Chapter 4 Matrices and Determinants

Ex 4.8

Answer 1e.

In the quadratic formula, the expression $b^2 - 4ac$ is called the discriminant of the associated equation $ax^2 + bx + c = 0$. The number and type of solutions depend on the discriminant of the quadratic equation.
Therefore, the given statement can be completed as
"You can use the discriminant of a quadratic equation to determine the equation's number and type of solutions."

Answer 1gp.

Write the equation in standard form before applying the quadratic formula.
Subtract $6x - 4$ from both the sides.

$$x^2 - 6x + 4 = 6x - 4 - 6x + 4$$

$$x^2 - 6x + 4 = 0$$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$, where a, b, c are real numbers and $a \neq 0$.

Substitute 1 for a, -6 for b, and 4 for c in the formula.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

Evaluate.

$$\begin{aligned} x &= \frac{6 \pm \sqrt{36 - 16}}{2} \\ &= \frac{6 \pm \sqrt{20}}{2} \\ &= \frac{6 \pm 2\sqrt{5}}{2} \\ &= 3 \pm \sqrt{5} \end{aligned}$$

Therefore, the solutions are $3 - \sqrt{5}$ and $3 + \sqrt{5}$.
The solutions can be checked using a graphing utility.

Answer 2e.

When an object is dropped, its height h above the ground after t seconds can be modeled by the function

$$h = -16t^2 + h_0$$

where h_0 is the object's initial height.

For an object that is launched or thrown, an extra term $v_0 t$ must be added to the model $h = -16t^2 + h_0$ to account for the object's initial velocity $v_0 t$.

So, for an object being launched or thrown the model can be defined by the function

$$h = -16t^2 + v_0 t + h_0$$

For example, a batsman in the cricket ground hits a ball into the air.

In this case, we do not use the model $h = -16t^2 + h_0$ but the model $h = -16t^2 + v_0 t + h_0$.

Answer 2gp.

We need to solve the following quadratic equation by using quadratic formula.

$$4x^2 - 10x = 2x - 9$$

$$\Rightarrow \quad 4x^2 - 12x + 9 = 0$$

By comparing with $ax^2 + bx + c = 0$. We get,

$a = 4, b = -12, c = 9$

Therefore the roots are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow \quad x = \frac{12 \pm \sqrt{144 - 4(4)(9)}}{2(4)}$$

$$\Rightarrow \quad x = \frac{12 \pm \sqrt{0}}{8}$$

$$\Rightarrow \quad x = \frac{12}{8}$$

$$\Rightarrow \quad x = \boxed{\frac{3}{2}, \frac{3}{2}}$$

Answer 3e.

The given equation is in standard form.
The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$, where a, b, c are real numbers and $a \neq 0$.

Substitute 1 for a, −4 for b, and −5 for c in the formula.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)}$$

Evaluate.

$$\begin{aligned} x &= \frac{4 \pm \sqrt{16 + 20}}{2} \\ &= \frac{4 \pm \sqrt{36}}{2} \\ &= \frac{4 \pm 6}{2} \\ &= \frac{10}{2} \text{ or } \frac{-2}{2} \\ &= 5 \text{ or } -1 \end{aligned}$$

Therefore, the solutions are 5 and −1.
The solutions can be checked using a graphing utility.

Answer 3gp.

Write the equation in standard form before applying the quadratic formula.
Subtract $2x + 3$ from both the sides.

$$\begin{aligned} 7x - 5x^2 - 4 - 2x - 3 &= 2x + 3 - 2x - 3 \\ -5x^2 + 5x - 7 &= 0 \end{aligned}$$

Divide both the sides by −1.

$$\begin{aligned} \frac{-5x^2 + 5x - 7}{-1} &= \frac{0}{-1} \\ 5x^2 - 5x + 7 &= 0 \end{aligned}$$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$, where a, b, c are real numbers and $a \neq 0$.

Substitute 5 for a, -5 for b, and 7 for c in the formula.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(5)(7)}}{2(5)}$$

Evaluate.

$$x = \frac{5 \pm \sqrt{25 - 140}}{10}$$

$$= \frac{5 \pm \sqrt{-115}}{10}$$

Use the imaginary unit to rewrite.

$$s = \frac{5 \pm i\sqrt{115}}{10}$$

Therefore, the solutions are $\frac{5 - i\sqrt{115}}{10}$ and $\frac{5 + i\sqrt{115}}{10}$.

The solutions can be checked using a graphing utility.

Answer 4e.

We need to solve the following quadratic equation by using quadratic formula.

$x^2 - 6x + 7 = 0$

By comparing with $ax^2 + bx + c = 0$. We get,

$a = 1, b = -6, c = 7$

Therefore the roots are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow \quad x = \frac{6 \pm \sqrt{36 - 28}}{2}$$

$$\Rightarrow \quad x = \frac{6 \pm \sqrt{8}}{2}$$

$$\Rightarrow \quad x = \frac{6 \pm 2\sqrt{2}}{2}$$

$$\Rightarrow \quad x = \boxed{3 \pm \sqrt{2}}$$

Answer 4gp.

We need to find the number and type of solutions of the following equation.

$2x^2+4x-4=0$

By comparing with $ax^2+bx+c=0$. We get,

$a=2, b=4, c=-4$

Then the deiscriminant is $D=b^2-4ac$

$$\Rightarrow \quad D = 4^2-4(2)(-4)$$
$$= 16+32$$
$$= 48$$
$$> 0$$

Since $D>0$, the given quadratic equation will have two real roots.

Answer 5e.

The given equation is in standard form.

The solutions of a quadratic equation of the form $ax^2+bx+c=0$ are

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 1 for a, 8 for b, 19 for c, and t for x in the formula.

$$t = \frac{-8 \pm \sqrt{8^2-4(1)(19)}}{2(1)}$$

Evaluate.

$$t = \frac{-8 \pm \sqrt{64-76}}{2}$$
$$= \frac{-8 \pm \sqrt{-12}}{2}$$

Use the imaginary unit to rewrite.

$$t = \frac{-8 \pm i\sqrt{12}}{2}$$
$$= \frac{-8 \pm 2\sqrt{3}i}{2}$$
$$= -4 \pm \sqrt{3}i$$

Therefore, the solutions are $-4-\sqrt{3}i$ and $-4+\sqrt{3}i$.

The solutions can be checked using a graphing utility.

Answer 5gp.

In the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression $b^2 - 4ac$ is called the discriminant.

Substitute 3 for a, 12 for b, and 12 for c in $b^2 - 4ac$ and evaluate.

$$\begin{aligned} b^2 - 4ac &= 12^2 - 4(3)(12) \\ &= 144 - 144 \\ &= 0 \end{aligned}$$

The discriminant of the given quadratic equation is 0.
Therefore, the equation has one real solution.

Answer 6e.

We need to solve the following quadratic equation by using quadratic formula.

$x^2 - 16x + 7 = 0$

By comparing with $ax^2 + bx + c = 0$. We get,

$a = 1, b = -16, c = 7$

Therefore the roots are $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \qquad x = \dfrac{16 \pm \sqrt{256 - 28}}{2}$

$\Rightarrow \qquad x = \dfrac{16 \pm \sqrt{228}}{2}$

$\Rightarrow \qquad x = \dfrac{16 \pm 2\sqrt{57}}{2}$

$\Rightarrow \qquad x = \boxed{8 \pm \sqrt{57}}$

Answer 6gp.

We need to find the number and type of solutions of the following equation.

$8x^2 = 9x - 11$

$\Rightarrow \qquad 8x^2 - 9x + 11 = 0$

By comparing with $ax^2 + bx + c = 0$. We get,

$a = 8, b = -9, c = 11$

Then the deiscriminant is $D = b^2 - 4ac$

$$\begin{aligned} \Rightarrow \qquad D &= (-9)^2 - 4(8)(11) \\ &= 81 - 352 \\ &= -271 \\ &< 0 \end{aligned}$$

Since $D < 0$, the given quadratic equation will have two imaginary roots.

Answer 7e.

The given equation is in standard form.
The solutions of a quadratic equation of the form $ax^2+bx+c=0$ are

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$, where a, b, c are real numbers and $a \neq 0$.

Substitute 8 for a, -8 for b, 2 for c, and w for x in the formula.

$$w = \frac{-(-8) \pm \sqrt{(-8)^2-4(8)(2)}}{2(8)}$$

Evaluate.

$$\begin{aligned} w &= \frac{8 \pm \sqrt{64-64}}{16} \\ &= \frac{8 \pm \sqrt{0}}{16} \\ &= \frac{8}{16} \\ &= \frac{1}{2} \end{aligned}$$

Therefore, the solution is $\frac{1}{2}$.

The solution can be checked using a graphing utility.

Answer 7gp.

Write the equation is standard form.
Subtract 5 from both the sides.

$$7x^2-2x-5 = 5-5$$

$$7x^2-2x-5 = 0$$

In the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, the expression b^2-4ac is called the discriminant.

Substitute 7 for a, -2 for b, and -5 for c in b^2-4ac and evaluate.

$$\begin{aligned} b^2-4ac &= (-2)^2-4(7)(-5) \\ &= 4+140 \\ &= 144 \end{aligned}$$

The discriminant of the given quadratic equation is greater than 0.
Therefore, the equation has two real solutions.

Answer 8e.

We need to solve the following quadratic equation by using quadratic formula.

$$5p^2-10p+24=0$$

By comparing with $ax^2+bx+c=0$. We get,

$a=5, b=-10, c=24$

Therefore the roots are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$p = \frac{10 \pm \sqrt{100 - 480}}{10}$$

$$\Rightarrow \quad p = \frac{10 \pm \sqrt{-380}}{10}$$

$$\Rightarrow \quad p = \frac{10 \pm 2\sqrt{95}i}{10}$$

$$\Rightarrow \quad p = \boxed{1 \pm \frac{1}{5}\sqrt{95}i}$$

Answer 8gp.

We need to find the number and type of solutions of the following equation.

$4x^2 + 3x + 12 = 3 - 3x$

$\Rightarrow \quad 4x^2 + 6x + 9 = 0$

By comparing with $ax^2 + bx + c = 0$. We get,

$a = 4, b = 6, c = 9$

Then the deiscriminant is $D = b^2 - 4ac$

$$\begin{aligned} \Rightarrow \quad D &= 6^2 - 4(4)(9) \\ &= 36 - 144 \\ &= -108 \\ &< 0 \end{aligned}$$

Since $D < 0$, the given quadratic equation will have two imaginary roots.

Answer 9e.

The given equation is in standard form.

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 4 for a, -8 for b, and 1 for c in the formula.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(1)}}{2(4)}$$

Simplify.

$$x = \frac{8 \pm \sqrt{64 - 16}}{8}$$
$$= \frac{8 \pm \sqrt{48}}{8}$$
$$= \frac{8 \pm 4\sqrt{3}}{8}$$
$$= \frac{2 \pm \sqrt{3}}{2}$$

Therefore, the solutions are $\frac{2 - \sqrt{3}}{2}$ and $\frac{2 + \sqrt{3}}{2}$.

The solutions can be checked using a graphing utility.

Answer 9gp.

Write the equation is standard form.

Subtract $6 - 7x$ from both the sides.

$$3x - 5x^2 + 1 - 6 + 7x = 6 - 7x - 6 + 7x$$
$$-5x^2 + 10x - 5 = 0$$

Divide both the sides by -1.

$$\frac{-5x^2 + 10x - 5}{-1} = \frac{0}{-1}$$
$$5x^2 - 10x + 5 = 0$$

In the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression $b^2 - 4ac$ is called the discriminant.

Substitute 5 for a, -10 for b, and 5 for c in $b^2 - 4ac$ and evaluate.

$$b^2 - 4ac = (-10)^2 - 4(5)(5)$$
$$= 100 - 100$$
$$= 0$$

The discriminant of the given quadratic equation is 0.

Therefore, the equation has one real solution.

Answer 10e.

We need to solve the following quadratic equation by using quadratic formula.

$6u^2+4u+11=0$

By comparing with $ax^2+bx+c=0$. We get,

$a=6, b=4, c=11$

Therefore the roots are $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\Rightarrow \quad u=\frac{-4\pm\sqrt{16-264}}{12}$

$\Rightarrow \quad u=\frac{-4\pm\sqrt{-248}}{12}$

$\Rightarrow \quad u=\frac{-4\pm 2\sqrt{62}i}{12}$

$\Rightarrow \quad u=\boxed{\frac{-2\pm\sqrt{62}i}{6}}$

Answer 10gp.

A juggler tosses a ball into air. The ball leaves the juggler's hand 4 feet above the ground and has an initial velocity of 50 feet per second.

The juggler catches the ball when it falls back to a height of 3 feet.

We need to find how long is the ball in the air.

Consider the equation $h=-16t^2+v_0t+h_0$

Where $h=3, v_0=50, h_0=4$

$\Rightarrow \quad 3=-16t^2+50t+4$

$\Rightarrow \quad 16t^2-50t-1=0$

$\Rightarrow \quad t=\frac{-(-50)\pm\sqrt{(-50)^2-4(16)(-1)}}{2(16)}$

$\Rightarrow \quad t=\frac{50\pm\sqrt{2564}}{32}$

$\Rightarrow \quad t=\frac{25\pm\sqrt{641}}{16}$

$\Rightarrow \quad t=\frac{25+\sqrt{641}}{16}$ (Since $t \not< 0$)

$\Rightarrow \quad t\approx\boxed{3.14}$

Answer 11e.

The given equation is in standard form.
The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$, where a, b, c are real numbers and $a \neq 0$.

Substitute 3 for a, -8 for b, -9 for c, and r for x in the formula.

$$r = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-9)}}{2(3)}$$

Simplify.

$$\begin{aligned} r &= \frac{8 \pm \sqrt{64 + 108}}{6} \\ &= \frac{8 \pm \sqrt{172}}{6} \\ &= \frac{8 \pm 2\sqrt{43}}{6} \\ &= \frac{4 \pm \sqrt{43}}{3} \end{aligned}$$

Therefore, the solutions are $\frac{4 - \sqrt{43}}{3}$ and $\frac{4 + \sqrt{43}}{3}$.

The solutions can be checked using a graphing utility.

Answer 12e.

We need to solve the following quadratic equation by using quadratic formula.

$2x^2 - 16x + 50 = 0$

$\Rightarrow \quad x^2 - 8x + 25 = 0$

By comparing with $ax^2 + bx + c = 0$. We get,

$a = 1, b = -8, c = 25$

Therefore the roots are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \frac{8 \pm \sqrt{64 - 100}}{2}$

$\Rightarrow \quad x = \frac{8 \pm \sqrt{-36}}{2}$

$\Rightarrow \quad x = \frac{8 \pm 6i}{2}$

$\Rightarrow \quad x = \boxed{4 \pm 3i}$

The answer is option (A).

Answer 13e.

Write the equation in standard form before applying the quadratic formula.
$3w^2 - 12w + 12 = 0$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 3 for a, -12 for b, 12 for c, and w for x in the formula.

$$w = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(12)}}{2(3)}$$

Evaluate.

$$\begin{aligned} w &= \frac{12 \pm \sqrt{144 - 144}}{6} \\ &= \frac{12 \pm \sqrt{0}}{6} \\ &= 2 \end{aligned}$$

Therefore, the solution is 2.
The solution can be checked using a graphing utility.

Answer 14e.

Solve the following quadratic equation by using quadratic formula.

$x^2 + 6x = -15$ Write original equation.

$\Rightarrow \quad x^2 + 6x + 15 = 0$ Write in standard form

By comparing with $ax^2 + bx + c = 0$. We get,
$a = 1, b = 6, c = 15$

Therefore the roots are

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic formula

$\Rightarrow \quad x = \frac{-6 \pm \sqrt{36 - 60}}{2}$ $a = 1, b = 6, c = 15$

$\Rightarrow \quad x = \frac{-6 \pm \sqrt{-24}}{2}$ Simplify

$\Rightarrow \quad x = \frac{-6 \pm 2\sqrt{6}i}{2}$

$\Rightarrow \quad x = -3 \pm \sqrt{6}i$

The solutions are $\boxed{-3 + \sqrt{6}i}$ and $\boxed{-3 - \sqrt{6}i}$

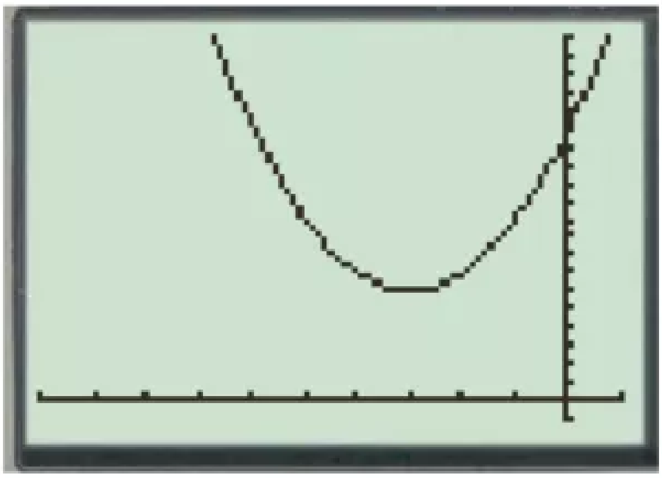
Check: Graph $y = x^2 + 6x + 15$. There are no x-intercepts. So the original equation has no real solutions. The algebraic check for the imaginary solution $-3 + \sqrt{6}i$ is shown.

$$\left(-3+\sqrt{6}i\right)^2 + 6\left(-3+\sqrt{6}i\right) \overset{?}{=} -15$$

$$\left(9-6-6\sqrt{6}i\right) + \left(-18+6\sqrt{6}i\right) \overset{?}{=} -15$$

$$-15 = -15$$

Graph of the equation



Answer 15e.

