Class-X Session 2022-23

Subject - Mathematics (Standard)

Sample Question Paper - 36

With Solution

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Ch. No.	Chapter Name	Per Unit Marks	Section-A (1 Mark)		Section-B (2 Marks)	Section-C (3 Marks)	Section-D (5 Marks)	Section-E (4 Marks)	Total Marks
			MCQ	A/R	VSA	SA	LA	Case-Study	
1	Real Number	6	2(Q1, 4)					1(Q36)	6
2	Polynomials	20				1(Q26)			3
3	Pair of Linear Equations in Two Variables							1(Q37)	4
4	Quadratic Equations						1(Q32)		5
5	Arithmetic Progression					1(Q27)	1(Q33)		8
6	Triangles	15	2(Q2, 5)	1(Q19)			1(Q34)		8
7	Circles		2(Q11, 14)		1(Q22)	1(Q30)			7
8	Coordinate Geometry	6	3(Q3, 6, 8)	1(Q20)	1(Q25)				6
9	Introduction to Trigonometry	12	3(Q7, 10, 13)		1(Q21)	1(Q28)			8
10	Some Applications of Trigonometry		1(Q9)			1(Q29)			4
11	Areas Related to Circles	10			1(Q23)	1(Q31)			5
12	Surface Areas and Volumes		1(Q12)					1(Q38)	5
13	Statistics	11	2(Q17, 18)				1(Q35)		7
14	Probability		2(Q16, 15)		1(Q24)				4
Total Marks (Total Questions)		80	18(18)	2(2)	10(5)	18(6)	20(4)	12(3)	80(38)

Note : The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

Time : 3 Hours **Max. Marks : 80**

General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 case based integrated units of assessment (4 marks each) with sub parts of values of 1, 1 and 2 marks each respectively.*

SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. The least number which is a perfect square and is divisible by each of 16, 20 and 24 is

(a) 240 (b) 1600 (c) 2400 (d) 3600

2. If the lengths of the diagonals of rhombus are 16 cm and 12 cm. Then, the length of the sides of the rhombus is

(a) 9 cm (b) 10 cm (c) 8 cm (d) 20 cm

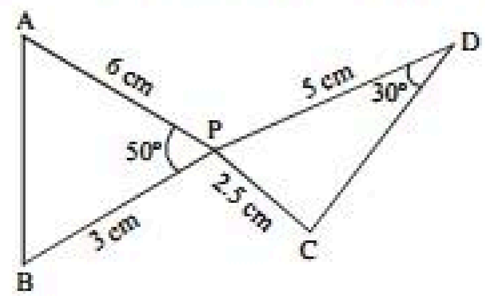
3. P, Q, R are three collinear points. The coordinates of P and R are (3, 4) and (11, 10) respectively and PQ is equal to 2.5 units. Coordinates of Q are

(a) (5, 11/2) (b) (11, 5/2) (c) (5, –11/2) (d) (–5, 11/2)

4. If $m = n^2 - n$, where n is an integer, then $m^2 - 2m$ is divisible by

(a) 20 (b) 24 (c) 30 (d) 16

5. In figure, two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, $\angle APB = 50°$ and $\angle CDP = 30°$. Then, $\angle PBA$ is equal to



(a) 50° (b) 30° (c) 60° (d) 100°

6. The coordinates of the point which is reflection of point (–3, 5) in x-axis are

(a) (3, 5) (b) (3, –5) (c) (–3, –5) (d) (–3, 5)

7. If cosec x + sin x = a and sec x + cos x = b, then

(a) $(a^2b)^{\frac{2}{3}} + (ab^2)^{\frac{2}{3}} = 1$ (b) $(ab^2)^{\frac{2}{3}} + (a^2b^2)^{\frac{2}{3}} = 1$ (c) $a^2 + b^2 = 1$ (d) $b^2 - a^2 = 1$

8. C is the mid-point of PQ, if P is $(4, x)$, C is $(y, -1)$ and Q is $(-2, 4)$, then x and y respectively are

(a) – 6 and 1 (b) – 6 and 2 (c) 6 and – 1 (d) 6 and – 2

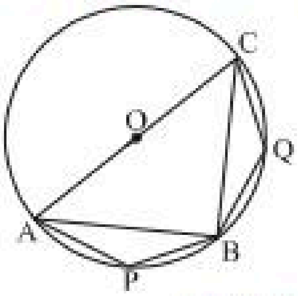
9. If the angle of depression of an object from a 75 m high tower is 30°, then the distance of the object from the tower is

(a) $25\sqrt{3}$ m (b) $50\sqrt{3}$ m (c) $75\sqrt{3}$ m (d) 150 m

10. $\dfrac{1+\tan^2 A}{1+\cot^2 A} = L$

(a) $\sec^2 A$ (b) -1 (c) $\cot^2 A$ (d) $\tan^2 A$

11. In the below diagram, O is the centre of the circle, AC is the diameter and if $\angle APB = 120^\circ$, then $\angle BQC$ is



(a) 30° (b) 150° (c) 90° (d) 120°

12. If four times the sum of the areas of two circular faces of a cylinder of height 8 cm is equal to twice the curve surface area, then diameter of the cylinder is

(a) 4 cm (b) 8 cm (c) 2 cm (d) 6 cm

13. $(\sec A + \tan A)(1 - \sin A) =$

(a) $\sec A$ (b) $\sin A$ (c) $\operatorname{cosec} A$ (d) $\cos A$

14. In two concentric circles, if chords are drawn in the outer circle which touch the inner circle, then

(a) all chords are of different lengths. (b) all chords are of same length.

