5.2 INTERFERENCE OF LIGHT

5.64 (a) In this case the net vibration is given by

$$x = a_1 \cos \omega t + a_2 \cos(\omega t + \delta)$$

where δ is the phase difference between the two vibrations which varies rapidly and randomly in the interval $(0, 2\pi)$. (This is what is meant by incoherence.)

Then $$x = (a_1 + a_2 \cos \delta) \cos \omega t + a_2 \sin \delta \sin \omega t$$

The total energy will be taken to be proportional to the time average of the square of the displacement.

Thus $$E = \left\langle (a_1 + a_2 \cos \delta)^2 + a_2^2 \sin^2 \delta \right\rangle = a_1^2 + a_2^2$$

as $\langle \cos \delta \rangle = 0$ and we have put $\langle \cos^2 \omega t \rangle = \langle \sin^2 \omega t \rangle = \frac{1}{2}$ and has been absorbed in the overall constant of proportionality.

In the same units the energies of the two oscillations are $\overline{a_1^2}$ and a_2^2 respectively so the proposition is proved.

(b) Here $\vec{r} = a_1 \cos \omega t \, \hat{i} + a_2 \cos(\omega t + \delta) \hat{j}$

and the mean square displacement is $\alpha \, a_1^2 + a_2^2$

if δ is fixed but arbitrary. Then as in (a) we see that $E = E_1 + E_2$.

5.65 It is easier to do it analytically.

$$\xi_1 = a \cos \omega t, \quad \xi_2 = 2a \sin \omega t$$

$$\xi_3 = \frac{3}{2} a \left(\cos \frac{\pi}{3} \cos \omega t - \sin \frac{\pi}{3} \sin \omega t \right)$$

Resultant vibration is

$$\xi = \frac{7a}{4} \cos \omega t + a \left(2 - \frac{3\sqrt{3}}{4} \right) \sin \omega t$$

This has an amplitude $= \frac{a}{4} \sqrt{49 + (8 - 3\sqrt{3})^2} = 1{\cdot}89\, a$

5.66 We use the method of complex amplitudes. Then the amplitudes are

$$A_1 = a, \; A_2 = a\, e^{i\varphi}, \; \ldots \; A_N = a\, e^{i(N-1)\varphi}$$

and the resultant complex amplitude is

$$A = A_1 + A_2 + \ldots + A_N = a\left(1 + e^{i\varphi} + e^{2i\varphi} + \ldots + e^{i(N-1)\varphi}\right)$$

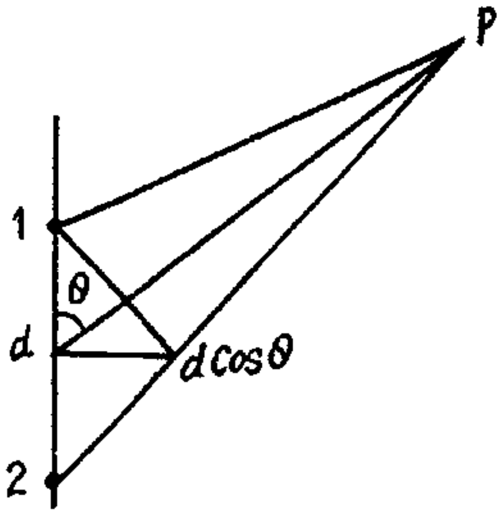
$$= a \frac{1 - e^{iN\varphi}}{1 - e^{i\varphi}}$$

The corresponding ordinary amplitude is

$$|A| = a\left|\frac{1-e^{iN\varphi}}{1-e^{i\varphi}}\right| = a\left[\frac{1-e^{iN\varphi}}{1-e^{i\varphi}}\times\frac{1-e^{-iN\varphi}}{1-e^{-i\varphi}}\right]^{1/2}$$

$$= a\left[\frac{2-2\cos N\varphi}{2-2\cos\varphi}\right]^{1/2} = a\frac{\sin\frac{N\varphi}{2}}{\sin\frac{\varphi}{2}}.$$

5.67 (a) With dipole moment $\perp^r$ to plane there is no variation with θ of individual radiation amplitude. Then the intensity variation is due to interference only.



In the direction given by angle θ the phase difference is

$$\frac{2\pi}{\lambda}(d\cos\theta)+\varphi = 2k\pi \quad \text{for maxima}$$

Thus $\quad d\cos\theta = \left(k-\frac{\varphi}{2\pi}\right)\lambda$

$$k = 0, \pm 1, \pm 2, \ldots$$

We have added φ to $\frac{2\pi}{\lambda}d\cos\theta$ because the extra path that the wave from 2 has to travel in going to P (as compared to 1) makes it lag more than it already is (due to φ).

