SOLVED PAPER (memory based)

VITEEE 2019

GENERAL INSTRUCTIONS

- This question paper contains total 125 questions divided into four parts :
  Part I : Physics Q. No - 1 to 40
  Part II : Chemistry Q. No - 41 to 80
  Part III : Mathematics Q. No - 81 to 120
  Part IV : English Q. No - 121 to 125
- All questions are multiple choice questions with four options, only one of them is correct.
- For each correct response, the candidate will get 1 mark.
- There is no negative marking for the wrong answer.
- The test is of 2½ hours duration.

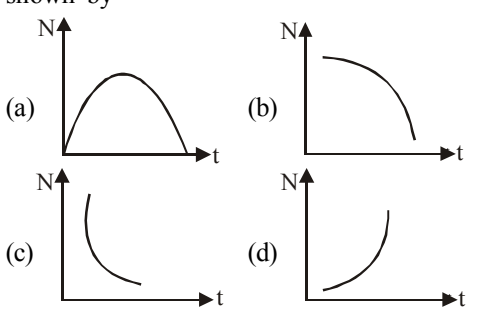
PART - I (PHYSICS)

1. The electric resistance of a certain wire of iron is R. If its length and radius are both doubled, then
(a) the resistance and the specific resistance, will both remain unchanged
(b) the resistance will be doubled and the specific resistance will be halved
(c) the resistance will be halved and the specific resistance will remain unchanged
(d) the resistance will be halved and the specific resistance will be doubled

2. Two thin lenses are in contact and the focal length of the combination is 80 cm. If the focal length of one lens is 20 cm, then the power of the other lens will be
(a) 1.66 D (b) 4.00 D (c) −100 D (d) −3.75 D

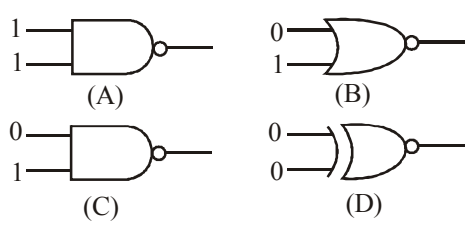
3. If the kinetic energy of a free electron doubles, it's de-Broglie wavelength changes by the factor
(a) 2 (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

4. Radioactive element decays to form a stable nuclide, then the rate of decay of reactant is shown by



5. The ratio of the energies of the hydrogen atom in its first to second excited states is
(a) 1/4 (b) 4/9 (c) 9/4 (d) 4

6. Which of the following gates will have an output of 1?



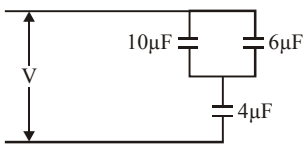
(a) D (b) A (c) B (d) C

7. A point charge q is rotated along a circle in the electric field generated by another point charge Q. The work done by the electric field on the rotating charge in one complete revolution is

(a) zero
(b) positive
(c) negative
(d) zero if the charge Q is at the centre and nonzero otherwise.

8. The equivalent capacitance of the combination of the capacitors is

(a) 3.20μF
(b) 7.80μF
(c) 3.90μF
(d) 2.16μF



9. The half-life period and the mean life period of a radioactive element are denoted respectively by T_h and T_m. Then

(a) $T_h = T_m$ (b) $T_h > T_m$
(c) $T_h < T_m$ (d) $T_h \geq T_m$

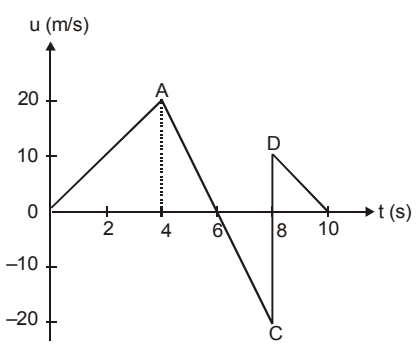
10. In a common base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be

(a) 49 (b) 50 (c) 51 (d) 48

11. The magnetic field at a distance r from a long wire carrying current i is 0.4 tesla. The magnetic field at a distance 2r is

(a) 0.2 tesla (b) 0.8 tesla
(c) 0.1 tesla (d) 1.6 tesla

12. The velocity-time graph of a body moving in a straight line is shown in fig. Find the displacement and distance travelled by the body in 10 seconds.



(a) 50m, 90m (b) 5m, 9m
(c) 9m, 5m (d) 90m, 50m

13. An electric dipole is kept in a uniform electric field. It experiences

(a) a force and a torque
(b) a force, but no torque
(c) a torque but no net force
(d) neither a force nor a torque

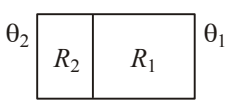
14. A person swims in a river aiming to reach exactly on the opposite point on the bank of a river. His speed of swimming is 0.5 m/s at an angle of 120° with the direction of flow of water. The speed of water is

(a) 1.0 m/s (b) 0.5 m/s
(c) 0.25 m/s (d) 0.43 m/s

15. A rain drop of radius 0.3 mm has a terminal velocity in air is 1 m/s. The viscosity of air is 8×10^{-5} poise. The viscous force on it is

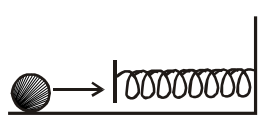
(a) 45.2×10^{-4} dyne (b) 101.73×10^{-5} dyne
(c) 16.95×10^{-4} dyne (d) 16.95×10^{-5} dyne

16. Consider a pair of insulating blocks with thermal resistances R_1 and R_2 as shown in the figure. The temperature θ at the boundary between the two blocks is



(a) $(\theta_1\theta_2\sqrt{R_1R_2}) / (\theta_1 + \theta_2)(R_1 + R_2)$
(b) $(\theta_1 R_1 + \theta_2 R_2) / (R_1 + R_2)$
(c) $[(\theta_1 + \theta_2) R_1 R_2] / (R_1^2 + R_2^2)$
(d) $(\theta_1 R_2 + \theta_2 R_1) / (R_1 + R_2)$

17. A mass of 0.5 kg moving with a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant k = 50 N/m. The maximum compression of the spring would be

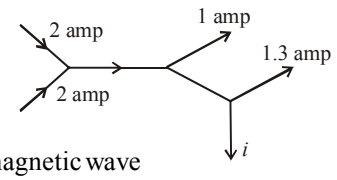


(a) 0.5m (b) 0.15m
(c) 0.12m (d) 1.5m

18. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected ?
(a) Angular velocity
(b) Angular momentum
(c) Moment of inertia
(d) Rotational kinetic energy

19. The Young's modulus of a perfectly rigid body is
(a) unity
(b) zero
(c) infinity
(d) some finite non-zero constant

20. The figure below shows currents in a part of electric circuit. The current *i* is
(a) 1.7 amp
(b) 3.7 amp
(c) 1.3 amp
(d) 1 amp



21. In an electromagnetic wave
(a) power is transmitted along the magnetic field
(b) power is transmitted along the electric field
(c) power is equally transferred along the electric and magnetic fields
(d) power is transmitted in a direction perpendicular to both the fields

22. A particle covers half of the circle of radius r. Then the displacement and distance of the particle are respectively
(a) $2\pi r, 0$ (b) $2r, \pi r$
(c) $\frac{\pi r}{2}, 2r$ (d) $\pi r, r$

23. A metal ring is held horizontally and bar magnet is dropped through the ring with its length along the axis of the ring. The acceleration of the falling magnet
(a) is equal to g
(b) is less than g
(c) is more than g
(d) depends on the diameter of ring and length of magnet

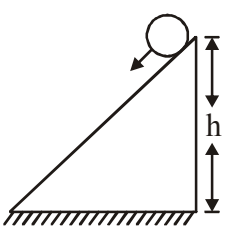
24. A particle having a mass 0.5 kg is projected under gravity with a speed of 98 m/sec at an angle of 60º. The magnitude of the change in momentum (in N-sec) of the particle after 10 seconds is $[g = 9.8\ m/sec^2]$
(a) 0.5 (b) 49
(c) 98 (d) 490

25. For a series RLC circuit $R = X_L = 2X_C$. The impedance of the circuit and phase difference between V and I respectively will be
(a) $\frac{\sqrt{5}R}{2}, \tan^{-1}(2)$ (b) $\frac{\sqrt{5}R}{2}, \tan^{-1}(1/2)$
(c) $\sqrt{5}X_C, \tan^{-1}(2)$ (d) $\sqrt{5}R, \tan^{-1}(1/2)$

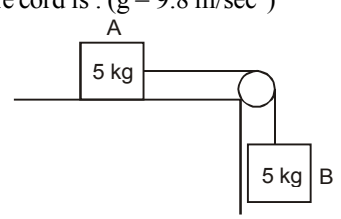
26. A man holding a rifle (mass of person and rifle together is 100kg) stands on a smooth surface and fires 10 shots horizontally in 5 sec. Each bullet has a mass 10 g with a muzzle velocity of 800 ms^{-1}. The velocity which the rifle man attains after firing 10 shots will be
(a) 8 ms^{-1} (b) 0.8 ms^{-1}
(c) 0.08 ms^{-1} (d) – 0.8 ms^{-1}

27. Escape velocity when a body of mass m is thrown vertically from the surface of the earth is v, what will be the escape velocity of another body of mass 4 m is thrown vertically
(a) v (b) 2v
(c) 4v (d) None of these

28. A solid cylinder of mass m and radius R rolls down inclined plane without slipping. The speed of its C.M. when it reaches the bottom is
(a) $\sqrt{2gh}$
(b) $\sqrt{4gh/3}$
(c) $\sqrt{3/4gh}$
(d) $\sqrt{4gh}$



29. A block of mass 5 kg resting on a horizontal surface is connected by a cord, passing over a light frictionless pulley to a hanging block of mass 5 kg. The coefficient of kinetic friction between the block and the surface is 0.5. Tension in the cord is : $(g = 9.8\ m/sec^2)$



(a) 49 N (b) Zero
(c) 36.75 N (d) 2.45 N

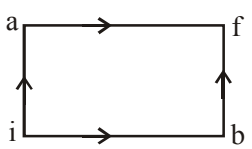
30. A and B are two wires. The radius of A is twice that of B. They are stretched by the same load. Then the stress on B is
(a) equal to that on A
(b) four times that on A
(c) two times that on A
(d) half that on A

31. A liquid is allowed to flow into a tube of truncated cone shape. Identify the correct statement from the following.
(a) The speed is high at the wider end and low at the narrow end.
(b) The speed is low at the wider end and high at the narrow end.
(c) The speed is same at both ends in a stream line flow.
(d) The liquid flows with uniform velocity in the tube.

