Class-X Session 2022-23
Subject - Mathematics (Standard)
Sample Question Paper - 41
With Solution

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Ch. No.	Chapter Name	Per Unit Marks	Section-A (1 Mark)		Section-B (2 Marks)	Section-C (3 Marks)	Section-D (5 Marks)	Section-E (4 Marks)	Total Marks
			MCQ	A/R	VSA	SA	LA	Case-Study	
1	Real Number	6	1(Q1)		1(Q21)	1(Q31)			6
2	Polynomials	20	1(Q2)					(Q36)	5
3	Pair of Linear Equations in Two Variables		1(Q3)			1(Q26)			4
4	Quadratic Equations		2(Q4, 8)			1(Q28)			5
5	Arithmetic Progression		3(Q5, 6, 9)			1(Q29)			6
6	Triangles	15	1(Q7)	1(Q19)			1(Q32)	(Q38)	7
7	Circles		2(Q14, 13)		1(Q22)				8
8	Coordinate Geometry	6	1(Q10)		1(Q23)	1(Q27)			6
9	Introduction to Trigonometry	12	1(Q11)		1(Q24)	1(Q30)			6
10	Some Applications of Trigonometry		1(Q12)				1(Q33)		6
11	Areas Related to Circles	10	1(Q15)					(Q37)	5
12	Surface Areas and Volumes						1(Q34)		5
13	Statistics	11	1(Q17)	1(Q20)			1(Q35)		7
14	Probability		2(Q16, 18)		1(Q25)				4
Total Marks (Total Questions)		80	18(18)	2(2)	10(5)	18(6)	20(4)	12(3)	80(38)

Note : The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

Time : 3 Hours | Max. Marks : 80

General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 case based/integrated units of assessment (4 marks each) with sub parts of values of 1, 1 and 2 marks each respectively.*

SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. When 2^{256} is divided by 17, then remainder would be

(a) 1 (b) 16 (c) 14 (d) None of these

2. If the polynomials $ax^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + a$ leave same remainder when divided by $(x - 3)$, find the value of a.

(a) -1 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

3. The perimeter of a rectangle is 40 cm. The ratio of its sides is 2 : 3. Find its length and breadth.

(a) $l = 10$ cm, b = 8 cm (b) $l = 12$ cm, b = 8 cm (c) $l = 12$ m, b = 8 m (d) $l = 40$ m, b = 30 m

4. If the product of roots of the equation $x^3 - 3x + k = 10$ is -2, then the value of k is

(a) -2 (b) -8 (c) 8 (d) 12

5. Sanjay starts his job with a certain monthly salary and earns a fixed increment every year. If his salary was ₹ 4500 after four years of service and ₹ 5400 after 10 years, find his initial salary and annual increment.

(a) 4000, 200 (b) 3900, 150 (c) 4500, 100 (d) 3800, 250

6. In an A.P. if $a = 5$, $a_n = 81$ and $S_n = 860$, then n is

(a) 10 (b) 15 (c) 20 (d) 25

7. If in two ΔABC and ΔPQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then

(a) ΔPQR ~ ΔCAB (b) ΔPQR ~ ΔABC (c) ΔCBA ~ ΔPQR (d) ΔBCA ~ ΔPQR

8. If the equation

$(1 + m^2)x^2 + (2mc)x + (c^2 - a^2) = 0$ has equal roots, then

(a) $c^2 - a^2 = 1 + m^2$ (b) $c^2 = a^2(1 + m^2)$ (c) $c^2a^2 = (1 + m^2)$ (d) $c^2 + a^2 = 1 + m^2$

9. If eight times the 8^{th} term of an A.P. is equal to 12 times the 12^{th} term of the A.P. then its 20^{th} term will be

(a) -1 (b) 1 (c) 0 (d) 2

10. The centroid of the triangle whose vertices are (3, –7), (–8, 6) and (5, 10) is

(a) (0, 9) (b) (0, 3) (c) (1, 3) (d) (3, 5)

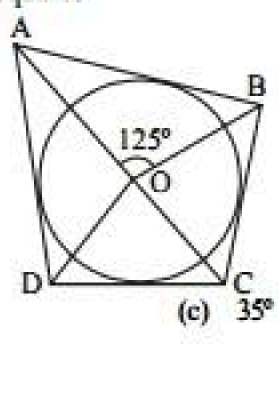
11. cos 1° . cos 2°. cos 3° .... cos 179° is equal to

(a) -1 (b) 0 (c) 1 (d) $1/\sqrt{2}$

12. A tree 6 m tall casts a 4 cm long shadow. At the same time, a flag pole casts a shadow 50 m long. How long is the flag pole?

