SURFACE AREA & VOLUMES

2
CHAPTER

CONTENTS

- Cuboid
- Cube
- Cylinder
- Cone
- Sphere & Hemisphere

➤ CUBOID

Let length (λ), breadth (b) and height (h).
Surface area = 2 (λb + bh + hλ)
Volume = base area × h
where base area = Breadth × length
So volume = λ × b × h
Length of its diagonals = $\sqrt{\lambda^2 + b^2 + h^2}$
and total length of its edges = 4 (λ + b + h)

❖ EXAMPLES ❖

Ex.1 The length, breadth and height of a cuboid are in the ratio 6 : 4 : 5. If the total surface area of the cuboid is 2368 cm^2 ; find its dimension.

Sol. Let length (λ) = 6x cm, breadth (b) = 4x cm and height (h) = 5x cm,

∴ Total surface area

= 2(λ × b + b × h + h × λ)

= 2(6x × 4x + 4x × 5x + 5x × 6x)cm^2

= 2(24x^2 + 20x^2 + 30x^2) cm^2

=148x^2 cm^2

Given : Total surface = 2368 cm^2

⇒ 148x^2 = 2368

⇒ $x^2 = \frac{2368}{148} = 16$

and $x = \sqrt{16} = 4$

∴ length = 6x cm = 6 × 4 cm = 24 cm,

breadth = 4x cm = 4 × 4 cm = 16 cm and

height = 5x cm = 5 × 4 cm = 20 cm

Ex.2 A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine :

(i) The area of the sheet required for making the box.

(ii) The cost of sheet for it, if a sheet measuring 1 m^2 costs Rs. 20.

Sol. Given. length (λ) = 1.5 m, breadth (b) = 1.25 m

and depth i.e., height (h) = 65 cm = 0.65 m.

(i) Since, the box is open it has five faces in which four faces are the walls forming lateral surface area and one face is the base.

∴ The area of the sheet required for making the box.

= Lateral surface area of the box + area of its base

= 2(λ + b) × h + λ × b

= 2(1.5m + 1.25m) × 0.65m + 1.5m × 1.25 m

= 2 × 2.75 m × 0.65 m + 1.875 m^2

= 3.575 m^2 + 1.875 m^2 = 5.45 m^2

(ii) Since, a sheet measuring 1 m^2 costs Rs.20

∴ The cost of sheet for the box

= 5.45 × Rs 20 = Rs. 109

Ex.3 The length, breadth and height of a room are 5m, 4m and 3m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of Rs. 7.50 per m^2.

Sol. Given. $\lambda = 5m$, $b = 4m$ and $h = 3m$

Since, the area of the walls of the room

$= $ its lateral surface area $= 2(\lambda + b) \times h$

And, the area of the ceiling of the room

$= \lambda \times b$

$\therefore$ Total area to be white washed

= Area of the walls + area of the ceiling

$= 2(\lambda + b) \times h + \lambda \times b.$

$= 2(5m + 4m) \times 3m + 5m \times 4m$

$= 2 \times 9m \times 3m + 20m^2$

$= 54m^2 + 20m^2 = 74m^2$

Since, the rate of white washing

= Rs. 7.50 per m^2

$\therefore$ Cost of white washing

= 74 × Rs.7.50 = Rs. 555

Ex.4 The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs. 10 per m^2 is Rs : 15,000, find the height of the hall.

Sol. We know, the perimeter of a rectangle

$= 2(\text{length} + \text{breadth}) = 2(\lambda + b)$

And, given the floor of a rectangular hall has perimeter 250 m

$\Rightarrow 2(\lambda + b) = 250$ m

Area of the four walls of the hall

= Lateral surface area of the hall

$= 2(\lambda + b) \times h = 250 \text{ m} \times h\text{m} = 250 \text{ hm}^2$

Since, the rate of painting four walls is Rs. 10 per m^2.

$\therefore$ Cost of painting 250 hm^2 = 250 h × Rs.10

According to the given statements :

250 h × Rs. 10 = Rs. 15,000

$\Rightarrow h = \frac{15{,}000}{250 \times 10} m = 6 \text{ m}$

$\therefore$ The height of the hall = 6 m

Ex.5 The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimension 22.5 cm × 10 cm × 7.5 cm can be painted out of this container.

[NCERT]

Sol. For each brick, $\lambda = 22.5$ cm, $b = 10$ cm and $h = 7.5$ cm.

$\therefore$ Surface area of each brick

$= 2(\lambda \times b + b \times h + h \times \lambda)$

$= 2(22.5 \times 10 + 10 \times 7.5 + 7.5 \times 22.5) \text{cm}^2$

$= 2(225 \times 75 + 168.75) \text{ cm}^2 = 937.5 \text{ cm}^2$

Since, the paint in the container is sufficient to paint an area.

$= 9.375 \text{ m}^2 = 9.375 \times 100 \times 100 \text{ cm}^2$

[1 m = 100 cm and 1 m^2 = 100 × 100 cm^2]

$= 93750 \text{ cm}^2$

$\therefore$ Number of bricks that can be painted

$$= \frac{\text{Total area which can be painted}}{\text{Surface area of the brick}}$$

$$= \frac{93750 \text{ cm}^2}{937.5 \text{ cm}} = 100$$

Ex.6 A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tap. It is 30 cm long, 25 cm wide and 25 cm high. **[NCERT]**

(i) What is the area of the glass?

(ii) How much of tape is needed for all the 12 edges?

Sol. The green house is a cuboid in shape in which $\lambda = 30$ cm, $b = 25$ cm and $h = 25$ cm

(i) Area of the glass

= Surface area of cubical green house.

$= 2(\lambda \times b + b \times h + h \times \lambda)$

$= 2(30 \times 25 + 25 \times 25 + 25 \times 30) \text{ cm}^2$

$= 2(750 + 625 + 750)\text{cm}^2 = 4250 \text{ cm}^2$

(ii) Length of the tape

= Length of the 12 edges of the cubical greenhouse $= 4(\lambda + b + h)$

$= 4(30 + 25 + 25)\text{cm} = 320 \text{ cm}$

Ex.7 Shanti sweets stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs. 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.

Sol. For each bigger box :

λ = 25 cm, b = 20 cm and h = 5 cm

$\therefore$ Surface area

$= 2(\lambda \times b + b \times h + h \times \lambda)$

$= 2(25 \times 20 + 20 \times 5 + 5 \times 25)cm^2$

$= 2(500 + 100 + 125)cm^2 = 1450\ cm^2$

For each smaller box :

λ = 15 cm, b = 12 cm and h = 5 cm

$\therefore$ Surface area

$= 2(\lambda \times b + b \times h + h \times \lambda)$

$= 2(15 \times 12 + 12 \times 5 + 5 \times 15)cm^2$

$= 2(180 + 60 + 75)\ cm^2 = 630\ cm^2$

Total surface area of 250 boxes of each kind

$= 250 \times 1450\ cm^2 + 250 \times 630\ cm^2$

$= 362500\ cm^2 + 157500\ cm^2$

$= 520000\ cm^2$

Cardboard required for overlaps

= 5% of 520000 cm^2

$= \frac{5}{100} \times 52000 cm^2 = 26000\ cm^2$

$\therefore$ Total area of the cardboard used

$= 520000\ cm^2 + 26000\ cm^2$

$= 546000\ cm^2$

Given, cost of 1000 cm^2 of cardboard = Rs.4

$\Rightarrow$ cost of 1 cm^2 of cardboard = Rs. $\frac{4}{1000}$

$\Rightarrow$ cost of 546000 cm^2 of cardboard

= Rs. $\frac{4}{1000} \times 546000$ = Rs. 2184

Ex.8 The volume of a cubical solid is 3240 cm^3, find, its

(i) height, if length =18 cm and breadth = 15cm

(ii) breadth, if length =24 cm and height = 10cm

(iii) length, if breadth = 9cm and height = 20cm

Sol. Volume = length × breadth × height

$\Rightarrow$ (i) height $= \frac{\text{Volume}}{\text{length} \times \text{breadth}}$

(ii) breadth $= \frac{\text{Volume}}{\text{length} \times \text{height}}$

(iii) height $= \frac{\text{Volume}}{\text{length} \times \text{breadth}}$

(i) Height $= \frac{\text{Volume}}{\text{length} \times \text{breadth}}$

$= \frac{3240\ cm^3}{18\ cm \times 15\ cm} = 12$ cm

(ii) Breadth $= \frac{\text{Volume}}{\text{length} \times \text{height}}$

$= \frac{3240\ cm^3}{24\ cm \times 10\ cm} = 13.5$ cm

(iii) Length $= \frac{\text{Volume}}{\text{breadth} \times \text{height}}$

$= \frac{3240\ cm^3}{9\ cm \times 20\ cm} = 18$ cm

Ex.9 A matchbox measures 6 cm × 4 cm × 2.5 cm. What will be the volume of a packet containing 24 such boxes ?

Sol. The shape of a matchbox is a cuboid

$\therefore$ Volume of 1 matchboxes

= its length × breadth × height

$= 6 \times 4 \times 2.5\ cm^2 = 60\ cm^3$

$\Rightarrow$ Volume of a packet containing 24 such boxes

= Volume of 24 matchboxes

$= 24 \times 60\ cm^3 = 1440\ cm^3$

Ex.10 A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many liters of water can it hold? ($1\ m^2 = 1000\ \lambda$)

Sol. Volume of water which tank can hold

= Volume of the tank

= Its length × width × height

$= 6\ m \times 5\ m \times 4.5\ m = 135\ m^3$

$= 135 \times 1000\ \lambda = 135000\ \lambda$

Ex.11 Find the cost of digging a cuboidal pit 10 m long, 7.5 m broad and 80 cm deep at the rate of Rs. 16 per m^3.