32. A planet revolves in an elliptical orbit around the sun. The semi-major and semi-minor axes are a and b. Then the square of time period, T is directly proportional to
(a) a^3 (b) b^3
(c) $\left(\frac{a+b}{2}\right)^3$ (d) $\left(\frac{a-b}{2}\right)^3$

33. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is (hc = 1240 eV.nm)
(a) 3.09 eV (b) 1.42 eV
(c) 1.51 eV (d) 1.68 eV

34. When a system is taken from state i to state f along the path iaf, it is found that Q = 50 cal and W = 20 cal. Along the path ibf Q = 36 cal. W along the path ibf is

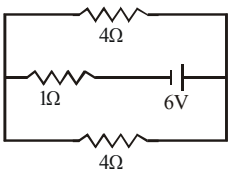


(a) 14 cal (b) 6 cal (c) 16 cal (d) 66 cal

35. If the differential equation for a simple harmonic motion is $\frac{d^2y}{dt^2} + 2y = 0$, the time-period of the motion is
(a) $\pi\sqrt{2}$ sec (b) $\frac{\sqrt{2}s}{\pi}$ sec
(c) $\frac{\pi}{\sqrt{2}}$ sec (d) 2π sec

36. Tube *A* has both ends open while tube *B* has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube *A* and *B* is
(a) 1 : 2 (b) 1 : 4 (c) 2 : 1 (d) 4 : 1

37. The current in the 1Ω resistor shown in the circuit is
(a) $\frac{2}{3}$ A
(b) 3 A
(c) 6 A
(d) 2 A



38. The work done by an uniform magnetic field, on a moving charge is
(a) zero because $\vec{F}$ acts parallel to $\vec{v}$
(b) positive because $\vec{F}$ acts perpendicular to $\vec{v}$
(c) zero because $\vec{F}$ acts perpendicular to $\vec{v}$
(d) negative because $\vec{F}$ acts parallel to $\vec{v}$

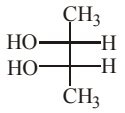
39. In Young's double slit experimental setup, if the wavelength alone is doubled, the bandwidth β becomes
(a) $\frac{\beta}{2}$ (b) 2β (c) 3β (d) β

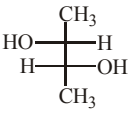
40. In Young's double slit experiment, the central point on the screen is
(a) bright
(b) dark
(c) first bright and then dark
(d) first dark and then bright

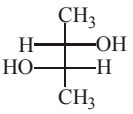
PART - II (CHEMISTRY)

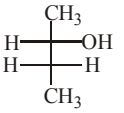
41. The work done in ergs for the reversible expansion of one mole of an ideal gas from a volume of 10 litres to 20 litres at 25°C is
(a) $2.303 \times 298 \times 0.082 \log 2$
(b) $298 \times 10^7 \times 8.31 \times 2.303 \log 2$
(c) $2.303 \times 298 \times 0.082 \log 0.5$
(d) $8.31 \times 10^7 \times 298 - 2.303 \log 0.5$

42. A Fischer projection of (2R, 3S)-2,3-butanediol is : -

(a) 

(b) 

(c) 

(d) 

43. Arrange the following particles in increasing order of values of e/m ratio : electron (e), proton (p), neutron (n) and α-particle (α) -

(a) n, p, e, α (b) n, α, p, e

(c) n, p, α, e (d) e, p, n, α

44. Acidified potassium dichromate is treated with hydrogen sulphide. In the reaction, the oxidation number of chromium

(a) Increases from + 3 to +6

(b) Decreases from +6 to +3

(c) Remains unchanged

(d) Decreases from +6 to +2

45. For the reaction $N_2 + 3H_2 \rightarrow 2NH_3$

if $\frac{\Delta[NH_3]}{\Delta t} = 2 \times 10^{-4}$ mol $l^{-1}s^{-1}$, then value of

$\frac{-\Delta[H_2]}{\Delta t}$ would be

(a) 1×10^{-4} mol $L^{-1}s^{-1}$

(b) 3×10^{-4} mol $L^{-1}s^{-1}$

(c) 4×10^{-4} mol $L^{-1}s^{-1}$

(d) 6×10^{-4} mol $L^{-1}s^{-1}$

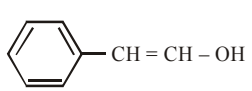
46. The value of K_c for the reaction :

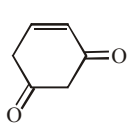
$A + 3B \rightleftharpoons 2C$ at 400°C is 0.5. Calculate the value of K_P

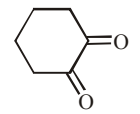
(a) 1.64×10^{-4} (b) 1.64×10^{-6}

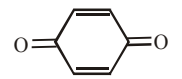
(c) 1.64×10^{-5} (d) 1.64×10^{-3}

47. Tautomerism is net exhibited by –

(a) 

(b) 

(c) 

(d) 

48. The electrode potentials for

$Cu^{2+}(aq) + e^- \longrightarrow Cu^+(aq)$

and $Cu^+(aq) + e^- \longrightarrow Cu(s)$

are + 0.15 V and + 0.50 V, respectively. The value of $E^\circ_{Cu^{2+}/Cu}$ will be :

(a) 0.500 V (b) 0.325 V

(c) 0.650 V (d) 0.150 V

49. Deep sea divers used to respirate a mixture of

(a) Oxygen and argon

(b) Oxygen and helium

(c) Oxygen and nitrogen

(d) Oxygen and hydrogen

50. The wavelengths of two photons are 2000Å and 4000Å respectively. What is the ratio of their energies?

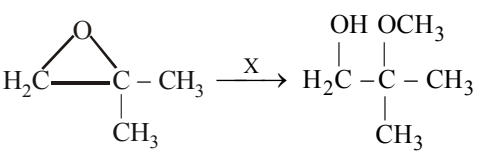
(a) 1/4 (b) 4 (c) 1/2 (d) 2

51. Ethylene glycol, on oxidation with per-iodic acid, gives

(a) Oxalic acid (b) Glycol

(c) Formaldehyde (d) Glycolic acid

52. What is X in the following change ?



(a) CH_3OH, H_2SO_4

(b) $CH_3OH, CH_3\bar{O}\overset{+}{N}a$

(c) H_2O / H_2SO_4 followed by CH_3OH

(d) $CH_3MgBr /\ H_3O^+$

53. In a closed insulated container, a liquid is stirred with a paddle to increase its temperature. In this process, which of the following is true ?

(a) $\Delta E = W = Q = 0$

(b) $\Delta E \neq 0, Q = W = 0$

(c) $\Delta E = W \neq 0, Q = 0$

(d) $\Delta E = Q \neq 0, W = 0$

54 At low pressure, the van der Waal's equation is reduced to

(a) $Z = \frac{PV_m}{RT} = 1 - \frac{a}{VRT}$

(b) $Z = \frac{PV_m}{RT} = 1 + \frac{b}{RT}P$

(c) $PV_m = RT$

(d) $Z = \frac{PV_m}{RT} = 1 - \frac{a}{RT}$

55. Copper sulphate solution reacts with KCN to give

(a) $Cu(CN)_2$ (b) CuCN

(c) $K_2[Cu(CN)_4]$ (d) $K_3[Cu(CN)_4]$

56. For the reaction

$$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l),\ \Delta H = -285.8\ kJ\ mol^{-1}$$

$\Delta S = -0.163\ kJ\ mol^{-1}K^{-1}$.

What is the value of free energy change at 27°C for the reaction ?

(a) $-236.9\ kJ\ mol^{-1}$ (b) $-281.4\ kJ\ mol^{-1}$

(c) $-334.7\ kJ\ mol^{-1}$ (d) $+334.7\ kJ\ mol^{-1}$

57. $A \xrightarrow[dil.H_2SO_4]{K_2Cr_2O_7} B \xrightarrow[H_2O]{CH_3MgI} CH_3-\underset{OH}{\overset{CH_3}{\underset{|}{\overset{|}{C}}}}-CH_3,$

the reactant A is

(a) $CH_3CHOHCH_3$ (b) CH_3COCH_3

(c) C_2H_5OH (d) CH_3COOH

58. Bragg's law is given by the which of the following equation?

(a) $n\lambda = 2\theta \sin\theta$ (b) $n\lambda = 2d \sin\theta$

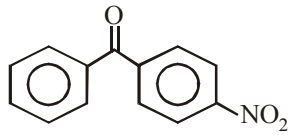
(c) $2n\lambda = d \sin\theta$ (d) $n\frac{\theta}{2} = \frac{d}{2}\sin\theta$

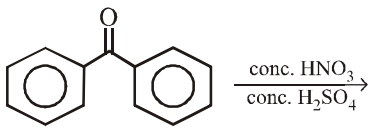
59. The reaction $2NO(g) + O_2(g) \rightarrow 2NO_2(g)$ is of first order. If volume of reaction vessel is reduced to 1/3, the rate of reaction would be

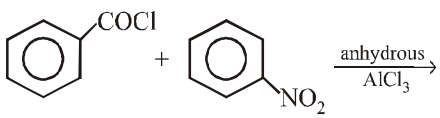
(a) 1/3 times (b) 2/3 times

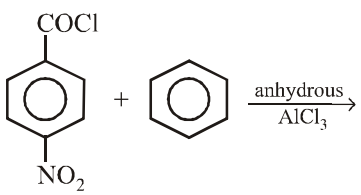
(c) 3 times (d) 6 times

60. Which are the starting materials for the preparation of?



(a) 

(b) 

(c) 

(d) Any of the three

61. Which element gives maximum number of oxides?

(a) V (b) Cr (c) Mn (d) Fe

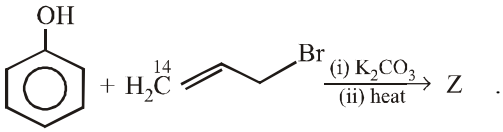
62. An organic compound 'A' has the molecular formula C_3H_6O, it undergoes iodoform test. When saturated with dil. HCl it gives 'B' of molecular formula $C_9H_{14}O$. A and B respectively are

(a) Propanal and mesitylene

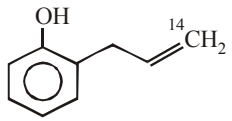
(b) Propanone and mesityl oxide

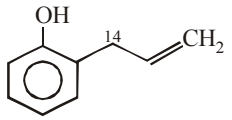
(c) Propanone and 2,6-dimethyl-2, 5-heptadien-4-one

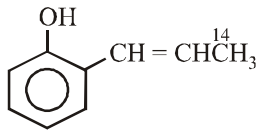
(d) Propanone and mesitylene oxide

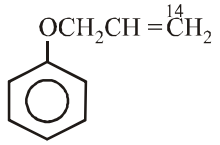
63. 

Here Z is

(a) 

(b) 

(c) 

(d) 

64. Calculate the entropy change in melting 1 mole of ice at 273K, $\Delta H_f^0 = 6.025$ kJ/mole-

(a) 11.2 JK^{-1} mol^{-1} (b) 22.1 JK^{-1} mol^{-1}

(c) 15.1 JK^{-1} mol^{-1} (d) 5.1 JK^{-1} mol^{-1}

65. The rate constant is doubled when temperature increases from 27 °C to 37 °C. Activation energy in kJ is

(a) 34 (b) 54 (c) 100 (d) 50

66. In the reaction, $C_6H_5OH \xrightarrow{NaOH} (A)$

$\xrightarrow[140°C,(4-7)atm]{CO_2} (B) \xrightarrow{HCl} (C)$, the compound (C) is

(a) Benzoic acid (b) Salicylaldehyde

(c) Chlorobenzene (d) Salicylic acid

67. Starting from propanoic acid, the following reactions were carried out

Propanoic acid

$\xrightarrow{SOCl_2} X \xrightarrow{NH_3} Y \xrightarrow{Br_2+KOH} Z$

What is the compound Z?

