) $b = -2$
30. Let $y = f(x)$ be the equation of parabola having its axis parallel to y axis, which is touched by the line $y = x$ at the point where $x = 1$. Then
 (a) $f'(0) = f'(1)$ (b) $f'(1) = 0$
 (c) $f(0) + f'(0) + f''(0) = 1$ (d) $2f(0) = 1 - f'(0)$
31. A point on the ellipse $4x^2 + 9y^2 = 36$ where the tangent is equally inclined to the axis is
 (a) $\left(\frac{9}{\sqrt{13}}, \frac{4}{\sqrt{13}}\right)$ (b) $\left(-\frac{9}{\sqrt{13}}, \frac{4}{\sqrt{13}}\right)$ (c) $\left(\frac{9}{\sqrt{13}}, -\frac{4}{\sqrt{13}}\right)$ (d) All of these
32. If $xy = a^2$ and $S = b^2x + c^2y$ where a, b and c are positive constants then the minimum value of S is
 (a) abc (b) $bc\sqrt{a}$ (c) $2abc$ (d) none of these
33. The global minimum value of $f(x) = x^4 - x^2 - 2x + 6$ is
 (a) 6 (b) 8
 (c) 4 (d) does not exist
34. The minimum value of $f(x) = 3\cos^2x + 4\sin^2x + \cos\frac{x}{2} + \sin\frac{x}{2}$ is
 (a) 4 (b) $3 + \sqrt{2}$ (c) $4 + \sqrt{2}$ (d) none of these
35. If $a > b > 0$, the minimum value of $a \sec \theta - b \tan \theta$ is
 (a) $b - a$ (b) $\sqrt{a^2 + b^2}$ (c) $\sqrt{a^2 - b^2}$ (d) $2\sqrt{a^2 - b^2}$
36. The number of values of x where the function $f(x) = \cos x + \cos(\sqrt{2}x)$ attains its maximum is
 (a) 0 (b) 1 (c) 2 (d) infinite
37. If $\theta + \phi = \frac{\pi}{3}$ then $\sin \theta \sin \phi$ has a maximum value if θ is
 (a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{4}$ (d) none of these

38. If $f(x) = a \log_e |x| + bx^2 + x$ has extremum at $x = 1$ and $x = 3$ then
 (a) $a = -\frac{3}{4}, b = -\frac{1}{8}$ (b) $a = \frac{3}{4}, b = -\frac{1}{8}$ (c) $a = -\frac{3}{4}, b = \frac{1}{8}$ (d) None of these
39. Let $f(x)$ be a function such that $f'(a) \neq 0$. Then at $x = a$, $f(x)$
 (a) cannot have a maximum
 (b) cannot have a minimum
 (c) must have neither a maximum nor a minimum
 (d) None of these
40. The function $f(x) = \sin^4 x + \cos^4 x$ increases if
 (a) $0 < x < \frac{\pi}{8}$ (b) $\frac{\pi}{4} < x < \frac{3\pi}{8}$ (c) $\frac{3\pi}{4} < x < \frac{5\pi}{8}$ (d) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$
41. If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, where $0 < x \leq 1$, then in the interval
 (a) both $f(x)$ and $g(x)$ are increasing functions
 (b) both $f(x)$ and $g(x)$ are decreasing functions
 (c) $f(x)$ is an increasing function.
 (d) $g(x)$ is an increasing function.
42. Let $h(x) = f(x) - \{f(x)\}^2 + \{f(x)\}^3$ for all real values of x . Then
 (a) h is increasing whenever $f(x)$ is increasing
 (b) h is increasing whenever $f'(x) < 0$
 (c) h is decreasing whenever f is increasing
 (d) nothing can be said in general
43. If $f(x) = \sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, then
 (a) $f(x)$ is increasing in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
 (b) $f\{f(x)\}$ is increasing in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
 (c) $\left[f(f(x))\right]$ is invertible in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
 (d) All of these
44. Let $f(x) = 2 \sin^3 x - 3 \sin^2 x + 12 \sin x + 5, 0 \leq x \leq \frac{\pi}{2}$. Then $f(x)$ is
 (a) decreasing in $\left[0, \frac{\pi}{2}\right]$
 (b) increasing in $\left[0, \frac{\pi}{2}\right]$
 (c) increasing in $\left[0, \frac{\pi}{4}\right]$ and decreasing in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$
 (d) none of these
45. Which of the following function is decreasing on $\left(0, \frac{\pi}{2}\right)$?
 (a) $\tan 2x$ (b) $\cos x$ (c) $\cos 3x$ (d) none of these
46. The tangent to the curve given by $x = e^t \cdot \cos t, y = e^t \cdot \sin t$ at $t = \frac{\pi}{4}$ makes with x -axis an angle:
 (a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

47. The equation of normal to the curve $3x^2 - y^2 = 8$ which is parallel to the line $x + 3y = 8$, is
 (a) $3x - y = 8$ (b) $3x + y + 8 = 0$ (c) $x + 3y \pm 8 = 0$ (d) $x + 3y = 0$
48. The equation of tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses x -axis, is
 (a) $x + 5y = 2$ (b) $x - 5y = 2$ (c) $5x - y = 2$ (d) $5x + y = 2$
49. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is
 (a) $[-1, \infty)$ (b) $[-2, -1]$ (c) $(-\infty, -2]$ (d) $[-1, 1]$
50. The function $f(x) = 4 \sin^3 x - 6 \sin^2 x + 12 \sin x + 100$ is strictly
 (a) Increasing in $\left(\pi, \frac{3\pi}{2}\right)$ (b) decreasing in $\left(\frac{\pi}{2}, \pi\right)$
 (c) decreasing in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (d) decreasing in $\left(0, \frac{\pi}{2}\right)$
51. The function $f(x) = \tan x - x$
 (a) always increases (b) always decreases
 (c) never increases (d) sometimes increases and sometimes decreases
52. If x is real, then the minimum value of $x^2 - 8x + 17$ is
 (a) -1 (b) 0 (c) 1 (d) 2
53. The smallest value of polynomial $x^3 - 18x^2 + 96x$ in $[0, 9]$ is
 (a) 126 (b) 0 (c) 135 (d) 160
54. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has
 (a) two points of local maximum (b) two points of local minimum
 (c) one maxima and one minima (d) no maxima or minima
55. The maximum value of $\sin x \cdot \cos x$ is
 (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) $2\sqrt{2}$
56. The maximum slope of curve $y = -x^3 + 3x^2 + 9x - 27$ is
 (a) 0 (b) 12 (c) 16 (d) 32
57. If $x + y = 8$ then the maximum value of xy is
 (a) 8 (b) 16 (c) 20 (d) 24
58. Find the slope of normal line to the curve: $x^2 - xy + 3y^2 - 5y = 0$ at $x = 2$.
 (a) $\left(\frac{13}{8}, 3\right)$ (b) $\left(\frac{3}{8}, \frac{-1}{3}\right)$ (c) $\left(\frac{-8}{3}, \frac{1}{3}\right)$ (d) $\left(\frac{8}{3}, 6\right)$
59. Find the point at which the normal to the curve: $y = 2x^2 - 2x + 7$ has a slope $\frac{1}{6}$.
 (a) $(-1, -11)$ (b) $(1, -11)$ (c) $(-1, 11)$ (d) $(-1, -9)$
60. Find the angle of intersection of the two curves $x^2y = 2$ and $xy^2 = 4$.
 (a) $\tan^{-1} \frac{3}{5}$ (b) $\tan^{-1} 3$ (c) $\tan^{-1} \frac{5}{3}$ (d) None of these
61. It is given that for the function f given by $f(x) = x^3 + bx^2 + ax$, $x \in [1, 3]$, then
 (a) $a = 11, b = -6$ (b) $a = -6, b = 11$ (c) $a = 6, b = 11$ (d) None of these
62. The slope of the curve $2y^2 = ax^2 + b$ at $(1, -1)$ is -1 . Find a, b .
 (a) $a = 2, b = 0$ (b) $a = 2, b = 1$ (c) $a = 0, b = 2$ (d) $a = 1, b = 2$

63. The angle between the curve $y^2 = x$ and $x^2 = y$ at $(1, 1)$ is
 (a) 90° (b) $\tan^{-1} \frac{3}{4}$ (c) $\tan^{-1} \frac{4}{3}$ (d) 45°
64. Find the maximum and minimum values of $f(x) = x + \sin 2x$ in the interval $[0, 2\pi]$.
 (a) Maximum value = 2π Minimum value = 0
 (b) Maximum value = 0 Minimum value = 2π
 (c) Maximum value = 0 Minimum value = 0
 (d) None of these
65. If the curve $ay + x^2 = 7$ and $x^3 = y$, cut orthogonally at $(1, 1)$, then the value of a is
 (a) 1 (b) 0 (c) -6 (d) 6
66. The total revenue received from the sale of x units of a product is given by: $R(x) = 5x^3 - 4x^2$, find the marginal revenue of $x = 20$.
 (a) 5860 (b) 5840 (c) 5000 (d) 5600
67. Find the equations of a tangent to the curve $x^3 - 2x^2y + xy^2 = 1$ at $(1, -1)$.
 (a) $2x - y = 3$ (b) $2x = 3$ (c) $2x + y = 3$ (d) $2x + y = -3$
68. The point on the curve $y = 12x - x^2$ where the slope of the tangent is zero will be
 (a) $(3, 9)$ (b) $(2, 16)$ (c) $(6, 36)$ (d) None of these
69. Find the intervals in which $f(x) = -x^2 - 2x + 15$ is increasing or decreasing
 (a) Increasing $(-\infty, -1)$ Decreasing $(-1, \infty)$
 (b) Increasing $(\infty, -2)$ Decreasing $(0, \infty)$
 (c) Increasing $(-\infty, -4)$ (d) None of these
 Decreasing $(-4, \infty)$
70. The maximum value of slope of the curve $y = -x^3 + 3x^2 + 12x - 5$ is
 (a) 15 (b) 12 (c) 9 (d) 0
71. It is given that at $x = 1$, the function $f(x) = x^4 - 62x^2 + ax + a$ attains its maximum value, on the interval $[0, 2]$. The value of a is
 (a) 20 (b) -120 (c) 120 (d) 52

