

Subject

EMT

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Title

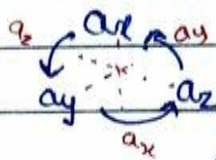
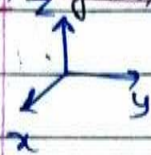
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Teacher's Signature/Remarks

Co-ordinate System

Cartesian System (x, y, z)

①



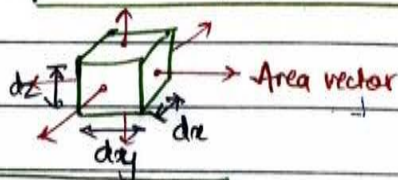
Use Right hand thumb rule to get 3rd axis
eg: $\vec{a}_z \times \vec{a}_x = \vec{a}_y$

② differential length:

③ differential surface

$$d\vec{l} = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z$$

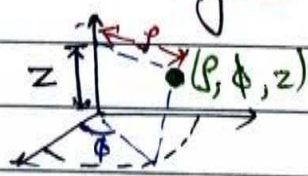
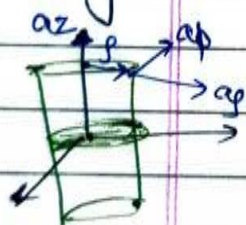
$$d\vec{s} = dx dy \hat{a}_z = dx dz \hat{a}_y \dots \text{etc.}$$



④ diff. volume,

$$dv = dx dy dz$$

Cylindrical Coordinate system (ρ, ϕ, z)



$$\begin{aligned} x &= \rho \cos \phi \\ y &= \rho \sin \phi \\ z &= z \end{aligned}$$

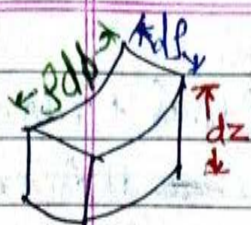
$$\vec{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Transpose

$\frac{d}{d\phi}$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$



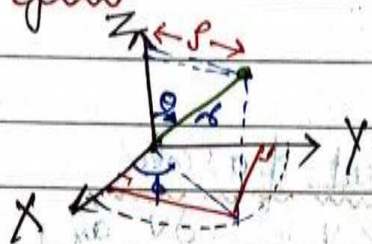
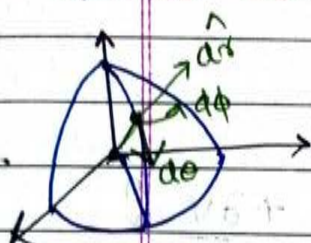
differential length

$$d\vec{r} = dr \hat{a}_r + r d\phi \hat{a}_\phi + dz \hat{a}_z$$

$$d\vec{s} = r dr d\phi \hat{a}_z = r d\phi dr \hat{a}_\phi = dr dz \hat{a}_\phi$$

$$dV = r dr d\phi dz$$

Spherical Co-ordinate system



$$x = r \cos \phi = r \sin \theta \cos \phi$$

$$y = r \sin \phi = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1} \left(\frac{z}{\sqrt{x^2 + y^2 + z^2}} \right)$$

$$\phi = \tan^{-1} \left(\frac{y}{x} \right)$$

Transformation of Vectors

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\ \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\ \cos \theta & -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

↓ Transpose

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$\nabla \cdot \vec{v} \Rightarrow * \frac{1}{\rho} \frac{\partial v_{\phi}}{\partial \phi} \Rightarrow \text{1 term me}$$

$$\therefore \nabla \cdot \vec{v} = \frac{1}{\rho} \frac{\partial (v_{\rho})}{\partial \rho} + \frac{1}{\rho} \frac{\partial v_{\phi}}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Ke term me
multiply &
divide hog

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 Date: / /

$$\rho d\phi a_{\hat{\phi}} + r d\phi a_{\hat{\phi}}$$

Similarly in spherical coordinates

$\frac{\partial}{\partial \theta}$ Ke bath $\frac{1}{r}$ extra $\Rightarrow \hat{a}_r$ Ke bath r mul & divide

