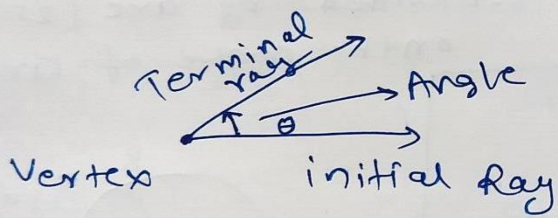
Class-11 Maths

Chapter-3 Trigonometric Functions

Exercise 3.1 * Basics of Angles

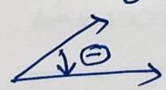
Angles: measurement of rotation between two arms (rays)



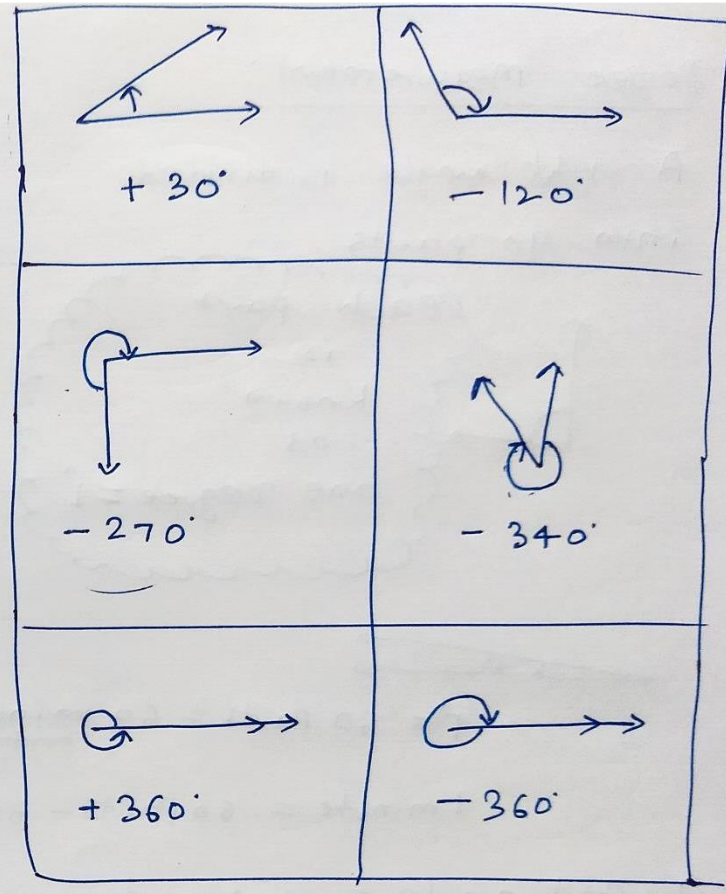
Angle

- Positive angle
- Negative Angle

anticlockwise



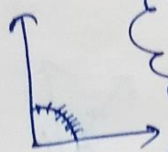
Clockwise



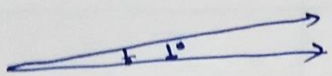
One complete revolution

Degree Measurement:

A right angle is divided into 90 parts.



each part is Known as one Degree $= 1^\circ$



$1^\circ = 60$ parts $= 60$ minutes $= 60'$

1 minute $= 60$ parts $= 60$ seconds $= 60''$

1 right angle $= 90^\circ = (90\times 60)' = (90\times 60\times 60)''$

General language — 1 right angle; Degree — 90°; minute — $(90\times 60)'$; seconds — $(90\times 60\times 60)''$

Radian Measurement:

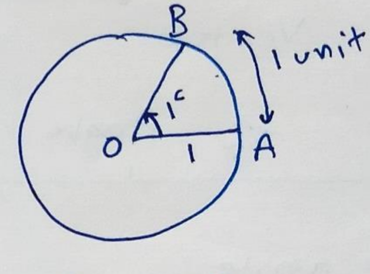
[Circular System]

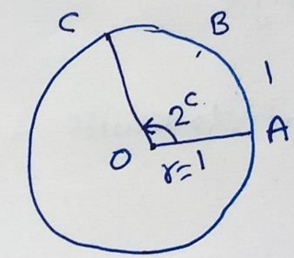
1 radian $= 1^c = 1$

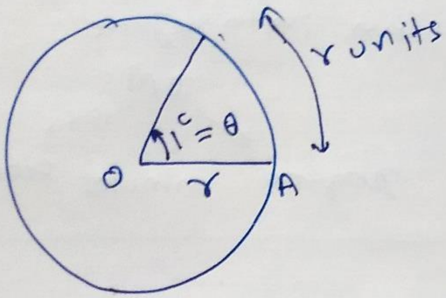
Definition of unit circle: $r = 1$ unit

Definition of one radian

In a circle ($r = 1$ unit), angle subtended by arc ($l = 1$ unit) on the centre of circle $=$







$$\boxed{\theta = \frac{l}{r}}$$

$l \to$ length of arc

$\theta \to$ Angle subtended by arc on the centre

$r \to$ radius of circle

(only in radian)

Note:

Radian ($\pi^c \to 180°$) | Degree

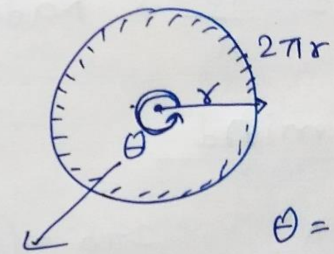
$\Updownarrow$

Real numbers

$\pi = 3.14.. \to \frac{22}{7}$

Relation b/w Degree & Radian

one complete revolution $= 360° = 2\pi^c$



$\theta = \frac{l}{r}$

$\theta = \frac{2\pi r}{r} = 2\pi$

$\theta = 2\pi = 2\pi$ radian $= 2\pi^c$

So Relation is

$360° = 2\pi^c$

$270° = \left(\frac{3\pi}{2}\right)^c$

$180° = \pi^c$

$90° = \left(\frac{\pi}{2}\right)^c$

$45° = \left(\frac{\pi}{4}\right)^c$

$\to 1° = \left(\frac{2\pi}{360}\right)^c$

$270° = \left(\frac{\pi}{180}\right) \cdot 270$

$\uparrow$ Degree $\qquad \uparrow$ Rad.

Formula For Conversion

$$\text{Radian} = \frac{\pi}{180} \times \text{Degree}$$

$$\text{Degree} = \frac{180}{\pi} \times \text{Radian}$$

Keep in mind

$360^\circ = 2\pi^c$ $\qquad 270^\circ = \left(\frac{3\pi}{2}\right)^c$

$180^\circ = \pi^c$ $\qquad 120^\circ = \left(\frac{2\pi}{3}\right)^c$

$150^\circ = \left(\frac{5\pi}{6}\right)^c$ $\qquad 135^\circ = \left(\frac{3\pi}{4}\right)^c$

$90^\circ = \left(\frac{\pi}{2}\right)^c$ $\qquad 60^\circ = \left(\frac{\pi}{3}\right)^c$

$45^\circ = \left(\frac{\pi}{4}\right)^c$ $\qquad 30^\circ = \left(\frac{\pi}{6}\right)^c$

$11 \,)\, 630^\circ \,(\, 57$

$\quad 3$

e.g. Convert 1 radian into degree.

DMS → Degree, minutes, seconds

Ans. 1 radian = ?? Degree

$$\text{Degree} = \frac{180}{\pi} \times \text{radian}$$

$$D = \left(\frac{180}{\left(\frac{22}{7}\right)} \times 1\right)^\circ = \left(\frac{180 \times 7}{22}\right)^\circ = \left(\frac{90 \times 7}{11}\right)^\circ$$

$$D = \left(\frac{630}{11}\right)^\circ = \left(57 + \frac{3}{11}\right)^\circ = 57^\circ + \left(\frac{3}{11}\right)^\circ$$

$$= 57^\circ + \left(\frac{3}{11} \times 60\right)' = 57^\circ + \left(\frac{180}{11}\right)'$$

$$= 57^\circ + \left(16 + \frac{4}{11}\right)' = 57^\circ + 16' + \left(\frac{4}{11}\right)'$$

$$= 57^\circ 16' + \left(\frac{4}{11} \times 60\right)'' = 57^\circ 16' + \underbrace{21''}_{\text{approx.}}$$

$$= 57^\circ 16' 21''$$

e.g.

Convert $40^\circ 30'$ into radian.

$$\text{radian} = \frac{\pi}{180} \times \text{Degree}$$

$$= \frac{\pi}{180} \times \left(40^\circ 30'\right)$$

$$= \frac{\pi}{180} \times \left(40^\circ + \frac{30}{60}^\circ\right)$$

$$= \frac{\pi}{180} \times \left(40 + \frac{1}{2}\right)^\circ$$

$$= \frac{\pi}{180} \times \left(\frac{81}{2}\right)^\circ$$

$$= \frac{\pi \times 9}{20 \times 2} = \frac{9\pi}{40} \text{ radian}$$

radian (π)
$\downarrow$
Degree (180°)

$$40^\circ 30' = \frac{9\pi}{40} \text{ radian}$$

Class - 11 Maths

Exercise 3.1

① (i) $25° = \frac{5\pi}{36}$

Radian $= \frac{\pi}{180} \times$ Degree

$= \frac{\pi}{\cancel{180}_{36}} \times \cancel{25}\,5$

$= \frac{5\pi}{36}$

(ii) $-(47°30')$

$= -\left(47° + \left(\frac{30}{60}\right)°\right)$

$= -\left(47 + \frac{1}{2}\right)°$

$= -\left(\frac{95}{2}\right)°$

Radian $= \frac{\pi}{\cancel{180}_{36}} \times \left(\frac{-\cancel{95}^{19}}{2}\right)$

$= \frac{-19\pi}{72}$

(iii) $240°$

Radian $= \frac{\pi}{\cancel{180}_{3}} \times \cancel{240}^{4} = \frac{4\pi}{3}$

(iv) $520°$

Radian $= \frac{\pi}{\cancel{180}_{9}} \times \cancel{520}^{26} = \frac{26\pi}{9}$

② (i) $\left(\frac{11}{16}\right)^c$

$$\text{Degree} = \left(\frac{180}{\pi} \times \text{Radian}\right)$$

$$\left(\pi = \frac{22}{7}\right)$$

$$= \frac{180}{\left(\frac{22}{7}\right)} \times \frac{11}{16}$$

$$= \frac{\cancel{180}^{45}}{\cancel{22}_{2}} \times 7 \times \frac{\cancel{11}}{\cancel{16}_{4}}$$

$$= \left(\frac{315}{8}\right)^\circ$$

$$8\overline{)315}(39$$
$$\underline{\ \ \ }$$
$$3$$

$$= \left(39\frac{3}{8}\right)^\circ$$

$$= 39^\circ\left(\frac{3}{8}\right)^\circ = 39^\circ + \left(\frac{3}{\cancel{8}_{2}} \times \cancel{60}^{15}\right)'$$

$$\Rightarrow 39^\circ + \left(\frac{45}{2}\right)'$$

$$= 39^\circ + \left(22\frac{1}{2}\right)'$$

$$= 39^\circ + 22' + \left(\frac{1}{2}\right)'$$

$$= 39^\circ + 22' + \left(\frac{1}{2} \times 60\right)''$$

$$= 39^\circ + 22' + 30''$$

$$= 39^\circ\, 22'\, 30''$$

(ii) $(-4)^c$ $\qquad \pi = \frac{22}{7}$

$$\text{Degree} = \frac{180}{\pi} \times (-4)$$

$$= \frac{180}{\cancel{22}_{11}} \times 7 \times (-\cancel{4})(-2)$$

$$= -\frac{1260}{11} \times (2)$$

$$= -\left(\frac{2520}{11}\right)^\circ$$

$$= -\left(\frac{2520}{11}\right)^\circ$$

$$= -\left(229\frac{1}{11}\right)^\circ$$

$$= -\left(229^\circ + \left(\frac{1}{11}\right)^\circ\right)$$

$$= -\left(229^\circ + \left(\frac{60}{11}\right)'\right)$$

$$= -\left(229^\circ + \left(5 + \frac{5}{11}\right)'\right)$$

$$= -\left(229^\circ + 5' + \left(\frac{5}{11}\right)'\right)$$

$$= -\left(229^\circ + 5' + \left(\frac{5}{11} \times 60\right)''\right)$$

$$= -\left(229^\circ + 5' + \left(\frac{300}{11}\right)''\right)$$

$$= -\left(229^\circ + 5' + 27''\right)$$

$$= \underline{-229^\circ 5' 27''}$$

③

In <u>one minute</u> $\Rightarrow$ 360 <u>rev^n.</u>

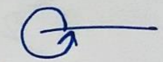
$\Rightarrow$ 60 sec. $\longrightarrow$ 360 rev^n

1 sec. $\longrightarrow$ $\left(\frac{36\cancel{0}}{6\cancel{0}}\right)$ rev^n

1 sec. $\longrightarrow$ 6 rev^n

$= (6 \times 2\pi)$ radian

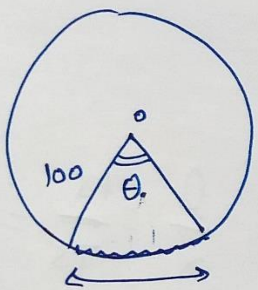
$= \underline{(12\pi)^c}$



1 rev^n $= (2\pi)^c$

Exercise 3.1 (Class 11 Maths)

(4)



$l = 22$ cm

$r = 100$ cm

radian

$\theta = \dfrac{l}{r}$

$\theta = \left(\dfrac{22}{100}\right)^c$

$$\text{Degree} = \frac{180}{\pi} \times (\text{Radian})$$

$$= \frac{18\cancel{0}}{\left(\frac{\cancel{22}}{7}\right)} \times \frac{\cancel{22}}{10\cancel{0}}$$

$$= \left(\frac{126}{10}\right)^\circ$$

$$= \left(\frac{126}{10}\right)^\circ$$

$$= (12.6)^\circ$$

$$= 12^\circ + \left(\frac{6}{10}\right)^\circ$$

$$= 12^\circ + \left(\frac{6}{10} \times 60\right)' = 12^\circ + (36)'$$

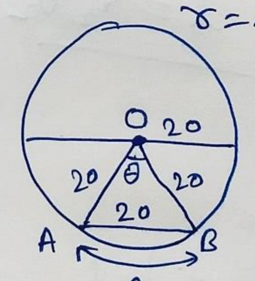
$$= 12^\circ 36'$$

(5)

$d = 40$ cm

$r = 20$ cm

$\widehat{AB} = ?$



radian

$\boxed{l = r\theta}$

20 ?