(c) only parallel chords are of same length. (d) only perpendicular chords are of same length.

15. A school has five houses A, B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and rest from house E. A single student is selected at random to be the class monitor. The probability that the selected student is not from A, B and C is

(a) $\frac{4}{23}$ (b) $\frac{6}{23}$ (c) $\frac{8}{23}$ (d) $\frac{17}{23}$

16. A box contains a number of marbles with serial number 18 to 38. A marble is picked at a random. Find the probability that it is a multiple of 3.

(a) $\frac{3}{5}$ (b) $\frac{7}{20}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$

17. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then the median of the new set

(a) is increased by 2 (b) is decreased by 2

(c) is two times the original median (d) remains the same as that of the original set

18. The mean of a group of eleven consecutive natural numbers is m. What will be the percentage change in the mean when next six consecutive natural numbers are included in the group ?

(a) m % (b) $\frac{m}{3}$ % (c) $\frac{m}{300}$ % (d) $\frac{300}{m}$ %

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false but R is true.***

19. **Assertion :** ABC is an isosceles, right triangle, right angled at C. Then $AB^2 = 3AC^2$.

Reason : In an isosceles triangle ABC if $AC = BC$ and $AB^2 = 2AC^2$, then $\angle C = 90^\circ$.

20. **Assertion :** The points $(k, 2 - 2k)$, $(-k + 1, 2k)$ and $(-4 - k, 6 - 2k)$ are collinear if $k = \frac{1}{2}$.

Reason : Three points A, B and C are collinear in same straight line, if $AB + BC = AC$.

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. If $\sin A + \operatorname{cosec} A = 3$ then the value of $\frac{\sin^4 A + 1}{\sin^2 A}$ is ________.

22. Find the length of the tangent drawn from a point, whose distance from the centre of a circle is 5 cm and radius of the circle is 3 cm.

23. An umbrella has 8 ribs which are equally spaced (see Fig.). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

24. 90% of the mangoes in a bag are good. If a mango is chosen randomly from the box, find the probability of getting a bad mango.

OR

A bag contains 40 coins, consisting of ₹ 2, ₹ 5 and ₹ 10 denominations. If a coin is drawn at random, the probability of drawing a ₹ 2 coin is $\frac{5}{8}$. If x ₹ 2 coins are removed from the bag and then a coin is drawn at random, the probability of drawing a ₹ 2 coin is $\frac{1}{2}$. Find the value of x.

25. Find 'k' so that the points (7, –2), (5, 1) and (3, k) are collinear.

OR

Find the value of a, if the distance between the points A(–3, –14) and B(a, –5) is 9 units.

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Verify whether 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$.

OR

Find the zeroes of the quadratic polynomial $5x^2 + 8x - 4$ and verify the relationship between the zeroes and the coefficients of the polynomial.

27. If p, q, r are in A.P. then find the value of $p^3 + r^3 - 8q^3$.

OR

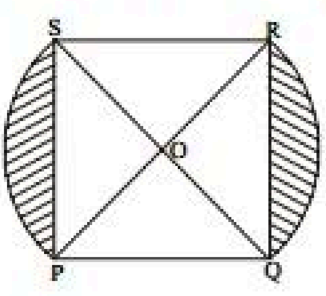
Find the number of terms of the AP 18, 15½, 13, ..., – 49½ and find the sum of all its terms.

28. If $a \cos\theta + b \sin\theta = m$ and $a \sin\theta - b\cos\theta = n$, prove that $a^2 + b^2 = m^2 + n^2$

29. Two ships are there in the sea on either side of a light house in such a way that the ships and the light house are in the same straight line. The angles of depression of two ships as observed from the top of the light house are 60° and 45°. If the height of the light house is 200m, find the distance between the two ships. [Use $\sqrt{3}$ = 1.73].

30. A circle touches all the four sides of a quadrilateral ABCD. Prove that AB + CD = BC + DA.

31. In figure, PQRS is a square lawn with side PQ = 42 metres. Two circular flower beds are there on the sides PS and QR with centre at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).



SECTION-D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Solve the equation : $\left(\frac{2x-3}{x-1}\right)-4\left(\frac{x-1}{2x-3}\right)=3,\ x \neq 1,\ 3/2$

OR

The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

33. If the sum of p terms of an A.P. is q and the sum of q terms is p then, show that sum of (p – q) terms is equal to $(p-q)\left(1+\frac{2q}{p}\right)$.