(b) Maximum for $\theta = \pi$ gives $-d = \left(k-\frac{\varphi}{2\pi}\right)\lambda$

Minimum for $\theta = 0$ gives $d = \left(k'-\frac{\varphi}{2\pi}+\frac{1}{2}\right)\lambda$

Adding we get $\quad \left(k+k'-\frac{\varphi}{\pi}+\frac{1}{2}\right)\lambda = 0$

This can be true only if $\quad k' = -k,\ \varphi = \frac{\pi}{2}$

since $0<\varphi<\pi$

Then $\quad -d = \left(k-\frac{1}{4}\right)\lambda$

Here $\quad k = 0, -1, -2, -3, \ldots$

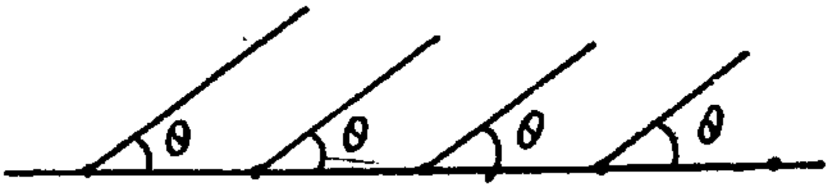
(Otherwise R.H.S. will become $+ve$).

Putting $k = -\bar{k}$, $\bar{k} = 0, +1, +2, +3, \ldots$

$$d = \left(\bar{k}+\frac{1}{4}\right)\lambda.$$

5.68 If $\Delta\varphi$ is the phase difference between neighbouring radiators then for a maximum in the direction θ we must have

$$\frac{2\pi}{\lambda}d\cos\theta + \Delta\varphi = 2\pi k$$



For scanning $\theta = \omega t + \beta$

Thus $$\frac{d}{\lambda}\cos(\omega t + \beta) + \frac{\Delta\varphi}{2\pi} = k$$

or $$\Delta\varphi = 2\pi\left[k - \frac{d}{\lambda}\cos(\omega t + \beta)\right]$$

To get the answer of the book, put $\beta = \alpha - \pi/2$.

5.69 From the general formula

$$\Delta x = \frac{l\lambda}{d}$$

we find that $$\frac{\Delta x}{\eta} = \frac{l\lambda}{d + 2\Delta h}$$

since d increases to $d + 2\Delta h$ when the source is moved away from the mirror plane by Δh.

Thus $$\eta d = d + 2\Delta h \quad \text{or} \quad d = 2\Delta h/(\eta - 1)$$

Finally $$\lambda = \frac{2\Delta h\,\Delta x}{(\eta - 1)\,l} = 0{\cdot}6\ \mu\text{m}.$$

5.70 We can think of the two coherent plane waves as emitted from two coherent point sources very far away. Then

$$\Delta x = \frac{l\lambda}{d} = \frac{\lambda}{d/l}$$

But $$\frac{d}{l} = \psi\ (\text{if } \psi \ll 1)$$

so $$\Delta x = \frac{\lambda}{\psi}.$$

5.71 (a) Here $S'S'' = d = 2r\alpha$

Then $\Delta x = \dfrac{(b + r)\lambda}{2\alpha}$

Putting $b = 1{\cdot}3$ metre, $r = {\cdot}1$ metre

$\lambda = 0{\cdot}55\ \mu\text{m}$, $\alpha = 12' = \dfrac{1}{5 \times 57}$ radian

we get $\Delta x = 1{\cdot}1$ mm

Number of possible maxima $= \dfrac{2b\alpha}{\Delta x} + 1 = 8{\cdot}3 + 1 = 9$

($2b\alpha$ is the length of the spot on the screen which gets light after reflection from both mirror. We add 1 above to take account of the fact that in a distance Δx there are two maxima).

(b) When the silt moves by δl along the arc of radius r the incident ray on the mirror rotates by $\frac{\delta l}{r}$; this is the also the rotation of the reflected ray. There is then a shift of the fringe of magnitude.

$$b\frac{\delta l}{r} = 13\text{ mm}.$$

(c) If the width of the slit is δ then we can imagine the slit to consist of two narrow slits with separation δ. The fringe pattern due to the wide slit is the superposition of the pattern due to these two narrow slits. The full pattern will not be sharp at all if the pattern due to the two narrow slits are $\frac{1}{2}\Delta x$ apart because then the maxima due to one will fill the minima due to the other. Thus we demand that

$$\frac{b\,\delta_{max}}{r} = \frac{1}{2}\Delta x = \frac{(b+r)\lambda}{4r\alpha}$$

or
$$\delta_{max} = \left(1+\frac{r}{b}\right)\frac{\lambda}{4\alpha} = 42\,\mu\text{ m}.$$