$\theta = 60^\circ$

~~θ = 180~~

$\theta = \dfrac{\pi}{180} \times 60$

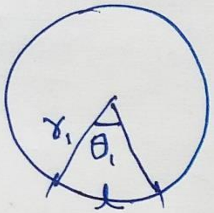
$= \dfrac{\pi}{3}$

$\Delta OAB \rightarrow$ equilateral Δ

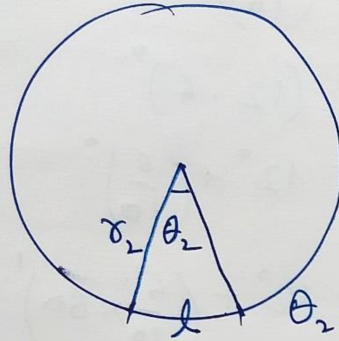
minor arc $= l = r\theta$

$\Rightarrow l = 20 \times \dfrac{\pi}{3} = \dfrac{20\pi}{3}$

(6)



$\theta_1 = 60°$



$\theta_2 = 75°$

$r_1 : r_2$

length of arc = same = l

$l = r\theta$

$l = r_1 \theta_1$ | $l = r_2 \theta_2$

$r_1 \theta_1 = r_2 \theta_2$

$\Rightarrow r_1 (60) = r_2 (75°)$

$\Rightarrow \dfrac{r_1}{r_2} = \dfrac{75}{60} = \dfrac{5}{4}$

$r_1 : r_2 = 5:4$

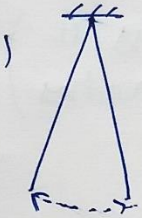
(7)

$r = 75$ cm

radian

$\theta = ?$

(i)

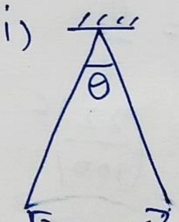


$l = 10$

radian

$\theta = \dfrac{l}{r} = \dfrac{10}{75} = \dfrac{2}{15}$

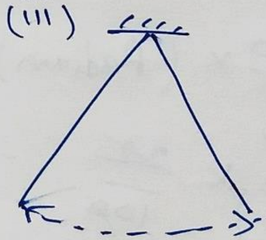
(ii)



$l = 15$

$\theta = \dfrac{l}{r} = \dfrac{15}{75} = \dfrac{1}{5}$

(iii)



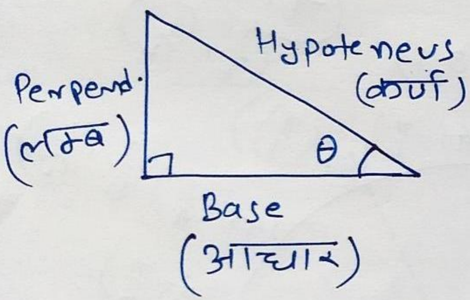
$l = 21$

$\theta = \dfrac{l}{r} = \dfrac{21}{75} = \dfrac{7}{25}$

Theory Before Exercise 3.2

Trigonometric Functions

[Circular Functions]



For English medium →

sin cos tan

$$\frac{P \quad B \quad P}{H \quad H \quad B}$$

cosec sec cot

For Hindi medium →

sin cos tan

$$\frac{L \quad A \quad L}{K \quad K \quad A}$$

cosec sec tan

Identities $\boxed{P^2 + B^2 = H^2}$

$\sin^2 x + \cos^2 x = 1$

$\sec^2 x = 1 + \tan^2 x \rightarrow \sec x + \tan x = \dfrac{1}{\sec x - \tan x}$

$\operatorname{cosec}^2 x = 1 + \cot^2 x \rightarrow$

Trigonometric Table

0°, 30°, 45°, 60°, 90°

↑ ↑ till 10^{th} class ↑ ↑

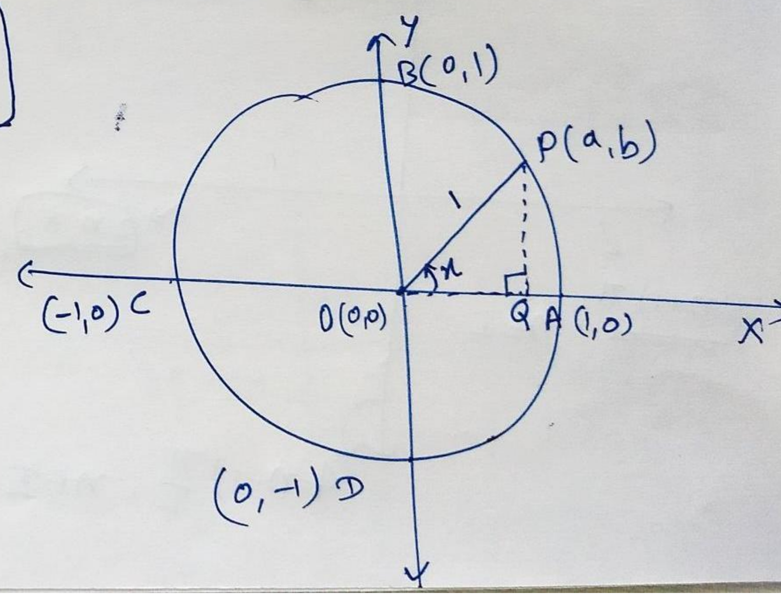
Now class-11, Trigonometric Fn.

Unit circle
$r = 1$
centre (0,0)

$OP = 1$

$PQ = b$

$OQ = a$

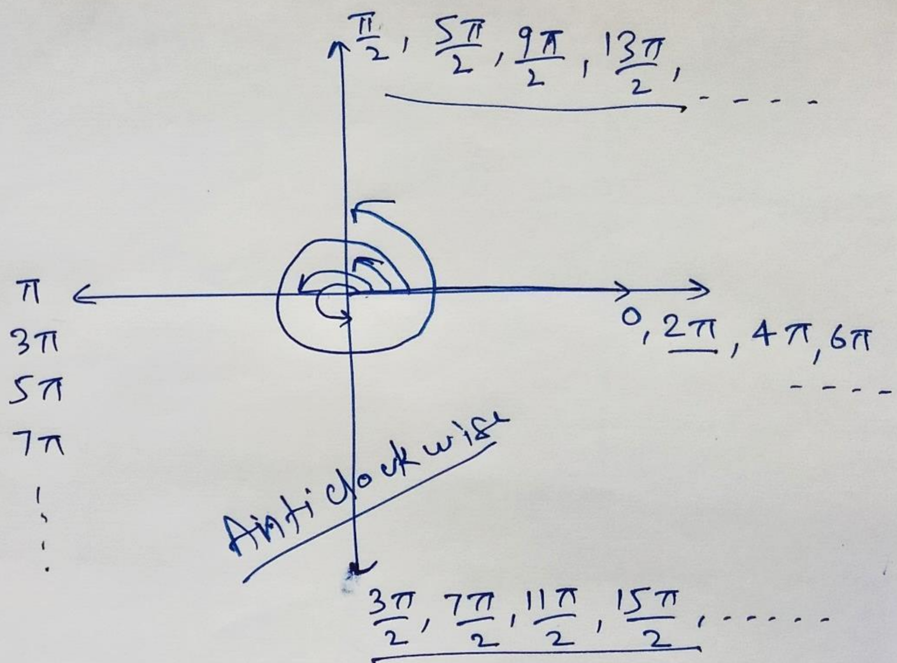


In Δ OPQ

$$\sin x = \frac{PQ}{OP} = \frac{b}{1} = b$$

$$\cos x = \frac{OQ}{OP} = \frac{a}{1} = a$$

$$P(a,b) \equiv P(\cos x, \sin x) \quad ☆$$



Note (1)

$$OP^2 = OQ^2 + PQ^2$$

$$\Rightarrow 1 = a^2 + b^2$$

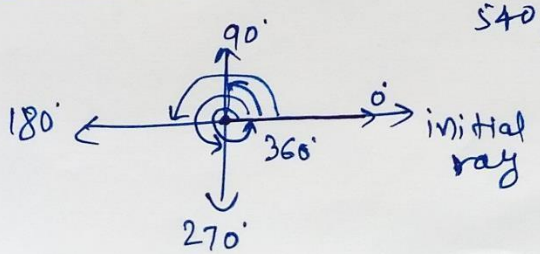
$$\Rightarrow \boxed{1 = \cos^2 x + \sin^2 x}$$

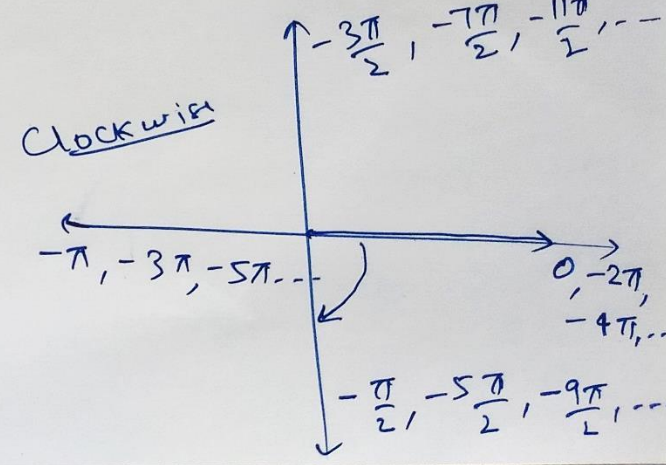
Note (2) Quadrantal Angles

90° के multiple = 0, 90, ... 360, 450°, 540°, ...

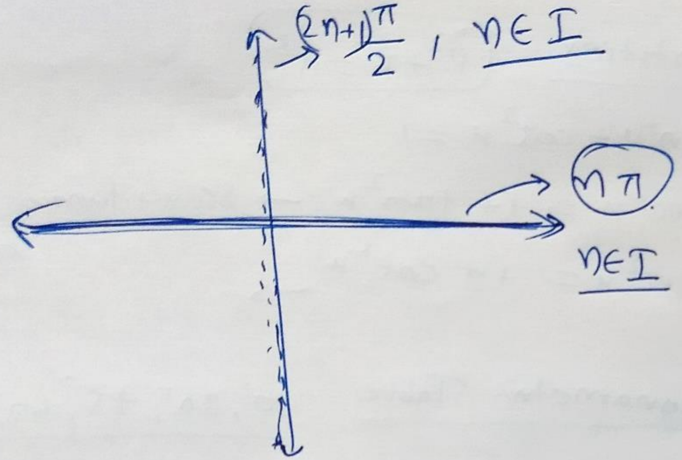
↓

$\frac{\pi}{2}$

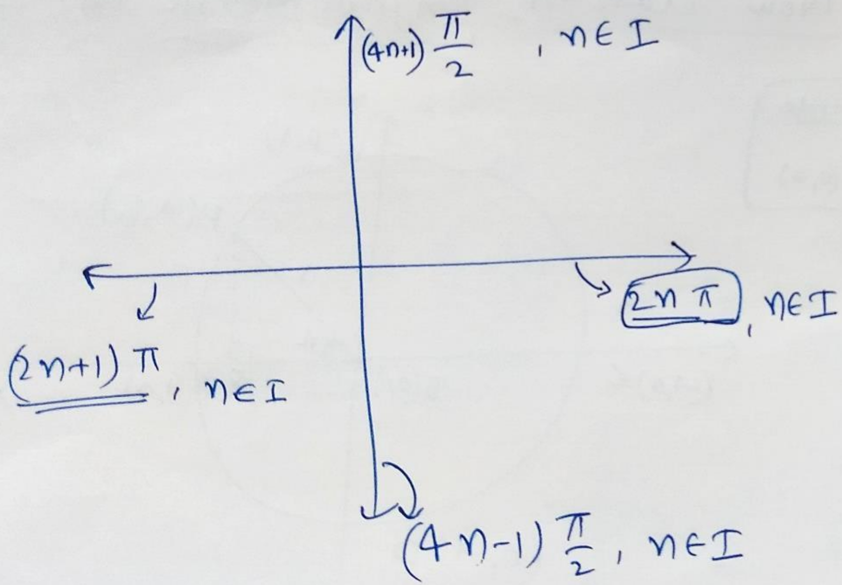




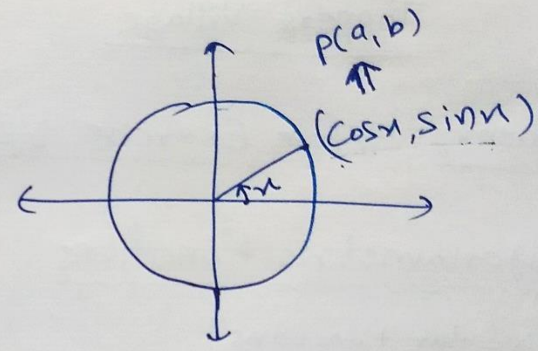
Note 3



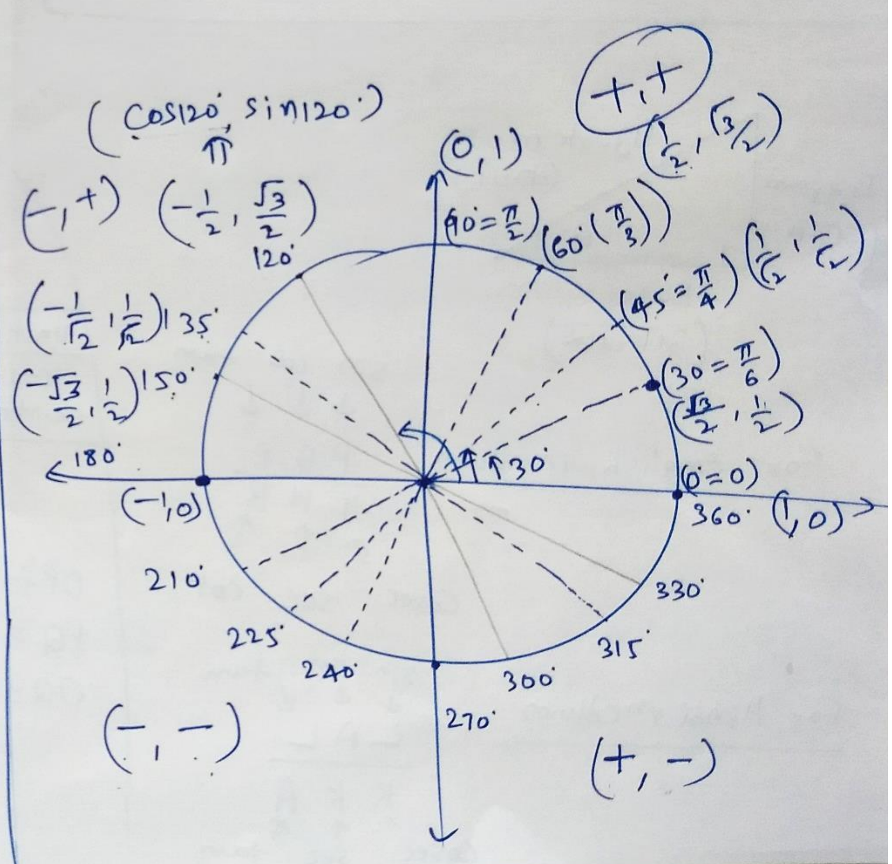
$(2n+1)\frac{\pi}{2}$, $n \in I$
$n\pi$
$n \in I$



