TOM

CHAPTER-1 Simple mechanisms

Types of relative motion

- Completely constraind motion [Self, desired motion]
- Successfully constraint motion [forcefully, desired motion]
- Incompletely constraint motion [unconstraint motion]

classification of Kinematic Pairs

Acc. to type of relative motion
- Turning Pair
- Sliding Pair
- Rolling Pair
- Screw Pair
- Spherical Pair

Acc. to type of motion
- Lower Pair (Surface contact)
- Higher Pair Line/Point contact
- Wrapping Pair

Acc. to type of closure
- Self closed Pair Permanent contact
- Forced closed Pair or open pair forced contact.

Degree of freedom (D.O.F)

$$F = 3(l-1) - 2j - h - F_r \quad \text{[Kutzback's equ}^n\text{]}$$

$F =$ D.O.F

$l =$ no. of links

$j =$ no. of Binary Joints

$h =$ no. of higher Pair

$F_r =$ no. of redundent link.

Grabler's equ^n $3l - 2j - 4 = 0$

here ; DOF = 1

& $h = 0$

- So minimum no. of links to form a mechanism is 4.

Simple mechanisms
- A → Four Bar mechanism
- B → Single slider crank mechanism
- C → Double slider crank mechanism

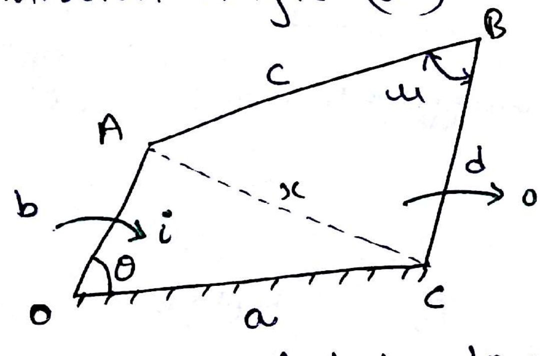
A Four Bar mechanism
- → Double crank mechanism
- → Crank rocker mechanism
- → Double rocker mechanism

] Inversions

Grashof's Law
$$s + l \leq p + q$$
- s is fixed = Double crank
- s is abjecent to fixed = Crank rocker
- s coupler = double rocker

If $s + l > p + q$ = Double rocker

Transmission angle (μ)



μ = mini at $\theta = 0°$

μ = max. at $\theta = 180°$

cosine rule of triangle ⇒ $x^2 = a^2 + b^2 - 2ab\cos\theta$

B Single slider crank mechanism
- → Cylinder fixed
  - compressor
  - engine
- → Crank fixed
  - → whitworth QRM
  - → Rotary IC engine [Gnome Engine]
- → Connecting Rod fixed
  - Crank & sloted lever QRM
  - Oscillatory cylinder engine
- → Piston/slider fixed
  - → Hand pump
  - → Pendulum pump

Slotted lever / crank & slotted lever QRMM

Stroke length = $\frac{2 \times (\text{length of slotted bar})(\text{crank length})}{\text{length of connecting rod.}}$

QRR = $\frac{\text{cutting time}}{\text{Return time}} = \frac{\beta}{\alpha}$

Whitworth ⇒ stroke = $2 \times (OC)$

QRR = β/α

Double slider crank mechanism

- → Slotted bar fixed • Elliptical Trammels
- → one slider fixed • Scotch yoke mechanism
- → Connecting link fixed • oldham's coupling

mechanical Advantage = MA = $\frac{F_{out}}{F_{in}} = \frac{T_{out}}{T_{in}} = \frac{V_{in}}{V_{out}} \cdot \eta_{mech.} = \frac{W_{in}}{W_{out}} \cdot \eta_{mech}$

• MA = ∞ at Toggle position

CHAPTER : MOTION ANALYSIS

no. of Instantenous center = ${}^{l}C_2 = \cancel{\frac{l(l-1)}{2}} = \frac{l(l-1)}{2}$

Kennedy's Theorem

• For relative motion b/w no. of links in a mechanism any three links, their I center must lie on a straight line