Write the equation in standard form before applying the quadratic formula.
Add 14 and $3s$ to both the sides.

$$s^2 + 14 + 3s = -14 - 3s + 14 + 3s$$

$$s^2 + 14 + 3s = 0$$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 1 for a, 3 for b, 14 for c, and s for x in the formula.

$$s = \frac{-3 \pm \sqrt{3^2 - 4(1)(14)}}{2(1)}$$

Evaluate.

$$s = \frac{-3 \pm \sqrt{9 - 56}}{2}$$
$$= \frac{-3 \pm \sqrt{-47}}{2}$$

Use the imaginary unit to rewrite.

$$s = \frac{-3 \pm i\sqrt{47}}{2}$$

Therefore, the solutions are $\frac{-3 - i\sqrt{47}}{2}$ and $\frac{-3 + i\sqrt{47}}{2}$.

The solutions can be checked using a graphing utility.

Answer 16e.

Solve the following quadratic equation by using quadratic formula.

$-3y^2 = 6y - 10$ Write original equation

$\Rightarrow 3y^2 + 6y - 10 = 0$ Write in standard form

By comparing with $ax^2 + bx + c = 0$. We get,

$a = 3, b = 6, c = -10$

Therefore the roots are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic formula

$$\Rightarrow \quad y = \frac{-6 \pm \sqrt{36 + 120}}{6}$$ $a = 3, b = 6, c = -10$

$$\Rightarrow \quad y = \frac{-6 \pm \sqrt{156}}{6}$$

$$\Rightarrow \quad y = \frac{-6 \pm 2\sqrt{39}}{6}$$ Simplify

$$\Rightarrow \quad y = -1 \pm \frac{1}{3}\sqrt{39}$$

The solutions are $\boxed{-1 + \frac{1}{3}\sqrt{39}}$ and $\boxed{-1 - \frac{1}{3}\sqrt{39}}$

Check: $-3y^2 = 6y - 10$

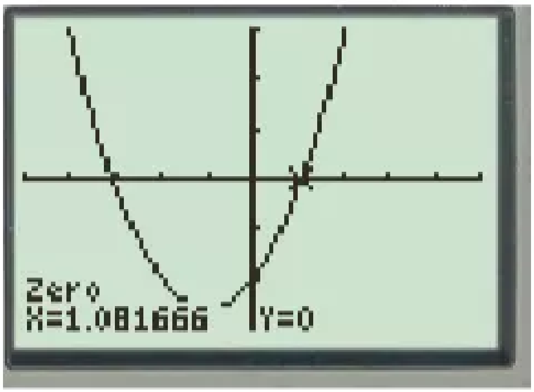
$$-3\left(-1+\frac{1}{3}\sqrt{39}\right)^2 \stackrel{?}{=} 6\left(-1+\frac{1}{3}\sqrt{39}\right)-10$$

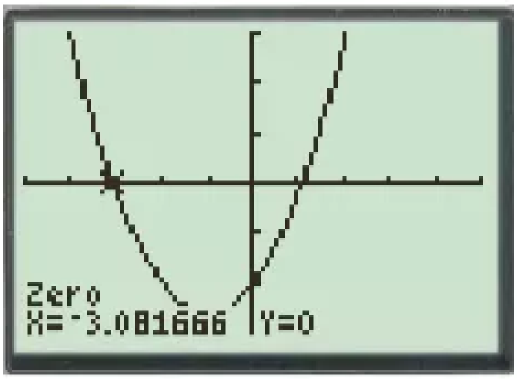
$$-3\left(1+\frac{13}{3}-\frac{2}{3}\sqrt{39}\right) \stackrel{?}{=} -6-2\sqrt{39}-10$$

$$-3-13-2\sqrt{39} \stackrel{?}{=} -16-2\sqrt{39}$$

$$-16-2\sqrt{39} \stackrel{?}{=} -16-2\sqrt{39}$$

Graph of the equation





Answer 17e.

Write the equation in standard form before applying the quadratic formula.
Subtract $2v$ to both the sides.

$$3 - 8v - 5v^2 - 2v = 2v - 2v$$

$$-5v^2 - 10v + 3 = 0$$

Divide both the sides by -1.

$$\frac{-5v^2 - 10v + 3}{-1} = \frac{0}{-1}$$

$$5v^2 + 10v - 3 = 0$$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$, where a, b, c are real numbers and $a \neq 0$.

Substitute 5 for a, 10 for b, -3 for c, and v for x in the formula.

$$v = \frac{-10 \pm \sqrt{10^2 - 4(5)(-3)}}{2(5)}$$

Evaluate.

$$v = \frac{-10 \pm \sqrt{100 + 60}}{10}$$

$$= \frac{-10 \pm \sqrt{160}}{10}$$

$$= \frac{-10 \pm 4\sqrt{10}}{10}$$

$$= \frac{-5 \pm 2\sqrt{10}}{5}$$

Therefore, the solutions are $\frac{-5 - 2\sqrt{10}}{5}$ and $\frac{-5 + 2\sqrt{10}}{5}$.

The solutions can be checked using a graphing utility.

Answer 18e.

Solve the following quadratic equation by using quadratic formula.

$7x - 5 + 12x^2 = -3x$ Write original equation

$\Rightarrow 12x^2 + 10x - 5 = 0$ Write in standard form

By comparing with $ax^2 + bx + c = 0$. We get,

$a = 12, b = 10, c = -5$

Therefore the roots are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic formula

$$\Rightarrow \quad x = \frac{-10 \pm \sqrt{100 + 240}}{24}$$ $a = 12, b = 10, c = -5$

$$\Rightarrow \quad x = \frac{-10 \pm \sqrt{340}}{24}$$ Simplify

$$\Rightarrow \quad x = \frac{-10 \pm 2\sqrt{85}}{24}$$

$$\Rightarrow \quad x = \frac{-5 \pm \sqrt{85}}{12}$$

The solutions are $\boxed{\frac{-5 + \sqrt{85}}{12}}$ and $\boxed{\frac{-5 - \sqrt{85}}{12}}$

Check: Graph $7x-5+12x^2=-3x$. The algebraic check for the solution $\frac{-5+\sqrt{85}}{12}$ is shown.

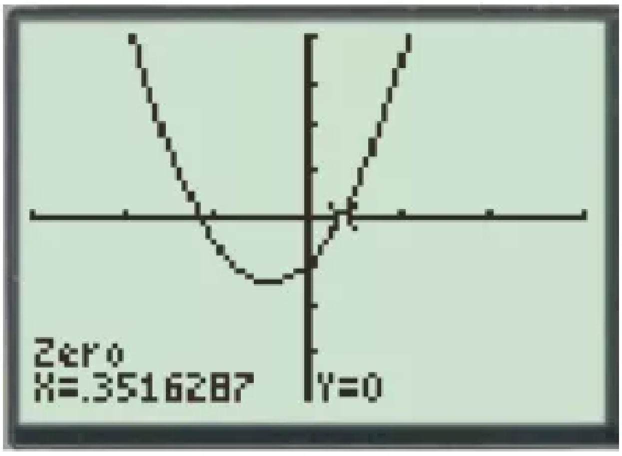
$$7\left(\frac{-5+\sqrt{85}}{12}\right)-5+12\left(\frac{-5+\sqrt{85}}{12}\right)^2\stackrel{?}{=}-3\left(\frac{-5+\sqrt{85}}{12}\right)$$

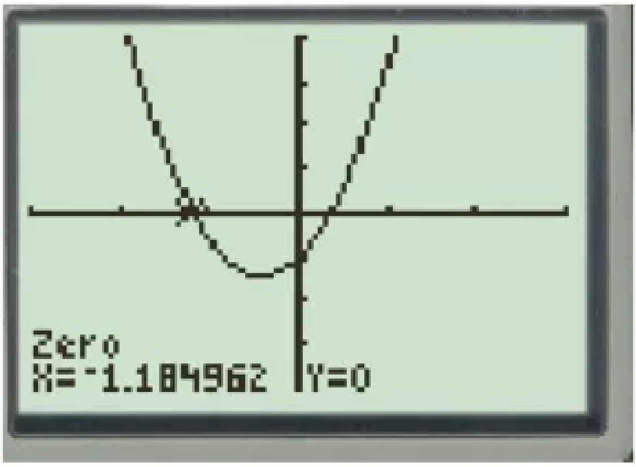
$$-\frac{35}{12}+\frac{7\sqrt{85}}{12}-5+12\left(\frac{25+85-10\sqrt{85}}{144}\right)\stackrel{?}{=}\frac{15}{12}-\frac{3\sqrt{85}}{12}$$

$$\frac{-95+7\sqrt{85}}{12}+\frac{110}{12}-\frac{10\sqrt{85}}{12}=\frac{15}{12}-\frac{3\sqrt{85}}{12}$$

$$\frac{15}{12}-\frac{3\sqrt{85}}{12}=\frac{15}{12}-\frac{3\sqrt{85}}{12}$$

Graph of the equation





Answer 19e.

Write the equation in standard form before applying the quadratic formula.
Add $7x$ to both the sides.

$$4x^2 + 3 + 7x = x^2 - 7x + 7x$$
$$4x^2 + 7x + 3 = x^2$$

Subtract x^2 from both the sides.

$$4x^2 + 7x + 3 - x^2 = x^2 - x^2$$
$$3x^2 + 7x + 3 = 0$$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 3 for a, 7 for b, and 3 for c in the formula.

$$x = \frac{-7 \pm \sqrt{7^2 - 4(3)(3)}}{2(3)}$$

Simplify.

$$x = \frac{-7 \pm \sqrt{49 - 36}}{6}$$
$$= \frac{-7 \pm \sqrt{13}}{6}$$

Therefore, the solutions are $\frac{-7 - \sqrt{13}}{6}$ and $\frac{-7 + \sqrt{13}}{6}$.

The solutions can be checked using a graphing utility.

Answer 20e.

Solve the following quadratic equation by using quadratic formula.

$6 - 2t^2 = 9t + 15$ Write original equation

$\Rightarrow 2t^2 + 9t + 9 = 0$ Write in standard form

By comparing with $ax^2 + bx + c = 0$. We get,

$a = 2, b = 9, c = 9$

Therefore the roots are

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic formula

$\Rightarrow t = \frac{-9 \pm \sqrt{81 - 72}}{4}$ $a = 2, b = 9, c = 9$

Continue the above

$\Rightarrow t = \frac{-9 \pm \sqrt{9}}{4}$ Simplify

$\Rightarrow t = \frac{-9 \pm 3}{4}$

$\Rightarrow t = -3, -\frac{3}{2}$

The solutions are $\boxed{-3}$ and $\boxed{-\frac{3}{2}}$

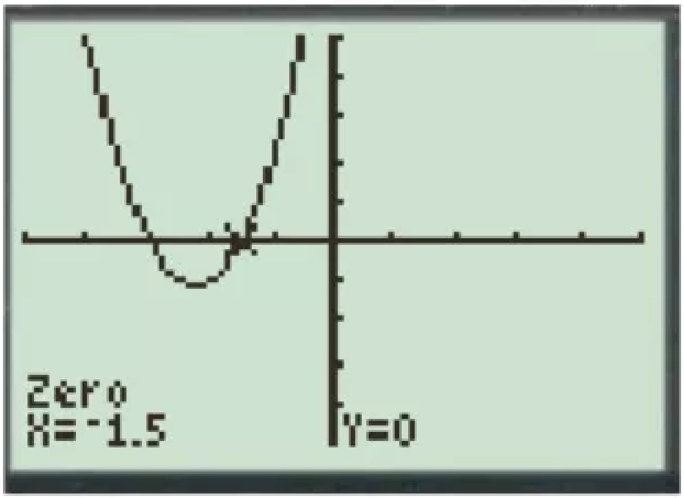
Check: Graph $6 - 2t^2 = 9t + 15$

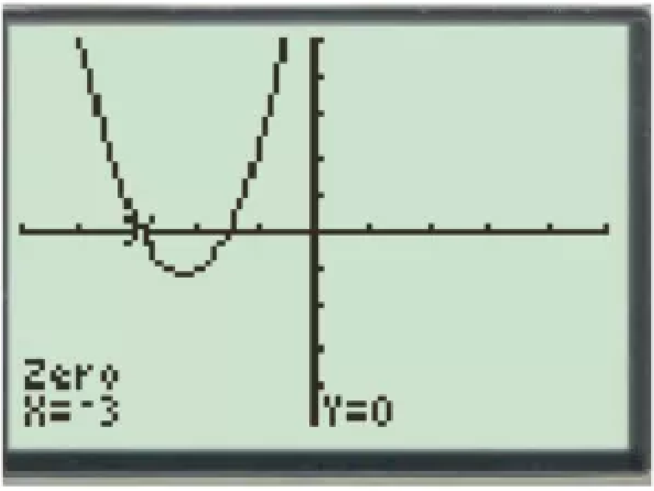
$6 - 2(-3)^2 = 9(-3) + 15$

$6 - 18 = -27 + 15$

$-12 = -12$

Graph of the solution





Answer 21e.

Write the equation in standard form before applying the quadratic formula.
Add $-2+n$ to both the sides.

$$4 + 9n - 3n^2 - 2 + n = 2 - n - 2 + n$$
$$-3n^2 + 10n + 4 = 0$$

Divide both the sides by -1.

$$\frac{-3n^2 + 10n + 4}{-1} = \frac{0}{-1}$$
$$3n^2 - 10n - 4 = 0$$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 3 for a, -10 for b, -2 for c, and n for x in the formula.

$$n = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(-2)}}{2(3)}$$

Simplify.

$$n = \frac{10 \pm \sqrt{100 + 24}}{6}$$
$$= \frac{10 \pm \sqrt{124}}{6}$$
$$= \frac{10 \pm 2\sqrt{31}}{6}$$
$$= \frac{5 \pm \sqrt{31}}{3}$$

Therefore, the solutions are $\dfrac{5 - \sqrt{31}}{3}$ and $\dfrac{5 + \sqrt{31}}{3}$.

The solutions can be checked using a graphing utility.

Answer 22e.

Solve the following quadratic equation by using quadratic formula.

$z^2 + 15z + 24 = -32$ Write original equation

$\Rightarrow z^2 + 15z + 56 = 0$ Write in standard form

By comparing with $ax^2 + bx + c = 0$. We get,

$a = 1, b = 15, c = 56$

Therefore the roots are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ Quadratic formula

$$\Rightarrow z=\frac{-15\pm\sqrt{225-224}}{2}$$ $a=1, b=15, c=56$

$$\Rightarrow z=\frac{-15\pm1}{2}$$ Simplify

$$\Rightarrow z=-8,-7$$

The solutions are $\boxed{-8,-7}$

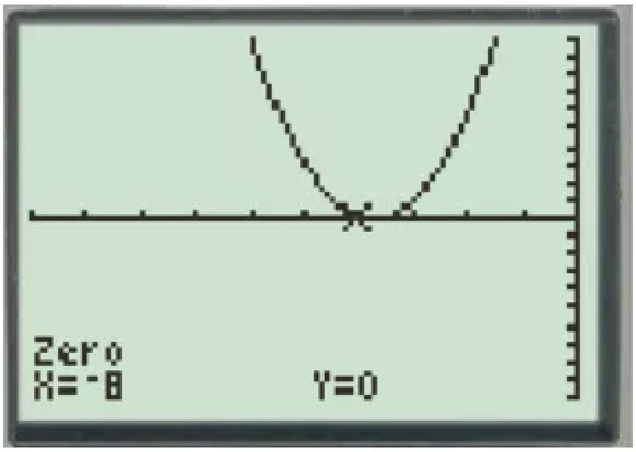
check:

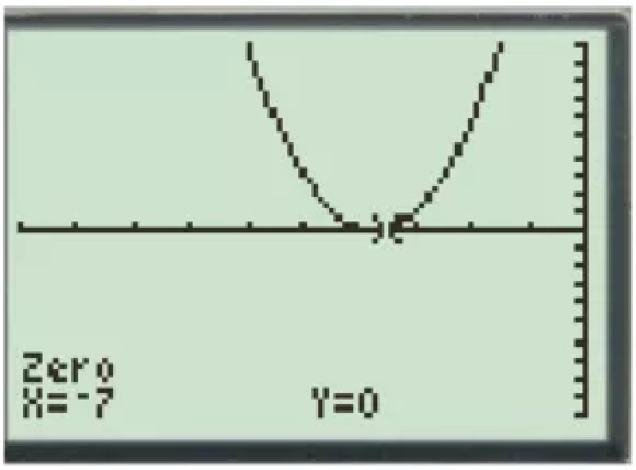
$$z^2+15z+24=-32$$
$$(-8)^2+15(-8)+24=-32$$
$$64-120+24=-32$$
$$-32=-32$$

Check for second root

$$(-7)^2+15(-7)+24=-32$$
$$49-105+24=-32$$
$$-32=-32$$

Graph of the quadratic equation





Answer 23e.