OR

If S_1, S_2, S_3, be the sum of n, 2n and 3n terms respectively of an A.P., prove that $S_3 = 3(S_2 - S_1)$

34. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then other two sides are divided in the same ratio.

35. On the sports day of a school, 300 students participated. Their ages are given in the following distribution:

Age (in years)	5 - 7	7 - 9	9 - 11	11 - 13	13 - 15	15 - 17	17 - 19
Number of students	67	33	41	95	36	13	15

Find the mode of the data.

SECTION-E

This section comprises of 3 case study/passage - based questions of 4 marks each with three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively.

36. Case - Study 1: Read the following passage and answer the questions given below.

Situation-1

H.C.F. × L.C.M. = Product of two integers.

(i) The H.C.F. of two numbers is 16 and their product is 3072. Find their L.C.M.

(ii) The sum of two numbers is 135 and their H.C.F. is 27. If their L.C.M. is 162, then what will be the numbers?

Situation-2

HCF of natural numbers is the largest factor which is common to all the number and LCM of natural numbers is the smallest natural number which is multiple of all the numbers.

(iii) The LCM and HCF of two rational numbers are equal. Which types of numbers satisfy the given condition ?

OR

If two positive integers a and b are expressible in the form $a = pq^2$ and $b = p^3q$; p, q being prime number, then what will be the LCM of (a, b) ?

37. Case - Study 2: Read the following passage and answer the questions given below.

A two digit number is obtained by either multiplying sum of the digits by 8 and adding 1 or by multiplying the difference of the digits by 13 and adding 2.

If x be the digit in ten's place and y be the digit at unit place with x > y, then answer the following questions.

(i) Find the equation corresponding to multiplying sum of the digits by 8 and adding 1.

(ii) Find the equation corresponding to multiplying the difference of the digits by 13 and adding 2.

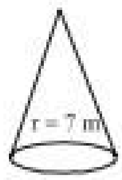
(iii) What is the value of y ?

OR

What is the value of x ?

38. Case - Study 3: Read the following passage and answer the questions given below.

Adventure camps are the perfect place for the children to practice decision making for themselves without parents and teachers guiding their every move. Some students of a school reached for adventure at Sakleshpur. At the camp, the waiters served some students with a welcome drink in a cylindrical glass and some students in a hemispherical cup whose dimensions are shown below. After that they went for a jungle trek. The jungle trek was enjoyable but tiring. As dusk fell, it was time to take shelter. Each group of four students was given a canvas of area $551 m^2$. Each group had to make a conical tent to accommodate all the four students. Assuming that all the stitching and wasting incurred while cutting, would amount to $1 m^2$, the students put the tents. The radius of the tent is 7 m.



(i) What will be the volume of cylindrical cup ?

(ii) What will be the volume of hemispherical cup ?

(iii) What is the height of the conical tent prepared to accommodate four students ?

OR

How much space on the ground is occupied by each student in the conical tent.

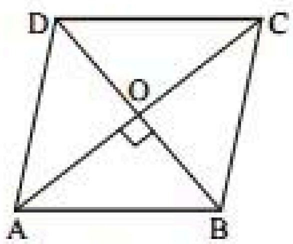
Solution

SAMPLE PAPER-5

1. (d) The L.C.M. of 16, 20 and 24 is 240. The least multiple of 240 that is a perfect square is 3600 and also we can easily eliminate choices (a) and (c) since they are not perfect number. Hence, the required least number which is also a perfect square is 3600 which is divisible by each of 16, 20 and 24.

2. (b) The diagonals of a rhombus are perpendicular bisector of each other.

Given, AC = 16 cm
and BD = 12 cm
∴ AO = 8 cm,
BO = 6 cm
and ∠AOB = 90°



In right angled ΔAOB,

$AB^2 = AO^2 + OB^2$ [by Pythagoras theorem]

$\Rightarrow AB^2 = 8^2 + 6^2$
$= 64 + 36 = 100 = 10^2$

∴ AB = 10 cm

3. (a) **Hint:** Use distance formula.

4. (b) ∵ $m = n^2 - n = n(n-1)$

Now, $m^2 - 2m = m(m-2)$
$= n(n-1)(n^2 - n - 2) = n(n-1)(n-2)(n+1)$

Since we know that product of any four consecutive integers is always divisible by 24.

∴ $m^2 - 2m$ is divisible by 24.

5. (d) In ΔAPB and ΔCPD,

∠APB = ∠CPD = 50° [vertically opposite angles]

Now, $\frac{AP}{PD} = \frac{6}{5}$ ...(i)

and $\frac{BP}{CP} = \frac{3}{2.5} = \frac{6}{5}$ ...(ii)

∴ $\frac{AP}{PD} = \frac{BP}{CP}$ From Eqs. (i) and (ii)

∴ ΔAPB ~ ΔDPC [by SAS similarity criterion]

∴ ∠A = ∠D = 30° [corresponding angles of similar triangles]

In ΔAPB, ∠A + ∠B + ∠APB = 180° [sum of angles of a triangle = 180°]

⇒ 30° + ∠B + 50° = 180°

⇒ ∠B = 180° − (50° + 30°) = 100°

i.e., ∠PBA = 100°

6. (c) For reflection of a point with respect to x-axis change sign of y-coordinate and with respect to y-axis change sign of x-coordinate.

