Assignment

Basic Limits

Basic Level

1. $\lim_{x\to\infty}\frac{(3x-1)(2x+5)}{(x-3)(3x+7)}$ is equal to

(a) 3 (b) 2 (c) –2 (d) 1

2. $\lim_{x\to\infty}\frac{2x^2+3x+4}{3x^2+3x+4}$ is equal to **[SCRA 1996; Rajasthan PET 1987 ; BIT Ranchi 1998; MP PET 1993)]**

(a) $\frac{2}{3}$ (b) 1 (c) 0 (d) ∞

3. $\lim_{x\to a} f(x).g(x)$ exists if **[Rajasthan PET 1995]**

(a) $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exists (b) $\lim_{x\to a} f(x)^{g(x)}$ exists

(c) $\lim_{x\to a}\frac{f(x)}{g(x)}$ exists (d) $\lim_{x\to a} f(x)g\left(\frac{1}{x}\right)$ exists

4. $\lim_{x\to\infty}\left[x-\sqrt{x^2+x}\right]=$ **[IIT 1975]**

(a) $\frac{1}{2}$ (b) 1 (c) $-\frac{1}{2}$ (d) 0

5. If $\lim_{x\to a}\frac{f(x)}{g(x)}$ exists then

(a) Both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ must exist (b) $\lim_{x\to a} f(x)$ not exist but $\lim_{x\to a} g(x)$ exists

(c) Neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists (d) $\lim_{x\to a} f(x)$ exist but $\lim_{x\to a} g(x)$ does not exist

6. Which of the following statement is not correct

(a) $\lim_{x\to c}[f(x)+g(x)]=\lim_{x\to c} f(x)+\lim_{x\to c} g(x)$ (b) $\lim_{x\to c}[f(x)-g(x)]=\lim_{x\to c} f(x)-\lim_{x\to c} g(x)$

(c) $\lim_{x\to c}[f(x).g(x)]=\lim_{x\to c} f(x).\lim_{x\to c} g(x)$ (d) $\lim_{x\to c}\frac{f(x)}{g(x)}=\frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$

7. $\lim_{x\to 3^-}\frac{|x-3|}{x-3}$ =

(a) 1 (b) –1 (c) 0 (d) Does not exist

8. $\lim_{x\to 3^+}\frac{|x-3|}{x-3}$ =

(a) 1 (b) –1 (c) 0 (d) Does not exist

9. If $\lim_{x\to a}\phi(x)=a^3, a\neq 0$ then $\lim_{x\to a}\phi\left(\frac{x}{a}\right)$ is equal to

(a) a^2 (b) $\frac{1}{a^2}$ (c) $\frac{1}{a^3}$ (d) a^3

10. If $\lim_{x\to a}\frac{a^x-x^a}{x^x-a^a}=-1$, then **[EAMCET 2003]**

(a) $a=1$ (b) $a=0$ (c) $a=e$ (d) None of these

11. $\lim_{x\to 4}\left[\frac{x^{3/2}-8}{x-4}\right]=$ **[DCE 1999]**

(a) $\frac{3}{2}$ (b) 3 (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

12. $\lim_{x\to 0}\left[\frac{\sqrt{a+x}-\sqrt{a-x}}{x}\right]=$ **[Karnataka CET 2001; Roorkee 1979; MP PET 1987]**

(a) 1 (b) 0 (c) $\sqrt{a}$ (d) $\frac{1}{\sqrt{a}}$

13. $\lim_{x\to\infty}\frac{\sqrt{4x^2+5x+8}}{4x+5}$ is equal to

(a) -1/2 (b) 0 (c) 1/2 (d) 1

14. $\lim_{x\to 4}\frac{3-\sqrt{5+x}}{x-4}$ is equal to **[Orissa JEE 1996]**

(a) 1/6 (b) -1/6 (c) 0 (d) 1

15. $\lim_{x\to\infty}\frac{\sqrt{x^2-1}}{2x+1}$ is equal to **[BIT Ranchi 1992]**

