

# CHAPTER

## 1.9

### MAGNETICALLY COUPLED CIRCUITS

#### Statement for Q.1-2:

In the circuit of fig. P1.9.1-2  $i_1 = 4 \sin 2t$  A, and  $i_2 = 0$ .

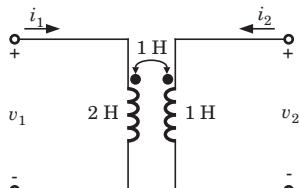


Fig. P1.9.1-2

1.  $v_1 = ?$

- (A)  $-16 \cos 2t$  V      (B)  $16 \cos 2t$  V  
(C)  $4 \cos 2t$  V      (D)  $-4 \cos 2t$  V

2.  $v_2 = ?$

- (A)  $2 \cos 2t$  V      (B)  $-2 \cos 2t$  V  
(C)  $8 \cos 2t$  V      (D)  $-8 \cos 2t$  V

#### Statement for Q.3-4:

Consider the circuit shown in Fig. P1.9.3-4

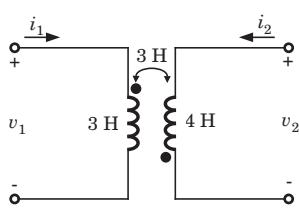


Fig. P1.9.3-4

3. If  $i_1 = 0$  and  $i_2 = 2 \sin 4t$  A, the voltage  $v_1$  is  
(A)  $24 \cos 4t$  V      (B)  $-24 \cos 4t$  V  
(C)  $1.5 \cos 4t$  V      (D)  $-1.5 \cos 4t$  V

4. If  $i_1 = e^{-2t}$  V and  $i_2 = 0$ , the voltage  $v_2$  is

- (A)  $-6e^{-2t}$  V      (B)  $6e^{-2t}$  V  
(C)  $1.5e^{-2t}$  V      (D)  $-1.5e^{-2t}$  V

#### Statement for Q.5-6:

Consider the circuit shown in fig. P19.5-6

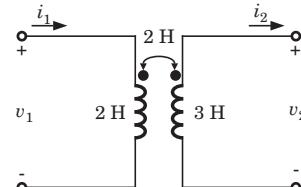


Fig. P1.9.5-6

5. If current  $i_1 = 3 \cos 4t$  A and  $i_2 = 0$ , then voltage  $v_1$  and  $v_2$  are

- (A)  $v_1 = -24 \sin 4t$  V,  $v_2 = -24 \sin 4t$  V  
(B)  $v_1 = 24 \sin 4t$  V,  $v_2 = -36 \sin 4t$  V  
(C)  $v_1 = 1.5 \sin 4t$  V,  $v_2 = \sin 4t$  V  
(D)  $v_1 = -1.5 \sin 4t$  V,  $v_2 = -\sin 4t$  V

6. If current  $i_1 = 0$  and  $i_2 = 4 \sin 3t$  A, then voltage  $v_1$  and  $v_2$  are

- (A)  $v_1 = 24 \cos 3t$  V,  $v_2 = 36 \cos 3t$  V  
(B)  $v_1 = 24 \cos 3t$  V,  $v_2 = -36 \cos 3t$  V  
(C)  $v_1 = -24 \cos 3t$  V,  $v_2 = 36 \cos 3t$  V  
(D)  $v_1 = -24 \cos 3t$  V,  $v_2 = -36 \cos 3t$  V

**Statement for Q.7-8:**

In the circuit shown in fig. P1.9.7-8,  $i_1 = 3\cos 3t$  A and  $i_2 = 4\sin 3t$  A.