Sol. Given :

Length of the pit = 10 m, breadth = 7.5 m and depth = 80 cm = 0.8 m

$\therefore$ Volume of cuboidal pit

= Its length × breadth × depth (or height)

= 10 m × 7.5 m × 0.8 m = 60 m^3

Rate of digging the pit = Rs. 16 per m^3

$\therefore$ Cost of digging 60 × Rs. 16 = Rs. 960

Ex.12 Wooden crates each measuring 1.5 m × 1.25 m × 0.5 m are to be stores in a godown. Find the largest number of wooden crates which can be stores in a godown measuring :

(i) 45 m × 25 m × 10m.

(ii) 40 m × 25 m × 10m

Sol. (i) Since, the length of godown is 45 m and the length of a crate is 1.5 m

$\therefore$ The largest no. of crates that can stores along the length of the godown

$= \frac{45\text{m}}{1.5\text{m}} = 30$

Similarly, the largest no. of crates stores along the width of the godown

$= \frac{25\text{m}}{1.25\text{m}} = 20$

And, the largest no. of crates stored along the length of the godown

$= \frac{10\text{m}}{0.5\text{m}} = 20$

$\therefore$ The largest no. of crates which can be stored in the godown

= 30 × 20 × 20

= 12000

(ii) No. of crates along the length

$= \frac{40\text{m}}{1.5\text{m}} = 26\frac{2}{3} = 26$

[NO. of crates can not be in fraction]

No. of crates along the width

$= \frac{25\text{m}}{1.25\text{m}} = 20$

No. of crates along the height

$= \frac{10\text{m}}{0.5\text{m}} = 20$

$\therefore$ The largest no. of crates which can be stored in the godown

= 26 × 20 × 20

= 10400

Ex.13 The capacity of a cuboidal tank is 50000 liters of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Sol. The capacity of cuboidal tank is 50000 liters of water.

$\Rightarrow$ Volume of the tank = 50000 liters

$= \frac{50000}{1000} m^3$ [$\because$ 1 m^3 = 1000 liters]

= 50 m^3

$\Rightarrow$ Length of the tank × its breadth × its height = 50 m^3

$\Rightarrow$ 2.5 m × breadth × 10 m = 50 m^3

$\Rightarrow$ Breadth $= \frac{50}{2.5 \times 10}$ m = 2m

➢ CUBE

Let length of each side is 'a' then
Surface area = 6 a^2
Volume = a^3
Lateral surface area = 4 a^2
Length of its diagonals = $a\sqrt{3}$
Total length of its edges = 12 a

❖ EXAMPLES ❖

Ex.14 If each edge (side) of a cube is 8 cm ; find its surface area and lateral surface area.

Sol. Given each side of the cube (a) = 8 cm

$\therefore$ Its surface area = $6a^2 = 6 \times 8^2$ sq. cm

= 6 × 64 cm^2 = 384 cm^2

Lateral surface area = $4a^2 = 4 \times 8^2$ sq. cm

= 4 × 64 cm^2 = 256 cm^2

Ex.15 A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high. **[NCERT]**

(i) Which box has the greater lateral surface area and by how much ?

(ii) Which box has the smaller total surface area and by how much ?

Sol.(i) For the cubical box :

Each edge = 10 cm i.e., a = 10 cm

$\therefore$ Lateral surface area of the cubical box

$= 4a^2 = 4 \times 10^2 \text{ cm}^2 = 400 \text{ cm}^2$

For the cuboidal box :

$\lambda = 12.5$ cm, b = 10 cm & h = 8 cm

$\therefore$ Lateral surface area of the cuboidal box

$= 2(\lambda + b) \times h$

$= 2(12.5 + 10) \times 8 \text{ cm}^2$

$= 2 \times 22.5 \times 8 \text{ cm}^2 = 360 \text{ cm}^2$

Clearly, cubical box has greater lateral surface area by

$400 \text{ cm}^2 - 360 \text{ cm}^2 = 40 \text{ cm}^2$

(ii) Total surface area of the cubical box:

$= 6a^2 = 6 \times 10^2$ sq. cm $= 600 \text{ cm}^2$

Total surface area of the cuboidal box

$= 2(\lambda \times b + b \times h + h \times \lambda)$

$= 2(12.5 \times 10 + 10 \times 8 + 8 \times 12.5) \text{ cm}^2$

$= 2(125 + 80 + 100) \text{ cm}^2 = 610 \text{ cm}^2$

Clearly, cubical box has smaller surface area by

$610 \text{ cm}^2 - 600 \text{ cm}^2 = 10 \text{ cm}^2$

Ex.16 Find the volume of a solid cube of side 12 cm. If this cube is cut into 8 identical cubes, find :

(i) Volume of each small cube.

(ii) Side of each small cube.

(iii) Surface area of each small cube.

Sol. Since, side (edge) of the given solid cube = 12 cm.

$\therefore$ Volume of given solid cube = (edge)^3

$= (12 \text{ cm})^3 = 1728 \text{ cm}^3$ Ans.

(i) As the given cube is cut into 8 identical cubes.

$\Rightarrow$ Vol. of 8 small cubes obtained

= Vol. of given cube $= 1728 \text{ cm}^3$

$\Rightarrow$ Volume of each small cube

$= \dfrac{1728 \text{ cm}^3}{8} = 216 \text{ cm}^3$

(ii) If edge (side) of each small cube = x cm

(edge)^3 = Volume

$\Rightarrow x^3 = 216 = 6 \times 6 \times 6 = 6^3 \Rightarrow x = 6$ cm

$\therefore$ Side of each small cube = 6 cm

(iii) Surface area of each small cube

$= 6 \times (\text{edge})^2$

$= 6 \times (6 \text{ cm})^2 = 216 \text{ cm}^2$

Ex.17 A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute ? **[NCERT]**

Sol. Volume of water that flows through a river, canal or pipe, etc., in unit time

= Area of cross-section × Speed of water through it.

$x \text{ km/hr} = x \times \dfrac{5}{18} \text{ m/s}$.

Reason : $1 \text{ km/hr} = \dfrac{1000\text{m}}{60 \times 60 \text{ sec}} = \dfrac{5}{18} \text{ m/s}$

Since, area of cross-section of the river

= Its depth × its width

$= 3\text{m} \times 40\text{m} = 120 \text{ m}^2$

And, speed of flow of water through the river

$= 2 \text{ km/hr} = 2 \times \dfrac{5}{18} \text{ m/s} = \dfrac{5}{9} \text{ m/s}$

$\therefore$ Vol. of water that flows through it in 1 sec.

= Area of cross-section × speed of water through it.

$= 120 \times \dfrac{5}{9} \text{ m}^3 = \dfrac{200}{3} \text{ m}^3$

$\Rightarrow$ Vol. of water that flows through it in 1 min. (60 sec.)

$= \dfrac{200}{3} \times 60 \, m^3 = 4000 \text{ m}^3$

$\Rightarrow$ Vol. of water that will fall into the sea in a minute. $= 4000 \text{ m}^3$

Ex.18 The volume of a cube is numerically equal to its surface area. Find the length of its one side.

Sol. Let length of each side is α unit.

Given: Volume of the cube = Surface area of the cube. [Numerically]

$\Rightarrow$ $a^3 = 6a^2$ $\Rightarrow$ $a = 6$

$\therefore$ The length of one side of the cube = 6 cm

Ex.19 A solid cuboid has square base and height 12 cm. If its volume is 768 cm^3, find :

(i) side of its square base.

(ii) surface area.

Sol.(i) Let side of the square base be x cm

i.e., $\lambda = b = x$ cm

$\lambda \times b \times h$ = volume

$\Rightarrow$ $x \times x \times 12 = 768$

[Given, height = 12 cm]

$\Rightarrow$ $x^2 = \frac{768}{12} = 64 \Rightarrow x = \sqrt{64}$ cm = 8 cm.

$\therefore$ Side of the square base = 8 cm

(ii) Now, λ = 8 cm, b = 8 cm and h = 12 cm

$\therefore$ Surface area = $2(\lambda \times b + b \times h + h \times \lambda)$

$= 2(8 \times 8 + 8 \times 12 + 12 \times 8)\ cm^2 = 512\ cm^2$

RIGHT CIRCULAR CYLINDER

Let radius of the base = r and height = h

Curved surface area = $2\pi rh$

Total surface area = $2\pi r(h + r)$

Volume = $\pi r^2 h$ where, $\pi = \frac{22}{7}$ or 3.14

❖ EXAMPLES ❖

Ex.20 The inner diameter of a circular well in 2 m and its depth is 10.5 m. Find :

(i) the inner curved surface area of the well.

(ii) the cost of plastering this curved surface area at the rate of Rs. 35 per m^2.

Sol. Given : Radius = $\frac{1}{2} \times 2m$ = 1m i.e., r = 1 m

and depth = 10.5 m i.e., h = 10.5 m

(i) The inner curved surface area of the well

$= 2\pi rh = 2 \times \frac{22}{7} \times 1 \times 10.5\ m^2 = 66\ m^2$

(ii) The cost of plastering

= Area to be plastered × Rate

= 66 × Rs. 35 = Rs. 2,310

Ex.21 In a hot water heating system there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system. **[NCERT]**

Sol. Given : Length of the cylindrical pipe = 28 m i.e., h = 2800 cm and,

its radius = $\frac{1}{2}$ × diameter = $\frac{1}{2}$ × 5 cm = 2.5 cm.