$(4n+1)\frac{\pi}{2}$, $n \in I$
$2n\pi$, $n \in I$
$(2n+1)\pi$, $n \in I$
$(4n-1)\frac{\pi}{2}$, $n \in I$



$p(a, b)$
$(\cos x, \sin x)$
x

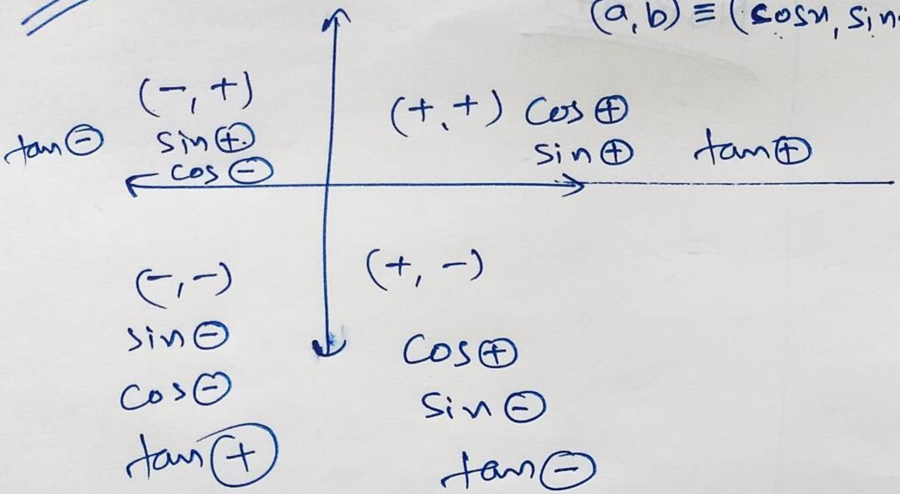


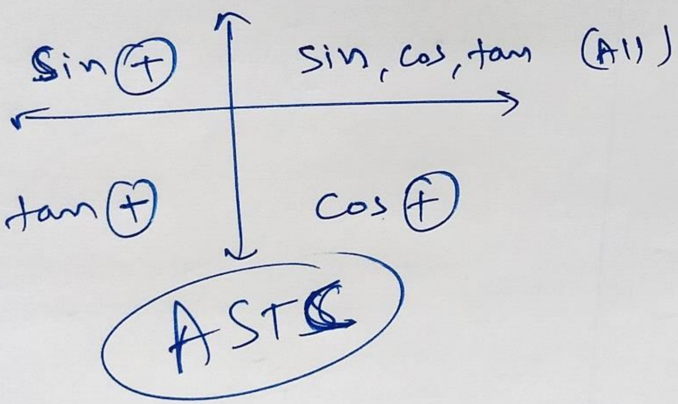
$(+, +)$
$(\cos 120°, \sin 120°)$
$(0, 1)$
$(\frac{1}{2}, \frac{\sqrt{3}}{2})$
$(-, +)$
$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$
$120°$
$90° = \frac{\pi}{2}$
$60° (\frac{\pi}{3})$
$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ $135°$
$(45° = \frac{\pi}{4})$ $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
$(-\frac{\sqrt{3}}{2}, \frac{1}{2})$ $150°$
$(30° = \frac{\pi}{6})$
$(\frac{\sqrt{3}}{2}, \frac{1}{2})$
$180°$
$30°$
$(-1, 0)$
$(0° = 0)$
$360°$ $(1, 0)$
$210°$
$330°$
$225°$
$315°$
$240°$
$300°$
$270°$
$(-, -)$
$(+, -)$

Note - 4

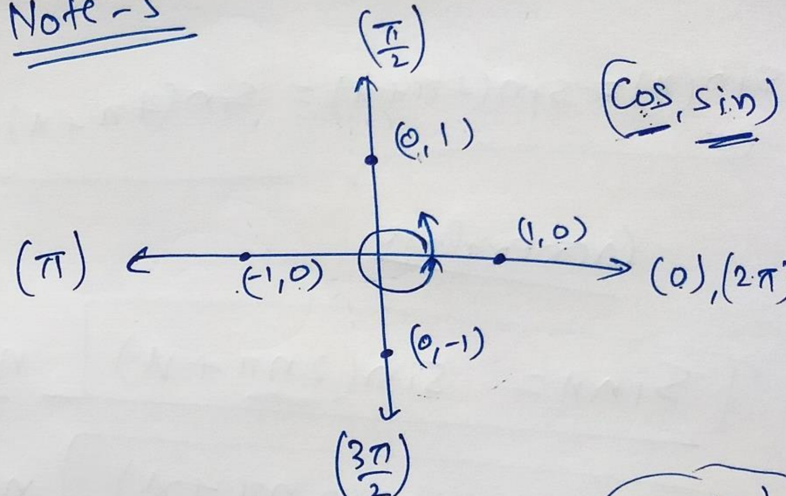
Point

$(a, b) \equiv (\cos x, \sin x)$





Note - 5



(Cos, sin)

$\sin 30° = \frac{1}{2}$

$\sin 390° = \frac{1}{2}$

$\cos 0 = 1 \leftarrow \sin 0 = 0$ ✓

$\cos \frac{\pi}{2} = 0 \rightarrow \sin \frac{\pi}{2} = 1$

$\cos \pi = -1 \leftarrow \sin \pi = 0$ ✓

$\cos \frac{3\pi}{2} = 0 \rightarrow \sin \frac{3\pi}{2} = -1$

$\cos 2\pi = 1 \leftarrow \sin 2\pi = 0$ ✓

$$\sin(x) = \sin(\underbrace{2\pi}_{\text{One complete revolution}} + x)$$

$$\underbrace{\sin(x) = \sin(2\pi + x) = \sin(4\pi + x)}\ldots$$

Generalise

$$\boxed{\sin x = \sin(2n\pi + x)} \quad n \in I.$$

$$\boxed{\cos x = \cos(2n\pi + x)} \quad n \in I.$$

Note: 6

$\sin x = 0$, $x = n\pi$, $n \in I$

$\sin n\pi = 0$

$\cos x = 0 \longrightarrow x = (2n+1)\frac{\pi}{2}$, $n \in I$

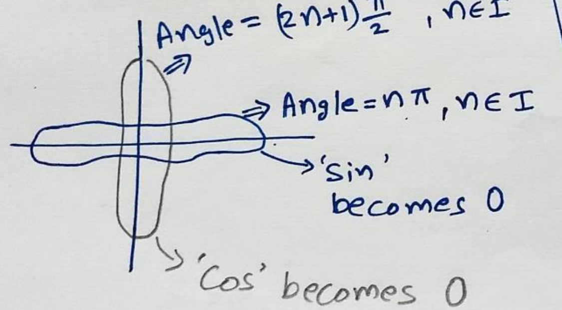
$\cos(2n+1)\frac{\pi}{2} = 0$

Class – 11 – Maths

Trigonometric Functions { Before Exercise 3.2 }

From Last Lecture

Sin ⊕	All ⊕
tan ⊕	Cos ⊕, sec ⊕



Complementary

Sin → Cos
Cos → sin
tan → cot
Cot → tan
sec → cosec
cosec → sec

How to find values of trigonometric ratios values for Big Angles ??

$\boxed{0°, 30°, 45°, 60°, 90°}$

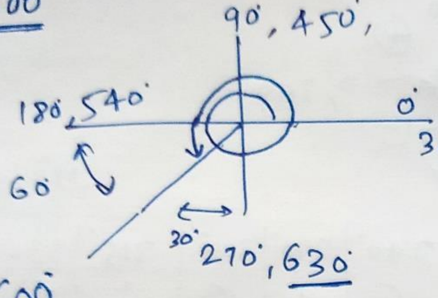
(i) quadrant → Final Answer
↓
Sign → ⊕
→ ⊖

(ii) Angle = $\underbrace{90 \times n \pm \theta}_{\text{Division Algorithm}} \approx \theta$

$90\overline{)\text{Ang.}}$

n → Even → No ~~complementary~~ Same
→ odd → Comlementary.

e.g. (i) $\sin 600^\circ$



600°

(III - quadrant)

$$90\,\big)\,600\,\big(\,6$$
$$-540$$
$$60$$

$$600^\circ = 90^\circ \times 6 + 60^\circ$$

$$\underbrace{\sin(600^\circ)}_{\text{III}} = \sin(\overbrace{90 \times \underset{\uparrow\ \text{Even}}{6} + 60^\circ}^{\text{III}})$$

$$= -(\sin 60^\circ)$$

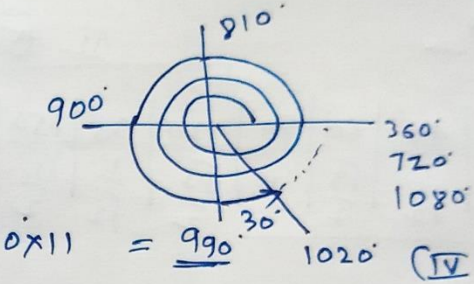
$$= -\frac{\sqrt{3}}{2}$$

$$\sin(600^\circ) = \sin(\overbrace{90 \times \overset{\text{odd}}{7} - 30^\circ}^{\text{III}})$$

$$= -\cos(30) = -\sqrt{3}/2$$

(ii) $\sec(1020^\circ)$

$$90\,\big)\,1020\,\big(\,11$$
$$-90$$
$$120$$
$$-90$$
$$30$$



$90 \times 11 = 990^\circ$ 1020° (IV)

$$\sec(1020^\circ) = \sec(\underbrace{90 \times \overset{\text{odd}}{11} + 30^\circ}_{\text{IV}})$$

$$= +\operatorname{cosec}(30^\circ)$$

$$\sec(1020^\circ) = +2$$

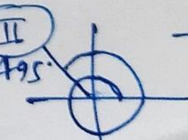
(iii) $\tan(495^\circ)$

$$90\,\big)\,495\,\big(\,5$$
$$-450$$
$$45$$

$$= \tan(90 \times \overset{\text{odd}}{5} + 45^\circ)$$

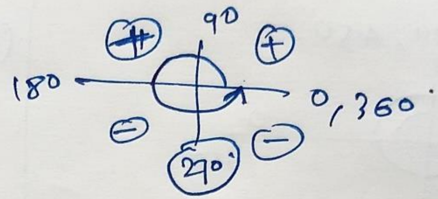
$$= -\cot(45^\circ)$$

II 495°



$$= -(1)$$

Graph + Domain + Range



① $y = \sin x$

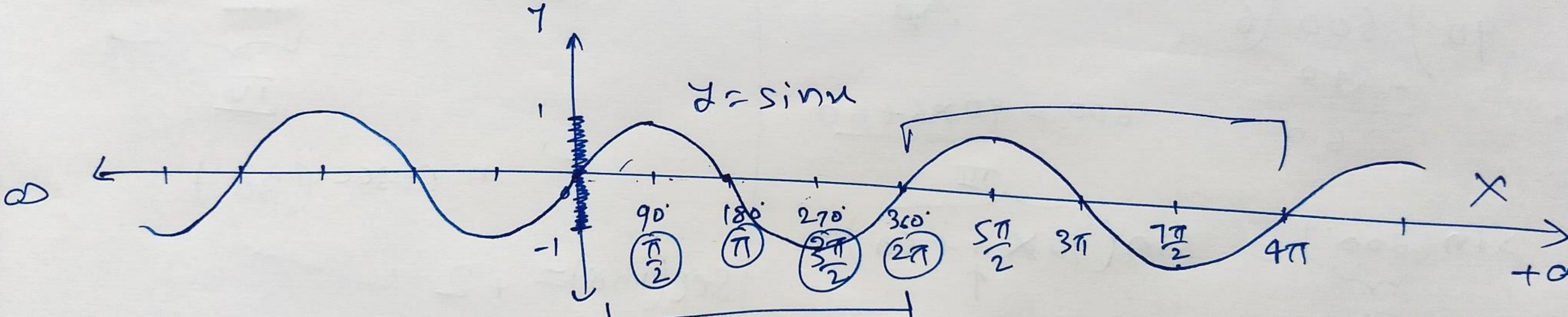
II (+)

IV = (−)

x	0	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
$\sin x = y$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0

I (+)

III (−)



