

BLUE PRINT

Time Allowed: 3 hours Maximum Marks: 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	1(1)	1(2)	1(3)	_	3(6)
2.	Inverse Trigonometric Functions	2(2)#	_	_	_	2(2)
3.	Matrices	2(2)#	_	_	1(5)*	3(7)
4.	Determinants	1(1)	1(2)	_	_	2(3)
5.	Continuity and Differentiability	1(1)	1(2)	2(6)#	_	4(9)
6.	Application of Derivatives	1(1)*	2(4)	1(3)	_	4(8)
7.	Integrals	1(1)*	2(4)#	1(3)*	_	4(8)
8.	Application of Integrals	1(1)	_	1(3)	_	2(4)
9.	Differential Equations	1(1)	1(2)	1(3)	_	3(6)
10.	Vector Algebra	1(1)* + 1(4)	_	_	_	2(5)
11.	Three Dimensional Geometry	2(2)	1(2)*	_	1(5)*	4(9)
12.	Linear Programming	_	-	_	1(5)*	1(5)
13.	Probability	2(2) + 1(4)	1(2)*	_	_	4(8)
	Total	18(24)	10(20)	7(21)	3(15)	38(80)

^{*}It is a choice based question.

[#]Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code: 041

MATHEMATICS

Time allowed: 3 hours

Maximum marks: 80

General Instructions:

- 1. This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.
- 2. Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.
- 3. Both Part-A and Part-B have internal choices.

Part - A:

- 1. It consists of two Sections-I and II.
- 2. Section-I comprises of 16 very short answer type questions.
- 3. Section-II contains 2 case study-based questions.

Part - B:

- 1. It consists of three Sections-III, IV and V.
- 2. Section-III comprises of 10 questions of 2 marks each.
- 3. Section-IV comprises of 7 questions of 3 marks each.
- 4. Section-V comprises of 3 questions of 5 marks each.
- 5. Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART - A

Section - I

1. Find the area of a parallelogram whose adjacent sides are represented by the vectors $2\hat{i} - 3\hat{k}$ and $4\hat{j} + 2\hat{k}$.

OR

Write the direction ratios of the vector $3\vec{a}+2\vec{b}$, where $\vec{a}=\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=2\hat{i}-4\hat{j}+5\hat{k}$.

- **2.** If *A* is a square matrix satisfying $A^2 = I$, then what is the inverse of *A*?
- 3. If the curve $ay + x^2 = 7$ and $x^3 = y$ cut orthogonally at (1, 1), then find the value of a.

OR

Find the slope of the tangent to the curves $x = a \sin t$, $y = a \left\{ \cos t + \log \left(\tan \frac{t}{2} \right) \right\}$ at the point 't'.

- **4.** A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, then find the probability that both are dead.
- 5. If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, then find x.

224 Class 12

Show that the matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a skew symmetric matrix.

- **6.** Solve: $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$
- 7. Evaluate: $\int \frac{dx}{x(x^5+3)}$

OR

Evaluate: $\int_{0}^{2} (x - [x]) dx$

- **8.** Find the area of the triangle formed by the straight lines y = 2x, x = 0 and y = 2 by integration.
- **9.** Evaluate: $\tan^{-1} \left\{ \sin \left(-\frac{\pi}{2} \right) \right\}$

OR

Write the domain of the function $\cos^{-1}(2x - 1)$.

- **10.** Find the distance of the plane $\vec{r} \cdot \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} \frac{6}{7}\hat{k}\right) = 1$ from the origin.
- 11. Find the principal value of $tan^{-1}(-1)$.
- **12.** If $y = \log_{10} x + \log_e y$, then find $\frac{dy}{dx}$.
- 13. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, then find the value of $|A^2 2A|$.
- **14.** If *A* and *B* are two events such that $P(B) = \frac{3}{5}$, $P(A \mid B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then find the value of P(A).
- **15.** If (1, 3), (2, 5) and (3, 3) are three elements of $A \times B$ and the total number of elements in $A \times B$ is 6, then write the remaining elements of $A \times B$.
- **16.** Find the direction cosines of the normal to the plane 3x 6y + 2z = 7.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

- 17. In a bilateral cricket series between India and England, the probability that India wins the first match is 0.5. If India wins any match, then the probability that it wins the next match is 0.4, otherwise the probability is 0.3. Also, it is given that there is no tie in any match. Based on the above answer the following:
 - (i) The probability that India losing the second match, if India has already won the first match is
 - (a) 0.5
- (b) 0.4



(c) 0.3

(d) 0.6

(ii) The probability that India winning the third match, if India has already loosed the first two matches is

(c) 0.4

(d) 0.6

(iii) The probability that India winning the first two matches is

(a) 0.1

(b) 0.2

(c) 0.4

(d) 0.01

(iv) The probability that India winning the first three matches is

(b) 0.6

(c) 0.04

(d) 0.08

(v) The probability that India winning exactly one of the first three matches is

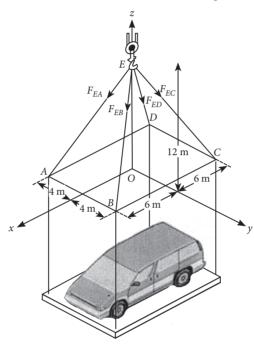
(a) 0.205

(b) 0.21

(c) 0.405

(d) 0.312

18. Consider the following diagram, where the forces in the cable are given.



Based on the above answer the following:

(i) The cartesian equation of line *EA* is

(a)
$$\frac{x}{-3} = \frac{y}{2} = \frac{z}{6}$$

(b)
$$\frac{x-6}{-3} = \frac{y}{2} = \frac{z}{6}$$

(c)
$$\frac{x}{-3} = \frac{y}{2} = \frac{z - 12}{6}$$

(b)
$$\frac{x-6}{-3} = \frac{y}{2} = \frac{z}{6}$$
 (c) $\frac{x}{-3} = \frac{y}{2} = \frac{z-12}{6}$ (d) $\frac{x}{3} = \frac{y}{6} = \frac{z-12}{6}$

(ii) The vector \overrightarrow{ED} is

(a) $6\hat{i} - 4\hat{j} + 12\hat{k}$

(b) $-6\hat{i} - 4\hat{j} + 12\hat{k}$ (c) $-6\hat{i} - 4\hat{j} - 12\hat{k}$ (d) $6\hat{i} + 4\hat{j} + 12\hat{k}$

(iii) The length of the cable *EB* is

(a) 14 units

(b) 16 units

(c) 17 units

(d) 15 units

(iv) The length of cable *EC* is equal to the length of

(c) ED

(d) All of these

(v) The sum of all vectors along the cables is

(a) $48 \hat{i}$

(b) $48\hat{i}$

(c) $-48 \hat{k}$

(d) $48 \,\hat{k}$

PART - B

Section - III

, then check whether function f(x) is continuous at x = 0 or not.

20. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Also, find the point of intersection of these lines.

OF

Find the equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 4$, $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

- **21.** Find domain and range of the function $\frac{x}{1+|x|}$.
- 22. Find the maximum profit that a company can make, if the profit function is given by $P(x) = 41 + 24x 18x^2$.
- 23. The probability that a student selected at random from a class will pass in Mathematics is $\frac{4}{5}$ and the probability that he/she passes in Mathematics and Computer Science is $\frac{1}{2}$. What is the probability that he/she will pass in Computer Science, if it is known that he/she passed in Mathematics?

OR

A speaks truth in 80% cases and *B* speaks truth in 90% cases. In what percentage of cases are they likely to agree with each other in stating the same fact?

24. Evaluate:
$$\int_{0}^{\pi/2} \frac{1+\sin x}{2+\sin x + \cos x} dx$$

25. If
$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & -2 & 0 \end{bmatrix}$$
 and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $M = AB$, then find M^{-1} .

- **26.** Find the solution of the differential equation $(x-y)\left(1-\frac{dy}{dx}\right) = e^x$.
- 27. Find the interval in which the function $x^4 \frac{1}{3}x^3$ is increasing.
- **28.** Find the value of integral $\int \frac{dx}{3\cos^2 x + 5}$.

OR

Evaluate: $\int \frac{\cos(x+a)}{\sin(x+b)} dx$

Section - IV

29. If
$$f(x) = \frac{\tan 2x}{\cot \left(\frac{\pi}{4} - x\right)}$$
, $x \neq \frac{\pi}{4}$, is continuous at $x = \frac{\pi}{4}$, then find the value of $f\left(\frac{\pi}{4}\right)$.

- **30.** Find the equation of the curve passing through origin if the slope of the tangant to the curve at any point (x, y) is equal to the square of the difference of the abscissa and ordinate of the point.
- 31. Evaluate: $\int_{\pi/6}^{\pi/3} \frac{1}{1 + \sqrt{\tan x}} dx$ OR

Evaluate: $\int (3\sin x + 4\csc x)^2 dx$

Mathematics 227

32. Let $f: \mathbb{R}^+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Is this function one one and onto?

- 33. Find the area of the region bounded by the parabola $y^2 = 4ax$, its axis and two ordinates x = 5 and x = 8.
- **34.** If $(x-a)^2 + (y-b)^2 = c^2$, for some c > 0, then prove that $\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is independent of a and b.

OR

If
$$y = \tan^{-1} \left[\frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}} \right]$$
, where $\pi < x < \frac{3\pi}{2}$, then find $\frac{dy}{dx}$.

35. Find the values of x, for which the function $f(x) = x^3 + 12x^2 + 36x + 6$ is increasing.

Section - V

36. Find the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 8\hat{k})$.

OR

Find the equation of the plane parallel to the line $\frac{x-2}{1} = \frac{y-1}{3} = \frac{z-3}{2}$, which contains the point (5, 2, -1) and passes through the origin.

37. Solve the following linear programming Problem (LPP) graphically.

Maximize Z = 20x + 30y

Subject to constraints:

$$2x + 3y \ge 100$$
; $x + 2y \le 80$; $x \ge 14$; $y \ge 16$, $x, y \ge 0$

OR

Solved the following linear programming problem (LPP) graphically.

Maximize Z = 0.6x + 0.4 y

Subject to constraints:

$$x + y \le 500$$
; $2x + 2y \le 800$; $x, y \ge 0$

38. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = O$.

OR

If
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$$
 and $6A^{-1} = A^2 + cA + dI$, then find the values of c and d .