$\therefore$ The total radiating surface in the system

= curved surface area of the pipe

$= 2\pi rh = 2 \times \frac{22}{7} \times 2.5 \times 2800\ cm^2$

$= 44000\ cm^2$

It is not clear from the question that how the cylindrical pipe is used. We can take the radiating surface of the system

= Total surface area of the pipe

$= 2\pi r\,(r + h) = 2 \times \frac{22}{7} \times 2.5\ (2.5 + 2800)\ cm^2$

$= 44039.29\ cm^2$

Ex.22 Find :

(i) the lateral or curved surface area of a cylindrical patrol storage tank that is 4.2 m in diameter and 4.5 m high.

(ii) how much steel was actually used if $\frac{1}{12}$ of the steel actually used was wasted in making the closed tank. **[NCERT]**

Sol. (i) Given : $r = \frac{4.2}{2}$ m = 2.1 m and h = 4.5 m

$\therefore$ Curved surface area of the tank

$= 2\pi rh = 2 \times \frac{22}{7} \times 2.1 \times 4.5\ m^2 = 59.4\ m^2$

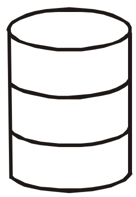
(ii) Let the steel actually used be x m^2

$\therefore$ $x - \frac{1}{12}x = 59.4$ $\Rightarrow$ $\frac{11}{12}x = 59.4$

$\Rightarrow$ $x = 59.4 \times \frac{12}{11} = 64.8\ m^2$

$\therefore$ Actual amount of steel used = 64.8 m^2

Ex.23 The figure, given alongside, shows the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame . Find how much cloth is required for covering the lampshade. **[NCERT]**



Sol. Given : The height of the lampshade = 30 cm.

Θ A margin of 2.5 cm is required for folding its over and bottom of the frame ; the resulting height of the cloth required in the shape of the cylinder = (30 + 2.5 + 2.5) cm = 35 cm.

∴ For the cloth, which must be in the shape of a cylinder with height 35 cm and radius $\frac{20}{2}$ cm = 10 cm.

i.e., h = 35 cm and r = 10 cm.

∴ Area of the cloth required for covering the lampshade.

$= 2\pi rh = 2 \times \frac{22}{7} \times 10 \times 35 \text{ cm}^2$

$= 2200 \text{ cm}^2$

Ex.24 The total surface area and the curved surface area of a cylinder are in the ratio 5 : 3. Find the ratio between its height and its diameter.-

Sol. Required = $\frac{\text{Height}}{\text{Diameter}} = \frac{h}{d} = \frac{h}{2r}$

According to the given statement :

$\frac{2\pi r(h+r)}{2\pi rh} = \frac{5}{3} \Rightarrow \frac{h+r}{h} = \frac{5}{3}$

$\Rightarrow$ 5h = 3h + 3r

i.e., 2h = 3r

$\Rightarrow \frac{h}{r} = \frac{3}{2}$

and $\frac{h}{2r} = \frac{3}{2\times 2} = \frac{3}{4}$ = 3 : 4

Ex.25 A soft drink is available in two packs of different shapes : (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

[NCERT]

Sol. (i) Volume of soft drink in the tin can

= Volume of the tin can

= Its length × breadth × height

$= 5 \text{ cm} \times 4 \text{ cm} \times 15 \text{ cm} = 300 \text{ cm}^3$

(ii) Volume of soft drink in the plastic cylinder

$= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10 \text{ cm}$

$[\Theta\ r = \frac{\text{diameter}}{2} = \frac{7}{2} \text{ cm}]$

$= 385 \text{ cm}^3$

Clearly, plastic container has greater capacity by

$385 \text{ cm}^3 - 300 \text{ cm}^3 = 85 \text{ cm}^3$

Ex.26 The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many liters of water can it hold? ($1000\text{cm}^3 = 1\lambda$)

Sol. Let the radius of the base of the cylindrical vessel be r cm.

∴ $2\pi r = 132$ [Θ circumference = $2\pi r$]

$\Rightarrow 2 \times \frac{22}{7} \times r = 132$

$\Rightarrow r = \frac{132 \times 7}{2 \times 22} \text{ cm} = 21 \text{ cm}$

Now, radius (r) = 21 cm and height (h) = 25 cm

$\Rightarrow$ Volume of the cylindrical vessel = $\pi r^2 h$

$= \frac{22}{7} \times 21 \times 21 \times 25 \text{ cm}^2 = 34560 \text{ cm}^3$

∴ Vol. of water which this vessel can hold

= Volume of the vessel = 34560 cm^3

$= \frac{34560}{1000} \lambda$ [Θ $1000 \text{ cm}^3 = 1\lambda$]

$= 34.650\ \lambda$

Ex.27 A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients? **[NCERT]**

Sol. Radius of cylindrical bowl

$= \frac{7}{2}$ cm i.e.,

$r = \frac{7}{2}$ cm

and the height of the soup in a bowl h = 4 cm

$\therefore$ Volume of soup required for 1 patient

$= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 4\,\text{cm}^3 = 154\ \text{cm}^3$.

$\Rightarrow$ Volume of soup required for 250 patients.

$= 250 \times 154\ \text{cm}^3 = 38500\ \text{cm}^3$

Ex.28 If the lateral surface of a cylinder is 94.2 cm^2 and its height is 5 cm, then find:

(i) radius of its base

(ii) its volume. (Use $\pi = 3.14$)

Sol. Given. $2\pi rh = 94.2\ \text{cm}^2$ and h = 5 cm

(i) $2\pi rh = 94.2\ \text{cm}^2$

$\Rightarrow 2 \times 3.14 \times r \times 5 = 94.2$

$\Rightarrow r = \frac{94.2}{2 \times 3.14 \times 5}\,\text{cm} = 3\ \text{cm}$.

(ii) Its volume $= \pi r^2 h$

$= 3.14 \times 3 \times 3 \times 5\ \text{cm}^2$

$= 141.3\ \text{cm}^3$

➤ HOLLOW CYLINDER

Let external radius = R, Internal radius = r, height = h. Then,
Outer curved surface area = $2\pi Rh$
Inner curved surface area = $2\pi rh$
Area of cross section = $\pi R^2 - \pi r^2$
Total surface area = $2\pi Rh + 2\pi rh + 2\pi (R^2 - r^2)$
Volume = $\pi(R^2 - r^2)h$

❖ EXAMPLES ❖

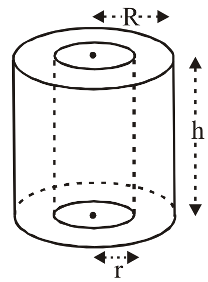
Ex.29 A hollow cylinder is 35 cm in length (height). Its internal and external diameters are 8 cm and 8.8 cm respectively. Find its :

(i) outer curved surface area

(ii) inner curved surface area

(iii) area of cross-section

(iv) total surface area.



Sol. The height of the cylinder h = 35 cm

The internal radius $r = \frac{8}{2}\,\text{cm} = 4\ \text{cm}$

The external radius $R = \frac{8.8}{2}\,\text{cm} = 4.4\ \text{cm}$

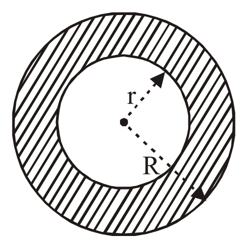
(i) Outer curved surface area = $2\pi Rh$

$= 2 \times \frac{22}{7} \times 4.4 \times 35\,\text{cm}^2 = 968\ \text{cm}^2$

(ii) Inner curved surface area = $2\pi rh$

$= 2 \times \frac{22}{7} \times 4 \times 35\,\text{cm}^2 = 880\ \text{cm}^2$

(iii) The cross-section of a hollow cylinder is like a ring with external radius R = 4.4 cm and internal radius r = 4 cm.



$\therefore$ Area of cross-section $= \pi R^2 - \pi r^2$

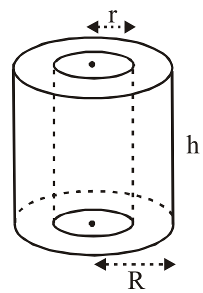
$= \pi(R^2 - r^2) = \frac{22}{7}(4.4^2 - 4^2)\,\text{cm}^2$

$= \frac{22}{7}(4.4 + 4)(4.4 - 4)\,\text{cm}^2$

$= \frac{22}{7} \times 8.4 \times 0.4\,\text{cm}^2$

$= 10.56\ \text{cm}^2$

Ex.30 Find the total surface area of a hollow cylindrical pipe of length 50 cm, external diameter 12 cm and internal diameter 9 cm.-



Sol. Given : Height (h) = 50 cm, external radius (R) = 6 cm and internal radius (r) = 4.5 cm

Θ Total surface area of the hollow cylinder

= External C.S.A + Internal C.S.A

+ 2 × Area of cross-section

$= 2\pi Rh + 2\pi rh + 2\pi (R^2 - r^2)$

$= 2\pi (R + r)h + 2\pi(R + r)(R - r)$

$= 2 \times \frac{22}{7} \times (6 + 4.5) \times 50$

$+ 2 \times \frac{22}{7} \times (6 + 4.5)(6 - 4.5)\ cm^2$

$= \frac{44}{7} \times 10.5 \times 50 + \frac{44}{7} \times 10.5 \times 1.5 cm^2$

$= 3300\ cm^2 + 99\ cm^2 = 3399\ cm^2$

Ex.31 The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has a mass of 0.6 g.

