Permutations and Combinations

Question 1.
It is required to seat 5 men and 4 women in a row so that the women occupy the even places. The number of ways such arrangements are possible are
(a) 8820
(b) 2880
(c) 2088
(d) 2808

Answer: (b) 2880
Total number of persons are 9 in which there are 5 men and 4 women
So total number of place = 9
Now women seat in even place
So total number of arrangement = 4! (_W_W_W_W_) (W-Woman)
Men sit in odd place
So total number of arrangement = 5! (MWMWMWMWM) (M-Man)
Now Total number of arrangement = $5! \times 4! = 120 \times 24 = 2880$

Question 2.
Six boys and six girls sit along a line alternately in x ways and along a circle (again alternatively in y ways), then
(a) $x = y$
(b) $y = 12x$
(c) $x = 10y$
(d) $x = 12y$

Answer: (d) $x = 12y$
Given, six boys and six girls sit along a line alternately in x ways and along a circle (again alternatively in y ways).
Now, $x = 6! \times 6! + 6! \times 6!$
$\Rightarrow x = 2 \times (6!)2$
and $y = 5! \times 6!$
Now, $x/y = \{2 \times (6!)2\}/(5! \times 6!)$

$\Rightarrow x/y = \{2 \times 6! \times 6!\}/(5! \times 6!)$
$\Rightarrow x/y = \{2 \times 6!\}/5!$
$\Rightarrow x/y = \{2 \times 6 \times 5!\}/5!$
$\Rightarrow x/y = 12$
$\Rightarrow x = 12y$

Question 3.
How many 3-letter words with or without meaning, can be formed out of the letters of the word, LOGARITHMS, if repetition of letters is not allowed
(a) 720
(b) 420
(c) none of these
(d) 5040

Answer: (a) 720
The word LOGARITHMS has 10 different letters.
Hence, the number of 3-letter words(with or without meaning) formed by using these letters
$= {}^{10}P_3$
$= 10 \times 9 \times 8$
$= 720$

Question 4.
A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of at least 3 girls
(a) 588
(b) 885
(c) 858
(d) None of these

Answer: (a) 588
Given number of boys = 9
Number of girls = 4
Now, A committee of 7 has to be formed from 9 boys and 4 girls.
Now, the committee consists of atleast 3 girls:
${}^4C_3 \times {}^9C_4 + {}^4C_4 \times {}^9C_3$
$= [\{4! / (3! \times 1!)\} \times \{9! / (4! \times 5!)\}] + 9C_3$
$= [\{(4 \times 3!) / 3!\} \times \{(9 \times 8 \times 7 \times 6 \times 5!) / (4! \times 5!)\}] + 9! / (3! \times 6!)$
$= [4 \times \{(9 \times 8 \times 7 \times 6) / 4!\}] + (9\times8\times7\times6!)/(3! \times 6!)$
$= [\{4 \times (9 \times 8 \times 7 \times 6)\} / (4 \times 3 \times 2 \times 1)] + (9 \times 8 \times 7)/3!$
$= (9 \times 8 \times 7) + (9 \times 8 \times 7)/(3 \times 2 \times 1)$

$= 504 + (504/6)$
$= 504 + 84$
$= 588$

Question 5.
In how many ways can 12 people be divided into 3 groups where 4 persons must be there in each group?
(a) none of these
(b) $12!/(4!)^3$
(c) Insufficient data
(d) $12!/\{3! \times (4!)^3\}$

Answer: (d) $12!/\{3! \times (4!)^3\}$
Number of ways in which
m × n”>
m × n distinct things can be divided equally into n
n”> groups
$= (mn)!/\{n! \times (m!)n\}$
Given, $12(3 \times 4)$ people needs to be divided into 3 groups where 4 persons must be there in each group.
So, the required number of ways $= (12)!/\{3! \times (4!)n\}$

Question 6.
How many factors are $2^5 \times 3^6 \times 5^2$ are perfect squares
(a) 24
(b) 12
(c) 16
(d) 22

Answer: (a) 24
Any factors of $2^5 \times 3^6 \times 5^2$ which is a perfect square will be of the form $2^a \times 3^b \times 5^c$
where a can be 0 or 2 or 4, So there are 3 ways
b can be 0 or 2 or 4 or 6, So there are 4 ways
a can be 0 or 2, So there are 2 ways
So, the required number of factors $= 3 \times 4 \times 2 = 24$