Answers

- | | | | | | |
|---------|---------|---------|---------|---------|---------|
| 1. (d) | 2. (a) | 3. (b) | 4. (b) | 5. (d) | 6. (c) |
| 7. (c) | 8. (a) | 9. (a) | 10. (b) | 11. (b) | 12. (b) |
| 13. (b) | 14. (b) | 15. (b) | 16. (c) | 17. (b) | 18. (b) |
| 19. (b) | 20. (c) | 21. (c) | 22. (b) | 23. (b) | 24. (c) |
| 25. (d) | 26. (c) | 27. (b) | 28. (a) | 29. (a) | 30. (d) |
| 31. (d) | 32. (c) | 33. (c) | 34. (c) | 35. (c) | 36. (b) |
| 37. (a) | 38. (a) | 39. (d) | 40. (b) | 41. (c) | 42. (a) |
| 43. (d) | 44. (b) | 45. (b) | 46. (d) | 47. (c) | 48. (a) |
| 49. (b) | 50. (b) | 51. (a) | 52. (c) | 53. (b) | 54. (c) |
| 55. (b) | 56. (b) | 57. (b) | 58. (b) | 59. (c) | 60. (a) |
| 61. (a) | 62. (a) | 63. (b) | 64. (a) | 65. (d) | 66. (b) |
| 67. (a) | 68. (c) | 69. (a) | 70. (a) | 71. (c) | |

CASE-BASED QUESTIONS

Choose the correct option in the following questions.

1. Read the following and answer any four questions from (i) to (v).



The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

Based on the above informations answer the following:

- (i) The rate of growth of the plant with respect to sunlight is

(a) $4x - \frac{1}{2}x^2$ (b) $4 - x$ (c) $x - 4$ (d) $x - \frac{1}{2}x^2$

- (ii) What is the number of days it will take for the plant to grow to the maximum height?

(a) 4 (b) 6 (c) 7 (d) 10

- (iii) What is the maximum height of the plant?

(a) 12 cm (b) 10 cm (c) 8 cm (d) 6 cm

- (iv) What will be the height of the plant after 2 days?

(a) 4 cm (b) 6 cm (c) 8 cm (d) 10 cm

- (v) If the height of the plant is $\frac{7}{2}$ cm, the number of days it has been exposed to the sunlight is

(a) 2 cm (b) 3 cm (c) 4 cm (d) 1 cm

Sol. (i) We have,

$$\begin{aligned} \text{the rate of growth} &= \frac{dy}{dx} \\ &= \frac{d\left(4x - \frac{1}{2}x^2\right)}{dx} \\ &= 4 - x \end{aligned}$$

\therefore Option (b) is correct.

- (ii) For height to be maximum or minimum

$$\begin{aligned} \frac{dy}{dx} = 0 &\Rightarrow 4 - x = 0 \Rightarrow x = 4 \\ \therefore \frac{d^2y}{dx^2} &= -1 < 0 \Rightarrow y \text{ will be maximum when } x = 4 \end{aligned}$$

\therefore Number of required days = 4

\therefore Option (a) is correct.

- (iii) We have, $y = 4x - \frac{1}{2}x^2$

\therefore When $x = 4$ the height of the plant will be maximum which is

$$y = 4 \times 4 - \frac{1}{2} \times (4)^2 = 16 - 8 = 8 \text{ cm}$$

\therefore Option (c) is correct.

(iv) Height of the plant after 2 days is given by

$$y = 4 \times 2 - \frac{1}{2} \times (2)^2 = 8 - 2 = 6 \text{ cm}$$

\therefore Option (b) is correct.

(v) Given height of the plant, $y = \frac{7}{2}$

$$\therefore \frac{7}{2} = 4x - \frac{1}{2}x^2 \Rightarrow 7 = 8x - x^2$$

$$\Rightarrow x^2 - 8x + 7 = 0 \Rightarrow x^2 - 7x - x + 7 = 0$$

$$\Rightarrow x(x - 7) - 1(x - 7) = 0$$

$$\Rightarrow (x - 1)(x - 7) = 0$$

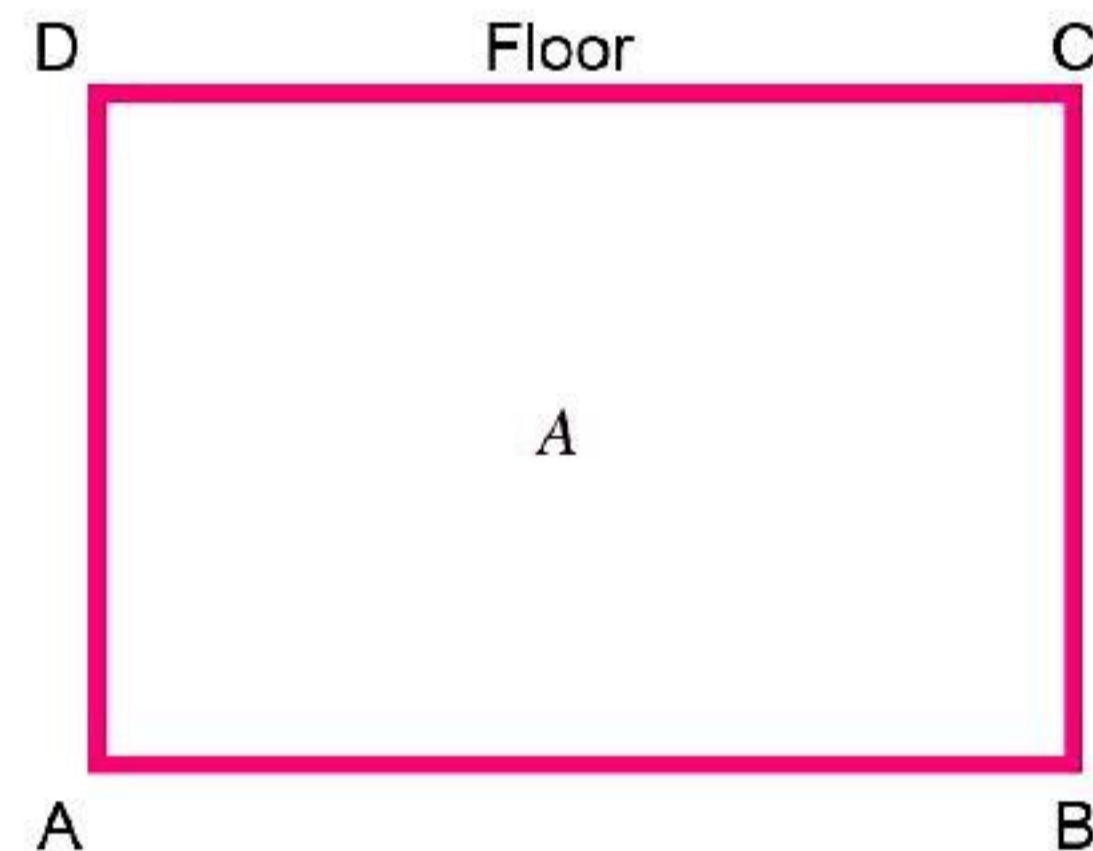
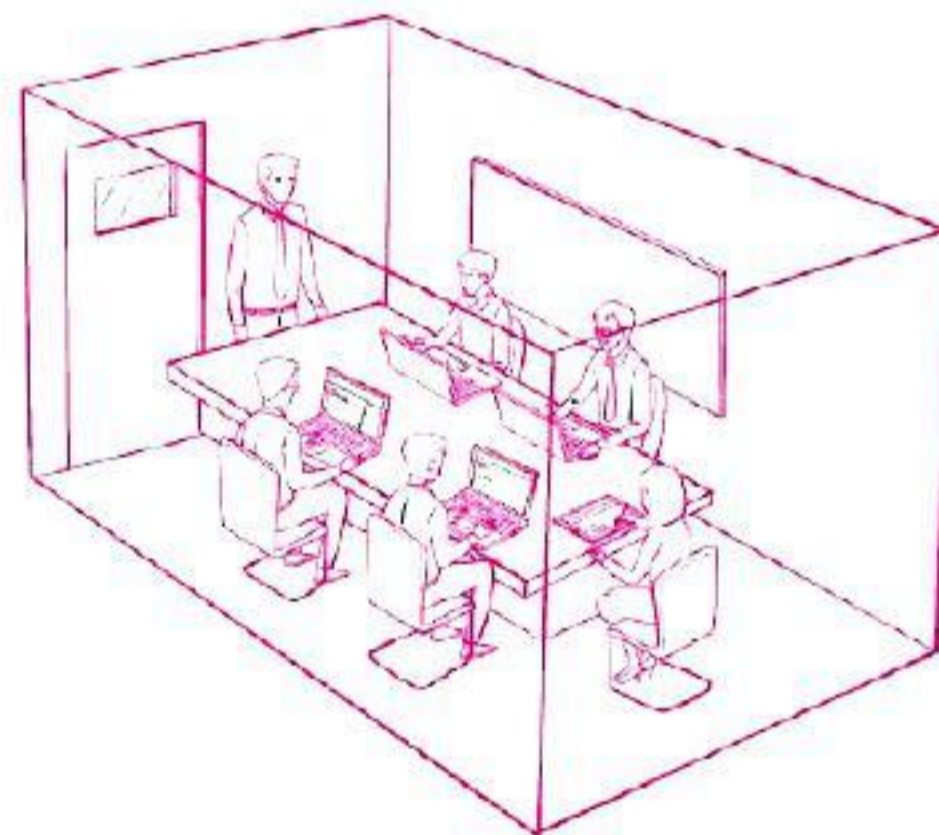
$$\Rightarrow x = 1 \text{ or } x = 7$$

$$x \neq 7$$

$$\therefore x = 1 \text{ cm}$$

\therefore Option (d) is correct.