Repeat

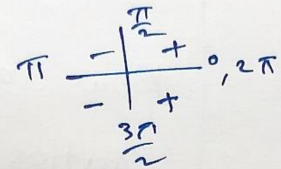
← 2π →

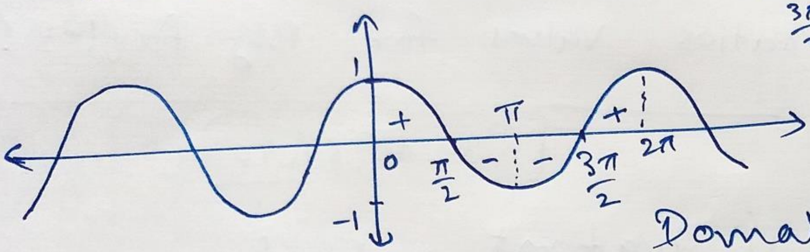
Domain = $(-\infty, \infty) = R$

Range = $[-1, 1]$

$$\boxed{-1 \leq \sin x \leq 1}$$ ☆

② $y = \cos x$

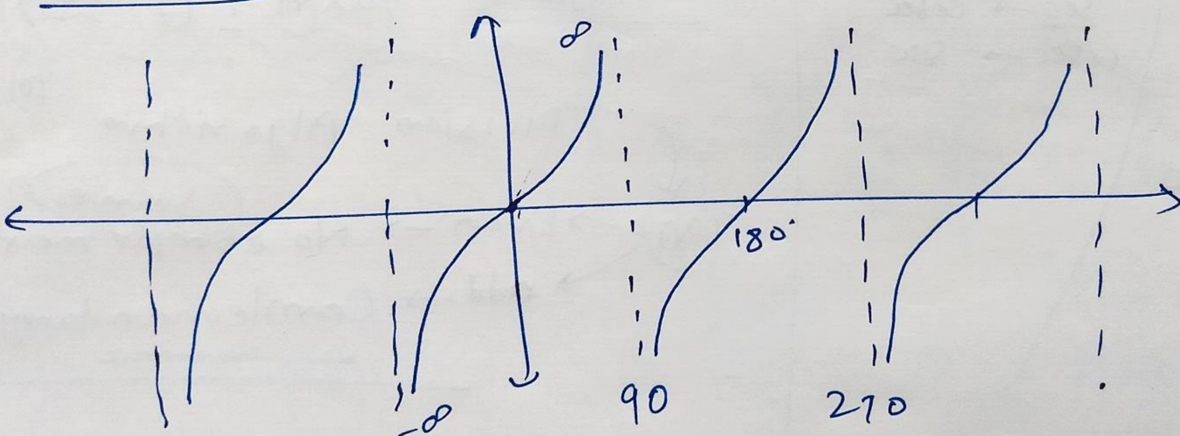




Domain $= (-\infty, \infty) = R$

Range $= [-1, 1]$

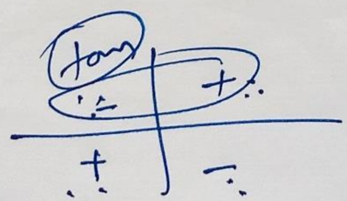
③ $y = \tan x = \frac{\sin x}{\cos x \neq 0}$ $\quad x \in R - \{a : a = (2n+1)\frac{\pi}{2}\}$



$n \in I$

Domain $= R - \{x : x = (2n+1)\frac{\pi}{2}\}$

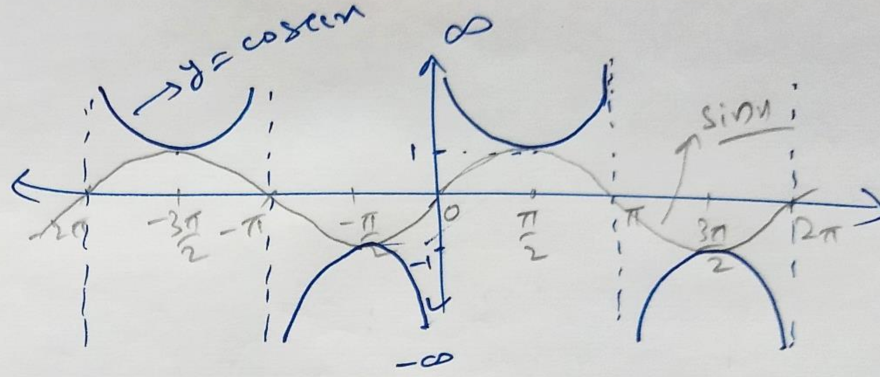
Range $= (-\infty, \infty) = R$



(4) $y = \operatorname{cosec} x = \frac{1}{\sin x} \neq 0$

Domain: $R - \{x : x = n\pi, n \in I\}$

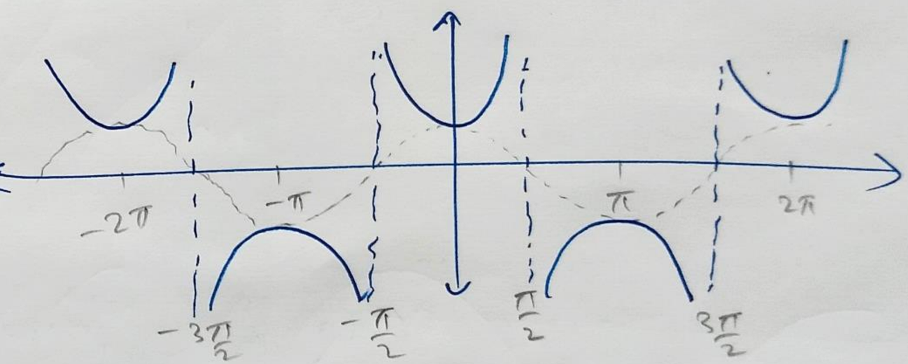
Range: $(-\infty, -1] \cup [1, \infty)$



(5) $y = \sec x = \frac{1}{\cos x} \neq 0$

Domain $= R - \{x : x = (2n+1)\frac{\pi}{2}\}$

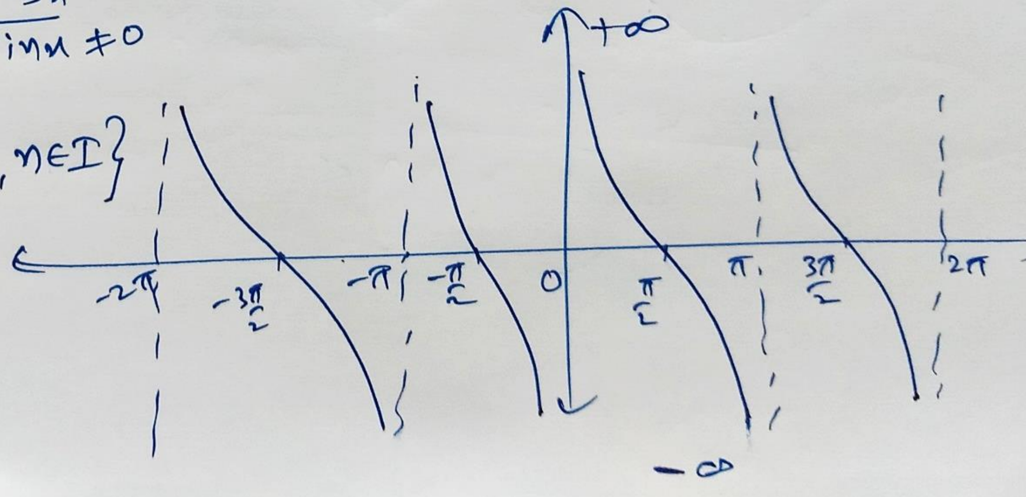
Range $= (-\infty, -1] \cup [1, \infty)$



(6) $y = \frac{1}{\tan x} = \cot x = \frac{\cos x}{\sin x} \neq 0$

Domain $= R - \{x : x = n\pi, n \in I\}$

Range $= (-\infty, \infty) = R$



Cot

−	+
+	−

Class - 11 - Maths

Exercise 3.2

(1) $\cos x = -\frac{1}{2}$, $x \to$ III quadrant

$\sin x$, $\tan x$, $\cot x$, $\sec x$, $\operatorname{cosec} x$

$\ominus$ $\oplus$ $\oplus$ $\ominus$ $\ominus$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin^2 x + \left(-\frac{1}{2}\right)^2 = 1$ $\quad \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

$\Rightarrow \sin^2 x = 1 - \frac{1}{4}$

$\Rightarrow \sin^2 x = \frac{4-1}{4} = \frac{3}{4}$

$\boxed{\sin x = \pm \frac{\sqrt{3}}{2}}$

+ II	I +
− III	IV −

$\sin x = -\frac{\sqrt{3}}{2}$, $\cos x = -\frac{1}{2}$

$\tan x = \frac{\sin x}{\cos x} = \frac{+\sqrt{3}/\cancel{2}}{+\frac{1}{\cancel{2}}} = \sqrt{3}$

$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$

$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$

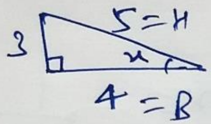
$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\sqrt{3}/2\right)} = -\frac{2}{\sqrt{3}}$

(2) $\sin x = \frac{3}{5}$, $x \to \text{II}$

sin	+
tan	cos

cos x, tan x, cot x, cosec x, sec x
− − − (+) −

$\sin x = \frac{3}{5} = \frac{P}{H}$



$\cos x = \frac{B}{H} = \frac{-4}{5}$

$\tan x = \frac{P}{B} = \frac{-3}{4}$

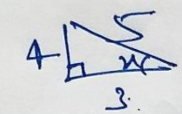
$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{5}{3}$

$\sec x = \frac{1}{\cos x} = -\frac{5}{4}$

~~tan x~~ $\cot x = \frac{1}{\tan x} = \frac{-4}{3}$

(3) $\cot x = \frac{3}{4}$ $x \to \text{III}$

$\cot x = \frac{3}{4} = \frac{B}{P}$



$\sin x = \frac{P}{H} = -\frac{4}{5}$

$\cos x = \frac{B}{H} = -\frac{3}{5}$

$\tan x = 4/3$

$\sec x = -5/3$

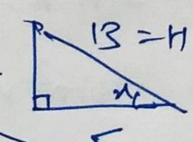
$\operatorname{cosec} x = -\frac{5}{4}$

(4) $\sec x = \frac{13}{5} = \frac{H}{B}$ $x \to \text{IV}$

$\cos x = \frac{5}{13}$

$\sin x = P/H = \frac{-12}{13} = -\frac{12}{13}$

$P = 12$



$\tan x = -\frac{12}{5}$

$\cot x = -5/12$

$\operatorname{cosec} x = \frac{-13}{12}$

(5) $\tan x = -\frac{5}{12} = \frac{P}{B}$ $x \to \text{II}$

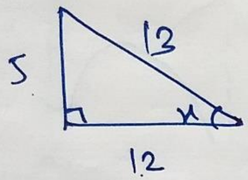
$\cot x = -\frac{12}{5}$

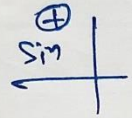
$\sin x = \frac{5}{13}$

$\cos x = -\frac{12}{13}$

$\sec x = -\frac{13}{12}$

$\operatorname{cosec} x = \frac{13}{5}$

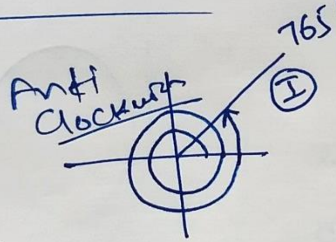




(6) $\sin(765^\circ)$

$765 = 90 \times n + \theta$ quadrant

$$90 \,)\, 765 \,(\, 8$$
$$-720$$
$$45$$



$765 = 90 \times \underset{\text{Even}}{8} + 45$

$\sin(765^\circ) = \sin(90^\circ \times 8 + 45^\circ)$

$= +\sin(45^\circ)$

$= \frac{1}{\sqrt{2}}$

(7) $\operatorname{cosec}(-1410^\circ)$

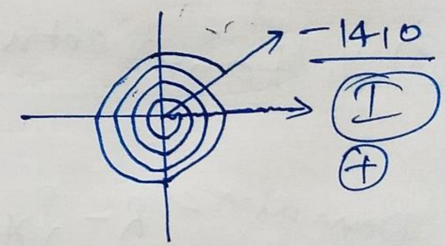
$90 \times n + \theta$ quadrant

$$90 \,)\, 1410 \,(\, 15$$
$$-90$$
$$510$$
$$-450$$
$$60$$

$1410 = 90 \times 15 + 60$

Angle $= -1410 = -90 \times 15 - 60$

Clock



$\operatorname{cosec}(-1410^\circ)$

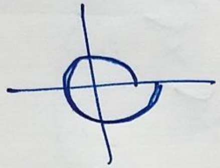
$= \operatorname{cosec}(-90 \times \underset{\text{odd}}{15} - 60^\circ)$

$= +\sec(60^\circ)$

$= 2$

2

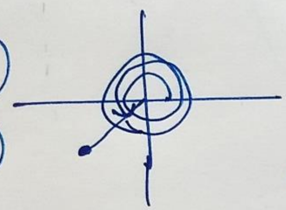
(8) $\tan\left(\frac{19\pi}{3}\right)$



$\pm 360^\circ \rightarrow 2\pi$

$\pm 720^\circ \rightarrow 4\pi$

$\pm 1080^\circ \rightarrow 6\pi$



X

No effect.

$\tan\left(\frac{19\pi}{3}\right) = \tan\left(6\pi + \frac{\pi}{3}\right)$

$= \tan\left(\frac{\pi}{3}\right)$

$= \sqrt{3}$

(9) $\sin\left(-\frac{11\pi}{3}\right) \longleftrightarrow$ Degree

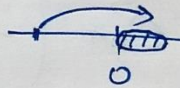
Radian

$\pi \rightarrow 180^\circ$

$=$

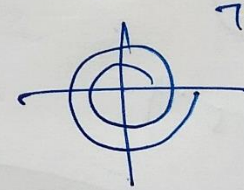
$= \sin\left(-\frac{11 \times \cancel{180}^{60}}{\cancel{3}}\right)$

$= \sin(-660^\circ)$



$= \sin(+720^\circ - 660^\circ)$

$720 = 2 \times 360^\circ$



$= \sin(60^\circ)$

$= \frac{\sqrt{3}}{2}$

(10) $\cot\left(\frac{-15\pi}{4}\right)$

$= \cot\left(\frac{-16\pi+\pi}{4}\right)$

$= \cot\left(-\frac{16\pi}{4}+\frac{\pi}{4}\right)$

$= \cot\left(-\frac{4\pi}{\cancel{4}}+\frac{\pi}{4}\right)$

$= \cot\left(\frac{\pi}{4}\right)$

$= \cot(45^\circ)$

$= 1$

$\pm 2\pi$ ✓ ✗

$\pm \underline{4\pi}$ ✓ ✗

$\pm 6\pi$ ✓ ✗

$\pm$ (even) π ✓ ✗

no effect

Remove

Chapter : 3 Trigonometric Functions (Class 11)

All formulae (Before Exercise 3.3)

+

Examples.

(1) Effect of Negative Angle.

$\sin(-x) = -\sin x$

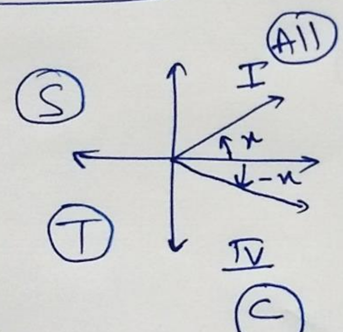
$\boxed{\cos(-x) = \cos x}$

$\tan(-x) = -\tan x$

$\operatorname{cosec}(-x) = -\operatorname{cosec} x$

$\boxed{\sec(-x) = \sec x}$

$\cot(-x) = -\cot x$



(2) ★ $\square\square \pm \square\square$

$\cos(x+y) = \cos x . \cos y - \sin x . \sin y$

$\cos(x-y) = \cos x . \cos y + \sin x . \sin y$

$\sin(x+y) = \sin x . \cos y + \cos x . \sin y$

$\sin(x-y) = \sin x . \cos y - \cos x . \sin y$

(3) Multiply $(\times)$ to Add / subtract $(+, -)$

- $2 \sin x . \cos y = \sin(x+y) + \sin(x-y)$

$2 \cos x . \sin y = \sin(x+y) - \sin(x-y)$

$2 \cos x . \cos y = \cos(x+y) + \cos(x-y)$

★ $2 \sin x . \sin y = \cos(x-y) - \cos(x+y)$

(3) cot $\frac{\pi}{6}$

(4) (+) or (−) ⟶ (×)

Add/subtract to Multiply.