7. (a) $\operatorname{cosec} x - \sin x = a$ & $\sec x - \cos x = b$

$\operatorname{cosec} x - \frac{1}{\operatorname{cosec} x} = a$ & $\sec x - \frac{1}{\sec x} = b$

$\Rightarrow \frac{\operatorname{cosec}^2 x - 1}{\operatorname{cosec} x} = a$ & $\frac{\sec^2 x - 1}{\sec x} = b$

$\Rightarrow \frac{\cot^2 x}{\operatorname{cosec} x} = a$ & $\frac{\tan^2 x}{\sec x} = b$

$\frac{\cos^2 x}{\sin x} = a$ & $\frac{\sin^2 x}{\cos x} = b$

Now, $a^2 b = \frac{\cos^4 x}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos x} = \cos^3 x$

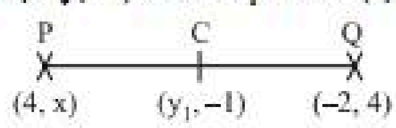
$\Rightarrow \cos x = (a^2b)^{1/3} \Rightarrow \cos^2 x = (a^2b)^{2/3}$

Similarly, $\sin^2 x = (ab^2)^{2/3}$

We know that, $\sin^2 x + \cos^2 x = 1$

$\Rightarrow (ab^2)^{2/3} + (a^2b)^{2/3} = 1$

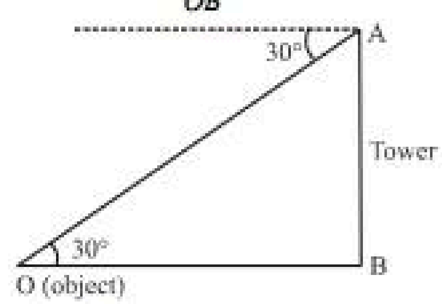
8. (a) Since, $C(y, -1)$ is the mid-point of $P(4, x)$ and $Q(-2, 4)$.



We have, $\frac{4-2}{2} = y$ and $\frac{4+x}{2} = -1$

∴ $y = 1$ and $x = -6$

9. (c) **Hint:** $\tan 30° = \frac{AB}{OB}$



$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{OB} \Rightarrow OB = 75\sqrt{3}$ m

10. (d) $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{(\sec^2 A - \tan^2 A) + \tan^2 A}{(\operatorname{cosec}^2 A - \cot^2 A) + \cot^2 A}$

$= \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\sin^2 A}{\cos^2 A} = \left(\frac{\sin A}{\cos A}\right)^2 = \tan^2 A.$

11. **(b)** (i) APBC is a cyclic quadrilateral.
(ii) $\angle ABC$ is an angle in a semi circle.
(iii) ABQC is a cyclic quadrilateral.

12. **(b)** **Hint:** C.S.A of cylinder

13. **(d)** $(\sec A + \tan A)(1 - \sin A)$

$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right) \times (1 - \sin A)$

$= \frac{(1+\sin A)(1-\sin A)}{\cos A}$

$= \frac{1-\sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$ $\quad (\because \cos^2 A = 1 - \sin^2 A)$

$= \cos A.$

14. **(b)** All chords are of same length.

15. **(b)** Given that, total number of students = 23

Number of students in house A, B and C

= 4 + 8 + 5 = 17

$\therefore$ Remains students = 23 – 17 = 6

Hence, probability that the selected student is not from A, B and C

$= \frac{\text{No. of possible outcomes}}{\text{Total no. of outcomes}} = \frac{6}{23}$

16. **(d)** Total number of marbles = 38 – 18 + 1 = 21

The multiples of 3 from 18 to 38 are 18, 21, 24, 27, 30, 33, 36.

These are 7 in numbers

$\therefore$ Required probability $= \frac{7}{21} = \frac{1}{3}$

17. **(d)** Since $n = 9$, then middle term $= \left(\frac{9+1}{2}\right)^{th} = 5^{th}$ term.

Now, last four observations are increased by 2.

$\because$ The median is 5^{th} observation, which is remains unchanged.

$\therefore$ There will be no change in median.