Theorem of Angular Velocity

$$\omega_m (I_{mn} I_{1m}) = \omega_n (I_{mn} I_{1n})$$

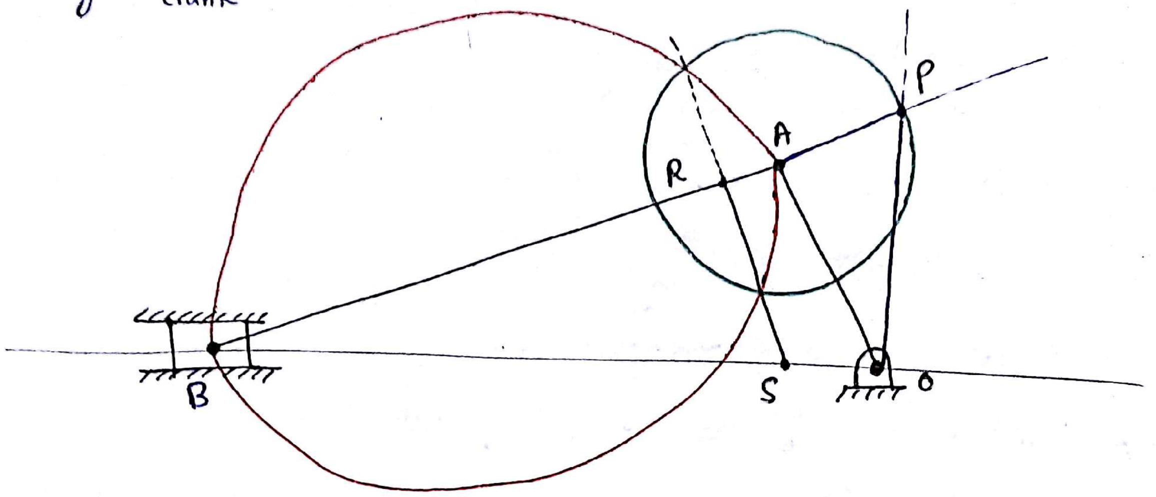
• If I_{1n} & I_{1m} both are on same side of I_{mn} then sense of rotation is same for ω_m & ω_n.

a^r = radial acceleration = $r\omega^2$ [always associated with circular motion]

a^t = tangential acceleration = $r\alpha$
⇓
comes into picture only in non-uniform circular motions.

Klein's Construction

• If $\alpha_{crank} = 0$



1. $\frac{V_A}{OA} = \frac{V_B}{OP} = \frac{V_{BA}}{AP} = \omega_{crank}$

2. $\frac{a_A}{OA} = \frac{a_B}{OS} = \frac{a^t_{BA}}{RS} = \frac{a^r_{BA}}{AR} = \omega^2_{crank}$

Coriolis Acceleration (a^c)

$a^c = 2v\omega$

v = velocity of slider

ω = angular velocity of link on which slider is sliding

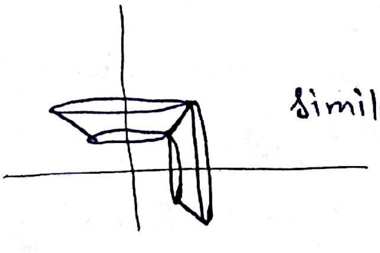
dirⁿ = Rotate the $\vec{v}$ in same sense as of $\vec{\omega}$