$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)$

$\sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cdot\cos\left(\frac{x+y}{2}\right)$

$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)$

$\cos x - \cos y = 2\sin\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{y-x}{2}\right)$

$= -2\sin\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)$

$\cot(x+y) = \frac{\cot x\cot y - 1}{\cot x + \cot y}$

$\cot(x-y) = \frac{\cot x\cot y + 1}{-\cot x + \cot y}$

(5) $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$

$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$

⑥ Multiple Angles

$$\sin 2x = 2\sin x.\cos x = \frac{2\sin x.\cos x}{1} = \frac{(2\sin x\cos x)/\cos^2 x}{(\sin^2 x+\cos^2 x)/\cos^2 x} = \frac{2\tan x}{1+\tan^2 x}$$

$$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$\cos^2 x = 1-\sin^2 x$

$\sin^2 x = 1-\cos^2 x$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos 2x = 1-2\sin^2 x$$

$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$\sin 3x = \sin(x+2x)$

$$\sin 3x = 3\sin x - 4\sin^3 x$$ Trick: 31 − 43

$$\cos 3x = 4\cos^3 x - 3\cos x$$ 43 − 31

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$$

$15 = 60 - 45$

Application.

$\sin(\frac{\pi}{2} - x) = \cos x$

$\cos(\frac{\pi}{2} - x) = \sin x$

$\vdots$

$\sin(\frac{\pi}{2} + x) = \cos x$

$\cos(\frac{\pi}{2} + x) = -\sin x$

$\vdots$

$\sin(\pi - x) = \sin x$

$\cos(\pi - x) = -\cos x$

$\vdots$

$\sin(\pi + x) = -\sin x$

$\cos(\pi + x) = -\cos x$

$\vdots$

$\sin(\frac{3\pi}{2} - x) = -\cos x$

$\cos(\frac{3\pi}{2} - x) = -\sin x$

$\vdots$

$\sin(2\pi - x) = -\sin x$

$\cos(2\pi - x) = +\cos x$

$\vdots$

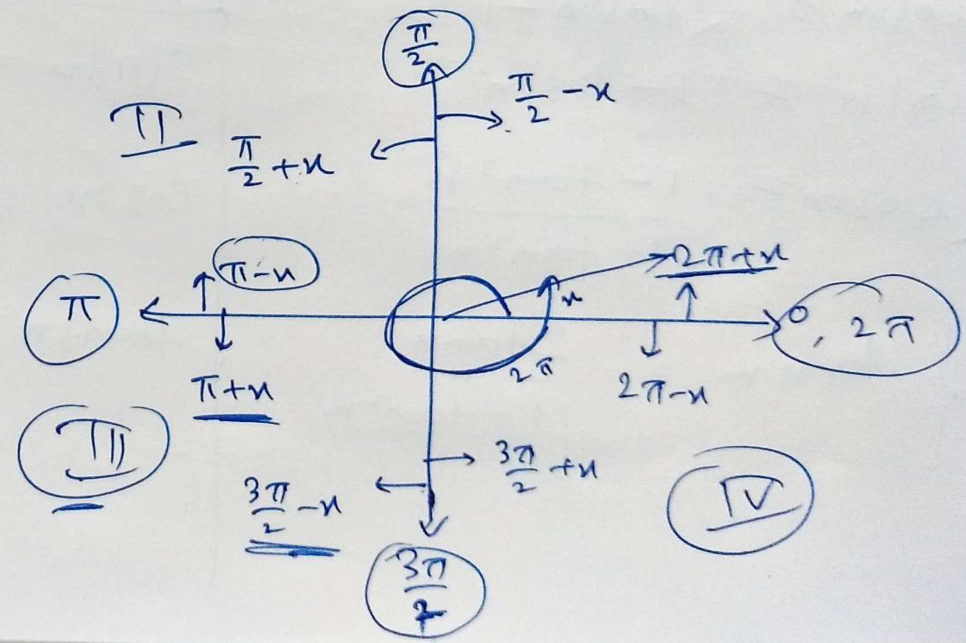
$\sin(2\pi + x) = \sin x$

$\cos(2\pi + x) = \cos x$

$\vdots$

$\sin(\frac{3\pi}{2} + x) = ?$

$\cos(\frac{3\pi}{2} + x) = ?$



(5) (i) sin 7 ... (ii) ...

Examples-

(11) $\sin 15^\circ$

$= \sin(\underbrace{45^\circ - 30^\circ}_{x\,-\,y})$

$= \sin(45^\circ)\cos(30^\circ) - \cos(45^\circ)\sin(30^\circ)$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

(14)* **Prove** $\boxed{\tan 3x \cdot \tan 2x \cdot \tan x = \tan 3x - \tan 2x - \tan x}$

$3x = (2x + x)$

tan

$\tan 3x = \tan(2x + x)$

$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \cdot \tan x}$

$\Rightarrow \tan 3x - \tan 3x \cdot \tan 2x \cdot \tan x = \tan 2x + \tan x$

$\Rightarrow \boxed{\tan 3x - \tan 2x - \tan x = \tan 3x \cdot \tan 2x \cdot \tan x}$

$2\sec^2 \frac{\pi}{} = 2(2)$ ... $= 10$

Example, 17

Prove that

$$\boxed{\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x}$$

$$LHS = \frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x}$$

$$= \frac{2\sin\left(\frac{6x}{2}\right).\cos\left(\frac{4x}{2}\right) - 2\sin 3x}{-2\sin\left(\frac{5x+x}{2}\right).\sin\left(\frac{5x-x}{2}\right)}$$

$$= \frac{\cancel{2}\,\cancel{\sin 3x}.\cos 2x - \cancel{2}\,\cancel{\sin 3x}}{-\cancel{2}\,\cancel{\sin 3x}.\sin 2x}$$

$$= \frac{\cos 2x - 1}{-\sin 2x} = \frac{1-\cos 2x}{\sin 2x}$$

$$= \frac{1-\cos 2x}{\sin 2x}$$

$\cos 2x \to \cos^2 x - \sin^2 x$, $2\cos^2 x - 1$, $1-2\sin^2 x$, $\frac{1-\tan^2 x}{1+\tan^2 x}$

$\sin 2x \to 2\sin x\cos x$, $\frac{2\tan x}{1+\tan^2 x}$

$$= \frac{1-(1-2\sin^2 x)}{2\sin x\cos x}$$

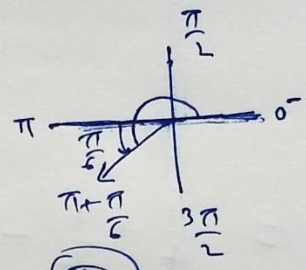
$$= \frac{\cancel{1}-\cancel{1}+2\sin^2 x}{2\sin x.\cos x}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x = RHS.$$

Class. 11. Maths. Exercise 3.3

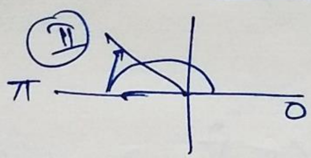
$$\operatorname{cosec}\frac{7\pi}{6} = \operatorname{cosec}\Big(\overbrace{\pi+\frac{\pi}{6}}^{\mathrm{III}}\Big) = -\operatorname{cosec}\Big(\frac{\pi}{6}\Big) = -2$$



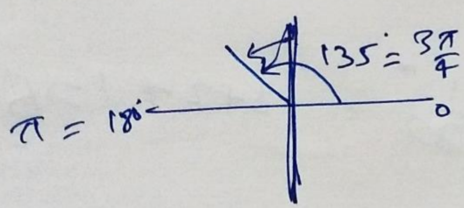
$$\sin\frac{\pi}{6} = \sin 30° = \frac{1}{2}$$

$$\operatorname{cosec}\frac{5\pi}{6}$$

$$\operatorname{cosec}\Big(\underbrace{\pi-\frac{\pi}{6}}\Big) = +\operatorname{cosec}\Big(\frac{\pi}{6}\Big) = 2$$



$$\sin\frac{3\pi}{4}$$



$$\sin\Big(\frac{3\pi}{4}\Big) = \sin\Big(\overbrace{\frac{\pi}{2}+\frac{\pi}{4}}^{\mathrm{II}}\Big) = +\cos\Big(\frac{\pi}{4}\Big) = \frac{1}{\sqrt{2}}$$

$$\sin\Big(\frac{\pi}{2}+x\Big)$$

$\operatorname{cosec}\frac{7\pi}{6} = -2$

$\operatorname{cosec}\frac{5\pi}{6} = +2$

$\sin\frac{3\pi}{4} = +\frac{1}{\sqrt{2}}$

$\frac{\pi}{6} = 30^\circ$ | $\frac{\pi}{4} = 45^\circ$ | $\frac{\pi}{3} = 60^\circ$

① $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = \frac{1}{4} + \frac{1}{4} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$

② $2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cdot\cos^2\frac{\pi}{3} = 2\left(\frac{1}{4}\right) + (-2)^2\cdot\left(\frac{1}{4}\right) = \frac{1}{2} + \cancel{4}\times\frac{1}{\cancel{4}} = \frac{3}{2}$

③ $\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = (\sqrt{3})^2 + 2 + \cancel{3}\left(\frac{1}{\cancel{3}}\right) = 3+2+1=6$

④ $2\sin^2\left(\frac{3\pi}{4}\right) + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 2\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right) + 2(4) =$

$= 1 + 1 + 8 = 10$

5 (i) $\sin 75^\circ$

$= \sin(45^\circ + 30^\circ)$

$\sin(x+y) = \sin x \cdot \cos y + \sin y \cdot \cos x$

$x = 45^\circ$, $y = 30^\circ$

$\sin(75^\circ) = \sin 45^\circ \cdot \cos 30^\circ + \sin 30^\circ \cdot \cos 45^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) $\tan 15^\circ$

$15 = 45 - 30$

$15 = 60 - 45$

$\tan(15^\circ) = \tan(45^\circ - 30^\circ)$

$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$

$x = 45$

$y = 30^\circ$

$\tan 15^\circ = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ}$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}}}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$

Exercise 3.3 Q6 — Q11 (पक्के Formulae)

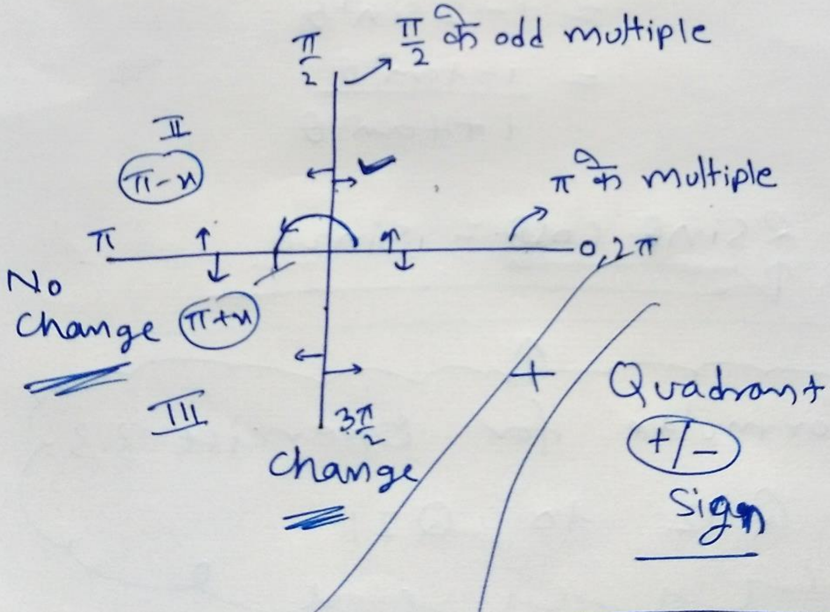
$$\cos A \cos B - \sin A \sin B = \cos(A+B)$$

$$\cos A \cos B + \sin A \sin B = \cos(A-B)$$

$$\sin A \cos B + \cos A \sin B = \sin(A+B)$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A . \tan B}$$



$$\boxed{\begin{aligned} \sin(-x) &= -\sin x \\ \cos(-x) &= \cos x \end{aligned}}$$

Class 11 Maths × Exercise 3.3

(6) $\cos\left(\frac{\pi}{4}-x\right)\cdot\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\left(\sin\left(\frac{\pi}{4}-y\right)\right)$

$= \sin(x+y)$

L.H.S. $= \underbrace{\cos\left(\frac{\pi}{4}-x\right).\cos\left(\frac{\pi}{4}-y\right)}_{A \quad B}-\underbrace{\sin\left(\frac{\pi}{4}-x\right).\sin\left(\frac{\pi}{4}-y\right)}_{A \quad B}$

$\cos A\cos B-\sin A\sin B=\cos(A+B)$

$=\cos\left(\underbrace{\frac{\pi}{4}-x}+\underbrace{\frac{\pi}{4}-y}\right)$

$=\cos\left(\frac{\pi}{2}-\underbrace{(x+y)}_{\theta}\right)$

$\left(\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta\right)$

$=\sin(x+y)$

$=$ RHS.

(7) $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

LHS $=\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$

$=\dfrac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}.\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}.\tan x}\right)}$

$\boxed{\tan\frac{\pi}{4}=\tan 45=1}$

$=\dfrac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)}=\dfrac{(1+\tan x)^2}{(1-\tan x)^2}$

$=$ RHS.