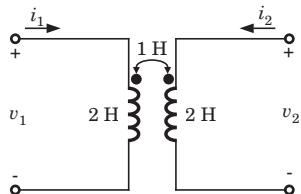


Fig. P1.9.7-8

7.  $v_1 = ?$

- (A)  $6(-2\cos t + 3\sin t)$  V      (B)  $6(2\cos t + 3\sin t)$  V  
 (C)  $-6(2\cos t + 3\sin t)$  V      (D)  $6(2\cos t - 3\sin t)$  V

8.  $v_2 = ?$

- (A)  $3(8\cos 3t - 3\sin t)$  V      (B)  $6(2\cos t + 3\sin t)$  V  
 (C)  $3(8\cos 3t + 3\sin 3t)$  V      (D)  $6(2\cos t - 3\sin t)$  V

**Statement for Q.9-10:**

In the circuit shown in fig. P1.9.9-10,  $i_1 = 5\sin 3t$  A and  $i_2 = 3\cos 3t$  A

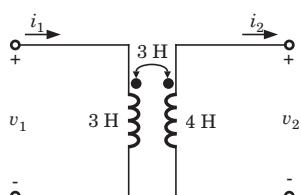


Fig. P1.9.9-10

9.  $v_1 = ?$

- (A)  $9(5\cos 3t + 3\sin 3t)$  V      (B)  $9(5\cos 3t - 3\sin 3t)$  V  
 (C)  $9(4\cos 3t + 5\sin 3t)$  V      (D)  $9(5\cos 3t - 3\sin 3t)$  V

10.  $v_2 = ?$

- (A)  $9(-4\sin 3t + 5\cos 3t)$  V      (B)  $9(4\sin 3t - 5\cos 3t)$  V  
 (C)  $9(-4\sin 3t - 5\cos 3t)$  V      (D)  $9(4\sin 3t + 5\cos 3t)$  V

11. In the circuit shown in fig. P1.9.11 if current  $i_1 = 5\cos(500t - 20^\circ)$  mA and  $i_2 = 20\cos(500t - 20^\circ)$  mA, the total energy stored in system at  $t = 0$  is

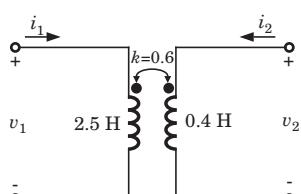
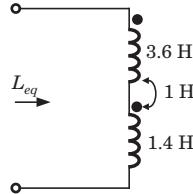


Fig. P1.9.11

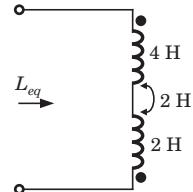
- (A)  $151.14 \mu J$       (B)  $45.24 \mu J$   
 (C)  $249.44 \mu J$       (D)  $143.46 \mu J$

12.  $L_{eq} = ?$



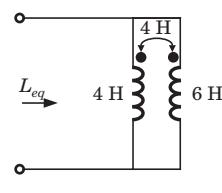
- Fig. P1.9.12  
 (A) 4 H      (B) 6 H  
 (C) 7 H      (D) 0 H

13.  $L_{eq} = ?$



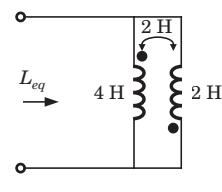
- Fig. P1.9.13  
 (A) 2 H      (B) 4 H  
 (C) 6 H      (D) 8 H

14.  $L_{eq} = ?$



- Fig. P1.9.14  
 (A) 8 H      (B) 6 H  
 (C) 4 H      (D) 2 H

15.  $L_{eq} = ?$



- Fig. P1.9.15  
 (A) 0.4 H      (B) 2 H  
 (C) 1.2 H      (D) 6 H

16. The equivalent inductance of a pair of a coupled inductor in various configuration are

- (a) 7 H after series adding connection  
 (b) 1.8 H after series opposing connection  
 (c) 0.5 H after parallel connection with dotted terminal connected together.