$\frac{\partial}{\partial \phi}$ Ke bath $\frac{1}{r \sin \theta}$ extra $\Rightarrow \hat{a}_{\phi}$ Ke bath r mul & divide
 a_{ϕ} Ke bath $\sin \theta$ mul & divide
 $= r dr d\theta d\phi \hat{a}_{\phi}$

$$+ r d\theta a_{\hat{\theta}} + r \sin \theta d\phi a_{\hat{\phi}}$$

$$d\theta d\phi \hat{a}_r$$

$$r d\phi \hat{a}_{\theta}$$

$$\hat{a}_{\phi}$$

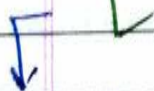
$$dV = r^2 \sin \theta dr d\theta d\phi$$

③ Gradient of a scalar

$$\nabla V = \frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z$$

$$= \frac{\partial V}{\partial \rho} \hat{a}_{\rho} + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \hat{a}_{\phi} + \frac{\partial V}{\partial z} \hat{a}_z$$

$$= \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{a}_{\theta} + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \hat{a}_{\phi}$$



To remember cylindrical



$$dV = dr d\phi dz \hat{a}_{\phi} + r d\phi dz \hat{a}_r + r dr dz \hat{a}_z$$

extra term

$$\nabla V = \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \phi} \hat{a}_{\phi} + \frac{\partial V}{\partial z} \hat{a}_z$$

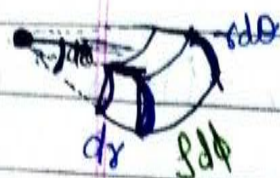
Similarly spherical

$$dV = dr r^2 d\theta \hat{a}_{\theta} + r^2 d\theta d\phi \hat{a}_{\phi} + r^2 \sin \theta dr d\phi \hat{a}_r$$

extra

$$\nabla V = \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{a}_{\theta} + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \hat{a}_{\phi}$$

∇V {gradient} gives max rate of change per unit distance and direction where max rate of change occurs.



$$d\vec{l} = dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin\theta d\phi \hat{a}_\phi$$

~~$$= dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin\theta d\phi \hat{a}_\phi$$~~

$$d\vec{l} = dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin\theta d\phi \hat{a}_\phi$$

$$\begin{aligned} d\vec{s} &= r^2 \sin\theta d\theta d\phi \hat{a}_r \\ &= r \sin\theta dr d\phi \hat{a}_\theta \\ &= r dr d\theta \hat{a}_\phi \end{aligned}$$

$$dV = r^2 \sin\theta dr d\theta d\phi$$

① Gradient of a scalar

$$\nabla V = \frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z$$

$$= \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{a}_\theta + \frac{1}{r \sin\theta} \frac{\partial V}{\partial \phi} \hat{a}_\phi$$

$$= \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{a}_\theta + \frac{1}{r \sin\theta} \frac{\partial V}{\partial \phi} \hat{a}_\phi$$



To remember cylindrical



$$d\vec{l} = dr \hat{a}_r + r d\phi \hat{a}_\phi + dz \hat{a}_z$$

$$\nabla V = \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \phi} \hat{a}_\phi + \frac{\partial V}{\partial z} \hat{a}_z$$

Similarly spherical

$$d\vec{l} = dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin\theta d\phi \hat{a}_\phi$$

$$\nabla V = \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{a}_\theta + \frac{1}{r \sin\theta} \frac{\partial V}{\partial \phi} \hat{a}_\phi$$

∇V {gradient} gives max rate of change per unit distance and direction where max rate of change occurs.

① Curl of a vector

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

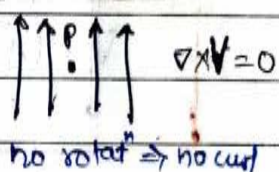
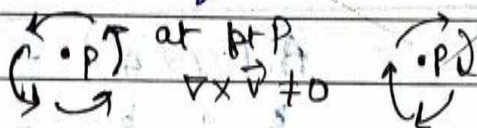
$$= \frac{1}{r} \begin{vmatrix} \hat{a}_r & r\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ v_r & r v_\phi & v_z \end{vmatrix}$$

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{a}_r & r\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ v_r & r v_\phi & v_z \end{vmatrix}$$

$$= \begin{vmatrix} \hat{a}_r & r\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ v_r & r v_\phi & v_z \end{vmatrix}$$

$$= \frac{1}{r^2 \sin \theta} \begin{vmatrix} a_r & r a_\theta & r \sin \theta a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ v_r & r v_\theta & r \sin \theta v_\phi \end{vmatrix}$$

* Curl is measure of how much a vector field rotates about point.