(8) $$\frac{\cos(\pi+x)\cdot\cos(-x)}{\sin(\pi-x)\ \cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$$

$$LHS = \frac{\cos(\pi+x)\cdot\cos(-x)}{\sin(\pi-x)\cdot\cos\left(\frac{\pi}{2}+x\right)}$$

10^{th} class $\cos(90-\theta)=\sin\theta$

$\cos\left(\frac{\pi}{2}-x\right) = \sin x$

III

$\cos(\pi+x) = -\cos x$

$\cos\left(\frac{\pi}{2}+x\right) = -\sin x$

$\sin(\pi-x) = +\sin x$

II

$$LHS = \frac{(\cancel{+}\cos x)\cdot(\cos x)}{(\sin x)\ (\cancel{+}\sin x)}$$

$$= \frac{\cos^2 x}{\sin^2 x} = \cot^2 x = RHS$$

(9)

$$\cos\left(\frac{3\pi}{2}+x\right)\cdot\cos(2\pi+x)\cdot\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$$

$\underbrace{\cos\left(\frac{3\pi}{2}+x\right)}_{\mathrm{IV}} = +\sin x$

$\cos(2\pi+x) = +\cos x$

$\underbrace{\cot\left(\frac{3\pi}{2}-x\right)}_{\mathrm{III}} = +\tan x$

$\cot(2\pi+x) = +\cot x$

$LHS = \sin x\cdot\cos x\,[\tan x+\cot x]$

$$= \sin x\cdot\cos x\left[\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right]$$

$$= \cancel{\sin x\cdot\cos x}\left[\frac{\sin^2 x+\cos^2 x}{\cancel{\cos x\cdot\sin x}}\right]$$

$= 1 = RHS.$

(10) $\sin(n+1)x \cdot \sin(n+2)x + \cos(n+1)x \cdot \cos(n+2)x = \cos x$

$$\text{LHS.} = \sin\underbrace{(n+1)x}_{A} \cdot \sin\underbrace{(n+2)x}_{B} + \cos\underbrace{(n+1)x}_{A} \cdot \cos\underbrace{(n+2)x}_{B}$$

$$\boxed{\sin A \cdot \sin B + \cos A \cdot \cos B = \cos(A-B)}$$

$$= \cos(\overset{(n+1)x}{A} - \overset{(n+2)x}{B})$$

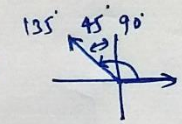
$$= \cos\left((n+1)x - (n+2)x\right) = \cos(\cancel{nx} + x - \cancel{nx} - 2x) = \cos(-x) = \cos x = \underline{\underline{RHS}}$$

---

(11) $\cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right) = -\sqrt{2}\,\sin x$

$\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

$\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$



$$\text{LHS.} = \underbrace{\cos\left(\frac{3\pi}{4}+x\right)}_{\cos(A+B)} - \underbrace{\cos\left(\frac{3\pi}{4}-x\right)}_{\cos(A-B)}$$

$$2 = \sqrt{2} \cdot \sqrt{2}$$

$$= \left[\cancel{\cos\frac{3\pi}{4} \cdot \cos x} - \underbrace{\sin\frac{3\pi}{4}}_{1/\sqrt{2}} \cdot \sin x\right] - \left[\cancel{\cos\frac{3\pi}{4} \cdot \cos x} + \sin\frac{3\pi}{4} \cdot \sin x\right]$$

$$= -\frac{1}{\sqrt{2}} \cdot \sin x - \frac{1}{\sqrt{2}} \cdot \sin x = -2\left(\frac{1}{\sqrt{2}} \cdot \sin x\right) = -\sqrt{2}\sin x = \text{RHS.}$$

Class - 11 - Maths × Exercise 3.3

(12) $\sin^2 6x - \sin^2 4x = \sin 2x \cdot \sin 10x$

LHS = $\sin^2 6x - \sin^2 4x$ $\quad (a^2 - b^2)$

$= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$ Apply $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

$\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cdot\cos\left(\frac{A+B}{2}\right)$

$= \left[2\sin\left(\frac{6x+4x}{2}\right)\cdot\cos\left(\frac{6x-4x}{2}\right)\right]\cdot\left[2\sin\left(\frac{6x-4x}{2}\right)\cdot\cos\left(\frac{6x+4x}{2}\right)\right]$

$= 4 \cdot \sin 5x \cdot \cos x \cdot \sin x \cdot \cos 5x$

$= (2\sin 5x \cdot \cos 5x)\cdot(2\sin x \cdot \cos x)$

$= (\sin 10x)\cdot(\sin 2x)$ $\quad$ (Here $2\sin\theta\cdot\cos\theta = \sin 2\theta$)

= RHS.

(13) $\cos^2 2x - \cos^2 6x = \sin 4x \cdot \sin 8x$

LHS $= \cos^2 2x - \cos^2 6x$ $\quad a^2 - b^2$

Apply $\cos A + \cos B = 2\cos\frac{A+B}{2} \cdot \cos\frac{A-B}{2}$

$\cos A - \cos B = -2\sin\frac{A+B}{2} \cdot \sin\frac{A-B}{2}$

$= (\cos 2x + \cos 6x)(\cos 2x - \cos 6x)$

$= \left[2\cos\left(\frac{2x+6x}{2}\right) \cdot \cos\left(\frac{2x-6x}{2}\right)\right] \cdot \left[-2 \cdot \sin\left(\frac{2x+6x}{2}\right) \cdot \sin\left(\frac{2x-6x}{2}\right)\right]$

$= -4\cos(4x) \cdot \cos(-2x) \cdot \sin(4x) \cdot \sin(-2x)$ $\quad \left\{ \begin{array}{l} \cos(-\theta) = \cos\theta \\ \sin(-\theta) = -\sin\theta \end{array} \right\}$

$= 4\cos 4x \cdot \cos 2x \cdot \sin 4x \cdot \sin 2x$

$= (2\cos 4x \cdot \sin 4x) \cdot (2\cos 2x \cdot \sin 2x)$

$= \sin 8x \cdot \sin 4x$

$=$ RHS.

(14) $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \cdot \sin 4x$

$LHS = \sin 2x + 2\sin 4x + \sin 6x$

$= (\sin 2x + \sin 6x) + 2\sin 4x$

$\boxed{\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)}$

$= 2\sin(4x) \cdot \cos(-2x) + 2\sin 4x$ $\quad (\cos(-\theta) = \cos\theta)$

$= 2\sin 4x \cdot \cos 2x + 2\sin 4x$

$= 2\sin 4x \cdot \{\cos 2x + 1\}$ $\quad (\because \cos 2\theta = 2\cos^2\theta - 1)$

$= 2\sin 4x \cdot \{2\cos^2 x - \cancel{1} + \cancel{1}\}$

$= 4\cos^2 x \cdot \sin 4x = RHS.$

Class - 11 - Maths   Exercise 3.3

(15) $\cot 4x\,(\sin 5x + \sin 3x) = \cot x\,(\sin 5x - \sin 3x)$

$\Rightarrow \dfrac{\cos 4x}{\sin 4x}\left(2\sin\left(\dfrac{5x+3x}{2}\right).\cos\left(\dfrac{5x-3x}{2}\right)\right) = \dfrac{\cos x}{\sin x}\left(2\sin\left(\dfrac{5x-3x}{2}\right).\cos\left(\dfrac{5x+3x}{2}\right)\right)$

$\Rightarrow \dfrac{\cos 4x}{\cancel{\sin 4x}}\left(2.\cancel{\sin 4x}.\cos x\right) = \dfrac{\cos x}{\cancel{\sin x}}.\left(2\cancel{\sin x}.\cos 4x\right)$

$\Rightarrow 2.\cos x.\cos 4x = 2\cos x.\cos 4x$

$\boxed{LHS = RHS}$

(16) $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\dfrac{\sin 2x}{\cos 10x}$

$LHS = \dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$= \dfrac{-\cancel{2}\cancel{\sin(7x)}.\sin(2x)}{\cancel{2}\cancel{\sin(7x)}.\cos(10x)}$

$= -\dfrac{\sin 2x}{\cos 10x} = RHS.$

Apply
$\cos A - \cos B =$
$\sin A - \sin B =$

(17) $$\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$$

$$LHS = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

(Apply
$\sin A + \sin B = \,=$
$\cos A + \cos B = \,=$)

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$

$$= \frac{\cancel{2}\sin 4x.\ \cancel{\cos(x)}}{\cancel{2}\cos 4x.\ \cancel{\cos x}}$$

$$= \tan 4x$$

(18) $$\frac{\sin x - \sin y}{\cos x + \cos y} = \tan\left(\frac{x-y}{2}\right)$$

$$LHS = \frac{\sin x - \sin y}{\cos x + \cos y}$$

(Apply $\sin A - \sin B =$
$\cos A + \cos B =$)

$$= \frac{\cancel{2}\sin\left(\frac{x-y}{2}\right).\cancel{\cos\left(\frac{x+y}{2}\right)}}{\cancel{2}\cancel{\cos\left(\frac{x+y}{2}\right)}.\cos\left(\frac{x-y}{2}\right)}$$

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)} = \tan\left(\frac{x-y}{2}\right) = RHS$$

Class 11 Maths   Exercise 3.3

(19)
$$\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$$

$$LHS = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$= \frac{\cancel{2}\sin\left(\frac{x+3x}{2}\right) \cdot \cancel{\cos\left(\frac{x-3x}{2}\right)}}{\cancel{2}\cos\left(\frac{x+3x}{2}\right) \cdot \cancel{\cos\left(\frac{x-3x}{2}\right)}}$$

$$= \tan 2x = \underline{RHS.}$$

(20)
$$\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$$

$$LHS = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} \longrightarrow \sin A - \sin B$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\ominus$$

$$-\cos 2x = -\cos^2 x + \sin^2 x$$

$$= 2\sin\left(\frac{A-B}{2}\right) \cdot \cos\left(\frac{A+B}{2}\right)$$

$$= \frac{2\sin\left(\frac{x-3x}{2}\right) \cdot \cos\left(\frac{x+3x}{2}\right)}{-\cos 2x}$$

$$= \frac{2\sin(-x) \cdot \cancel{\cos 2x}}{-\cancel{\cos 2x}} = \frac{-2\sin x}{-1}$$

$$= 2\sin x = RHS.$$

(21) ☆

$$\boxed{\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x}$$

$\cos(A) + \cos B$

$= 2\cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$

$2\cos\frac{7x}{2} \cdot \cos\frac{x}{2} +$

$\cos 2x$

$$\text{LHS} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos(3x) \cdot \cos x + \cos 3x}{2\sin 3x \cdot \cos x + \sin 3x}$$

$$= \frac{\cos 3x\,[\cancel{2\cos x + 1}]}{\sin 3x\,[\cancel{2\cos x + 1}]} = \cot 3x = \text{RHS}.$$

$$\boxed{a^2-b^2=(a+b)(a-b)}$$

$$\left\{\begin{aligned}
\sin A+\sin B &= 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)\\
\sin A-\sin B &= 2\sin\left(\frac{A-B}{2}\right)\cdot\cos\left(\frac{A+B}{2}\right)\\
\cos A+\cos B &= 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)\\
\cos A-\cos B &= -2\sin\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)
\end{aligned}\right.$$

$$\begin{aligned}
\cos 2\theta &= \cos^2\theta-\sin^2\theta\\
&= 2\cos^2\theta-1\\
&= 1-2\sin^2\theta\\
&= \frac{1-\tan^2\theta}{1+\tan^2\theta}
\end{aligned}$$

$$\underline{2\sin\theta\cdot\cos\theta} = \sin\underline{2\theta}$$

Formulae for Exercise 3.3:
Q 12 to Q 21

Class-11-Maths Exercise 3.3

Q.22

$\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x . \cot x = 1$

Proof: $3x = (2x + x)$

By taking 'tan' both sides

$\Rightarrow \tan 3x = \tan(2x+x)$

$\Rightarrow \tan 3x = \dfrac{\tan 2x + \tan x}{1 - \tan 2x . \tan x}$

tan $\rightarrow$ $\dfrac{1}{\cot}$

$\Rightarrow \dfrac{1}{\cot 3x} = \dfrac{\dfrac{1}{\cot 2x} + \dfrac{1}{\cot x}}{\dfrac{1}{1} - \dfrac{1}{\cot 2x} . \dfrac{1}{\cot x}}$

$\Rightarrow \dfrac{1}{\cot 3x} = \dfrac{\dfrac{\cot x + \cot 2x}{\cancel{\cot x . \cot 2x}}}{\dfrac{\cot x . \cot 2x - 1}{\cancel{\cot x . \cot 2x}}}$

$\Rightarrow \dfrac{1}{\cot 3x} = \dfrac{\cot x + \cot 2x}{\cot x . \cot 2x - 1}$

$\Rightarrow \cot x . \cot 2x - 1 = \cot 3x . \cot x + \cot 3x . \cot 2x$

$\Rightarrow \cot x . \cot 2x - \cot 3x . \cot x - \cot 3x . \cot 2x = 1$

H.P.

Q.23

$$\tan 4x = \frac{4\tan x.(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$$

LHS. $= \tan 4x$

$= \tan(2(2x))$

$$\tan 2\theta \Downarrow \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\tan(2x)}{1-\tan^2(2x)}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{\frac{1}{1}-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\frac{4\tan x}{\cancel{1-\tan^2 x}}}{\frac{(1-\tan^2 x)^2-(2\tan x)^2}{(1-\tan^2 x)^{\cancel{2}}}}$$

$$= \frac{4\tan x.(1-\tan^2 x)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

$$= \frac{4\tan x.(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$$

$=$ RHS.

Q 22, Q 23

$$\cot(x+y) = \frac{\cot x\cot y-1}{\cot x+\cot y}$$

$$\tan(x+y) = \frac{\tan x+\tan y}{1-\tan x.\tan y}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Class - 11 - Maths. Exercise 3.3

Q.24

Prove that

$$\boxed{\cos 4x = 1 - 8\sin^2 x \cdot \cos^2 x}$$

Proof:

$LHS = \cos 4x$ $\quad \left(\because \cos 2\theta = 1 - 2\sin^2\theta\right)$ $\quad \theta = 2x$

$= 1 - 2\sin^2(2x)$

$= 1 - 2(\sin 2x)^2$ $\quad \sin 2x = 2\sin x \cdot \cos x$

$= 1 - 2(2\sin x \cdot \cos x)^2$

$= 1 - 2(4\sin^2 x \cdot \cos^2 x)$

$= 1 - 8\sin^2 x \cdot \cos^2 x$

$= RHS.$

$\cos 2x = \begin{cases} \cos^2 x - \sin^2 x \\ 2\cos^2 x - 1 \\ 1 - 2\sin^2 x \\ \dfrac{1 - \tan^2 x}{1 + \tan^2 x} \end{cases}$

Q24 Q25

$\sin 2x = \begin{cases} 2\sin x \cos x \\ \dfrac{2\tan x}{1 + \tan^2 x} \end{cases}$ X

$\cos 3x = 4\cos^3 x - 3\cos x$

$43 - 31$

Q.25 Prove that

$$\boxed{\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1}$$

$LHS. = \cos 6x$

$= \cos 3(2x)$ $\quad \boxed{\cos 3\theta = 4\cos^3\theta - 3\cos\theta}$

$= 4\cos^3(2x) - 3\cos(2x)$

$= 4(\cos 2x)^3 - 3(\cos 2x)$

$= 4(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1)$

$\boxed{(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3}$

$= 4\left[(2\cos^2 x)^3 - 3\cdot(2\cos^2 x)^2\cdot 1 + 3(2\cos^2 x)\cdot 1^2 - 1^3\right]$

$- 6\cos^2 x + 3$

$= 4(8\cos^6 x - 12\cos^4 x + 6\cos^2 x - 1)$

$- 6\cos^2 x + 3$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Trigonometric Equations.

