

CHAPTER-2

RELATIONS AND FUNCTIONS

ONE MARK QUESTIONS:

1. Define Cartesian product of two nonempty sets. (K)
2. If the set A has 3 elements and the set $B = \{3, 4, 5\}$, then find the number of elements in $(A \times B)$. (U)
3. Define a relation. (K)
4. Define domain of a relation. (K)
5. Define range of a relation. (K)
6. $A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$. Define a relation R from A to B by
 $R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$. Write R in roster form. (U)
7. Define a function. (K)
8. The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$. Find the value of C, when $t(C) = 212$. (A)
9. Find the range of the function $f(x) = 2 - 3x, x \in \mathbb{R}, x > 0$. (U)
10. Find the range of the function $f(x) = x^2 + 2, x$ is a real number. (U)
11. Find the range of the function $f(x) = x, x$ is a real number. (U)
12. Find the domain of the function $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$ (U)
13. If $(x + 1, y - 2) = (3, 1)$, find the values of x and y. (U)
14. If $P = \{m, n\}$ and $Q = \{n, m\}$, form the set $P \times Q$. (U)
15. Find the domain of the function $f(x) = \frac{x^2+3x+5}{x^2-5x+4}$ (U)

TWO MARKS QUESTIONS:

1. If $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$. Find $A \times (B \cap C)$. (U)
2. If $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$. Find $(A \times B) \cap (A \times C)$. (U)
3. If $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$. Find $A \times (B \cup C)$. (U)
4. If $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$. Find $(A \times B) \cup (A \times C)$. (U)
5. If $P = \{1, 2\}$, form the set $P \times P \times P$. (U)
6. If $A \times B = \{(p, q), (p, r), (m, q), (m, r)\}$, find A and B. (U)
7. If $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of x and y. (U)

8. If $G = \{7, 8\}$ and $H = \{5, 4, 2\}$, find $G \times H$ and $H \times G$. (U)
9. If $A = \{-1, 1\}$, find $A \times A \times A$. (U)
10. If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$. Find A and B . (U)
11. Let $A = \{1, 2, 3, 4, 5, 6\}$. Define a relation R from A to A by $R = \{(x, y) : y = x + 1\}$. Write down the domain and range of R . (U)
12. Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Find the number of relations from A to B . (S)
13. Write the relation $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$ in roster form. (U)
14. Let $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of relations from A to B . (S)
15. State whether the relation $R = \{(2, 2), (2, 4), (3, 3), (4, 4)\}$ is a function or not. Justify your answer. (U)
16. State whether the relation $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)\}$ is a function or not. Justify your answer. (U)
17. State whether the relation $R = \{(1, 3), (1, 5), (2, 5)\}$ is a function or not. Justify your answer. (U)
18. Find the domain and range of the function $f(x) = -|x|$ (U)
19. Find the domain and range of the function $f(x) = \sqrt{9-x^2}$ (U)
20. Let \mathbb{R} be the set of real numbers. Define the real function $f: \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = x + 10$. Sketch the graph of this function. (S)
21. Draw the graph of the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x + 5$, where \mathbb{R} is the set of real numbers. (S)
22. The relation f is defined by $f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$
- The relation g is defined by $g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$
- Show that f is a function and g is not a function. (U)
23. If $f(x) = x^2$, find $\frac{f(1.1) - f(1)}{1.1 - 1}$ (U)
24. Find the domain and range of the function f defined by $f(x) = \sqrt{x - 1}$ (U)
25. Find the domain and range of the function f defined by $f(x) = |x - 1|$ (U)
26. Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a function from \mathbb{Z} to \mathbb{Z} defined by $f(x) = ax + b$, for some integers a, b . Determine a, b . (U)

27. Let $A = \{1, 2, 3, 4\}$, $B = \{1, 5, 9, 11, 15, 16\}$ and $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$. Are the following are true?

(i) f is a relation from A to B (ii) f is a function from A to B .

Justify your answer in each case. (U)

28. Let $A = \{9, 10, 11, 12, 13\}$ and let $f: A \rightarrow \mathbb{N}$ be defined by $f(n) =$ the highest prime factor of n .