Question 7.
If ${}^nC_{15} = {}^nC_6$ then the value of ${}^nC_{21}$ is
(a) 0
(b) 1

(c) 21
(d) None of these

Answer: (b) 1
We know that
if ${}^nC_{r1} = {}^nC_{r2}$
⇒ $n = r_1 + r_2$
Given, ${}^nC_{15} = {}^nC_6$
⇒ $n = 15 + 6$
⇒ $n = 21$
Now, ${}^{21}C_{21} = 1$

Question 8.
If ${}^{n+1}C_3 = 2\ {}^nC_2$, then the value of n is
(a) 3
(b) 4
(c) 5
(d) 6

Answer: (d) 6
Given, ${}^{n+1}C_3 = 2\ {}^nC_2$
⇒ $[(n + 1)!/\{(n + 1 – 3) \times 3!\}] = 2n!/\{(n – 2) \times 2!\}$
⇒ $[\{n \times n!\}/\{(n – 2) \times 3!\}] = 2n!/\{(n – 2) \times 2\}$
⇒ $n/3! = 1$
⇒ $n/6 = 1$
⇒ $n = 6$

Question 9.
There are 15 points in a plane, no two of which are in a straight line except 4, all of which are in a straight line. The number of triangle that can be formed by using these 15 points is
(a) ${}^{15}C_3$
(b) 490
(c) 451
(d) 415

Answer: (c) 451
The required number of triangle = ${}^{15}C_3 – {}^4C_3 = 455 – 4 = 451$

Question 10.
In how many ways in which 8 students can be sated in a circle is
(a) 40302
(b) 40320
(c) 5040
(d) 50040

Answer: (c) 5040
The number of ways in which 8 students can be sated in a circle = (8 – 1)!
= 7!
= 5040

Question 11.
Let R = {a, b, c, d} and S = {1, 2, 3}, then the number of functions f, from R to S, which are onto is
(a) 80
(b) 16
(c) 24
(d) 36

Answer: (d) 36
Total number of functions = $3^4 = 81$
All the four elements can be mapped to exactly one element in 3 ways, and exactly 3 elements in $3(2^4 - 2) = 3(16 - 2) = 3 \times 14 = 42$
Thus the number of onto functions = 81 – 42 -3 = 81 – 45 = 36

Question 12.
If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots\ldots\ldots + C_n x^n$, then the value of $C_0^2 + C_1^2 + C_2^2 + \ldots\ldots\ldots\ldots + C_n^n = {}^{2n}C_n$ is
(a) (2n)!/(n!)
(b) (2n)!/(n! × n!)
(c) (2n)!/(n! × n!)2
(d) None of these

Answer: (b) (2n)!/(n! × n!)
Given, $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots\ldots\ldots + C_n x^n \ldots\ldots\ldots\ 1$
and $(1 + x)^n = C_0 x^n + C_1 x^{n-1} + C_2 x^{n-2} + \ldots\ldots\ldots\ldots C_r x^{n-r} + \ldots\ldots\ldots + C_{n-1} x + C_n \ldots\ldots\ldots\ 2$
Multiply 1 and 2, we get
$(1 + x)^{2n} = (C_0 + C_1 x + C_2 x^2 + \ldots\ldots\ldots\ldots + C_n x^n) \times (C_0 x^n + C_1 x^{n-1} + C_2 x^{n-2} + \ldots\ldots\ldots\ldots C_r$

x^{n-r} + ………. + C_{n-1} x + C_n)
Now, equating the coefficient of xn on both side, we get
$C_0^2 + C_1^2 + C_2^2 + \ldots\ldots\ldots\ldots + C_n^n = {}^{2n}C_n = (2n)!/(n! \times n!)$

Question 13.
The total number of 9 digit numbers of different digits is
(a) 99!
(b) 9!
(c) 8 × 9!
(d) 9 × 9!