(a) 1 (b) 0 (c) -1 (d) $\frac{1}{2}$

16. $\lim_{x\to 2}\frac{x^2-3x+2}{x^2+x-6}$ equals **[IIT 1970; IIT 1976)]**

(a) 1/5 (b) 2/5 (c) 1 (d) 5

17. The value of $\lim_{\theta\to 0}\left(\frac{\sin\theta/4}{\theta}\right)$ is

(a) 0 (b) $\frac{1}{4}$ (c) 1 (d) Note in existence

18. $\lim_{x\to 0}\frac{x^2-2x}{2\sin x}$ equals **[Rajasthan PET 1985]**

(a) 1 (b) -1 (c) 0 (d) None of these

19. $\lim_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x}=$

(a) sin 2 (b) 2 sin 2 (c) 2 cos 2 (d) 2

20. $\lim_{n\to\infty}(3^n+4^n)^{\frac{1}{n}}=$ **[Karnataka CET 2003]**

(a) 3 (b) 4 (c) ∞ (d) e

21. True statement for $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{2+3x}-\sqrt{2-3x}}$ is **[Ranchi BIT 1982; Haryana 1996)]**

(a) Does not exist (b) Lies between 0 and $\frac{1}{2}$ (c) Lies between $\frac{1}{2}$ and 1

22. $\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}=$ **[IIT 1977]**

(a) $-\frac{1}{10}$ (b) $\frac{1}{10}$ (c) $-\frac{1}{8}$ (d) None of these

23. $\lim_{x\to\infty}(\sqrt{x^2+8x+3}-\sqrt{x^2+4x+3})=$ **[MP PET 1997; Rajasthan PET 1995)]**

(a) 0 (b) ∞ (c) 2 (d) $\frac{1}{2}$

24. $\lim_{x\to 0}\left(\frac{x^o}{\sin x^o}\right)$ equals **[AMU 1991]**

(a) 1 (b) $\frac{\pi}{180}$ (c) $\frac{180}{\pi}$ (d) None of these

Advance Level

25. $\lim_{n\to\infty}\left[\frac{1^3+2^3+3^3+.....+n^3}{n^4}\right]=$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) None of these

26. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+.....+\frac{1}{3n}\right]=$ **[Karnataka CET 1999]**

(a) 0 (b) $\log_e 4$ (c) $\log_e 3$ (d) $\log_e 2$

27. $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n^2+k^2}=$ **[Roorkee 1999]**

(a) $\left(\frac{1}{2}\right)\log 2$ (b) $\log 2$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

28. The $\lim_{x\to 0}(\cos x)^{\cot x}$ is **[Rajasthan PET 1999]**

(a) 0 (b) 1 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

29. $\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}=$ **[IIT 1999]**

(a) 2 (b) – 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

30. If $f(x)=\sqrt{\frac{x-\sin x}{x+\cos^2 x}}$, then $\lim_{x\to\infty} f(x)$ is **[DCE 2000; EAMCET 1997)]**

(a) 0 (b) ∞ (c) 1 (d) Not exist

31. If $f(x)=\begin{cases} x & ; \ x<0 \\ 1 & ; \ x=0, \\ x^2 & ; \ x>0 \end{cases}$ then, $\lim_{x\to 0} f(x)=$ **[DCE 2000]**

(a) 0 (b) 1 (c) 2 (d) Does not exist

32. If $f(x)=\begin{cases}\sin x & , x\neq n\pi \\ 0 & , \text{other wise}\end{cases}, n\in Z$ $g(x)=\begin{cases} x^2+1 & , \ x\neq 0,2 \\ 4 & , \ x=0 \\ 5 & , \ x=2 \end{cases}$, then $\lim_{x\to 0} g\{f(x)\}=$ **[Karnataka CET 2000]**

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

33. $\lim_{n\to\infty}\frac{1^p+2^p+3^p+..... +n^p}{n^{p+1}}=$ **[AIEEE 2002]**

(a) $\frac{1}{p+1}$ (b) $\frac{1}{1-p}$ (c) $\frac{1}{p}-\frac{1}{(p-1)}$ (d) $\frac{1}{p+2}$

34. $\lim_{n\to\infty} \frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots\ldots+\frac{1}{2^n}$ equals **[Rajasthan PET 1996]**

(a) 2 (b) –1 (c) 1 (d) 3

35. $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{\sin^{-1} x} =$

(a) 2 (b) 1 (c) –1 (d) None of these

36. $\lim_{x\to 0} \cos\frac{1}{x}$ **[UPSEAT 2002]**

(a) Is continuous at $x=0$ (b) Differentiable at $x=0$ (c) Does not exist (d) None of these