CHAPTER : GEARS

Positive drive → slip is not possible eg:- gears

Negative drive → slip is possible eg:- Belt, rope, chain

Mitre Gear



Similar gears at 90°

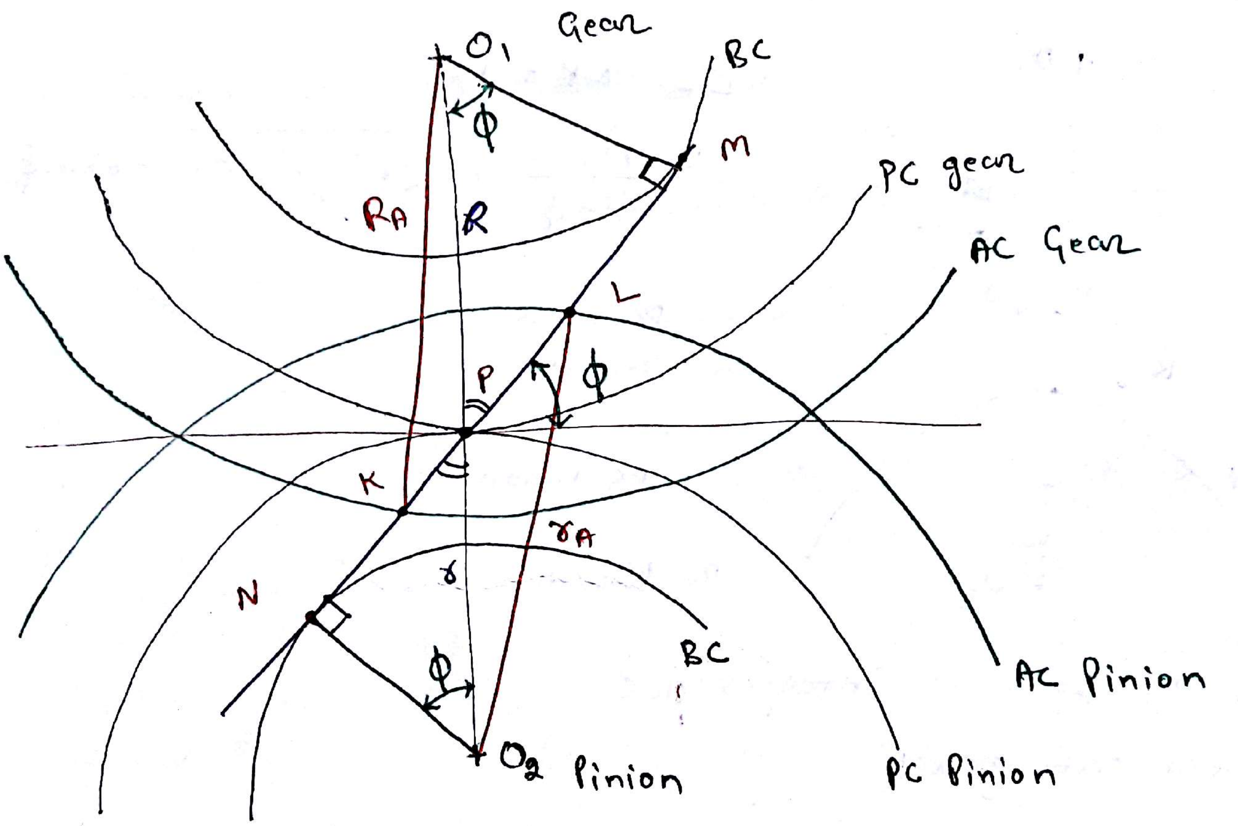
$\frac{\omega_1}{\omega_2} = 1$

• Circular Pitch = $P_c = \frac{\pi D}{T}$

• module = $m = \frac{D}{T}$ [module is same for mating gears]

• Diametrical Pitch = $P_d = \frac{T}{D}$

* $P_d \cdot P_c = \frac{T}{D} \cdot \frac{\pi D}{T} = \pi$
* working depth = Addendum + Dedendum − Clearance
  = Sum of addendum of both mating gears
* Tooth Space − Tooth thickness of mating gear = Backlash
* Law of gearing: line of action must always pass through the fixed point (pitch point) on the line joining the centers of rotation of gears.
* Sliding velocity = $V_s = (\omega_1 + \omega_2)\, QP$
* Involute Gear



Path of contact = Path of Approach + Path of recess

KL = KP + PL

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\phi} = \frac{KL}{\cos\phi}$$

- Angle turned by Pinion $= \frac{\text{Arc of contact}}{r}$ radian
- Angle turned by Gear $= \frac{\text{Arc of contact}}{R}$ radian

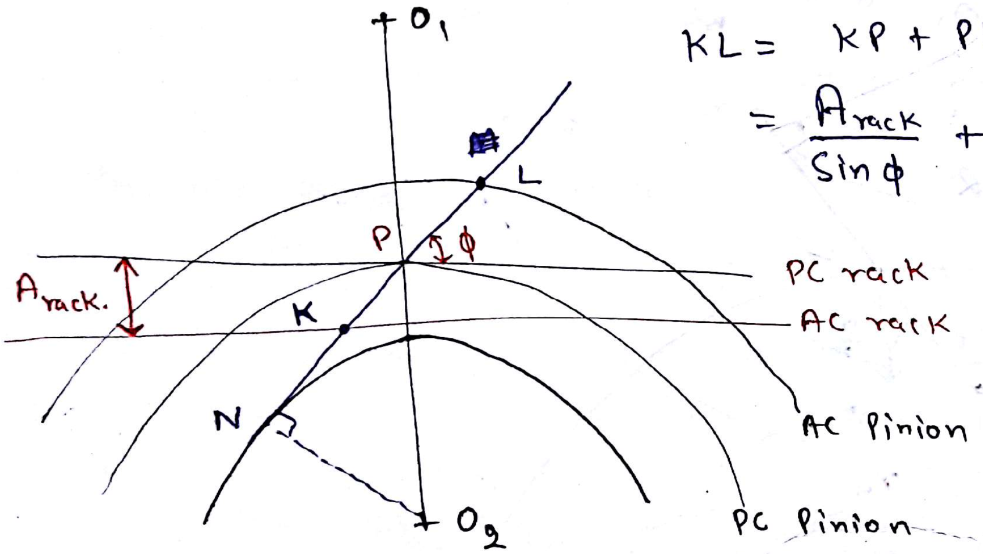
[In one engagement period.

* Contact Ratio $= \frac{\text{Arc of contact}}{\text{Circular Pitch } (P_c)} \geq 1$

* Gear Ratio $= G = \frac{T}{t}$; Velocity ratio $\Rightarrow \frac{\omega_p}{\omega_g} = \frac{T}{t} > 1$

$\Rightarrow \frac{\omega_g}{\omega_p} = \frac{t}{T} < 1$

* Path of contact in Rack & Pinion



$KL = KP + PL$

$= \frac{A_{rack}}{\sin\phi} + \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

* methods to prevent interference
- Under cut gears
- increase ϕ
- By stubbing the teeth
- Increasing no. of teeth.