Find the range of f . (U)

29. Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, find A and B , where x, y and z are distinct elements. (S)

30. Let $f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in \mathbb{R} \right\}$ be a function from \mathbb{R} into \mathbb{R} . Determine the range of f . (U)

31. Let f be the subset of $\mathbb{Z} \times \mathbb{Z}$ defined by $f = \{(ab, a + b) : a, b \in \mathbb{Z}\}$. Is f a function from \mathbb{Z} to \mathbb{Z} ? Justify your answer. (U)

THREE MARKS QUESTIONS:

1. If $P = \{a, b, c\}$ and $Q = \{r\}$, form the sets $P \times Q$ and $Q \times P$. Are these two products equal? (U)

2. If $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$ and $C = \{5, 6\}$. Verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. (U)

3. If $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify that $A \times C$ is a subset of $B \times D$. (U)

4. Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Write $A \times B$. How many subsets will $A \times B$ have? List them. (U)

5. The Cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $A \times A$. (S)

6. Let $A = \{1, 2, \dots, 14\}$. Define a relation R from A to A by $R = \{(x, y) : 3x - y = 0, \text{ where } x, y \in A\}$.

Write down its domain, codomain and range. (U)

7. Define a relation R on the set \mathbb{N} of natural numbers by

$R = \{(x, y) : y = x + 5, x \text{ is a natural number less than } 4, x, y \in \mathbb{N}\}$. Depict this relationship using roster form. Write down the domain and range. (U)

8. Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by $R = \{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$.

Write R in roster form. Find the domain and range of R . (U)

9. Determine the domain and range of the relation R defined by $R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$. (U)

10. Let R be the relation on \mathbb{Z} defined by $R = \{(a, b) : a, b \in \mathbb{Z}, a - b \text{ is an integer}\}$. Find the domain and range of R . (U)

11. Let \mathbb{N} be the set of natural numbers and the relation R be defined on \mathbb{N} such that $R = \{(x, y) : y = 2x, x, y \in \mathbb{N}\}$. What is the domain and range of R ? Is this relation a function? (U)
12. Let $f(x) = x^2$ and $g(x) = 2x + 1$ be two real functions. Find $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$. (U)
13. Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two real functions defined over the set of non-negative real numbers. Find $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$. (U)
14. Is the relation $R = \{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$ a function? Give reason. If it is a function determine its domain and range. (U)
15. Is the relation $R = \{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$ a function? Give reason. If it is a function determine its domain and range. (U)
16. A function f is defined by $f(x) = 2x - 5$. Write down the values of (i) $f(0)$, (ii) $f(7)$, (iii) $f(3)$. (U)
17. The function ' t ' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$. Find (i) $t(0)$, (ii) $t(28)$, (iii) $t(-10)$. (A)
18. Let R be a relation from \mathbb{Q} to \mathbb{Q} defined by $R = \{(a, b) : a, b \in \mathbb{Q} \text{ and } a - b \in \mathbb{Z}\}$. Show that
 (i) $(a, a) \in R$ for all $a \in \mathbb{Q}$
 (ii) $(a, b) \in R$ implies that $(b, a) \in R$
 (iii) $(a, b) \in R$ and $(b, c) \in R$ implies that $(a, c) \in R$ (U)
19. Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a linear function from \mathbb{Z} into \mathbb{Z} . Find $f(x)$. (A)
20. The function f defined by $f(x) = \begin{cases} 1 - x, & x < 0 \\ 1, & x = 0 \\ x + 1, & x > 0 \end{cases}$
 Draw the graph of $f(x)$. (S)
21. Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and $\frac{f}{g}$. (U)
22. Let R be a relation from \mathbb{N} to \mathbb{N} defined by $R = \{(a, b) : a, b \in \mathbb{N} \text{ and } a = b^2\}$. Are the following true?
 (i) $(a, a) \in R$, for all $a \in \mathbb{N}$ (ii) $(a, b) \in R$, implies $(b, a) \in R$
 (iii) $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$. (S)
- FIVE MARKS QUESTIONS:**
1. Define Identity function. Draw the graph of it. Also write its domain and range. (S)
2. Define Signum function. Draw the graph of it. Also write its domain and range. (S)
3. Define Greatest integer function. Draw the graph of it. Also write its domain and range. (S)

4. Define Constant function. If the function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 3$ for each $x \in \mathbb{R}$, draw the graph of it.

Also write its domain and range. (S)

5. Define Rational function. If the real valued function: $\mathbb{R} - \{0\} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{1}{x}$, draw the graph of it.

Also write its domain and range. (S)

6. Define polynomial function. . If the function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2$, draw the graph of it. Also write its domain and range. (S)

7. Define Modulus function. Draw the graph of it. Also write its domain and range. (S)

8. Draw the graph of the function $f(x) = x^3$. Write its domain and range. (S)