The value of  $L_1$ ,  $L_2$  and  $M$  are

- (A) 3 H, 1.6 H, 1.2 H      (B) 1.6 H, 3 H, 1.4 H  
 (C) 3.7 H, 0.7 H, 1.3 H      (D) 2 H, 3 H, 3 H

**17.**  $L_{eq} = ?$

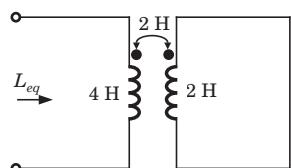


Fig. P1.9.17

- (A) 0.2 H      (B) 1 H  
 (C) 0.4 H      (D) 2 H

**18.**  $L_{eq} = ?$

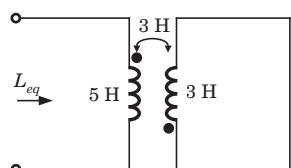


Fig. P1.9.18

- (A) 1 H      (B) 2 H  
 (C) 3 H      (D) 4 H

**19.** In the network of fig. P1.9.19 following terminal are connected together

- (i) none      (ii) A to B  
 (iii) B to C      (iv) A to C

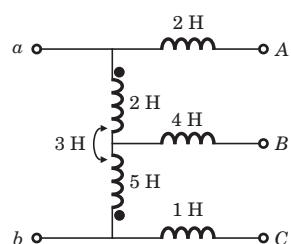


Fig. P1.9.19

The correct match for equivalent induction seen at terminal  $a - b$  is

- |          |         |       |          |
|----------|---------|-------|----------|
| (i)      | (ii)    | (iii) | (iv)     |
| (A) 1 H  | 0.875 H | 0.6 H | 0.75 H   |
| (B) 13 H | 0.875 H | 0.6 H | 0.75 H   |
| (C) 13 H | 7.375 H | 6.6 H | 2.4375 H |
| (D) 1 H  | 7.375 H | 6.6 H | 2.4375 H |

**20.**  $L_{eq} = ?$

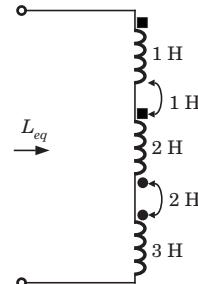


Fig. P1.9.20

- (A) 1 H      (B) 2 H  
 (C) 3 H      (D) 4 H

**21.**  $L_{eq} = ?$

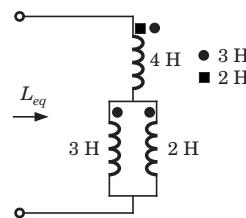


Fig. P1.9.21

- (A)  $\frac{41}{5}$  H      (B)  $\frac{49}{5}$  H  
 (C)  $\frac{51}{5}$  H      (D)  $\frac{39}{5}$  H

#### Statement for Q.22-24:

Consider the circuit shown in fig. P1.9.22-24.

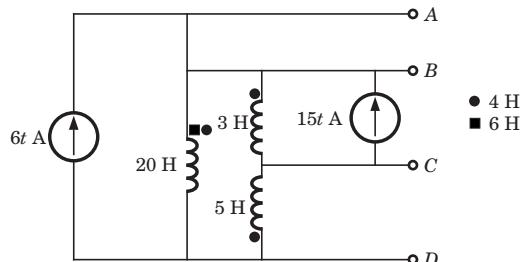


Fig. P1.9.22-24

**22.** The voltage  $V_{AG}$  of terminal AD is

- (A) 60 V      (B) -60 V  
 (C) 180 V      (D) 240 V

**23.** The voltage  $v_{BG}$  of terminal BD is

- (A) 45 V      (B) 33 V  
 (C) 69 V      (D) 105 V

**24.** The voltage  $v_{CG}$  of terminal CD is

- (A) 30 V      (B) 0 V  
 (C) -36 V      (D) 36 V

- 33.** In the circuit of fig. P1.9.33 the  $\omega = 2$  rad/s. The resonance occurs when  $C$  is

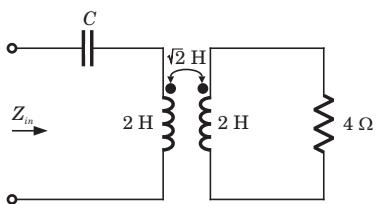


Fig. P1.9.33



- 34.** In the circuit of fig. P1.9.34, the voltage gain is zero at  $\omega = 333.33 \text{ rad/s}$ . The value of  $C$  is

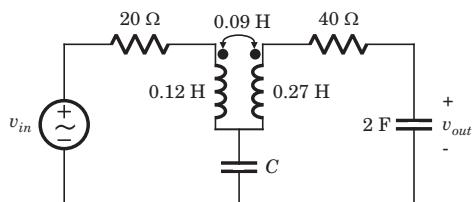


Fig. P1.9.34

- (A)  $100 \mu\text{F}$       (B)  $75 \mu\text{F}$   
 (C)  $50 \mu\text{F}$       (D)  $25 \mu\text{F}$