2. A rectangular hall is to be developed for a meeting of farmers in an agriculture college to aware them for new technique in cultivation. It is given that the floor has a fixed perimeter P as shown below.



Based on the above information answer the following.

(i) If x and y represents the length and breadth of the rectangular region, then the relation between the variables is

- (a) $x + y = P$ (b) $x^2 + y^2 = P$ (c) $2(x + y) = P$ (d) $x + 2y = P$

(ii) The area of the rectangular region 'A' expressed as a function of x is

- (a) $\frac{1}{2}(P + x^2)$ (b) $\frac{1}{2}(Px - 2x^2)$ (c) $\frac{1}{2}(Px + 2x^2)$ (d) $Px - 2x^2$

(iii) Principal of agriculture college is interested in maximizing the area of floor 'A'. For this to happen the value of x should be

- (a) P (b) $\frac{P}{2}$ (c) $\frac{2P}{3}$ (d) $\frac{P}{4}$

(iv) To maximizing the area of floor 'A'. For this to happen the value of y should be

- (a) $\frac{P}{2}$ (b) $\frac{P}{4}$ (c) $\frac{P}{3}$ (d) $\frac{2P}{5}$

(v) The maximum value of area of floor 'A' is

(a) $\frac{P^2}{8}$

(b) $\frac{2P}{9}$

(c) $\frac{P}{10}$

(d) $\frac{P^2}{16}$

Sol. (i) Perimeter of rectangle $ABCD = 2(l + b) = 2(x + y)$

Option (c) is correct.

$$(ii) \because A = x.y \quad \left[\begin{array}{l} \text{Given} \\ \text{Perimeter} = P \\ \Rightarrow 2(x + y) = P \\ \Rightarrow y = \frac{P}{2} - x \\ \quad = \frac{P - 2x}{2} \end{array} \right]$$

$$= x \cdot \frac{P - 2x}{2}$$

$$= \frac{Px - 2x^2}{2}$$

Option (b) is correct.

$$(iii) \because A = \frac{Px - 2x^2}{2}$$

$$\Rightarrow \frac{dA}{dx} = \frac{P - 4x}{2}$$

For maximum or minimum value of x

$$\frac{dA}{dx} = 0 \Rightarrow \frac{P - 4x}{2} = 0$$

$$\Rightarrow P - 4x = 0 \Rightarrow x = \frac{P}{4}$$

$$\text{Also, } \left. \frac{d^2A}{dx^2} \right|_{x=\frac{P}{4}} = -2 \text{ (-ve)}$$

Option (d) is correct.

$$(iv) \text{ Putting } x = \frac{P}{4} \text{ in } y = \frac{P - 2x}{2}$$

$$\Rightarrow y = \frac{P - 2 \times \frac{P}{4}}{2}$$

$$= \frac{4P - 2P}{8} = \frac{2P}{8} = \frac{P}{4}$$

Option (b) is correct.

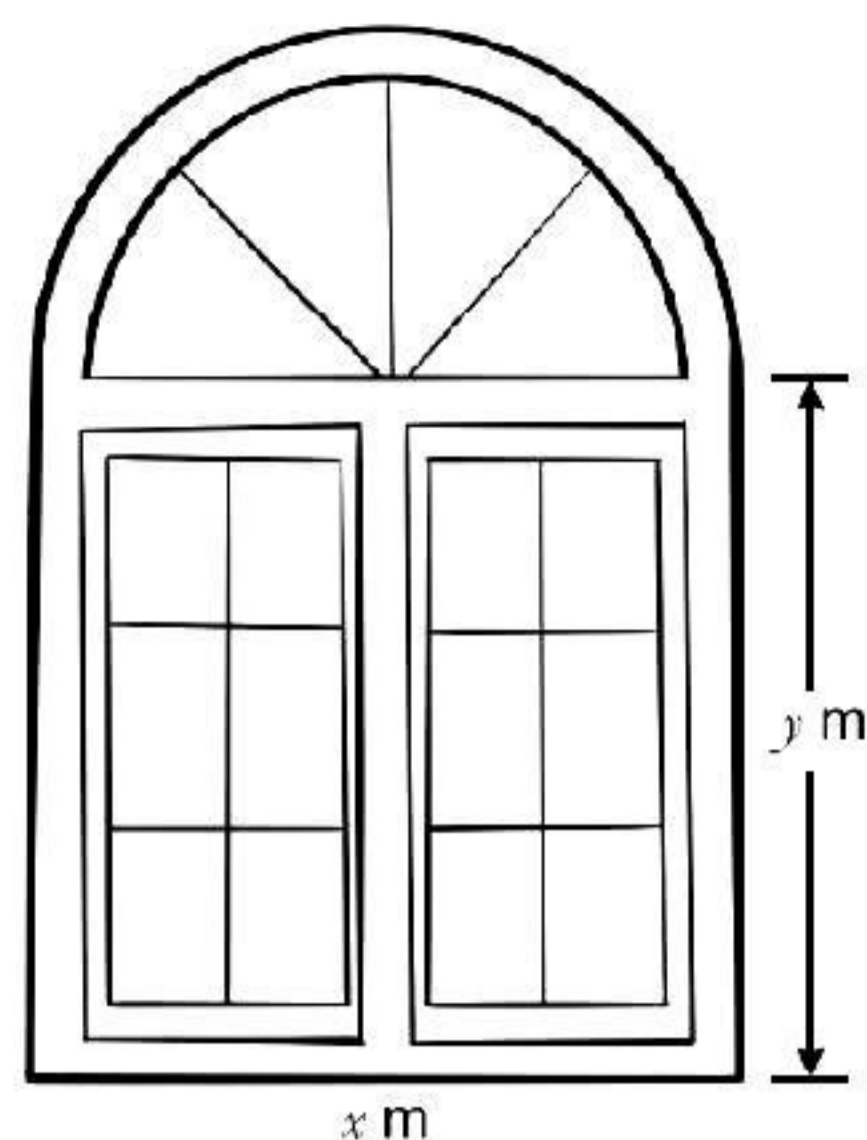
(v) Since $A = \text{length} \times \text{breadth}$

$$= x \times y$$

$$= \frac{P}{4} \times \frac{P}{4} = \frac{P^2}{16}$$

Option (d) is correct.

3. Dr. Ritam residing in Delhi went to see an apartment of 3 BHK in Noida. The window of the house was in the form of a rectangle surmounted by a semicircular opening having a perimeter of the window 10 m as shown in figure.



Based on above information answer the following.

(i) If x and y represents the length and breadth of the rectangular region, then the relation between the variables is

(a) $x + y + \frac{x}{2} = 10$ (b) $x + 2y + \frac{x}{2} = 10$ (c) $x + 2y + \pi \frac{x}{2} = 10$ (d) $2x + 2y = 10$

(ii) The area of the window (A) expressed as a function of x is

(a) $A = x - \frac{\pi x^3}{8} - \frac{x^2}{2}$ (b) $A = 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}$
 (c) $A = 5x - \frac{x^2}{2} - \frac{3x^2}{8}$ (d) $A = 5x + \frac{x^2}{2} + \frac{\pi x^2}{8}$

(iii) Dr. Ritam is interested in maximising the area of the whole window. For this to happen the value of x should be

(a) $\frac{20}{\pi}$ (b) $\frac{20}{4 - \pi}$ (c) $\frac{20}{2 + \pi}$ (d) $\frac{20}{4 + \pi}$

(iv) For maximum value of A, the breadth of rectangular part of window is

(a) $\frac{20}{4 + \pi}$ (b) $\frac{20}{\pi}$ (c) $\frac{10}{4 + \pi}$ (d) $\frac{5}{2 + \pi}$

(v) The maximum area of window is

(a) $\frac{100}{(4 + \pi)^2}$ sq.m (b) $\frac{10\pi}{(4 + \pi)^2}$ sq.m (c) $\frac{800}{(4 + \pi)^2}$ sq.m (d) $\frac{200 + 50\pi}{(4 + \pi)^2}$ sq.m

Sol. (i) Since perimeter of window = $x + y + y$ + perimeter of semicircle

$$= x + 2y + \frac{1}{2} \times 2\pi \times \frac{x}{2} \quad \left[\text{Here radius of semicircle is } \frac{x}{2} \right]$$

$$= x + 2y + \frac{\pi x}{2}$$

Option (c) is correct.

(ii) $A = x \times y + \frac{1}{2} \pi \left(\frac{x}{2} \right)^2$

$$= x \times y + \frac{\pi x^2}{8} = x \left(5 - \frac{x}{2} - \frac{\pi x}{4} \right) + \frac{\pi x^2}{8}$$

$$= 5x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8} = 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}$$

Option (b) is correct.

$$\left[\begin{array}{l} \because 10 = x + 2y + \frac{\pi x}{2} \\ \Rightarrow y = \left(5 - \frac{x}{2} - \frac{\pi x}{4} \right) \end{array} \right]$$

(iii) For maximum value of A

$$\frac{dA}{dx} = 0$$

$$\begin{aligned} \Rightarrow 5 - x - \frac{\pi x}{4} &= 0 & \Rightarrow x + \frac{\pi x}{4} &= 5 \\ \Rightarrow 4x + \pi x &= 20 & \Rightarrow x(4 + \pi) &= 20 \\ \Rightarrow x &= \frac{20}{4 + \pi} \end{aligned}$$

Option (d) is correct.

$$\begin{aligned} (iv) \because y &= 5 - \frac{x}{2} - \frac{\pi x}{4} \\ &= 5 - \left(\frac{2x + \pi x}{4} \right) = 5 - x \left(\frac{2 + \pi}{4} \right) \\ &= 5 - \frac{20}{4 + \pi} \cdot \frac{2 + \pi}{4} = \frac{20(4 + \pi) - 20(2 + \pi)}{4(4 + \pi)} \\ &= \frac{80 + 20\pi - 40 - 20\pi}{4(4 + \pi)} = \frac{40}{4(4 + \pi)} \\ &= \frac{10}{4 + \pi} \end{aligned}$$