Sol. Since, inner diameter = 24 cm

⇒ inner radius (r) = $\frac{24}{2}$ cm = 12 cm

Since, outer diameter = 28 cm

⇒ outer radius (R) = $\frac{28}{2}$ cm = 14 cm

Also, given that height (h) = 35 cm

∴ Volume of wood in the pipe = $\pi(R^2 - r^2)h$

$= \frac{22}{7}(14^2 - 12^2) \times 35\ cm^3 = 5720\ cm^3$

Since, mass of 1 cm^3 of wood = 0.6 gm

⇒ Mass of 5720 cm^3 of wood = 0.6 × 5720 gm

= 3432 gm

∴ Mass of the pipe = mass of wood

= 3432 gm

= 3.432 kg

Ex.32 A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

[NCERT]

Sol. Clearly, for wooden part, which is the form of a hollow cylinder :

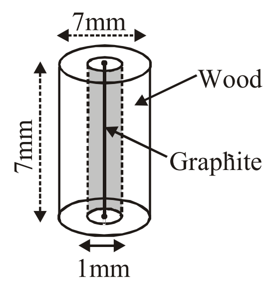
External radius (R) = $\frac{7}{2}$ mm = 3.5 mm

= 0.35 cm

Internal radius (r) = $\frac{1}{2}$ mm = 0.5 mm

= 0.05 cm

And, height (length) = 14 cm.



∴ Volume of the wood = $\pi(R^2 - r^2)h$

$= \frac{22}{7} \times [(0.35)^2 - (0.05)^2] \times 14\ cm^3$

$= \frac{22}{7} \times 0.12 \times 14\ cm^3 = 5.28\ cm^3$

And, volume of graphite = $\pi r^2 h$

$= \frac{22}{7} \times (0.05)^2 \times 14\ cm^3$

$= 0.11\ cm^3$

Ex.33 The internal radius of a hollow cylinder is 8 cm and thickness of its wall is 2 cm. Find the volume of material in the cylinder, if its length is 42 cm.

Sol. Since, internal radius = 8 cm ⇒ r = 8 cm,

thickness of the wall of the cylinder = 2 cm.

∴ Its external radius = 8 cm + 2 cm = 10 cm i.e., R = 10 cm.

Also, length (height) of the cylinder = 42 cm i.e., h = 42 cm.

∴ Volume of material in the hollow cylinder.

$= \pi(R^2 - r^2)h$

$= \frac{22}{7}[(10)^2 - (8)^2] \times 42\ cm^3$

$= \frac{22}{7} \times 36 \times 42\ cm^3 = 4752\ cm^3$

Ex.34 The radii of two right circular cylinders are in the ratio 3 : 4 and their heights are in the ratio 6 : 5. Find the ratio between their curved (lateral) surface areas.

Sol. If the radii of two cylinders be r_1 and r_2, let $r_1 = 3x$ and $r_2 = 4x$.

Similarly, if the heights of two cylinders be h_1 and h_2, let $h_1 = 6y$ and $h_2 = 5y$.

Ratio between their C.S.A.

$= \frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \frac{2\pi \times 3x \times 6y}{2\pi \times 4x \times 5y} = \frac{9}{10}$

= 9 : 10

➤ RIGHT CIRCULAR CONE

Let λ, h and r are the slant height, height and radius of a cone then

$\lambda^2 = h^2 + r^2$

Area of base = πr^2

Curved (lateral) surface area = $\pi r \lambda$

Total surface area = $\pi r(\lambda + r)$

Volume = $\frac{1}{3}\pi r^2 h$.

❖ EXAMPLES ❖

Ex.35 The height of a cone is 48 cm and the radius of its base is 36 cm. Find the curved surface area and the total surface area of the cone. (Take π = 3.14).

Sol. Given : h = 48 cm and r = 36 cm.

∴ $\lambda^2 = h^2 + r^2$

⇒ $\lambda^2 = 48^2 + 36^2 = 2304 + 1296 = 3600$

⇒ $\lambda = \sqrt{3600}$ cm = 60 cm

∴ The curved surface area = $\pi r \lambda$

$= 3.14 \times 36 \times 60\ cm^2 = 6782.4\ cm^2$

And, the total surface area of the cone

$= \pi r\lambda + \pi r^2 = \pi r(\lambda + r)$

$= 3.14 \times 36 \times (60 + 36) cm^2$

$= 10851.84\ cm^2$.

Ex.36 Curved surface area of a cone is 2200 cm^2.

It its slant height is 50 cm, find :

(i) radius of the base.

(ii) total surface area.

(iii) height of the cone.

Sol. (i) Given : $\pi r \lambda = 2200\ cm^2$ and λ = 50 cm

⇒ $\frac{22}{7} \times r \times 50 = 2200$

i.e., $r = \frac{2200 \times 7}{22 \times 50}$ cm = 14 cm

(ii) Total surface area = $\pi r\lambda(\lambda + r)$

$= \frac{22}{7} cm \times 14 \times (50 + 14)\ cm^2$

$= 2816\ cm^2$ Ans.

(iii) $\lambda^2 = h^2 + r^2$ ⇒ $h^2 = \lambda^2 - r^2$

$= 50^2 - 14^2 = 2500 - 196 = 2304$

∴ $h = \sqrt{2304}$ cm = 48 cm

Ex.37 A conical tent is 10 m high and radius of its base is 24 m. Find **[NCERT]**

(i) slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs. 70.

Sol. (i) Given : h = 10 m and r = 24 m

∴ $\lambda^2 = h^2 + r^2$

$\Rightarrow\ \lambda^2 = 10^2 + 24^2 = 100 + 576 = 676$

$\Rightarrow\ \lambda = \sqrt{676}\ \text{m} = 26\ \text{m}$

(ii) Area of canvas required

= Curved surface area of the tent

$= \pi r\lambda = \frac{22}{7} \times 24 \times 26\text{m} = \frac{13728}{7}\text{m}^2$

Θ Cost of 1 m^2 canvas is Rs. 70

$\therefore$ Total cost of canvas required

$= \frac{13728}{7} \times \text{Rs.}70 = \text{Rs. } 1{,}37{,}280$

Ex.38 What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use $\pi = 3.14$).

[NCERT]

Sol. For the tent : h = 8 m and r = 6 m

$\lambda^2 = h^2 + r^2 \quad \Rightarrow \quad \lambda^2 = 8^2 + 6^2$

$= 64 + 36 = 100$ and $\lambda = \sqrt{100}$ m = 10m

Curved surface area of the tent = $\pi r\lambda$

$= 3.14 \times 6 \times 10\ \text{m}^2 = 188.4\ \text{m}^2$

$\Rightarrow$ Area of tarpaulin used = 188.4 m^2

$\Rightarrow$ Length of tarpaulin × its width = 188.4 m^2

$\Rightarrow$ Length of tarpaulin × 3 m = 188.4 m^2

$\Rightarrow$ Length of tarpaulin = $\frac{188.4}{3}$m = 62.8m

Θ Extra length of tarpaulin required

= 20 cm = 0.2 m

$\therefore$ Total length of tarpaulin required

= 62.8 m + 0.2 m = 63 m

Ex.39 A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs. 12 per m^2, what will be the cost of painting all these cones?

(Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$)

[NCERT]

Sol. For each cone :

$r = \frac{40}{2}$ cm = 20 cm = 0.2 m and h = 1 m

$\therefore\ \lambda^2 = h^2 + r^2 \Rightarrow \lambda^2 = (1)^2 + (0.2)^2$

= 1 + 0.04 = 1.04

$\Rightarrow\ \lambda = \sqrt{1.04}$ m = 1.02 m

C.S.A of each cone = $\pi r\lambda$

$= 3.14 \times 0.2 \times 1.02\ \text{m}^2 = 0.64056\ \text{m}^2$

$\Rightarrow$ C.S.A of 50 cones = 50 × 0.64056 m^2

= 30.028 m^2 = Area to be painted

Θ The cost of painting is Rs. 12 per m^2

$\therefore$ The total cost of painting outer sides of 50 cones

= 30.028 × Rs. 12 = Rs. 384.34

Ex.40 The radius and the slant height of a cone are in the ratio 3 : 5. If its curved surface area is 2310 cm^2, find its height.

Sol. Given : $r : \lambda = 3 : 5$

$\Rightarrow$ if r = 3x cm, λ = 5x cm

C.S.A. = $\pi r\lambda \Rightarrow \frac{22}{7} \times 3x \times 5x = 2310$

$\Rightarrow\ x^2 = \frac{2310 \times 7}{22 \times 3 \times 5} = 49 \Rightarrow x = 7$

$\therefore$ r = 3x = 3 × 7 cm = 21 cm

and λ = 5x = 5 × 7 cm = 35 cm,

$\lambda^2 = h^2 + r^2 \Rightarrow h^2 = 35^2 - 21^2$

= 1225 – 441 = 784

$\therefore$ Height (h) = $\sqrt{784}$cm = 28 cm

Ex.41 A circus tent is in the shape of a cylinder, upto a height of 8 m, surmounted by a cone of the same radius 28 m. If the total height of the tent is 13 m, find:

(i) total inner curved surface area of the tent.

(ii) cost of painting its inner surface at the rate of Rs. 3.50 per m^2.

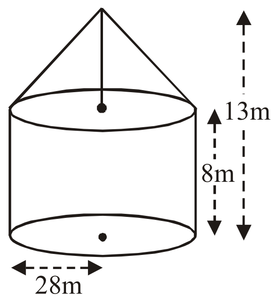
Sol. According to the given statement, the rough sketch of the circus tent will be as shown:

(i) For the cylindrical portion :

r = 28 and h = 8 m

$\therefore$ Curved surface area = $2\pi rh$

$= 2 \times \frac{22}{7} \times 28 \times 8\ m^2 = 1408\ m^2$



For conical portion :

r = 28 m and h = 13 m – 8 m = 5 m

$\therefore$ $\lambda^2 = h^2 + r^2 \Rightarrow \lambda^2 = 5^2 + 28^2 = 809$

$\Rightarrow \lambda = \sqrt{809}$ m = 28.4 m

$\therefore$ Curved surface area = $\pi r\lambda$

$= \frac{22}{7} \times 28 \times 28.4\ m^2 = 2499.2\ m^2$

$\therefore$ Total inner curved surface area of the tent.