37. If $x_n = \frac{1-2+3-4+5-6+\ldots..-2n}{\sqrt{n^2+1}+\sqrt{4n^2-1}}$, then $\lim_{n\to\infty} x_n$ is equal to

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 1

38. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{\sqrt{n^2+n}}+\frac{1}{\sqrt{n^2+2n}}+\ldots..+\frac{1}{\sqrt{n^2+(n-1)n}}\right]$ is equal to **[Rajasthan PET 2000]**

(a) $2+2\sqrt{2}$ (b) $2\sqrt{2}-2$ (c) $2\sqrt{2}$ (d) 2

39. $\lim_{n\to\infty} \frac{1}{1^3+n^3}+\frac{4}{2^3+n^3}+\ldots.+\frac{1}{2n}$ is equal to **[Rajasthan PET 1995]**

(a) $\frac{1}{3}\log_e 3$ (b) $\frac{1}{3}\log_e 2$ (c) $\frac{1}{3}\log_e \frac{1}{3}$ (d) None of these

40. The value of $\lim_{n\to\infty}\left[\frac{n}{1+n^2}+\frac{n}{4+n^2}+\frac{n}{9+n^2}+\ldots.+\frac{1}{2n}\right]$ is equal to

(a) e (b) $\frac{1}{e}$ (c) $\frac{\pi}{4}$ (d) $\frac{4}{\pi}$

41. $\lim_{x\to\infty} \frac{x^n}{e^x}=0$ for

(a) No value of n (b) n is any whole number (c) n = 0 only (d) n = 2 only

Exponential and Logarithmic Limits

Basic Level

42. $\lim_{x\to 0}\left[\tan\left(\frac{\pi}{4}+x\right)\right]^{1/x}$ is equal to **[Rajasthan PET 2001]**

(a) e^{-1} (b) e (c) e^2 (d) $\sqrt{e}$

43. $\lim_{x\to 0}\left(\frac{3^x-1}{x}\right)$ equals **[Rajasthan PET 1998]**

(a) log 3 (b) 3 log 3 (c) 2 log 3 (d) None of these

44. $\lim_{x\to 0} \frac{\cos(\sin x)-1}{x^2} =$ **[Orissa JEE 2003]**

(a) 1 (b) – 1 (c) 1/2 (d) – 1/2

45. $\lim_{x\to 0} \frac{e^{1/x}}{\frac{1}{e^x}+1} =$ **[DCE 1999]**

(a) 0 (b) 1 (c) Does not exist (d) None of these

46. $\lim_{x\to 0}\frac{a^{\sin x}-1}{b^{\sin x}-1}=$ **[Karnataka CET 2000]**

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) $\frac{\log a}{\log b}$ (d) $\frac{\log b}{\log a}$

47. $\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)=$ **[Karnataka CET 2001]**

(a) $\frac{1}{2}$ (b) ∞ (c) 1 (d) 0

48. $\lim_{x\to\infty}\left(\frac{x+3}{x+1}\right)^{x+1}=$ **[Rajasthan PET 2003]**

(a) e^2 (b) e^3 (c) e (d) e^{-1}

49. $\lim_{x\to 0}(1-ax)^{\frac{1}{x}}=$ **[Karnataka CET 2003]**

(a) e (b) e^{-a} (c) 1 (d) e^a

Advance Level

50. $\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ is equal to **[IIT 2000]**

(a) e (b) e^{-1} (c) e^{-5} (d) e^5

51. $\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$ is **[IIT 1996]**

(a) e^2 (b) e (c) e^{-1} (d) None of these

52. The value of $\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$ is equal to

(a) e^2 (b) e^{-2} (c) e^6 (d) None of these

53. $\lim_{x\to\infty}\left[1+\frac{1}{mx}\right]^x$ equal to **[Haryana CEE 1998]**

(a) $e^{1/m}$ (b) $e^{-1/m}$ (c) e^m (d) m^e

54. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^x=$ **[AIEEE 2002]**

(a) e^4 (b) e^2 (c) e^3 (d) e

55. $\lim_{x\to 0}\frac{x.2^x-x}{1-\cos x}$ is equal to **[IIT 1980; BIT Ranchi 1983; Rajasthan PET 1999, 2001]**

(a) $\log 2$ (b) $\log 4$ (c) 0 (d) None of these

56. $\lim_{x\to 0}\left(\frac{a^x-b^x}{x}\right)=$ **[EAMCET 1988; Rajasthan PET 1995]**