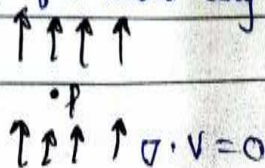
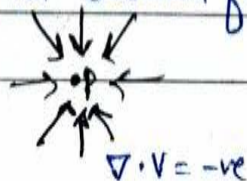
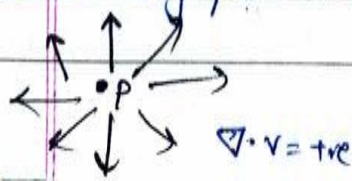
• Divergence of a vector

$$\nabla \cdot \vec{V} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$= \frac{1}{r} \frac{\partial (r v_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (v_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial (v_\phi \sin \theta)}{\partial \phi}$$

$$= \frac{1}{r^2} \frac{\partial (r^2 v_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (v_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial (v_\phi \sin \theta)}{\partial \phi}$$

* It is a measure of how much vector field diverges about any pt. { net outward flow of field about any pt. }



Stoke's theorem: $\oint \vec{A} \cdot d\vec{l} = \int (\nabla \times \vec{A}) \cdot d\vec{S}$

Divergence theorem: $\oint \vec{D} \cdot d\vec{S} = \int (\nabla \cdot \vec{D}) dv$

~~① $\nabla \cdot \vec{D} \neq 0$
 $(\nabla \times \vec{D}) = 0$~~ ~~$\text{div}(\text{curl } \vec{V}) \neq 0$
 $\nabla \cdot (\nabla \times \vec{V}) \neq 0$~~

① Divergenceless or solenoidal vector $\Rightarrow \nabla \cdot \vec{D} = 0$

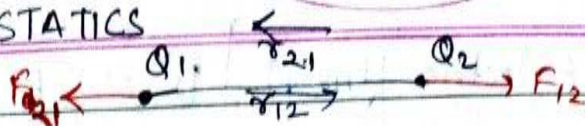
② Irrotational or conservative vector $\Rightarrow \nabla \times \vec{V} = 0$

③ $\nabla \cdot (\nabla \times \vec{V}) = 0$ and $\nabla \times (\nabla \cdot \vec{V}) = 0$

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

a b c b a b - b a c

ELECTROSTATICS



① Coulomb's Law:

$$\vec{F}_{12} = \frac{K Q_1 Q_2}{|\vec{r}_{12}|^3} \vec{r}_{12}$$

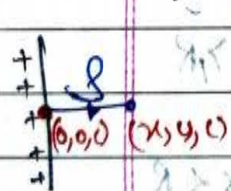
$$K = 9 \times 10^9 \text{ m/F}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

Electric field Intensity $\vec{E} = \frac{\vec{F}}{q}$

$$= \begin{cases} \frac{KQ}{R^2} \hat{a}_R \\ \frac{K \rho_L dl}{R^2} \hat{a}_R \text{ (line charge)} \\ \frac{K \rho_S ds}{R^2} \hat{a}_R \text{ (surface charge)} \\ \frac{K \rho_V dv}{R^2} \hat{a}_R \text{ (volume charge)} \end{cases}$$

(i) Infinite long line charge

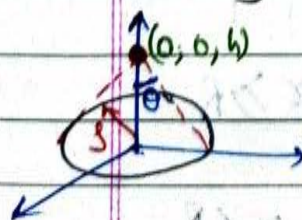


$$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0} \hat{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0 \rho^2} \hat{a}_\rho$$

$$\left\{ \begin{aligned} \vec{E} \cdot 2\pi \rho l &= \frac{\rho_L l}{\epsilon_0} \\ \text{Derivation} \\ \text{Gauss law} \end{aligned} \right\}$$

ρ = perpendicular distance from line charge to point charge

(ii) Circular ring



$$\vec{E} = \frac{\rho_L (\sin\theta)}{2\epsilon_0} = \frac{\rho_L (\sin\theta \cos\theta)}{2\sqrt{\rho^2 + h^2} \epsilon_0}$$

$$E = \frac{\rho_L h}{2\epsilon_0 (\rho^2 + h^2)^{3/2}} \hat{a}_h$$

(iii) Circular disk

$$E = \frac{\rho_S}{2\epsilon_0} \left[1 - \frac{h}{\sqrt{h^2 + R^2}} \right] \hat{a}_h$$