Write the equation in standard form before applying the quadratic formula.
$x^2 - 5x + 6 = 0$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 1 for a, -5 for b, and 6 for c in the formula.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

Simplify.

$$\begin{aligned} x &= \frac{5 \pm \sqrt{25 - 24}}{2} \\ &= \frac{5 \pm \sqrt{1}}{2} \\ &= \frac{5 \pm 1}{2} \\ &= 3 \text{ or } 2 \end{aligned}$$

Therefore, the solutions are 2 and 3.

Check:

Factor $x^2 - 5x + 6 = 0$.
$(x - 2)(x - 3) = 0$
Use the zero-product property.
$x - 2 = 0 \quad \text{or} \quad x - 3 = 0$
Solve for s
$x = 2 \quad \text{or} \quad x = 3$

The solution checks.

Answer 24e.

Solve the following quadratic equation by using quadratic formula.

$m^2 + 5m - 99 = 3m$ Write original equation

$\Rightarrow m^2 + 2m - 99 = 0$ Write in standard form

By comparing with $ax^2 + bx + c = 0$. We get,
$a = 1, b = 2, c = -99$

Therefore the roots are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ Quadratic formula

$$\Rightarrow m=\frac{-2\pm\sqrt{4+396}}{2}$$ $a=1, b=2, c=-99$

$$\Rightarrow m=\frac{-2\pm\sqrt{400}}{2}$$ Simplify

$$\Rightarrow m=\frac{-2\pm 20}{2}$$

$$\Rightarrow m=-11,9$$

The solutions are $\boxed{-11,9}$

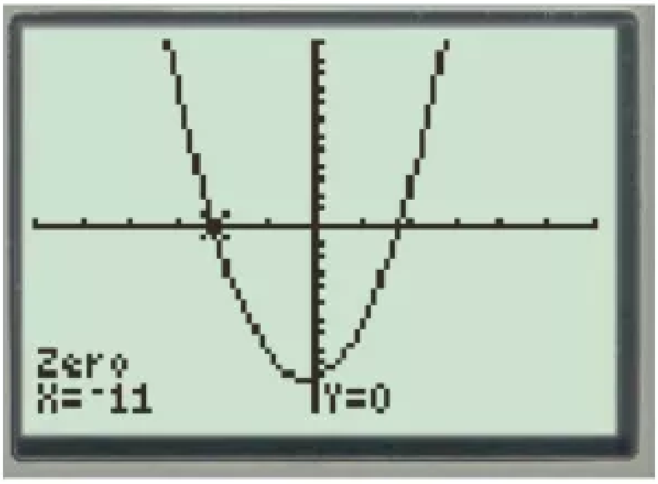
check:

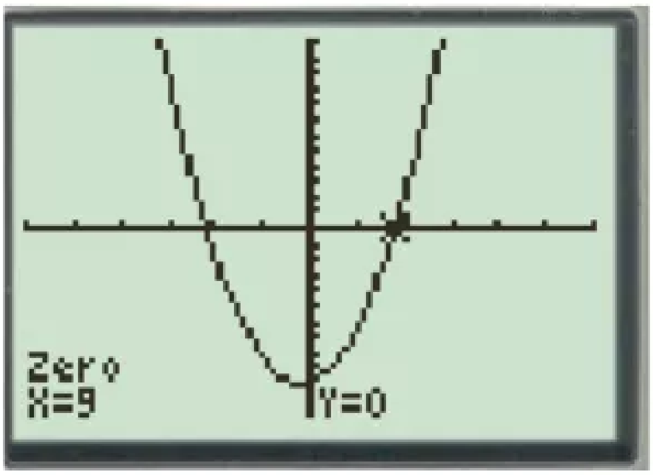
$$m^2+5m-99=3m$$
$$(-11)^2+5(-11)-99=3(-11)$$
$$121-55-99=-33$$
$$-33=-33$$

Check for second root

$$(9)^2+5(9)-99=27$$
$$81+45-99=27$$
$$27=27$$

Graph of the quadratic solutions





Answer 25e.

Write the equation in standard form before applying the quadratic formula.
$s^2 - 2s - 3 = 0$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$, where a, b, c are real numbers and $a \neq 0$.

Substitute 1 for a, -2 for b, -3 for c, and s for x in the formula.

$$s = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$$

Simplify.

$$s = \frac{2 \pm \sqrt{4 + 12}}{2}$$
$$= \frac{2 \pm \sqrt{16}}{2}$$
$$= \frac{2 \pm 4}{2}$$
$$= \frac{2 - 4}{2} \text{ or } \frac{2 + 4}{2}$$
$$= \frac{-2}{2} \text{ or } \frac{6}{2}$$
$$= -1 \text{ or } 3$$

Therefore, the solutions are -1 and 3.

Check:

Factor $s^2 - 2s - 3 = 0$.
$(s - 3)(s + 1) = 0$
Use the Zero product property.
$s - 3 = 0$ or $s + 1 = 0$
Solve for s
$s = 3$ or $s = -1$

The solution checks.

Answer 26e.

Solve the following quadratic equation by using quadratic formula.

$r^2 - 4r + 8 = 5r$ Write original equation

$\Rightarrow r^2 - 9r + 8 = 0$ Write in standard form

By comparing with $ax^2 + bx + c = 0$. We get,
$a = 1, b = -9, c = 8$

Therefore the roots are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ Quadratic formula

$$\Rightarrow r=\frac{9\pm\sqrt{81-32}}{2}$$ $a=1, b=-9, c=8$

$$\Rightarrow r=\frac{9\pm\sqrt{49}}{2}$$ Simplify

$$\Rightarrow r=\frac{9\pm7}{2}$$

$$\Rightarrow r=1,8$$

The solutions are $\boxed{1,8}$

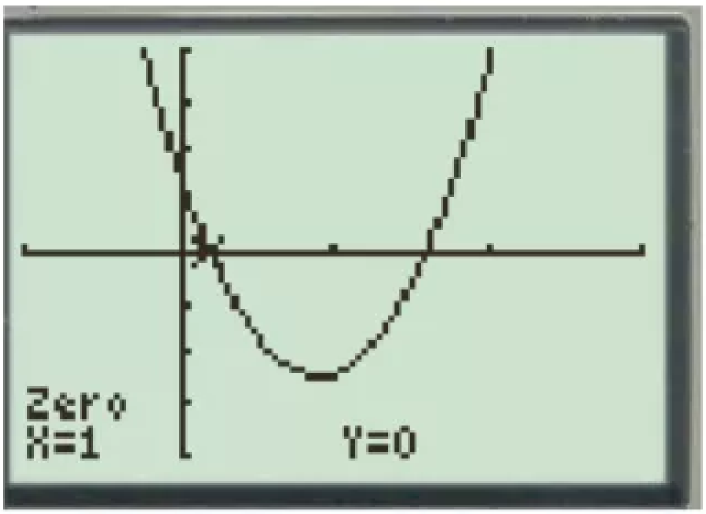
check:

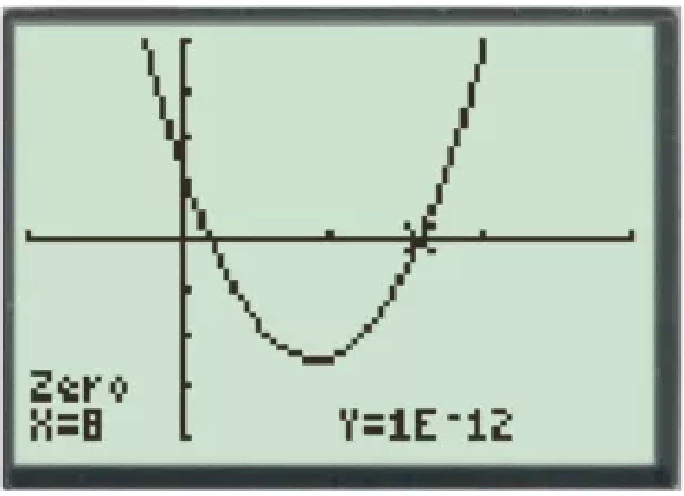
$$r^2-4r+8=5r$$
$$(1)^2-4(1)+8=5(1)$$
$$1-4+8=5(1)$$
$$5=5$$

Check for second root

$$(8)^2-4(8)+8=5(8)$$
$$64-32+8=40$$
$$40=40$$

Graph of the quadratic solution





Answer 27e.

Write the equation in standard form before applying the quadratic formula.
Subtract $13x$ from both the sides.

$$3x^2 + 7x - 24 - 13x = 13x - 13x$$
$$3x^2 + 6x - 24 = 0$$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 3 for a, −6 for b, and −24 for c in the formula.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-24)}}{2(3)}$$

Simplify.

$$x = \frac{6 \pm \sqrt{36 + 288}}{6}$$
$$= \frac{6 \pm \sqrt{324}}{6}$$
$$= \frac{6 \pm 18}{6}$$
$$= \frac{6 - 18}{6} \text{ or } \frac{6 \pm 18}{6}$$
$$= \frac{-12}{6} \text{ or } \frac{24}{6}$$
$$= -2 \text{ or } 4$$

Therefore, the solutions are −2 and 4.

Check:

Factor $3x^2 - 6x - 24 = 0$.
$3(x - 4)(x + 2) = 0$
Use the Zero product property.
$x - 4 = 0$ or $x + 2 = 0$
Solve for x
$x = 4$ or $x = -2$

The solution checks.

Answer 28e.

Solve the following quadratic equation by using quadratic formula.

$45x^2 + 57x + 1 = 5$ — Write original equation

$\Rightarrow 45x^2 + 57x - 4 = 0$ — Write in standard form

By comparing with $ax^2 + bx + c = 0$. We get,
$a = 45, b = 57, c = -4$

Therefore the roots are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ Quadratic formula

$$\Rightarrow x=\frac{-57\pm\sqrt{3249+720}}{90}$$ $a=45, b=57, c=-4$

$$\Rightarrow x=\frac{-57\pm\sqrt{3969}}{90}$$ Simplify

$$\Rightarrow x=\frac{-57\pm 63}{90}$$

$$\Rightarrow x=-\frac{4}{3},\frac{1}{15}$$

The solutions are $\boxed{-\frac{4}{3},\frac{1}{15}}$

check:

$$45x^2+57x+1=5$$

$$45\left(-\frac{4}{3}\right)^2+57\left(-\frac{4}{3}\right)+1=5(1)$$

$$80-76+1=5$$

$$5=5$$

Check for second root

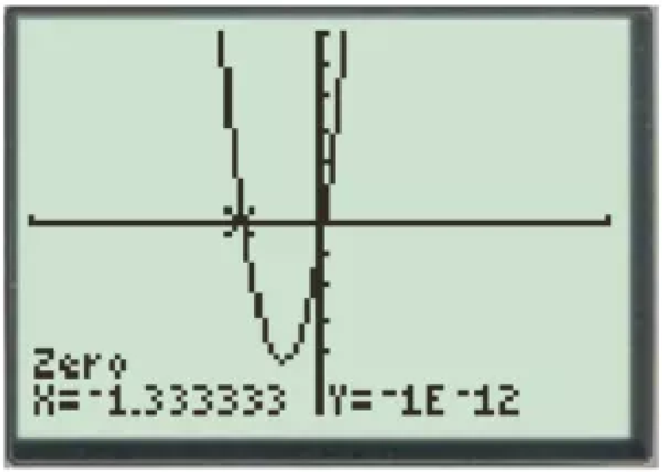
$$45\left(\frac{1}{15}\right)^2+57\left(\frac{1}{15}\right)+1=5$$

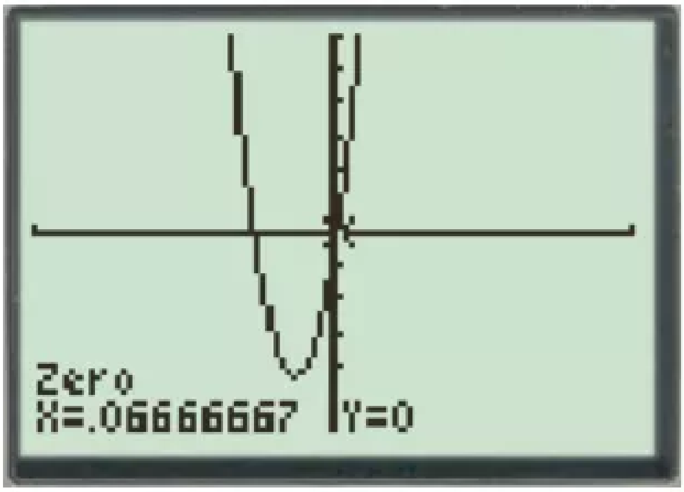
$$\left(\frac{1}{5}\right)+57\left(\frac{1}{15}\right)+1=5$$

$$\frac{75}{5}=5$$

$$5=5$$

Graph of the solution





Answer 29e.

Write the equation in standard form before applying the quadratic formula.
Subtract 25 from both the sides.

$$5p^2 + 40p + 100 - 25 = 25 - 25$$
$$5p^2 + 40p + 75 = 0$$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 5 for a, 40 for b, 75 for c, and p for x in the formula.

$$p = \frac{-40 \pm \sqrt{40^2 - 4(5)(75)}}{2(5)}$$

Simplify.

$$p = \frac{-40 \pm \sqrt{1600 - 1500}}{10}$$
$$= \frac{-40 \pm \sqrt{100}}{10}$$
$$= \frac{-40 \pm 10}{10}$$
$$= \frac{-40 - 10}{10} \text{ or } \frac{-40 + 10}{10}$$
$$= \frac{-50}{10} \text{ or } \frac{-30}{10}$$
$$= -5 \text{ or } -3$$

Therefore, the solutions are -5 and -3.

Check:

Factor $5p^2 + 40p + 75$.
$5(p+5)(p+3) = 0$
Use the Zero product property.
$p + 5 = 0$ or $p + 3 = 0$
Solve for p.
$p = -5$ or $p = -3$

The solution checks.

Answer 30e.

Solve the following quadratic equation by using quadratic formula.

$9n^2 - 42n - 162 = 21n$ Write original equation

$\Rightarrow 9n^2 - 63n - 162 = 0$

$\Rightarrow n^2 - 7n - 18 = 0$ Write in standard form

By comparing with $ax^2 + bx + c = 0$. We get,
$a = 1, b = -7, c = -18$

Therefore the roots are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic formula

$$\Rightarrow n = \frac{7 \pm \sqrt{49 + 72}}{2}$$ $a = 1, b = -7, c = -18$

$$\Rightarrow n = \frac{7 \pm \sqrt{121}}{2}$$ Simplify

$$\Rightarrow n = \frac{7 \pm 11}{2}$$

$$\Rightarrow n = -2, 9$$

The solutions are $\boxed{-2, 9}$

check:

$9n^2 - 42n - 162 = 21n$

$9(-2)^2 - 42(-2) - 162 = 21(-2)$

$36 + 84 - 162 = -42$

$-42 = -42$

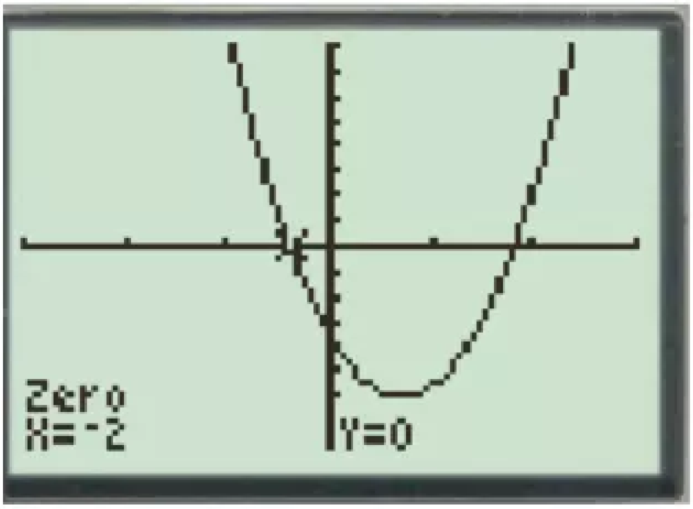
Check for second root

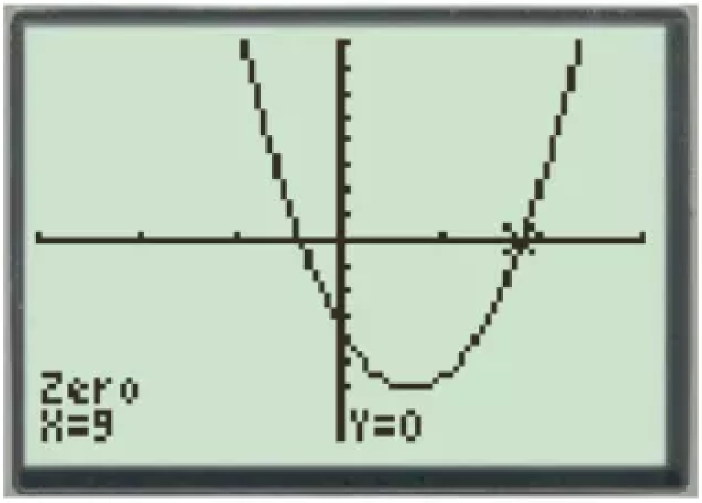
$9(9)^2 - 42(9) - 162 = 21(9)$

$729 - 378 - 162 = 189$

$189 = 189$

Graph of the solution





Answer 31e.