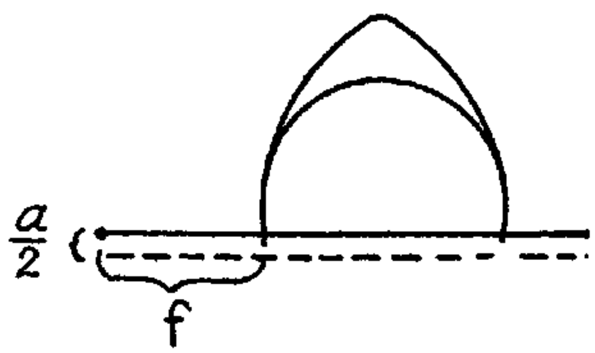
5.72 To get this case we must let $r \to \infty$ in the formula for Δx of the last example.

So
$$\Delta x = \frac{(b+r)\lambda}{2\alpha r} \longrightarrow \frac{\lambda}{2\alpha}.$$

(A plane wave is like light emitted from a point source at ∞).

Then
$$\lambda = 2\alpha\Delta x = 0{\cdot}64\,\mu\text{ m}.$$

5.73



(a) We show the upper half or the lens. The emergent light is at an angle $\frac{a}{2f}$ from the axis. Thus the divergence angle of the two incident light beams is

$$\psi = \frac{a}{f}$$

When they interfere the fringes produced have a width

$$\Delta x = \frac{\lambda}{\psi} = \frac{f\lambda}{a} = 0{\cdot}15\text{ mm.}$$

The patch on the screen illuminated by both light has a width $b\,\psi$ and this contains

$$\frac{b\,\psi}{\Delta x} = \frac{b\,a^2}{f^2\lambda}\text{ fringes} = 13\text{ fringes}$$

(if we ignore 1 in comparis on to $\frac{b\,\psi}{\Delta x}$ (if 5.71 (a))

(b) We follow the logic of (5.71 c). From one edge of the slit to the other edge the distance is of magnitude δ $\left(\text{i.e. } \frac{a}{2} \text{ to } \frac{a}{2}+\delta\right)$.

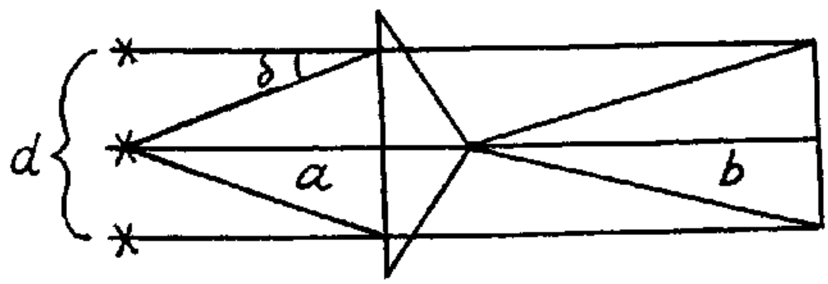
If we imagine the edge to shift by this distance, the angle $\psi/2$ will increase by $\frac{\Delta\psi}{2} = \frac{\delta}{2f}$

and the light will shift $\pm\, b\frac{\delta}{2f}$

The fringe pattern will therefore shift by $\frac{\delta\cdot b}{f}$

Equating this to $\frac{\Delta x}{2} = \frac{f\lambda}{2a}$ we get $\delta_{max} = \frac{f^2\lambda}{2ab} = 37{\cdot}5\,\mu\text{m}$.

5.74 $\Delta x = \frac{l\lambda}{d}$



$l = a+b$

$d = 2(n-1)\theta a$

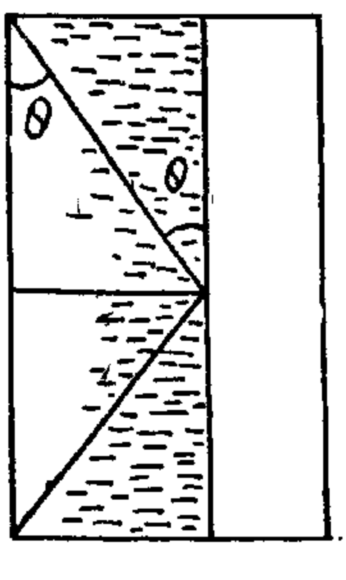
$\delta = (n-1)\theta$

$d = 2\delta\cdot a$

$n =$ R.I. of glass

Thus $$\lambda = \frac{2(n-1)\theta a\Delta x}{a+b} = 0{\cdot}64\,\mu\text{m}.$$

5.75 It will be assumed that the space between the biprism and the glass plate filled with benzene constitutes complementary prisms as shown.