(iv) Infinite sheet ($R \rightarrow \infty$)

$$E = \frac{\rho_S}{2\epsilon_0} \hat{a}_n = \frac{\rho_S}{2\epsilon_0} [1 - \cos\theta] \hat{a}_n$$

Electric flux (Ψ) $\boxed{\vec{D} = \epsilon \vec{E}}$ C/m²

$\boxed{\Psi = \int_S \vec{D} \cdot d\vec{s}}$ C max flux $\Psi = \int D \cos 0^\circ ds$
 $= D \times A_{\text{proj}}$

min flux $\Psi = \int D \cos 90^\circ ds$
 $= 0$

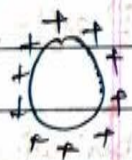
Gauss law :- $\boxed{\oint \vec{D} \cdot d\vec{s} = \int_V \rho_v dv}$

$\oint \vec{D} \cdot d\vec{s} = \int_V (\nabla \cdot \vec{D}) dv = \int_V \rho_v dv$

$\boxed{\nabla \cdot \vec{D} = \rho_v}$ { Maxwell 1st eqⁿ (Integral form) }
 { Maxwell 1st eqⁿ (Differential or point form) }

(i) Electric field due to point charge: $\boxed{E = \frac{KQ}{R^2}}$

(ii) Electric field due to conducting sphere



$E = \begin{cases} \frac{KQ}{r^2} & r \geq R \\ 0 & r < R \end{cases}$

(iii) Electric field due to non-conducting sphere



$E = \begin{cases} \frac{KQ}{r^2} & r \geq R \\ \frac{KQ r}{R^3} & r < R \end{cases}$

$= \frac{KQ}{r^2} \left(\frac{r^3}{R^3} \right)$

$\hookrightarrow V = \frac{4\pi}{3} r^3$

Potential difference

$$* V_{AB} = - \int_A^B \vec{E} \cdot d\vec{l}$$

$$* V_{AB} = \frac{KQ}{r_B} - \frac{KQ}{r_A} = V_B - V_A$$

* If $r_B = r$, $r_A = \infty$, $V_{AB} = \text{absolute pot. at } B$

KVL: * $\oint \vec{E} \cdot d\vec{l} = 0$ { Maxwell's 2nd eqⁿ Integral form }

from Stokes then $\oint \vec{E} \cdot d\vec{l} = \int_S (\nabla \times \vec{E}) \cdot d\vec{s} = 0$

$$\boxed{(\nabla \times \vec{E}) = 0} \quad \text{Maxwell's 2nd eqⁿ differential form}$$

$$* \vec{E} = -\nabla V = -\text{grad}(V)$$

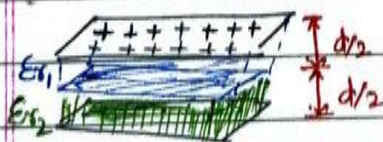
Capacitance

(1) Parallel Plate Capacitor

$$\odot C = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$\odot W_E = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$

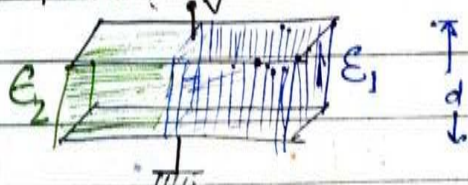
Series connection



$$C_1 = \frac{\epsilon_1 A}{d/2} \quad C_2 = \frac{\epsilon_2 A}{d/2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

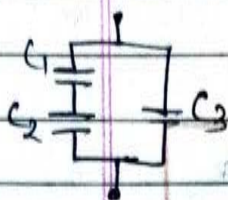
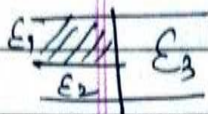
Parallel connection



$$C_1 = \frac{\epsilon_1 A_1}{d}$$

$$C_2 = \frac{\epsilon_2 A_2}{d}$$

$$C = C_1 + C_2$$



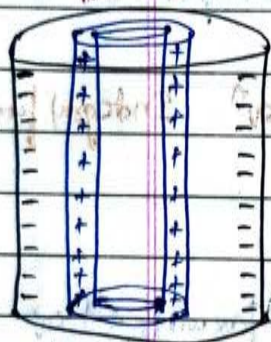
① In series connectⁿ $Q = \text{constant}$

$$C_1 V_1 = C_2 V_2$$

② In shunt connection $V = \text{constant}$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