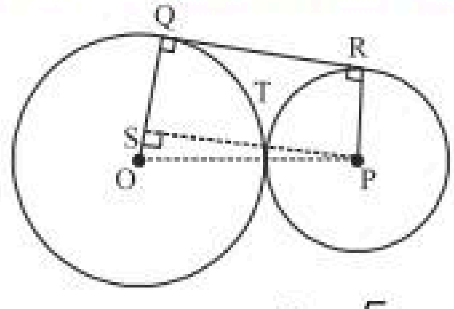
(a) 75 m (b) 100 m (c) 150 m (d) 50 m

13. **2.** In figure, if ∠AOB = 125°, then ∠COD is equal to



(a) 62.5° (b) 45° (c) 35° (d) 55°

14. Two circles with centres O and P, and radii 8 cm and 4 cm touch each other externally. Find the length of their common tangent QR.



(a) 8 cm (b) 7 cm (c) $8\sqrt{2}$ cm (d) $7\sqrt{3}$ cm

15. A drain cover is made from a square metal plate of side 40 cm having 441 holes of diameter 1 cm each drilled in it. Find the area of the remaining square plate.

(a) 1250.5 cm^2 (b) 1253.5 cm^2 (c) 1240.2 cm^2 (d) 1260.2 cm^2

16. Two fair dice are thrown. Find the probability that both dice show different numbers.

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$ (c) $\frac{32}{36}$ (d) $\frac{29}{36}$

17. In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. The lower limit of the class is

(a) 6 (b) 7 (c) 8 (d) 12

18. A factory has 120 workers in January, 90 of them are female workers. In February, another 15 male workers were employed. A worker is then picked at random. Calculate the probability of picking a female worker.

(a) $\frac{3}{4}$ (b) $\frac{4}{9}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false but R is true.***

19. **Assertion :** If in a ΔABC, a line $DE \parallel BC$, intersects AB in D and AC in E, then $\frac{AB}{AD} = \frac{AC}{AE}$.

Reason : If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.

20. **Assertion :** If the number of runs scored by 11 players of a cricket team of India are 5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27 then median is 30.

Reason : Median $= \left(\frac{n+1}{2}\right)^{th}$ value, if n is odd.

SECTION-B

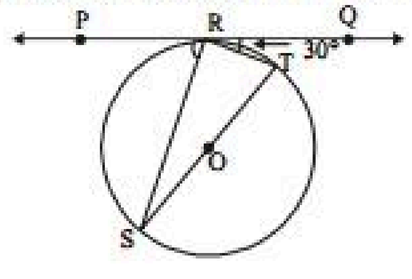
This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find how many integers between 200 and 500 are divisible by 8.

OR

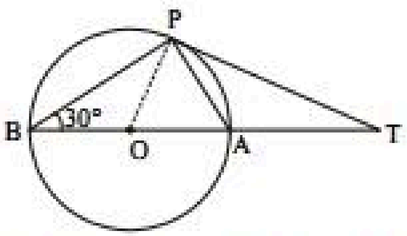
Show that one and only one out of n, $n+2$, $n+4$ is divisible by 3, where n is any positive integer.

22. In the figure, PQ is tangent at a point R of the circle with centre O. If $\angle TRQ = 30°$, find $\angle PRS$.



OR

In figure, BOA is a diameter of a circle and the tangent at a point P meets BA extended at T. If $\angle PBO = 30°$, then what is the measure of $\angle PTA$?



23. In what ratio is the line segment joining the points (3, 5) & (–4, 2) divided by y–axis?

24. Solve : $\sec^2\theta + \tan^2\theta = \frac{5}{3}; \theta < 90°$

25. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, Find the number of blue balls in the bag.

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test if Yash attempted every question?

OR

One hundred men in 10 days do one third of a piece of work. The work is then required to be completed in another 13 days. On the next day (the eleventh day) 50 more men are employed, and on the day after that, another 50. How many men must be relieved at the end of the 18th day so that the rest of the men, working for the remaining time, will just complete the work ?