Then the two prisms being oppositely placed, the net deviation produced by them is

$\delta = (n-1)\theta - (n'-1)\theta = (n-n')\theta$

Hence as in the previous problem

$d = 2a\delta = 2a\theta(n-n')$

So $$\Delta x = \frac{(a+b)\lambda}{2a\theta(n-n')}$$

For plane incident wave we let $a \to \infty$

so $$\Delta x = \frac{\lambda}{2\theta(n-n')} = 0{\cdot}2\text{ mm}.$$

5.76 Extra phase difference introduced by the glass plate is

$$\frac{2\pi}{\lambda}(n-1)h$$

This will cause a shift equal to $(n-1)\frac{h}{\lambda}$ fringe widths

i.e. by $$(n-1)\frac{h}{\lambda}\times\frac{l\lambda}{d} = \frac{(n-1)hl}{d} = 2\text{ mm}.$$

The fringes move down if the lower slit is covered by the plate to compensate for the extra phase shift introduced by the plate.

5.77 No. of fringes shifted $= (n'-n)\frac{l}{\lambda} = N$

so $$n' = n + \frac{N\lambda}{l} = 1{\cdot}000377\,.$$

5.78 (a) Suppose the vector $\vec{E}$, $\vec{E'}$, $\vec{E''}$ correspond to the incident, reflected and the transmitted wave. Due to the continuity of the tangential component of the electric field across the interface, it follows that

$$E_\tau + E'_\tau = E''_\tau \tag{1}$$

where the subscript τ means tangential.

The energy flux density is $\vec{E}\times\vec{H} = \vec{S}$.

Since $$H\sqrt{\mu\mu_0} = E\sqrt{\varepsilon\varepsilon_0}$$

$$H = E\sqrt{\frac{\varepsilon_0}{\mu_0}}\sqrt{\varepsilon/\mu} = n\sqrt{\frac{\varepsilon_0}{\mu_0}}\,E$$

Now $S \sim nE^2$ and since the light is incident normally

$$n_1 E_\tau^2 = n_1 E_\tau'^2 + n_2 E_\tau''^2 \tag{2}$$

or $$n_1(E_\tau^2 - E_\tau'^2) = n_2 E_\tau''^2$$

so $$n_1(E_\tau - E_\tau') = n_2 E_\tau'' \tag{3}$$

so $$E_\tau'' = \frac{2n_1}{n_1+n_2}E_\tau$$

Since E_τ'' and E_τ have the same sign, there is no phase change involved in this case.

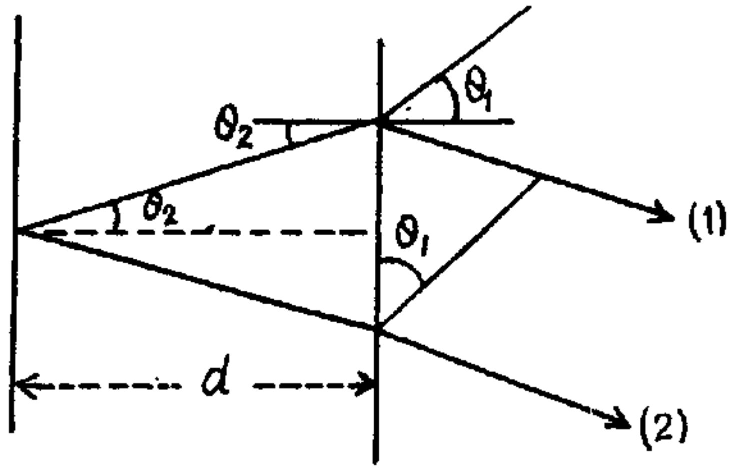
(b) From (1) & (3)

$$(n_2+n_1)E_\tau' + (n_2-n_1)E_\tau = 0$$

or $$E_\tau' = \frac{n_1-n_2}{n_1+n_2}E_\tau\,.$$

If $n_2 > n_1$, then E_τ' & E_τ have opposite signs. Thus the reflected wave has an abrupt change of phase by π if $n_2 > n_1$ i.e. on reflection from the interface between two media when light is incident from the rarer to denser medium.

5.79



Path difference between (1) & (2) is

$$2\,n\,d\sec\theta_2 - 2\,d\tan\theta_2\sin\theta_1$$

$$= 2\,d\,\frac{n - \dfrac{\sin^2\theta_1}{n}}{\sqrt{1 - \dfrac{\sin^2\theta_1}{n^2}}} = 2\,d\sqrt{n^2 - \sin^2\theta_1}$$

For bright fringes this must equal $\left(k + \frac{1}{2}\right)\lambda$ where $\frac{1}{2}$ comes from the phase change of π for (1).