* $T_{min} = \frac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$; $t_{min} = \frac{2A_p}{\sqrt{1+G(G+2)\sin^2\phi} - 1}$

Gear // T_{min} ; Pinion // t_{min}

- Always make gear safe 1st if $A_g = A_p$

* For rack & pinion ; $t_{min} = \dfrac{2A_R}{\sin^2\phi}$

In all, A_g, A_p, A_R] these are fractional addendum

i.e. $mA_p = A$ of pinion

$mA_g = A$ of gear

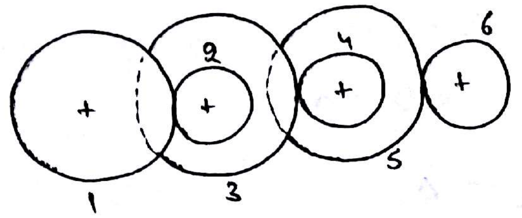
$mA_R = A$ of rack

CHAPTER : GEAR TRAIN

Speed Ratio = S.R. = $\dfrac{\omega_1}{\omega_2} = \dfrac{\omega_{Driver}}{\omega_{Driven}}$

Train value = $\dfrac{1}{S.R.}$

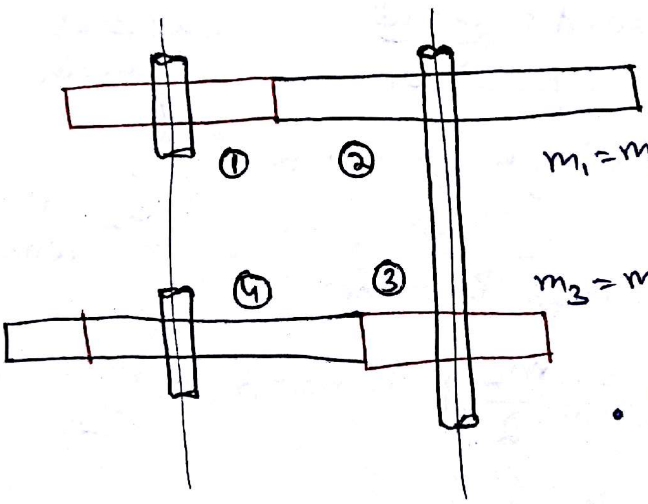
Compound gear train



$$S.R. = \frac{\omega_1}{\omega_6} = \frac{T_2 \cdot T_4 \cdot T_6}{T_1 \cdot T_3 \cdot T_5}$$

Reverted Gear Train.

$$S.R. = \frac{\omega_1}{\omega_4} = \frac{T_2 \cdot T_4}{T_1 \cdot T_3}$$



$m_1 = m_2 = m$

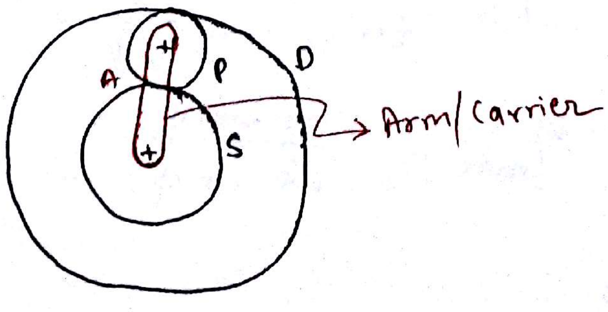
$m_3 = m_4 = m'$

$$r_1 + r_2 = r_3 + r_4$$

$$m\left(\frac{t_1}{\cancel{2}} + \frac{t_2}{\cancel{2}}\right) = m'\left(\frac{t_3}{\cancel{2}} + \frac{t_4}{\cancel{2}}\right)$$

- If speed reduction is same then, $\dfrac{T_2}{T_1} = \dfrac{T_4}{T_3}$

Planetary Gear train [epi-cyclic]



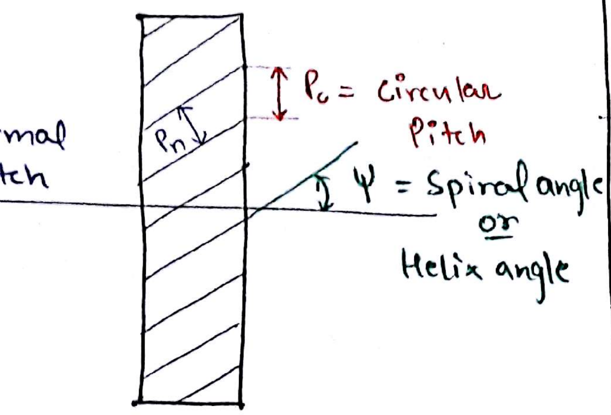
$$r_S + 2r_P = r_D$$

$$T_S + 2T_P = T_D$$

$T_{input} + T_{output} + T_{Fixing} = 0$

$\eta_{GT} \cdot (T_{in} \cdot \omega_{in}) + T_{out} \cdot \omega_{out} = 0$

<u>Helical & Spiral Gear</u>



$P_c \cos(\psi) = P_n$

$m \cos(\psi) = m_n$

$m_{n_1} = m_{n_2}$ for mating gears

→ $\theta = \psi_1 + \psi_2$ for same hand / if not given

→ $\theta = \psi_1 - \psi_2$ oppo. hand

→ Velocity Ratio ⇒ $VR = \frac{T_1}{T_2} = \frac{\omega_2}{\omega_1}$

→ Center Distance

$$= \frac{m_n}{2}\left[\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right]$$