27. Find the distance between each of the following pair of points :

(a) P(6, 8) and Q(–9, –12) (b) A(–6, –1) and B(–6, 11)

28. Roots of the quadratic equation $36x^2 - 12ax + (a^2 - b^2) = 0$ are $\frac{a+b}{c}$ and $\frac{a-b}{c}$. Then, find the value of c.

OR

Find the real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$.

29. If the ratio of the sum of first n terms of two A.P's is (7n + 1) : (4n + 27), find the ratio of their mth terms.

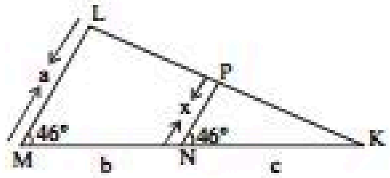
30. If $\tan\theta = \frac{x\sin\phi}{1 - x\cos\phi}$ and $\tan\phi = \frac{y\sin\theta}{1 - y\cos\theta}$, then the value of $\frac{x}{y}$ is _______ .

31. Given that HCF (306, 657) = 9, find LCM (306, 657).

SECTION-D

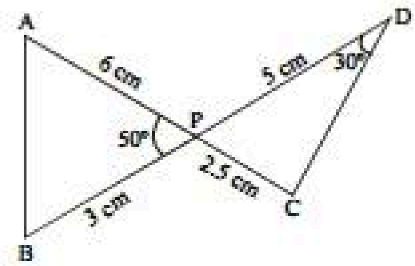
This section comprises of long answer-type questions (LA) of 5 marks each.

32. From given fig. express 'x' in terms of a, b, c.



OR

In figure, two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, $\angle APB = 50°$ and $\angle CDP = 30°$. Then, find the value of $\angle PBA$



33. A boy on horizontal plane finds bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of 20 metre high building, finds the angle of elevation of the same bird to be 45°. Both the boy and the girl are on opposite sides of the bird. Find the distance of bird from the girl.

OR

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After sometime, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

34. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.

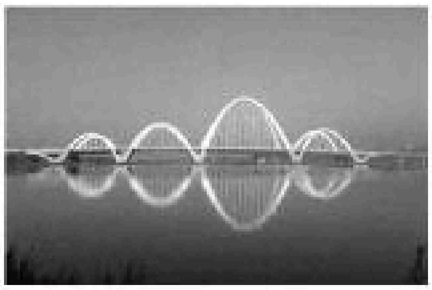
35. Find the mean of the following frequency distribution by Assumed Mean Method.

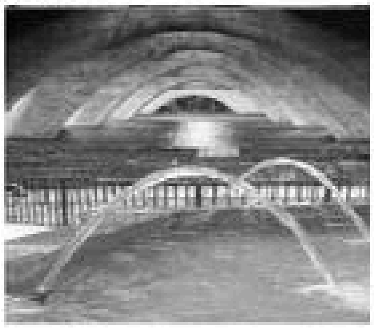
Class Interval	Frequency
0–4	6
4–8	3
8–12	6
12–16	16
16–20	3
20–24	14
24–28	10
28–32	8

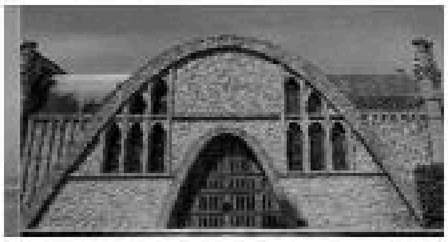
SECTION-E

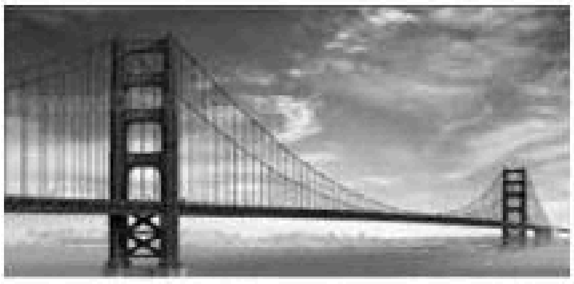
This section comprises of 3 case study/passage - based questions of 4 marks each with three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively.

36. **Case - Study 1:** Read the following passage and answer the questions given below.
The below picture are few natural examples of parabolic shape which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms.









(i) Write in the standard form of quadratic polynomial.
(ii) If two zeros are 2 and 3 then find polynomial.