- 35.** In the circuit of fig. P1.9.35 at  $\omega = 333.33 \text{ rad/s}$ , the voltage gain  $v_{out}/v_{in}$  is zero. The value of  $C$  is

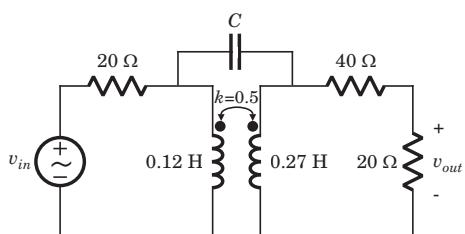


Fig. P1.9.35



- 36.** The Thevenin equivalent at terminal  $ab$  for the network shown in fig. P1.9.36 is

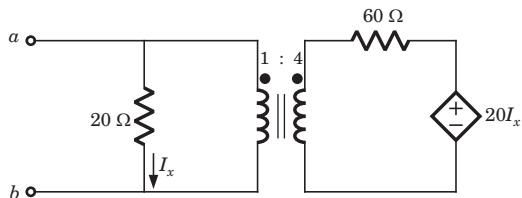


Fig. P1.9.36

- (A) 6 V, 10  $\Omega$       (B) 6 V, 4  $\Omega$   
 (C) 0 V, 4  $\Omega$       (D) 0 V, 10  $\Omega$

- 37.** In the circuit of fig. P1.9.37 the maximum power delivered to  $R_1$  is

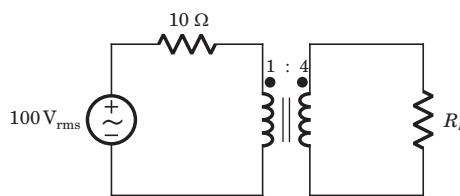


Fig. P1.9.37



- 38.** The average power delivered to the  $8\ \Omega$  load in the circuit of fig. P1.9.38 is

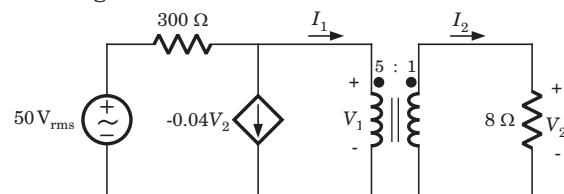


Fig. P1.9.38



- 39.** In the circuit of fig. P1.9.39 the ideal source supplies 1000 W, half of which is delivered to the  $100\ \Omega$  load. The value of  $a$  and  $b$  are

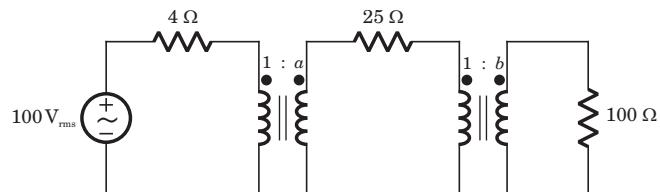


Fig. P1.9.39



- 40.**  $I_a = ?$

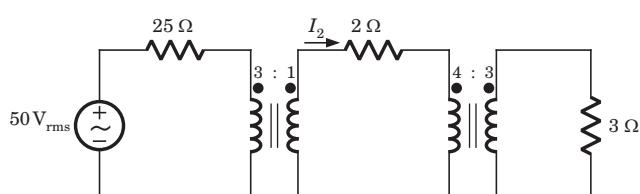


Fig. P1.9.40

- (A)  $1.65 \text{ A}_{\text{rms}}$       (B)  $0.18 \text{ A}_{\text{rms}}$   
 (C)  $0.66 \text{ A}_{\text{rms}}$       (D)  $5.90 \text{ A}_{\text{rms}}$