(a) CH_3-CH_2-Br

(b) $CH_3-CH_2-NH_2$

(c) $CH_3-CH_2-\overset{\overset{O}{||}}{C}-Br$

(d) $CH_3-CH_2-CH_2-NH_2$

68. Calculate the energy needed to convert three moles of sodium atoms in the gaseous state to sodium ions. The ionization energy of sodium is 495 kJ mol^{-1}–

(a) 1485 kJ (b) 495 kJ

(c) 148.5 kJ (d) None

69. The uncertainty in position and velocity of a particle are 10^{-10} m and 5.27×10^{-24} ms^{-1} respectively. Calculate the mass of the particle is ($h = 6.625 \times 10^{-34}$ Js) –

(a) 0.099 kg (b) 0.99 g

(c) 0.92 kg (d) None

70. Choose the complex which is paramagnetic

(i) $[Fe(H_2O)_6]^{2+}$ (ii) $K_3[Cr(CN)_6]$

(iii) $K_3[Fe(CN)_6]$ (iv) $K_2[Ni(CN)_4]$

(a) (i), (ii) and (iii) (b) (i), (iii) and (iv)

(c) (ii), (iii), and (iv) (d) (i), (ii) and (iv)

71. Phosphine is generally prepared in the laboratory :

(a) By heating phosphorus in a current of hydrogen

(b) By heating white phosphorus with an aqueous solution of caustic potash

(c) By decomposition of P_2H_4 at 110°C

(d) By heating red phosphorus with an aqueous solution of caustic soda

72. In a hydrogen – oxygen fuel cell, combustion of hydrogen occurs to :

(a) Produce high purity water

(b) Create potential difference between the two electrodes

(c) Generate heat

(d) Remove adsorbed oxygen from electrode surfaces

73. Among the following molecule

(i) XeO_3 (ii) $XeOF_4$ (iii) XeF_6

Those having same number of lone pairs on Xe are

(a) (i) and (ii) only (b) (i) and (iii) only

(c) (ii) and (iii) only (d) (i), (ii) and (iii)

74. An alkene of molecule formula C_9H_{18} on ozonolysis gives 2, 2-dimethylpropanal & 2-butanone, then the alkene is

(a) 2, 2, 4-trimethyl -3-hexene

(b) 2, 2, 6-trimethyl -3-hexene

(c) 2, 3, 4-trimethyl -2-hexene

(d) 2, 2-dimethyl -2-heptene

75. Glucose when heated with CH_3OH in presence of dry HCl gas gives α- and β-methyl glucosides because it contains:

(a) An aldehyde group

(b) A - CH_2OH group

(c) A ring structure

(d) Five hydroxyl groups

76. When acetylene is passed into methanol at 160-200ºC in the presence of a small amount of potassium methoxide under pressure, the following is formed –

(a) Polyvinyl alcohol

(b) Divinyl ether

(c) Dimethyl ether

(d) Methyl vinyl ether

77. Which compound can exist in a dipolar (zwitter ion) state ?

(a) $C_6H_5CH_2CH(N{=}CH_2)COOH$

(b) $(CH_3)_2CH.CH(NH_2)COOH$

(c) $C_6H_5CONHCH_2COOH$

(d) $HOOC.CH_2CH_2COCOOH$

78. A coordination complex compound of cobalt has the molecular formula containing five ammonia molecules, one nitro group and two chlorine atoms for one cobalt atom. One mole of this compound produces three mole ions in an aqueous solution; on reacting with excess of $AgNO_3$, AgCl is precipitated. The ionic formula for this complex would be:

(a) $[Co(NH_3)_5(NO_2)]Cl_2$

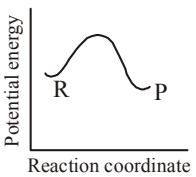
(b) $[Co(NH_3)_5\,Cl]\,[Cl(NO_2)]$

(c) $[Co(NH_3)_4\,(NO_2)\,Cl]\,[(NH_3)Cl]$

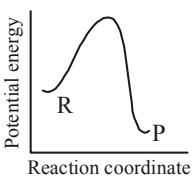
(d) $[Co(NH_3)_5]\,[NO_2)_2Cl_2]$

79. An endothermic reaction with high activation energy for the forward reaction is given by the diagram.

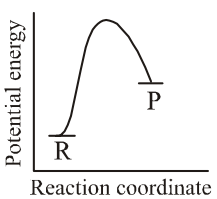
(a)



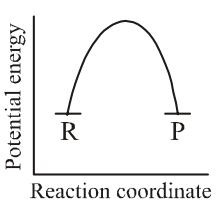
(b)



(c)



(d)



80 . The volume of a closed reaction vessel in which the following equilibrium reaction occurs is halved :

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$$

As a result,

(a) The rates of forward and backward reactions will remain the same.

(b) The equilibrium will not shift.

(c) The equilibrium will shift to the right.

(d) The rate of forward reaction will become double that of reverse reaction and the equilibrium will shift to the right.

PART - III (MATHEMATICS)

81. If $\tan\theta = \sqrt{n}$ for some non-square natural number n, then $\sec 2\theta$ is :

(a) a rational number
(b) an irrational number
(c) a positive number
(d) none of the above

82. If $z = x + iy$, $z^{1/3} = a - ib$, then $\frac{x}{a} - \frac{y}{b} = k(a^2 - b^2)$ where k is equal to

(a) 1 (b) 2 (c) 3 (d) 4

83. If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are (–3, 2), then the coordinates at the other end are

(a) (5, 3) (b) (6, 2) (c) (1, –8) (d) (11, 2)

84. The system of linear equations : $x + y + z = 0$, $2x + y - z = 0$, $3x + 2y = 0$ has :

(a) no solution
(b) a unique solution
(c) an infinitely many solution
(d) None of these

85. If $\omega = \frac{-1+\sqrt{3}i}{2}$ then $(3 + \omega + 3\omega^2)^4$ is

(a) 16 (b) -16 (c) 16ω (d) $16\omega^2$

86. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then radius of the circle is

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{5}{2}$

87. The position vector of A and B are

$2\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{i} + 4\hat{j} + 4\hat{k}$

The length of the internal bisector of $\angle BOA$ of triangle AOB is

(a) $\sqrt{\frac{136}{9}}$ (b) $\frac{\sqrt{136}}{9}$

(c) $\frac{20}{3}$ (d) $\sqrt{\frac{217}{9}}$

88. If $y = \tan^{-1}\frac{4x}{1+5x^2} + \tan^{-1}\frac{2+3x}{3-2x}$, then $\frac{dy}{dx} =$

(a) $\frac{1}{1+25x^2} + \frac{2}{1+x^2}$

(b) $\frac{5}{1+25x^2} + \frac{2}{1+x^2}$

(c) $\frac{5}{1+25x^2}$

(d) $\frac{1}{1+25x^2}$

89. If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, then AB is equal to

(a) B (b) A (c) O (d) I

90. A circle has radius 3 and its centre lies on the line $y = x - 1$. The equation of the circle, if it passes through (7, 3), is

(a) $x^2 + y^2 + 8x - 6y + 16 = 0$

(b) $x^2 + y^2 - 8x + 6y + 16 = 0$

(c) $x^2 + y^2 - 8x - 6y - 16 = 0$

(d) $x^2 + y^2 - 8x - 6y + 16 = 0$

91. At how many points between the interval $(-\infty, \infty)$ is the function $f(x) = \sin x$ is not differentiable.

(a) 0 (b) 7 (c) 9 (d) 3

92. If vector equation of the line

$\frac{x-2}{2} = \frac{2y-5}{-3} = z + 1$, is

$\vec{r} = \left(2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - \frac{3}{2}\hat{j} + p\hat{k}\right)$ then p is equal to

(a) 0 (b) 1 (c) 2 (d) 3

93. Evaluate $\lim_{x\to 2} \frac{\sqrt{(x+7)} - 3\sqrt{(2x-3)}}{\sqrt[3]{(x+6)} - 2\sqrt[3]{(3x-5)}}$.

(a) $\frac{17}{9}$ (b) $\frac{17}{18}$ (c) $\frac{34}{23}$ (d) $\frac{26}{7}$

94. The radius of a right circular cylinder increases at the rate of 0.1 cm/min, and the height decreases at the rate of 0.2 cm/min. The rate of change of the volume of the cylinder, in cm^3/min, when the radius is 2 cm and the height is 3 cm is

(a) -2π (b) $-\frac{8\pi}{5}$ (c) $-\frac{3\pi}{5}$ (d) $\frac{2\pi}{5}$

95. If a, b, c are in A. P., then the value of

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$ is:

(a) 3 (b) -3

(c) 0 (d) None of these

96. The solution of $\sin x = -\frac{\sqrt{3}}{2}$ is

(a) $x = n\pi + (-1)^n \frac{4\pi}{3}$, where $n \in Z$

(b) $x = n\pi + (-1)^n \frac{2\pi}{3}$, where $n \in Z$

(c) $x = n\pi + (-1)^n \frac{3\pi}{3}$, where $n \in Z$

(d) None of the these

97. The equation $y^2 + 3 = 2(2x + y)$ represents a parabola with the vertex at

(a) $\left(\frac{1}{2}, 1\right)$ and axis parallel to y-axis

(b) $\left(1, \frac{1}{2}\right)$ and axis parallel to x-axis

(c) $\left(\frac{1}{2}, 1\right)$ and focus at $\left(\frac{3}{2}, 1\right)$

(d) $\left(1, \frac{1}{2}\right)$ and focus at $\left(\frac{3}{2}, 1\right)$

98. If $\sin y = x \sin(a + y)$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{\sin\sqrt{a}}{\sin(a+y)}$ (b) $\frac{\sin^2(a+y)}{\sin a}$

(c) $\sin(a + y)$ (d) None of these

99. $\int (27e^{9x} + e^{12x})^{1/3} dx$ is equal to

(a) $(1/4)(27 + e^{3x})^{1/3} + C$

(b) $(1/4)(27 + e^{3x})^{2/3} + C$

(c) $(1/3)(27 + e^{3x})^{4/3} + C$

(d) $(1/4)(27 + e^{3x})^{4/3} + C$

100. The area under the curve $y = |\cos x - \sin x|$, $0 \le x \le \frac{\pi}{2}$, and above x-axis is :

(a) $2\sqrt{2}$ (b) $2\sqrt{2} - 2$

(c) $2\sqrt{2} + 2$ (d) 0

101. The conic represented by $x = 2(\cos t + \sin t)$, $y = 5(\cos t - \sin t)$ is

(a) a circle (b) a parabola

(c) an ellipse (d) a hyperbola

102. If $\left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a}.\vec{b}\right)^2 = 676$ and $\left|\vec{b}\right| = 2$, then $\left|\vec{a}\right|$ is equal to

(a) 13 (b) 26

(c) 39 (d) None of these

103. The equation of the plane which bisects the angle between the planes $3x - 6y + 2z + 5 = 0$ and $4x - 12y + 3z - 3 = 0$ which contains the origin is

(a) $33x - 13y + 32z + 45 = 0$

(b) $x - 3y + z - 5 = 0$

(c) $33x + 13y + 32z + 45 = 0$

(d) None of these

104. A wire 34 cm long is to be bent in the form of a quadrilateral of which each angle is 90°. What is the maximum area which can be enclosed inside the quadrilateral?