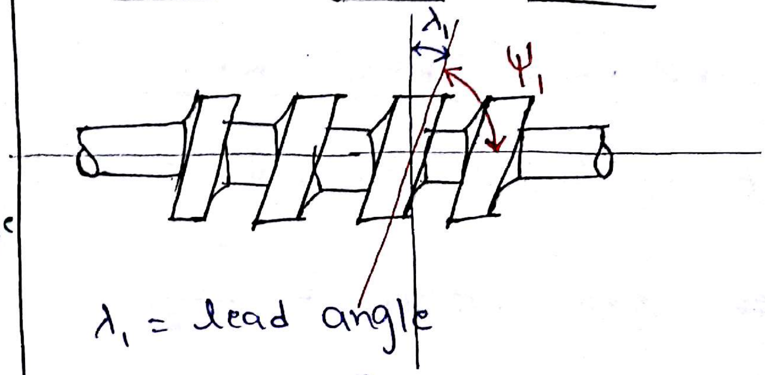
→ Efficiency

$$\eta = \frac{\cos(\psi_2 + \phi) \cdot \cos\psi_1}{\cos(\psi_2 - \phi) \cdot \cos\psi_2}$$

$$\eta_{max} = \frac{1 + \cos(\theta + \phi)}{1 - \cos(\theta - \phi)}$$

$\theta = \psi_1 + \psi_2$; $\psi_1 = \frac{\theta + \phi}{2}$; $\psi_2 = \frac{\theta - \phi}{2}$

<u>worm & worm wheel</u>



λ_1 = lead angle

$\lambda_1 + \psi_1 = 90°$

here $\theta = 90°$

- $\psi_1 = 90 - \lambda_1$
- $\psi_2 = \lambda_1$
- $\phi = \tan^{-1}(\mu)$ [∵ μ = coefficient of friction
- $\tan\lambda = \frac{lead}{\pi d_1}$

→ Velocity Ratio ⇒ $VR = \frac{l}{\pi d_2}$

(l ⇒ lead of worm; d_2 ⇒ dia of wheel)

→ Center Distance

$$= \frac{m_2}{2}\left[T_1 \cot\lambda_1 + T_2\right]$$

→ Efficiency

$$\eta = \frac{\tan(\lambda_1)}{\tan(\lambda_1 + \phi)}$$

$$\eta_{max} = \frac{1 - \sin\phi}{1 + \sin\phi}$$

CHAPTER : MECHANICAL VIBRATIONS

(A) Natural Vib.
- mass
- stiffness

Ideal system

(B) Damped Vib.
- mass
- stiffness
- friction (Damping)

non-running m/c

(C) Unbalanced force Vib.
- mass
- stiffness
- friction (Damping)
- $F_{unbal.}$

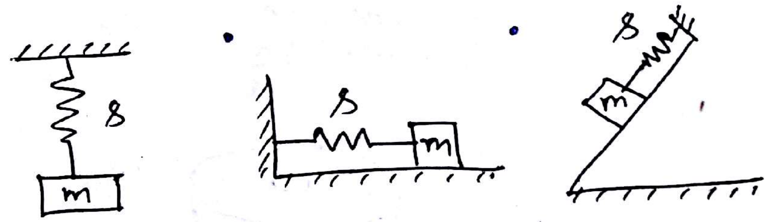
Running m/c

A Natural Vibrations

$$\ddot{x} + \left(\frac{s}{m}\right) x = 0$$

$$\omega_n = \sqrt{\frac{s}{m}} \text{ rad/s} \quad ; \quad f_n = \frac{\omega_n}{2\pi}$$

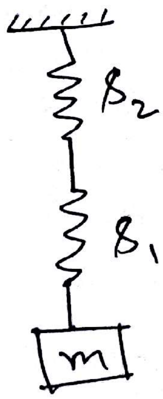
$$T_n = \frac{2\pi}{\omega_n} \quad ; \quad \ddot{x} + (\omega_n)^2 x = 0$$



In all case

$$\omega_n = \sqrt{\frac{s}{m}} \text{ rad/s}$$

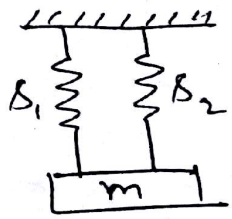
- Springs in Series



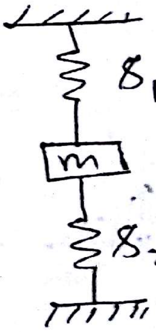
$$\frac{1}{s} = \frac{1}{s_1} + \frac{1}{s_2}$$

$$\omega_n = \sqrt{\frac{s}{m}} \text{ rad/s}$$

- Springs in parallel



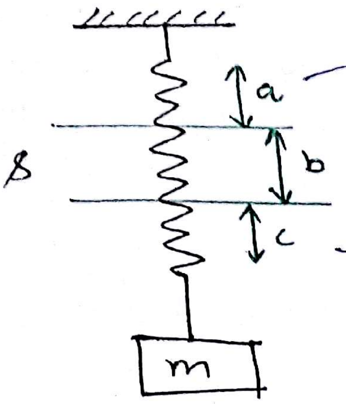
or



$$s = s_1 + s_2$$

$$\omega_n = \sqrt{\frac{s}{m}} \text{ rad/s}$$

* Cutting of springs:



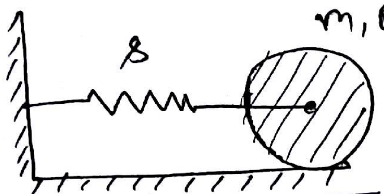
$$\left(\frac{a+b+c}{a}\right) \times s$$

$$\left(\frac{a+b+c}{b}\right) \times s$$

$$\left(\frac{a+b+c}{c}\right) \times s$$

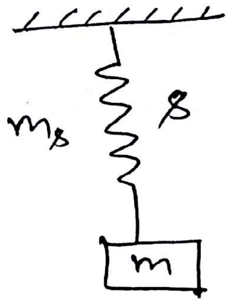
Special cases

1.



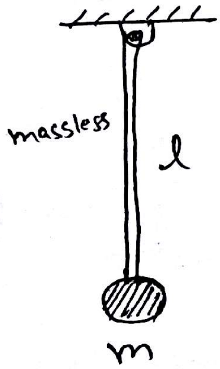
$$\omega_n = \sqrt{\frac{2s}{3m}}$$

2.



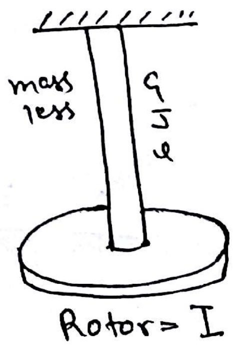
$$\omega_n = \sqrt{\frac{s}{m + \frac{m_s}{3}}}$$

3.



$$\omega_n = \sqrt{\frac{g}{l}}$$

4.



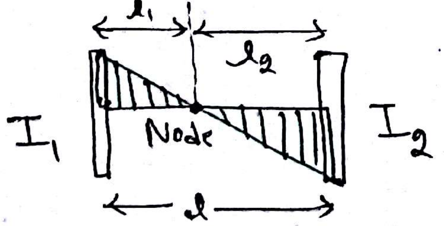
$$\omega_n = \sqrt{\frac{s_t}{I}}$$

$$\omega_n = \sqrt{\frac{s}{I + \frac{I_r}{3}}}$$

⇓

if mass of rod is considered

Two rotor system



$$l_1 + l_2 = l$$

$$l_1 I_1 = l_2 I_2$$

- no. of rotor = n

  then no. of node points = n-1

Rayleigh's method

- If Δ is the static deflection of mass in a spring mass system, then

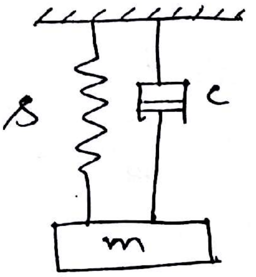
$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{g}{\Delta}}$$

B Damped System

→ Friction b/w dry surfaces [coulomb's damping] very high

→ Viscous Damping [very less]

- C = Coefficient of damping
- ζ = damping factor / damping ratio



$$m\ddot{x} + c\dot{x} + sx = 0$$

$$\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + \left(\frac{s}{m}\right)x = 0$$

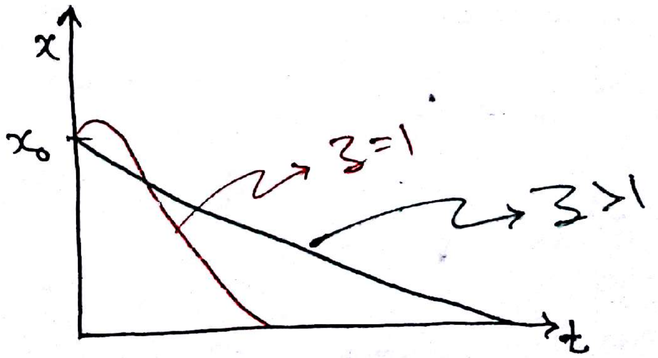
$$\ddot{x}\ (2\zeta\omega_n)\dot{x} + (\omega_n)^2 = 0$$

- $2\zeta\omega_n = \frac{c}{m}$

- $\alpha_{1,2} = \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right).\omega_n$

- $\zeta > 1 \Rightarrow$ over damping] no. vib.
  $\zeta = 1 \Rightarrow$ Critical damping]
  $\zeta < 1 \Rightarrow$ underdamping — vib.