Here $$k = 0, 1, 2, \ldots$$

Thus $$4\,d\sqrt{n^2 - \sin^2\theta_1} = (2\,k + 1)\,\lambda$$

or $$d = \frac{\lambda(1 + 2\,k)}{4\sqrt{n^2 - \sin^2\theta_1}} = 0{\cdot}14\,(1 + 2\,k)\,\mu\,\text{m}\,..$$

5.80 Given

$$2\,d\sqrt{n^2 - 1/4} = \left(k + \frac{1}{2}\right) \times 0{\cdot}64\,\mu\,\text{m} \quad (\text{bright fringe})$$

$$2\,d\sqrt{n^2 - 1/4} = k' \times 0{\cdot}40\,\mu\,\text{m} \qquad (\text{dark fringe})$$

where k, k' are integers.

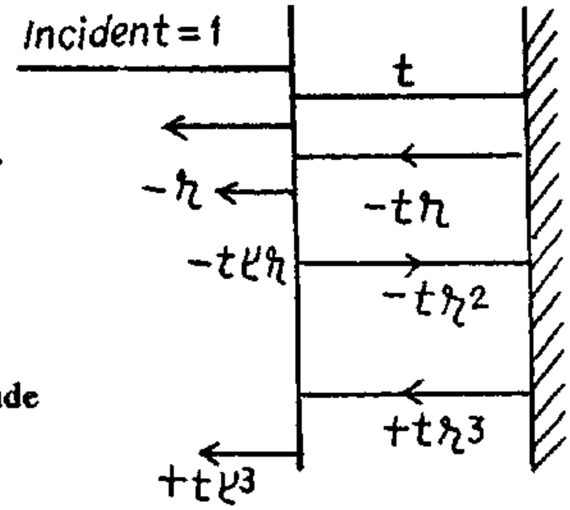
Thus $$64\left(k + \frac{1}{2}\right) = 40\,k' \quad \text{or} \quad 4\,(2\,k + 1) = 5\,k'$$

This means, for the smallest integer solutions

$$k = 2,\; k' = 4$$

Hence $$d = \frac{4 \times 0{\cdot}40}{2\sqrt{n^2 - 1/4}} = 0{\cdot}65\,\mu\,\text{m}\,.$$

5.81 When the glass surface is coated with a material of R.I. $n' = \sqrt{n}$ (n = R.I. of glass) of appropriate thickness, reflection is zero because of interference between various multiply reflected waves. We show this below.



Let a wave of unit amplitude be normally incident from the left. The reflected amplitude is $-r$ where

$$r = \frac{\sqrt{n}-1}{\sqrt{n}+1}$$

Its phase is $-ve$ so we write the reflected wave as $-r$. The transmitted wave has amplitude t

$$t = \frac{2}{1+\sqrt{n}}$$

This wave is reflected at the second face and has amplitude $-tr$

$$\left(\text{because } \frac{n-\sqrt{n}}{n+\sqrt{n}} = \frac{\sqrt{n}-1}{\sqrt{n}+1}.\right)$$

The emergent wave has amplitude $-t\,t'\,r$.

We prove below that $-tt' = 1 - r^2$. There is also a reflected part of emplitude $t\,r\,r' = -t\,r^2$, where r' is the reflection coefficient for a ray incident from the coating towards air. After reflection from the second face a wave of amplitude

$$+tt'r^3 = +(1-r^2)r^3$$

emerges. Let δ be the phase of the wave after traversing the coating both ways.

Then the complete reflected wave is

$$-r-(1-r^2)re^{i\delta}+(1-r^2)r^3e^{2i\delta}$$
$$-(1-r^2)r^5e^{3i\delta}\ \dots\dots$$
$$= -r-(1-r^2)re^{i\delta}\frac{1}{1+r^2e^{i\delta}}$$
$$= -r\left[1+r^2e^{i\delta}+(1-r^2)e^{i\delta}\right]\frac{1}{1+r^2e^{i\delta}}$$
$$= -r\frac{1+e^{i\delta}}{1+r^2e^{i\delta}}$$