Before exercise $\boxed{3.4}$

Algebraic equations

↓

No. of Solutions → Fixed.

$2x - 1 = 0 \longrightarrow x = \frac{1}{2}$

$x^2 - 5x + 6 = 0 \longrightarrow x = 2, 3$

Trigonometric equations. → No. of solutions → ∞

↓

Unknown ↓ Angle

$\sin x = \frac{1}{2}$

↑ Angle

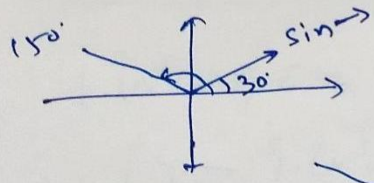
$\theta = 30° = \frac{\pi}{6}$

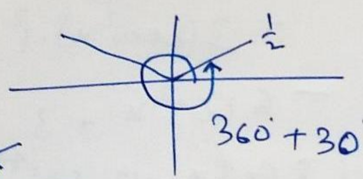
$\theta = 150° = \frac{5\pi}{6}$

$\theta = 390° =$

$\sin 30° = \frac{1}{2}$

$\sin 150° = \frac{1}{2}$





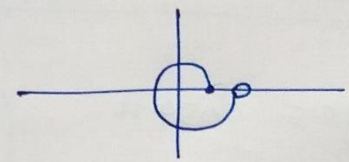
$360° + 30° = 390°$

$\theta = 750°$

Principal solution.

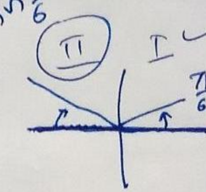
Angle → $[0, 2\pi)$

$0 \leq \text{Angle} < 2\pi$



e.g.

$\sin x = +\frac{1}{2} \to \sin\frac{\pi}{6}$



$x = \frac{\pi}{6}$ (I)

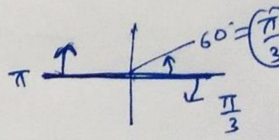
$x = \frac{5\pi}{6} = \pi - \frac{\pi}{6}$ (II)

Pri. sol^n. $= \frac{\pi}{6}, \frac{5\pi}{6} \in [0, 2\pi)$

$\tan x = -\sqrt{3}$

II ↓ $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

IV → $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$



General Solutions of Trigonometric Equations

unknown, known

$\sin x = \sin\theta$ $\operatorname{cosec} x = \operatorname{cosec}\theta$	$x = n\pi + (-1)^n\theta$	$n \in I$
$\cos x = \cos\theta$ $\sec x = \sec\theta$	$x = 2n\pi \pm \theta$	
$\tan x = \tan\theta$ $\cot x = \cot\theta$	$x = n\pi + \theta$	

$(x-1)(x-2)=0$

$x=1 \quad x=2$

?

Proof: (I) $\sin x = \sin\theta$

$\Rightarrow \sin x - \sin\theta = 0$

$\Rightarrow 2\sin\left(\frac{x-\theta}{2}\right) \cdot \cos\left(\frac{x+\theta}{2}\right) = 0$

0 | 0

$\sin\left(\frac{x-\theta}{2}\right) = 0$

$\Rightarrow \frac{x-\theta}{2} = n\pi$

$\Rightarrow \boxed{x = 2n\pi + \theta}$

Even $\oplus$

$\cos\left(\frac{x+\theta}{2}\right) = 0$

$\frac{x+\theta}{\cancel{2}} = (2n+1)\frac{\pi}{\cancel{2}}$

$\boxed{x = (2n+1)\pi - \theta}$

odd $\ominus$

$x = 2n\pi + \theta \qquad x = (2n+1)\pi - \theta$

$\boxed{x = n\pi + (-1)^n\theta}$

$n =$ even

$n =$ odd

(II) $\cos x = \cos\theta$

$\Rightarrow \cos x - \cos\theta = 0$

$\Rightarrow -2\sin\left(\frac{x+\theta}{2}\right) \cdot \sin\left(\frac{x-\theta}{2}\right) = 0$

$\sin\left(\frac{x+\theta}{2}\right) = 0$

$\frac{x+\theta}{2} = n\pi$

$\boxed{x = 2n\pi - \theta}$

$\sin\left(\frac{x-\theta}{2}\right) = 0$

$\frac{x-\theta}{2} = n\pi$

$\boxed{x = 2n\pi + \theta}$

$\boxed{x = 2n\pi \pm \theta}$

(iii) $\tan x = \tan\theta$

$\Rightarrow \tan x - \tan\theta = 0$

$\Rightarrow \dfrac{\sin x}{\cos x} - \dfrac{\sin\theta}{\cos\theta} = 0$

$\Rightarrow \dfrac{\sin x.\cos\theta - \cos x.\sin\theta}{\cos x.\cos\theta} = 0$

$\Rightarrow \underbrace{\sin x.\cos\theta - \cos x.\sin\theta = 0}$

$\Rightarrow \sin(x-\theta) = 0$

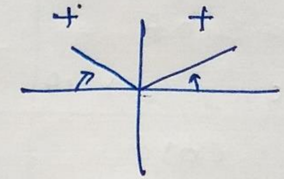
$\Rightarrow x - \theta = n\pi$

$\boxed{x = n\pi + \theta}$

Note,

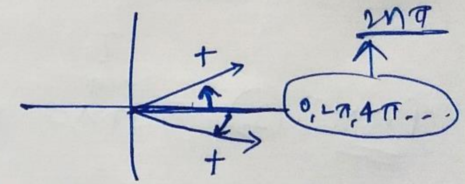
$\sin x = \sin\theta$

$\boxed{x = n\pi + (-1)^n\theta}$



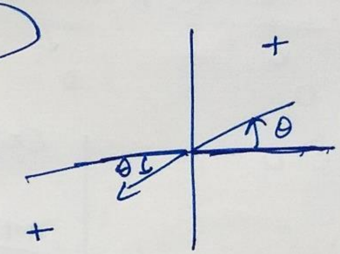
$\cos x = \cos\theta$

$\boxed{x = 2n\pi \pm \theta}$



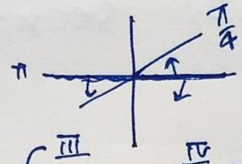
$\tan x = \tan\theta$

$\boxed{x = n\pi + \theta}$



Example

General Sol^n:



$\sin x = -\frac{1}{\sqrt{2}} = \sin\left(\pi + \frac{\pi}{4}\right)$ or $\sin\left(2\pi - \frac{\pi}{4}\right)$

$\sin x = \sin\left(\frac{5\pi}{4}\right)$

$x = n\pi + (-1)^n \theta \quad n \in I$

General Sol^n:

$$x = n\pi + (-1)^n \cdot \frac{5\pi}{4}$$

Formula

$\sin x = \sin \frac{7\pi}{4}$

$$x = n\pi + (-1)^n \cdot \frac{7\pi}{4} \quad n \in I$$

Both are Correct.

But Look Different

$n = 0$

$x = \frac{5\pi}{4}$ ✓

$n = 3$

$x = \frac{3\pi}{1} - \frac{7\pi}{4} = \frac{12\pi - 7\pi}{4} = \frac{5\pi}{4}$

Class - 11 - Maths × Exercise 3.4

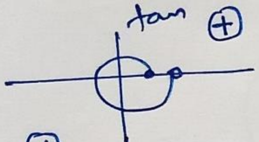
Principal & General Solutions

Trigonometric Equations

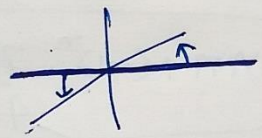
Q.1 $\tan x = \sqrt{3}$

$x = \frac{\pi}{3} = 60^\circ$

tan ⊕



⊕



Principal solⁿ: $[0, 2\pi)$

$\mathrm{I} \rightarrow \frac{\pi}{3} = \frac{\pi}{3}$

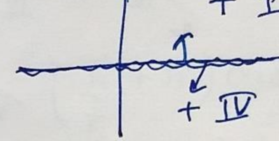
$\mathrm{III} \rightarrow \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

General solⁿ: $\rightarrow$ $x = n\pi + \frac{\pi}{3}$

$\sqrt{3} = \tan x = \tan\frac{\pi}{3}$

Q.2 $\sec x = 2 = \sec\frac{\pi}{3}$

$\cos x = \frac{1}{2} \rightarrow x = \frac{\pi}{3}$



$+\ \mathrm{I} \longrightarrow \frac{\pi}{3}$ ✓

$+\ \mathrm{IV} \longrightarrow 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Principal solⁿ: $= \frac{\pi}{3}, \frac{5\pi}{3}$.

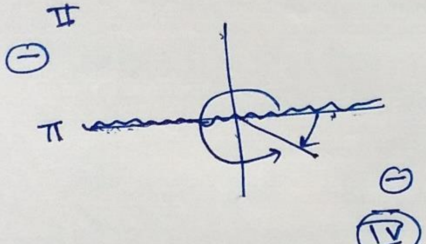
General solⁿ:

$x = 2n\pi \pm \theta, \quad n \in I$

$\boxed{x = 2n\pi \pm \frac{\pi}{3}}$

Q.3 $\cot x = -\sqrt{3} = \cot\frac{5\pi}{6} = \cot\frac{11\pi}{6}$

$30^\circ, \frac{\pi}{6}$



Principal $\longrightarrow \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\longrightarrow 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

General sol^n.

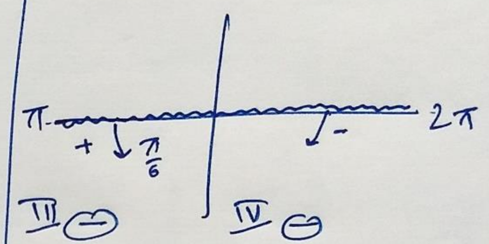
$x = n\pi + \theta$, $n \in I$.

? ✓

$$\boxed{x = n\pi + \frac{5\pi}{6}}$$ ✓

Q.4 $\operatorname{cosec} x = -2$

$\frac{\pi}{6} \rightarrow 30^\circ$



Principal sol^n. $= \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ III

$\longrightarrow 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ IV

General.

$x = n\pi + (-1)^n \cdot \theta$

$$\boxed{x = n\pi + (-1)^n \frac{7\pi}{6}}$$ ✓

Class - 11 - Maths   Exercise 3.4

Q.5

General solutions

$\cos 4x = \cos 2x$

$\cos x = \cos \theta$
$\downarrow$
$x = 2n\pi \pm \theta$

$n \in I$

$\Rightarrow \quad 4x = 2n\pi \pm 2x$

(+)

$4x = 2n\pi + 2x$

$\Rightarrow 4x - 2x = 2n\pi$

$\Rightarrow \cancel{2}x = \cancel{2}n\pi$

$\boxed{x = n\pi}, \ n \in I$

(−)

$4x = 2n\pi - 2x$

$\Rightarrow 4x + 2x = 2n\pi$

$\Rightarrow 6x = 2n\pi$

$\Rightarrow \boxed{x = \frac{n\pi}{3}} \quad n \in I$

(6) $\underbrace{\cos 3x + \cos x}{} - \cos 2x = 0$

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2}$

$\Rightarrow 2\cos\frac{3x+x}{2}\cdot\cos\frac{3x-x}{2} - \cos 2x = 0$

$\Rightarrow 2\cos 2x\cdot\cos x - \cos 2x = 0$

$\Rightarrow \cos 2x\,[2\cos x - 1] = 0$

$\cos 2x = 0$

$\Rightarrow 2x = (2n+1)\frac{\pi}{2}$

$\Rightarrow \boxed{x = (2n+1)\frac{\pi}{4}}$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

General sol^n

$\boxed{x = 2n\pi \pm \frac{\pi}{3}}$

(7) $\sin 2x + \cos x = 0$

$\Rightarrow 2\sin x\cdot\cos x + \cos x = 0$

$\Rightarrow \cos x\,[2\sin x + 1] = 0$

$\cos x = 0$

$\boxed{x = (2n+1)\frac{\pi}{2}}$

$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

$\sin x = \sin\left(\pi + \frac{\pi}{6}\right)$

$\sin x = \sin\frac{7\pi}{6}$

$\boxed{x = n\pi + (-1)^n\frac{7\pi}{6}}$

Class - 11 - Maths Exercise 3.4

Q.8 $\sec^2 2x = 1 - \tan 2x$

$\boxed{\sec^2\theta = 1 + \tan^2\theta}$

$\Rightarrow \cancel{1} + \tan^2 2x = \cancel{1} - \tan 2x$

$\Rightarrow \tan^2 2x + \tan 2x = 0$

$\Rightarrow \tan 2x . [\tan 2x + 1] = 0$

$\tan 2x = 0$

$\Rightarrow 2x = n\pi$

$\Rightarrow \boxed{x = \frac{n\pi}{2}}$

$\tan 2x = -1$

$\tan 2x = \tan\left(\pi - \frac{\pi}{4}\right)$

$\tan 2x = \tan\left(\frac{3\pi}{4}\right)$

$2x = n\pi + \frac{3\pi}{4}$

$\boxed{x = \frac{n\pi}{2} + \frac{3\pi}{8}}$

(9) $\sin x + \sin 3x + \sin 5x = 0$

$\boxed{\sin A + \sin B = 2\sin\frac{A+B}{2} . \cos\left(\frac{A-B}{2}\right)}$

$2x \quad x$

$\Rightarrow \sin x + \sin 5x + \sin 3x = 0$

$\Rightarrow 2\sin\frac{6x}{2} . \cos\frac{4x}{2} + \sin 3x = 0$

$\Rightarrow 2\sin 3x . \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x [2\cos 2x + 1] = 0$