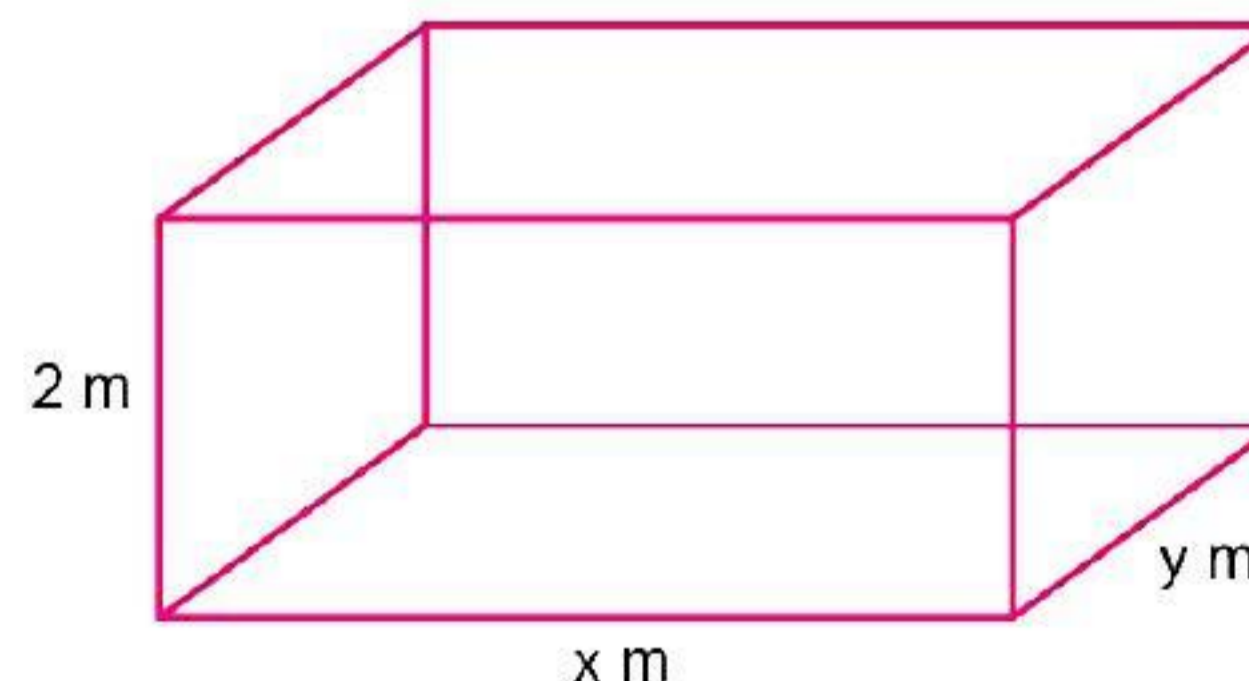
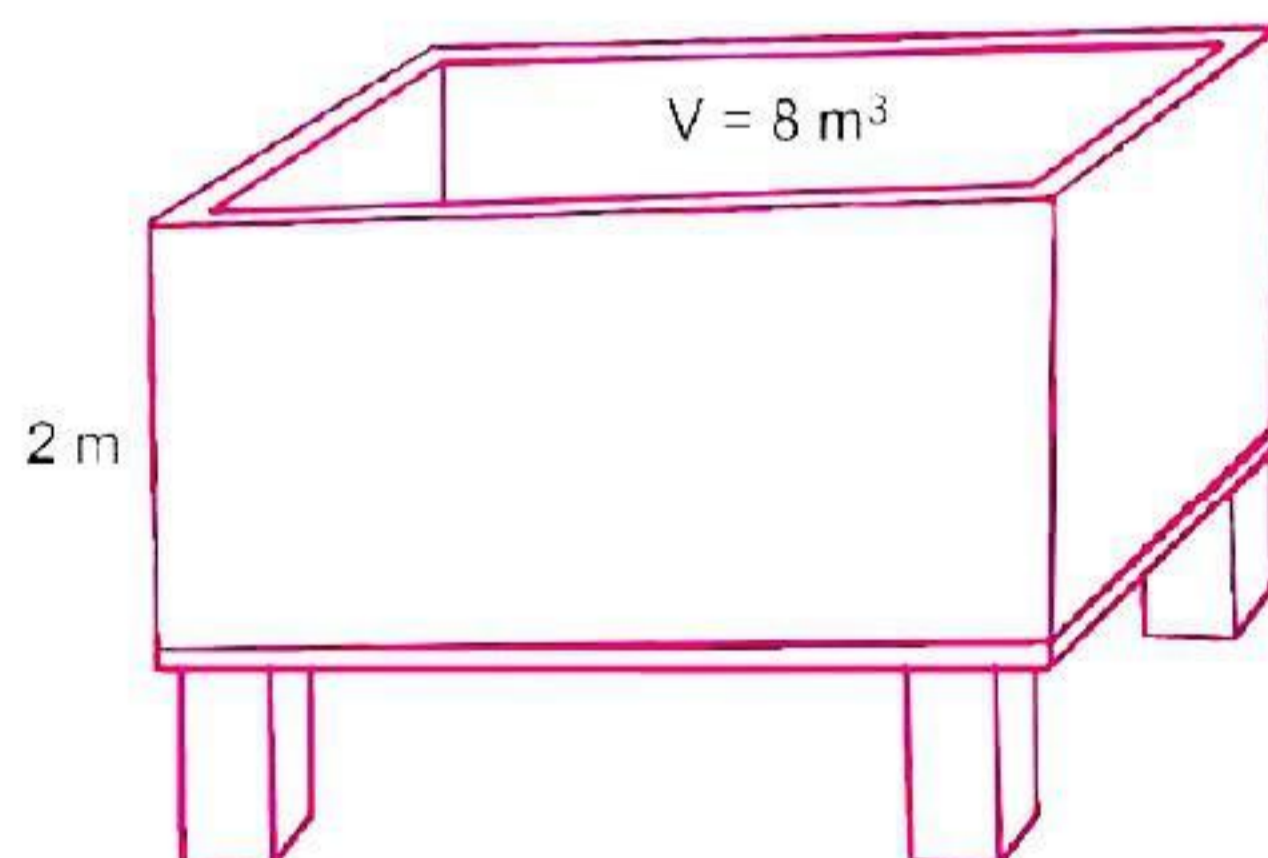
Option (c) is correct.

(v) Area of window = ar(rectangular part) + ar(semi circular part)

$$\begin{aligned} &= \frac{20}{4 + \pi} \cdot \frac{10}{4 + \pi} + \frac{1}{2} \pi \left(\frac{x}{2} \right)^2 \\ &= \frac{200}{(4 + \pi)^2} + \frac{\pi x^2}{8} = \frac{200}{(4 + \pi)^2} + \frac{\pi}{8} \left(\frac{20}{4 + \pi} \right)^2 \\ &= \frac{200}{(4 + \pi)^2} + \frac{400\pi}{8(4 + \pi)^2} = \frac{200 + 50\pi}{(4 + \pi)^2} \end{aligned}$$

Option (d) is correct.

4. On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m³ as shown below:



Based on the above information answer the following questions:

(i) If x and y represent the length and breadth of its rectangular base, then the relation between the variables is

- (a) $x + y = 8$ (b) $x \cdot y = 4$ (c) $x + y = 4$ (d) $\frac{x}{y} = 4$

(ii) If construction of tank cost ₹70 per sq. metre for the base and ₹45 per square metre for sides, then making cost 'C' expressed as a function of x is

$$(a) C = 80 + 80\left(x + \frac{4}{x}\right)$$

$$(b) C = 280x + 280\left(x + \frac{4}{x}\right)$$

$$(c) C = 280 + 180\left(x + \frac{4}{x}\right)$$

$$(d) C = 70x + 70\left(x + \frac{x}{4}\right)$$

(iii) The owner of a construction agency is interested in minimizing the cost 'C' of whole tank, for this to happen the value of x should be

$$(a) 4 \text{ m}$$

$$(b) 3 \text{ m}$$

$$(c) 1 \text{ m}$$

$$(d) 2 \text{ m}$$

(iv) For minimum cost 'C' the value of y should be

$$(a) 1 \text{ m}$$

$$(b) 3 \text{ m}$$

$$(c) 2 \text{ m}$$

$$(d) 4 \text{ m}$$

(v) The Pradhan of village wants to know minimum cost. The minimum cost is

$$(a) ₹2000$$

$$(b) ₹4000$$

$$(c) ₹11,000$$

$$(d) ₹1000$$

Sol. (i) Volume of tank = length × breadth × height (Depth)

$$8 = x \cdot y \cdot 2$$

$$\Rightarrow 2xy = 8 \quad \Rightarrow xy = 4$$

Option (b) is correct.

(ii) Since 'C' is cost of making tank

$$\therefore C = 70xy + 45 \times 2(2x + 2y)$$

$$\Rightarrow C = 70xy + 90(2x + 2y)$$

$$\Rightarrow C = 70xy + 180(x + y)$$

$$\Rightarrow C = 70x \times \frac{4}{x} + 180\left(x + \frac{4}{x}\right)$$

$$\Rightarrow C = 280 + 180\left(x + \frac{4}{x}\right)$$

$$\left[\begin{array}{l} \because 2 \cdot x \cdot y = 8 \\ \Rightarrow y = \frac{8}{2x} \\ \Rightarrow y = \frac{4}{x} \end{array} \right]$$

Option (c) is correct.