This vanishes if $\delta = (2k+1)\pi$. But

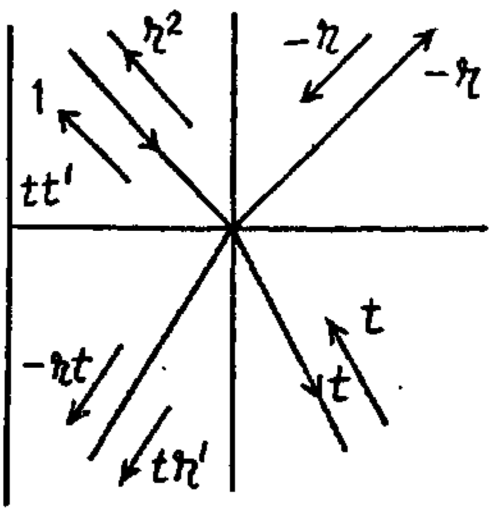
$$\delta = \frac{2\pi}{\lambda}2\sqrt{n}\,d \text{ so}$$

$$d = \frac{\lambda}{4\sqrt{n}}(2k+1)$$

We now deduce $tt' = 1 - r^2$ and $r' = +r$. This follows from the principle of reversibility of light path as shown in the figure below.

$$tt' + r^2 = 1$$
$$-rt + r't = 0$$
$$\therefore \; tt' = 1 - r^2$$
$$r' = +r.$$

($-r$ is the reflection ratio for the wave entering a denser medium).



5.82 We have the condition for maxima

$$2d\sqrt{n^2 - \sin^2\theta_1} = \left(k + \frac{1}{2}\right)\lambda$$

This must hold for angle $\theta \pm \dfrac{\delta\theta}{2}$ with successive values of k. Thus

$$2d\sqrt{n^2 - \sin^2\left(\theta + \frac{\delta\theta}{2}\right)} = \left(k - \frac{1}{2}\right)\lambda$$

$$2d\sqrt{n^2 - \sin^2\left(\theta - \frac{\delta\theta}{2}\right)} = \left(k + \frac{1}{2}\right)\lambda$$

Thus
$$\lambda = 2d\left\{\sqrt{n^2 - \sin^2\theta + \delta\theta\sin\theta\cos\theta} - \sqrt{n^2 - \sin^2\theta - \delta\theta\sin\theta\cos\theta}\right\}$$
$$= 2d\,\frac{\delta\theta\sin\theta\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

Thus
$$d = \frac{\sqrt{n^2 - \sin^2\theta}\,\lambda}{\sin 2\theta\,\delta\theta} = 15{\cdot}2\ \mu\text{m}$$

5.83 For small angles θ we write for dark fringes

$$2d\sqrt{n^2 - \sin^2\theta} = 2d\left(n - \frac{\sin^2\theta}{2n}\right) = (k + 0)\lambda$$

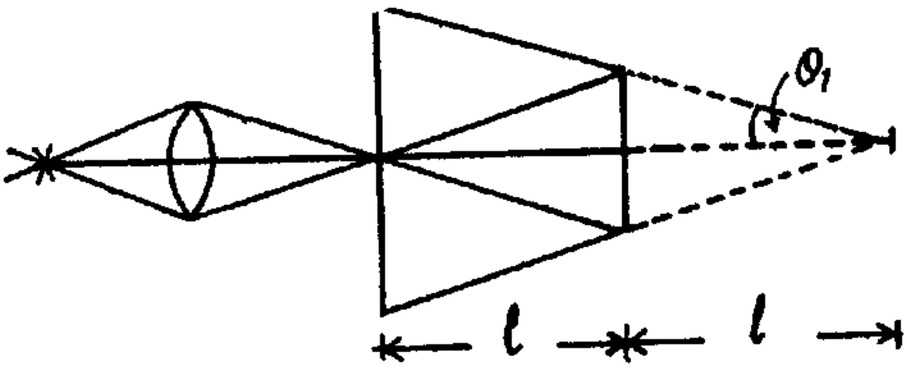
For the first dark fringe $\theta \approx 0$ and