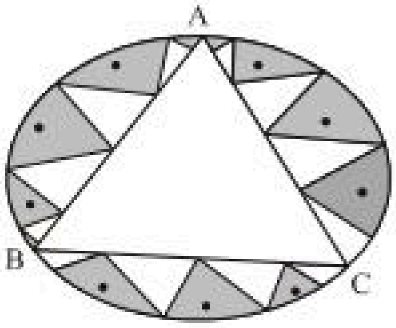
(iii) If α and $\frac{1}{\alpha}$ are the zeroes of the quadratic polynomial

$2x^2 - x + 8k$, then find k.

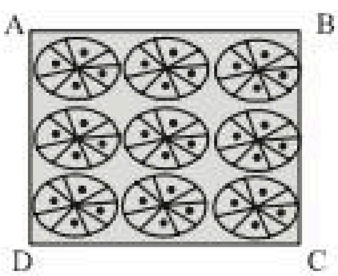
OR

If the sum of the roots is $-p$ and product of the roots is $-\frac{1}{p}$, then find the quadratic polynomial.

37. Case - Study 2: Read the following passage and answer the questions given below.
Pookalam is the flower bed or flower pattern designed during Onam in Kerala. It is similar as Rangoli in North India and Kolam in Tamil Nadu.
During the festival of Onam, your school is planning to conduct a Pookalam competition. Your friend who is a partner in competition , suggests two designs given below.
Observe these carefully.



I



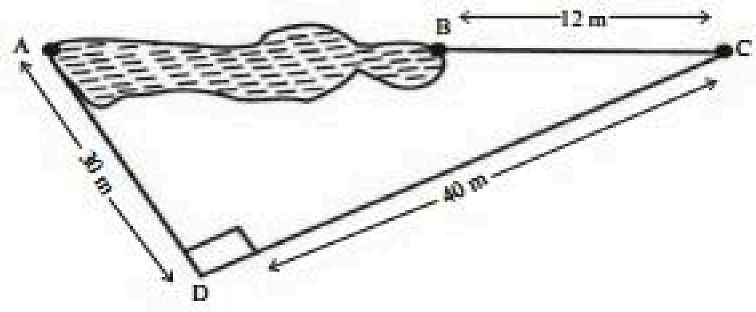
II

Design I: This design is made with a circle of radius 32cm leaving equilateral triangle ABC in the middle as shown in the given figure.
Design II: This Pookalam is made with 9 circular design each of radius 7cm.
Refer Design I:
(i) Find the side of equilateral triangle.
(ii) Find the altitude of the equilateral triangle is
Refer Design II:
(iii) Find the area of square is

OR

Find area of each circular design.

38. Rohan wants to measure the distance of a pond during the visit to his native. He marks points A and B on the opposite edges of a pond as shown in the figure below. To find the distance between the points, he makes a right-angled triangle using rope connecting B with another point C are a distance of 12m, connecting C to point D at a distance of 40m from point C and the connecting D to the point A which is are a distance of 30m from D such the $\angle ADC = 90^\circ$.



(i) What is the distance AC?

(ii) Which is the following does not form a Pythagoras triplet?

(iii) Find the length AB?

OR

Find the length of the rope used.

Solution

SAMPLE PAPER-8

1. (a) When 2^{256} is divided by 17 then, $\frac{2^{256}}{2^4+1} = \frac{(2^4)^{64}}{(2^4+1)}$

By remainder theorem when $f(x)$ is divided by $x + a$ the remainder $= f(-a)$

Here, $f(a) = (2^4)^{64}$ and $x = 2^4$ and $a = 1$

$\therefore$ Remainder $= f(-1) = (-1)^{64} = 1$

2. (a) Substitute $x = 3$ in polynomial $ax^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + a$ to obtain remainder and equate.

$a(3)^3 + 4(3)^2 + 3(3) - 4 = (3)^3 - 4(3) + a \Rightarrow a = -1$

3. (b) Let length and breadth be x cm and y cm respectively.

According to problem,

$2(x + y) = 40$ ...(i)

and $\frac{y}{x} = \frac{2}{3}$ ...(ii)

on solving, $x = 12, y = 8$

$\therefore$ length = 12 cm and breadth = 8cm.

4. (c) Given equation is $x^2 - 3x + (k - 10) = 0$.

$\therefore$ Product of roots $= (k - 10)$.

So, $k - 10 = -2 \Rightarrow k = 8$.