In the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression $b^2 - 4ac$ is called the discriminant.

Substitute 1 for a, −8 for b, and 16 for c in $b^2 - 4ac$ and evaluate.

$$\begin{aligned} b^2 - 4ac &= (-8)^2 - 4(1)(16) \\ &= 64 - 64 \\ &= 0 \end{aligned}$$

The discriminant of the given quadratic equation is 0.
Therefore, the equation has one real solution.

Answer 32e.

Consider the following quadratic equation.

$s^2 + 7s + 11 = 0$

By comparing the above quadratic equation with the standard form of quadratic equation $ax^2 + bx + c = 0$. We get,

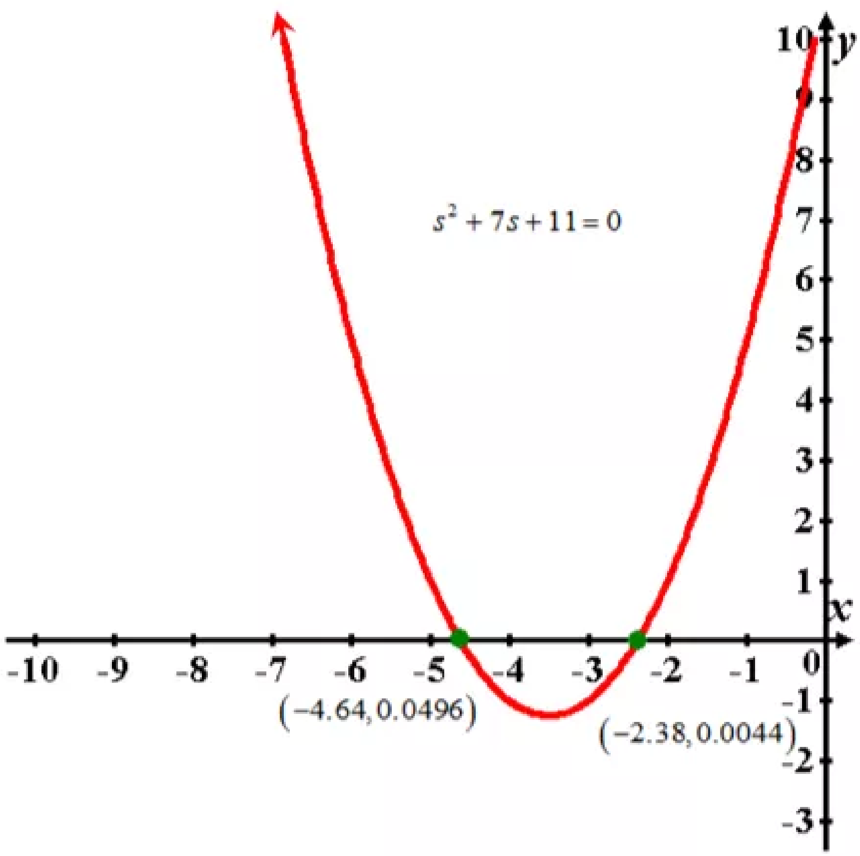
$a = 1, b = 7, c = 11$

The discriminant is $D = b^2 - 4ac$

$$\begin{aligned} \Rightarrow D &= 7^2 - 4(1)(11) \\ &= 49 - 44 \\ &= 5 \\ &> 0 \end{aligned}$$

Since $D > 0$, the given quadratic equation will have two real solution.

Graph of the equation



Answer 33e.

In the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression $b^2 - 4ac$ is called the discriminant.

Substitute 8 for a, 8 for b, and 3 for c in $b^2 - 4ac$ and evaluate.

$$\begin{aligned} b^2 - 4ac &= 8^2 - 4(8)(3) \\ &= 64 - 96 \\ &= -32 \end{aligned}$$

The discriminant of the given quadratic equation is less than 0.
Therefore, the equation has two imaginary solutions.

Answer 34e.

Consider the following quadratic equation.

$-4w^2 + w - 14 = 0$

Compare the above quadratic equation with the standard form of quadratic equation $aw^2 + bw + c = 0$.

Then, $a = -4, b = 1, c = -14$

then, the discriminant is $D = b^2 - 4ac$

$$\begin{aligned} D &= 1^2 - 4(-4)(-14) \\ &= 1 - 224 \\ &= -223 \\ &< 0 \end{aligned}$$

Since, $D < 0$, the given quadratic equation will have two imaginary solutions.

The solutions of $-4w^2 + w - 14 = 0$ are

$$\begin{aligned} x &= \frac{-1 \pm \sqrt{1-224}}{2(-4)} \\ &= \boxed{\frac{-1 \pm \sqrt{223}i}{-8}} \end{aligned}$$

Answer 35e.

In the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression $b^2 - 4ac$ is called the discriminant.

Substitute 5 for a, 20 for b, and 21 for c in $b^2 - 4ac$ and evaluate.

$$\begin{aligned} b^2 - 4ac &= 20^2 - 4(5)(21) \\ &= 400 - 420 \\ &= -20 \end{aligned}$$

The discriminant of the given quadratic equation is less than 0.
Therefore, the equation has two imaginary solutions.

Answer 36e.

Consider the following quadratic equation.

$8z-10=z^2-7z+3$

$z^2-15z+13=0$

Compare the above quadratic equation with the standard form of quadratic equation $az^2+bz+c=0$.

$a=1, b=-15, c=13$

then, the discriminant is $D=b^2-4ac$

$$\begin{aligned} d &= (-15)^2-4(1)(13) \\ &= 225-52 \\ &= 173 \\ &> 0 \end{aligned}$$

Since $D>0$, the given quadratic equation will have two real solutions.

The solutions of $z^2-15z+13=0$ are

$$\boxed{z=\frac{15\pm\sqrt{173}}{2(1)}}.$$

Answer 37e.

Write the equation is standard form.
Subtract $5n + 11$ from both the sides.

$$\begin{aligned} 8n^2-4n+2-5n+11 &= 5n-11-5n+11 \\ 8n^2-9n+13 &= 0 \end{aligned}$$

In the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$, the expression b^2-4ac is called the discriminant.
Substitute 8 for a, -9 for b, and 13 for c in b^2-4ac and evaluate.

$$\begin{aligned} b^2-4ac &= (-9)^2-4(8)(13) \\ &= 81-416 \\ &= -335 \end{aligned}$$

The discriminant of the given quadratic equation is less than 0. Therefore, the equation has two imaginary solutions.

Answer 38e.

Consider the quadratic equation.

$5x^2 + 16x = 11x - 3x^2$

$8x^2 + 5x = 0$ Add $-11x, 3x^2$ on both sides

Compare the above quadratic equation with the standard form of quadratic equation $ax^2 + bx + c = 0$.

Then $a = 8, b = 5, c = 0$

then, the discriminant is $D = b^2 - 4ac$

$$\begin{aligned} D &= (5)^2 - 4(8)(0) \\ &= 25 \\ &> 0 \end{aligned}$$

Since, $D > 0$, the given quadratic equation will have two real solution.

The solutions of quadratic equation $8x^2 + 5x = 0$ are

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-5 \pm \sqrt{25}}{2(8)} \\ &= \frac{-5 \pm 5}{16} \\ &= 0, -\frac{10}{16} \\ &= \boxed{0, -\frac{5}{8}}. \end{aligned}$$

Answer 39e.

Write the equation is standard form.

Subtract $2r + 9r^2$ from both the sides.

$$7r^2 - 5 - 2r - 9r^2 = 2r + 9r^2 - 2r - 9r^2$$

$$-2r^2 - 2r - 5 = 0$$

Divide both the sides by -1.

$$\frac{-2r^2 - 2r - 5}{-1} = \frac{0}{-1}$$

$$2r^2 + 2r + 5 = 0$$

In the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression $b^2 - 4ac$ is called the discriminant.

Substitute 2 for a, 2 for b, and 5 for c in $b^2 - 4ac$ and evaluate.

$$\begin{aligned} b^2 - 4ac &= 2^2 - 4(2)(5) \\ &= 4 - 40 \\ &= -36 \end{aligned}$$

The discriminant of the given quadratic equation is less than 0.
Therefore, the equation has two imaginary solutions.

Answer 40e.

Consider the quadratic equation.

$16t^2 - 7t = 17t - 9$

$16t^2 - 24t + 9 = 0$ Add -17t,9 on both sides

$(4t)^2 - 2(4t)(3) + 3^2 = 0$ Write left side terms as square terms

$(4t-3)^2 = 0$ Write left side as a perfect square

Solve the above quadratic equation by taking square roots.

$4t - 3 = 0$

$4t = 3$

$\boxed{t = \dfrac{3}{4}}$ is the only solution of the given quadratic equation.

Answer 41e.

Write the equation is standard form.
Subtract $11x - 3x^2$ from both the sides.

$$\begin{aligned} 7x - 3x^2 - 85 - 2x^2 - 2x &= 85 + 2x^2 + 2x - 85 - 2x^2 - 2x \\ -5x^2 + 5x - 85 &= 0 \end{aligned}$$

Divide both the sides by -1.

$$\begin{aligned} \frac{-5x^2 + 5x - 85}{-1} &= \frac{0}{-1} \\ 5x^2 - 5x + 85 &= 0 \end{aligned}$$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 5 for a, -5 for b, and 85 for c in the formula.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(5)(85)}}{2(5)}$$

Simplify.

$$x = \frac{5 \pm \sqrt{25 - 1700}}{10}$$
$$= \frac{5 \pm \sqrt{-1675}}{10}$$
$$= \frac{5 \pm 5\sqrt{67}\,i}{10}$$
$$= \frac{1 \pm \sqrt{67}\,i}{2}$$
$$= \frac{1 - \sqrt{67}\,i}{2} \text{ or } \frac{1 + \sqrt{67}\,i}{2}$$

Therefore, the solutions are $\frac{1 - \sqrt{67}\,i}{2}$ and $\frac{1 + \sqrt{67}\,i}{2}$.

Answer 42e.

Consider the quadratic equation.

$4(x-1)^2 = 6x+2$

$4(x^2 - 2x + 1) = 6x + 2$ Expand

$4x^2 - 8x + 4 = 6x + 2$ Multiply left side terms by 4

$4x^2 - 8x + 4 - 6x - 2 = 0$ Add -6x, −2 on both sides

$4x^2 - 14x + 2 = 0$ Simplify

$2x^2 - 7x + 1 = 0$ Divide by 2

Solve the quadratic equation $2x^2 - 7x + 1 = 0$ by using quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

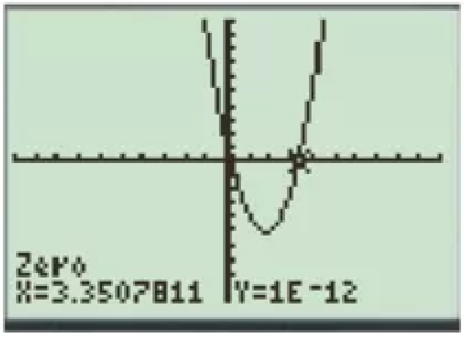
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(1)}}{2(2)}$$ $a = 2, b = -7$ and $c = 1$

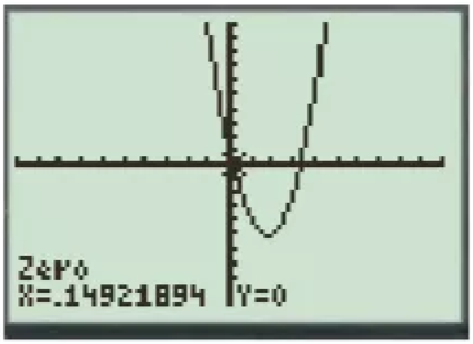
$$x = \frac{7 \pm \sqrt{41}}{4}$$

Therefore solutions of equation $2x^2 - 7x + 1 = 0$ are

$x = \frac{7 + \sqrt{41}}{4} \approx \boxed{3.35}$ and $x = \frac{7 - \sqrt{41}}{4} \approx \boxed{0.15}$

Check:
Graph of $y = 2x^2 - 7x + 1$ is shown below:





From the graph x- intercepts are about 3.35 and 0.15

Answer 43e.

Write the equation is standard form.
Use the distributive property to remove the parentheses.
$25 - 16v^2 = 12v^2 + 60v$

Subtract $12v^2 + 60v$ from both the sides.
$$25 - 16v^2 - 12v^2 - 60v = 12v^2 + 60v - 12v^2 - 60v$$
$$-28v^2 - 60v + 25 = 0$$

Divide both the sides by -1.
$$\frac{-28v^2 - 60v + 25}{-1} = \frac{0}{-1}$$
$$28v^2 + 60v - 25 = 0$$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.
Substitute 28 for a, 60 for b, -25 for c and v for x in the formula.
$$v = \frac{-60 \pm \sqrt{60^2 - 4(28)(-25)}}{2(28)}$$

Simplify.

$$v = \frac{-60 \pm \sqrt{3600 + 2800}}{56}$$

$$= \frac{-60 \pm \sqrt{6400}}{56}$$

$$= \frac{-60 \pm 80}{56}$$

$$= \frac{-60 - 80}{56} \text{ or } \frac{-60 + 80}{56}$$

$$= -\frac{5}{2} \text{ or } \frac{5}{14}$$

Therefore, the solutions are $-\frac{5}{2}$ and $\frac{5}{14}$.

Answer 44e.

Consider the quadratic equation.

$$\frac{3}{2}y^2 - 6y = \frac{3}{4}y - 9$$

$$\frac{3}{2}y^2 - 6y - \frac{3}{4}y + 9 = 0 \qquad \text{Add } -\frac{3}{4}y, 9 \text{ on both sides}$$

$$\frac{3}{2}y^2 - \frac{27}{4}y + 9 = 0 \qquad \text{Simplify}$$

$$6y^2 - 27y + 36 = 0 \qquad \text{Multiply by 4}$$

$$2y^2 - 9y + 12 = 0 \qquad \text{Divide by 3}$$

Solve the above quadratic equation $2y^2 - 9y + 12 = 0$ by using quadratic formula.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

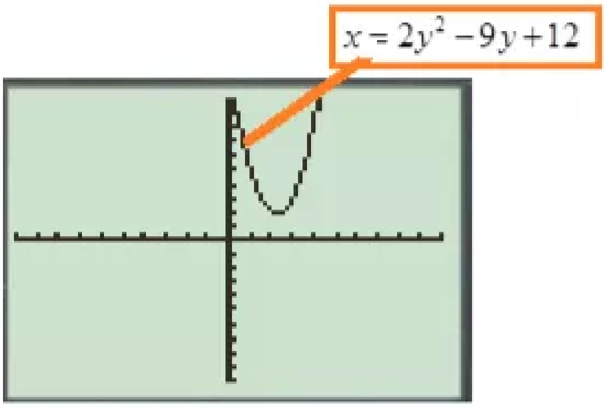
$$y = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(12)}}{2(2)} \qquad a = 2, b = -9, c = 12$$

$$y = \frac{9 \pm \sqrt{-15}}{4}$$

$$y = \frac{9 \pm i\sqrt{15}}{4}$$

The solutions of the quadratic equation are $y = \boxed{\frac{9 + i\sqrt{15}}{4}}$ and $y = \boxed{\frac{9 - i\sqrt{15}}{4}}$.

Check:
The graph of $x = 2y^2 - 9y + 12$ is shown below:



There are no x-intercepts. It has no real solution.

Answer 45e.

Write the equation is standard form.

Subtract $5x + \frac{3}{4}$ from both the sides.

$$3x^2 + \frac{9}{2}x - 4 - 5x - \frac{3}{4} = 5x + \frac{3}{4} - 5x - \frac{3}{4}$$

$$3x^2 - \frac{1}{2}x - \frac{19}{4} = 0$$

Multiply both the sides by the least common denominator, 4.

$$4\left(3x^2 - \frac{1}{2}x - \frac{19}{4}\right) = 2(0)$$

$$12x^2 - 2x - 19 = 0$$

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$, where a, b, c are real numbers and $a \neq 0$.

Substitute 12 for a, -2 for b, and -19 for c in the formula.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(12)(-19)}}{2(12)}$$

Simplify.

$$x = \frac{2 \pm \sqrt{4+912}}{24}$$
$$= \frac{2 \pm \sqrt{916}}{24}$$
$$= \frac{2 \pm 2\sqrt{229}}{24}$$
$$= \frac{2 - 2\sqrt{229}}{24} \text{ or } \frac{2 + 2\sqrt{229}}{24}$$
$$= \frac{1 - \sqrt{229}}{12} \text{ or } \frac{1 + \sqrt{229}}{12}$$

Therefore, the solutions are $\frac{1-\sqrt{229}}{12}$ and $\frac{1+\sqrt{229}}{12}$.

Answer 46e.