(iii) For maximum or minimum

$$\frac{dC}{dx} = 0$$

$$\frac{d}{dx}\left(280 + 180\left(x + \frac{4}{x}\right)\right) = 0 \quad \Rightarrow 180\left(1 + 4\left(-\frac{1}{x^2}\right)\right) = 0$$

$$\Rightarrow 180\left(1 - \frac{4}{x^2}\right) = 0 \quad \Rightarrow 1 - \frac{4}{x^2} = 0$$

$$\Rightarrow \frac{4}{x^2} = 1 \quad \Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$$\Rightarrow x = 2 \text{ (length can never be negative)}$$

$$\text{Now, } \frac{d^2C}{dx^2} = 180\left(+\frac{8}{x^3}\right)$$

$$\Rightarrow \left.\frac{d^2C}{dx^2}\right|_{x=2} = 180 \times \frac{8}{8} = 180 = +ve$$

Hence, to minimize C , $x = 2\text{m}$

Option (d) is correct.

$$(iv) \therefore \begin{aligned} xy &= 4 \\ \Rightarrow y &= \frac{4}{x} \end{aligned} \quad \Rightarrow \quad y = \frac{4}{2} \quad \Rightarrow \quad y = 2\text{ m}$$

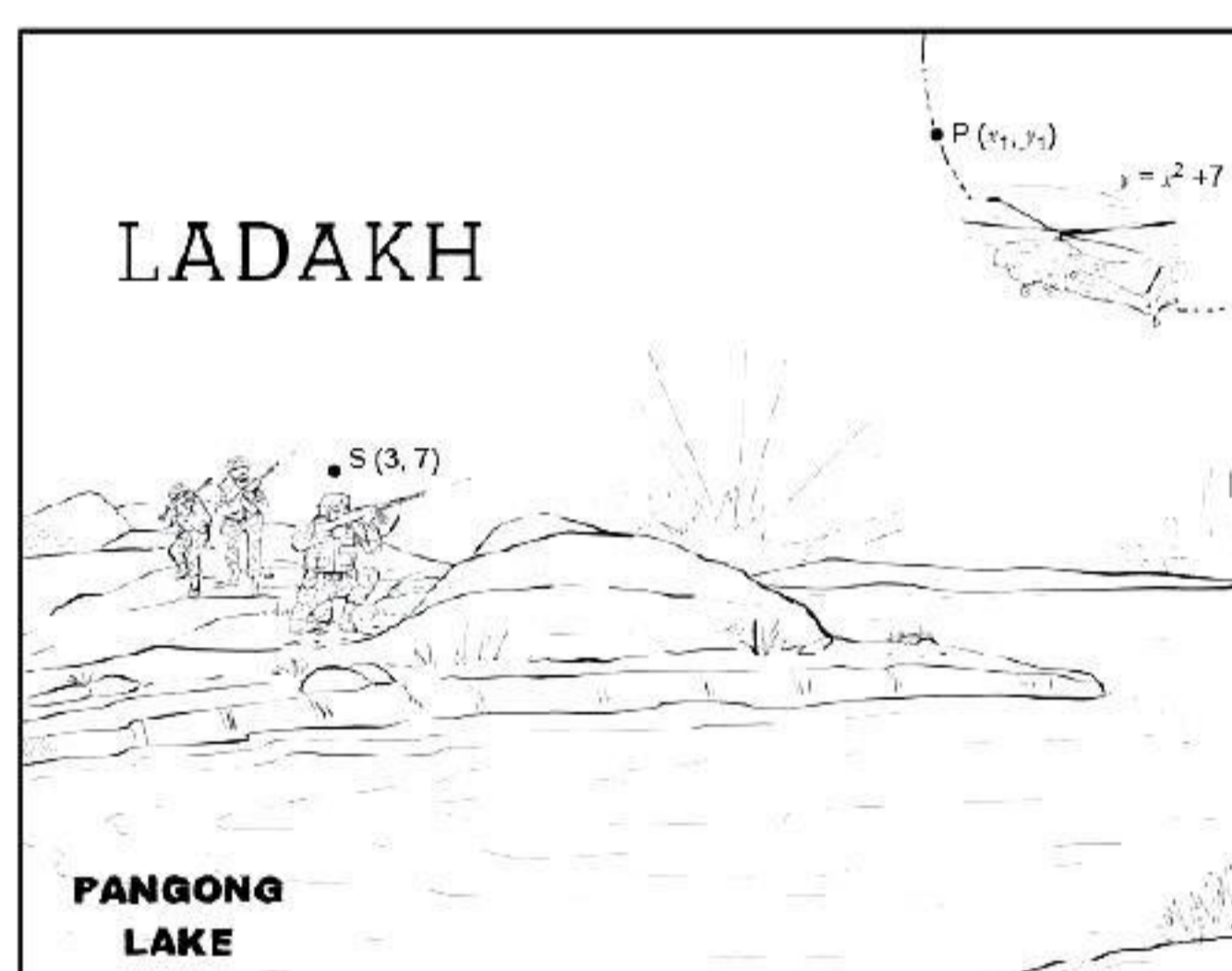
Option (c) is correct.

$$(v) \therefore C = 280 + 180\left(x + \frac{4}{x}\right) = 280 + 180(2 + 2) \\ = 280 + 180 \times 4 = 280 + 720 = ₹1000$$

Option (d) is correct.

5. These days chinese and Indian troops are engaged in aggressive melee, face-offs skirmishes at locations near the disputed Pangong Lake in Ladakh.

One day a helicopter of enemy is flying along the curve represented by $y = x^2 + 7$. A soldier placed at $(3, 7)$ wants to shoot down the helicopter when it is nearest to him.



Based on above information answer the following questions:

- (i) If (x_1, y_1) represents the position of helicopter on the curve $y = x^2 + 7$, when the distance D from soldier placed at $S(3, 7)$ is minimum, then the relation between x_1, y_1 is

(a) $x_1 = y_1^2 + 7$ (b) $y_1 = x_1^2 + 7$ (c) $y_1 + x_1^2 = 7$ (d) $y_1^2 + x_1 = 7$

- (ii) The distance 'D' expressed as a function of x_1 is

(a) $D = x_1^2 - 6x_1 + x_1^4$ (b) $D = x_1^2 - 6x_1 + 9 + x_1^4$ (c) $D^2 = x_1^2 - 6x_1 + 9 + x_1^4$ (d) $D^2 = x_1^2 + 6x_1 - 9 + x_1^4$

- (iii) The soldier at S wants to know when the enemy helicopter is nearest to soldier, then the value y_1 should be

(a) 4 (b) 3 (c) 8 (d) 5

- (iv) When the enemy helicopter is nearest to soldier, then the value of D should be

(a) 4 units (b) 5 units (c) $\sqrt{5}$ units (d) $\sqrt{7}$ units

- (v) The nearest position of helicopter from soldier is

(a) $(1, \sqrt{5})$ (b) $(1, 8)$ (c) $(1, 7)$ (d) $(1, \sqrt{7})$

Sol. (i) $\therefore (x_1, y_1)$ lie on curve $y = x^2 + 7$
 $\Rightarrow (x_1, y_1)$ satisfy the equation of curve
 $\Rightarrow y_1 = x_1^2 + 7$

Option (b) is correct.

- (ii) Here, since soldier is at $(3, 7)$

$$D = \sqrt{(x_1 - 3)^2 + (y_1 - 7)^2} \Rightarrow D^2 = (x_1 - 3)^2 + (y_1 - 7)^2$$

$$\because (x_1, y_1) \text{ lie on curve } y = x^2 + 7$$

$$\Rightarrow y_1 = x_1^2 + 7$$

$$\therefore D^2 = (x_1 - 3)^2 + (x_1^2 + 7 - 7)^2$$

$$D^2 = x_1^2 - 6x_1 + 9 + x_1^4$$

Option (c) is correct.

(iii) We have $D^2 = x_1^2 - 6x_1 + 9 + x_1^4$

$$\therefore \frac{d(D^2)}{dx_1} = 2x_1 - 6 + 4x_1^3$$

For minimum value of D i.e. D^2

$$\frac{d(D^2)}{dx_1} = 0$$

$$\Rightarrow 2x_1 + 4x_1^3 - 6 = 0$$

$$\Rightarrow 4x_1^2(x_1 - 1) + 4x_1(x_1 - 1) + 6(x_1 - 1) = 0$$

$$\Rightarrow (x_1 - 1)(4x_1^2 + 4x_1 + 6) = 0$$

$$\Rightarrow x_1 - 1 = 0$$

$$[\because 4x_1^2 + 4x_1 + 6 = 0, \text{ have no real roots i.e., real value of } x_1 \text{ is not possible.}]$$

$$\Rightarrow x_1 = 1$$

\Rightarrow For minimum points, (distance) or nearest distance $x_1 = 1$

$$\text{Also } \frac{d^2(D^2)}{dx_1^2} = 2 + 12x_1^2 \Rightarrow \left. \frac{d^2(D^2)}{dx_1^2} \right|_{x=1} = +ve$$

Since (x_1, y_1) lie on curve $y = x^2 + 7$

$$\Rightarrow y_1 = x_1^2 + 7$$

$$\Rightarrow y_1 = 1^2 + 7 \quad [\text{For nearest value of } D, x_1 = 1]$$

$$= 1 + 7 = 8$$

Option (c) is correct.

(iv) For nearest distance D i.e. minimum value of D , $x_1 = 1$

$$\because D^2 = x_1^2 - 6x_1 + 9 + x_1^4$$

$$\Rightarrow D^2 = 1^2 - 6(1) + 9 + (1)^4$$

$$\Rightarrow D^2 = 1 - 6 + 9 + 1$$

$$\Rightarrow D^2 = 5$$

$$\Rightarrow D = \sqrt{5} \text{ units}$$

Option (c) is correct.

(v) For minimum value of D

$$x_1 = 1 \text{ and } y_1 = 8$$

$$\therefore \text{Nearest position of helicopter is } (1, 8).$$

Option (b) is correct.

ASSERTION-REASON QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false and R is also false.

1. **Assertion (A):** The rate of change of area of a circle with respect to its radius r when $r = 6$ cm is $12\pi \text{ cm}^2/\text{cm}$.

Reason (R): Rate of change of area of a circle with respect to its radius r is $\frac{dA}{dr}$, where A is the area of the circle.

2. **Assertion (A):** $f(x) = \tan x - x$ always increases.

Reason (R): Any function $y = f(x)$ is increasing if $\frac{dy}{dx} > 0$.

