

Series : ABCD4/3

SET – 2



प्रश्न-पत्र कोड
Q.P. Code

65/3/2

रोल नं.
Roll No.

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परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 8 हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 14 प्रश्न हैं ।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains 8 printed pages.
- Q.P. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 14 questions.
- Please write down the Serial Number of the question in the answer-book before attempting it.
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

*



गणित

MATHEMATICS



निर्धारित समय : 2 घण्टे

Time allowed : 2 hours

अधिकतम अंक : 40

Maximum Marks : 40

65/3/2

310 B

Page 1 of 8

P.T.O.



सामान्य निर्देश :

- (i) इस प्रश्न पत्र के तीन खण्ड – खण्ड क, ख तथा ग हैं।
- (ii) प्रत्येक खण्ड अनिवार्य है।
- (iii) खण्ड-क में 6 लघु उत्तर-I प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।
- (iv) खण्ड-ख में 4 लघु उत्तर-II प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।
- (v) खण्ड-ग में 4 दीर्घ उत्तरीय प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।
- (vi) कुछ प्रश्नों में आंतरिक विकल्प दिए गए हैं।
- (vii) प्रश्न 14 एक प्रकरण अध्ययन आधारित प्रश्न है, जिसमें दो भाग हैं, जिनमें प्रत्येक के 2 अंक हैं।

खण्ड – क

प्रश्न संख्या 1 से 6 तक प्रत्येक प्रश्न के 2 अंक हैं।

1. सदिश $(\vec{b} + \vec{c})$ का सदिश \vec{a} पर प्रक्षेप ज्ञात कीजिए, जहाँ $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ और $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ हैं। 2
2. अवकल समीकरण : $\log \left(\frac{dy}{dx} \right) = ax + by$ का व्यापक हल ज्ञात कीजिए। 2
3. ताश के 52 पत्तों की एक सुमिश्रित गड्डी से यादृच्छया दो पत्ते उत्तरोत्तर प्रतिस्थापना के साथ निकाले जाते हैं। हुकुम के पत्तों की संख्या का प्रायिकता बंटन ज्ञात कीजिए। 2
4. पासों का एक जोड़ा उछाला जाता है और पासों पर प्राप्त संख्याओं का योगफल 7 है। कम-से-कम एक पासे पर संख्या 5 प्राप्त होने की प्रायिकता ज्ञात कीजिए। 2

अथवा

A द्वारा लक्ष्य पर निशाना लगाने की प्रायिकता $\frac{1}{3}$ और B द्वारा निशाना लगाने की प्रायिकता $\frac{2}{5}$ है। यदि दोनों, स्वतंत्र रूप से, लक्ष्य पर निशाना साधते हैं, तो लक्ष्य पर निशाना लगने की प्रायिकता ज्ञात कीजिए।

5. यदि बिन्दु $(1, 1, 1)$ की समतल $x - y + z + \lambda = 0$ से दूरी $\frac{5}{\sqrt{3}}$ है, तो λ के मान ज्ञात कीजिए। 2
6. ज्ञात कीजिए : $\int \frac{dx}{x^2 - 6x + 13}$ 2





General Instructions :

- (i) This question paper contains **three** Sections – Section **A**, **B** and **C**.
- (ii) Each Section is compulsory.
- (iii) Section–**A** has **6** short answer type-**I** questions of **2** marks each.
- (iv) Section–**B** has **4** short answer type-**II** questions of **3** marks each.
- (v) Section–**C** has **4** long answer type questions of **4** marks each.
- (vi) There is an internal choice in some questions.
- (vii) Q. **14** is a case study based problem with **2** sub-parts of **2** marks each.

SECTION – A

Question Nos. **1** to **6** carry **2** marks each.

1. Write the projection of the vector $(\vec{b} + \vec{c})$ on the vector \vec{a} , where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. **2**
2. Find the general solution of the differential equation : $\log \left(\frac{dy}{dx} \right) = ax + by$. **2**
3. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards. **2**
4. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die. **2**

OR

The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probability that the target is hit.