= C.S.A. of cylindrical portion + C.S.A. of the conical portion

$1408\ m^2 + 2499.2\ m^2 = 3907.2\ m^2$

(ii) Cost of painting the inner surface

= 3907.2 × Rs. 3.50 = Rs. 13675.20

Ex.42 The height of a cone is 30 cm and its volume is 3140 cm^3. Taking $\pi = 3.14$, find :

(i) radius of the base.

(ii) area of the base.

Sol. (i) Given : h = 30 cm and volume = 3140 cm^3

Volume = $\frac{1}{3}\pi r^2 h$

$\Rightarrow 3140 = \frac{1}{3} \times 3.14 \times r^2 \times 30$

$\Rightarrow r^2 = \frac{3140 \times 3}{3.14 \times 30} = 100$ and r = 10cm

(ii) Area of the base = πr^2

$= 3.14 \times 10^2\ cm^2 = 314\ cm^2$

Alternative Method :

$\frac{1}{3} \times$ area of base × height = volume

$\Rightarrow \frac{1}{3} \times$ area of base × 30 = 3140

$\Rightarrow$ Area of base = $\frac{3140 \times 3}{30} cm^2$

$= 314\ cm^2$

Ex.43 A right triangle ABC has sides 5 cm, 12 cm and 13 cm. Find the : **[NCERT]**

(i) volume of solid obtained by revolving Δ ABC about the side 12 cm.

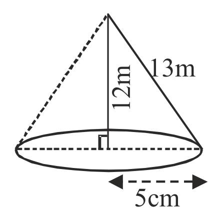
(ii) volume of solid obtained by revolving Δ ABC about side 5 cm.

(iii) difference between the volumes of the solids obtained in step (i) and step (ii).

Sol. $\because$ $5^2 + 12^2 = 13^2 \Rightarrow$ Angle opp. to 13 cm is right angle

(i) When the Δ is revolved about the side of 12 cm, for the cone formed :

h = 12 cm and r = 5.



$\therefore$ Volume of solid obtained = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12\ cm^3$

$= 314.29\ cm^3$

(ii) When the Δ is revolved about the side of 5 cm, for the cone formed :

h = 5 cm, and r = 12 cm.

$\therefore$ Volume of solid obtained = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 5\ cm^3 = 754.29\ cm^3$

(iii) Required difference

$= 754.29\ cm^3 - 314.29\ cm^3 = 440\ cm^3$

Ex.44 The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find:

(i) height of the cone.

(ii) slant height of the cone

(iii) curved surface area of the cone.

Sol. Given : volume of cone = 9856 cm^3

and radius (r) = $\frac{28}{2}$ cm = 14cm

(i) $\text{Volume} = \frac{1}{3}\pi r^2 h \Rightarrow 9856 = \frac{1}{3}\times\frac{22}{2}\times 14\times 14\times h$

$\Rightarrow \quad h = \frac{9856\times 3\times 7}{22\times 14\times 14}\text{cm} = 48 \text{ cm}$

(ii) $\lambda^2 = h^2 + r^2 \Rightarrow \lambda^2 = 48^2 + 14^2$

$= 2304 + 196 = 2500$

$\Rightarrow \lambda = \sqrt{2500}$ cm = 50 cm

$\therefore$ Slant height = 50 cm

(iii) Curved surface area = $\pi r\lambda$

$= \frac{22}{7}\times 14 \times 50 \text{ cm}^2 = 2200 \text{ cm}^2$

Ex.45 A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Sol. For the conical heap :

Radius (r) = $\frac{10.5}{2}$ m = 5.25 m

and height (h) = 3m.

$\therefore$ Volume = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times 5.25 \times 3 \text{ m}^3 = 86.625 \text{ m}^3$

Now, $\lambda^2 = h^2 + r^2 \Rightarrow \lambda^2 = (3)^2 + (5.25)^2$

$= 9 + 27.5625 = 36.5625$

$\Rightarrow \lambda = \sqrt{36.5625}$ m = 6.047 m

$\therefore$ Area of canvas required

= curved surface area of the conical heap

$= \pi r\lambda = \frac{22}{7}\times 5.25\times 6.047\text{m}^2 = 99.7755 \text{ m}^2$

Ex.46 A cylinder and a cone have same base area. But the volume of cylinder is twice the volume of cone. Find the ratio between their heights.

Sol. Since, the base areas of the cylinder and the cone are the same.

$\Rightarrow$ their radius are equal (same).

Let the radius of their base be r and their heights be h_1 and h_2 respectively.

Clearly, volume of the cylinder = $\pi r^2 h_1$

and, volume of the cone = $\frac{1}{3}\pi r^2 h_2$

Given :

Volume of cylinder = 2 × volume of cone

$\Rightarrow \pi r^2 h_1 = 2 \times \frac{1}{3}\pi r^2 h_2$

$\Rightarrow h_1 = \frac{2}{3} h_2$

$\Rightarrow \frac{h_1}{h_2} = \frac{2}{3}$

i.e., $h_1 : h_2 = 2 : 3$

SPHERE & HEMISPHERE

Let radius of sphere = r

Surface area = $4\pi r^2$

Curved surface area of a hemisphere = $2\pi r^2$

Total surface area of a hemisphere = $3\pi r^2$

Volume of the sphere = $\frac{4}{3}\pi r^3$

Volume of the hemisphere = $\frac{2}{3}\pi r^3$.

❖ EXAMPLES ❖

Ex.47 Find the total surface area of the hemisphere of radius 20 cm. (Take π = 3.14).

Sol. Total surface area of the hemisphere

$= 3\pi r^2$

$= 3 \times 3.14 \times (20)^2 \text{ cm}^2$

[Given : r = 20 cm]

$= 3768 \text{ cm}^2$

Ex.48 The area of the flat surface of a hemisphere is 154 cm^2. Find its total surface area.

Sol. Given : $\pi r^2 = 154$

$\Rightarrow \frac{22}{7} r^2 = 154$

$\Rightarrow r^2 = 154 \times \frac{7}{22} = 49$

$\Rightarrow r = 7$ cm

$\therefore$ Its total surface are = $3\pi r^2$

$= 3 \times \frac{22}{7} \times 7 \times 7 cm^2 = 462 cm^2$

Alternative method :

Total surface area of the hemisphere = $3\pi r^2$

$= 3 \times 154$ cm^2 [Given : $\pi r^2 = 154$]

$= 462$ cm^2

Ex.49 The radius of a spherical balloon increases from 10 cm to 15 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Sol. Required ratio

$$= \frac{\text{Surface area of the balloon in 1st case}}{\text{Surface area of the balloon in 2nd case}}$$

$$= \frac{4\pi r^2 \text{ in 1st case}}{4\pi r^2 \text{ in 2nd case}} = \frac{4 \times \pi \times 10 \times 10}{4 \times \pi \times 15 \times 15} = \frac{4}{9}$$

$= 4 : 9$

Ex.50 A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin plating it on the inside at the rate of Rs. 16 per 100 cm^2. **[NCERT]**

Sol. Θ Inner diameter = 10.5 cm

$\Rightarrow$ Inner radius (r) = $\frac{10.5}{2}$ cm = 5.25 cm

The area of tin plating = Inner curved surface area of the bowl = $2\pi r^2$

$= 2 \times \frac{22}{7} \times 5.25 \times 5.25$ cm^2

$= 173.25$ cm^2

Θ Cost of 100 cm^2 tin-plating = Rs.16

$\Rightarrow$ Cost of 1 cm^2 tin-plating = $\frac{16}{100}$

$\Rightarrow$ Cost of 173.25 cm^2 tin-plating

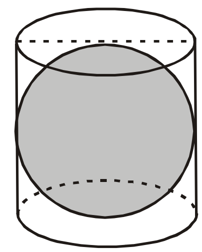
= Rs. $\frac{16}{100} \times 173.25$ = Rs. 27.72

Ex.51 A right circular cylinder just encloses a sphere of radius r (see the given figure.). Find: **[NCERT]**

(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in (i) and (ii).



Sol. (i) Surface area of the sphere = $4\pi r^2$

(ii) Since, the height of the cylinder = diameter of the sphere

$\Rightarrow h = 2r$ $\therefore$ C.S.A. of the cylinder

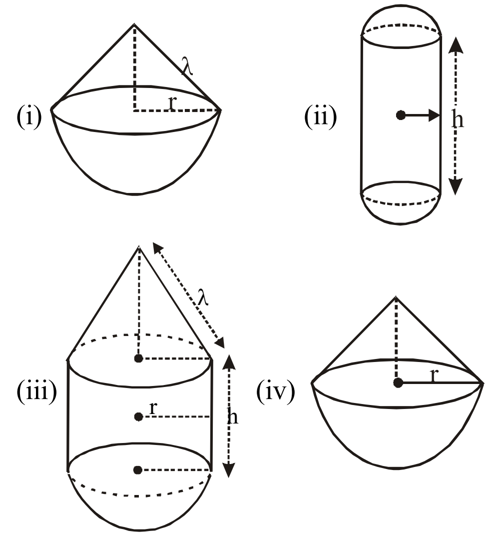
$= 2\pi rh = 2\pi r \times 2r = 4\pi r^2$

(iii) Required ratio = $\frac{4\pi r^2}{4\pi r^2} = 1 = 1 : 1$

If a cylinder just encloses a sphere, the surface area of the sphere is always same as the curved surface area of the cylinder.

In other words, if a sphere and a cylinder have the same radius and same height, there curved surface areas are also equal.