(a) $\log\left(\frac{b}{a}\right)$ (b) $\log\left(\frac{a}{b}\right)$ (c) $\frac{a}{b}$ (d) $\log a^b$

57. $\lim_{x\to\infty} \frac{\log x^n - [x]}{[x]}, n \in N$, ($[x]$ denotes greatest integer less than or equal to x) **[AIEEE 2002]**

(a) Has value –1 (b) Has value 0 (c) Has value 1 (d) Does not exist

Trigonometric Limits

Basic Level

58. $\lim_{x\to 0} \frac{\sin(\pi \cos^2 x)}{x^2} =$ **[MP PET 2002; UPSEAT 2001; IIT Screening 2001]**

(a) $-\pi$ (b) π (c) $\frac{\pi}{2}$ (d) 1

59. $\lim_{x\to 0}\left\{\frac{\sin x - x + \frac{x^3}{6}}{x^5}\right\} =$ **[MNR 1985; MNR 1986)]**

(a) $\frac{1}{120}$ (b) $-\frac{1}{120}$ (c) $\frac{1}{20}$ (d) None of these

60. $\lim_{x\to 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x} =$ **[Rajasthan PET 2001; AIEEE 2002]**

(a) 1 (b) – 1 (c) 0 (d) Does not exist

61. $\lim_{x\to\pi/4} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} =$ **[BIT Ranchi 1989; IIT 1990]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{2\sqrt{2}}$ (d) 1

62. $\lim_{x\to a} \frac{\cos x - \cos a}{\cot x - \cot a} =$ **[BIT Ranchi 1987]**

(a) $\frac{1}{2}\sin^3 a$ (b) $\frac{1}{2}\operatorname{cosec}^2 a$ (c) $\sin^3 a$ (d) $\operatorname{cosec}^3 a$

Advance Level

63. $\lim_{x\to\infty} \sqrt{\frac{x+\sin x}{x-\cos x}} =$ **[Roorkee 1994]**

(a) 0 (b) 1 (c) –1 (d) None of these

64. The value of $\lim_{n\to\infty} \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)....\cos\left(\frac{x}{2^n}\right)$ is

(a) 1 (b) $\frac{\sin x}{x}$ (c) $\frac{x}{\sin x}$ (d) None of these

65. If x is a real number in [0,1], then the value of $\lim_{m\to\infty}\lim_{n\to\infty}[1+\cos^{2m}(n!\pi x)]$ is given by

(a) 2 or 1 according as x is rational or irrational (b) 1 or 2 according as x is rational or irrational

(c) 1 for all x (d) 2 or 1 for all x

66. $\lim_{x\to 0}\frac{1-\cos 5x}{2^x-3^x}$ is equal to

(a) 0 (b) 1 (c) 5 (d) None of these

67. If $\lim_{x\to 0}\frac{x^n-\sin x^n}{x-\sin^n x}$ is non zero definite, then n must be

(a) 1 (b) 2 (c) 3 (d) None of these

68. The values of a and b such that $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1,$ are

(a) $\frac{5}{2},\frac{3}{2}$ (b) $\frac{5}{2},-\frac{3}{2}$ (c) $-\frac{5}{2},-\frac{3}{2}$ (d) None of these

69. $\lim_{h\to 0}\frac{2\left[\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)\right]}{\sqrt{3}h(\sqrt{3}\cos h-\sin h)}=$ **[BIT Ranchi 1987]**

(a) $-\frac{2}{3}$ (b) $-\frac{3}{4}$ (c) $-2\sqrt{3}$ (d) $\frac{4}{3}$

70. If $f(x)=\int\frac{2\sin x-\sin 2x}{x^3}dx,$ then $\lim_{x\to 0}f'(x)$ equals **[IIT Screening 1997]**

(a) 0 (b) 1 (c) –1 (d) 1/2

L'- Hospital Rule

Basic Level

71. $\lim_{x\to 0}\frac{x}{\tan^{-1}(2x)}$ is equal to **[IIT 1992; Rajasthan PET 2001]**

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) None of these

72. $\lim_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x-x}$ is equal to **[Rajasthan PET 2001]**