5. If the distance of the point (1, 1, 1) from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ . **2**
6. Find : $\int \frac{dx}{x^2 - 6x + 13}$ **2**





खण्ड – ख

प्रश्न संख्या 7 से 10 तक प्रत्येक प्रश्न के 3 अंक हैं।

7. यदि \vec{a} , \vec{b} , \vec{c} ऐसे तीन सदिश हैं जिनके लिए $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ और $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq 0$ हो, तो दर्शाइए कि $\vec{b} = \vec{c}$. 3

अथवा

यदि $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 4$ तथा $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ हो, तो $(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$ का मान ज्ञात कीजिए।

8. मान ज्ञात कीजिए : $\int_{-1}^2 |x^3 - x| dx$ 3

9. दर्शाइए कि निम्न रेखाएँ सहतलीय हैं : 3
 $\frac{1-x}{2} = \frac{y-3}{4} = \frac{z}{-1}$ और $\frac{x-4}{3} = \frac{2y-2}{-4} = z-1$

10. अवकल समीकरण $x \frac{dy}{dx} - y = x^2 \cdot e^x$ का विशिष्ट हल ज्ञात कीजिए, $y(1) = 0$ दिया गया है। 3

अथवा

अवकल समीकरण $x \frac{dy}{dx} = y(\log y - \log x + 1)$ का व्यापक हल ज्ञात कीजिए।

खण्ड – ग

प्रश्न संख्या 11 से 14 तक प्रत्येक प्रश्न के 4 अंक हैं।

11. ज्ञात कीजिए : $\int \frac{x^2}{(x^2 + 1)(3x^2 + 4)} dx$ 4

अथवा

मान ज्ञात कीजिए : $\int_{-2}^1 \sqrt{5 - 4x - x^2} dx$





SECTION – B

Question Nos. 7 to 10 carry 3 marks each.

7. If \vec{a} , \vec{b} , \vec{c} are three vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq 0$, then show that $\vec{b} = \vec{c}$. 3

OR

If $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 4$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$.

8. Evaluate : $\int_{-1}^2 |x^3 - x| dx$ 3

9. Show that the lines : 3
 $\frac{1-x}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{2y-2}{-4} = z-1$ are coplanar.

10. Find the particular solution of the differential equation $x \frac{dy}{dx} - y = x^2 \cdot e^x$, given $y(1) = 0$. 3

OR

Find the general solution of the differential equation $x \frac{dy}{dx} = y(\log y - \log x + 1)$.

SECTION – C

Question Nos. 11 to 14 carry 4 marks each.

11. Find : $\int \frac{x^2}{(x^2+1)(3x^2+4)} dx$ 4

OR

Evaluate : $\int_{-2}^1 \sqrt{5-4x-x^2} dx$





12. समाकलन के प्रयोग से, प्रथम चतुर्थांश में वक्रों $x^2 + y^2 = 4$, $x = \sqrt{3}y$ तथा x -अक्ष से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए । 4

13. बिंदु $(1, -2, 9)$ से रेखा $\vec{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ और समतल $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10$ के प्रतिच्छेदन बिंदु की दूरी ज्ञात कीजिए । 4

प्रकरण अध्ययन आधारित प्रश्न :

14. एक दुकानदार तीन प्रकार के फूल-बीज A1, A2, A3 बेचता है । वह इन बीजों को एक मिश्रण के रूप में बेचता है जिसमें इन तीन प्रकारों के बीज क्रमशः 4 : 4 : 2 के अनुपात में मिश्रित हैं । इन तीनों बीजों की अंकुरण-दर क्रमशः 45%, 60% और 35% हैं ।



उपरोक्त पर आधारित होकर, निम्न प्रश्नों के उत्तर दीजिए :