Ex.52 Find the formula for the total surface area of each figure given bellow :



Sol. (i) Required surface area

= C.S.A. of the hemisphere

+ C.S.A. of the cone

$= 2\pi r^2 + \pi r\lambda = \pi r\,(2r + \lambda)$

(ii) Required surface area

= 2 × C.S.A. of a hemisphere

+ C.S.A. of the cylinder

$= 2 \times 2\pi r^2 + 2\pi rh = 2\pi r\,(2r + h)$.

(iii) Required surface area

= C.S.A. of the hemisphere

+ C.S.A. of the cylinder + C.S.A. of the cone

$= 2\pi r^2 + 2\pi rh + \pi r\lambda = \pi r\,(2r + 2h + \lambda)$

(iv) If slant height of the given cone be λ

$= \lambda^2 = h^2 + r^2$

$\Rightarrow \lambda = \sqrt{h^2 + r^2}$

And, required surface area

$= 2\pi r^2 + \pi r\lambda = \pi r\,(2r + \lambda)$

$= \pi r\left(2r + \sqrt{h^2 + r^2}\right)$

Ex.53 The radius of a sphere increases by 25%. Find the percentage increase in its surface area.

Sol. Let the original radius be r.

$\Rightarrow$ Original surface area of the sphere $= 4\pi r^2$

Increase radius = r + 25% of r

$= r + \frac{25}{100}r = \frac{5r}{4}$

$\Rightarrow$ Increased surface area

$= 4\pi\left(\frac{5r}{4}\right)^2 = \frac{25\pi r^2}{4}$

Increased in surface area

$= \frac{25\pi r^2}{4} - 4\pi r^2$

$= \frac{25\pi r^2 - 16\pi r^2}{4} = \frac{9\pi r^2}{4}$

and, percentage increase in surface area

$= \frac{\text{Increase in area}}{\text{Original aera}} \times 100\%$

$= \frac{\frac{9\pi r^2}{4}}{4\pi r^2} \times 100\% = \frac{9}{16} \times 100\%$

= 56.25%

Alternative Method :

Let original radius = 100

$\Rightarrow$ Original C.S.A. $= \pi(100)^2 = 10000\pi$

Increased radius = 100 + 25% of 100 = 125

$\Rightarrow$ Increased C.S.A. $= \pi(125)^2 = 15625\pi$

Increase in C.S.S. $= 15625\pi - 10000\pi$

$= 5625\pi$

$\therefore$ Percentage increase in C.S.A.

$= \frac{\text{Increase in C.S.A.}}{\text{Original C.S.A}} \times 100\%$

$= \frac{5625\pi}{1000\pi} \times 100\% = 56.25\%$

Conversely, if diameter decreases by 20%, the radius also decreases by 20%.

Ex.54 The diameter of a solid metallic ball is 8.4 cm. Find its mass, if density of its material is 6.8 gm per cm^3.

Sol. Since, diameter of the ball = 8.4 cm,

its radius (r) $= \frac{8.4}{2}$ cm = 4.2 cm

Volume of material in the ball

= Volume of the ball $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2\ cm^3 = 310.464\ cm^3$

Since, mass = Volume × density

$\therefore$ Mass of the ball

= 310.464 × 6.8 gm

= 2111.1552 gm = 2.111 kg (App.)

Ex.55 The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon ? **[NCERT]**

Sol. Given : The diameter of the moon

$= \frac{1}{4} \times$ the diameter of the earth

⇒ The radius of the moon

$= \frac{1}{4} \times$ the radius of the earth.

$\Rightarrow R_m = \frac{1}{4} \times R_e$

Now, $\frac{\text{the volume of the moon}}{\text{the volume of the earth}} = \frac{\frac{4}{3}\pi R_m^3}{\frac{4}{3}\pi R_e^3}$

$= \frac{R_m^3}{R_e^3} = \frac{\left(\frac{1}{4}R_e\right)^3}{R_e^3} = \frac{1}{64}$

∴ The volume of the moon

$= \frac{1}{64}$ times the volume of earth.

Ex.56 Twenty seven solid iron spheres, each of radius r and surface area S, are melted to form a sphere with surface area S′. Find the- **[NCERT]**

(i) radius r′ of the new sphere,

(ii) ratio of S and S′.

Sol. (i) Vol. of bigger solid sphere formed = 24 × vol. of each solid sphere melted.

$\Rightarrow \frac{4}{3}\pi(r')^3 = 27 \times \frac{4}{3}\pi r^3$

$\Rightarrow (r')^3 = 27r^3 = (3r)^3 \quad \Rightarrow r' = 3r$

(ii) Θ S = surface area of each sphere melted = $4\pi r^2$

And, S′ = Surface area of the sphere formed

$= 4\pi (r')^2 = 4\pi (3r)^2 = 36\pi r^2$

∴ Ratio of S and S′ = $\frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9} = 1 : 9$

Ex.57 A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank. **[NCERT]**

Sol. Since, the inner radius (r) = 1 m = 100 cm and thickness of sheet = 1cm

∴ External radius (R) = 100 cm + 1 cm = 101 cm

∴ The volume of the iron used to make the hemispherical tank

= Its external volume – Its internal volume

$= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \ (R^3 - r^3)$

$= \frac{2}{3} \times \frac{22}{7} \times (101^3 - 100^3)\ cm^3 = 63487.81\ cm^3$

Ex.58 A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of Rs. 498.96. If the cost of white-washing is Rs. 2.00 per square meter, find the **[NCERT]**

(i) inside surface area of the dome,

(ii) volume of the air inside the dome.

Sol. (i) Cost of white-washing = Rate of white-washing × Surface area of the dome.

⇒ Rs. 498.96 = Rs. 2 × Surface area of the dome.

⇒ Surface area of the dome

$= \frac{498.96}{2} m^2 = 249.48\ m^2$

(ii) Let radius of the hemispherical dome = r m

$\therefore 2\pi r^2 = 249.48$

$\Rightarrow 2 \times \frac{22}{7} \times r^2 = 249.48$

$\Rightarrow r^2 = \frac{249.48 \times 7}{2 \times 22} m^2 = 39.69$

⇒ r = 6.3 m.

∴ Volume of air inside the dome

$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 6.3 \times 6.3 \times 6.3\ m^3$

$= 523.908\ m^3$

Ex.59 The radii of two spheres are in the ratio 3 : 2. Find the ratio between their volumes.

Sol. Given :

Ratio between the radii of two spheres = 3 : 2

⇒ If radius of one sphere = 3r,

radius of the other = 2r

Required ratio = $\frac{\text{Volume of one sphere}}{\text{Volume of other sphere}}$

$\frac{\frac{4}{3} \times \pi \times (3r)^3}{\frac{4}{3} \times \pi \times (2r)^3} = \frac{27}{8} = 27 : 8$ Ans.

Ex.60 Three solid spheres of radii 1 cm, 6 cm and 8 cm are melted and recasted into a single sphere. Find the radius of the sphere obtained.

Sol. Let radius of the sphere obtained = R cm.

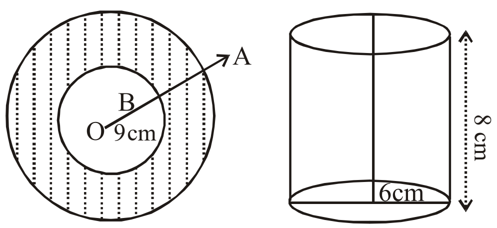
$$\therefore \ \frac{4}{3}\times\pi R^3 = \frac{4}{3}\pi(1)^3 + \frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3.$$

$R^3 = 1 + 216 + 512$

$R = (729)^{1/3}$

$R = 9$ cm.

Ex.61 A spherical shell of lead, whose external diameter is 18 cm, is melted and recast into a right circular cylinder, whose height is 8 cm and diameter 12 cm. Find the internal diameter of the shell.



Sol. We have, height of the right circular cylinder = h = 8cm and radius of the base of it = R = 6cm

So, volume of it = $\pi R^2 h$

$$\left(\frac{22}{7}\times 6\times 6\times 8\right) \text{ cm}^3 \quad \text{....(i)}$$

It is given that the external diameter of the spherical shell = 18 cm

$\Rightarrow$ The external radius of it = $r_1 = \frac{18}{2} = 9$ cm

Let the internal radius of it be r_2 cm.

Since the spherical shell is melted and recast into a right circular cylinder, we have the volume of the solid of spherical shell = volume of the cylinder

$$\Rightarrow \frac{4}{3}\pi(r_1^3 - r_2^3) = \frac{22}{7} \times 6 \times 6 \times 8$$

[Using equation (i)]

$$\Rightarrow \frac{4}{3} \times \frac{22}{7}(9^3 - r_2^3) = \frac{22}{7} \times 36 \times 8$$

$$\Rightarrow \frac{4}{3}(9^3 - r_2^3) = 36 \times 8 \Rightarrow 9^2 - r_2^3 = \frac{108\times 8}{4}$$

$$\Rightarrow r_2^3 = 9^3 - 108 \times 2 = 729 - 216 \Rightarrow r_2^3 = 513$$

$$\Rightarrow r_2 = \text{(Internal radius of the shell)} = \sqrt[3]{513} \cong 8\text{cm}$$

$\Rightarrow$ Internal diameter of the shall

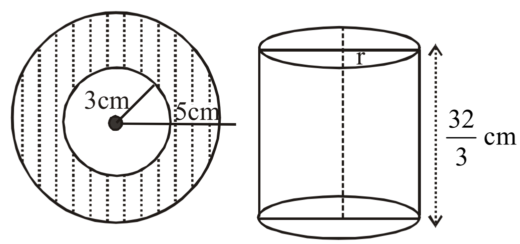
$= 8 \text{ cm} \times 2 \cong 16$ cm

Ex.62 The radius of the internal and external surface of a metallic spherical shell are 3 cm and 5 cm respectively. It is melted and recast into a solid right circular cylinder of height $10\frac{2}{3}$ cm. Find the diameter of the base of the cylinder.