2) Co-axial Capacitor



from Gauss law $\oint \vec{D} \cdot d\vec{s} = Q$

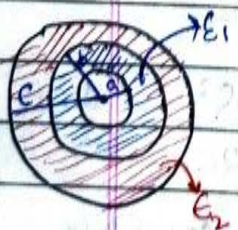
step ① $E = \frac{1}{\epsilon} \left(\frac{Q}{2\pi a l} \right)$

step ② $V_{21} = - \int_2^1 E \cdot \hat{a}_r \cdot dr$

step ③ $C = \frac{Q}{V}$

$$C = \frac{2\pi \epsilon l}{\ln(b/a)}$$

Series Connection:

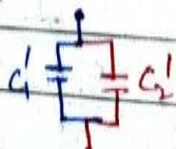
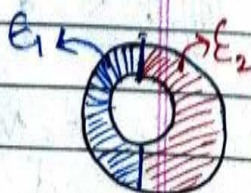


$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_1 = \frac{2\pi \epsilon_1 l}{\ln(b/a)}$$

$$C_2 = \frac{2\pi \epsilon_2 l}{\ln(b/a)}$$

Shunt connection:

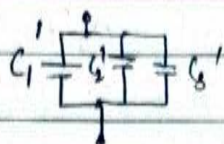
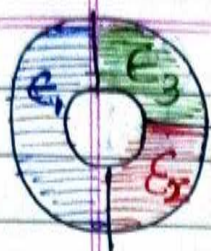


$$C' = \frac{1}{2} \left(\frac{2\pi \epsilon_1 l}{\ln(b/a)} \right); C_2' = \frac{1}{2} \left(\frac{2\pi \epsilon_2 l}{\ln(b/a)} \right)$$

$$C = C_1' + C_2'$$

To remember $C = \frac{\epsilon_0 A}{d}$

Capacitance $K\epsilon_0$ unit area A ϵ_0 K unit Area A ϵ_0 $\therefore A \rightarrow \frac{1}{2} \rightarrow C \rightarrow \frac{1}{2}$



$$C_1' = \frac{1}{2} \left(\frac{2\pi\epsilon_1 l}{\ln(b/a)} \right)$$

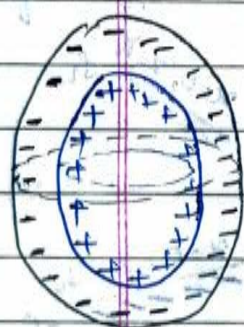
$$C_2' = \frac{1}{4} \left(\frac{2\pi\epsilon_2 l}{\ln(b/a)} \right)$$

$$C_3' = \frac{1}{4} \left(\frac{2\pi\epsilon_3 l}{\ln(b/a)} \right)$$

radius change $\rightarrow$ series

medium change $\rightarrow$ shunt

(3) Spherical capacitor



Step (1)

$$E = \frac{Q}{4\pi\epsilon r^2}$$

(2)

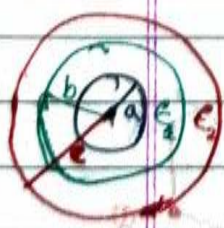
$$V = - \int_a^b E \cdot dl$$

$$(3) C = \frac{Q}{V}$$

$$C = \frac{4\pi\epsilon}{\left(\frac{1}{a} - \frac{1}{b} \right)}$$

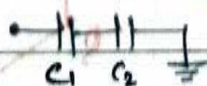
If outer conducting sphere at $\infty \Rightarrow$ isolated sphere
 $b \rightarrow \infty \Rightarrow C = 4\pi\epsilon a$

Series connection:



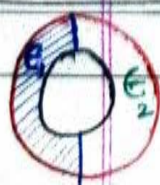
$$C_1 = \frac{4\pi\epsilon_1}{\frac{1}{a} - \frac{1}{b}}$$

$$C_2 = \frac{4\pi\epsilon_2}{\frac{1}{b} - \frac{1}{a}}$$



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Shunt connection:



$$C_1 = \frac{1}{2} \left(\frac{4\pi\epsilon_1}{\frac{1}{a} - \frac{1}{b}} \right)$$

$$C_2 = \frac{1}{2} \left(\frac{4\pi\epsilon_2}{\frac{1}{a} - \frac{1}{b}} \right)$$