Consider the quadratic equation.

$$1.1(3.4x-2.3)^2 = 15.5$$

$$(3.4x-2.3)^2 = \frac{15.5}{1.1} \quad \text{Divide by 1.1}$$

$$(3.4x-2.3)^2 = \frac{155}{11} \quad \text{multiply numerator and denominator by 10}$$

Solve the above quadratic equation $(3.4x-2.3)^2 = \frac{155}{11}$ by taking square roots.

$$3.4x - 2.3 = \pm\sqrt{\frac{155}{11}}$$

$$3.4x = 2.3 \pm \sqrt{\frac{155}{11}} \quad \text{Add 2.3 on both sides}$$

$$34x = 23 \pm 10\sqrt{\frac{155}{11}} \quad \text{Multiply both sides by 10}$$

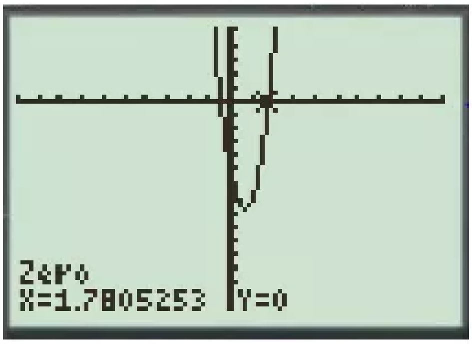
$$x = \frac{23}{34} \pm \frac{5}{17}\sqrt{\frac{155}{11}}$$

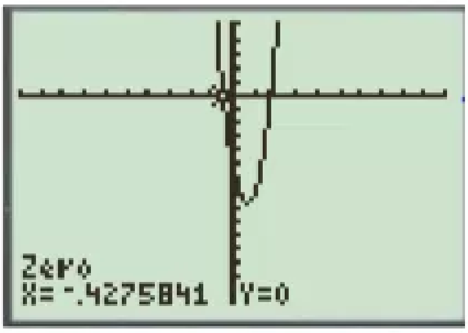
The solutions of the quadratic equation are $x = \frac{23}{34} + \frac{5}{17}\sqrt{\frac{155}{11}} \approx \boxed{1.78}$ and

$x = \frac{23}{34} - \frac{5}{17}\sqrt{\frac{155}{11}} \approx \boxed{-0.43}$.

Check:

The graph of $1.1(3.4x-2.3)^2=15.5$ is shown below:





From the graph x-intercepts are -0.43 and 1.78.

Answer 47e.

Write the equation is standard form.
Use the perfect square trinomial to factor $(2r-1.75)^2$.

$19.25=-8.5(4r^2-7r+3.0625)$

Use the distributive property to remove the parentheses.

$19.25=-34r^2+59.5r-26.03125$

Subtract 19.25 from both the sides.

$$19.25-19.25 = -34r^2+59.5r-26.03125-19.25$$
$$0 = -34r^2+59.5r-45.28125$$

Divide both the sides by -1.

$$\frac{0}{-1} = \frac{-34r^2+59.5r-45.28125}{-1}$$
$$0 = 34r^2-59.5r+45.28125$$

The solutions of a quadratic equation of the form $ax^2+bx+c=0$ are

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, where a, b, c are real numbers and $a\neq 0$.

Substitute 34 for a, -59.5 for b, 45.28125 for c, and r for x in the formula.

$$r = \frac{-(-59.5)\pm\sqrt{(-59.5)^2-4(34)(45.28125)}}{2(34)}$$

Simplify.

$$r = \frac{59.5 \pm \sqrt{3540.25 - 6158.25}}{68}$$
$$= \frac{59.5 \pm \sqrt{-2618}}{68}$$
$$= \frac{59.5 \pm i\sqrt{2618}}{68}$$
$$= 0.875 \pm 0.752i$$

Therefore, the solutions are $0.875 - 0.7525i$ and $0.875 + 0.7525i$.

Answer 48e.

Consider the quadratic equation.

$$4.5 = 1.5(3.25 - s)^2$$
$$(s - 3.25)^2 = \frac{4.5}{1.5} \qquad \text{Divide by 1.5}$$
$$(s - 3.25)^2 = 3$$

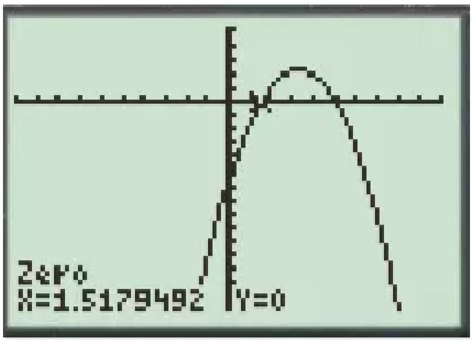
Solve the above quadratic equation by taking square roots.

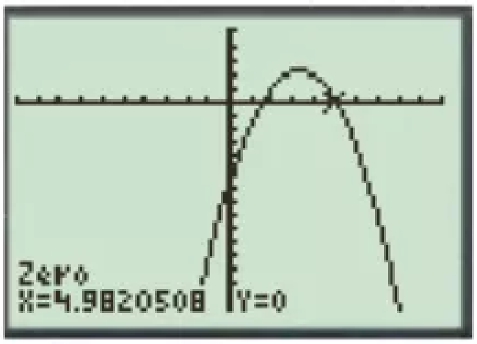
$$s - 325 = \pm\sqrt{3}$$
$$s = 3.25 \pm \sqrt{3}$$

The solutions of the given quadratic equation are $s = 3.25 + \sqrt{3} = \boxed{4.98}$ and $s = 3.25 - \sqrt{3} = \boxed{1.52}$.

Check:

The graph of $4.5 = 1.5(3.25 - s)^2$ is shown below:





From the graph, the x-intercepts are 1.52 and 4.98 .

Answer 49e.

The square root of a negative number cannot be a real number. It should be an imaginary number. The error is that the value of $\sqrt{-144}$ is written as 12.

The value of $\sqrt{-144}$ is $12i$.

Substitute $12i$ for $\sqrt{-144}$ and simplify.

$$\begin{aligned} x &= \frac{-6 \pm \sqrt{-144}}{6} \\ &= \frac{-6 \pm 12i}{6} \\ &= -1 \pm 2i \end{aligned}$$

Answer 50e.

In solving the equation $x^2+6x+8=2$, error occurred because the quadratic formula was used without rewriting the given equation in standard form.
Hence, the correct way solving the equation is

$x^2+6x+8=2$

$x^2+6x+6=0$ Add -2 on both sides

Compare the above quadratic equation with the standard form of quadratic equation $ax^2+bx+c=0$.

Then, $a=1, b=6, c=6$

The roots are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-6\pm\sqrt{36-24}}{2}$$

$$x=\frac{-6\pm\sqrt{12}}{2}$$

$$x=\frac{-6\pm2\sqrt{3}}{2}$$

$$x=-3\pm\sqrt{3}$$

The solutions are $x=-3+\sqrt{3}\approx\boxed{-1.268}$ and $x=-3-\sqrt{3}=\boxed{-4.73}$

Answer 51e.

The solutions of a quadratic equation of the form $ax^2+bx+c=0$ are

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a},$$ where a, b, c are real numbers and $a \neq 0$.

Add the solutions first to find the mean.

$$\begin{aligned} \frac{-b-\sqrt{b^2-4ac}}{2a} + \frac{-b+\sqrt{b^2-4ac}}{2a} &= \frac{-b-\sqrt{b^2-4ac}-b+\sqrt{b^2-4ac}}{2a} \\ &= \frac{-2b}{2a} \\ &= -\frac{b}{a} \end{aligned}$$

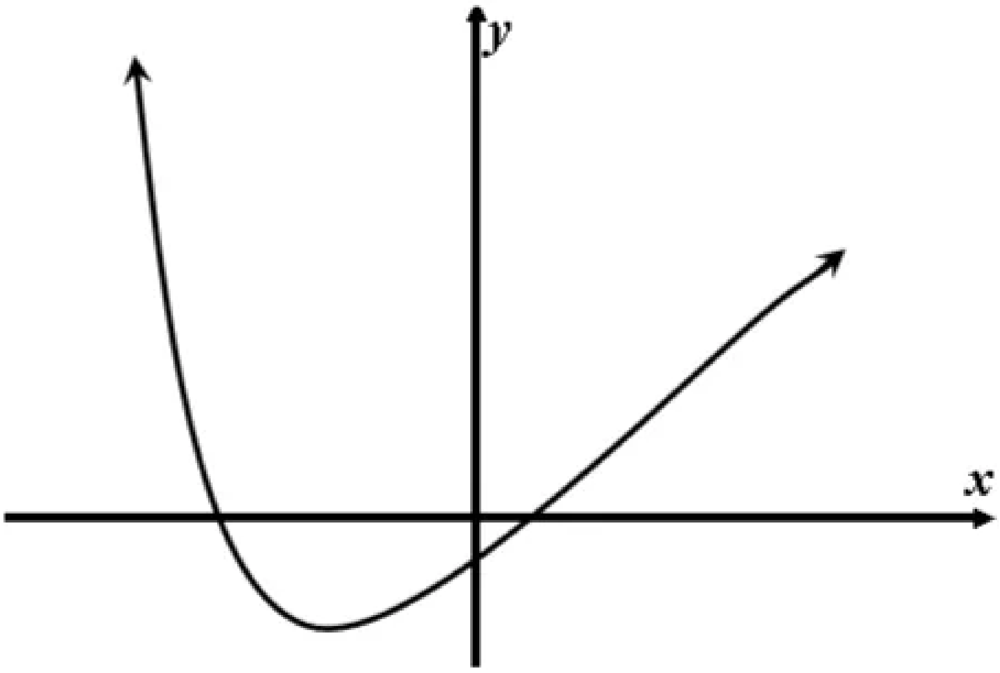
Divide the sum by 2.

$$\frac{-\frac{b}{a}}{2} = -\frac{b}{2a}$$

The mean of the solutions is $-\frac{b}{2a}$, which is the formula for the axis of symmetry.

Answer 52e.

Consider the following graph:



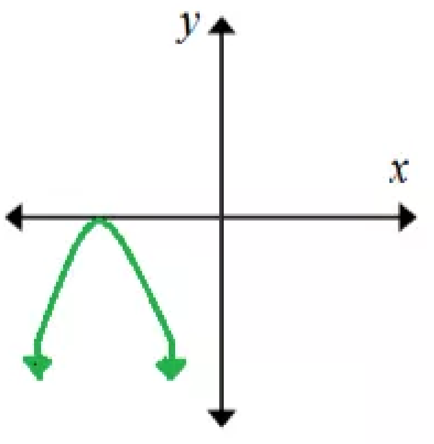
From the graph of the given quadratic function, it is clear that the quadratic function has two real zeros.
Therefore, the discriminant of its corresponding quadratic equation must be positive.

Answer 53e.

The given graph has no x-intercept. Thus, there are two imaginary solutions and $b^2 - 4ac$ is less than 0.
Therefore, the discriminant of $ax^2 + bx + c = 0$ is negative.

Answer 54e.

Consider the graph of a quadratic function,



From the graph of the quadratic function, it is clear that the quadratic function has only one real zero.
Therefore, the discriminant of its corresponding quadratic function must be zero.

Answer 55e.

In the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression $b^2 - 4ac$ is called the discriminant.
Substitute 2 for a, and 5 for b in $b^2 - 4ac$.
$b^2 - 4ac = 5^2 - 4(2)(c)$

Evaluate.
$5^2 - 4(2)(c) = 25 - 8c$

The discriminant is given as -23.
$25 - 8c = -23$

Subtract 25 from both the sides.

$$25 - 8c - 25 = -23 - 25$$
$$-8c = -48$$

Divide both the sides by -8.

$$\frac{-8c}{-8} = \frac{-48}{-8}$$
$$c = 6$$

The correct answer is choice **C**.

Answer 56e.

Consider the quadratic equation,

$$x^2 - 4x + c = 0$$

By comparing the quadratic equation with the standard form of quadratic equation $ax^2 + bx + c = 0$, we get

$$a = 1, b = -4, c = c$$

Then, the discriminant is

$$D = b^2 - 4ac$$

Substituting the values of $a = 1, b = -4, c = c$, we get

$$D = (-4)^2 - 4(1)(c)$$
$$= 16 - 4c$$

(a)

Let us find the value of c such that the quadratic equation will have two real solutions.

Assume $D > 0$, then

$16 - 4c > 0$

$16 > 4c$ Add both sides by 4c

$4c < 16$ Write in reverse order

$c < 4$ Divide each side by 4

$c \in (-\infty, 4)$ Write in interval notation

Thus, the values of c is

$c \in \boxed{(-\infty, 4)}$.

(b)

Now, find c such that the given quadratic equation will have only one real solution.

$D = 0$

$\Rightarrow 16 - 4c = 0$

$\Rightarrow 4c = 16$ Add both sides by 4c

$\Rightarrow c = 4$ Divide each side by 4

Hence, the only real solution is

$c = \boxed{\{4\}}$.

(c)

Also find c such that the given quadratic equation will have two imaginary solutions.

$D < 0$

$\Rightarrow 16 - 4c < 0$

$\Rightarrow 16 < 4c$ Add both sides by 4c

$\Rightarrow 4c > 16$ Write in reverse order

$\Rightarrow c > 4$ Divide each side by 4

$\Rightarrow c \in (4, \infty)$ Write in interval notation

Hence, the solution set of c is

$c \in \boxed{(4, \infty)}$.

Answer 57e.

a) The discriminant is greater than zero if the equation has two real solutions.

$b^2 - 4ac > 0$

Substitute 1 for a and 8 for b in $b^2 - 4ac$.

$8^2 - 4(1)c > 0$

Simplify.

$64 - 4c > 0$

Subtract 64 from both the sides.

$64 - 4c - 64 > 0 - 64$

$-4c > -64$

Divide both the sides by –4.

$\frac{-4c}{-4} < \frac{-64}{-4}$

$c < 16$

b) The discriminant is zero if the equation has one real solution.

$b^2 - 4ac = 0$

Substitute 1 for a, and 8 for b in $b^2 - 4ac$.

$8^2 - 4(1)c = 0$

Simplify.

$64 - 4c = 0$

Subtract 64 from both the sides.

$64 - 4c - 64 = 0 - 64$

$-4c = -64$

Divide both the sides by –4.

$\frac{-4c}{-4} = \frac{-64}{-4}$

$c = 16$

c) The discriminant is less than zero if the equation has two imaginary solutions.

$b^2 - 4ac < 0$

Substitute 1 for a and 8 for b in $b^2 - 4ac$.

$8^2 - 4(1)c < 0$

Simplify.

$64 - 4c < 0$

Subtract 64 from both the sides.

$64 - 4c - 64 < 0 - 64$

$-4c < -64$

Divide both the sides by –4.

$\frac{-4c}{-4} > \frac{-64}{-4}$

$c > 16$

Answer 58e.

Consider the quadratic equation,

$$-x^2+16x+c=0$$

By comparing the quadratic equation with the standard form of quadratic equation $ax^2+bx+c=0$, we get

$$a=-1, b=16, c=c$$

Then, the discriminant is

$$D=b^2-4ac$$

Insert the values of $a=-1, b=16, c=c$, we obtain

$$D=16^2-4(-1)(c)$$
$$=256+4c$$

(a)

Let us find the value of c such that the quadratic equation will have two real solutions.

Assume $D>0$, then

$$256+4c>0$$
$$\Rightarrow \quad 4c>-256$$
$$\Rightarrow \quad c>-64$$

Hence, the solution set of c is

$$c\in\boxed{(-64,\infty)}.$$

(b)

Now, find c such that the given quadratic equation will have only one real solution.

$$D=0$$
$$256+4c=0$$
$$\Rightarrow \quad 4c=-256$$
$$\Rightarrow \quad c=-64$$

Hence, the only solution set of c is

$$c=\boxed{\{-64\}}.$$

(c)

Also find c such that the given quadratic equation will have two imaginary solutions.

$$D<0$$
$$256+4c<0$$
$$\Rightarrow \quad 4c<-256$$
$$\Rightarrow \quad c<-64$$
$$\Rightarrow \quad c\in(-\infty,-64)$$

Hence, the solution set of c is

$$c\in\boxed{(-\infty,-64)}.$$

Answer 59e.

a) The discriminant is greater than zero if the equation has two real solutions.

$b^2 - 4ac > 0$

Substitute 3 for a, and 24 for b in $b^2 - 4ac$.

$24^2 - 4(3)c > 0$

Simplify.

$576 - 12c > 0$

Subtract 576 from both the sides.

$576 - 12c - 576 > 0 - 576$

$-12c > -576$

Divide both the sides by -12.

$$\frac{-12c}{-12} < \frac{-576}{-12}$$

$c < 48$

b) The discriminant is zero if the equation has one real solution.

$b^2 - 4ac = 0$

Substitute 3 for a and 24 for b in $b^2 - 4ac$.

$24^2 - 4(3)c = 0$

Simplify.

$576 - 12c = 0$

Subtract 576 from both the sides.

$576 - 12c - 576 = 0 - 576$

$-12c = -576$

Divide both the sides by -12.

$$\frac{-12c}{-12} = \frac{-576}{-12}$$

$c = 48$

c) The discriminant is less than zero if the equation has two imaginary solutions.

$b^2 - 4ac < 0$

Substitute 1 for a, and 8 for b in $b^2 - 4ac$.

$24^2 - 4(3)c < 0$

Simplify.

$576 - 12c < 0$

Subtract 576 from both the sides.

$$576 - 12c - 576 < 0 - 576$$
$$-12c < -576$$

Divide both the sides by -12.

$$\frac{-12c}{-12} > \frac{-576}{-12}$$
$$c > 48$$

Answer 60e.