5. (b) Let the annual increment be ₹ y and initial salary be ₹ x

$\therefore$ $x + 4y = 4500$ ... (i)

and $x + 10y = 5400$ .... (ii)

Solving eqs. (i) and (ii), we get

$x = 3900$ and $y = 150$

$\therefore$ Initial salary = ₹ 3900

and increment = ₹ 150

6. (c) $S_n = (a + a_n)$

$\Rightarrow 860 = (5 + 81)$

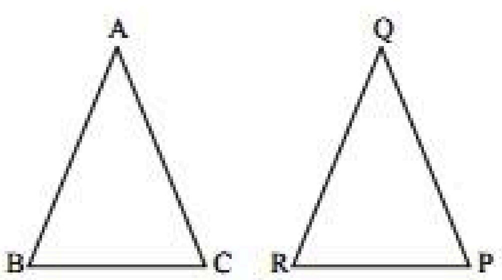
$n = 860 \div 43 = 20$

7. (a) Given that, ΔABC and ΔPQR,

$$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$$

Therefore sides of one triangle are proportional to the side of the other triangle, then their corresponding angles are also equal. Hence, by SSS similarity, triangles are similar.

$\Rightarrow$ $\Delta CAB \sim \Delta PQR$



8. (b) Since the equation has two equal roots, D = 0

$\Rightarrow (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$\Rightarrow 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$

$\Rightarrow -4c^2 + 4a^2 + 4m^2a^2 = 0 \Rightarrow 4c^2 = 4a^2 + 4m^2a^2$

$\Rightarrow 4c^2 = 4a^2(1 + m^2) \Rightarrow c^2 = a^2(1 + m^2)$

9. (c) $t_8 = a + 7d, t_{12} = a + 11d$

According to question, $8t_8 = 12t_{12}$ (given)

$\Rightarrow 8(a + 7d) = 12(a + 11d)$

$\Rightarrow 8a + 56d = 12a + 132d$

$\Rightarrow 8a - 12a + 56d - 132d = 0$

$\Rightarrow -4a - 76d = 0$

$\Rightarrow a + 19d = 0$ ...(i)

$\therefore$ $t_{20} = a + 19d = 0$ using (i)

$\therefore$ $t_{20} = 0$

10. (b) Centroid is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

i.e. $= \left(\frac{3 + (-8) + 5}{3}, \frac{-7 + 6 + 10}{3}\right) = \left(\frac{0}{3}, \frac{9}{3}\right) = (0, 3)$

11. (b)

12. (a) Let h be the length of the pole.

By the given condition $\frac{6}{4} = \frac{h}{50}$

$\Rightarrow h = \frac{6 \times 50}{4} = 75$m

13. (d) The opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

$\therefore$ $\angle AOB + \angle COD = 180^\circ$

$\Rightarrow$ $\angle COD = 180^\circ - \angle AOB$

$= 180^\circ - 125^\circ = 55^\circ$

14. (c) Join O to P and Q. Join P to R. Draw SP $\perp$ OQ.

Now SP = QR, as they are opposite sides of rectangle PRQS.

OP = 8 cm + 4 cm = 12 cm; OS = 8 cm – 4 cm = 4 cm

In right triangle POS,

$SP = \sqrt{OP^2 - OS^2} = \sqrt{12^2 - 4^2} = 8\sqrt{2}$ cm

$\therefore$ QR $= 8\sqrt{2}$ cm

15. (b) We have,

Area of square metal plate $= 40 \times 40 = 1600$ cm^2

Area of each hole $= \pi r^2 = \frac{22}{7} \times \left(\frac{1}{2}\right)^2 = \frac{11}{14}$cm^2

$\therefore$ Area of 441 holes $= 441 \times \frac{11}{14} = 346.5$cm^2

Hence, area of the remaining square plate

$= (1600 - 346.5) = 1253.5$ cm^2

16. **(b)** S = {(1, 1), ..., (1, 6), (2, 1), ..., (2, 6), (3, 1), ..., (3, 6), (4, 1), ..., (4, 6), (5, 1), ..., (5, 6), (6, 1), ..., (6, 6)}

n(S) = 36

Let E be the event that both dice show different numbers.

E {(1, 2), (1, 3),...., (1, 6), (2, 1), (2, 3), (2, 4),...., (2, 6), (3, 1), (3, 2), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (4, 5)}

n(E) = 30

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{30}{36} = \frac{5}{6}$

17. **(b)** Let x be the upper limit and y be the lower limit.