41.  $V_2 = ?$

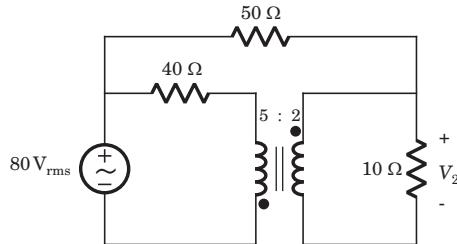


Fig. P1.9.41

- (A) -12.31 V      (B) 12.31 V  
 (C) -9.231 V      (D) 9.231 V

42. The power being dissipated in  $400 \Omega$  resistor is

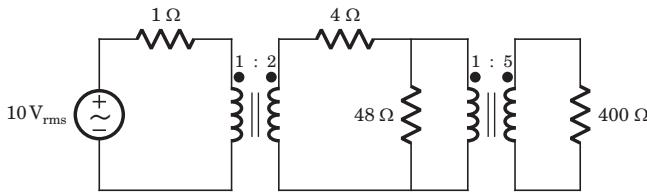


Fig. P1.9.42

- (A) 3 W      (B) 6 W  
 (C) 9 W      (D) 12 W

43.  $I_x = ?$

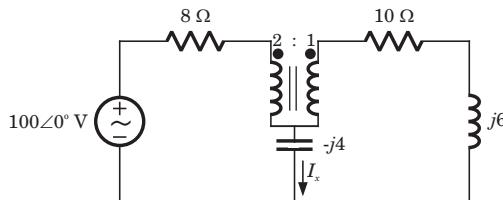


Fig. P1.9.43

- (A)  $1.921\angle57.4^\circ$  A      (B)  $2.931\angle59.4^\circ$  A  
 (C)  $1.68\angle43.6^\circ$  A      (D)  $1.79\angle43.6^\circ$  A

44.  $Z_{in} = ?$

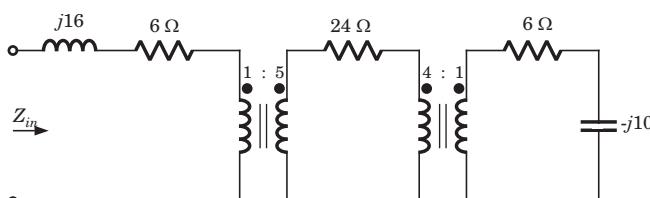


Fig. P1.9.44

- (A)  $46.3 + j6.8 \Omega$       (B)  $432.1 + j0.96 \Omega$   
 (C)  $10.8 + j9.6 \Omega$       (D)  $615.4 + j0.38 \Omega$

# SOLUTIONS

1. (B)  $v_1 = 2 \frac{di_1}{dt} + 1 \frac{di_2}{dt} = 2 \frac{di_1}{dt} = 16 \cos 2t \text{ V}$

2. (C)  $v_2 = (1) \frac{di_2}{dt} + (1) \frac{di_1}{dt} = \frac{di_1}{dt} = 8 \cos 2t \text{ V}$

3. (B)  $v_1 = 3 \frac{di_1}{dt} - 3 \frac{di_2}{dt} = -3 \frac{di_2}{dt} = -24 \cos 4t \text{ V}$

4. (C)  $v_2 = 4 \frac{di_2}{dt} - 3 \frac{di_1}{dt} = -3 \frac{di_1}{dt} = 6e^{-2t} \text{ V}$

5. (A)  $v_1 = 2 \frac{di_1}{dt} - 2 \frac{di_2}{dt} = 2 \frac{di_1}{dt} = -24 \sin 4t \text{ V}$

$v_2 = -3 \frac{di_2}{dt} + 2 \frac{di_1}{dt} = 2 \frac{di_1}{dt} = -24 \sin 4t \text{ V}$

6. (D)  $v_1 = 2 \frac{di_1}{dt} - 2 \frac{di_2}{dt} = -2 \frac{di_2}{dt} = -24 \cos 3t \text{ V}$

$v_2 = 3 \frac{di_2}{dt} + 2 \frac{di_1}{dt} = -3 \frac{di_2}{dt} = -36 \cos 3t \text{ V}$

7. (A)  $v_1 = 2 \frac{di_1}{dt} + 1 \frac{di_2}{dt}$   
 $= -18 \sin t + 12 \cos t = 6(2 \cos t - 3 \sin t) \text{ V}$