- (a) यादृच्छया चुने एक बीज के अंकुरित होने की प्रायिकता ज्ञात कीजिए । 2
- (b) यदि यह दिया गया है कि एक यादृच्छया चुना बीज अंकुरित होता है, तो इस बीज का प्रकार A2 होने की प्रायिकता ज्ञात कीजिए । 2





12. Using integration, find the area of the region bounded by the curves $x^2 + y^2 = 4$, $x = \sqrt{3}y$ and x -axis lying in the first quadrant. 4
13. Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line $\vec{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10$. 4

Case Study Problem :

14. A shopkeeper sells three types of flower seeds A1, A2, A3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.



Based on the above information :

- (a) Calculate the probability that a randomly chosen seed will germinate; 2
- (b) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates. 2





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Strictly Confidential: (For Internal and Restricted use only)
Senior Secondary School Term II Examination, 2022
Marking Scheme – MATHEMATICS (SUBJECT CODE — 041)
(PAPER CODE — 65/3/2)

General Instructions: -

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2. **“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its’ leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under IPC.”**
3. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.**
4. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
5. Evaluators will mark (✓) wherever answer is correct. For wrong answer ‘X’ be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
6. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totalled up and written in the left-hand margin and encircled. This may be followed strictly.
7. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
8. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
9. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

10. A full scale of marks ____0 to 40_____ (example 0-40 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
11. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 30 answer books per day in main subjects and 35 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
12. Ensure that you do not make the following common types of errors committed by the Examiner in the past :-
- Leaving answer or part thereof unassessed in an answer book.
 - Giving more marks for an answer than assigned to it.
 - Wrong totalling of marks awarded on a reply.
 - Wrong transfer of marks from the inside pages of the answer book to the title page.
 - Wrong question wise totalling on the title page.
 - Wrong totalling of marks of the two columns on the title page.
 - Wrong grand total.
 - Marks in words and figures not tallying.
 - Wrong transfer of marks from the answer book to online award list.
 - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
 - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
13. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
14. Any unassessed portion, non-carrying over of marks to the title page, or totalling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
15. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
16. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totalled and written in figures and words.
17. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

MARKING SCHEME

Senior Secondary School Examination TERM–II, 2022

MATHEMATICS (Subject Code — 041)

[Paper Code — 65/3/2]

Maximum Marks: 40

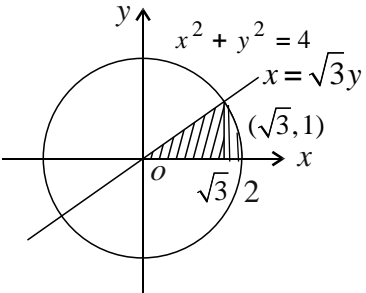
Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION—A	
1.	Write the projection of the vector $(\vec{b} + \vec{c})$ on the vector \vec{a} , where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.	2
Sol.	$\vec{b} + \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$ $\text{Projection of } \vec{b} + \vec{c} \text{ on } \vec{a} = \frac{(\vec{b} + \vec{c}) \cdot \vec{a}}{ \vec{a} }$ $= \frac{(3\hat{i} + \hat{j} + 2\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{9}}$ $= \frac{6 - 2 + 2}{3} = \frac{6}{3} = 2$	<div>1/2</div> <div>1</div> <div>1/2</div> <div>2</div>
2.	Find the general solution of the differential equation : $\log \left(\frac{dy}{dx} \right) = ax + by$.	2
Sol.	$\log \left(\frac{dy}{dx} \right) = ax + by$ $\frac{dy}{dx} = e^{ax+by}$ $\frac{dy}{dx} = e^{ax} \cdot e^{by}$ $\int \frac{dy}{e^{by}} = \int e^{ax} dx$ $\frac{-e^{-by}}{b} = \frac{e^{ax}}{a} + C$	<div>1/2</div> <div>1</div> <div>1/2</div> <div>2</div>
3.	Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards.	2