$$C = C_1 + C_2$$



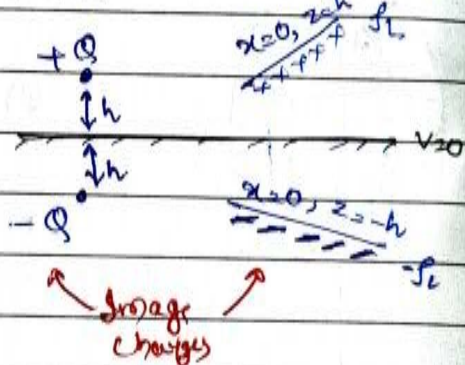
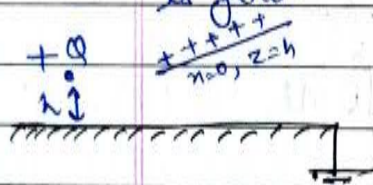
$$C_1 = \frac{4\pi\epsilon_1}{\frac{1}{a} - \frac{1}{b}}$$

$$C_2 = \frac{4\pi\epsilon_2}{\frac{1}{a} - \frac{1}{b}}$$

Method of Images

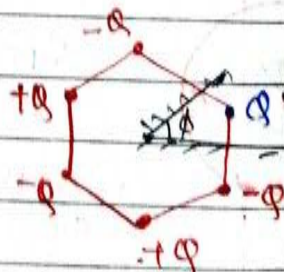
Condition necessary:

- ① The image charges must be located in conducting region.
- ② The image charges must be located such that the potential on the conducting surface is zero.



** when method of images is used for system consisting of point charge b/w two semi-infinite perfect conducting grounded plane inclined at an angle of ϕ then no. of image is given by

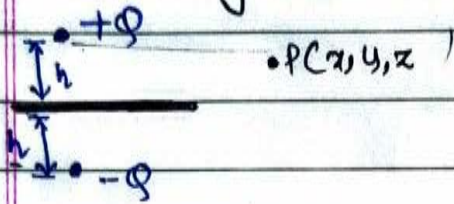
$$N = \frac{360^\circ}{\phi^\circ} - 1$$



No. of sides = $N + 1 = n$

$$\theta = \left(\frac{n-2}{n} \right) 180^\circ$$

- Q A point charge placed above perfect conducting grounded plane.



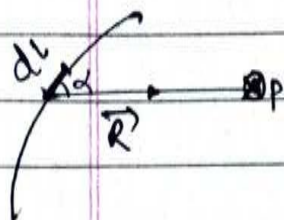
to be continued...

1 pages do write it.

Magnetostatics

Page: _____
Date: / /

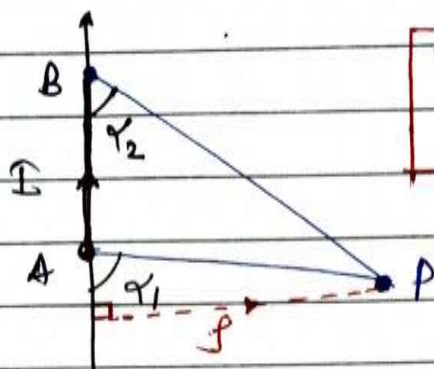
① Biot-Savart law:



$$\vec{H} = \int_L \frac{1}{4\pi} \frac{I d\vec{l} \times \vec{R}}{R^3} \quad \text{A/m}$$

$$\boxed{B = \mu H}$$

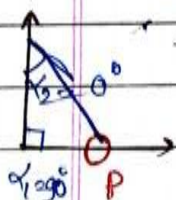
(1) Magnetic field due to straight conductor



$$\boxed{\vec{H} = \frac{I}{4\pi r} (\cos \alpha_2 - \cos \alpha_1) \hat{a}_\phi}$$

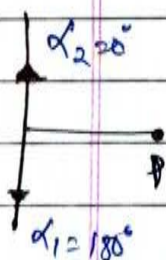
$$\hat{a}_\phi = \hat{a}_i \times \hat{a}_r$$