Since, the mid value of the class is 10.

$\therefore \frac{x+y}{2} = 10 \quad \Rightarrow \quad x + y = 20 \qquad$ .....(i)

and $x - y = 6$ (width of the class = 6) .....(ii)

By solving equations (i) and (ii), we get $y = 7$.

Hence, lower limit of the class is 7.

18. **(c)** Initial number of workers = 120

When 15 male workers are added, then the total number of workers = 120 + 15 = 135

Number of female workers = 90

$\therefore$ Probability of female workers $= \frac{90}{135} = \frac{2}{3}$

19. **(a)** Reason is true. [This is Thale's Theorem]

For Assertion

Since $DE \parallel BC$ $\therefore$ by Thale's Theorem

Figure

$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$

$\Rightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$

$\Rightarrow \frac{AD + DB}{AD} = \frac{AE + EC}{AE}$

$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$

$\therefore$ Assertion is true.

Since reason gives Assertion.

20. **(d)** Arranging the terms in ascending order,

0, 5, 11, 19, 21, 27, 30, 36, 42, 50, 52

median value $= \left(\frac{11+1}{2}\right)^{\text{th}} = 6^{\text{th}}$ value = 27

21. First number divisible by 8 between 200 and 500 is 208.

It forms an A.P. = 208, 216, 224, ..., 496.

Here, $a_n = 496$, $a = 208$, $d = 8$

$a_n = a + (n-1)d \Rightarrow 496 = 208 + (n-1)\,8$ [1 Mark]

$(n-1)8 = 496 - 208$

$(n-1) = \frac{288}{8} \Rightarrow n - 1 = 36 \quad \Rightarrow \quad n = 37$ [1 Mark]

OR

Let q be the quotient and r be the remainder when n is divided by 3. [1 Mark]

Therefore, $n = 3q + r$, where r = 0, 1 , 2

$\Rightarrow \quad n = 3q$ or $n = 3q + 1$ or $3q + 2$

Case(i) if $n = 3q$, then n is divisible by 3, $n + 2$ and $n + 4$ are not divisible by 3.

Case (ii) if $n = 3q + 1$ then $n + 2 = 3q + 3$ $= 3(q + 1)$, which is divisible by 3 and $n + 4 = 3q + 5$, which is not divisible by 3.

So, only $(n + 2)$ is divisible by 3.

Case (iii) if $n = 3q + 2$, then $n + 2 = 3q + 4$, which is not divisible by 3 and $(n + 4) = 3q + 6$ $= 3(q + 2)$, which is divisible by 3.

So, only $(n + 4)$ is divisible by 3. [1 Mark]

Hence one and only one out of n, $(n + 2)$, $(x + 4)$ is divisible by 3.

22. **(d)** Since ST is a diameter of the circle with centre O, So $\angle SRT = 90°$. [Angle in a semicircle] [1 Mark]

Now, $\angle PRS + \angle SRT + \angle TRQ = 180°$ [Linear pair]

$\angle PRS + 90° + 30° = 180°$

$\angle PRS = 180° - 120° = 60°$ [1 Mark]

True. As $\angle BPA = 90°$, $\angle PAB = \angle OPA = 60°$. Also, $OP \perp PT$. Therefore, $\angle APT = 30°$ and [1 Mark]

$\angle PTA = 60° - 30° = 30°$. [1 Mark]

23. **(b)** Let the required ratio be K : 1

$\therefore$ The co–ordinates of the required point on the y–axis is

$x = \frac{K(-4) + 3(1)}{K+1}$; $y = \frac{K(2) + 5(1)}{K+1}$

Since, it lies on y – axis

$\therefore$ Its x–cordinates = 0

$\therefore \frac{-4K + 3}{K + 1} = 0 \Rightarrow -4K + 3 = 0$

$\Rightarrow K = \frac{3}{4}$

$\Rightarrow$ Required ratio $= \frac{3}{4} : 1$ [1 Mark]

$\therefore$ ratio = 3 : 4

24. $\sec^2\theta + \tan^2\theta = \frac{5}{3}$ or $1 + \tan^2\theta + \tan^2\theta = \frac{5}{3}$

$2\tan^2\theta = \frac{2}{3}$ or $\tan^2\theta = \frac{1}{3}$ or $\tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan 30°$ [1 Mark]