Consider the quadratic equation,

$$-4x^2 - 10x + c = 0$$

By comparing the quadratic equation with the standard form of quadratic equation $ax^2 + bx + c = 0$, we get

$$a = -4, b = -10, c = c$$

Then, the discriminant is

$$D = b^2 - 4ac$$

Substituting the values of $a = -4, b = -10, c = c$, we get

$$D = (-10)^2 - 4(-4)(c)$$
$$D = 100 + 16c$$

(a)

Let us find the value of c such that the quadratic equation will have two real solutions.

Assume $D > 0$, then

$$100 + 16c > 0$$

$\Rightarrow \quad 16c > -100$ Subtract 100 to each side

$\Rightarrow \quad c > -\frac{25}{4}$ Divide both sides by 16

$\Rightarrow \quad c \in \left(-\frac{25}{4}, \infty\right)$ Write in interval notation

Hence, the solution set of c is

$$c \in \boxed{\left(-\frac{25}{4}, \infty\right)}.$$

(b)

Now, find c such that the given quadratic equation will have only one real solution.

$$D = 0$$

$\Rightarrow 100 + 16c = 0$

$\Rightarrow \quad 16c = -100$ Subtract 100 to each side

$\Rightarrow \quad c = -\frac{25}{4}$ Divide both sides by 16

Hence, the only the only real solution is

$$c = \boxed{\left\{-\frac{25}{4}\right\}}.$$

(c)

Also find c such that the given quadratic equation will have two imaginary solutions.

$$D<0$$
$$\Rightarrow 100+16c<0$$
$$\Rightarrow \quad 16c<-100 \qquad \text{Subtract 100 to each side}$$
$$\Rightarrow \quad c<-\frac{25}{4} \qquad \text{Divide both sides by 16}$$
$$\Rightarrow c\in\left(-\infty,-\frac{25}{4}\right) \qquad \text{Write in interval notation}$$

Hence, the solution set of c is

$$c\in\boxed{\left(-\infty,-\frac{25}{4}\right)}.$$

Answer 61e.

a) The discriminant is greater than zero if the equation has two real solutions.

$b^2-4ac > 0$

Substitute 1 for a and -1 for b in b^2-4ac.

$(-1)^2-4(1)c > 0$

Simplify.

$1-4c > 0$

Subtract 576 from both the sides.

$1-4c-1 > 0-1$

$-4c > -1$

Divide both the sides by -4.

$$\frac{-4c}{-4} < \frac{-1}{-4}$$

$$c < \frac{1}{4}$$

b) The discriminant is zero if the equation has one real solution.

$b^2-4ac = 0$

Substitute 1 for a and -1 for b in b^2-4ac.

$(-1)^2-4(1)c = 0$

Simplify.

$1-4c = 0$

Subtract 576 from both the sides.

$1-4c-1 = 0-1$

$-4c = -1$

Divide both the sides by -12.

$$\frac{-4c}{-4} = \frac{-1}{-4}$$

$$c = \frac{1}{4}$$

c) The discriminant is less than zero if the equation has two imaginary solutions.

$b^2 - 4ac < 0$

Substitute 1 for a and -1 for b in $b^2 - 4ac$.

$(-1)^2 - 4(1)c < 0$

Simplify.

$1 - 4c < 0$

Subtract 576 from both the sides.

$1 - 4c - 1 < 0 - 1$

$-4c < -1$

Divide both the sides by -4.

$$\frac{-4c}{-4} > \frac{-1}{-4}$$

$$c > \frac{1}{4}$$

Answer 62e.

Consider the quadratic equation,

$$13x^2 + 4x + \frac{1}{2} = 0$$

By comparing the quadratic equation with the standard form of quadratic equation $ax^2 + bx + c = 0$, we get.

$$a = 13, b = 4, c = \frac{1}{2}$$

Then, the discriminant is

$$D = b^2 - 4ac$$

Insert the values of $a = 13, b = 4, c = \frac{1}{2}$, we get

$$D = 4^2 - 4(13)\left(\frac{1}{2}\right)$$

$$= 16 - 26$$

$$= -10$$

Hence, the quadratic equation whose discriminant -10 is

$$\boxed{13x^2 + 4x + \frac{1}{2} = 0}.$$

Answer 63e.

It is given that -4 and 3 are the two solutions of the equation in the form $ax^2 + bx + c = 0$.
The solutions can be written as
$x = -4$ and $x = 3$.

Rewrite the equations with 0 on the right side.
$x + 4 = 0$ and $x - 3 = 0$

Use the Zero product property.
$(x + 4)(x - 3) = 0$

Remove the parentheses using FOIL method.

$$x^2 - 3x + 4x - 12 = 0$$
$$x^2 - x - 12 = 0$$

The equation obtained has the constant term -12. But it should be 4.

Multiply both the sides of the equation by $-\frac{1}{3}$.

$$\frac{1}{3}\left(x^2 - x - 12\right) = \frac{1}{3}(0)$$
$$-\frac{1}{3}x^2 - \frac{1}{3}x + 4 = 0$$

Therefore, the quadratic equation that has the given solutions is $-\frac{1}{3}x^2 - \frac{1}{3}x + 4 = 0$.

Answer 64e.

Consider the solutions of a quadratic equation,

$$-\frac{4}{3} \text{ and } -1$$

Write a quadratic equation whose roots $-\frac{4}{3}$ and -1:

Since the quadratic equation whose roots are p and q is

$$x^2 - (p+q)x + pq = 0$$

Let $p = -\frac{4}{3}$ and $q = -1$

Insert the values $p = -\frac{4}{3}$ and $q = -1$, we get

$$x^2 - \left(-\frac{4}{3} - 1\right)x + \left(-\frac{4}{3}\right)(-1) = 0$$
$$x^2 + \frac{7}{3}x + \frac{4}{3} = 0$$
$$3x^2 + 7x + 4 = 0$$

Hence, the required quadratic equation is

$$\boxed{3x^2 + 7x + 4 = 0}.$$

Answer 65e.

The given solutions can be written as
$x = -1 + i$ and $x = -1 - i$.

Rewrite the equations with 0 on the right side.
$x + 1 - i = 0$ and $x + 1 + i = 0$

Use the Zero product property.
$(x + 1 - i)(x + 1 + i) = 0$

Remove the parentheses by multiplying.

$$x^2 + x + ix + x + 1 + i - ix - i - i^2 = 0$$
$$x^2 + 2x + 1 - i^2 = 0$$

We know that the value of i^2 is -1.
Substitute the value of i^2 in the equation and simplify.

$$x^2 + 2x + 1 - (-1) = 0$$
$$x^2 + 2x + 2 = 0$$

The equation obtained has the constant term 2. It is given that the constant term should be 4.
Multiply both the sides of the equation by 2.

$$2(x^2 + 2x + 2) = 2(0)$$
$$2x^2 + 4x + 4 = 0$$

Therefore, the quadratic equation that has the given solutions is $2x^2 + 4x + 4 = 0$.

Answer 66e.

Consider the quadratic equation,

$$ax^2 + bx + c = 0$$

Where a, b and c are real constants.

Let us show there is no quadratic equation $ax^2 + bx + c = 0$ such that are real numbers and $3i, -2i$ are solutions.

Since the sum of the roots of $ax^2 + bx + c = 0$ is

$$-\frac{b}{a}$$

Now the sum of the roots of $3i, -2i$ is

$$3i + (-2i) = -\frac{b}{a}$$ Since sum of the roots is $-\frac{b}{a}$

$$i = -\frac{b}{a}$$ Simplify

$$\frac{b}{a} = -i \notin \mathbb{R}$$ This is a contradiction

Hence, there is **no quadratic equation** $ax^2 + bx + c = 0$ such that are real numbers and $3i, -2i$ are solutions.

Answer 67e.

Write the left side in the form $ax^2 + bx$.
Subtract c from both the sides of $ax^2 + bx + c = 0$.

$$ax^2 + bx + c - c = 0 - c$$
$$ax^2 + bx = -c$$

Divide each side by the coefficient of x^2, a.

$$\frac{ax^2 + bx}{a} = \frac{-c}{a}$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Square half the coefficient of x.

$$\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

Add $\frac{b^2}{4a^2}$ to each side of $x^2 + \frac{b}{a}x = -\frac{c}{a}$.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

Write the left side as a binomial squared.

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

Take the square root of each side.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}}$$
$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

Subtract $\frac{b}{2a}$ from each side and solve for x.

$$x + \frac{b}{2a} - \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$$
$$x = \pm\frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$$
$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Answer 68e.

Because the ball is batted, we use the model $h = -16t^2 + v_0 t + h_0$.

To find how long do the defensive player's teammates have to intercept the ball before it hits the ground, we solve for t when $h = 0$.

$h = -16t^2 + v_0 t + h_0$ [Write height model]

$0 = -16t^2 + (-50)t + 7$ [Substitute -50 for v_0 and 7 for h_0]

$0 = -16t^2 - 50t + 7$ [Simplify]

$t = \dfrac{-(-50) \pm \sqrt{(-50)^2 - 4(-16)(7)}}{2(-16)}$ [Quadratic formula]

$t = \dfrac{50 \pm \sqrt{2948}}{-32}$ [Simplify]

$t \approx -3.26$ or $t \approx 0.134$ [Use a calculator]

We reject the solution -3.26 because the ball's time cannot be negative.

Hence, the defensive player's teammates have to intercept the ball before it hits the ground for about

$\boxed{0.134 \text{ seconds}}$.

Answer 69e.

The number S is given in thousands. Substitute 50,000 for S in the given model.

$50{,}000 = 858t^2 + 1412t + 4982$

Subtract 50,000 from both the sides.

$$50{,}000 - 50{,}000 = 858t^2 + 1412t + 4982 - 50{,}000$$
$$0 = 858t^2 + 1412t - 45{,}018$$

Use the quadratic formula, $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find the value of t.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1412 \pm \sqrt{(1412)^2 - 4(858)(-45018)}}{2(858)}$$
$$= \frac{-1412 \pm \sqrt{1{,}993{,}744 + 154{,}501{,}776}}{1716}$$
$$= \frac{-1412 \pm \sqrt{156{,}495{,}520}}{1716}$$

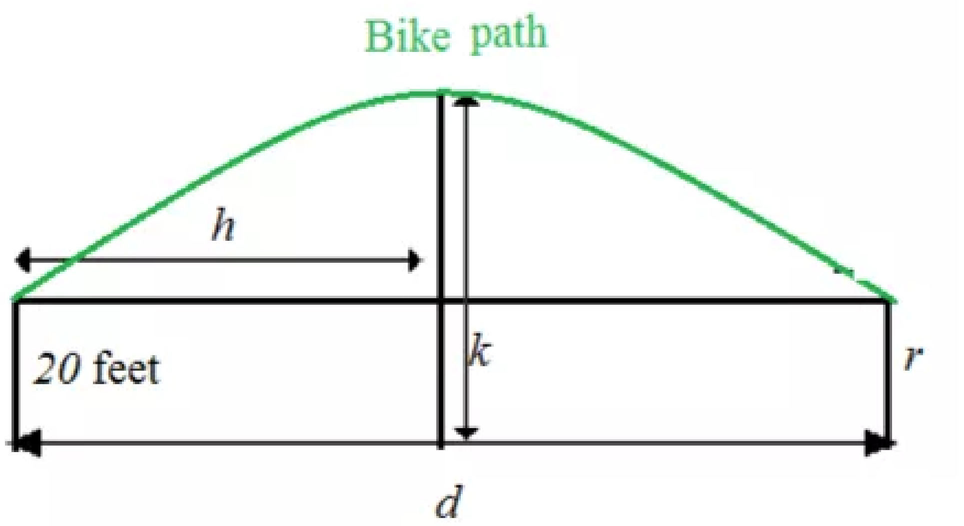
Use a calculator to evaluate.
$t \approx 6$ and $t \approx -8$
Since t represents the number of years since 1990, it cannot be negative.

Add 6 to 1990 to find the number of subscribers that reached 50 million.
$1990 + 6 = 1996$

Therefore, the number of subscribers reached 50 million in 1996.

Answer 70e.

Consider the diagram,



From the diagram, we observe that a stunt motorcyclist makes a jump from one ramp 20 feet off the ground to other ramp 20 feet off the ground.
The jumps between the ramps can be modeled by

$$y = -\frac{1}{640}x^2 + \frac{1}{4}x + 20$$

Where x the horizontal is distance (in feet) and y is the height above the ground (in feet).

(a)
The motorcycle's height r when it lands on the ramp is 20 feet which are same as the height of the ramp.
Hence,

$$r = \boxed{20 \text{ feet}}.$$

(b)

To find the distance between the ramps d,

Let us solve $y = 20$

$$y = 20$$

$$-\frac{1}{640}x^2 + \frac{1}{4}x + 20 = 20 \quad \text{Substitute } y = 20$$

$$\frac{1}{640}x^2 = \frac{1}{4}x \quad \text{Cancel out the similar values}$$

$$x^2 = 160x \quad \text{Simplify}$$

$$x(x-16) = 0 \quad \text{Factor out}$$

$$x = 0 \text{ or } x - 16 = 0 \quad \text{Set the factors equals to zero}$$

$$x = 16$$

Hence,

$$d = \boxed{160 \text{ feet}}.$$

(c)

Let us rewrite the function $y = -\frac{1}{640}x^2 + \frac{1}{4}x + 20$ in vertex form.

$$y = 20 - \frac{1}{640}(x^2 - 160x)$$

$$y = 20 - \frac{1}{640}\left(x^2 - 160x + \left(-\frac{160}{2}\right)^2\right) + \frac{1}{640}\left(-\frac{160}{2}\right)^2$$

$$y = 30 - \frac{1}{640}(x-80)^2$$

$$y = -\frac{1}{640}(x-80)^2 + 30$$

Thus, the horizontal distance where the motorcycle has reached maximum height is

$$h = \boxed{80 \text{ feet}}.$$

(d)

From the equation $y = -\frac{1}{640}(x-80)^2 + 30$, we get

The maximum height k above the ground is

$$k = \boxed{30 \text{ feet}}.$$

Answer 71e.

Substitute 10 for S in the given model.

$10 = -0.000013E^2 + 0.042E - 21$

Subtract 10 from both the sides.

$$10 - 10 = -0.000013E^2 + 0.042E - 21 - 10$$

$$0 = -0.000013E^2 + 0.042E - 31$$

Use the quadratic formula, $E = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find the value of E.

$$E = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-0.042 \pm \sqrt{(0.042)^2 - 4(-0.000013)(-31)}}{2(-0.000013)}$$

$$= \frac{-0.042 \pm \sqrt{0.001764 - 0.001612}}{-0.000026}$$

$$= \frac{-0.042 \pm \sqrt{0.000152}}{-0.000026}$$

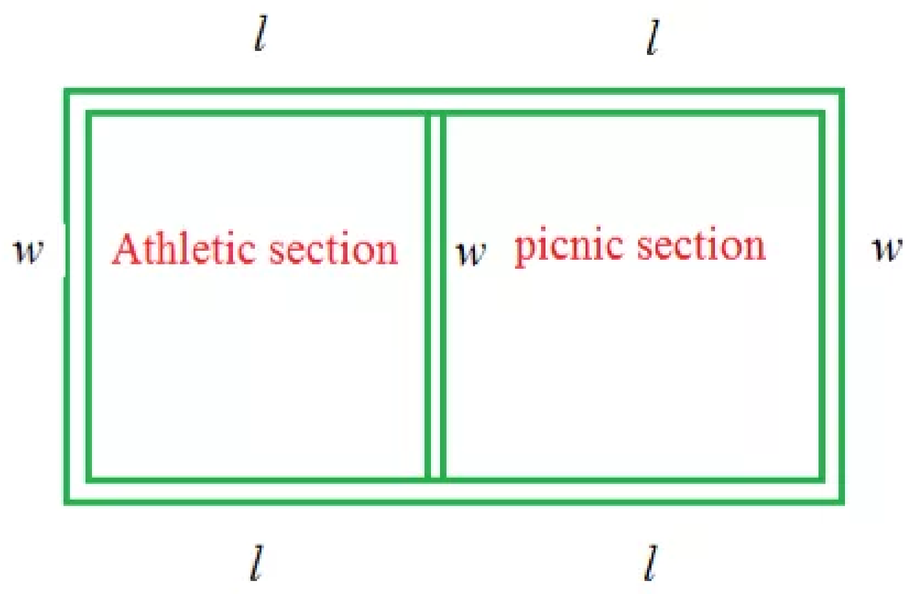
Use a calculator to evaluate.

$E \approx 1141$ or $E \approx 2090$

Therefore, the expected elevation is about 1141 meters or about 2090 meters.

Answer 72e.