Case 1: Semi-infinite long conductor



$$\boxed{\vec{H} = \frac{I}{4\pi r} \hat{a}_\phi}$$

Case 2: Infinite long conductor



$$\boxed{\vec{H} = \frac{I}{2\pi r} \hat{a}_\phi}$$

Also derivable from Ampere's Circuital law

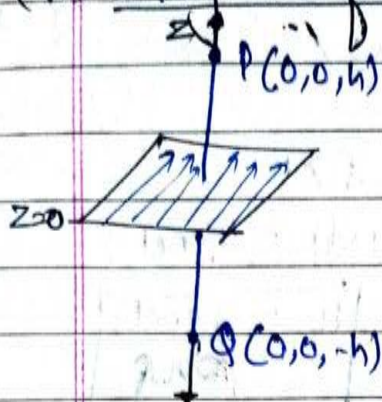
Current distribution

$$I d\vec{l} = \vec{K} d\vec{s} = \vec{J} dv$$

↓
Surface current density

↓
Volume current density

(ii) Magnetic field due to infinite current sheet

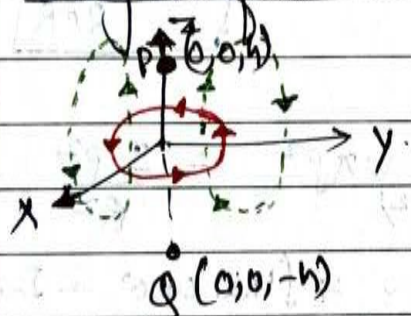


$\vec{K}$ = surface current density

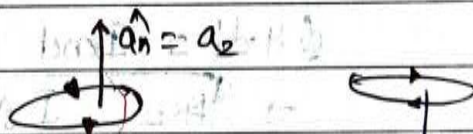
$$\vec{H} = \frac{1}{2} (\vec{K} \times \hat{n})$$

for pt. P $\hat{n} = \hat{a}_z$
for pt. Q $\hat{n} = -\hat{a}_z$

(iii) Magnetic field due to Circular conductor

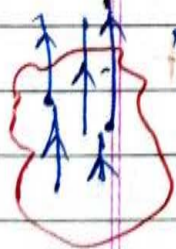


$$\vec{H} = \frac{I a^2}{2 (a^2 + h^2)^{3/2}} \hat{n}$$



*** directⁿ of $\hat{n}$ from RHT R & not from posⁿ of $\hat{n} = -\hat{a}_z$ but.

Ampere's Law



$$\oint \vec{H} \cdot d\vec{l} = I_{enc}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\oint \vec{H} \cdot d\vec{l} = \int \vec{J}_c \cdot d\vec{s}$$

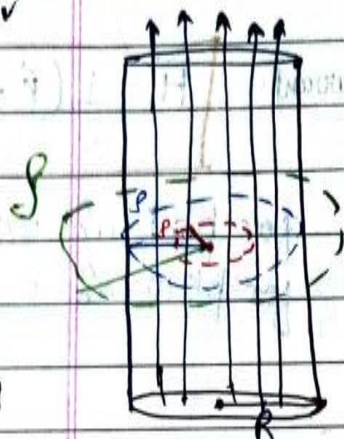
$$\Rightarrow \int_V (\nabla \times \vec{H}) d\vec{s} = \int_V \vec{J}_c d\vec{s}$$

Maxwell's 3rd eqⁿ
Integral form

$$\nabla \times \vec{H} = \vec{J}_c$$

Maxwell's 3rd eqⁿ differential form.

(iv) Magnetic field due to current carrying conductor of radius R .



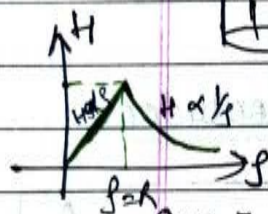
Case 1 $r > R$

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc}$$

$$\boxed{H = \frac{I_{enc}}{2\pi r}}$$

Case 2 $r = R$

$$\boxed{H = \frac{I_{enc}}{2\pi R}}$$



Case 3 $r < R$

$$I_{enc} = I_c \cdot A = \left(\frac{I}{\pi R^2} \right) \times \pi r^2 = \frac{I r^2}{R^2}$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc}$$

$$\Rightarrow \int H dl \cos 0^\circ = I_{enc}$$

$$\Rightarrow \boxed{H = \frac{I r}{2\pi R^2}} = \frac{\left(\frac{I r^2}{R^2} \right)}{2\pi r}$$