Sol.	Let X denotes the number of spades									
	$p = \frac{13}{52} = \frac{1}{4}, q = \frac{3}{4}$									
	<table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$</td><td>$2 \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{6}{16}$</td><td>$\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$</td></tr></table>	X	0	1	2	P(X)	$\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$	$2 \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{6}{16}$	$\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$	$\frac{1}{2}$ $1\frac{1}{2}$
	X	0	1	2						
P(X)	$\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$	$2 \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{6}{16}$	$\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$							
		2								
4.(a)	A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die.	2								
Sol.	A: sum is 7 B: 5 has appeared at least on one die $A \cap B = \{(2,5), (5,2)\}$ $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{2/36}{6/36} = \frac{1}{3}$	$\frac{1}{2}$ $\frac{1}{2}$ 1 2								
	OR									
4.(b)	The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probability that the target is hit.	2								
Sol.	$E_1 = A$ hits the target $E_2 = B$ hits the target $P(\text{target is hit}) = 1 - P(\text{target is not hit})$ $= 1 - \left(\frac{2}{3} \cdot \frac{3}{5}\right) = \frac{3}{5}$	1 1 2								
5.	If the distance of the point (1, 1, 1) from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ .	2								
Sol.	Distance of point (1, 1, 1) from $x - y + z + \lambda = 0$ is $\frac{ 1+\lambda }{\sqrt{3}}$ $\frac{ 1+\lambda }{\sqrt{3}} = \frac{5}{\sqrt{3}} \Rightarrow 1 + \lambda = \pm 5$ $\lambda = -6 \text{ or } +4$	$\frac{1}{2}$ 1 $\frac{1}{2}$								


		2
6.	Find : $\int \frac{dx}{x^2 - 6x + 13}$	2
Sol.	$\int \frac{dx}{x^2 - 6x + 13}$ $= \int \frac{dx}{(x-3)^2 + (2)^2}$ $= \frac{1}{2} \tan^{-1} \frac{(x-3)}{2} + C$	1 1 2
SECTION—B		
7.(a)	If \vec{a} , \vec{b} , \vec{c} are three vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq 0$, then show that $\vec{b} = \vec{c}$.	3
Sol.	$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ $\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$ $\Rightarrow \vec{a} = 0, \vec{b} = \vec{c} \text{ or } \vec{a} \perp (\vec{b} - \vec{c}) \quad (1)$ <p>but $a \neq 0 \Rightarrow \vec{b} = \vec{c} \text{ or } \vec{a} \perp (\vec{b} - \vec{c})$</p> $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ $\vec{a} \times (\vec{b} - \vec{c}) = 0$ $\vec{a} = 0, \vec{b} = \vec{c} \text{ or } \vec{a} \parallel (\vec{b} - \vec{c}) \quad (2)$ <p>From (1) and (2)</p> $\vec{a} \neq 0 \text{ given and } \vec{a} \text{ cannot be both } \perp \text{ and } \parallel \text{ to } (\vec{b} - \vec{c})$ $\therefore \vec{b} = \vec{c}$	1/2 1 1 1/2 3
OR		
7.(b)	If $ \vec{a} = 3$, $ \vec{b} = 5$, $ \vec{c} = 4$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$.	3
Sol.	$\vec{a} + \vec{b} + \vec{c} = 0 \text{ gives } \vec{a} + \vec{b} + \vec{c} ^2 = 0$ $\Rightarrow \vec{a} ^2 + \vec{b} ^2 + \vec{c} ^2 + 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a} = 0$ $\Rightarrow 9 + 25 + 16 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$ $\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -25$	1/2 1 1 1/2 3