$\Rightarrow \theta = 30°$ [1 Mark]

25. Let the number of blue balls = x

$\therefore$ Total number of balls = 5 + x

P (blue ball) = $\frac{x}{5+x}$; P (red ball) = $\frac{5}{5+x}$ [1 Mark]

Given that P (blue) = 2 × P (red)

$\frac{x}{5+x} = 2\times\frac{5}{5+x} \Rightarrow \frac{x}{5+x} = \frac{10}{5+x}$

On solving we get x = 10 [1 Mark]

26. Let Yash make x correct and y wrong answers then $3x - y = 40$ and $4x - 2y = 50$.

So, equations are $3x - y - 40 = 0$ ...(i)

and $4x - 2y - 50 = 0$ [1 Mark]

or $2x - y - 25 = 0$ ...(ii)

Subtracting (ii) from (i), $3x - 2x - 40 + 25 = 0$ or $x - 15 = 0$ [1 Mark]

$\Rightarrow x = 15$ and putting $x = 15$ in eq. (i)

$3 \times 15 - y - 40 = 0 \Rightarrow -y + 5 = 0$. So, $y = 5$

$\therefore$ Total number of questions in the test = $x + y = 15 + 5 = 20$ [1 Mark]

OR

100 men do rd of work in 10 days. So, 100 men do complete work in 30 days.

So man-days for complete work = 100 × 30. Same work is completed by 100 men for 10 days + 150 men for 1 day + 200 men for 7 days + x men for 5 days. [1 Mark]

where x is the number of men who work from 19th to 23rd day.

So, $100 \times 10 + 150 \times 1 + 200 \times 7 + x \times 5 = 100 \times 30$

$\Rightarrow 5x = 2000 - 1400 - 150 \Rightarrow 5x = 450 \Rightarrow x = 90$

Hence, 200 – 90 = 110 men should be relieved. [1 Mark]

27. (a) Here the points are $P(6, 8)$ and $Q(-9, -12)$.

By using distance formula, we have

$$PQ = \sqrt{(-9-6)^2+(-12-8)^2}$$

$$= \sqrt{15^2+20^2} = \sqrt{225+400} = \sqrt{625} = 25$$

Hence, $PQ = 25$ units. [1½ Marks]

(b) Here the points are $A(-6, -1)$ and $B(-6, 11)$

By using distance formula, we have

$$AB = \sqrt{\{-6-(-6)\}^2+\{11-(-1)\}^2} = \sqrt{0^2+12^2} = 12$$

Hence, $AB = 12$ units. [1½ Marks]

28. Given equation is $36x^2 - 12ax + (a^2 - b^2) = 0$

$D = (-12a)^2 - 4(36)(a^2 - b^2)$ [$\because D = b^2 - 4ac$] [1 Mark]

$= 144a^2 - 144(a^2 - b^2) = 144b^2$ [½ Mark]

Now, $= \frac{12a \pm 12b}{72} = \frac{a \pm b}{6}$

Hence, $c = 6$ [½ Mark]

OR

The given equation is $x^{2/3} + x^{1/3} - 2 = 0$

Put $x^{1/3} = y$, then $y^2 + y - 2 = 0 \Rightarrow y^2 + 2y - y - 2 = 0$ [½ Mark]

$\Rightarrow y(y+2) - 1(y+2) = 0 \Rightarrow (y-1)(y+2) = 0$

$\Rightarrow y = 1$ or $y = -2$ [½ Mark]

$\Rightarrow x^{1/3} = 1$ or $x^{1/3} = -2$ [½ Mark]

$\therefore x = (1)^3$ or $x = (-2)^3 = -8$

Hence, the real roots of the given equations are 1, –8. [½ Mark]

29. Suppose a and a' be the first terms and d and d' be the common difference of the two AP's.

Now, $S_n = \frac{n}{2}[2a+(n-1)d]$ and $S'_n = \frac{n}{2}[2a'+(n-1)d']$ [½ Mark]

$$\therefore \quad S_n : S'_n = \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a'+(n-1)d']} = \frac{2a+(n-1)d}{2a'+(n-1)d'}$$ [½ Mark]

Now, $\frac{S_n}{S'_n} = \frac{7n+1}{4n+27}$ (Given)

$$\Rightarrow \quad \frac{2a+(n-1)d}{2a'+(n-1)d'} = \frac{7n+1}{4n+27} \quad ...(i)$$

We replace n by (2m – 1) in (i).

$$\therefore \quad \frac{2a+(2m-2)d}{2a'+(2m-2)d'} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\Rightarrow \quad \frac{a+(m-1)d}{a'+(m-1)d'} = \frac{14m-6}{8m+23}$$ [1 Mark]

So, the ratio of the m^th terms of the two AP's is (14m – 6) : (8m + 23).