$$2dn = (k_0 + 0)\lambda$$

For the i^{th} dark fringe

$$2d\left(n - \frac{\sin^2\theta_i}{2n}\right) = (k_0 - i + 1)\lambda$$

or
$$\sin^2\theta_i = \frac{n\lambda}{d}(i - 1) = \frac{r_i^2}{4l^2}$$



Finally $$\frac{n\lambda}{d}(i-k) = \frac{r_i^2 - r_k^2}{4l^2}$$

so $$\lambda = \frac{d(r_i^2 - r_k^2)}{4l^2 n(i-k)}$$

5.84 We have the usual equation for maxima

$$2h_k\alpha\sqrt{n^2 - \sin^2\theta_1} = \left(k + \frac{1}{2}\right)\lambda$$

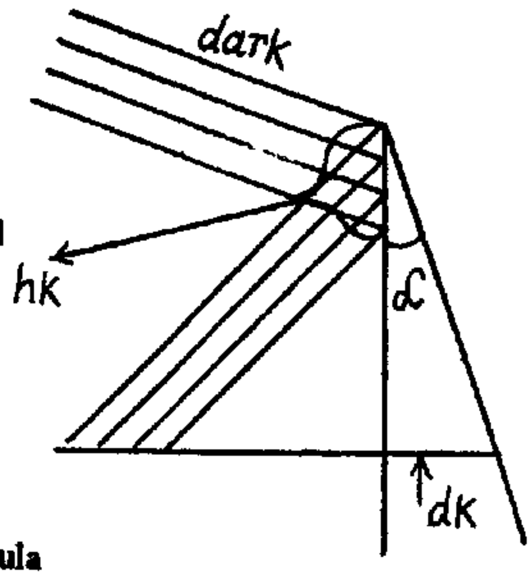
Here h_k = distance of the fringe from top

$h_k\alpha = d_k$ = thickness of the film

Thus on the screen placed at right angles to the reflected light

$$\Delta x = (h_k - h_{k-1})\cos\theta_1$$

$$= \frac{\lambda\cos\theta_1}{2\alpha\sqrt{n^2 - \sin^2\theta_1}}$$



5.85 (a) For normal incidence we have using the above formula

$$\Delta x = \frac{\lambda}{2n\alpha}$$

so $$\alpha = \frac{\lambda}{2n\Delta x} = 3' \text{ on putting the values}$$

(b) In a distance l on the wedge there are $N = \dfrac{l}{\Delta x}$ fringes.

If the fringes disappear there, it must be due to the fact that the maxima due to the component of wavelength λ coincide with the mainima due to the component of wavelength $\lambda + \Delta\lambda$. Thus

$$N\lambda = \left(N - \frac{1}{2}\right)(\lambda + \Delta\lambda) \quad \text{or} \quad \Delta\lambda = \frac{\lambda}{2N}$$

so $$\frac{\Delta\lambda}{\lambda} = \frac{1}{2N} = \frac{\Delta x}{2l} = \frac{0{\cdot}21}{30} = 0{\cdot}007.$$

The answer given in the book is off by a factor 2.

5.86 We have

$$r^2 = \frac{1}{2} k \lambda R$$

So for k differing by 1 ($\Delta k = 1$)

$$2 r \Delta r = \frac{1}{2} \Delta k \lambda R = \frac{1}{2} \lambda R$$

or

$$\Delta r = \frac{\lambda R}{4 r}.$$

5.87 The path traversed in air film of the wave constituting the k^{th} ring is

$$\frac{r^2}{R} = \frac{1}{2} k \lambda$$

when the lens is moved a distance Δh the ring radius changes to r' and the path length becomes

$$\frac{r'^2}{R} + 2 \Delta h = \frac{1}{2} k \lambda$$

Thus

$$r' = \sqrt{r^2 - 2 R \Delta h} = 1{\cdot}5 \text{ mm}.$$

5.88 In this case the path difference is $\frac{r^2 - r_0^2}{R}$ for $r > r_0$ and zero for $r \le r_0$.

This must equals $(k - 1/2)\lambda$ (where $k = 6$ for the sixth bright ring.)

Thus $r = \sqrt{r_0^2 + \left(k - \frac{1}{2}\right) \lambda R} = 3{\cdot}8$ mm

5.89 From the formula for Newton's rings we derive for dark rings

$$\frac{d_1^2}{4} = k_1 R \lambda, \quad \frac{d_2^2}{4} = k_2 R \lambda$$

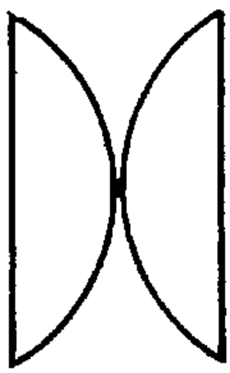
so

$$\frac{d_2^2 - d_1^2}{4 (k_2 - k_1) R} = \lambda$$

Substituting the values, $\lambda = 0{\cdot}5 \, \mu$ m.

5.90 Path difference between waves reflected by the two convex surfaces is

$$r^2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$



Taking account of the phase change at the 2nd surface we write the condition of bright rings as

$$r^2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{2k + 1}{2} \lambda$$

$k = 4$ for the fifth bright ring.

Thus $$\frac{1}{R_1}+\frac{1}{R_2}=\frac{9}{2}\lambda\cdot\frac{4}{d^2}=\frac{18\,\lambda}{d^2}$$

Now $$\frac{1}{f_1}=(n-1)\frac{1}{R_1},\ \frac{1}{f_2}=(n-1)\frac{1}{R_2}$$

so $$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=(n-1)\frac{18\,\lambda}{d^2}=\Phi=2{\cdot}40\,\mathrm{D}$$

Here n = R.I. of glass = 1·5.