$3x = n\pi$

$\boxed{x = \frac{n\pi}{3}}$

$2\cos 2x + 1 = 0$

$\Rightarrow \boxed{\cos 2x = -\frac{1}{2}} = \cos\frac{2\pi}{3}$

$\cancel{2}x = \cancel{2}n\pi \pm \frac{\cancel{2}\pi}{3}$

$\boxed{x = n\pi \pm \frac{\pi}{3}}$

Miscellaneous Exercise on Chapter 3

Trigonometric Functions

① $2\cos\frac{\pi}{13}\cdot\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Prove

LHS =

$= 2\cos\frac{\pi}{13}\cdot\cos\frac{9\pi}{13} + 2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cdot\cos\left(\frac{\frac{5\pi}{13}-\frac{3\pi}{13}}{2}\right)$

$= 2\cos\frac{\pi}{13}\cdot\cos\frac{9\pi}{13} + 2\cos\left(\frac{4\pi}{13}\right)\cdot\cos\frac{\pi}{13}$

$= 2\cos\frac{\pi}{13}\cdot\left[\cos\frac{9\pi}{13}+\cos\frac{4\pi}{13}\right]$

⊖ ⊕

$\frac{9\pi}{13}$ $\frac{4\pi}{13}$

$\frac{9\pi}{13}+\frac{4\pi}{13}=\pi$

$\frac{9\pi}{13}=\pi-\frac{4\pi}{13} \Rightarrow \cos\left(\frac{9\pi}{13}\right)=\cos\left(\pi-\frac{4\pi}{13}\right)$

$\Rightarrow \cos\frac{9\pi}{3} = -\cos\frac{4\pi}{13}$

$= 2\cos\frac{\pi}{13}\cdot\left\{-\cancel{\cos\frac{4\pi}{13}}+\cancel{\cos\frac{4\pi}{13}}\right\}$

$= 0 = RHS.$

$\cos A+\cos B = 2\cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2}$

$\sin A+\sin B = 2\sin\frac{A+B}{2}\cdot\cos\frac{A-B}{2}$

$\cos A-\cos B = -2\sin\frac{A+B}{2}\cdot\sin\frac{A-B}{2}$

$\sin A-\sin B = 2\sin\frac{A-B}{2}\cdot\cos\frac{A+B}{2}$

+

OBSERVA

② $(\sin 3x + \sin x).\sin x + (\cos 3x - \cos x).\cos x = 0$

Prove

$$LHS = (\sin 3x + \sin x).\sin x + (\cos 3x - \cos x).\cos x$$

$$= \left(2\sin\frac{3x+x}{2}.\cos\frac{3x-x}{2}\right)\sin x + \left(-2\sin\frac{3x+x}{2}.\sin\frac{3x-x}{2}\right).\cos x$$

$$= 2\sin 2x.\cos x.\sin x - 2\sin 2x.\sin x.\cos x$$

$$= 0$$

Class-11- Maths

Misc. Ex. 3

Q.3

$$(\cos x+\cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2\left(\frac{x+y}{2}\right)$$

$\downarrow$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\sin\frac{A-B}{2}\cdot\cos\frac{A+B}{2}$$

$$LHS = (\cos x+\cos y)^2 + (\sin x - \sin y)^2$$

$$= \left(2\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2}\right)^2 + \left(2\sin\frac{x-y}{2}\cdot\cos\frac{x+y}{2}\right)^2$$

$$= \underline{4\cos^2\left(\frac{x+y}{2}\right)}\cdot\cos^2\left(\frac{x-y}{2}\right) + \underline{4}\sin^2\left(\frac{x-y}{2}\right)\cdot\underline{\cos^2\left(\frac{x+y}{2}\right)}$$

$$= 4\cos^2\left(\frac{x+y}{2}\right)\cdot\left\{\underbrace{\cos^2\left(\frac{x-y}{2}\right)+\sin^2\left(\frac{x-y}{2}\right)}_{1}\right\}$$

$$= 4\cos^2\left(\frac{x+y}{2}\right)\cdot 1$$

$$= RHS.$$

Q.4 $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\left(\frac{x-y}{2}\right)$

$LHS = (\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$\cos A - \cos B = -2\sin\frac{A+B}{2}\cdot\sin\frac{A-B}{2}$, $\sin A - \sin B = 2\sin\frac{A-B}{2}\cdot\cos\frac{A+B}{2}$

$= \left(-2\sin\frac{x+y}{2}\cdot\sin\frac{x-y}{2}\right)^2 + \left(2\sin\frac{x-y}{2}\cdot\cos\frac{x+y}{2}\right)^2$

$= 4\sin^2\left(\frac{x+y}{2}\right)\cdot\sin^2\left(\frac{x-y}{2}\right) + 4\cdot\sin^2\left(\frac{x-y}{2}\right)\cdot\cos^2\left(\frac{x+y}{2}\right)$

$= 4\sin^2\left(\frac{x-y}{2}\right)\cdot\left\{\sin^2\left(\frac{x+y}{2}\right) + \cos^2\left(\frac{x+y}{2}\right)\right\}$ $\rightarrow 1$

$\Rightarrow 4\sin^2\left(\frac{x-y}{2}\right) = RHS.$

Miscellaneous Exercise on Chapter 3

Q. 5

To Prove

$\sin x + \sin 3x + \sin 5x + \sin 7x$

$= 4\cos x \cdot \cos 2x \cdot \sin 4x$

$$LHS = \sin x + \sin 3x + \sin 5x + \sin 7x$$

$$= 2\sin\frac{4x}{2} \cdot \cos\frac{2x}{2} + 2\sin\frac{12x}{2} \cdot \cos\frac{2x}{2}$$

$$= 2\cos x \cdot \{\sin 2x + \sin 6x\}$$

$$= 2\cos x \cdot \{2\sin 4x \cdot \cos 2x\}$$

$$= RHS$$

Q. 6

$RHS = \tan 6x$

$$LHS = \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$$

$$= \frac{(\cancel{2}\sin 6x \cdot \cos x) + (\cancel{2}\sin 6x \cdot \cos 3x)}{(\cancel{2}\cos 6x \cdot \cos x) + (\cancel{2}\cos 6x \cdot \cos 3x)}$$

$$= \frac{\sin 6x \cdot \cancel{(\cos x + \cos 3x)}}{\cos 6x \cdot \cancel{(\cos x + \cos 3x)}}$$

$$= \tan 6x = RHS.$$

Q. 7

$\sin 3x + \sin 2x - \sin x = 4 \sin x . \cos \frac{x}{2} . \cos \frac{3x}{2}$

LHS $= \sin 3x + \sin 2x - \sin x$

$= (\sin 3x - \sin x) + \sin 2x$

$\sin 2x = 2 \sin x . \cos x$

$= 2 \sin x . \cos 2x + \sin 2x$

$= 2 \sin x . \cos 2x + 2 \sin x . \cos x$

$= 2 \sin x . \{ \cos 2x + \cos x \}$

$= 2 \sin x . \left\{ 2 \cos \frac{2x+x}{2} . \cos \frac{2x-x}{2} \right\}$

~~$= 2 \sin x . \{$~~ $= 4 \left\{ \sin x . \cos \frac{3x}{2} . \cos \frac{x}{2} \right\} =$ RHS.

Miscellaneous Exercise on Chapter 3

$\boxed{Q8,\ Q9,\ Q10}$

$$\# \begin{cases} \cos 2\theta = 2\cos^2\theta - 1 \longrightarrow \cos x = 2\cos^2\frac{x}{2} - 1 \longrightarrow \boxed{2\cos^2\frac{x}{2} = \cos x + 1} \\ \cos 2\theta = 1 - 2\sin^2\theta \longrightarrow \cos x = 1 - 2\sin^2\frac{x}{2} \longrightarrow \boxed{2\sin^2\frac{x}{2} = 1 - \cos x} \end{cases}$$

\#

x	$x/2$
$\sin x$	$\sin \frac{x}{2}$
$\cos x$	$\cos \frac{x}{2}$
$\tan x$	$\tan \frac{x}{2}$
Ques.	Ans.

Q.8

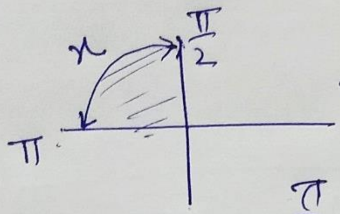
$\tan x = -\frac{4}{3}$ $\rightarrow$ II quadrant

$\rightarrow$ $\sin\frac{x}{2}$? $+$

$\cos\frac{x}{2}$? $+$

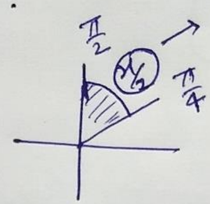
$\tan\frac{x}{2}$? $+$

I quadrant

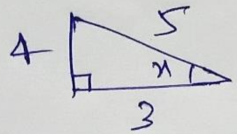


$x \in \left(\frac{\pi}{2}, \pi\right)$

$\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$ (I)



$\tan x = -\frac{4}{3} = \frac{P}{B}$



$\cos x = \frac{B}{H} = \frac{-3}{5}$ $\rightarrow$ II

$\boxed{2\cos^2\frac{x}{2} = 1 + \cos x}$

$\Rightarrow 2\left(\cos\frac{x}{2}\right)^2 = 1 + \left(-\frac{3}{5}\right)$

$\Rightarrow \cancel{2}\left(\cos\frac{x}{2}\right)^2 = \frac{\cancel{2}}{5} \Rightarrow \cos\frac{x}{2} = \pm\frac{1}{\sqrt{5}} = +\frac{1}{\sqrt{5}}$ ✓

$2\sin^2\frac{x}{2} = 1 - \cos x$

$\cancel{2}\sin^2\frac{x}{2} = 1 + \frac{3}{5} = \frac{\cancel{8}\,4}{5}$

$\sin\frac{x}{2} = \pm\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}}$ ✓

$\cos\frac{x}{2} = \frac{1}{\sqrt{5}}$ ✓

$\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$ ✓

$\tan\frac{x}{2} = 2$ ✓

Q.9 $\cos x = -\frac{1}{3}$

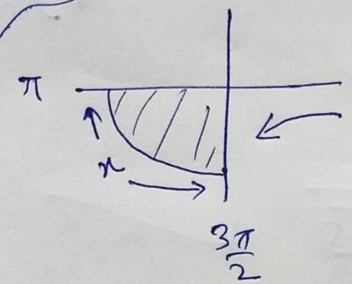
$\downarrow$

III

✓ $\sin\frac{x}{2} = \sqrt{\frac{2}{3}}$

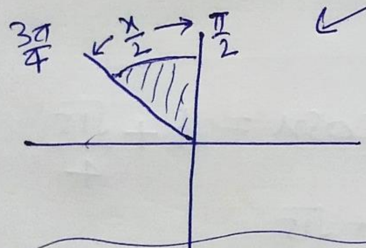
✓ $\cos\frac{x}{2} = -\frac{1}{\sqrt{3}}$

✓ $\tan\frac{x}{2} = -\sqrt{2}$



$\pi < x < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$



$x \to \text{III}$

$\frac{x}{2} \to \text{II}$

$2\cos^2\frac{x}{2} = 1+\cos x$

$2\cos^2\frac{x}{2} = 1-\frac{1}{3}$

$2.\cos^2\frac{x}{2} = \frac{2}{3}$

$\cos\frac{x}{2} = -\frac{1}{\sqrt{3}}$

$\cos^2\frac{x}{2} = \frac{1}{3}$

$\cos\frac{x}{2} = \pm\frac{1}{\sqrt{3}}$ → + ✗, − ✓

II

$2\sin^2\frac{x}{2} = 1-\cos x$

$\cancel{2}\sin^2\frac{x}{2} = 1+\frac{1}{3} = \frac{\cancel{4}\;2}{3}$

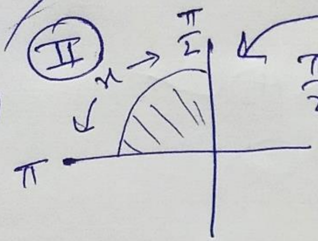
$\sin\frac{x}{2} = \pm\sqrt{\frac{2}{3}}$ → + ✓, − ✗

II

$\sin\frac{x}{2} = +\sqrt{\frac{2}{3}}$

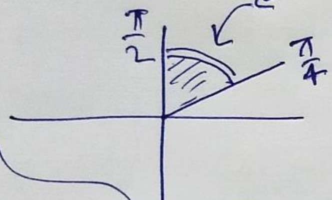
Q 10 $\sin x = \frac{1}{4}$ → II - quadrant

$$\boxed{\begin{aligned}\sin\frac{x}{2} &= \sqrt{\frac{4+\sqrt{15}}{8}}\\ \cos\frac{x}{2} &= \sqrt{\frac{4-\sqrt{15}}{8}}\\ \tan\frac{x}{2} &= 4+\sqrt{15}\end{aligned}}$$



$\frac{\pi}{2} < x < \pi$

$\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

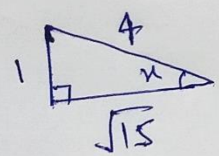


I

$x \to \text{II}$

$\frac{x}{2} \to \text{I}$

$\frac{P}{H} = \sin x = \frac{1}{4}$ ($x \to \text{II}$)



$\cos x = -\frac{\sqrt{15}}{4}$

$$4+\sqrt{15} = \sqrt{\frac{(4+\sqrt{15})^2}{16-15}}$$

$$2\cos^2\frac{x}{2} = 1+\cos x = 1+\left(-\frac{\sqrt{15}}{4}\right)$$

$$\cos^2\frac{x}{2} = \frac{4-\sqrt{15}}{8}$$

$$\cos\frac{x}{2} = +\sqrt{\frac{4-\sqrt{15}}{8}}$$

I-quad.

$$2\sin^2\frac{x}{2} = 1-\cos x = 1+\frac{\sqrt{15}}{4}$$

$$\Rightarrow 2\sin^2\frac{x}{2} = \frac{4+\sqrt{15}}{4}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{4+\sqrt{15}}{8}$$

$$\sin\frac{x}{2} = +\sqrt{\frac{4+\sqrt{15}}{8}}$$

I

$$\tan\frac{x}{2} = \frac{\sin x/2}{\cos x/2} = \frac{\sqrt{\frac{4+\sqrt{15}}{8}}}{\sqrt{\frac{4-\sqrt{15}}{8}}} = \sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}} \times \frac{4+\sqrt{15}}{4+\sqrt{15}}}$$