8.	Evaluate : $\int_{-1}^2 x^3 - x dx$	3
Sol.	Here, $x^3 - x \geq 0$ on $[-1, 0]$, $x^3 - x \leq 0$ on $[0, 1]$ and $x^3 - x \geq 0$ on $(1, 2)$	$1\frac{1}{2}$
	So, $I = \int_{-1}^0 (x^3 - x)dx + \int_0^1 (x - x^3)dx + \int_1^2 (x^3 - x)dx$	$\frac{1}{2}$
	$= \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 + \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 + \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_1^2$	$\frac{1}{2}$
	$= \frac{1}{4} + \frac{1}{4} + \frac{9}{4}$ $= \frac{11}{4}$	$\frac{1}{2}$
9.	Show that the lines : $\frac{1-x}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{2y-2}{-4} = z-1$ are coplanar.	3
Sol.	Equations of given lines in standard form are $\frac{x-1}{-2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{1}$	$\frac{1}{2}$
	Lines are coplanar if $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$	$\frac{1}{2}$
	LHS = $\begin{vmatrix} 3 & -2 & 1 \\ -2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix}$ $= 3(4-2) + 2(-2+3) + 1(4-12)$ $= 6 + 2 - 8$ $= 0$ $\Rightarrow \text{Coplanar.}$	$\frac{1}{2}$
		1
		3
10.(a)	Find the particular solution of the differential equation $x \frac{dy}{dx} - y = x^2 \cdot e^x$, given $y(1) = 0$.	3

Sol.	$\frac{dy}{dx} - \frac{y}{x} = \left(\frac{x^2}{x}\right)e^x = xe^x$	1/2
	$I \cdot F = e^{\int -\frac{1}{x} dx} = e^{-\log x} = e^{\log x^{-1}} = \frac{1}{x}$	1
	Solution is $y \times \frac{1}{x} = \int e^x dx$	1/2
	$\frac{y}{x} = e^x + C$ $y = xe^x + Cx$ $C = -e$	1/2
	Particular solution is $y = xe^x - ex$	1/2
		3
	OR	
10.(b)	Find the general solution of the differential equation $x \frac{dy}{dx} = y(\log y - \log x + 1)$.	3
Sol.	$x \frac{dy}{dx} = y[\log y - \log x + 1]$	
	$\frac{dy}{dx} = \frac{y}{x} \left[\log \frac{y}{x} + 1 \right]$	1/2
	$y = vx$	
	$\frac{dy}{dx} = v + x \frac{dv}{dx}$	1/2
	$v + x \frac{dv}{dx} = v[\log v + 1]$	
	$x \frac{dv}{dx} = v \log v$	1/2
	$\int \frac{dv}{v \log v} = \int \frac{1}{x} dx$	1/2
	$\log \log v = \log x + \log C$	1/2
	$\log \left(\frac{y}{x}\right) = Cx$	1/2
		3
	SECTION—C	
11.(a)	Find : $\int \frac{x^2}{(x^2 + 1)(3x^2 + 4)} dx$	4

Sol.	$\int \frac{x^2 dx}{(x^2+1)(3x^2+4)}$ <p>Let $x^2 = t$</p> $\frac{t}{(t+1)(3t+4)} = \frac{-1}{t+1} + \frac{4}{3t+4} \quad (\text{by Partial fraction})$ $\int \frac{x^2}{(x^2+1)(3x^2+4)} dx = \int -\frac{1}{x^2+1} dx + \int \frac{4}{3x^2+4} dx$ $= -\tan^{-1} x + \frac{4}{3} \times \frac{\sqrt{3}}{2} \tan^{-1} \frac{\sqrt{3}x}{2} + C$ $= -\tan^{-1} x + \frac{2}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}x}{2} + C$	<div>1/2</div> <div>1</div> <div>1/2</div> <div>1 1/2</div> <div>1/2</div> <div>4</div>
	OR	
11.(b)	Evaluate : $\int_{-2}^1 \sqrt{5-4x-x^2} dx$	<div>4</div>
Sol.	$\int_{-2}^1 \sqrt{5-4x-x^2} dx$ $= \int_{-2}^1 \sqrt{9-(x+2)^2} dx$ $= \left[\frac{x+2}{2} \sqrt{9-(x+2)^2} + \frac{9}{2} \sin^{-1} \frac{x+2}{3} \right]_{-2}^1$ $= 0 + \frac{9}{2} \cdot \frac{\pi}{2} - 0$ $= \frac{9\pi}{4}$	<div>1 1/2</div> <div>1 1/2</div> <div>1</div> <div>4</div>
12.	Using integration, find the area of the region bounded by the curves $x^2 + y^2 = 4$, $x = \sqrt{3}y$ and x-axis lying in the first quadrant.	<div>4</div>
Sol.	<div style="text-align: center;">  </div> <p style="text-align: right;">Correct Figure</p>	<div>1</div>