18. **(d)** Since, the mean of a group of eleven consecutive natural numbers is m, then

$\frac{x + x + 1 + ... + x + 10}{11} = m$

$11x + 55 = 11m; x + 5 = m; x = m - 5$

Let n be the mean when next six consecutive natural numbers are included in the group then

$\frac{x + x + 1 + ... + x + 16}{17} = n;$

$17x + \frac{16 \times 17}{2} = 17n$

$17x + 8 \times 17 = 17n$

$m - 5 + 8 = n$

$\Rightarrow \quad n = m + 3 \qquad (\therefore x = m - 5)$

Hence, required percentage change in the mean

$= \frac{n-m}{m} \times 100 = \frac{m+3-m}{m} \times 100 = \frac{300}{m}\%$

19. **(d)** In right angled ΔABC,

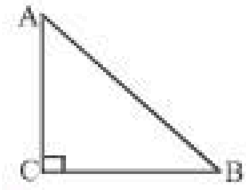
$AB^2 = AC^2 + BC^2$ (By Pythagorus Theorem)

$= AC^2 + AC^2 \quad [\because BC = AC]$

$= 2AC^2$

$\therefore \quad AB^2 = 2AC^2$

$\therefore$ Assertion is false.



Again since

$AB^2 = 2AC^2 = AC^2 + AC^2$

$= AC^2 + BC^2 \, (\because AC = BC$ given)

$\therefore \angle C = 90°$ (By converse of Pythagoras Theorem)

$\therefore$ Reason is true.

20. **(a)** Both assertion and reason are correct. Reason is correct explanation of assertion.

21. $\sin A + \operatorname{cosec} A = 3$

$\Rightarrow \sin A + \frac{1}{\sin A} = 3 \Rightarrow \frac{\sin^2 A + 1}{\sin A} = 3$ [1 Mark]

or $\sin^2 A + 1 = 3 \sin A$

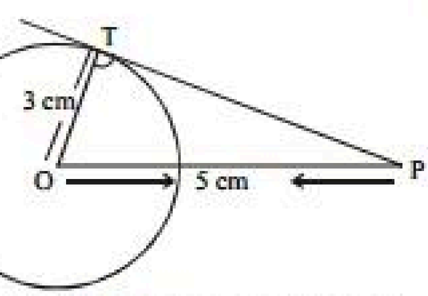
Squaring both sides, we get

$1 + \sin^4 A + 2\sin^2 A = 9\sin^2 A$

$\Rightarrow 1 + \sin^4 A = 7\sin^2 A \Rightarrow \frac{1+\sin^4 A}{\sin^2 A} = 7$ [1 Mark]

22. Let OT be a radius and OP be a tangent.

In a ΔOPT. [1 Mark]



$OT^2 + PT^2 = OP^2$ (By Pythagoras theorem)

$\Rightarrow \quad (3)^2 + PT^2 = (5)^2 \Rightarrow PT^2 = 25 - 9$

$\Rightarrow \quad PT^2 = 16 \Rightarrow PT = 4$ cm. [1 Mark]

23. Here, $r = 45$ cm

θ = sector angle between two consecutive ribs.

$= \frac{360}{8}$ degree $\quad (\because$ there are 8 sectors of same size)

= 45 degrees [1 Mark]

Therefore, the area between two consecutive ribs of the umbrella

= The area of one sector $= \frac{45}{360} \times \pi r^2$

$\left[\because \text{area of sector} = \frac{\theta}{360} \times \pi r^2\right]$

$= \frac{11}{28} \times 45 \times 45 \text{cm}^2 = \frac{22275}{28} \text{cm}^2$ [1 Mark]

24. Percentage of bad mangoes
= 100% – percentage of good mangoes
= 100% – 90% = 10% [1 Mark]

$\therefore$ probability of bad magoes = $\frac{10}{100}=\frac{1}{10}$ [1 Mark]

OR

Total number of coins = 40
let number of ₹ 2 coins is C.

$\because$ P(drawing a ₹ 2 coin) = $\frac{5}{8} \Rightarrow \frac{C}{40}=\frac{5}{8} \Rightarrow C = 25$

[1 Mark]

Now, $\frac{25-x}{40-x}=\frac{1}{2} \Rightarrow x=10$ [1 Mark]

25. Since points are collinear,

Distance AB = $\sqrt{13}$ [1 Mark]

Distance BC = $\sqrt{4+k^2+1-2k}$

Put AB = BC
Then, k = 4 [1 Mark]

OR

Given that
Distance between A(–3, –14) and B(a, –5), AB = 9
$\therefore$ using distance formula

$$\sqrt{(a+3)^2+(-5+14)^2}=9$$ [½ Mark]

$$\Rightarrow \sqrt{(a+3)^2+(9)^2}=9$$ [½ Mark]

On squaring both the sides,
$(a+3)^2+81=81$ [½ Mark]
$\Rightarrow (a+3)^2=0$
$\Rightarrow a=-3$ [½ Mark]

26. $p(x)=2x^3-11x^2+17x-6$

$p(2)=2(2)^3-11(2)^2+17(2)-6$

$=16-44+34-6$

$=50-50=0$

Hence 2 is the zero of $p(x)$.