Answer: (d) 9 × 9!
Given digit in the number = 9
1st place can be filled = 9 ways = 9 (from 1-9 any number can be placed at first position)
2nd place can be filled = 9 ways (from 0-9 any number can be placed except the number which is placed at the first position)
3rd place can be filled = 8 ways
4th place can be filled = 7 ways
5th place can be filled = 6 ways
6th place can be filled = 5 ways
7th place can be filled = 4 ways
8th place can be filled = 3 ways
9th place can be filled = 2 ways
So total number of ways = 9 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2
= 9 × 9!

Question 14.
The number of ways in which 6 men add 5 women can dine at a round table, if no two women are to sit together, is given by
(a) 30
(b) 5 ! × 5 !
(c) 5 ! × 4 !
(d) 7 ! × 5 !

Answer: (b) 5 ! × 5 !
Again, 6 girls can be arranged among themselves in 5! ways in a circle.
So, the number of arrangements where boys and girls sit attentively in a circle = 5! × 5!

Question 15.
There are 15 points in a plane, no two of which are in a straight line except 4, all of which are in a

straight line. The number of triangle that can be formed by using these 15 points is
(a) $^{15}C_3$
(b) 490
(c) 451
(d) 415

Answer: (c) 451
The required number of triangle = $^{15}C_3 - {}^{4}C_3 = 455 - 4 = 451$

Question 16.
The number of 6-digit numbers can be formed from the digits 0, 1, 3, 5, 7 and 9 which are divisible by 10 and no digit is repeated are
(a) 110
(b) 120
(c) 130
(d) 140

Answer: (b) 120
A number is divisible by 10 if the unit digit of the number is 0.
Given digits are 0, 1, 3, 5, 7, 9
Now we fix digit 0 at unit place of the number.
Remaining 5 digits can be arranged in 5! ways
So, total 6-digit numbers which are divisible by 10 = 5! = 120

Question 17.
6 men and 4 women are to be seated in a row so that no two women sit together. The number of ways they can be seated is
(a) 604800
(b) 17280
(c) 120960
(d) 518400

Answer: (a) 604800
6 men can be sit as
× M × M × M × M × M × M ×
Now, there are 7 spaces and 4 women can be sit as $^{7}P_4 = {}^{7}P_3 = 7!/3! = (7 \times 6 \times 5 \times 4 \times 3!)/3!$
$= 7 \times 6 \times 5 \times 4 = 840$
Now, total number of arrangement = $6! \times 840$
$= 720 \times 840$
$= 604800$

Question 18.
A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of exactly 3 girls
(a) 540
(b) 405
(c) 504
(d) None of these

Answer: (c) 504
Given number of boys = 9
Number of girls = 4
Now, A committee of 7 has to be formed from 9 boys and 4 girls.
Now, If in committee consist of exactly 3 girls:
${}^4C_3 \times {}^9C_4$
$= \{4! / (3! \times 1!)\} \times \{9! / (4! \times 5!)\}$
$= \{(4\times3!) /3!\} \times \{(9 \times 8 \times 7 \times 6 \times 5!) / (4! \times 5!)\}$
$= 4 \times \{(9 \times 8 \times 7 \times 6) / 4!\}$
$= \{4 \times (9 \times 8 \times 7 \times 6)\} / (4 \times 3 \times 2 \times 1)$
$= 9 \times 8 \times 7$
$= 504$

Question 19.
How many factors are $2^5 \times 3^6 \times 5^2$ are perfect squares
(a) 24
(b) 12
(c) 16
(d) 22

Answer: (a) 24
Any factors of $2^5 \times 3^6 \times 5^2$ which is a perfect square will be of the form $2^a \times 3^b \times 5^c$
where a can be 0 or 2 or 4, So there are 3 ways
b can be 0 or 2 or 4 or 6, So there are 4 ways
a can be 0 or 2, So there are 2 ways
So, the required number of factors = $3 \times 4 \times 2 = 24$

Question 20.
The value of $2 \times P(n, n-2)$ is
(a) n
(b) 2n

(c) n!
(d) 2n!

Answer: (c) n!
Given, $2 \times P(n, n - 2)$
$= 2 \times \{n!/(n - (n - 2))\}$
$= 2 \times \{n!/(n - n + 2)\}$
$= 2 \times (n!/2)$
$= n!$
So, $2 \times P(n, n - 2) = n!$