	<p>For Point of intersection $x^2 + \frac{x^2}{3} = 4$</p> <p>$x^2 = 3 \Rightarrow x = \sqrt{3}$</p> <p>$I = \int_0^{\sqrt{3}} \frac{1}{\sqrt{3}} x dx + \int_{\sqrt{3}}^2 \sqrt{4-x^2} dx$</p> <p>$= \frac{1}{\sqrt{3}} \left[\frac{x^2}{2} \right]_0^{\sqrt{3}} + \left[\frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_{\sqrt{3}}^2$</p> <p>$= \frac{\sqrt{3}}{2} + 2 \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{3}$</p> <p>$= \pi - \frac{2\pi}{3} = \frac{\pi}{3}$</p>	<p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p>
		4
13.	Find the distance of the point (1, -2, 9) from the point of intersection of the line $\vec{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10$.	4
Sol.	<p>Given line is $\vec{r} = (4\hat{i} + 2\hat{j} + 7\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$</p> <p>Any point on the line is (4 + 3λ, 2 + 4λ, 7 + 2λ)</p> <p>This point lies on the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10$</p> <p>$\Rightarrow 4 + 3\lambda - 2 - 4\lambda + 7 + 2\lambda = 10$</p> <p>$\Rightarrow \lambda + 9 = 10 \Rightarrow \lambda = 1$</p> <p>$\therefore$ Point of intersection is (7, 6, 9)</p> <p>Distance of (7, 6, 9) from (1, -2, 9) is</p> <p>$d = \sqrt{(7-1)^2 + (6+2)^2 + 0}$</p> <p>$= \sqrt{36 + 64} = 10$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>
		4

14.	<p>Case Study Problem :</p> <p>A shopkeeper sells three types of flower seeds A₁, A₂, A₃. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.</p>  <p>Based on the above information :</p> <p>(a) Calculate the probability that a randomly chosen seed will germinate;</p> <p>(b) Calculate the probability that the seed is of type A₂, given that a randomly chosen seed germinates.</p>	<p>2</p> <p>2</p>
Sol.	<p> E_1 : Seed is of type A_1 E_2 : Seed is of type A_2 E_3 : Seed is of type A_3 A : Seed germinates </p> <p> $P(E_1) = \frac{4}{10}, P(E_2) = \frac{4}{10}, P(E_3) = \frac{2}{10}$ $P(A E_1) = \frac{45}{100}, P(A E_2) = \frac{60}{100}, P(A E_3) = \frac{35}{100}$ </p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
	<p>(a) $P(A) = P(E_1)P(A E_1) + P(E_2)P(A E_2) + P(E_3)P(A E_3)$</p> $= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$ $= \frac{180 + 240 + 70}{1000}$ $= \frac{490}{1000} = \frac{49}{100}$	<p>1</p> <p>$\frac{1}{2}$</p>
	<p>(b) $P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P(A E_2)}{P(A)}$</p>	

	$= \frac{\frac{4 \times 60}{10 \times 100}}{\frac{490}{1000}} = \frac{24}{49}$	$\frac{1}{2} + \frac{1}{2}$
		4

* * *