5.91 Here $\Phi=(n-1)\left(\frac{2}{R_1}-\frac{2}{R_2}\right)$

so $$\frac{1}{R_1}-\frac{1}{R_2}=\frac{\Phi}{2(n-1)}.$$

As in the previous example, for the dark rings we have

$$r_k^2\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{\Phi}{2(n-1)}r_k^2=k\lambda$$

$k=0$ is dark spot; excluding it, we take $k=10$ hre.

Then $$r=\sqrt{\frac{20\,\lambda\,(n-1)}{\Phi}}=3{\cdot}49\text{ mm}.$$

(b) Path difference in water film will be

$$n_0\bar{r}^2\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

where $\bar{r}$ = new radius of the ring. Thus

$$n_0\bar{r}^2=r^2$$

or $$\bar{r}=r/\sqrt{n_0}=3{\cdot}03\text{ mm}.$$

Where n_0 = R.I. of water = 1·33.

5.92 The condition for minima are

$$\frac{r^2}{R}n_2=\left(k+\frac{1}{2}\right)\lambda,$$

(There occur phase changes at both surfaces on reflection, hence minima when path difference is half integer multiple of λ).

In this case $k=4$ for the fifth dark ring

(Counting from $k=0$ for the first dark ring).

Thus, we can write

$$r=\sqrt{(2K-1)\lambda R/2n_2}\ ,\ K=5$$

Substituting we get $r=1{\cdot}17$ mm.

5.93 Sharpness of the fringe pattern is the worst when the maxima and minima intermingle :-

$$n_1\lambda_1=\left(n_1-\frac{1}{2}\right)\lambda_2$$

or putting $\lambda_1 = \lambda, \ \lambda_2 = \lambda + \Delta\lambda$

we get $$n_1 \Delta\lambda = \frac{\lambda}{2}$$

or $$n_1 = \frac{\lambda}{2\Delta\lambda} = \frac{\lambda_1}{2(\lambda_2 - \lambda_1)} = 140.$$

5.94 Interference pattern vanishes when the maxima due to one wavelength mingle with the minima due to the other. Thus

$$2\Delta h = k\lambda_2 = (k+1)\lambda_1$$

where Δh = displacement of the mirror between the sharpest patterns of rings

Thus $$k(\lambda_2 - \lambda_1) = \lambda_1$$

or $$k = \frac{\lambda_1}{\lambda_2 - \lambda_1}$$

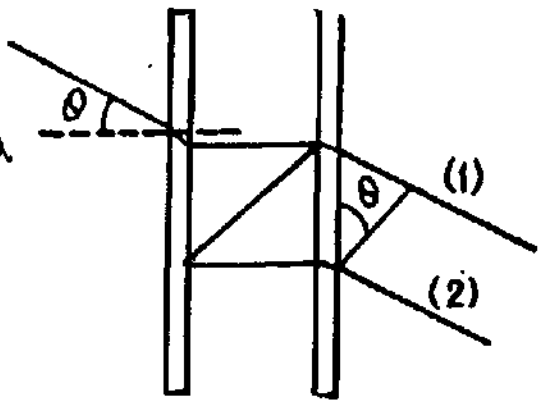
So $$\Delta h = \frac{\lambda_1 \lambda_2}{2(\lambda_2 - \lambda_1)} = \frac{\lambda^2}{2\Delta\lambda} = \cdot 29 \text{ mm}.$$

5.95 The path difference between (1) & (2) can be seen to be

$$\Delta = 2d\sec\theta - 2d\tan\theta\sin\theta$$
$$= 2d\cos\theta = k\lambda$$

for maxima. Here k = half-integer.

The order of interference decreases as θ increases i.e. as the radius of the rings increases.



(b) Differentiating

$$2d\sin\theta\,\delta\theta = \lambda$$

on putting $\delta k = -1$. Thus

$$\delta\theta = \frac{\lambda}{2d\sin\theta}$$

$\delta\theta$ decreases as θ increases.

5.96 (a) We have $k_{max} = \frac{2d}{\lambda}$· for $\theta = 0 .= 10^5$.

(b) We must have

$$2d\cos\theta = k\lambda = (k-1)(\lambda + \Delta\lambda)$$

Thus $\frac{1}{k} = \frac{\lambda}{2d}$· and $\Delta\lambda = \frac{\lambda}{k} = \frac{\lambda^2}{2d} = 5$ pm. on putting the values.