8. (A)  $v_2 = 2 \frac{di_2}{dt} + 1 \frac{di_1}{dt}$   
 $= 24 \cos 3t - 9 \sin 3t = 3(8 \cos 3t - 3 \sin 3t) \text{ V}$

9. (A)  $v_1 = 3 \frac{di_1}{dt} - 3 \frac{di_2}{dt}$   
 $= 45 \cos 3t + 27 \sin 3t = 9(5 \cos 3t + 3 \sin 3t) \text{ V}$

10. (D)  $v_2 = -4 \frac{di_2}{dt} + 3 \frac{di_1}{dt}$   
 $= 36 \sin 3t + 45 \cos 3t = 9(4 \sin 3t + 5 \cos 3t) \text{ V}$

11. (A)  $W = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$

At  $t = 0$ ,  $i_1 = 4 \cos(-20^\circ) = 4.7 \text{ mA}$

$i_2 = 20 \cos(-20^\circ) = 18.8 \text{ mA}$ ,

$M = 0.6 \sqrt{2.5 \times 0.4} = 0.6$

$W = \frac{1}{2}(2.5)(4.7)^2 + \frac{1}{2}(0.4)(18.8)^2 + 0.6(4.7)(18.8)$   
 $= 151.3 \mu\text{J}$

12. (C)  $L_{eq} = L_1 + L_2 + 2M = 7 \text{ H}$

\*\*\*\*\*

**13. (A)**  $L_{eq} = L_1 + L_2 - 2M = 4 + 2 - 2 \times 2 = 2$  H

**14. (C)**  $L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{24 - 16}{6 + 4 - 8} = 4$  H

**15. (A)**  $L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{8 - 4}{6 + 4} = 0.4$  H

**16. (C)**  $L_1 + L_2 + 2M = 7$ ,  $L_1 + L_2 - 2M = 1.8$

$$\Rightarrow L_1 + L_2 = 4.4, M = 1.3$$

$$\frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = 0.5, L_1 L_2 - 1.3^2 = 0.5 \times 1.8$$

$$L_1 L_2 = 2.59, (L_1 - L_2)^2 = 4.4^2 - 4 \times 2.59 = 9$$

$$L_1 - L_2 = 3, L_1 = 3.7, L_2 = 0.7$$

**17. (D)**  $L_{eq} = L_1 - \frac{M^2}{L_2} = 4 - \frac{4}{2} = 2$  H

**18. (B)**  $L_{eq} = L_1 - \frac{M^2}{L_2} = 5 - \frac{9}{3} = 2$  H

**19. (A)**

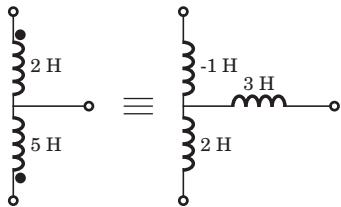


Fig. S.1.9.19

**20. (D)**  $V_{L1} = 1sI + 1sI = 2sI$

$$V_{L2} = 2sI + 1sI - 2sI = sI,$$

$$V_{L3} = 3sI - 2sI = sI$$

$$V_L = V_{L1} + V_{L2} + V_{L3} = 4sI \Rightarrow L_{eq} = 4$$
 H

**21. (B)** Let  $I_1$  be the current through 4 H inductor and  $I_2$  and  $I_3$  be the current through 3 H, and 2 H inductor respectively