Consider the diagram,



(a)
From the diagram, there is 900 feet of fencing available.
Thus, the total area is 900feet
Now,

$$l+l+l+l+w+w+w=900 \quad \text{Add the total area}$$
$$4l+3w=900 \quad \text{Simplify}$$
$$\frac{4}{3}l+w=300 \quad \text{Divide each side by 3}$$
$$w=300-\frac{4l}{3}. \quad \text{Subtract each side by } \frac{4l}{3}$$

Hence,

$$w=\boxed{300-\frac{4l}{3}}.$$

(b)
The area of each section is $12,000$ square feet.
Since the area of a rectangle is

$$A=lw$$

Now,

$$lw=12000$$
$$l\left(300-\frac{4}{3}l\right)=12000 \quad \text{Substitute } w=300-\frac{4l}{3}$$
$$-\frac{4}{3}l^2+300l=12000 \quad \text{Multiply}$$
$$4l^2-900l+36000=0 \quad \text{Simplify}$$
$$l^2-225l+9000=0 \quad \text{Divide each side by 4}$$

On comparing the quadratic equation $l^2-225l+9000=0$ with $ax^2+bx+c=0$, we get

$$a=1, b=-225, \text{ and } c=9000$$

Using the quadratic formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Substitute the values in quadratic formula, we get

$$l=\frac{225\pm\sqrt{14625}}{2}$$
$$l=\frac{225\pm15\sqrt{65}}{2}$$

Thus, the possible dimensions are

When $l=\dfrac{225+15\sqrt{65}}{2}$

$$w=300-\frac{4}{3}\left(\frac{225+15\sqrt{65}}{2}\right)$$ Since $w=300-\dfrac{4l}{3}$

$$=300-2\left(75+5\sqrt{65}\right)$$

$$=150-10\sqrt{65}$$

Similarly:

When $l=\dfrac{225-15\sqrt{65}}{2}$, then

$$w=150+10\sqrt{65}$$

Hence, the required dimensions are

$l=\dfrac{225+15\sqrt{65}}{2}$, $w=150-10\sqrt{65}$ and $l=\dfrac{225-15\sqrt{65}}{2}$, $w=150+10\sqrt{65}$.

Answer 73e.

a. Substitute 0 for t in $x=20t$ and solve for x.

$$x = 20(0)$$
$$= 0$$

Substitute 0 for t in $y=-16t^2+21t+6$ and solve for y.

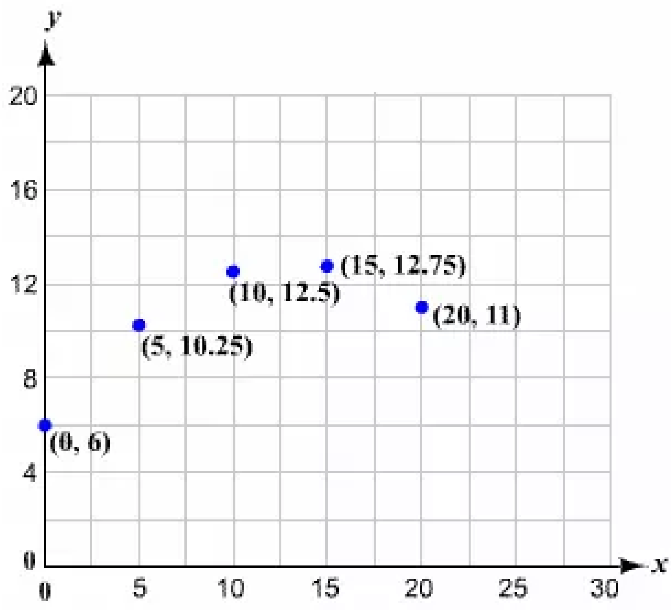
$$y = -16(0)^2+21(0)+6$$
$$= 6$$

When $t=0$, the point (x, y) is at $(0, 6)$.

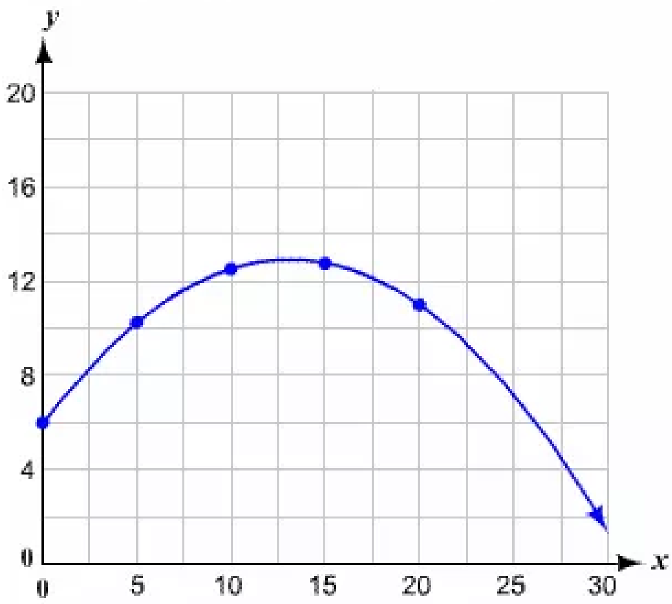
Similarly, we can find (x, y) for the remaining given values of t.
Organize the results in a table.

t	$x = 20t$	$y = -16t^2+21t+6$	(x, y)
0	$20(0)=0$	$-16(0)^2+21(0)+6=6$	$(0, 6)$
0.25	$20(0.25)=5$	$-16(0.25)^2+21(0.25)+6=10.25$	$(5, 10.25)$
0.5	$20(0.5)=10$	$-16(0.5)^2+21(0.5)+6=12.5$	$(10, 12.5)$
0.75	$20(0.75)=51$	$-16(0.75)^2+21(0.75)+6=12.75$	$(51, 12.75)$
1	$20(1)=20$	$-16(1)^2+21(1)+6=11$	$(20, 11)$

b. Plot the points (0, 6), (5, 10.25), (10, 12.5), (51, 12.75), and (20, 11) on a coordinate plane.



Join the points with a smooth curve.



c. From the table obtained in part (a), the y-value is 12.75 ft when the value of x is 15. It should be 10 ft to make a free throw.
Since the height of the ball is above the backboard, the player does not make the free throw.

Answer 74e.

The height h (in feet) of a rider on the Big Shot can be modeled by

$$h = -16t^2 + v_0 t + 921$$

where t is the elapsed time (in seconds) after launch and v_0 is the initial vertical velocity (in feet per second).

a.

We have to find v_0 using the fact that the maximum value of h is $921+160=1081$ feet.

We are going to try rewriting the model of the height $h=-16t^2+v_0t+921$ as a vertex form so that we use the fact that the maximum value of h is $921+160=1081$ feet.

$$h=-16t^2+v_0t+921 \qquad \text{[The given model]}$$

$$h=-16\left(t^2-v_0t+10\right)+1081 \qquad \text{[Adjust the right side]}$$

$$h=-16\left(t^2-2\sqrt{10}t+10\right)+1081 \qquad \begin{bmatrix}\text{For making a quadratic equation} \\ \text{we must substitute for } v_0\end{bmatrix}$$

$$h=-16\left(t-\sqrt{10}\right)^2+1081 \qquad \text{[Vertex form for the height]}$$

The vertex form $h=-16\left(t-\sqrt{10}\right)^2+1081$ for the height shows that the maximum height of h is $921+160=1081$ feet .

Hence, the initial vertical velocity, v_0 , is

$\boxed{2\sqrt{10} \text{ feet per second or about 6.325 feet per second}}$.

b.

We substitute 2 seconds for v_0 in the model $h=-16t^2+v_0t+921$.

$$h=-16t^2+v_0t+921 \qquad \text{[The given model]}$$

$$h=-16t^2+2t+921 \qquad \text{[Substitute 2 for } v_0\text{]}$$

$$h=-16\left(t^2-\frac{1}{8}t+\left(\frac{1}{16}\right)^2\right)+\frac{14737}{16} \qquad \text{[Adjust the right side]}$$

$$h\approx-16\left(t^2-\frac{1}{8}t+\left(\frac{1}{16}\right)^2\right)+921.0625 \qquad \text{[Simplify]}$$

$$h=-16\left(t-\frac{1}{16}\right)^2+921.0625 \qquad \text{[Vertex form for the height]}$$

The vertex form $h=-16\left(t-\frac{1}{16}\right)^2+921.0625$ for the height shows that the maximum height of h is 921.0625 feet which $\boxed{\text{is not same}}$ as we did in the part (a).

Answer 75e.

Find the ratio of the vertical change to the horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (2, −7) for (x_1, y_1), and (4, 9) for (x_2, y_2) and evaluate.

$$\begin{aligned} m &= \frac{9-(-7)}{4-2} \\ &= \frac{9+7}{4-2} \\ &= \frac{16}{2} \\ &= 8 \end{aligned}$$

The slope of the line that passes through (2, −7) and (4, 9) is 8.

The slope of the given line is positive.
Therefore, the line passing through the given points rises from left to right.

Answer 76e.

Consider the points,

$$(-8,3) \text{ and } (4,-5)$$

Let us find the slope of the line passing through the points.
Since the slope line joining the two points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Insert the points, we get
The slope of the line is

$$\begin{aligned} m &= \frac{-5-3}{4-(-8)} \\ &= \frac{-8}{12} \\ &= -\frac{2}{3} \\ &< 0 \end{aligned}$$

Therefore, the line passing through the given two points falls.

Answer 77e.

Find the ratio of the vertical change to the horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (−3, −2) for (x_1, y_1), and (6, −2) for (x_2, y_2) and evaluate.

$$\begin{aligned} m &= \frac{-2-(-2)}{6-(-3)} \\ &= \frac{-2+2}{6+3} \\ &= \frac{0}{9} \\ &= 0 \end{aligned}$$

The slope of the line that passes through (−3, −2) and (6, −2) is 0.

The slope of the given line is 0.
Therefore, the line passing through the given points is horizontal.

Answer 78e.

Consider the points,

$\left(\frac{3}{4}, 2\right)$ and $\left(\frac{1}{2}, \frac{5}{4}\right)$

Find the slope of the line passing through the points $\left(\frac{3}{4}, 2\right)$ and $\left(\frac{1}{2}, \frac{5}{4}\right)$

Here, $(x_1, y_1) = \left(\frac{3}{4}, 2\right)$ and $(x_2, y_2) = \left(\frac{1}{2}, \frac{5}{4}\right)$

Then, slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\frac{5}{4} - 2}{\frac{1}{2} - \frac{3}{4}}$$ Substitute the points $\left(\frac{3}{4}, 2\right)$ and $\left(\frac{1}{2}, \frac{5}{4}\right)$

$$= \frac{-\frac{3}{4}}{-\frac{1}{4}}$$

$= 3$ Cancel out common terms

> 0

Therefore the line passing through the given two points **rises.**

Answer 79e.

Find the ratio of vertical change to horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (−1, 0) for (x_1, y_1), and (−1, 5) for (x_2, y_2) and evaluate.

$$\begin{aligned} m &= \frac{5 - 0}{-1 - (-1)} \\ &= \frac{5 - 0}{-1 + 1} \\ &= \frac{5}{0} \\ &= \text{undefined} \end{aligned}$$

The slope of the line that passes through (−1, 0) and (−1, 5) is undefined.

The slope of the given line is undefined.
Therefore, the line passing through the given points is vertical.

Answer 80e.

Consider the points,

$\left(\frac{1}{3}, \frac{7}{3}\right)$ and $\left(4, \frac{2}{3}\right)$

Find the slope of the line passing through the points $\left(\frac{1}{3}, \frac{7}{3}\right)$ and $\left(4, \frac{2}{3}\right)$.

Here, $(x_1, y_1) = \left(\frac{1}{3}, \frac{7}{3}\right)$ and $(x_2, y_2) = \left(4, \frac{2}{3}\right)$

Then, slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\frac{2}{3} - \frac{7}{3}}{4 - \frac{1}{3}} \quad \text{Substitute the points } \left(\frac{3}{4}, 2\right) \text{ and } \left(\frac{1}{2}, \frac{5}{4}\right)$$

$$= \frac{-\frac{5}{3}}{\frac{11}{3}} \quad \text{Simplify}$$

$$= -\frac{5}{11}$$

$$< 0$$

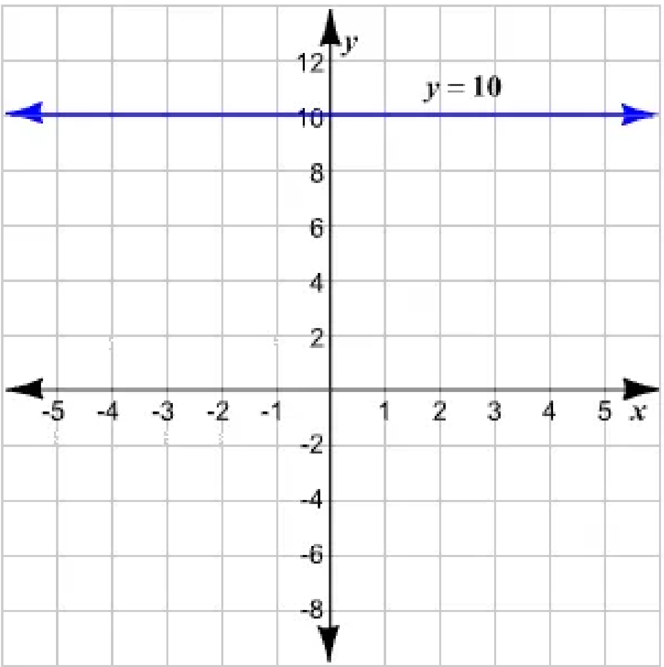
Therefore, the line passing through the given two points **falls.**

Answer 81e.

STEP 1 **Graph the boundary line of the inequality.**

In order to obtain the boundary line, replace the inequality sign with = sign. Then, we get an equation of the form $y = 10$.

Graph $y = 10$ on a coordinate plane. Since $\leq$ is the inequality sign used, draw a solid line.

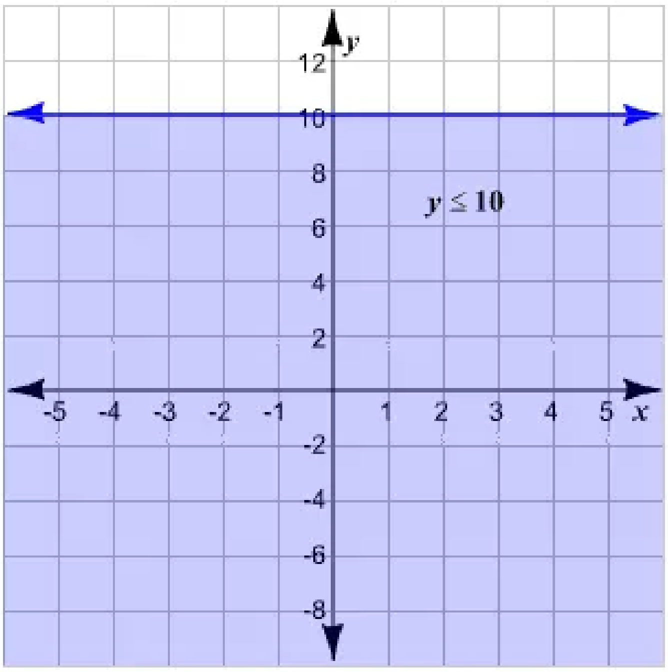


STEP 2 **Test a point.**

Let us take a test point (0, 0) which does not lie on the boundary line. Substitute 0 for y. Check if the test point satisfies the given inequality.

$0 \overset{?}{\leq} 10$ TRUE

The test point is a solution. Shade the half-plane that contains the point (0, 0).



Answer 82e.

Consider the inequality,

$$8x - 4y < -16$$

Sketch the graph of the inequality,

$$8x - 4y < -16$$

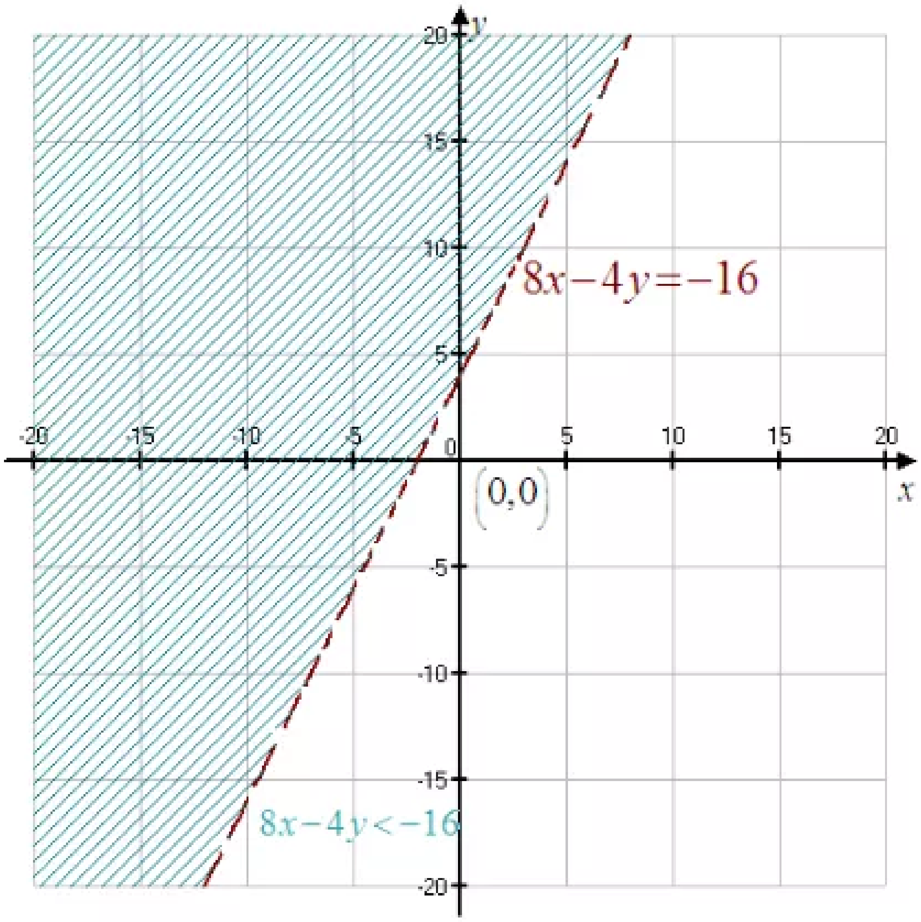
Draw a boundary line $8x - 4y = -16$

Sketch boundary line as dashed line, because the symbol is $<$

the point $(0,0)$ is not solution of the inequality.