$p(3)=2(3)^3-11(3)^2+17(3)-6$

$=54-99+51-6$

$=105-105=0$ [1 Mark]

Hence, 3 is the zero of $p(x)$.

$$p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3-11\left(\frac{1}{2}\right)^2+17\left(\frac{1}{2}\right)-6$$

$$=\frac{1}{4}-\frac{11}{4}+\frac{17}{2}-6=0$$

Hence, $\frac{1}{2}$ is also the zero of $p(x)$. [1 Mark]

OR

Suppose $p(x)=5x^2+8x-4$ [1 Mark]
$=5x^2+10x-2x-4$
$=5x(x+2)-2(x+2)$
$=(x+2)(5x-2)$ [1 Mark]

Hence, zeroes of the quadratic polynomial

$5x^2+8x-4$ are -2 and $\frac{2}{5}$. [1 Mark]

Verification:

Sum of zeroes = $-2+\frac{2}{5}=\frac{-8}{5}$

Product of zeroes = $(-2)\times\left(\frac{2}{5}\right)=\frac{-4}{5}$

Again sum of zeroes = $-\frac{\text{Coeff. of } x}{\text{Coeff. of } x^2}=\frac{-8}{5}$

Product of zeroes = $\frac{\text{Constant term}}{\text{Coeff. of } x^2}=\frac{-4}{5}$

27. Since p, q, r are in A.P
$\therefore\ q-p=r-q$
$\Rightarrow 2q=r+p$ ... (i) [1 Mark]
$p^3+r^3-8q^3=(p^3+r^3)-(2q)^3$
$=(p+r)(p^2+r^2-rp)-(2q)^3$
$=(p+r)(p^2+r^2-rp)-(r+p)^3$ [From (i), $2q=r+p$]
[1 Mark]
$=(r+p)[(p^2+r^2-rp)-(r+p)^2]$
$=(r+p)[p^2+r^2-rp-r^2-p^2-2pr]$
$=(r+p)(-3pr)=(2q)(-3pr)=-6pqr$ [From (i), $2q=r+p$]
[1 Mark]

OR

The given AP is 18, $15\frac{1}{2}$, 13 ...., $-49\frac{1}{2}$.

Suppose the number of terms in given AP is n.
As, last term of an AP, $l=a+(n-1)d$ [1 Mark]

So, $-49\frac{1}{2}=18+(n-1)\left(15\frac{1}{2}-18\right)$

$\Rightarrow -\frac{99}{2}=18+(n-1)\left(-\frac{5}{2}\right)$ [½ Mark]

$\Rightarrow 99=-36+(n-1)5 \Rightarrow 5(n-1)=99+36=135$

$\Rightarrow n-1=\frac{135}{5}=27 \Rightarrow n=27+1=28$ [½ Mark]

Therefore, the number of terms in given AP is 28.
And, the sum of all 28terms

$=\frac{28}{2}\left(18-49\frac{1}{2}\right)=14\left(\frac{36-99}{2}\right)=-441$ [1 Mark]

Hence, the number of terms in given AP is 28 and the sum of all its terms is – 441.

28. $m=a\cos\theta+b\sin\theta$
$(m)^2=(a\cos\theta+b\sin\theta)^2$
$m^2=a^2\cos^2\theta+b^2\sin^2\theta+2ab\cos\theta\sin\theta$ ...(i)
[1 Mark]

also, $n = a\sin\theta - b\cos\theta$; $(n)^2 = (a\sin\theta - b\cos\theta)^2$

$n^2 = a^2\sin^2\theta + b^2\cos^2\theta - 2ab\cos\theta\sin\theta$ ...(ii)

[1 Mark]

On adding equation (i) and (ii), we get

$m^2 + n^2 = a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta$

$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

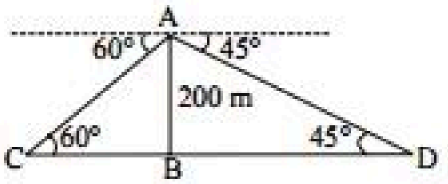
$m^2 + n^2 = a^2 + b^2$ [1 Mark]

(Hence proved.)

29. Let AB is the height of light house = 200m

Two ships are at points C and D on either side of AB (light house)

In ΔABC,



[1 Mark]

$\tan 60° = \frac{AB}{BC}$

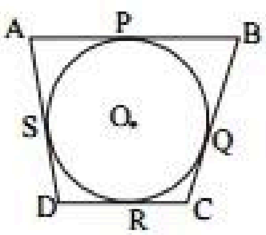
$\Rightarrow BC = \frac{200\sqrt{3}}{3} = \frac{200\times1.73}{3} = 115.33$ m [1 Mark]

In ΔABD, $\tan 45° = \frac{AB}{BD} \Rightarrow BD = 200$ [1 Mark]

Distance between both ships

= BC + BD = 115.33 + 200 = 315.33 m [1 Mark]

30. Let there be a circle with centre O whereas AB, BC, CD and DA are tangents at P, Q, R and S respectively.



[1 Mark]

Here,

AP = AS ...(i)

BP = BQ ...(ii)

CR = CQ ...(iii)

DR = DS ..(iv)

[Tangents drawn from a point (outside the circle) on a given circle are equal in lengths]