(a) 0 (b) 1 (c) e (d) $\frac{1}{e}$

73. $\lim_{x\to\pi/2}(\sec x-\tan x)$ equals **[Rajasthan PET 1998]**

(a) 0 (b) 1 (c) –1 (d) None of these

74. $\lim_{x\to 0}\frac{2\sin^2 3x}{x^2}=$ **[Roorkee 1982; DCE 1999]**

(a) 0 (b) 1 (c) 18 (d) 36

75. $\lim_{x\to 0}\frac{\sin 2x}{x}=$ **[MNR 1990; UPSEAT 2000]**

(a) 0 (b) 1 (c) 1/2 (d) 2

76. If $f(1)=1$ and $f'(1)=4,$ then the value of $\lim_{x\to 1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ is **[DCE 2001]**

(a) 9 (b) 4 (c) 12 (d) 1

77. $\lim_{x\to 0}\frac{\log_e(1+x)}{3^x-1}=$ **[MP PET 2002]**

(a) $\log e\ 3$ (b) 0 (c) 1 (d) $\log_3 e$

78. $\lim_{x\to 0}\frac{e^x-e^{-x}}{\sin x}=$ **[Haryana CEE 2002]**

(a) 0 (b) 1 (c) 2 (d) Non existent

79. $\lim_{x\to -2}\frac{\sin^{-1}(x+2)}{x^2+2x}=$ **[Orissa JEE 2002]**

(a) 0 (b) ∞ (c) $-\frac{1}{2}$ (d) None of these

80. $\lim_{x\to a}\frac{(x^{-1}-a^{-1})}{x-a}=$ **[MP PET 1994]**

(a) $\frac{1}{a}$ (b) $-\frac{1}{a}$ (c) $\frac{1}{a^2}$ (d) $-\frac{1}{a^2}$

81. $\lim_{x\to 1}\frac{\log x}{x-1}$ is equal to **[Rajasthan PET 1996; MP PET 1996]**

(a) 1 (b) 0 (c) –1 (d) 1/2

82. $\lim_{x\to 1}\frac{1-x^{-1/3}}{1-x^{-2/3}}=$

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

Advance Level

83. The value of $\lim_{x\to 0}\frac{x\cos x-\log(1+x)}{x^2}$ is **[Rajasthan PET 1999]**

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) None of these

84. $\lim_{x\to\frac{\pi}{4}}\left(\frac{1-\tan x}{1-\sqrt{2}\sin x}\right)$ equals **[SCRA 1999]**

(a) 0 (b) 1 (c) – 2 (d) 2

85. $\lim_{x\to 0}\frac{2^x-1}{\sqrt{1+x}-1}$ equals **[Karnataka CET 1999, IIT 1983]**

(a) $\log 2$ (b) $\log 4$ (c) $\log 3$ (d) None of these

86. $\lim_{x\to 0}\left[\frac{\sin(x+a)+\sin(a-x)-2\sin a}{x\sin x}\right]$ is equal to **[UPSEAT 1998]**

(a) $\sin a$ (b) $-\sin a$ (c) 1 (d) 0

87. $\lim_{x\to\pi/2}\frac{\int_{\pi/2}^{x}t\,dt}{\sin(2x-\pi)}$ is equal to **[MP PET 1998]**

(a) ∞ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

88. $\lim_{y\to 0}\frac{(x+y)\sec(x+y)-x\sec x}{y}=$

(a) $\sec x(x\tan x+1)$ (b) $x\tan x+\sec x$ (c) $x\sec x+\tan x$ (d) None of these

89. $\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(\frac{\pi}{3}-x\right)}{2\cos x-1}$ is equal to **[AMU 1991]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\frac{2}{\sqrt{3}}$

90. $\lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$ is equal to **[AMU 1990]**

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) None of these

91. $\lim_{x \to 0} \frac{x \cos x - \sin x}{x^2 \sin x}$ equals **[SCRA 1999]**

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) 3 (d) –3

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Answer Sheet

Assignment (Basic & Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	a	a	c	a	d	b	a	d	a	b	d	c	b	d	a	b	b	c	b
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
b	a	c	a	c	c	a	b	c	c	d	a	a	c	b	c	b	b	b	c
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
b	c	a	d	d	c	c	a	b	c	a	a	a	a	b	b	a	b	a	d
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
b	c	b	b	a	a	a	c	d	b	c	b	a	c	d	b	d	c	c	d
81	82	83	84	85	86	87	88	89	90	91									
a	b	a	d	b	b	c	a	b	a	b									