3. **Assertion (A):** $f(x) = x^4$ is decreasing in the interval $(0, \infty)$.

Reason (R): Any function $y = f(x)$ is decreasing if $\frac{dy}{dx} < 0$.

4. **Assertion (A):** The slope of the tangent to the curve $y = x^3$ where it cuts x -axis, is 0.

Reason (R): Slope of tangent to the curve $y = f(x)$ at point (x_0, y_0) is $\frac{dy}{dx}$ at (x_0, y_0) .

Answers

1. (a) 2. (a) 3. (d) 4. (a)

HINTS/SOLUTIONS OF SELECTED MCQS

1. We have, $f(x) = x^2 e^{-x}$

$$\rightarrow f'(x) = -x^2 e^{-x} + 2x e^{-x} = x e^{-x} (2 - x)$$

for $f(x)$ to be strictly increasing, $f'(x) > 0$

$$\Rightarrow x e^{-x} (2 - x) > 0 \Rightarrow x (2 - x) > 0$$

$$\Rightarrow x(x - 2) < 0 \Rightarrow 0 < x < 2$$

$$\therefore x \in (0, 2)$$

Option (d) is correct.

2. Let (x_1, y_1) be the point on the given curve $3y = 6x - 5x^3$ at which the normal passes through the origin. Then we have $\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 2 - 5x_1^2$. Again the equation of the normal at (x_1, y_1) passing through the origin gives $2 - 5x_1^2 = \frac{-x_1}{y_1} = \frac{-3}{6 - 5x_1^2}$. Since $x_1 = 1$ satisfies the equation, therefore, correct answer is (a).

3. We have, $f(x) = x^x$

$$\text{Let } y = x^x$$

and $\log y = x \log x$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1 \quad [\text{Differentiate both sides}]$$

$$\Rightarrow \frac{dy}{dx} = (1 + \log x) \cdot x^x$$

$$\therefore \frac{dy}{dx} = 0 \Rightarrow (1 + \log x) \cdot x^x = 0 \quad [\because x^x \neq 0]$$

$$\Rightarrow \log x = -1 \Rightarrow \log x = \log e^{-1}$$

$$\Rightarrow x = e^{-1} \Rightarrow x = \frac{1}{e}$$

Hence, $f(x)$ has a stationary point at $x = \frac{1}{e}$.

Option (b) is correct.

4. From first equation of the curve, we have $3x^2 - 3y^2 - 6xy \frac{dy}{dx} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 - y^2}{2xy} = (m_1) \text{ say and second equation of the curve gives}$$

$$6xy + 3x^2 \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{-2xy}{x^2 - y^2} = (m_2) \text{ say}$$

Since $m_1 \cdot m_2 = -1$. Therefore, the cut at right angles

Option (b) is correct.

6. $\frac{dy}{dx} = \cos x$. Therefore, slope of normal $= \left(\frac{-1}{\cos x} \right)_{x=0} = -1$.

Hence, the equation of normal is $y - 0 = -1(x - 0)$ or $x + y = 0$.

Therefore, correct answer is (c).

7. Here, $y = e^{2x}$

$$\Rightarrow \frac{dy}{dx} = 2e^{2x} \quad \Rightarrow \quad \left. \frac{dy}{dx} \right|_{(0,1)} = 2 \times e^0 = 2 \times 1 = 2$$

\therefore Slope of tangent to the curve $y = e^{2x}$ at $(0, 1) = 2$

\Rightarrow Equation of tangent to the curve $y = e^{2x}$ at $(0, 1)$ is

$$\frac{y-1}{x-0} = 2 \quad \Rightarrow \quad y = 2x + 1$$

For meeting point with x -axis, putting $y = 0$, we get

$$0 = 2x + 1 \quad \Rightarrow \quad x = -\frac{1}{2}$$

Hence required point is $\left(-\frac{1}{2}, 0\right)$.

Option (c) is correct.

11. Let $f(x) = x^{1/x}$

$$\therefore f'(x) = x^{1/x} \left(\frac{1 - \log x}{x^2} \right)$$

For critical point

$$f'(x) = 0 \quad \Rightarrow \quad 1 - \log x = 0$$

$$\Rightarrow \log x = 1 \quad \Rightarrow \quad x = e \text{ (critical point)}$$

Now, $f''(x) \big|_{x=e} = -ve$

$$\Rightarrow x = e \text{ is maxima of } f(x) = x^{1/x}.$$

$$\Rightarrow e^{1/e} \text{ is greatest value of } f(x) = x^{1/x}.$$

$$\text{Also, } f(4) = 4^{1/4} = 2^{2 \times 1/4} = 2^{1/2} = f(2)$$

[\because By Rolle's theorem $f(a) = f(b) \therefore \exists c(a, b)$ s.t. $f'(c) = 0$]

And $1 < 2 < e < 3 < 4 < 5 < 6 < 7$

Hence, $3^{1/3}$ is the greatest number.

Option (b) is correct.

13. Given, $y = x^2 + ax + 25 \Rightarrow \frac{dy}{dx} = 2x + a \dots(i)$

The curve (i) touches the x -axis implies that x -axis is tangent to curve at meeting point.

$$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow 2x + a = 0$$

$$\Rightarrow x = -\frac{a}{2}$$

\Rightarrow The co-ordinate of meeting point are $\left(-\frac{a}{2}, 0\right)$, therefore it satisfies the curve (i)

$$\Rightarrow \left(-\frac{a}{2}\right)^2 + a\left(-\frac{a}{2}\right) + 25 = 0$$

$$\Rightarrow \frac{a^2}{4} - \frac{a^2}{2} + 25 = 0 \Rightarrow -a^2 + 100 = 0$$

$$\Rightarrow a = \pm 10$$

Option (b) is correct.

16. $f(x) = 2x + \cos x$

$$f'(x) = 2 - \sin x > 0 \forall x \in R$$

$$\Rightarrow f(x) \text{ is an increasing function}$$

Option (c) is correct.

17. $f(x) = 2 \sin 3x + 3 \cos 3x$

$$\Rightarrow f'(x) = 6 \cos 3x - 9 \sin 3x$$

$$\Rightarrow f''(x) = -18 \sin 3x - 27 \cos 3x$$

$$\therefore f''\left(\frac{5\pi}{6}\right) = -18 \sin\left(\frac{5\pi}{2}\right) - 27 \cos\left(\frac{5\pi}{2}\right)$$

$$\Rightarrow f''\left(\frac{5\pi}{6}\right) = -18 \sin\left(2\pi + \frac{\pi}{2}\right) - 27 \cos\left(2\pi + \frac{\pi}{2}\right)$$

$$= -18 \sin \frac{\pi}{2} - 27 \cos \frac{\pi}{2} = -18 < 0$$

$$\therefore \text{At } x = \frac{5\pi}{6}, f(x) \text{ is minimum.}$$

Option (b) is correct.

18. $y = x^{1/2} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$$\therefore \frac{dy}{dx} \Big|_{(0,0)} = \infty$$

$$\Rightarrow \text{Tangent of the curve } y = x^{1/2} \text{ at } (0, 0) \text{ is parallel to } y\text{-axis.}$$

Option (b) is correct.

21. $f(x) = \sin x - ax + b$

$$\Rightarrow f'(x) = \cos x - a$$

For increasing function

$$f'(x) \geq 0$$

$$\cos x - a \geq 0 \Rightarrow \cos x \geq a$$

$$\text{i.e. } a \leq \cos x \quad a \leq \min(\cos x) = -1$$

$$\therefore a \in (-\infty, -1).$$

So option (c) is correct.

22. $f(x) = \frac{2x^2 - 1}{x^4}$

$$\begin{aligned} f'(x) &= \frac{x^4(4x) - (2x^2 - 1)(4x^3)}{(x^4)^2} = \frac{4x^5 - 8x^5 + 4x^3}{x^8} \\ &= \frac{-4x^5 + 4x^3}{x^8} = \frac{-4x^2 + 4}{x^5} \end{aligned}$$

f is decreasing if $f'(x) \leq 0$

$$\Rightarrow \frac{-4x^2 + 4}{x^5} \leq 0 \Rightarrow \frac{4x^2 - 4}{x^5} \geq 0$$

$$\Rightarrow x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow |x| \geq 1 \text{ and } x > 0$$

$$\Rightarrow x \in [1, \infty)$$

Option (b) is correct.

25. We have $e^y = 1 + x^2$

$$\Rightarrow e^y \frac{dy}{dx} = 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x}{e^y} = \frac{2x}{1 + x^2} \quad [\because e^y = 1 + x^2]$$

$$\Rightarrow m = \frac{2x}{1 + x^2} \text{ or } |m| = \frac{2|x|}{1 + |x|^2}$$

$$\text{As } 1 + |x|^2 - 2|x| = (1 - |x|)^2 \geq 0$$

$$\Rightarrow 1 + |x|^2 \geq 2|x| \quad \Rightarrow \quad 1 \geq \frac{2|x|}{1 + |x|^2} = |m|$$

$$\Rightarrow |m| \leq 1$$

Option (d) is correct.

26. We have

$$y = x^3 - ax^2 + x + 1 \Rightarrow \frac{dy}{dx} = 3x^2 - 2ax + 1$$

$$\text{From question } \frac{dy}{dx} \geq 0 \Rightarrow 3x^2 - 2ax + 1 \geq 0 \quad \forall x$$

$$\therefore D \leq 0 \Rightarrow 4a^2 - 12 \leq 0 \Rightarrow 4a^2 \leq 12 \Rightarrow a^2 \leq 3$$

$$\Rightarrow |a| \leq \sqrt{3} \Rightarrow -\sqrt{3} \leq a \leq \sqrt{3}$$

Option (c) is correct.