- Critical damping Response is much fast than over Damping

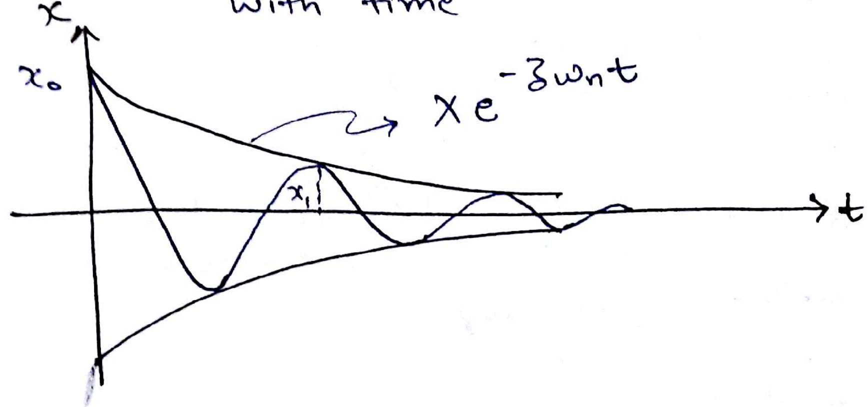


Underdamping ($\zeta < 1$)

$$x = \underbrace{X e^{-\zeta \omega_n t}}_{\Downarrow} \cdot \left[\sin \left((\omega_d t) + \phi \right) \right]$$

Amplitude decrease with time

$\sin(\omega_d t)$ → Vib. with frequency ω_d.



- Decrement ratio $= \frac{x_o}{x_1} = \frac{x_1}{x_2} = \frac{x_2}{x_3} ---- = e^{\zeta \omega_n T_d}$ = const.

- logrithamic decrement $= \delta = \zeta \omega_n T_d$

$$\delta = \frac{2\pi \zeta}{\sqrt{1-\zeta^2}}$$

- $\omega_d = \sqrt{1-\zeta^2} \times \omega_n$

- Critical damping coefficient (C_c) $\Rightarrow \zeta = \frac{c}{C_c}$

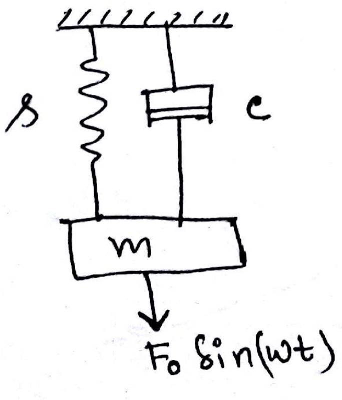
$$2\zeta \omega_n = \frac{c}{m} \Rightarrow C_c = 2\omega_n m$$

Vibrations Caused by F_{un}.

$$F_{un} = F_o \sin(\omega t)$$

$F_o = m_{rotor} \cdot e \cdot \omega^2$ if rotating rotor is given

$= m_{reci.} \cdot r \cdot \omega^2$ if slider crank is given



$$m\ddot{x} + c\dot{x} + sx - F_o \sin \omega t = 0$$

$$\ddot{x} + (2\zeta \omega_n)\dot{x} + (\omega_n)^2 x = F_o \sin(\omega t)$$

F_o = max value of unbalanced force

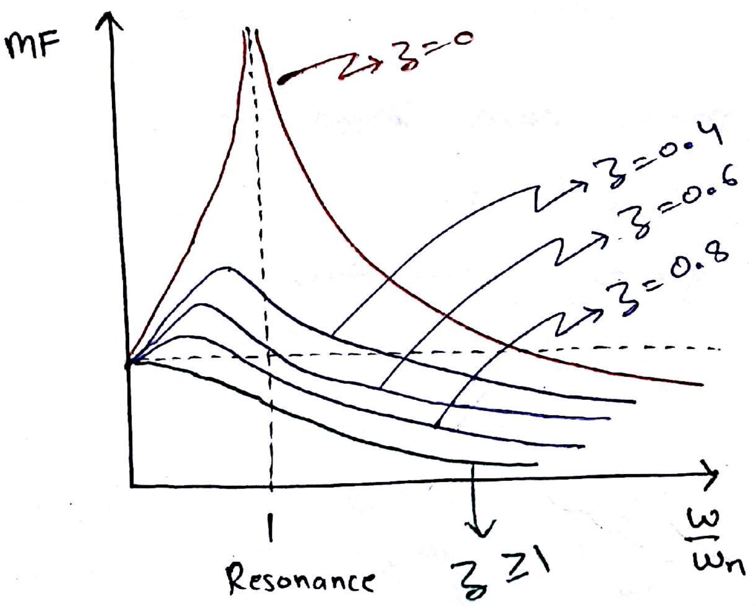
ω = forced frequency.