Sol. Here, radius of the internal and external surfaces of a metallic spherical shell are 3 cm and 5 cm respectively.

So, its volume = $\left[\frac{4}{3}\pi(5^3 - 3^3)\right]$ cm^3

$$= \left[\frac{4}{3}\pi\times(125-27)\right]\text{cm}^3 = \left(\frac{4}{3}\pi\times 98\right)\text{cm}^3$$



Let r be the radius of the right circular cylinder of height $\frac{32}{3}$ cm.

Its volume = $\pi r^2 h = \left(\pi\times r^2\times\frac{32}{3}\right)$cm^3

We have

Volume of the spherical shell = volume of the right circular cylinder

$$\Rightarrow \frac{4}{3}\pi \times 98 = \pi \times r^2 \times \frac{32}{3}$$

$$\Rightarrow 392 = 32r^2 \Rightarrow r^2 = \frac{392}{32} = \frac{49}{4}$$

$$\Rightarrow r = \sqrt{\frac{49}{2}} = \frac{7}{2} = 3.5 \text{ cm}$$

Hence, diameter of the right circular cylinder

$= 2r = 2 \times 3.5$ cm $= 7$ cm

Ex.63 A spherical ball of lead 3cm in diameter is melted and recast into three spherical balls. The diameters of two of these are 1 cm and 1.5cm. Find the diameter of the third ball.

Sol. It is given that

The diameter of a spherical ball = 3cm

$\Rightarrow$ radius of it = 1.5 cm = $\frac{3}{2}$ cm

So, volume of it $= \left[\frac{4}{3}\pi\times\left(\frac{3}{2}\right)^3\right]$ cm^3

$= \left(\frac{4}{3}\pi\times\frac{3}{2}\times\frac{3}{2}\times\frac{3}{2}\right)$ cm^3 = $\frac{108\pi}{24}$ cm^3 = $\frac{9\pi}{2}$ cm^2

This spherical ball is melted and recast into three small spherical balls. The diameters of two of these are 1 cm and 1.5 cm respectively.

So, volume of the two spherical balls.

$= \left[\frac{4}{3}\pi\times\left\{\left(\frac{1}{2}\right)^3+\left(\frac{3}{4}\right)^3\right\}\right]$ cm^3

$= \left[\frac{4}{3}\pi\left(\frac{1}{8}+\frac{27}{64}\right)\right]$ cm^3

$= \left(\frac{4}{3}\pi\times\frac{35}{64}\right)$ cm^3 = $\frac{140\pi}{192}$ cm^3

Let r be the radius of the third small spherical ball.

Thus, volume of the third ball = volume of the big spherical ball – sum of volume of two small spherical balls.

$\Rightarrow \frac{4}{3}\pi r^3 = \frac{9\pi}{2} - \frac{140\pi}{192}$

$\Rightarrow \frac{4}{3}r^3 = \frac{9}{2}-\frac{140}{192} = \frac{864-140}{192} = \frac{724}{192}$

$\Rightarrow r^3 = \frac{724\times3}{4\times192} = \frac{181}{64} \quad \Rightarrow r = \sqrt[3]{\frac{181}{64}}$ cm

Hence, diameter of the 3rd spherical ball = 2r

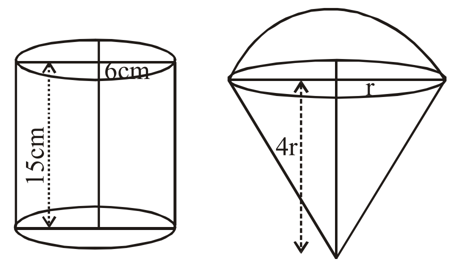
$= 2\times\frac{\sqrt[3]{181}}{\sqrt[3]{64}} = 2\times\frac{\sqrt[3]{181}}{4} = \frac{\sqrt[3]{181}}{2}$ cm

Ex.64 A cylindrical container is filled with ice-cream whose diameter and the height are 12cm and 15cm respectively. The whole ice-cream is distributed to 10 children in equal inverted cones having hemispherical tops. Find the diameter of the ice-cream, if the height of the conical part is twice the diameter of its base.

Sol. We have radius of the cylindrical container = r = $\frac{12}{2}$ = 6cm and height of it (h) = 15cm.

So, its volume = $\pi r^2 h = (\pi\times 6^2\times 15)$ cm^3

$= (\pi\times 36\times 15)$ cm^3 = 540 πcm^3



Let radius of the hemispherical part of the ice-cream = radius of the base of the conical part of the ice-cream = r

So, height of the conical part of the icecream = 4r

So, the volume of one ice-cream = volume of the hemispherical part + volume of the conical part.

$= \left(\frac{2}{3}\pi r^3+\frac{1}{3}\pi r^2\times 4r\right)$ cm^3

$= \left(\frac{2}{3}\pi r^3+\frac{4}{3}\pi r^3\right)$ cm^3 = $(2\pi r^3)$ cm^3

Volume of 10 ice-cream

$= (10\times 2\pi r^3)$cm^3 = $(20\pi r^3)$ cm^3

Here, volume of 10 ice-cream = volume of the cylindrical container

$\Rightarrow 20\pi r^3 = 540\pi \Rightarrow 20r^3 = 540$

$\Rightarrow r^3 = \frac{540}{20} = 27 \Rightarrow r = \sqrt[3]{27} = 3$ cm

Hence, the required diameter of the ice-cream

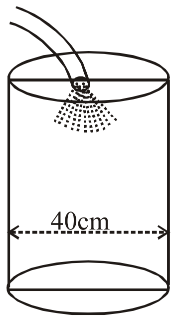
= 2r = 2 × 3 = 6cm

Ex.65 Water flows out through a circular pipe, whose internal diameter is 2cm, at the rate of 0.7m/sec into a cylindrical tank, the radius of whose base is 40cm. By how much will the level of water rise in half an hour ?

Sol. We have volume of water flows out through a circular pipe in 1 second = volume of a cylinder of the base of radius 1 cm (r = $\frac{2}{2}$ = 1 cm) and height 70 cm (h = 0.7m = 70 cm)

$$= \pi r^2 h = \left(\frac{22}{7} \times 1^2 \times 70\right) \text{ cm}^3 = 220 \text{ cm}^3$$

So, volume of water passed through the pipe into the cylindrical tank in 1800 seconds $\left(\frac{1}{2}\text{hour} = \frac{3600}{2} = 1800 \sec\right)$



$$= (220 \times 1800) \text{ cm}^3 = 396000 \text{ cm}^3$$

Thus, rise in the level of water in 1800 sec or half an hour

$$= \frac{\text{Total volume of water poured into the cylindrical tank}}{\text{Area of base of the cylindrical tank}}$$

$= \frac{396000 \text{ cm}^3}{\pi \times 40^2 \text{ cm}^2}$ (radius of the base of the cylindrical tank = 40 cm)

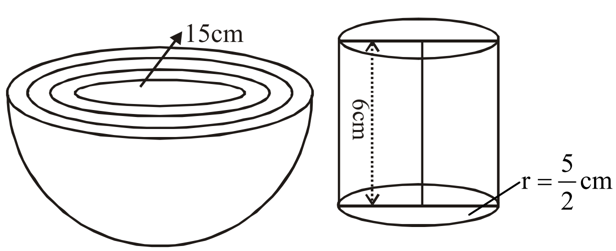
$$= \left(\frac{396000 \text{ cm}^3}{\frac{22 \times 1600}{7} \text{cm}^2}\right) = \left(\frac{396000 \times 7}{22 \times 1600}\right) \text{cm}$$

$= 78.75$ cm $\cong$ 79 cm

Hence, water rise upto 79 cm in half an hour

Ex.66 A hemispherical bowl of internal radius 15cm is full of a liquid. The liquid is to be filled into some bottles of cylindrical in shape whose diameters and heights are 5 cm and 6 cm respectively. Find the number of bottles necessary to empty the bowl.

Sol. We have internal radius of the hemispherical bowl = R = 15 cm.



So, its volume = $\frac{2}{3} \pi \times R^3$

$$= \left[\frac{2}{3} \times \pi \times (15)^3\right] \text{cm}^3 = \left(\frac{2}{3} \times \pi \times 15 \times 15 \times 15\right) \text{cm}^3$$

$= 10 \times 15 \times 15$ πcm^3 = 2250 πcm^3

So, volume of the entire liquid = 2250 πcm^3

The liquid is to be filled into some bottles of cylindrical in shape whose diameters and heights are 5 cm and 6 cm respectively. So, radius of the cylindrical bottle = $\frac{5}{2}$ cm and height of it = 6cm

So, volume of one cylindrical bottle = $\pi r^2 h$

$$= \left(\pi \times \frac{5}{2} \times \frac{5}{2} \times 6\right) \text{cm}^3 = \left(\frac{75\pi}{2}\right) \text{cm}^3$$

So, the number of bottles necessary to empty the hemispherical bowl

$$= \frac{\text{Volume of the entire liquid in the bowl}}{\text{Volume of one cylindrical bottle}}$$

$$= \frac{2250 \text{cm}^3}{\frac{75\pi}{2} \text{cm}^3} = \frac{2250 \times 2}{75} = 60$$

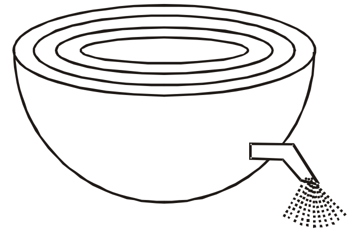
Ex.67 A hemispherical tank of radius $1\frac{3}{4}$ m is full of water. It is connected with a pipe which empties it at the rate of 7 lt/sec. How much time will it take to empty the tank completely ?