Therefore, expect that point the shaded region represent the graph of the inequality,

$$8x - 4y < -16$$



Answer 83e.

STEP 1 **Graph the boundary line of the inequality.**

In order to obtain the boundary line, replace the inequality sign with "=."Then, we get an equation of the form $\frac{1}{2}x + 3y = 8$.

Substitute 0 for y in above equation and solve for x.

$$\frac{1}{2}x + 3(0) = 8$$

$$\frac{1}{2}x = 6$$

$$x = 12$$

The x-intercept is 12. A point that can be plotted on the graph is (12, 0).

Next, replace x with -2 and solve for y.

$$\frac{1}{2}(-2) + 3y = 8$$

$$-1 + 3y = 8$$

$$3y = 9$$

$$y = 3$$

Since the y-intercept is 3, another point that can be plotted on the graph is (–2, 3).

We can find a third point by replacing x with 4.

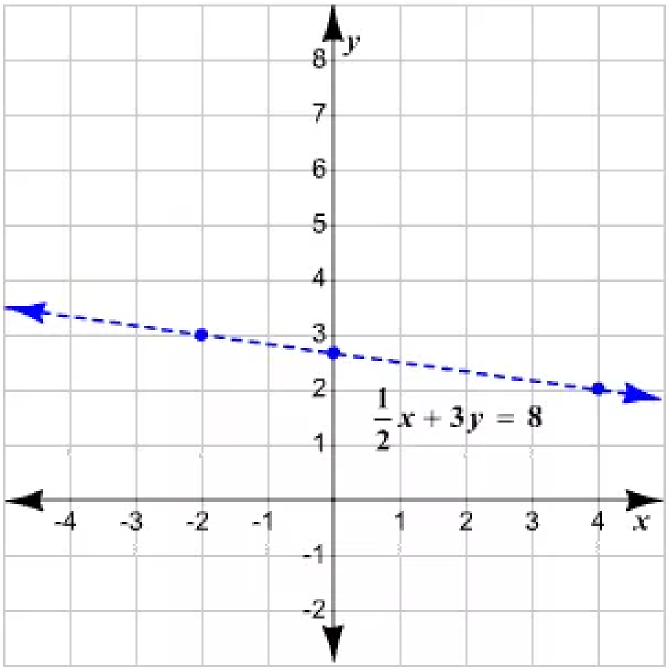
$$\frac{1}{2}(4) + 3y = 8$$

$$2 + 3y = 8$$

$$3y = 6$$

$$y = 2$$

Plot (–2, 3), (12, 0), and (4, 2) on the graph and draw a line passing through them. Since > is the inequality sign used, draw a dashed line.



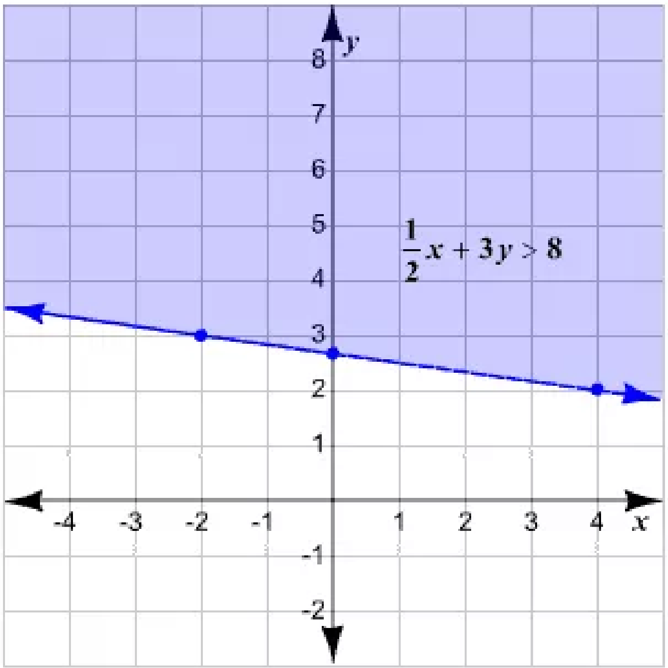
STEP 2 **Test a point.**

Let us take a test point which does not lie on the boundary line, say, (0, 0). Substitute 0 for y, and 0 for x. Check if the test point satisfies the given inequality.

$$\frac{1}{2}(0)+3(0) \overset{?}{>} 8$$

$$0 > 8 \qquad \text{FALSE}$$

The test point is not a solution to the inequality. Shade the half-plane that does not contain the point (0, 0).



Answer 84e.

Consider the inequality,

$$y \geq -\frac{4}{9}x-7$$

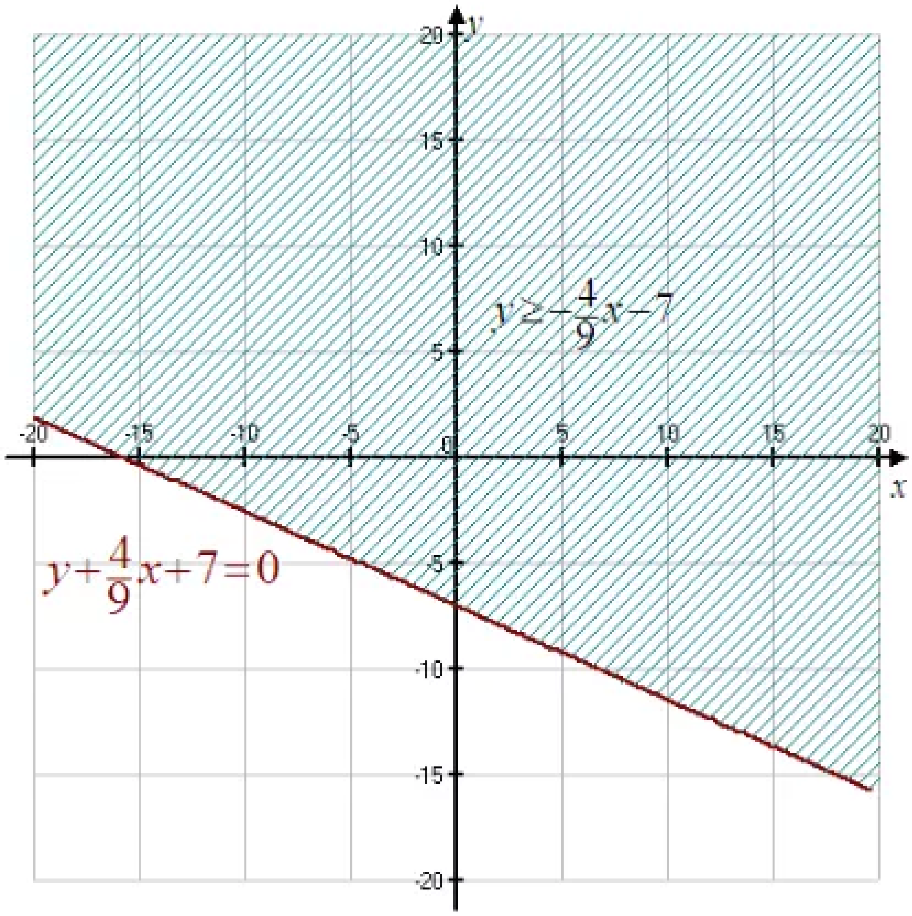
Sketch the graph of the inequality,

$$y \geq -\frac{4}{9}x-7$$

Draw the boundary line $y=-\frac{4}{9}x-7$ as a solid line,

Because, the inequality has symbol $\geq$

In the graph the shaded region with the boundary line represents the graph of the inequality, $y \geq -\frac{4}{9}x - 7$



Answer 85e.

STEP 1 The intercept form of a quadratic function is $y = a(x - p)(x - q)$, where p and q are the x-intercepts and $x = \frac{p + q}{2}$ is the axis of symmetry.

In order to graph the given function, first we have to identify the x-intercepts.
On comparing the given equation with the intercept form, we find that $a = 3, p = -1$, and $q = -2$. Thus, the x-intercepts occur at $(-1, 0)$ and $(-2, 0)$. Since $a > 0$, the parabola opens up.

STEP 2 Then, find the coordinates of the vertex. Substitute for p and q in $x = \frac{p+q}{2}$ and evaluate.

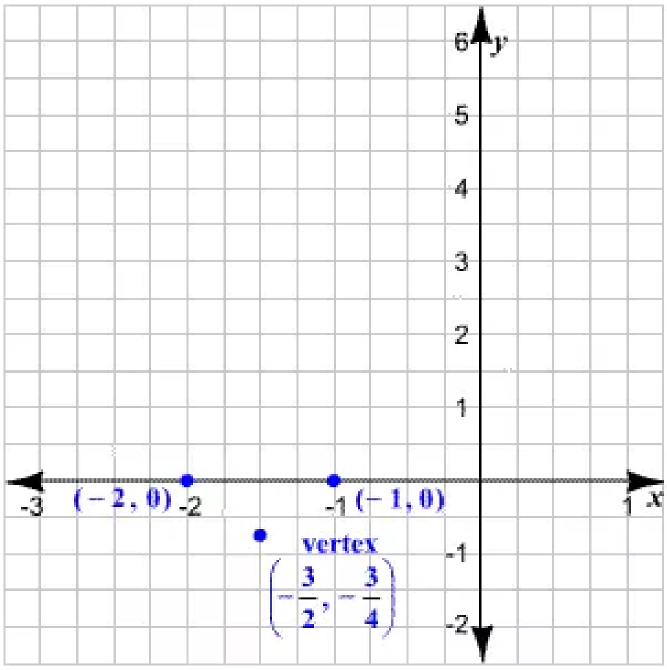
$$x = \frac{-1+(-2)}{2} = -\frac{3}{2}$$

Substitute $-\frac{3}{2}$ for x in the given function and evaluate y.

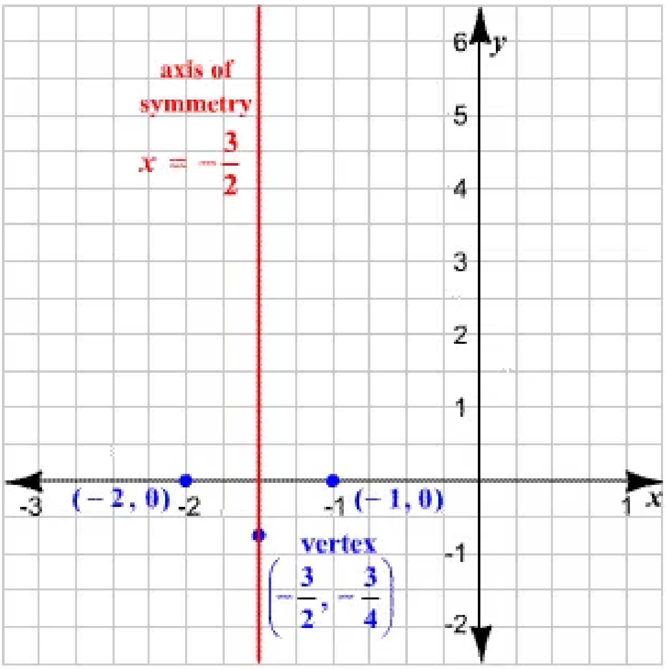
$$y = 3\left(-\frac{3}{2}+1\right)\left(-\frac{3}{2}+2\right)$$
$$= 3\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$$
$$= -\frac{3}{4}$$

Thus, the vertex is $\left(-\frac{3}{2}, -\frac{3}{4}\right)$.

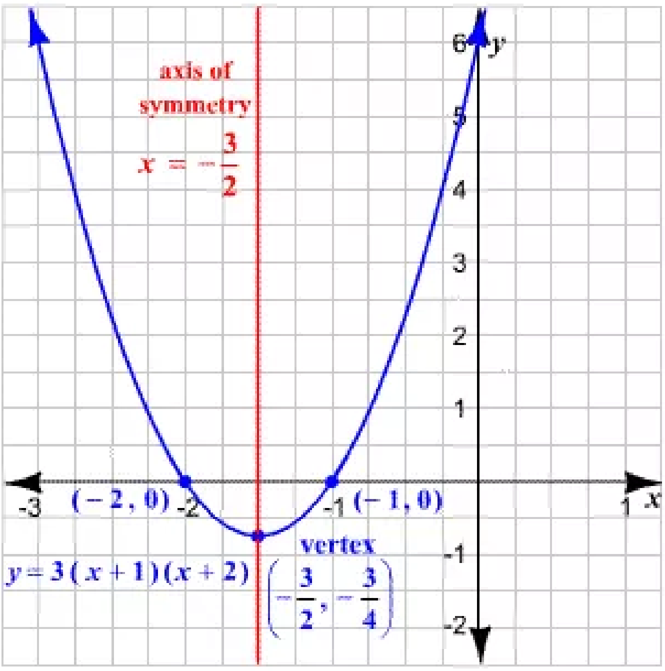
STEP 3 Now, plot the vertex and the points where the x-intercepts occur on a coordinate plane.



Draw the axis of symmetry $x = -\dfrac{3}{2}$ on the same coordinate plane.



Draw a parabola through the points plotted.



Answer 86e.

Consider the equation,

$$y = -2(x-3)(x-1)$$

Sketch the graph of the following equation.

$y = -2(x-3)(x-1)$

STEP 1:

The equation of the form

$y = a(x-p)(x-q)$

Identify the $x-$ intercepts.

Because $p = 3$ and $q = 1$, the $x-$ intercepts occur at the points $(3,0)$ and $(1,0)$

STEP 2:

Find the coordinates of the vertex.

$$x = \frac{p+q}{2}$$

$$= \frac{3+1}{2}$$ Substitute the values of *p*, *q*

$$= \frac{4}{2}$$

$$= 2$$

$y = -2(x-3)(x-1)$

$y = -2(2-3)(2-1)$ Substitute the value of *x*

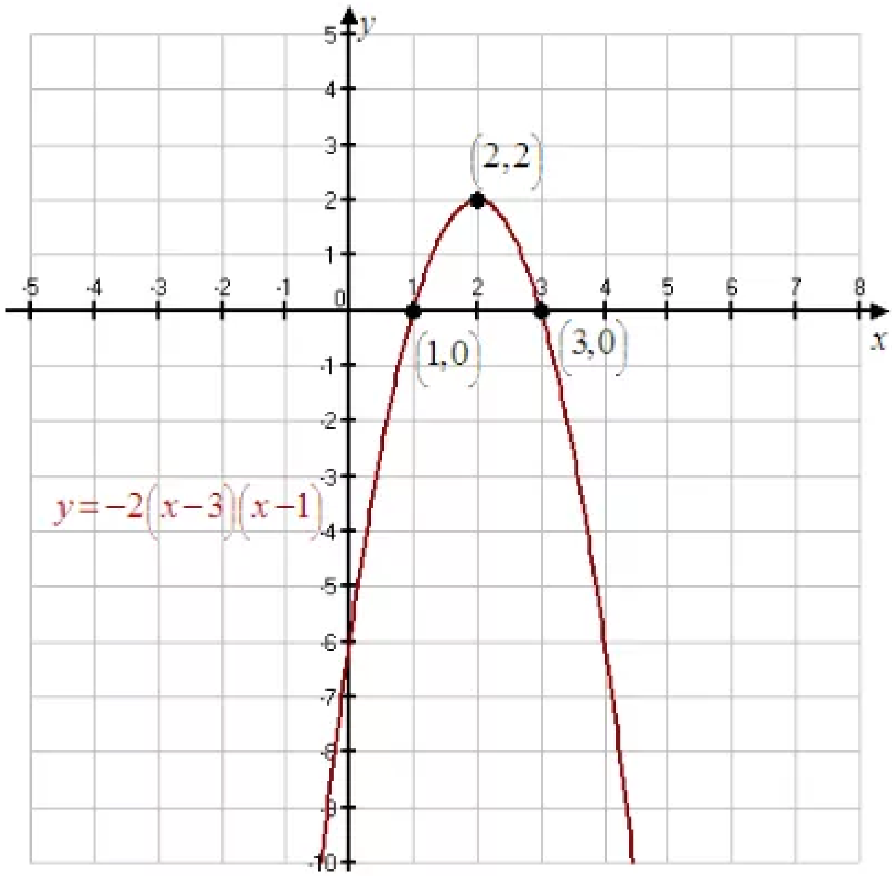
$$= -2(-1) \cdot 1$$

$$= 2$$

Therefore, the vertex is $\boxed{(2,2)}$

STEP 3:

Draw a parabola through the vertex and the points where the $x-$ intercepts occur.



Answer 87e.

Substitute 0 for t in the given model and simplify.

$$\begin{aligned} a &= 2000 - 250(0) \\ &= 2000 - 0 \\ &= 2000 \end{aligned}$$

The height of the hang glider prior to the descent is 2000 feet.

Substitute 0 for a in the given model and solve for t.

$$\begin{aligned} 0 &= 2000 - 250t \\ -2000 &= -250t \\ 8 &= t \end{aligned}$$

The hang glider takes 8 minutes to reach the ground.