27. We have $y = x^2 - x \Rightarrow \frac{dy}{dx} = 2x - 1$

Slope of tangent $m = \frac{dy}{dx} = 2x - 1 \quad \dots(i)$

Since the line $y = 2$ cuts the curve $y = x^2 - x$

$$\Rightarrow 2 = x^2 - x \Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow x^2 + x - 2x - 2 = 0 \Rightarrow x(x+1) - 2(x+1) = 0$$

$$\Rightarrow (x+1)(x-2) = 0$$

$$\Rightarrow x = -1 \text{ or } 2$$

Point of intersection of the line $y = 2$ and the curve

$$y = x^2 - x \text{ are } (-1, 2), (2, 2)$$

As point $(2, 2)$ lies in first quadrant

$$\therefore \text{Slope of tangent at } (2, 2) \text{ from (i) is } m = 2 \times 2 - 1 = 3$$

Option (b) is correct.

29. $y = \cos^{-1}(\cos x)$

$$\Rightarrow \cos y = \cos x$$

$$\Rightarrow -\sin y \frac{dy}{dx} = -\sin x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin x}{\sin y}$$

$$\left. \frac{dy}{dx} \right|_{x=-\frac{\pi}{4}} = \frac{\sin\left(-\frac{\pi}{4}\right)}{\sin\left(-\frac{\pi}{4}\right)} = \frac{-\frac{1}{\sqrt{2}}}{-1/\sqrt{2}} = 1$$

Option (a) is correct.

32. $S = b^2x + c^2y$ and $xy = a^2$

$$\Rightarrow S = b^2x + \frac{a^2c^2}{x}$$

$$\therefore \frac{dS}{dx} = b^2 - \frac{c^2a^2}{x^2} = 0$$

$$\rightarrow b^2 - \frac{c^2a^2}{x^2} = 0 \rightarrow x^2 = \frac{c^2a^2}{b^2} \rightarrow x = \pm \frac{ca}{b}$$

$$\frac{d^2S}{dx^2} = \frac{2c^2a^2}{x^3}$$

$$\therefore \left. \frac{d^2S}{dx^2} \right|_{x=\frac{ca}{b}} = \frac{2c^2a^2}{\frac{c^3a^3}{b^3}} = \frac{2b^3}{ca} > 0$$

$$\therefore \text{Minimum value of } S = b^2 \times \frac{ca}{b} + \frac{c^2a^2}{ca/b} = abc + abc = 2abc$$

Option (c) is correct.

33. $f(x) = x^4 - x^2 - 2x + 6$

$$\Rightarrow f'(x) = 4x^3 - 2x - 2$$

$$\therefore f'(x) = 0 \Rightarrow 4x^3 - 2x - 2 = 0$$

$$\Rightarrow 2x^3 - x - 1 = 0$$

$$\Rightarrow 2x^3 - 2x^2 + 2x^2 - x - 1 = 0$$

$$\Rightarrow 2x^3 - 2x^2 + 2x^2 - 2x + x - 1 = 0$$

$$\Rightarrow 2x^2(x-1) + 2x(x-1) + 1(x-1) = 0$$

$$\Rightarrow (x-1)(2x^2 + 2x + 1) = 0$$

$$\Rightarrow x = 1 \text{ as } 2x^2 + 2x + 1 \neq 0 \text{ for any real } x.$$

$$\therefore f''(x) = 12x^2 - 2$$

$$\therefore f''(1) = 12 - 2 = 10 > 0$$

$$\text{Global minimum value of } f(x) = 1^4 - 1^2 - 2 \times 1 + 6 = 4.$$

Option (c) is correct.

36. $f(x) = \cos x + \cos(\sqrt{2}x)$

$$\therefore f(x) = 2 \cos \frac{\sqrt{2}+1}{2}x \cos \frac{\sqrt{2}-1}{2}x \leq 2$$

and it is 2 when $\cos \frac{\sqrt{2}+1}{2}x$ and $\cos \frac{\sqrt{2}-1}{2}x$ are both equal to 1 for a value of x . This is possible only when $x = 0$.

Option (b) is correct.

39. Let $f(x) = |x|$ is not differential

$$\because f'(0) \neq 0 \text{ but } f(x) \text{ has a minimum at } x = 0$$

Option (d) is correct.

40. $f(x) = \sin^4 x + \cos^4 x \Rightarrow f'(x) = 4 \sin^3 x \cos x - 4 \cos^3 x \sin x$

$f(x)$ is increasing if $f'(x) > 0$

$$\Rightarrow 4 \sin^3 x \cos x - 4 \cos^3 x \sin x > 0$$

$$\Rightarrow 4 \sin x \cos x (\sin^2 x - \cos^2 x) > 0$$

$$\Rightarrow -2(2 \sin x \cos x)(\cos^2 x - \sin^2 x) > 0$$

$$\Rightarrow -2 \sin 2x \cos 2x > 0$$

$$\Rightarrow -\sin 4x > 0$$

$$\therefore -\sin 4x > 0 \Rightarrow \sin 4x < 0$$

$$\Rightarrow \pi < 4x < 2\pi \Rightarrow \frac{\pi}{4} < x < \frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{4} < x < \frac{3\pi}{8}$$

Option (b) is correct.

41. $f'(x) = \frac{\sin x - x \cos x}{\sin^2 x}, g'(x) = \frac{\tan x - x \sec^2 x}{\tan^2 x}$

$$\text{Now } \frac{d}{dx}(\sin x - x \cos x) = \cos x + x \sin x - \cos x$$

$$= x \sin x > 0 \text{ for } 0 \leq x < 1$$

$\therefore \sin x - x \cos x$ is an increasing function.

But at $x = 0$, $x \sin x$ is 0

$$\therefore \text{In } 0 < x \leq 1, \sin x - x \cos x > 0$$

$$\therefore f'(x) > 0 \text{ for } 0 < x \leq 1$$

So $f(x)$ is increasing in the interval $0 < x \leq 1$.

$$\text{Again } \frac{d}{dx}(\tan x - x \sec^2 x) = \sec^2 x - 2x \sec^2 x \tan x - \sec^2 x$$

$$= -2x \sec^2 x \tan x < 0 \text{ for } 0 \leq x \leq 1$$

$\therefore g(x)$ is decreasing in $0 < x \leq 1$

Option (c) is correct.

$$\begin{aligned}
 42. \quad h(x) &= f(x) - \{f(x)\}^2 + \{f(x)\}^3 \\
 h'(x) &= f'(x) - 2\{f(x)\}f'(x) + 3\{f(x)\}^2f'(x) \\
 &= \{1 - 2\{f(x)\} + 3\{f(x)\}^2\}f'(x) \\
 &= 3\left\{\frac{1}{3} - \frac{2}{3}f(x) + (f(x))^2\right\}f'(x) \\
 &= \left(3\left[(f(x))^2 - \frac{2}{3}f(x) + \frac{1}{9}\right] + \frac{2}{3}\right)f'(x) \\
 &= \left[3\left\{f(x) - \frac{1}{3}\right\}^2 + \frac{2}{3}\right]f'(x)
 \end{aligned}$$

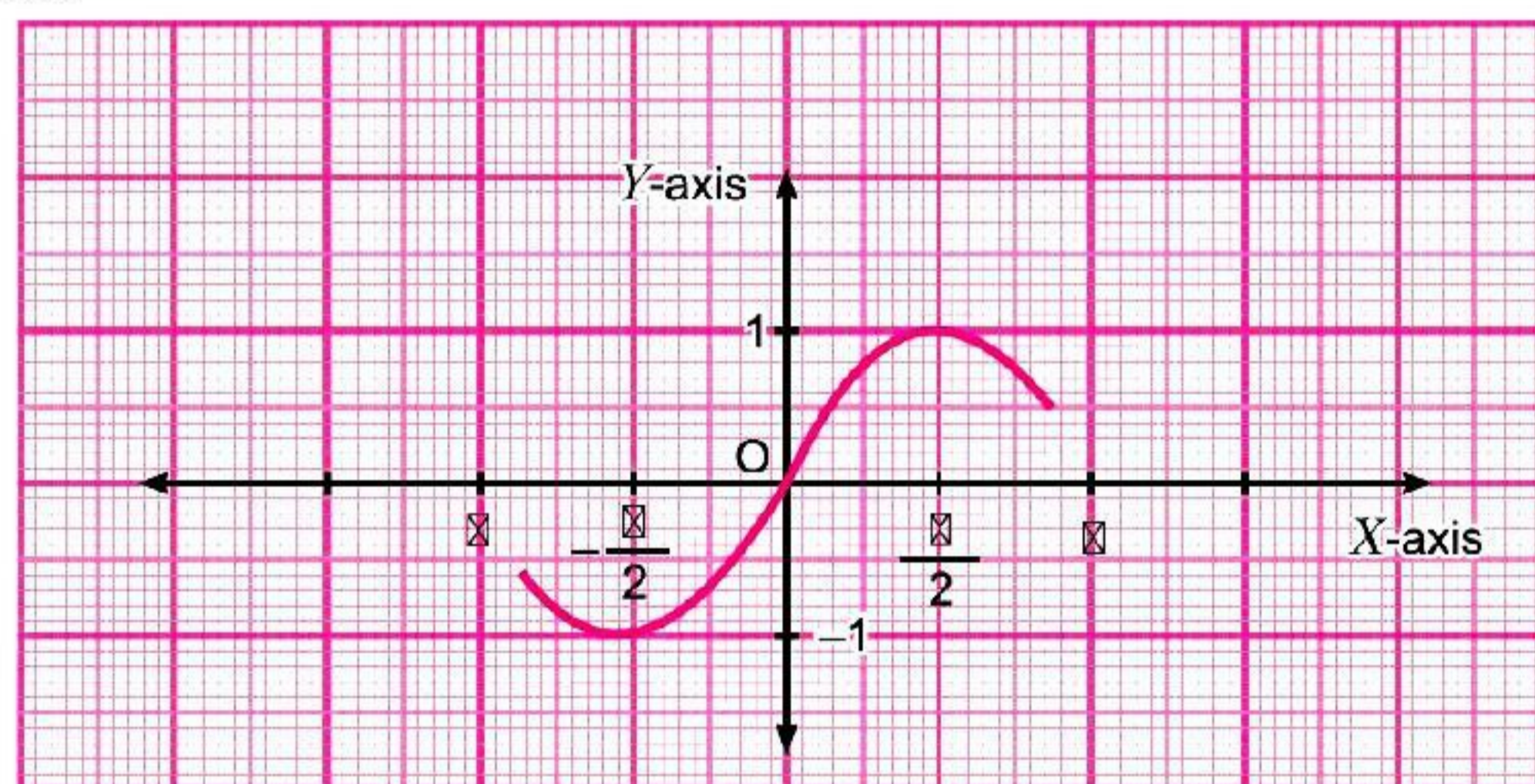
If $f(x)$ is increasing, $f'(x) > 0$ and therefore $h'(x) > 0$

i.e., $h(x)$ is increasing

If $f(x)$ is decreasing, $f'(x) < 0$ and $h'(x) < 0$ i.e., $h(x)$ is decreasing.