$$I_1 = I_2 + I_3, V_2 = V_3$$

$$3sI_2 + 3sI_1 = 2sI_3 + 2sI_1$$

$$\Rightarrow 3I_2 + I_1 = 2I_3 \Rightarrow 4I_2 = I_3$$

$$\Rightarrow I_2 = \frac{I_1}{5}, I_3 = \frac{4}{5}I_1$$

$$V = 4sI_1 + 3sI_2 + 2sI_3 + 3sI_2 + 3sI_1$$

$$= 7sI_1 + \frac{6s}{5}I_1 + \frac{2 \times 4s}{5}I_1$$

$$V = \frac{49}{5}sI_1, L_{eq} = \frac{49}{5}$$
 H

**22. (C)**  $v_{AG} = 20 \frac{d(6t)}{dt} + 4 \frac{d(15t)}{dt} = 180$  V

**23. (B)**  $v_{BG} = 3 \frac{d(15t)}{dt} + 4 \frac{d(6t)}{dt} - 6 \frac{d(6t)}{dt} = 33$  V

**24. (C)**  $v_{CG} = -6 \frac{d(6t)}{dt} = -36$  V

**25. (B)**  $Z = Z_{11} + \frac{\omega^2 M^2}{Z_{22}}$

$$= 4 + j(50)\left(\frac{1}{10}\right) + \frac{(50)^2\left(\frac{1}{5}\right)^2}{5 + j(50)\frac{1}{2}} \\ = 4.77 + j1.15 \Omega$$

**26. (B)**  $V_s = j(0.8)10(1.2\angle 0) - j(0.2)(10)(2\angle 0)$

$$= 9.6 + j21.6 = 26.64\angle 66.04^\circ$$
 V

**27. (A)**  $[j(100\pi)(2) + 10]I_2 + j(100\pi)(0.4)(2\angle 0) = 0$

$$\Rightarrow I_2 = -0.4 - j0.0064,$$

$$V_o = 10I_2 = -4 - j0.064$$

$$= 4\angle -179.1^\circ$$

$$\Rightarrow v_o = 4 \cos(100\pi t - 179.1^\circ)$$
 V

**28. (B)**  $30\angle 30^\circ = I(-j6 + j8 - j4 + j12 - j4 + 10)$

$$\Rightarrow I = \frac{30\angle 30^\circ}{(10 + j6)} = 2.57 - j0.043$$

$$V_o = I(j12 - j4 + 10)$$

$$= (2.57 - j0.043)(10 + 8j)$$

$$= 26.067 + j20.14 = 32.9\angle 37.7^\circ$$
 V

**29. (A)**

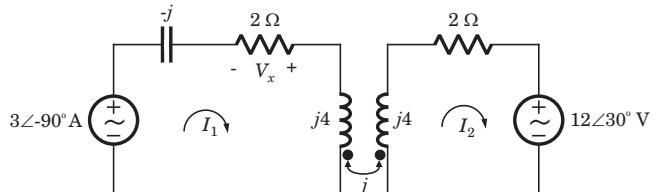


Fig. S.1.9.29

$$(-j + 2 + j4)I_1 - jI_2 = -j3$$

$$(j4 + 2)I_2 - jI_1 = -12\angle 30^\circ$$
 V

$$I_1 = -1.45 - j0.56,$$

$$V_x = -2I_1 = 2.9 + j1.12$$

$$= 3.11\angle 21.12^\circ$$
 V

30. (D)

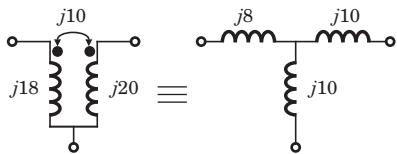


Fig. S.1.9.30

$$Z_{eq} = 10 + j8 + \frac{(j14)(j10 + 2 - j6)}{j14 + j10 + 2 - j6} \\ = 11.2 + j11.2 \Omega$$

31. (C)  $Z_{in} = (-j6) \parallel (Z_o)$

$$Z_o = j20 + \frac{(12)^2}{(j30 + j5 - j2 + 4)} = 0.52 + j15.7$$

$$Z_{in} = \frac{(-j6)(0.52 + j15.7)}{(-j6 + 0.52 + j15.7)} = 0.20 - j9.7 \Omega$$

32. (D)  $M = k\sqrt{L_1 L_2}$ ,  $M^2 = 160 \times 10^{-12}$

$$Z_{in} = j\omega L_1 + \frac{\omega^2 M^2}{Z_L + j\omega L_2} \\ = j250 \times 10^3 \times 2 \times 10^{-6} + \frac{(250 \times 10^3)^2 \times 160 \times 10^{-12}}{2 + j10 + j \times 250 \times 10^3 \times 80 \times 10^{-6}}$$

$$Z_{in} = 0.02 + j0.17 \Omega$$

33. (D)