From equations (i), (ii), (iii) and (iv), [1 Mark]

(AP + BP) + (CR + DR) = (BQ + CQ) + (DS + AS)

AB + CD = BC + DA (Hence proved.) [1 Mark]

31. Since, radius of circle with centre O is OR.

Suppose OR = x

$\therefore\ x^2 + x^2 = (42)^2$ [By Pythagoras theorem]

$x = 21\sqrt{2}$m [1 Mark]

So, area of one flower bed = Area of segment of circle – Area of triangle POS

$= \pi r^2 \times \frac{90°}{360°} - \frac{1}{2}\times OR \times OR$

$= \frac{22}{7}\times 21\sqrt{2}\times 21\sqrt{2}\times\frac{90°}{360°} - \frac{1}{2}\times 21\sqrt{2}\times 21\sqrt{2}$ [1 Mark]

$= \frac{22}{7}\times 21\times 21\times 2\times\frac{1}{4} - \frac{1}{2}\times 21\times 21\times 2$

$= 33\times 21 - 21\times 21 = 21[33-21] = 21\times 12 = 252\text{m}^2$

[½ Mark]

Hence, area of two flower beds = 2 × 252 = 504 m^2

[½ Mark]

32. Let $\frac{2x-3}{x-1} = y$ then the given equation becomes

$\therefore y - 4\times\frac{1}{y} = 3 \Rightarrow y^2 - 4 = 3y$ [1 Mark]

or $y^2 - 3y - 4 = 0 \Rightarrow y^2 - 4y + y - 4 = 0$

$\Rightarrow y(y-4) + 1(y-4) = 0 \Rightarrow (y+1)(y-4) = 0$ [2 Marks]

Either $y + 1 = 0 \Rightarrow y = -1$ or $y - 4 = 0 \Rightarrow y = 4$

Putting the value of y = – 1 then

$\frac{2x-3}{x-1} = -1 \Rightarrow 2x - 3 = -x + 1$

or $2x + x - 3 - 1 = 0 \Rightarrow 3x = 4 \Rightarrow x = \frac{4}{3}$ [1 Mark]

Putting the value of y = 4 in $\frac{2x-3}{x-1} = 4 \Rightarrow 2x - 3 = 4x - 4$

or $2x - 4x - 3 + 4 = 0$ or $-2x + 1 = 0 \Rightarrow -2x = -1$

$\Rightarrow x = 1/2$ [1 Mark]

x = {4/3, 1/2}

OR

Let the present age of Rehman be x years.

Rehman's age, 3 years ago = $(x - 3)$ years [1 Mark]

Rehman's age, 5 years later = $(x + 5)$ years

According to the question

$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$\Rightarrow 6x + 6 = x^2 + 2x - 15 \Rightarrow x^2 - 4x - 21 = 0$

On comparing with $ax^2 + bx + c = 0$,

we get $a = 1, b = -4, c = -21$

$D = b^2 - 4ac = (-4)^2 - 4(1)(-21) = 16 + 84 = 100$

$\Rightarrow \sqrt{D} = \sqrt{100} = 10$

$\therefore x = \frac{-b\pm\sqrt{D}}{2a} = \frac{4\pm10}{2} = \frac{14}{2}, \frac{-6}{2}$ [2 Marks]

$\Rightarrow x = 7, -3$ [1 Mark]

We reject $x = -3$ ($\because$ age cannot be negative)

$\therefore$ Rehman's present age = 7 years [1 Mark]

33. $\because$ Sum of p terms = q

$\therefore \frac{p}{2}[2a+(p-1)d]=q$ ...(i) [½ Mark]

Now, sum of q terms = p

$\therefore \frac{q}{2}[2a+(q-1)d]=p$ ...(ii) [½ Mark]

After solving (i) and (ii), we get

$d=\frac{-2(p+q)}{pq}$ ...(iii) [1 Mark]

$S_{p-q}=\frac{p-q}{2}[2a+(p-q-1)d]$

$=\frac{p-q}{2}[2a+(p-1)d-qd]$

$=\frac{p-q}{2}\left[\frac{2q}{p}-q\left\{\frac{-2(p+q)}{pq}\right\}\right]$ [From (i) and (iii)] [2 Marks]

$=(p-q)\left[\frac{q+(p+q)}{p}\right]$ [From (i) and (iii)]

$=(p-q)\left[\frac{p}{p}+\frac{2q}{p}\right]=(p-q)\left[1+\frac{2q}{p}\right]$

(Hence Proved). [1 Mark]

OR

Let 'a' be the 1st term and 'd' the common difference of an A.P.

$S_1=$ Sum of n terms of the A.P. $=\frac{n}{2}[2a+(n-1)d]$

[½ Mark]

$S_2=$ Sum of 2n terms of the A.P.

$=\frac{2n}{2}[2a+(2n-1)d]=n[2a+(2n-1)d]$ [½ Mark]

$S_3=$ Sum of 3n terms of the A.P.