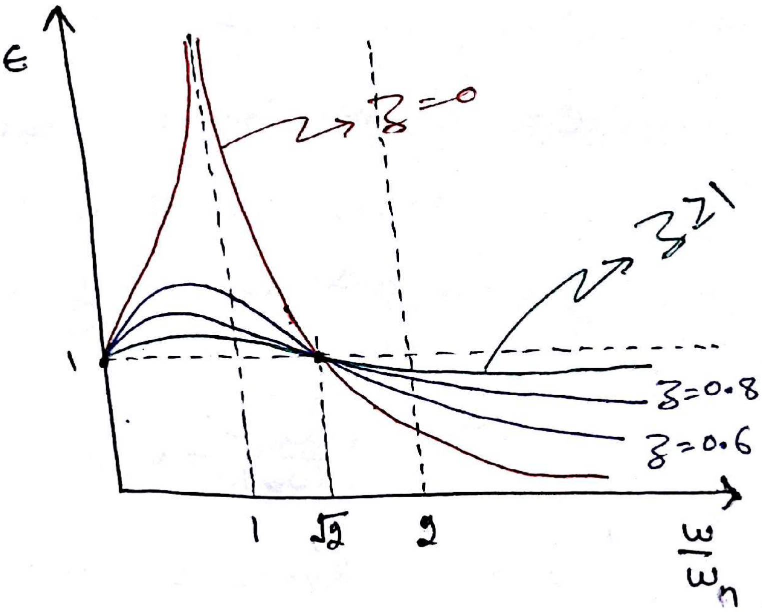
- $x = \dfrac{F_0/s}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}} \cdot \sin[(\omega t) - \phi]$

- $x = A \sin(\omega t - \phi)$

- A = Amplitude $= \dfrac{F_0/s}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$.

- Magnification factor $= MA = \dfrac{A}{F_0/s} = \dfrac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$





- A_{max} at

  $\frac{\omega}{\omega_n} < 1$ for underdamping

  $\frac{\omega}{\omega_n} = 1$ no damping

  $\frac{\omega}{\omega_n} = 0$ critical/over damping

- $A_{Reso.} = \dfrac{F_0/s}{2\zeta}$

Transmissibility (ϵ)

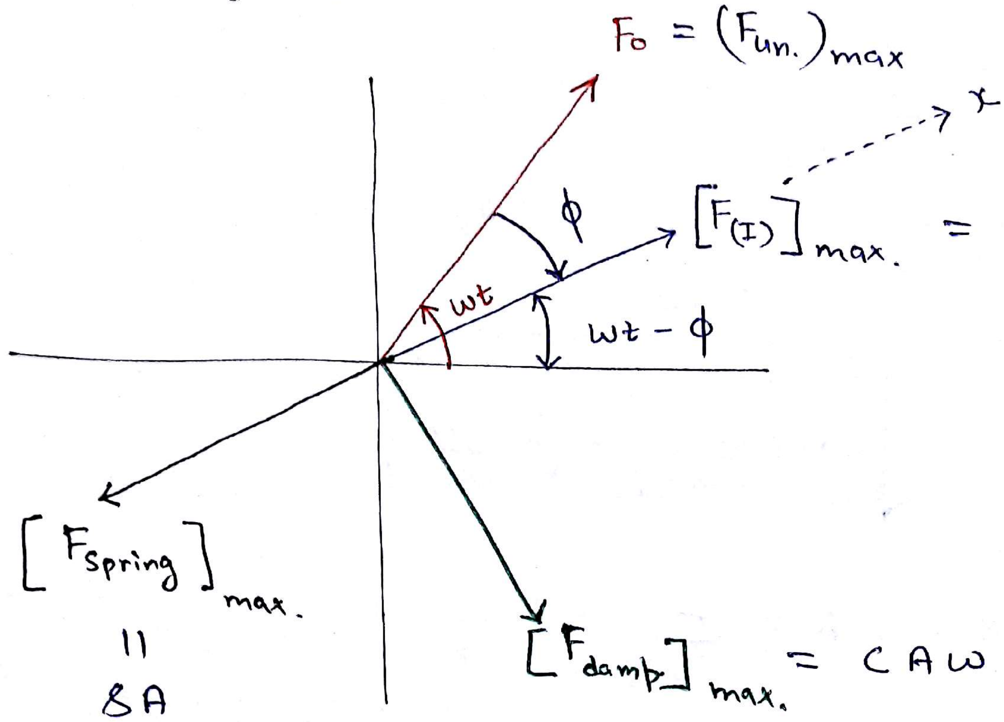
$$\epsilon = \frac{\sqrt{1 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

- $\epsilon = 1$ when $\frac{\omega}{\omega_n} = \sqrt{2}, 0$

- $\epsilon < 1$ when $\frac{\omega}{\omega_n} > \sqrt{2}$

Phase diagram

$x = A \sin(\omega t - \phi)$



$$\boxed{\tan\phi = \frac{2\zeta \frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}}$$

Whirling of shaft

e = eccentricity of rotor mass from shaft Axis

y = Transverse displacement of shaft.

$y + e$ = net offset of cm of rotor from Axis

$$y = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}$$

whirling speed $= \omega = \omega_n = \sqrt{\frac{s}{m}}$ rad/s