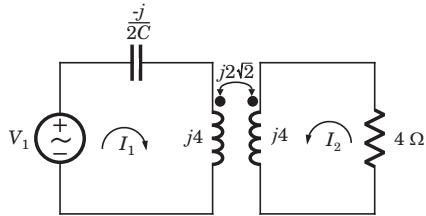


Fig. S.1.9.33

$$V_1 = -\frac{jI_1}{2C} + j4I_1 + j2\sqrt{2}I_2$$

$$0 = (4 + 4j)I_2 + j2\sqrt{2}I_1$$

$$\Rightarrow I_2 = \frac{-j\sqrt{2}I_1}{2(1+j)}$$

$$\frac{V_1}{I_1} = \frac{-j}{2C} + j4 + \frac{2}{1+j} = \frac{-j + j8C + 2C - j2C}{2C}$$

$$Z_{in} = \frac{-j + j8C + 2C - j2C}{2C}$$

$$\text{Im}(Z_{in}) = 0 \Rightarrow -j + j8C - j2C = 0$$

$$\Rightarrow C = \frac{1}{6}$$

34. (A)  $j30 - \frac{3j}{1000C} = 0, C = 100 \mu F$

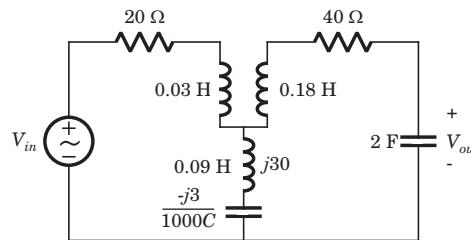


Fig. P1.9.34

35. (D) The  $\pi$  equivalent circuit of coupled coil is shown in fig. S1.9.35

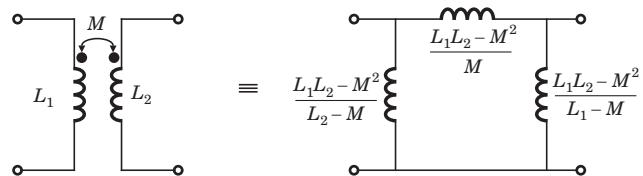


Fig. S1.9.35

$$\frac{L_1 L_2 - M^2}{M} = \frac{\sqrt{L_1 L_2}(1 - k^2)}{k} = \frac{\sqrt{0.12 \times 0.27}(1.05^2)}{0.5} = 0.27$$

Output is zero if  $\frac{-j}{0.27\omega} + jC\omega = 0$

$$C = \frac{1}{0.27\omega^2} = 33.33 \mu F$$

36. (C) Applying 1 V test source at ab terminal,

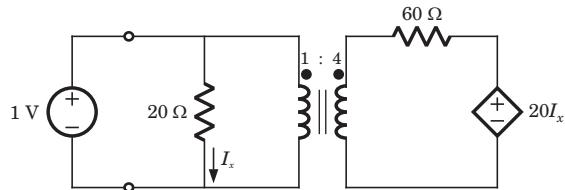


Fig. S1.9.36

$$V_{ab} = 1 \text{ V}, I_x = \frac{1}{20} = 0.05 \text{ A}, V_2 = 4 \text{ V},$$

$$4 = 60I_2 + 20 \times 0.05 \Rightarrow I_2 = 0.05 \text{ A}$$

$$I_{in} = I_x + I_1 = I_x + 4I_2 = 0.25 \text{ A}$$

$$R_{TH} = \frac{1}{I_{in}} = 4 \Omega, V_{TH} = 0$$

37. (A) Impedance seen by  $R_L = 10 \times 4^2 = 160 \Omega$

For maximum power  $R_L = 160 \Omega, Z_o = 10 \Omega$

$$P_{Lmax} = \left( \frac{100}{10 + 10} \right)^2 \times 10 = 250 \text{ W}$$

38. (B)  $I_2 = \frac{V_2}{8}, I_1 = \frac{I_2}{5} = \frac{V_2}{40}, V_1 = 5V_2$