(v) Magnetic field due to co-axial condⁿ

Apply Ampere's circuital law & get



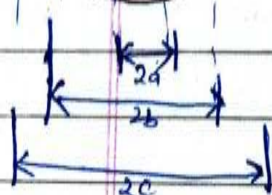
$$\underline{r > c} : \oint \mathbf{H} \cdot d\mathbf{l} = I - I = 0$$

$$\boxed{H = 0}$$

$$b < r < c : H = \frac{I}{2\pi r} \left(\frac{c^2 - r^2}{c^2 - b^2} \right)$$

$$a < r < b : H = \frac{I}{2\pi r}$$

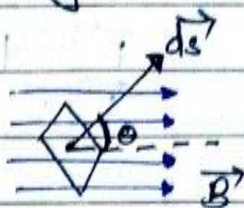
$$r < a : H = \frac{I r}{2\pi a^2}$$



Magnetic flux & flux density

$$\textcircled{1} \vec{B} = \mu \vec{H}$$

$$\textcircled{2} \psi = \int_S \vec{B} \cdot d\vec{s}$$



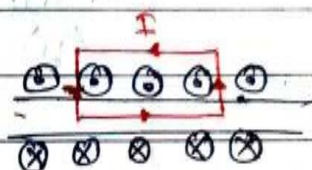
$\textcircled{3}$ Total flux passing through a closed surface is zero $\oint \vec{B} \cdot d\vec{s} = 0$

$$\text{or } \int (\nabla \cdot \vec{B}) dV = 0 \quad \text{or } \boxed{\nabla \cdot \vec{B} = 0}$$

$\textcircled{1}$ Solenoid:

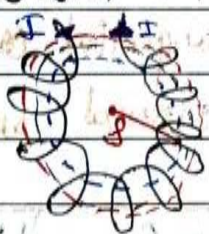
$$B \cdot l = \mu_0 NI$$

$$\boxed{B = \frac{\mu_0 NI}{l}}$$



* $\vec{B}$ exist only inside loop
 $(\vec{B} \cdot \vec{B} = 0)$

$\textcircled{2}$ Toroid:



$$\boxed{B = \frac{\mu_0 NI}{2\pi r}}$$

$$L = \frac{N\psi}{I} = \frac{NBA}{I} \quad \text{with } \begin{cases} \frac{\mu_0 N^2 A}{l} & \text{Solenoid} \\ \frac{\mu_0 N^2 A}{2\pi r} & \text{Toroid} \end{cases}$$

Faraday Law:

$$\oint \vec{E} \cdot d\vec{l} = -N \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad \left| \begin{array}{l} \text{Maxwell's} \\ \text{2nd eqn} \\ \text{(Integral form)} \end{array} \right.$$

Stokes theorem:

$$\oint \vec{A} \cdot d\vec{l} = \int (\nabla \times \vec{A}) \cdot d\vec{s}$$

$$\boxed{\nabla \times \vec{E} = -N \frac{\partial \vec{B}}{\partial t}}$$

Lorentz Force equation

$$\vec{F}_E = Q\vec{E} \quad \vec{F}_M = Q(\vec{v} \times \vec{B})$$

$$\vec{F} = \vec{F}_E + \vec{F}_M$$

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$$

Magnetic Boundary conditions

$$\textcircled{1} B_{n1} = B_{n2}$$

$$\textcircled{2} H_{t1} = H_{t2}$$

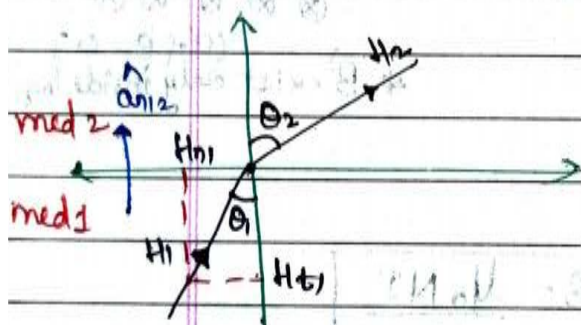
$$(\vec{H}_1 - \vec{H}_2) \times \hat{a}_{n12} = \vec{K}$$

$$\text{if } \vec{K} = 0$$

$$\text{if } \vec{K} \neq 0$$

where $\vec{K}$: Current density

$\hat{a}_{n12}$: from med 1 to med 2



H_t : Tangential component

H_n : Normal component

$$\boxed{\frac{\tan \theta_1}{\tan \theta_2} = \frac{H_{t1}}{H_{n1}}}$$

$$\boxed{\tan \theta_2 = \frac{H_{t2}}{H_{n2}}}$$

$$\boxed{\frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_{r1}}{\mu_{r2}}}$$