(a) 68 cm^2 (b) 70 cm^2

(c) 71.25 cm^2 (d) 72. 25 cm^2

105. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B\log\sin(x-\alpha) + C$, then value of (A, B) is

(a) $(-\cos\alpha, \sin\alpha)$ (b) $(\cos\alpha, \sin\alpha)$

(c) $(-\sin\alpha, \cos\alpha)$ (d) $(\sin\alpha, \cos\alpha)$

106. The equation of the chord of the hyperbola $25x^2 - 16y^2 = 400$ that is bisected at point (5, 3) is:

(a) $135x - 48y = 481$ (b) $125x - 48y = 481$

(c) $125x - 4y = 48$ (d) None of these

107. Value of $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\cot x}} dx$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$

(c) $\frac{12}{\pi}$ (d) None of these

108. The domain of the function $f(x) = \sqrt{\frac{1}{|x-2|-(x-2)}}$ is :

(a) $(-\infty, 2]$ (b) $(2, \infty)$

(c) $(-\infty, 2)$ (d) $[2, \infty)$

109. Six identical coins are arranged in a row. The number of ways in which the number of tails is equal to the number of heads is

(a) 20 (b) 9 (c) 120 (d) 40

110. Let f be the function defined by

$$f(x) = \begin{cases} \frac{x^2-1}{x^2-2|x-1|-1}, & x \neq 1 \\ 1/2, & x = 1 \end{cases}$$

(a) The function is continuous for all values of x

(b) The function is continuous only for x > 1

(c) The function is continuous at x = 1

(d) The function is not continuous at x = 1

111. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$,

(a) $f(x)$ is a strictly increasing function

(b) $f(x)$ has local maxima

(c) $f(x)$ is a strictly decreasing function

(d) $f(x)$ is bounded

112. The solution of the differential equation $\left\{1 + x\sqrt{\left(x^2+y^2\right)}\right\} dx + \left\{\sqrt{\left(x^2+y^2\right)} - 1\right\} y\, dy = 0$ is

(a) $x^2 + \frac{y^2}{2} + \frac{1}{3}\left(x^2+y^2\right)^{3/2} = C$

(b) $x - \frac{y^2}{3} + \frac{1}{2}\left(x^2+y^2\right)^{1/2} = C$

(c) $x - \frac{y^2}{2} + \frac{1}{3}\left(x^2+y^2\right)^{3/2} = C$

(d) None of these

113. The integrating factor of $\frac{xdy}{dx} - y = x^4 - 3x$ is

(a) x (b) log x (c) $\frac{1}{x}$ (d) –x

114. It is given that the events A and B are such that $P(A) = \frac{1}{4}, P(A \mid B) = \frac{1}{2}$ and $P(B \mid A) = \frac{2}{3}$. Then $P(B)$ is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

115. Let $f : R \to R$, $g : R \to R$ be two functions such that $f(x) = 2x - 3, g(x) = x^3 + 5$. The function $(fog)^{-1}(x)$ is equal to

(a) $\left(\frac{x+7}{2}\right)^{1/3}$ (b) $\left(x - \frac{7}{2}\right)^{1/3}$

(c) $\left(\frac{x-2}{7}\right)^{1/3}$ (d) $\left(\frac{x-7}{2}\right)^{1/3}$

116. The inverse of the statement $(p \wedge \sim q) \to r$ is

(a) $\sim(p \vee \sim q) \to \sim r$ (b) $(\sim p \wedge q) \to \sim r$

(c) $(\sim p \vee q) \to \sim r$ (d) None of these

117. The value of x in the interval [4, 9] at which the function $f(x) = \sqrt{x}$ satisfies the mean value theorem is

(a) $\frac{13}{4}$ (b) $\frac{17}{4}$ (c) $\frac{21}{4}$ (d) $\frac{25}{4}$

118. The value of the integral $\int_{-1}^{3}(|x|+|x-1|)dx$ is

(a) 4 (b) 9 (c) 2 (d) $\frac{9}{2}$

119. Let a, b, c, be in A.P. with a common difference d. Then $e^{1/c}$, $e^{b/ac}$, $e^{1/a}$ are in :

(a) G.P. with common ratio e^d
(b) G.P with common ratio $e^{1/d}$
(c) G.P. with common ratio $e^{d/(b^2-d^2)}$
(d) A.P.

120. If the second term in the expansion $\left(\sqrt[13]{a}+\frac{a}{\sqrt{a^{-1}}}\right)^n$ is $14a^{5/2}$, then $\frac{{}^nC_3}{{}^nC_2}$ =

(a) 4 (b) 3 (c) 12 (d) 6

PART - IV (ENGLISH)

121. In the given sentence, find out which part has an error. The letter of that part will be your answer. If there is no error, mark (d) as your answer.
She is a brilliant teacher (a)/ but of her three children (b)/ neither has any merit. (c)/ No error (d)

122. Find the synonym of the word IMPECCABLE
(a) Remarkable (b) Unbelievable
(c) Flawless (d) Displeasing

123. Find the antonym of the word AMELIORATE
(a) Improve (b) Depend
(c) Soften (d) Worsen

124. Find the meaning of the given idiom
A bolt from the blue
(a) An unpleasant event
(b) An inexplicable event
(c) A delayed event
(d) An unexpected event

125. Read the passage and answer the given question
There seems to be no chilly distance existing between the German students and the professor, but, on the contrary, a companionable intercourse, the opposite of chilliness and reserve. When the professor enters a beer hall in the evening where students are gathered together, these rise up and take off their caps and invite the old gentleman to sit with them and partake. He accepts, and the pleasant talk and the beer flow for an hour or two, and by and by the professor, properly charged and comfortable, gives a cordial good night, while the students stand bowing and uncovered, and then he moves on his happy way homeward with all his vast cargo of learning afloat in his hold. Nobody finds fault or feels outraged. No harm has been done.
What does the author mean by the phrase 'no chilly distance'?
(a) Professor's home is not very far from the beer hall.
(b) Students and the professor are very friendly with each other.
(c) The weather is not very chilly in Germany.
(d) The professor being very strict scares the students quite a few times as in the beer hall.

SOLUTIONS

PART - I (PHYSICS)

1. **(c)** $R_1 = \frac{\rho \ell_1}{A_1}$, now $\ell_2 = 2\ell_1$ and $r_2 = 2r_1$

$A_1 = \pi r_1^2$

$A_2 = \pi(r_2)^2 = \pi(2r_1)^2 = 4\pi r_1^2 = 4A_1$

$R_2 = \frac{\rho \ell_2}{A_2}$

$\therefore \quad R_2 = \frac{\rho(2\ell_1)}{4A_1} = \frac{\rho \ell_1}{2A_1} = \frac{R_1}{2}$

$\therefore$ Resistance is halved, but specific resistance remains the same.

2. **(d)** $P_2 = P - P_1 = \frac{100}{80} - \frac{100}{20} = -3.75$ D

3. **(d)** de-Broglie wavelength,

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2.m.(K.E)}}$

$\therefore \quad \lambda \propto \frac{1}{\sqrt{K.E}}$

If K.E is doubled, then de-Broglie wavelength becomes $\frac{\lambda}{\sqrt{2}}$

4. **(c)** No. of nuclide at time t is given by

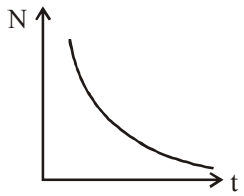
$N = N_o e^{-\lambda t}$

Where, N_o = initial nuclide

Thus this equation is equivalent to

$y = ae^{-kx}$

So correct graph is in option (c)



5. **(c)** Ist excited state corresponds to n = 2
2nd excited state corresponds to n = 3

As $E \propto \frac{1}{n^2}$

$\therefore \quad \frac{E_1}{E_2} = \frac{n_2^2}{n_1^2} = \frac{3^2}{2^2} = \frac{9}{4}$

6. **(d)** (A) is a NAND gate so output is

$\overline{1 \times 1} = \bar{1} = 0$

(B) is a NOR gate so output is

$\overline{0 + 1} = \bar{1} = 0$

(C) is a NAND gate so output is

$\overline{0 \times 1} = \bar{0} = 1$

(D) is a XOR gate so output is

$0 \oplus 0 = 0$

7. **(a)** Work done in displacement of charge, $W = q\Delta V$
where ΔV = potential difference between the displacement of charge
Hence, work done in complete rotation is zero.

8. **(a)** Equivalent capacitance of two parallel capacitors 10 μF and 6 μF = (10 + 6)μF = 16 μF
This 16 μF capacitor is in series combination with 4μF capacitor,
$\therefore$ Equivalent capacitance of the entire combination $= \frac{16 \times 4}{16 + 4} = \frac{64}{20} = 3.2$ μF

9. **(c)** Half life, $T_h = \frac{0.693}{\lambda}$, $T_m = \frac{1}{\lambda}$

Clearly, $T_h < T_m$.

10. **(a)** $I_c = 5.488$ mA, $I_e = 5.6$ mA

$$\alpha = \frac{I_c}{I_e}$$

$$\alpha = \frac{5.488}{5.6}$$

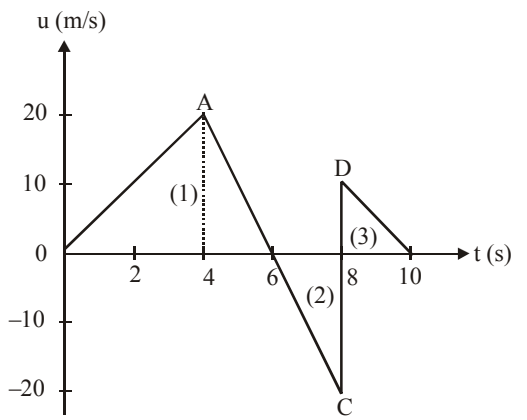
$$\beta = \frac{\alpha}{(1-\alpha)} = 49$$

11. **(a)** Magnetic field due to long wire,

$$B = \frac{\mu_0 i}{2\pi r} \text{ or } B \propto \frac{1}{r}$$

When r is doubled, the magnetic field becomes half, i.e., now the magnetic field will be 0.2 T.

12. **(a)**



Total distance covered in 10 s = Area 1 + Area 2 + Area 3

$$\Rightarrow \frac{1}{2}\times 6\times 20 + \frac{1}{2}\times 2\times 20 + \frac{1}{2}\times 2\times 10 = 90\,\text{m}$$

Total displacement in 10s = Area 1 – Area 2 + Area 3

$$\Rightarrow \frac{1}{2}\times 6\times 20 - \frac{1}{2}\times 2\times 20 + \frac{1}{2}\times 2\times 10 = 50\,m$$

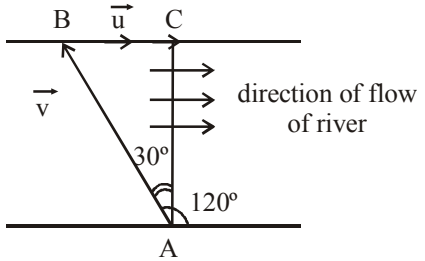
13. **(c)** In a uniform electric field, net force = 0, but torque $\neq 0$.

14. **(c)** Let the speed of water = $\vec{u}$

Speed of swimmer = $\vec{v} = 0.5$ m/sec

Angle between $\vec{v}$ and $\vec{u}$ is 120°. Then

$$\sin\theta = \frac{\vec{u}}{\vec{v}} \Rightarrow \frac{u}{0.5} = \frac{1}{2} \text{ or } u = 0.25\ \text{ms}^{-1}$$



15. **(a)** Radius of drop, r = 0.3 mm = 0.03 cm

Terminal velocity, v = 1 m/s = 100 cm/sec

Viscosity of air, $\eta = 8 \times 10^{-5}$ poise

Viscous force, $f = 6\pi\eta rv$

$F = 6 \times 3.14 \times (8\times10^{-5}) \times 0.03 \times 100 = 4.52 \times 10^{-3}$ dyne

16. **(d)** Rate of transmission of heat

$$= \frac{\text{Temperature difference}}{\text{Thermal Resistance}}$$

$$\therefore \quad \frac{dQ}{dt} = \frac{d\theta}{R}$$

Here, $\frac{dQ}{dt} = \frac{(\theta - \theta_2)}{R_2} = \frac{\theta_1 - \theta}{R_1}$

$$\Rightarrow \frac{\theta - \theta_2}{R_2} = \frac{\theta_1 - \theta}{R_1}$$

$$\Rightarrow \quad R_1\theta - R_1\theta_2 = R_2\theta_1 - R_2\theta$$

$$\Rightarrow \quad \theta(R_1 + R_2) = R_2\theta_1 + R_1\theta_2$$

$$\therefore \quad \theta = \frac{(R_2\theta_1 + R_1\theta_2)}{(R_1 + R_2)}$$

17. (b) Apply law of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$\Rightarrow mv^2 = kx^2$ or $0.5 \times (1.5)^2 = 50 \times x^2$

$\therefore x = 0.15\,m$

18. (b) Taking the same mass of sphere if radius is increased then moment of inertia, angular velocity and rotational kinetic energy will change but according to law of conservation of momentum, angular momentum will not change.