$=\frac{3n}{2}[2a+(3n-1)d]$ .....(i) [1 Mark]

Now, R.H.S. $=3(S_2-S_1)$

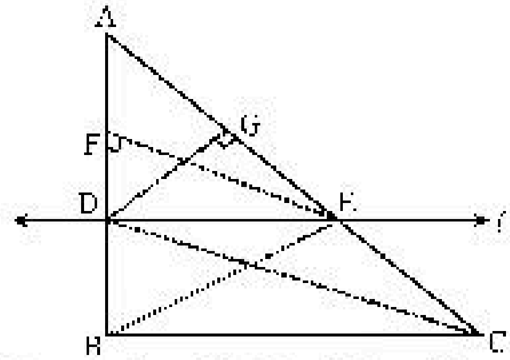
$=3\left[n\{2a+2(n-1)d\}-\frac{n}{2}\{2a+(n-1)d\}\right]$ [2 Marks]

$3\left[an+\frac{3n^2d}{2}-\frac{nd}{2}\right]=3n\left[\frac{2a+3nd-d}{2}\right]$

$=\frac{3n}{2}[2a+(3n-1)d]=S_3$ (from (i)) = L.H.S. [1 Mark]

34. In ΔABC, ℓ is drawn parallel to BC which intersects AB and AC at D and E respectively.

To prove : $\frac{AD}{DB}=\frac{AE}{EC}$



[1 Mark]

Construction : Join BE and CD and draw $EF\perp AB$ and $DG\perp AC$. [½ Mark]

Proof : $\Delta\,DBE$ and $\Delta\,CDE$ are on the same base DE and between the same parallel lines DE and BC.

$\therefore$ Area ($\Delta\,BDE$) = Area ($\Delta\,CDE$) .... (i)

[½ Mark]

Since, EF $\perp$ AB therefore the height of both triangles is EF.

$\therefore \frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta BDE)}=\frac{\frac{1}{2}\times AD\times EF}{\frac{1}{2}\times BD\times EF}=\frac{AD}{BD}$ ...... (ii)

[1 Mark]

Similarly,

$\frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta CDE)}=\frac{\frac{1}{2}\times AE\times DG}{\frac{1}{2}\times EC\times DG}=\frac{AE}{EC}$ ...... (iii)

[1 Mark]

Hence from (i), (ii) and (iii), we get $\frac{AD}{DB}=\frac{AE}{EC}$ [1 Mark]

35. Here, maximum frequency = 95,

So modal class = 11 – 13 [1 Mark]

$l=11,\ f_1=95,\ f_0=41,\ f_2=36,\ h=2$ [1 Mark]

Mode $=l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

$=11+\left(\frac{95-41}{190-41-36}\right)\times 2=11+\frac{54}{113}\times 2$ [2 Marks]

$\therefore$ Mode = 11 + 0.95 = 11.95 [1 Mark]

36. (i) (c) H.C.F. = 16 and Product = 3072

L.C.M. $=\frac{\text{Product}}{\text{H.C.F.}}=\frac{3072}{16}=192$ [1 Mark]

(ii) (c) H.C.F. of two numbers is 27

So let the numbers are 27a and 27b

Now 27a + 27b = 135

$\Rightarrow$ a + b = 5 ...(i)

Also 27a × 27b = 27 × 162.

$\Rightarrow$ ab = 6 ...(ii)

$(a-b)^2 = (a+b)^2 - 4ab$

$\Rightarrow \quad a - b = 1$

Solving (i) and (iii), we get

$a = 3, b = 2$

So numbers are 27×3, 27×2 i.e., 81, 54 [1 Mark]

(iii) LCM = HCF

Let a_1 and a_2 be two numbers.

LCM $(a_1, a_2) = x$

HCF $(a_1, a_2) = x$

$\Rightarrow$ Then, two numbers are equal. [2 Marks]

OR

Clearly, LCM = (LCM of p and p^3)

(LCM of q^2 and q) = p^3q^2 [2 Marks]

37. **(i)** **Hint:** Solve $(10x+y)$ [1 Mark]

(ii) **Hint:** Solve $(10x+y)=(x-y)\,13+2$ [1 Mark]

(iii) Use elimination method to solve the equations. [2 Marks]

38. **(i)** Volume of cylindrical cup = $\pi r^2 h$

$\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10.5 = 404.25 \text{ cm}^3$ [1 Mark]

(ii) Volume of hemispherical cup

$= \frac{2}{3}\pi r^3 = \frac{3}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3 = 89.83 \text{ cm}^3$ [1 Mark]

(iii) Curved surface area of cone = 551 $\Rightarrow \pi r l = 441$

$\Rightarrow \frac{22}{7} \times 7 \times l = 551$

$\Rightarrow \quad l = 25.045$

$\therefore \; h = \sqrt{l^2 - r^2} = 24 \text{ m}$ [2 Marks]

OR

Space occupied by each student

$= \frac{\pi r^2}{4} = 38.5 \text{ m}^2$ [2 Marks]