Option (a) is correct.

$$43. \quad f(x) = \sin x$$



It is clear from the graph that $f(x)$ is increasing in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$f(f(x)) = \sin(\sin x)$$

$$\therefore \frac{d}{dx}\{f(f(x))\} = \cos(\sin x) \cos x \geq 0, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\therefore f\{f(x)\} \text{ is increasing in } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\text{As } f\{f(x)\} \text{ is strictly increasing in } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{ and } f\left(f\left(-\frac{\pi}{2}\right)\right) = \sin\left(\sin\left(-\frac{\pi}{2}\right)\right)$$

$$= \sin(-1) = -\sin 1$$

$$\text{and } f\left(f\left(\frac{\pi}{2}\right)\right) = \sin\left(\sin\left(\frac{\pi}{2}\right)\right) = \sin 1$$

and the values $\pm \sin 1$ are not attained at any point in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, we find that $f(f(x))$ is invertible in

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

Option (d) is correct.

$$\begin{aligned}
 44. \quad f(x) &= 2 \sin^3 x - 3 \sin^2 x + 12 \sin x + 5 \\
 f'(x) &= 6 \sin^2 x \cos x - 6 \sin x \cos x + 12 \cos x
 \end{aligned}$$

$$\begin{aligned}
&= 6 \cos x \{ \sin^2 x - \sin x + 2 \} \\
&= 6 \cos x \left\{ \sin^2 x - 2 \sin x \times \frac{1}{2} + \frac{1}{4} - \frac{1}{4} + 2 \right\} \\
&= 6 \cos x \left\{ \left(\sin x - \frac{1}{2} \right)^2 + \frac{7}{4} \right\} \geq 0 \quad \forall x \in \left[0, \frac{\pi}{2} \right]
\end{aligned}$$

$\therefore f(x)$ is increasing in $\left[0, \frac{\pi}{2} \right]$

Option (b) is correct.

46. $x = e^t \cos t, \frac{dx}{dt} = e^t \cos t - e^t \sin t = e^t (\cos t - \sin t)$

$$y = e^t \sin t, \frac{dy}{dt} = e^t \sin t + e^t \cos t = e^t (\sin t + \cos t)$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t + \sin t}{\cos t - \sin t}$$

$$\therefore \text{Slope of tangent to curve at } \frac{\pi}{4} = \infty$$

Tangent makes an angle of $\frac{\pi}{2}$ with the x -axis.

Option (d) is correct.

49. $f(x) = 2x^3 + 9x^2 + 12x - 1$

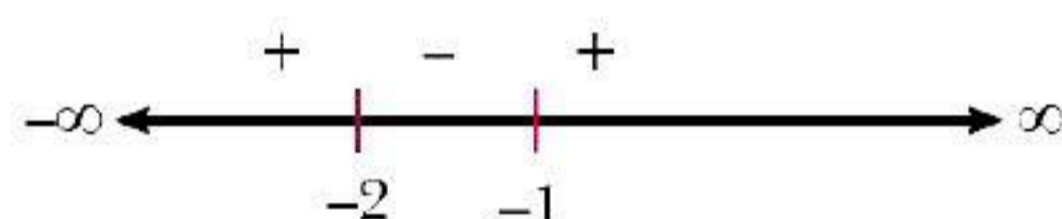
$$f'(x) = 6x^2 + 18x + 12$$

f is decreasing so $f'(x) \leq 0$

$$\text{i.e., } 6x^2 + 18x + 12 \leq 0 \Rightarrow x^2 + 3x + 2 \leq 0$$

$$\Rightarrow x^2 + x + 2x + 2 \leq 0 \Rightarrow x(x+1) + 2(x+1) \leq 0$$

$$\Rightarrow (x+1)(x+2) \leq 0$$



$\therefore f(x)$ is decreasing in $[-2, -1]$.

Option (b) is correct.

50. $f(x) = 4 \sin^3 x - 6 \sin^2 x + 12 \sin x + 100$

$$\Rightarrow f'(x) = 12 \sin^2 x \cos x - 12 \sin x \cos x + 12 \cos x$$

$$\Rightarrow f'(x) < 0 \text{ if } x \in \left(\frac{\pi}{2}, \pi \right)$$

Option (b) is correct.

51. $f(x) = \tan x - x$

$$\Rightarrow f'(x) = \sec^2 x - 1 \geq 0$$

\therefore Range $(\sec x) = (-\infty, -1] \cup [1, \infty)$, hence $f(x)$ always increases.

Option (a) is correct.

52. $f(x) = x^2 - 8x + 17$

$$\Rightarrow f'(x) = 2x - 8 = 0 \Rightarrow x = 4$$

$$f''(x) = 2 > 0$$

$x = 4$ point of minima

$$\text{Minimum value of } f(x) = 16 - 32 + 17 = 1$$

Option (c) is correct.

53. $f(x) = x^3 - 18x^2 + 96x$

$$f(x) \geq 0 \quad \forall x \in [0, 9]$$

\Rightarrow Minimum value of $f(x)$ in $[0, 9]$ is 0.

Option (b) is correct.

55. $f(x) = \sin x \cos x = \frac{1}{2} \sin 2x$

$$\therefore -1 \leq \sin 2x \leq 1.$$

$$\therefore -\frac{1}{2} \leq \frac{1}{2} \sin 2x \leq \frac{1}{2}$$

Hence maximum value of f is $\frac{1}{2}$.

Option (b) is correct.

56. $y = -x^3 + 3x^2 + 9x - 27$

$$y'(x) = -3x^2 + 6x + 9 = \text{slope of the curve} = m$$

$$\therefore m'(x) = -6x + 6 = 0 \Rightarrow x = 1$$

$$m''(x) = -6 \text{ and } m''(1) = -6 < 0$$

Hence $x = 1$ is the point where slope is minimum.

Hence minimum value of slope

$$m(1) = -3 + 6 + 9 = 12$$

Option (b) is correct.

57. We have $x + y = 8 \Rightarrow y = 8 - x$

$$\text{Let } f(x) = xy = x(8 - x) = 8x - x^2$$

$$\Rightarrow f'(x) = 8 - 2x = 0 \Rightarrow x = 4$$

$$\Rightarrow y = 4$$

$$f''(x) = -2$$

$$f''(4) = -2 < 0$$

$$\therefore f(x) \text{ is minimum when } x = 4$$

$$\therefore \text{Maximum value of } f(x) = xy = 16$$

Option (b) is correct.

59. Given curve $y = 2x^2 - 2x + 7$

$$\therefore \text{Slope of tangent} = \frac{dy}{dx} = 4x - 2$$

$$\therefore \text{Slope of normal} = -\frac{1}{4x - 2}$$

$$\text{Given } -\frac{1}{4x - 2} = \frac{1}{6} \Rightarrow -6 = 4x - 2$$

$$\Rightarrow 4x = -4 \Rightarrow x = -1$$

$$\text{When } x = -1, y = 2 + 2 + 7 = 11$$

$$\therefore \text{Required point} = (-1, 11)$$

Option (c) is correct.

62. We are given curve $2y^2 = ax^2 + b \dots (A)$
and point $(1, -1)$ is $2 = a + b, a + b = 2 \dots (i)$

Differentiating (A), w.r.t x we get

$$4y \frac{dy}{dx} = 2ax \Rightarrow \frac{dy}{dx} = \frac{ax}{2y}$$

$$\therefore \frac{dy}{dx} \Big|_{(1, -1)} = -\frac{a}{2} \quad \text{Given } \frac{dy}{dx} \Big|_{(1, -1)} = -1$$

$$\Rightarrow a = 2$$

$$(i) \Rightarrow b = 2 - 2 = 0$$

$$\therefore a = 2, b = 0$$

Option (a) is correct.

64. $f(x) = x + \sin 2x$
 $f(0) = 0$ and $f(2\pi) = 2\pi$

Hence $f(x)$ has maximum value 2π and minimum value is 0.

Option (a) is correct.

65. $R(x) = 5x^3 - 4x^2, R'(x) = 15x^2 - 8x$
 $R'(x) \Big|_{x=20} = 6000 - 160 = 5840$

Option (b) is correct.

68. $y = 12x - x^2 \Rightarrow \frac{dy}{dx} = 12 - 2x$
 \therefore Slope of tangent = 0
 $\Rightarrow 12 - 2x = 0 \Rightarrow x = 6$
 $\therefore x = 6 \Rightarrow y = 72 - 36 = 36$
 \therefore Required point is $(6, 36)$.

Option (c) is correct.

69. $f(x) = -x^2 - 2x + 15$
 $f'(x) = -2x - 2 = -2(x + 1) > 0$
if $x < -1$ i.e., in $(-\infty, -1)$
 $f'(x) < 0$ if $x > -1$ i.e., in $(-1, \infty)$
Hence $f(x)$ is increasing in $(-\infty, -1)$ and decreasing in $(-1, \infty)$

Option (a) is correct.

71. $f'(x) = 4x^3 - 62 \times 2x + a$
 $f'(x) = 4x^3 - 124x + a$

As function attains maximum at $x = 1 \in [0, 2]$

$$f'(1) = 0$$

$$\Rightarrow 4 - 124 + a = 0 \Rightarrow a = 120$$

Option (c) is correct.