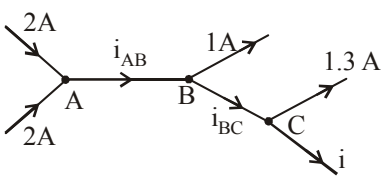
19. (c) For a perfectly rigid body strain produced is zero for the given force applied, so

Y = stress/strain = Infinite (∞)

20. (a) According to Kirchhoff's first law,

At junction A, $i_{AB} = 2 + 2 = 4\,A$

At junction B, $i_{AB} = i_{BC} + 1 \Rightarrow i_{BC} = 4 - 1 = 3A$

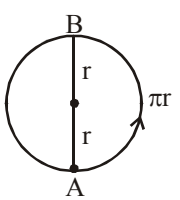


At junction C, $i = i_{BC} - 1.3 = 3 - 1.3 = 1.7A$

21. (d)

22. (b) When a particle cover half of circle of radius r, then displacement is AB = 2r

Distance = half of circumference of circle = πr



23. (b) As the magnet falls, the magnetic flux linked with the ring increases. This induces emf in the ring which opposes the motion of the falling magnet, so a < g.

24. (b) Change in momentum = F × t

$= mg \times 10$

$\Rightarrow 0.5 \times 9.8 \times 10$

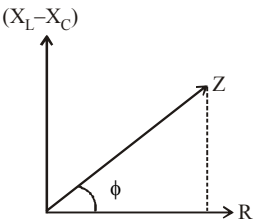
$\Rightarrow 49$ N-sec

25. (b) Given, $R = X_L = 2X_C$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(2X_C)^2 + (2X_C - X_C)^2}$$

$$= \sqrt{4X_C^2 + X_C^2}$$



$$= \sqrt{5}X_C = \frac{\sqrt{5}R}{2} \qquad [\because R = 2X_C]$$

$$\tan\phi = \frac{(X_L - X_C)}{R} = \frac{(2X_C - X_C)}{2X_C}$$

$$\tan\phi = \frac{1}{2} \;;\; \phi = \tan^{-1}\left(\frac{1}{2}\right)$$

26. (d) According to the law of conservation of momentum,

$P_{(initial)} = P_{(final)}$

$\Rightarrow mu_1 + Mu_2 = mv_1 + Mv_2$

$\Rightarrow 0 + 0 = 10 \times 10 \times 10^{-3} \times 800 + [100 - 10 \times 10 \times 10^{-3}]v$

$$\Rightarrow v = \frac{-800}{999} \Rightarrow -0.8\,m/sec$$

Negative sign indicates that the velocity is opposite to the direction of bullets.

27. (a) Escape velocity does not depend upon the mass of the body.

28. (b) By energy conservation

$$(K.E)_i + (P.E)_i = (K.E)_f + (P.E)_f$$

$$(K.E)_i = 0, (P.E)_i = mgh, (P.E)_f = 0$$

$$(K.E)_f = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}mv_{cm}^2$$

For solid cylinder, moment of inertia, $I = \tfrac{1}{2}mR^2$

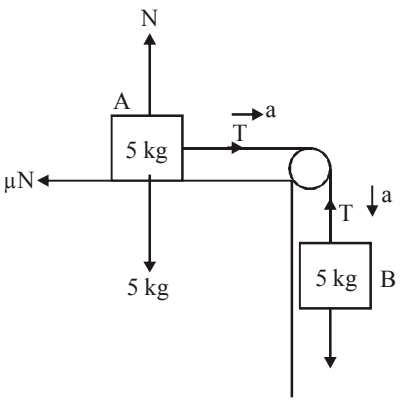
$$(K.E)_i + (P.E)_i = (K.E)_f + (P.E)_f$$

so, $mgh = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v_{cm}^2}{R^2}\right) + \frac{1}{2}mv_{cm}^2$

$\Rightarrow v_{cm} = \sqrt{4gh/3}$

29. (c) For block A,

$T - \mu N = 5a$ and $N = 5g$



For block B,

$5g - T = 5a$

$\Rightarrow T = 36.75N,\ a = 2.45\ m/sec^2$

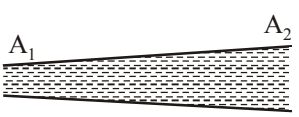
30. (b) $\text{Stress} = \frac{\text{Force}}{\text{Area}} \therefore \text{Stress} \propto \frac{1}{\pi r^2}$

$$\frac{S_B}{S_A} = \left(\frac{r_A}{r_B}\right)^2 = (2)^2 \Rightarrow S_B = 4S_A$$

31. (b) The theorem of continuity is valid.

$\therefore A_1v_1\rho = A_2v_2\rho$

As the density of the liquid can be taken as uniform.



$\therefore A_1v_1 = A_2v_2$

$\Rightarrow$ Smaller the area, greater the velocity.

32. (a) The square of the period of planet is directly proportional to the cube of the semi-major axis of the elliptical orbit.

$\Rightarrow T^2 \propto a^3$

33. (b) Energy of incident light, $E = \frac{12400}{\lambda(\text{Å})}$

$E = \frac{1240}{400} = 3.1\ eV$

$\because E = W_0 + K_{max}$

$\Rightarrow 3.1 = W_0 + 1.68$

$\Rightarrow W_0 = 1.42\ eV$

34. (b) For path iaf,

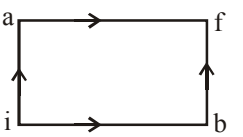
$Q = 50$ cal

$W = 20$ cal

By first law of thermodynamics,

$\Delta U = Q - W = 50 - 20 = 30$ cal.

For path ibf



$Q' = 36$ cal

$W' = ?$

or, $W' = Q' - \Delta U'$

Since, the change in internal energy does not depend on the path, therefore

$\Delta U' = 30$ cal

$\therefore \quad W' = Q' - \Delta U' = 36 - 30 = 6$ cal.

35. (a) The differential equation of simple harmonic motion is

$$\frac{d^2y}{dt^2} + 2y = 0 \text{ or } \frac{d^2y}{dt^2} = -2y \qquad ...(i)$$

Standard equation of simple harmonic motion is

$$\frac{d^2y}{dt^2} = -\omega^2 y \qquad ...(ii)$$

Comparing eq. (i) and (ii),

$\omega^2 = 2$ or $\omega = \sqrt{2}$

As we know, $\omega = \frac{2\pi}{T}$

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{2}} = \pi\sqrt{2}$ sec

36. (c) Fundamental freq. for,

Tube A $\Rightarrow f_A = \frac{v}{2L}$

Tube B $\Rightarrow f_B = \frac{v}{4L}$

Now,

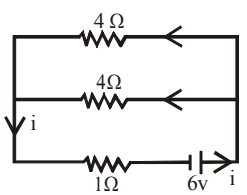
$\frac{f_A}{f_B} = \frac{v}{2L} \times \frac{4L}{v} = \frac{2}{1}$

$f_A : f_B = 2 : 1$

37. (d) Two 4Ω resistors are in parallel combination. Their equivalent resistance

$= \frac{4\times4}{4+4} = \frac{16}{8} = 2\Omega$

$\therefore$ Total resistance of the



network $= 2 + 1 = 3\Omega$

$\therefore$ Current through 1Ω resistor, $i = \frac{6}{3} = 2A$

38. (c) Force on moving charge while moving in magnetic field is; $\vec{F} = q(\vec{v} \times \vec{B})$ where $\vec{F}$ is perpendicular to $\vec{v}$.

Work done/sec $= \vec{F}.\vec{v} = Fv\cos 90° = 0$

39. (b) Fringe width, $\beta = \frac{\lambda D}{d}$

When λ alone is doubled then fringe width becomes

$\beta' = \frac{(2\lambda)D}{d} = 2\beta$

40. (a) In the normal adjustment of YDSE, Path difference between the waves at central location is always zero. So maxima is obtained at central position. Central point on the screen is bright.

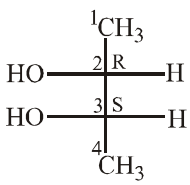
PART - II (CHEMISTRY)

41. (b) $W = 2.303\, nRT \log \frac{V_2}{V_1}$

$= 2.303 \times 1 \times 8.314 \times 10^7 \times 298 \log \frac{20}{10}$

$= 298 \times 10^7 \times 8.31 \times 2.303 \log 2$

42. (a)



43. (b)

	Electron	Proton	Neutron	α -particle
e	1 unit	1 unit	zero	2 units
m	1/1837 unit	1 unit	1 unit	4-units
e/m	1837	1	zero	1/2

44. (b) Decreases from +6 to +3.

$K_2Cr_2O_7 + 4H_2SO_4 \rightarrow K_2SO_4 + Cr_2(SO_4)_3 + 4H_2O + 3O$

$[H_2S + [O] \rightarrow S + H_2O] \times 3$

$\overset{+6}{K_2Cr_2O_7} + 4H_2SO_4 + 3H_2S \rightarrow$

$K_2SO_4 + \overset{+3}{Cr_2}(SO_4)_3 + 7H_2O + 3S$

45. (b) $N_2 + 3H_2 \rightarrow 2NH_3$

$$\frac{-\Delta[N_2]}{\Delta t} = -\frac{1}{3}\frac{\Delta[H_2]}{\Delta t} = \frac{1}{2}\frac{\Delta[NH_3]}{\Delta t}$$

$$\therefore \frac{-\Delta[H_2]}{\Delta t} = \frac{3}{2} \times \frac{\Delta[NH_3]}{\Delta t} = \frac{3}{2} \times 2 \times 10^{-4}$$

$$= 3 \times 10^{-4} \text{ mol } L^{-1}S^{-1}$$

46. (a) $K_p = K_c [RT]^{\Delta n_g}$ $\Delta n = 2 - 4 = -2$

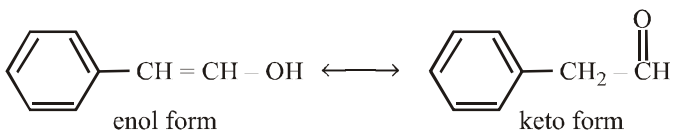
$T = 673K$, $K_c = 0.5$,

$R = 0.082 \text{ L. atm. mol}^{-1} K^{-1}$

$K_p = 0.5 \times (0.082 \times 673)^{-2}$

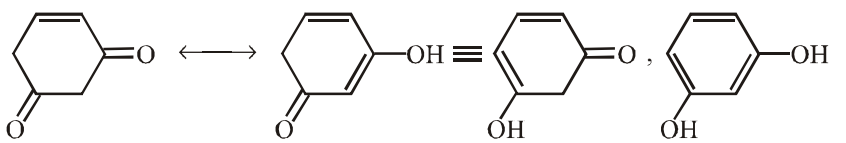
$= 1.64 \times 10^{-4}$ atm.

47. (d) (a)



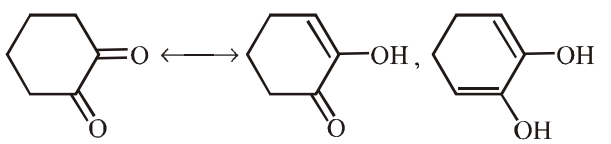
enol form keto form

(b)



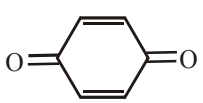
keto form enol form

(c)



keto form enol form

(d)



cannot tautomerise

48. (b) $Cu^{2+} + 1e^- \rightarrow Cu^+ \quad ; \Delta G_1^o = -n_1 E_1^o F$]

$Cu^+ + 1e^- \rightarrow Cu \quad ; \Delta G_2^o = -n_2 E_2^o F$

$Cu^{2+} + 2e^- \rightarrow Cu \quad \Delta G^\circ = \Delta G_1^o + \Delta G_2^o$

$-nE^\circ F = -1\, n_1 \quad E_1^o F + (-1) n_2 \, E_2^o F$

$-nE^\circ F = -F (n_1 \, E_1^o + n_2 \, E_2^o F)$

$$E^\circ = \frac{n_1 E_1^o + n_2 E_2^o}{n} = \frac{0.15 \times 1 + 0.50 \times 1}{2} = 0.325$$

49. (b) An oxygen- helium mixture is used for artificial respiration in deep sea diving instead of air because nitrogen present in air dissolves in blood under high pressure when sea diver goes into deep sea. When he comes to the surface nitrogen bubbles out of the blood due to decrease in pressure, causing pain. This disease is called "bends".