Sol. We have radius of the hemispherical tank

$= 1\frac{3}{4} = \frac{7}{4}$ m. It is full of water.

So, volume of entire water in the hemispherical tank

$$= \left[\frac{4}{3} \pi \times \left(\frac{7}{4}\right)^3\right] \text{m}^3 = \left(\frac{4}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4}\right) \text{m}^3$$



This tank is connected with a pipe which empties it at the rate of 7lt/sec.

So, volume of water flows out in 1 sec = 7 litre

$= (7 \times 1000)cm^3 = 7000\ cm^3$

$= \left(\frac{7000}{100\times100\times100}\right)m^3$

Thus, total time will be taken to empty the tank full of water

$= \left(\frac{4}{3}\times\frac{22}{7}\times\frac{7}{4}\times\frac{7}{4}\times\frac{7}{4}\right) \div \left(\frac{7000}{100\times100\times100}\right)$

$= \left(\frac{22\times49}{48}\div\frac{7}{1000}\right)sec$

$= \frac{22\times49\times1000}{48\times7}\ sec = \frac{19250}{12}\ sec$

$= \left(\frac{19250}{12\times60}\right)min = \frac{1925}{72}\ min = 26.73\ minutes$

Hence, the required time is 26.73 minutes.

Ex.68 A hemispherical bowl of internal radius 9cm is full of liquid. This liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and height 4 cm. How many bottles are necessary to empty the bowl ?

Sol. Volume of the hemispherical bowl

$= \frac{2}{3}\pi R^3 = \frac{2}{3} \times \frac{22}{7} \times (9)^3$

(R = Internal radius of the hemispherical bowl = 9 cm)

$= \left(\frac{2}{3}\times\frac{22}{7}\times9\times9\times9\right)cm^3$

So, volume of the liquid in the bowl

$= \left(\frac{2}{3}\times\frac{22}{7}\times9\times9\times9\right)cm^3$

Volume of a bottle = $\pi r^2 h = \frac{22}{7} \times \left(\frac{3}{2}\right)^2 \times 4$

(r = radius of the cylindrical bottle = $\frac{3}{2}$ cm and height (h) = 4 cm)

$= \frac{22}{7} \times \frac{9}{4} \times 4 = \frac{198}{7}\ cm^3.$

Number of bottles required to empty the bowl

$= \frac{\text{Volume of the liquid in the bowl}}{\text{volume of the one bottle}}$

$= \frac{2}{3} \times \frac{22}{7} \times 9 \times 9 \times 9 \div \frac{198}{7}$

$= \frac{2}{3} \times \frac{22}{7} \times 9 \times 9 \times 9 \times \frac{7}{198} = 54$

Hence, the required number of bottle necessary to empty the bowl is 54.

Ex.69 Water in a canal, 30 dm wide and 12 dm deep is flowing with velocity of 10 km per hour. How much area will it irrigate in 30 minutes, if 8cm of standing water is required for irrigation ?

Sol. We have

$30\ dm = \frac{30}{10}\ m,\ 12dm = \frac{12}{10}\ m$

$10\ km = 10 \times 1000\ m$

Volume of water flowing in canal in 1 hour

$= \frac{30}{10} \times \frac{12}{10} \times 10 \times 1000 = 36000\ m^3.$

Volume of water flowing in canal in 30 minutes

$= \left(\frac{1}{2}\text{hour}\right) = \frac{36000}{2} = 18000\ m^3.$

Then Area that will be irrigated in $\frac{1}{2}$ hour

$= \frac{\text{volume}}{\text{height}} = \frac{18000m^3}{8m}$

$= \left(\frac{18000\times100}{8}\right)m^2 = 225000\ m^2$

Hence, the required amount of standing water needed is 225000 m^2.

Ex.70 Water flows at the rate of 10m per minutes through a cylindrical pipe having its diameter as 5 mm. How much time will it take to fill a conical vessel whose diameter of base is 40 cm and depth 24 cm ?

Sol. We have diameter = 5mm

$\Rightarrow$ radius $= \frac{5}{2}\ mm = \frac{5}{10\times2}\ cm$

Also 10m = (10 × 100) cm

Volume of water that flows through the cylindrical pipe in 1 minute

$$= \left[\frac{22}{7}\times\left(\frac{5}{10\times 2}\right)^2 \times 10\times 100\right]\text{cm}^3$$

$$= \left[\frac{22}{7}\times\frac{5\times 5}{20\times 20}\times 1000\right]\text{cm}^3 = \frac{1375}{7}\text{ cm}^3$$

Volume of the conical vessel with radius 20 cm $\left(\frac{40}{2}=20\text{cm}\right)$ and depth 24 cm

$$= \left[\frac{1}{3}\times\frac{22}{7}\times(20)^2\times 24\right]\text{cm}^3$$

$$= \left(\frac{1}{3}\times\frac{22}{7}\times 20\times 20\times 24\right)\text{cm}^3$$

Time taken to fill the conical vessel

$$= \left(\frac{1}{3}\times\frac{22}{7}\times 20\times 20\times 24\right) \div \frac{1375}{7}$$

$$= \frac{1}{3}\times\frac{22}{7}\times\frac{20\times 20\times 24\times 7}{1375}$$

$$= \frac{1478400}{28875} = 51.2\text{ minutes}$$

Hence, the required time needed is 51.2 minutes.

Ex.71 A right triangle with sides 3 cm and 4 cm is revolved around its hypotenuse. find the volume of the double cone thus generated.

Sol. Hypotenuse BC of the right triangle BAC, right-angled at A $= \sqrt{3^2+4^2} = \sqrt{9+16} = 5\text{cm}$

Since the triangle is revolved around the hypotenuse, therefore AO is the radius of the common base of the double cone so formed.

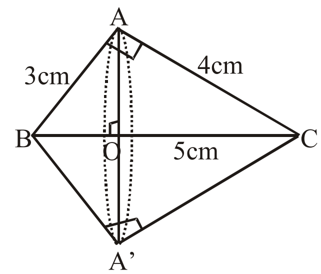
Height of the cone BAA' is BO and its slant height is 3cm. Height of the cone CAA' is CO and its slant height is 4cm

In the right triangles AOB and BAC, we have

So, ∠B = ∠B (common)

∠ BOA = ∠BAC = 90º

Thus, by AA – criterion of similarity, we have



$\Delta AOB \sim \Delta BAC$

$$\frac{AO}{AC} = \frac{AB}{BC}$$

$$\Rightarrow \frac{AO}{4} = \frac{3}{5} \Rightarrow AO = \frac{3}{5}\times 4 = \frac{12}{5}\text{ cm}$$

and $\frac{BO}{AB} = \frac{AB}{BC}$

$$\Rightarrow \frac{BO}{3} = \frac{3}{5} \qquad \Rightarrow BO = \frac{3\times 3}{5} = \frac{9}{5}\text{ cm}$$

Now, $CO = BC - BO = 5 - \frac{9}{5} = \frac{16}{5}$ cm.

Volume of cone BAA'

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times(AO)^2\times BO$$

$$= \frac{1}{3}\times\frac{22}{7}\times\frac{12}{5}\times\frac{12}{5}\times\frac{9}{5} = \frac{9504}{875}\text{ cm}^3$$

Volume of conc CAA' $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times(AO)^2\times CO$$

$$= \frac{1}{3}\times\frac{22}{7}\times\frac{12}{5}\times\frac{12}{5}\times\frac{16}{5}$$

$$= \frac{16896}{875}\text{ cm}^3$$

So, volume of the double cone thus formed

$$= \frac{9504}{875} + \frac{16896}{875} = \frac{26400}{875}$$

$$= \frac{1056}{35} = 30\frac{6}{35}\text{ cm}^3$$

Hence, the required volume is $30\frac{6}{35}$ cm^3.

IMPORTANT POINTS TO BE REMEMBERED

1. If λ, b and h denote respectively the length, breadth and height of a cuboid, then -

(i) total surface area of the cuboid = $2(\lambda b+bh+\lambda h)$ square units.

(ii) Volume of the cuboid = Area of the base × height = λbh cubic units.

(iii) Diagonal of the cuboid = $\sqrt{\lambda^2+b^2+h^2}$ units.

(iv) Area of four walls of a room = $2(\lambda + b)h$ sq. units.

2. If the length of each edge of a cube is 'a' units, then-

(i) Total surface area of the cube = $6a^2$ sq. units.

(ii) Volume of the cube = a^3 cubic units

(iii) Diagonal of the cube = $\sqrt{3}$ a units.

3. If r and h denote respectively the radius of the base and height of a right circular cylinder, then -

(i) Area of each end = πr^2

(ii) Curved surface area = $2\pi rh$

(iii) Total surface area = $2\pi r(h + r)$ sq. units.

(iv) Volume = $\pi r^2 h$ = Area of the base × height

4. If R and r denote respectively the external and internal radii of a hollow right circular cylinder, then -

(i) Area of each end = $\pi(R^2 - r^2)$

(ii) Curved surface area of hollow cylinder = $2\pi(R + r)h$

(iii) Total surface area = $2\pi(R + r)(R + h - r)$

(iv) Volume of material = $\pi h(R^2 - r^2)$

5. If r, h and λ denote respectively the radius of base, height and slant height of a right circular cone, then-

(i) $\lambda^2 = r^2 + h^2$

(ii) Curved surface area = $\pi r\lambda$

(iii) Total surface area = $\pi r^2 + \pi r\lambda$

(iv) Volume = $\frac{1}{3}\pi r^2 h$

6. For a sphere of radius r, we have

(i) Surface area = $4\pi r^2$

(ii) Volume = $\frac{4}{3}\pi r^3$