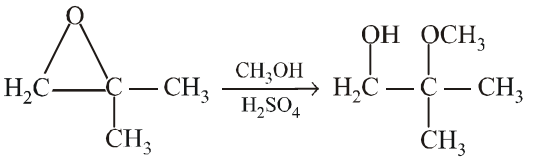
50. (d) $E_1 = h. \frac{c}{\lambda_1}$; $E_2 = h. \frac{c}{\lambda_2}$

$$\frac{E_1}{E_2} = \frac{hc}{\lambda_1} \times \frac{\lambda_2}{hc} = \frac{\lambda_2}{\lambda_1} = \frac{4000}{2000} = 2$$

51. (c)

$$\begin{array}{l} CH_2OH \\ | \\ CH_2OH \end{array} + HIO_4 \rightarrow 2HCHO + HIO_3 + H_2O$$

52. (a)



53. (c) As the system is closed and insulated, no heat enters. or leaves the system, i.e. $Q = 0$;

$\therefore \Delta E = Q + W = W.$

54. (a) When pressure is low

$$\left[P + \frac{a}{V^2}\right](V - b) = RT$$

or $$PV = RT + Pb - \frac{a}{V} + \frac{ab}{V^2}$$

or $$\frac{PV}{RT} = 1 - \frac{a}{VRT}$$

$$Z = -\frac{a}{VRT}$$

$$\left(\because \frac{PV}{RT} = Z\right)$$

55. (d) $CuSO_4 + 2KCN \rightarrow Cu(CN)_2 + K_2SO_4$

$2Cu(CN)_2 \rightarrow Cu_2(CN)_2$

or $2CuCN + \underset{\text{cyanogen}}{CN - CN}$

$CuCN + 3KCN \rightarrow K_3[Cu(CN)_4]$

colourless

56. (a) $\Delta G = \Delta H - T\Delta S,\ T = 27 + 273 = 300\ K$

$\Delta G = (-285.8) - 300\,(-0.163) = -236.9\ kJ\ mol^{-1}$

57. (a) $\underset{A}{CH_3CHOHCH_3} \xrightarrow[dil.H_2SO_4]{K_2Cr_2O_7} \underset{B}{CH_3COCH_3}$

$$\xrightarrow[H_2O]{CH_3MgI} (CH_3)_3COH$$

58. (b) $n\lambda = 2d\sin\theta$

59. (c) For following reaction, $2NO(g) + O_2(g) \rightarrow 2NO_2(g)$

When the volume of vessel is reduced to $\frac{1}{3}$ then concentration of reactant becomes three times.

The rate of reaction for first order reaction is proportional to the concentration. So rate of reaction will increase three times.

60. (c) (a) Nitration is difficult to be carried out, further the $-NO_2$ group will go to *m*-position to the > C = O group

(b) Nitrobenzene, being deactivated toward electrophilic substitution will not undergo Friedal Craft reaction.

(c) Benzene easily undergoes Friedal Craft reaction forming the required product.

61. (c) Among the transition metals, Mn forms maximum no. of oxides.

MnO	Mn_3O_4	Mn_2O_3	MnO_2	Mn_2O_7
basic	amphoteric	amphoteric	amphoteric	acidic

62. (c) The compound A with formula C_3H_6O gives iodoform test, it is propanone. It forms a compound B having carbon atoms three times to the number of carbon atoms in propanone, it is 2, 6-dimethyl-2, 5-heptadien-4-one.

$$\underset{CH_3}{CH_3 - \overset{|}{C} = O} + \underset{O}{CH_3 - \overset{\|}{C} - CH_3} + \underset{CH_3}{O = \overset{|}{C} - CH_3} \xrightarrow{dil.\,HCl}$$

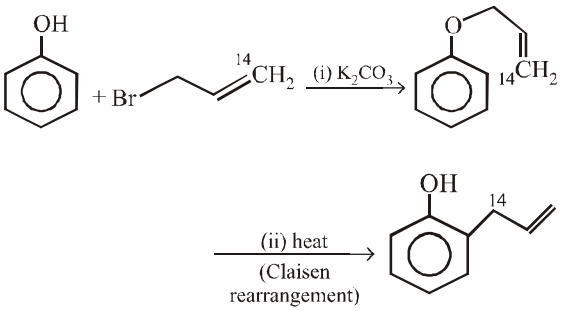
(A)

$$CH_3 - \underset{CH_3}{\overset{|}{C}} = CH - \underset{O}{\overset{\|}{C}} - CH = \underset{CH_3}{\overset{|}{C}} - CH_3$$

(B)

2, 6-Dimethyl-2, 5-heptadien-4-one

63. (b)



64. (b) $\Delta S_f = \dfrac{\Delta H_f}{T} = \dfrac{6025\,J\,mol^{-1}}{273\,K} = 22.1\ JK^{-1}\ mol^{-1}$

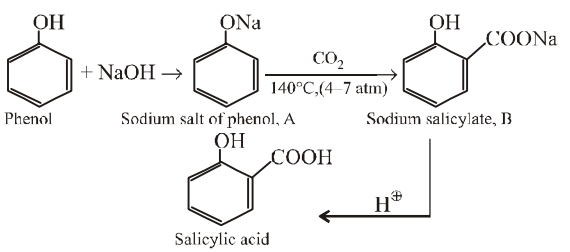
65. (b) $\log\dfrac{K_2}{K_1} = \dfrac{E_a}{2.303R}\left[\dfrac{1}{T_1} - \dfrac{1}{T_2}\right]$; If $\dfrac{K_2}{K_1} = 2$

$$\log 2 = \frac{E_a}{2.303\times 8.314}\left[\frac{1}{300} - \frac{1}{310}\right]$$

$$E_a = 0.3010 \times 2.303 \times 8.314 \left(\frac{300\times 310}{10}\right)$$

$$= 53598.59\ Jmol^{-1} = 54\ kJ$$

66. (d) Treatment of sodium salt of phenol with CO_2 under pressure brings about substitution of the –COOH group for the hydrogen of the ring. This is called as Kolbe's reaction



67. (b) $CH_3CH_2COOH \xrightarrow{SOCl_2} CH_3CH_2COCl + SO_2 + HCl$

$$CH_3CH_2COCl + NH_3 \rightarrow \underset{(X)}{CH_3\overset{}{CH_2}}CONH_2 + HCl$$

$$CH_3CH_2CONH_2 + Br_2/NaOH \rightarrow \underset{\text{Ethylamine, (Z)}}{CH_3CH_2NH_2} + CO_2$$

68. (a) Energy needed to convert 1 mole of sodium (g) to sodium ions = 495 kJ

$\therefore$ Energy needed to convert 3 moles of Na (g) to Na^+ ions $= 495 \times 3 = 1485$ kJ

69. (a) According to Heisenberg's uncertainty principle,

$$\Delta x \,.\, m\Delta v = \frac{h}{4\pi}$$

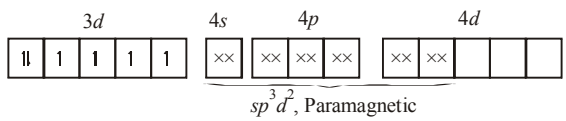
$$\text{or } m = \frac{h}{4\pi \Delta x.\ \Delta v}$$

$$= \frac{6.625\times 10^{-34}\,Js}{4\times 3.143\times 10^{-10}\,m \times 5.27\times 10^{-24}\,ms^{-1}}$$

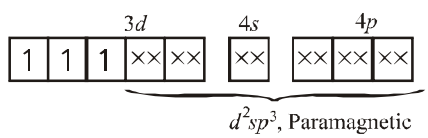
$\because [J = Kgm^2s^2]$

$= 0.099$ kg.

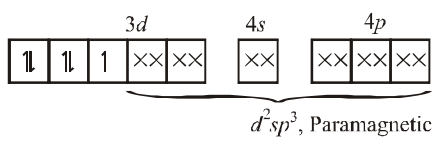
70. (a) (i) $[Fe(H_2O)_6]^{2+}$



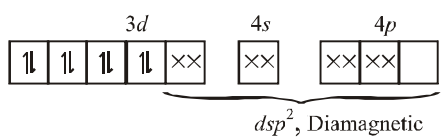
(ii) $[Cr(CN)_6]^{3-}$



(iii) $[Fe(CN)_6]^{3-}$



(iv) $[Ni(CN)_4]^{2-}$



Thus complexes (i), (ii) and (iii) are paramagnetic species.

71. **(b)** $\underset{\text{White}}{P_4} + 3NaOH + 3H_2O \rightarrow \underset{\text{Phosphine}}{PH_3} + 3NaH_2PO_2$

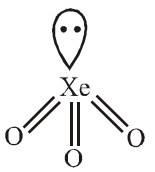
72. **(b)** In hydrogen–oxygen fuel cell, following reactions take place to create potential difference between the two electrodes.

$2H_2(g) + 4OH^-(aq) \rightarrow 4H_2O(l) + 4e^-$

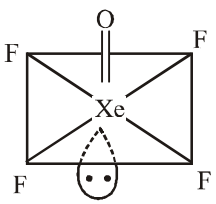
$$\frac{O_2(g) + 2H_2O(l) + 4e^- \rightarrow 4OH^-(aq)}{\text{Overall reaction} = 2H_2(g) + O_2(g) \rightarrow 2H_2O(l)}$$

the net reaction is the same as burning (combustion) of hydrogen to form water.

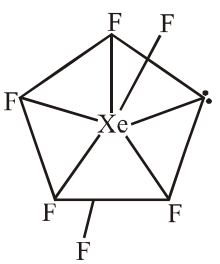
73. **(d)** XeO_3:



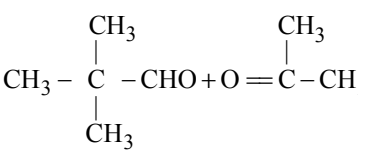
$XeOF_4$:

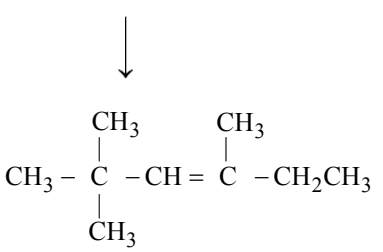


XeF_6:



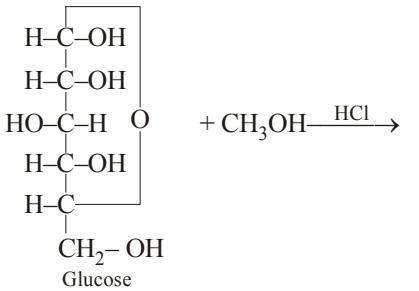
74. **(a)**

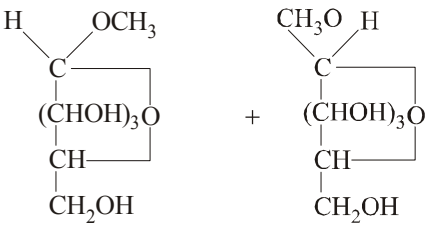




2,2,4 trimethyl-3-hexene

75. **(c)** A ring structure





α-Methyl glucoside β-Methyl glucoside

76. **(d)** When acetylene is passed into methanol at 160-200ºC in the presence of a small amount (1-2%) of potassium methoxide and under pressure just high enough to prevent boiling, methyl vinyl ether is formed.

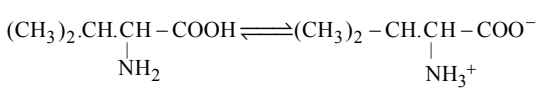
The mechanism is believed to involve nucleophildic attack in the first step.

$$HC \equiv CH + CH_3O^- \rightarrow C^-H = CHOCH_3$$

$$\xrightarrow{CH_3OH} CH_2 = CHOCH_3 + CH_3O^-$$

Methyl vinyl ether

77. **(b)**



78. **(a)** The most probable complex which gives three mole ions in aqueous solution may be $[Co(NH_3)_5NO_2]Cl_2$.

79. **(c)** Endothermic reactions are those which involve absorption of heat. High activation energy means potential energy of product must be much greater than reactants.

80. (d) $K_c = \dfrac{[SO_3]^2}{[SO_2]^2[O_2]}$

If volume is reduced to half, K_c will decrease to half. Thus, to maintain the equilibrium, the reaction should shift in the forward direction, i.e. towards right. Also, to attain equilibrium back, the rate of forward direction will become double the rate of backward direction.

PART - III (MATHEMETICS)

81. (a) $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-n}{1+n} \Rightarrow \sec 2\theta = \dfrac{1+n}{1-n}$

Since, n is non-square natural number $\therefore n \neq 1$

$\therefore \sec 2\theta = \dfrac{1+n}{1-n} =$ rational number

82. (d) $z^{1/3} = a - ib \Rightarrow z = (a-ib)^3$

$\therefore x + iy = a^3 + ib^3 - 3ia^2b - 3ab^2$. Then

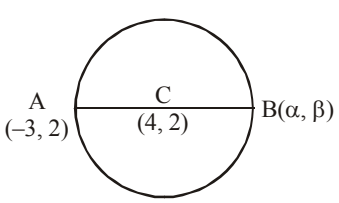
$x = a^3 - 3ab^2 \Rightarrow \dfrac{x}{a} = a^2 - 3b^2$

$y = b^3 - 3a^2b \Rightarrow \dfrac{y}{b} = b^2 - 3a^2$

So, $\dfrac{x}{a} - \dfrac{y}{b} = 4(a^2 - b^2)$

83. (d) The centre of the given circle is $C \equiv (4, 2)$

Let $A \equiv (-3, 2)$



If (α, β) are the coordinates of the other end of the diameter, then, as the middle ploint of the diameter is the centre,

$\therefore \quad \dfrac{\alpha - 3}{2} = 4$ and $\dfrac{\beta + 2}{2} = 2 \Rightarrow \alpha = 11, \beta = 2$

Thus, the coordinates of the other end of diameter are (11, 2)

84. (c) The system is homogenuous system.

$\therefore$ it has either unique solution or infinite many solution depend on |A|

$$\therefore \quad |A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & 0 \end{vmatrix} = 2 \times 1 + 1(-3) + (4-3)$$

$$= 2 - 3 + 1 = 0$$

85. (c) Let $\omega = \dfrac{-1 + \sqrt{3}i}{2}$

Now, $1 + \omega + \omega^2 = 0$, and $\omega^3 = 1$

then $(3 + \omega + 3\omega^2)^4 = (3 + \omega + 3\omega^2 + 2\omega - 2\omega)^4$

$= (3 + 3\omega + 3\omega^2 - 2\omega)^4 = [3(1 + 2 + \omega^2) - 2\omega]^4$

$= (0 - 2\omega)^4 = 16\omega^4 = 16\omega \quad (\because \omega^3 = 1)$

86. (a) The diameter of the circle is perpendicular distance between the parallel lines (tangents) $3x - 4y + 4 = 0$ and

$3x - 4y - \dfrac{7}{2} = 0$ and so it is equal to

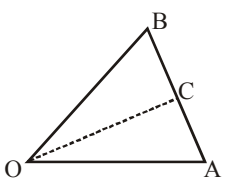
$\dfrac{4}{\sqrt{9+16}} + \dfrac{7/2}{\sqrt{9+16}} = \dfrac{3}{2}$. Hence radius is

$\dfrac{3}{4}$.

87. (a) We have,

$OA = |2\hat{i} + 2\hat{j} + \hat{k}| = 3$; $OB = |2\hat{i} + 4\hat{j} + 4\hat{k}| = 6$

Since the internal bisector divides opposite side in the



ratio of adjacent sides

$$\therefore \frac{AC}{BC} = \frac{3}{6} = \frac{1}{2}$$

where OC is the bisector of $\angle BOA$.

$\therefore$ Position vector of C is

$$\frac{2(2\hat{i}+2\hat{j}+\hat{k})+(2\hat{i}+4\hat{j}+4\hat{k})}{2+1} = 2\hat{i}+\frac{8}{3}\hat{j}+2\hat{k}$$

$$\therefore OC = \left|2\hat{i}+\frac{8}{3}\hat{j}+2\hat{k}\right| = \sqrt{\frac{136}{9}}$$

88. (c) $y = \tan^{-1}\frac{4x}{1+5x^2} + \tan^{-1}\frac{2+3x}{3-2x}$

$$= \tan^{-1}\frac{5x-x}{1+5x\cdot x} + \tan^{-1}\frac{\frac{2}{3}+x}{1-\frac{2}{3}\cdot x}$$

$$= \tan^{-1}5x - \tan^{-1}x + \tan^{-1}\frac{2}{3} + \tan^{-1}x$$

$$\Rightarrow \frac{dy}{dx} = \frac{5}{1+25x^2}$$

89. (c) $AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

$$AB = \begin{bmatrix} abc-abc & b^2c-b^2c & bc^2-bc^2 \\ -a^2c+a^2c & -abc+abc & -ac+ac \\ a^2b-a^2b & ab^2-ab^2 & abc-abc \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

90. (d) Let the centre of the circle be (h, k).

Since the centre lies on the line $y = x - 1$

$\therefore\ k = h - 1$ ...(i)

Since the circle passes through the point (7, 3), therefore, the distance of the centre from this point is the radius of the circle.

$$\therefore \quad 3 = \sqrt{(h-7)^2+(k-3)^2}$$

$$\Rightarrow \quad 3 = \sqrt{(h-7)^2+(h-1-3)^2}$$

[using (i)]

$\Rightarrow\ h = 7$ or $h = 4$

For $h = 7$, we get $k = 6$ and for $h = 4$, we get $k = 3$

Hence, the circles which satisfy the given conditions are:

$$(x-7)^2+(y-6)^2 = 9$$

or $x^2+y^2-14x+12y+76 = 0$

and $(x-4)^2+(y-3)^2 = 9$ or

$$x^2+y^2-8x-6y+16 = 0$$

91. (a) The function f (x) = sin x is differentiable for all x $\in$ R. Therefore the number of points in the interval $(-\infty, \infty)$ where the function is not differentiable are zero.

92. (a) The given line is $\frac{x-2}{2} = \frac{2y-5}{-3} = z+1$,

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y-\frac{5}{2}}{-\frac{3}{2}} = \frac{z+1}{0}$$

This shows that the given line passes through the point $\left(2, \frac{5}{2}, -1\right)$ and has direction ratios $\left(2, \frac{-3}{2}, 0\right)$. Thus, given line passes through the point having position vector $\vec{a} = 2\hat{i}+\frac{5}{2}\hat{j}-\hat{k}$ and is parallel to the vector $\vec{b} = \left(2\hat{i}-\frac{3}{2}\hat{j}-0\hat{k}\right)$. So, its vector equation is

$$\vec{r} = \left(2\hat{i}+\frac{5}{2}\hat{j}-\hat{k}\right)+\lambda\left(2\hat{i}-\frac{3}{2}\hat{j}-0\hat{k}\right).$$

Hence, p = 0

93. **(c)** We have

$$L = \lim_{x\to 2} \frac{\sqrt{(x+7)} - 3\sqrt{(2x-3)}}{\sqrt[3]{(x+6)} - 2\sqrt[3]{(3x-5)}} \left(\frac{0}{0} \text{ form}\right)$$

Let $x-2=t$ such that when $x \to 2$, $t \to 0$. Then

$$L = \lim_{t\to 0} \frac{(t+9)^{\frac{1}{2}} - 3(2t+1)^{\frac{1}{2}}}{(t+8)^{\frac{1}{3}} - 2(3t+1)^{\frac{1}{3}}} \qquad \left(\frac{0}{0} \text{ form}\right)$$

$$= \frac{3}{2}\lim_{t\to 0} \frac{\left(1+\frac{t}{9}\right)^{\frac{1}{2}} - (2t+1)^{\frac{1}{2}}}{\left(1+\frac{t}{8}\right)^{\frac{1}{3}} - (3t+1)^{\frac{1}{3}}} \qquad \left(\frac{0}{0} \text{ form}\right)$$

$$= \frac{3}{2}\lim_{t\to 0} \frac{\frac{1}{2}\frac{t}{9} - (2t)\frac{1}{2}}{\frac{t}{8}\frac{1}{3} - (3t)\frac{1}{3}} = \frac{3}{2}\frac{\frac{1}{18}-1}{\frac{1}{24}-1} = \frac{34}{23}.$$

94. **(d)** Given $V=\pi r^2 h$.
Differentiating both sides, we get

$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + 2r\frac{dr}{dt}h\right) = \pi r\left(r\frac{dh}{dt} + 2h\frac{dr}{dt}\right)$$

$$\frac{dr}{dt} = \frac{1}{10} \text{ and } \frac{dh}{dt} = -\frac{2}{10}$$

$$\frac{dV}{dt} = \pi r\left(r\left(-\frac{2}{10}\right) + 2h\left(\frac{1}{10}\right)\right) = \frac{\pi r}{5}(-r+h)$$

Thus, when $r=2$ and $h=3$,

$$\frac{dV}{dt} = \frac{\pi(2)}{5}(-2+3) = \frac{2\pi}{5}.$$

95. **(c)** Given a, b, c are in A.P.

$\therefore \quad 2b = a + c$ ....(i)

Now, $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

[Applying $R_2 \to 2R_2$]

$$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 2x+4 & 2x+6 & 2x+2b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 2x+4 & 2x+6 & 2x+(a+c) \\ x+3 & x+4 & x+c \end{vmatrix}$$

[using equation (i)]

$$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 0 & 0 & 0 \\ x+3 & x+4 & x+c \end{vmatrix} = \frac{1}{2}.0 = 0$$

[Applying $R_2 \to R_2 - (R_1 + R_3)$]

96. **(a)** We have, $\sin x = -\frac{\sqrt{3}}{2} = -\sin\frac{\pi}{3}$

Hence, $\sin x = \sin\frac{4\pi}{3}$, which gives

$x = n\pi + (-1)^n \frac{4\pi}{3}$, where $n \in Z$.

97. **(c)** The given equation can be rewritten as

$(y-1)^2 = 4\left(x-\frac{1}{2}\right)$ which is a parabola

with its vertex $\left(\frac{1}{2}, 1\right)$ axis along the line

$y=1$, hence axis parallel to x-axis.

Its focus is $\left(\frac{1}{2}+1, 1\right)$, i.e., $\left(\frac{3}{2}, 1\right)$.

98. **(b)** Given, $\sin y = x \sin(a+y)$

$$\Rightarrow \quad x = \frac{\sin y}{\sin(a+y)}$$

On differentiating w.r. to y, we get

$$\frac{dx}{dy} = \frac{d}{dy}\left[\frac{\sin y}{\sin(a+y)}\right]$$

$$= \frac{\sin(a+y)\cos y - \sin y\cos(a+y)}{\sin^2(a+y)}$$

$$= \frac{\sin a}{\sin^2(a+y)} \Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}.$$

99. (d) Let $I = \int (27e^{9x} + e^{12x})^{1/3} dx$

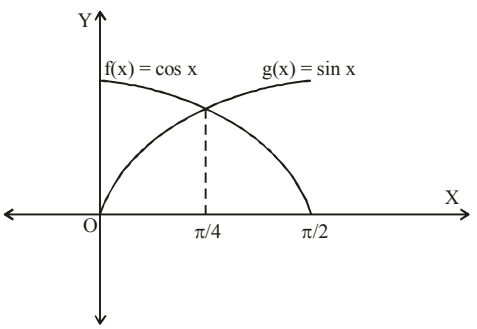
$= \int e^{3x}(27 + e^{3x})^{1/3} dx$

Put $27 + e^{3x} = t \Rightarrow 3e^{3x} dx = dt$

$\therefore \quad I = \frac{1}{3}\int t^{1/3} dt = \frac{1}{3} \times \frac{t^{4/3}}{4/3} + C$

$= \frac{1}{4}(27 + e^{3x})^{4/3} + C$

100. (b) $y = |\cos x - \sin x|$



Required area $= 2\int_0^{\pi/4} (\cos x - \sin x)\, dx$

$= 2\left[\sin x + \cos x\right]_0^{\pi/4} = 2\left[\frac{2}{\sqrt{2}} - 1\right]$

$= (2\sqrt{2} - 2)$ sq. units

101. (c) From given equations

$$\frac{x}{2} = \cos t + \sin t \qquad \ldots(i)$$

$$\frac{y}{5} = \cos t - \sin t \qquad \ldots(ii)$$

Eliminating t from (i) and (ii), we have

$\frac{x^2}{4} + \frac{y^2}{25} = 2 \Rightarrow \frac{x^2}{8} + \frac{y^2}{50} = 1$, which is an ellipse.

102. (a) Since, $\left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a}.\vec{b}\right)^2 = 676$

$\Rightarrow \left(|\vec{a}|.|\vec{b}|\sin\theta\hat{n}\right)^2 + \left(|\vec{a}|.|\vec{b}|\cos\theta\right)^2 = 676$

$\Rightarrow |\vec{a}|^2.|\vec{b}|^2\left(\sin^2\theta + \cos^2\theta\right) = 676$

$\Rightarrow |\vec{a}|^2(2)^2 = 676 \Rightarrow |\vec{a}|^2 = 169$

$\Rightarrow |\vec{a}| = 13$

103. (d) Equation of plane bisecting the angle containing origin is (making constant term of same sign)

$$\frac{-3x + 6y - 2z - 5}{\sqrt{3^2 + 6^2 + 2^2}} = +\left[\frac{4x - 12y + 3z - 3}{\sqrt{4^2 + 12^2 + 3^2}}\right]$$

or $\frac{-3x + 6y - 2z - 5}{7} = \frac{4x - 12y + 3z - 3}{13}$

or $67x - 162y + 47z + 44 = 0$.

104. (d) Let one side of quadrilateral be x and another side be y so, $2(x + y) = 34$

or, $(x + y) = 17 \qquad \ldots(i)$

We know from the basic principle that for a given perimeter square has the maximum area, so, x = y and putting this value in equation (i)

$x = y = \frac{17}{2}$

Area $= x \,.\, y = \frac{17}{2} \times \frac{17}{2} = \frac{289}{4} = 72.25$

105. (b) $\int \frac{\sin x}{\sin(x - \alpha)} dx = \int \frac{\sin(x - \alpha + \alpha)}{\sin(x - \alpha)} dx$

$= \int \frac{\sin(x - \alpha)\cos\alpha + \cos(x - \alpha)\sin\alpha}{\sin(x - \alpha)} dx$

$= \int \{\cos\alpha + \sin\alpha \cot(x-\alpha)\}dx$

$= (\cos\alpha)x + (\sin\alpha)\log\sin(x-\alpha) + C$

$\therefore$ A = cosα, B = sin α

106. (b) The equation of the chord, having mid-point as (x_1, y_1), of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is given by

$T = S_1$ ..... (i)

where, $T = \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1$ and

$S_1 = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$

According to the question,
$(x_1, y_1) = (5, 3)$ and $a^2 = 16, b^2 = 25$
as $25x^2 - 16y^2 = 400$

$\Rightarrow \frac{x^2}{16} - \frac{y^2}{25} = 1 \therefore \frac{5x}{16} - \frac{3y}{25} = \frac{25}{16} - \frac{9}{25}$

[Using (i)]

$\Rightarrow \quad 125x - 48y = 625 - 144 \Rightarrow 125x - 48y = 481$

107. (b) $I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\cot x}}dx$

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx$ ...(i)

Then, $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}}dx$

$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}dx$ ...(ii)

Adding (i) and (ii), we get

$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}dx$

$\Rightarrow 2I = \int_{\pi/6}^{\pi/3} 1.dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} \Rightarrow I = \pi/12$

108. (c) $|x-2| = \begin{cases} x-2, & x \geq 2 \\ 2-x, & x < 2 \end{cases}$

$\Rightarrow \quad |x-2| - (x-2) = \begin{cases} 0, & x \geq 2 \\ 4-2x, & x < 2 \end{cases}$

$\Rightarrow$ given expression is defined for $(-\infty, 2)$

109. (a) Required number of ways

$= \frac{6!}{3!3!} = \frac{720}{6 \times 6} = 20$

110. (d) For $x < 1$, $f(x) = \frac{x^2 - 1}{x^2 + 2x - 3} = \frac{x+1}{x+3}$

$\therefore \lim_{x \to 1^-} f(x) = \frac{1}{2}$

For $x > 1$, $f(x) = \frac{x^2 - 1}{x^2 - 2x + 1} = \frac{x+1}{x-1}$

$\therefore \lim_{x \to 1^+} f(x) = \infty$

$\therefore$ the function is not continuous at x = 1.

111. (a) $f(x) = x^3 + bx^2 + cx + d$, $0 < b^2 < c$

$\therefore \quad f'(x) = 3x^2 + 2bx + c$

Discriminant $= 4b^2 - 12c = 4(b^2 - 3c) < 0$

$\therefore f'(x) > 0 \ \forall \ x \in R$

Thus, $f(x)$ is strictly increasing $\forall \ x \in R$

112. (c) Rearranging the equation, we have

$dx - y\,dy + \sqrt{\left(x^2 + y^2\right)}\ (xdx + y\,dy) = 0$

$\Rightarrow \quad dx - y\,dy + \frac{1}{2}\sqrt{\left(x^2 + y^2\right)}\,d(x^2 + y^2) = 0$

On integrating, we get

$$x - \frac{y^2}{2} + \frac{1}{2}\int \sqrt{t}\ dt = C,\ \left\{t = \sqrt{\left(x^2 + y^2\right)}\right\}$$

or $\quad x - \frac{y^2}{2} + \frac{1}{3}\left(x^2 + y^2\right)^{3/2} = C$

113. (c) Since $x\frac{dy}{dx} - y = x^4 - 3x$

$$\therefore \quad \frac{dy}{dx} - \frac{y}{x} = x^3 - 3$$

Hence

$$IF = e^{\int P dx} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$$

114. (b) $P(A) = 1/4, P(A/B) = \frac{1}{2}, P(B/A) = 2/3$

By conditional probability,

$P(A \cap B) = P(A)\,P(B/A) = P(B)P(A/B)$

$$\Rightarrow \frac{1}{4} \times \frac{2}{3} = P(B) \times \frac{1}{2} \Rightarrow P(B) = \frac{1}{3}$$

115. (d) We have, $f(x) = 2x - 3, g(x) = x^3 + 5$

$(fog)x = f(x^3 + 5) = 2(x^3 + 5) - 3 = 2x^3 + 7$

Let $y = (fog)x = 2x^3 + 7$

$$\Rightarrow x = \left(\frac{y-7}{2}\right)^{1/3} \Rightarrow (fog)^{-1} x = \left(\frac{x-7}{2}\right)^{1/3}$$

116. (c) The inverse of the proposition $(p \wedge \sim q) \rightarrow r$ is

$\sim (p \wedge \sim q) \rightarrow \sim r \equiv \sim p \vee \sim(\sim q) \rightarrow \sim r$

$\equiv \sim p \vee q \rightarrow \sim r$

117. (d) (i) $f(x) = \sqrt{x}$ is continuous in [4, 9]

(ii) $f'(x) = \frac{1}{2\sqrt{x}}$

Thus f(x) is differentiable in (4, 9)

(iii) $f(4) \neq f(9)$. All the three conditions of LMV theorem satisfied then there exist at least one $c \in (4, 9)$ such that.

$$f'(c) = \frac{f(b) - f(a)}{b - a} \Rightarrow \frac{1}{2\sqrt{c}} = \frac{1}{5} \Rightarrow c = \frac{25}{4}$$

118. (b) We have

$$|x| + |x-1| = \begin{cases} -x - (x-1) = -2x + 1, & \text{if } x \le 0 \\ x - (x-1) = 1, & \text{if } 0 \le x \le 1 \\ x + x - 1 = 2x - 1, & \text{if } x \ge 1 \end{cases}$$

$$\therefore \int_{-1}^{3} (|x| + |x-1|)dx$$

$$= \int_{-1}^{0} (-2x + 1)dx + \int_{0}^{1} 1dx + \int_{1}^{3} (2x - 1)dx$$

$$= \left[-x^2 + x\right]_{-1}^{0} + \left[x\right]_{0}^{1} + \left[x^2 - x\right]_{1}^{3} = 9$$

119. (c) a, b, c are in A.P. $\Rightarrow$ $2b = a + c$

Now,

$$e^{1/c} \times e^{1/a} = e^{(a+c)/ac} = e^{2b/ac} = (e^{b/ac})^2$$

$\therefore$ $e^{1/c}$, $e^{b/ac}$, $e^{1/a}$ in G.P. with common ratio

$$= \frac{e^{b/ac}}{e^{1/c}} = e^{(b-a)/ac} = e^{d/(b-d)(b+d)}$$

$$= e^{d/(b^2 - d^2)}$$

[$\because$ a, b, c are in A.P. with common difference d $\therefore$ b – a = c – b = d]

120. (a) We have $T_2 = 14\ a^{\frac{5}{2}}$

$$\Rightarrow {}^{n}C_1 (a^{\frac{1}{13}})^{n-1} (a^{\frac{3}{2}}) = 14a^{\frac{5}{2}}$$

$$\Rightarrow n a^{\frac{n-1}{13} + \frac{3}{2}} = 14a^{\frac{5}{2}} \Rightarrow n = 14$$

$$\Rightarrow \frac{{}^{n}C_3}{{}^{n}C_2} = \frac{{}^{14}C_3}{{}^{14}C_2} = \frac{12}{3} = 4$$

PART - IV (ENGLISH)

121. (c) 'Neither' is used for two things. For more than two things, 'none' is used.

122. (c) Impeccable means 'in accordance with the highest standard; faultless'.

123. (d) Ameliorate means 'make (something bad or unsatisfactory) better.

124. (d) The idiom 'a bolt from the blue' means 'an event or a piece of news which is sudden and unexpected; a complete surprise'.

125. (b) The second part of the first sentence clearly states that the German students and the professor are quite friendly with each other and no strict relation whatsoever is seen between them which is generally seen between a professor and his students.