30. $x = \frac{\tan\theta}{\sin\phi+\tan\theta\cos\phi}$, $y = \frac{\tan\phi}{\sin\theta+\tan\phi\cos\theta}$ [1 Mark]

$$\therefore \frac{x}{y} = \frac{\tan\theta}{\tan\phi}\left[\frac{\sin\phi\cos\theta+\sin\theta\cos\phi}{\sin\phi\cos\theta+\sin\theta\cos\phi}\right]\times\frac{\cos\theta}{\cos\phi}$$

$$= \frac{\sin\theta}{\cos\theta}\times\frac{\cos\phi}{\sin\phi}\times\frac{\cos\theta}{\cos\phi} = \frac{\sin\theta}{\sin\phi}$$ [1 Mark]

31. We know that

HCF (a, b) × LCM (a, b) = a × b

$\Rightarrow$ LCM(a, b) = $\frac{a \times b}{\text{HCF}(a, b)}$ [1½ Marks]

$\therefore$ LCM (306, 657) = $\frac{306 \times 657}{\text{HCF}(306, 657)}$

$= \frac{306 \times 657}{9} = 34 \times 657 = 22338$ [1½ Marks]

32. ΔPNK ~ ΔLMK

[($\because$ ∠LMK = ∠PNK (each 46°) and ∠K = ∠K (common)] [½ Mark]

∴ Corresponding sides are proportional

$\Rightarrow \frac{PN}{LM} = \frac{NK}{MK}$ [½ Mark]

$\Rightarrow \frac{x}{a} = \frac{c}{b+c} \Rightarrow x(b+c) = ac \Rightarrow x = \frac{ac}{b+c}$ [2 Marks]

OR

In ΔAPB and ΔCPD,

$\angle APB = \angle CPD = 50^\circ$ [vertically opposite angles]

Now, $\frac{AP}{PD} = \frac{6}{5}$ ...(i) [1 Mark]

and $\frac{BP}{CP} = \frac{3}{2.5} = \frac{6}{5}$ ...(ii) [1 Mark]

$\therefore \frac{AP}{PD} = \frac{BP}{CP}$ [From Eqs. (i) and (ii)] [1 Mark]

$\therefore \Delta APB \sim \Delta DPC$ [by SAS similarity criterion] [1 Mark]

$\therefore \angle A = \angle D = 30^\circ$ [corresponding angles of similar triangles]

In ΔAPB, $\angle A + \angle B + \angle APB = 180^\circ$ [sum of angles of a triangle = 180°]

$\Rightarrow 30^\circ + \angle B + 50^\circ = 180^\circ$

$\Rightarrow \angle B = 180^\circ - (50^\circ + 30^\circ) = 100^\circ$ [1 Mark]

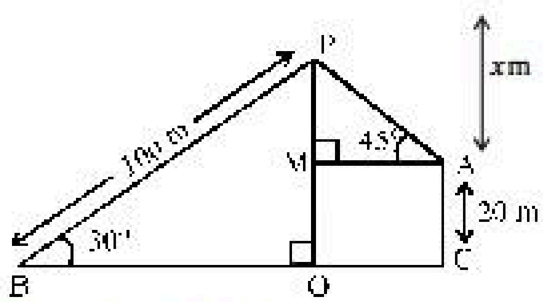
i.e., $\angle PBA = 100^\circ$

33. Let P, B and A be the positions of the bird, boy and girl respectively.

Given PB = 100 m, AC = 20 m, Let PM = x m.

In right-angled ΔPBO, we have,

$\sin 30^\circ = \frac{OP}{BP} \Rightarrow \frac{1}{2} = \frac{OP}{100} \Rightarrow OP = \frac{100}{2} = 50\text{ m}$ [1 Mark]



[4 Marks]

Now, OP = OM + MP

$\Rightarrow 50 = 20 + x$ [$\because$ OM = AC = 20m]

$\Rightarrow x = 30\text{ m}$ [1 Mark]

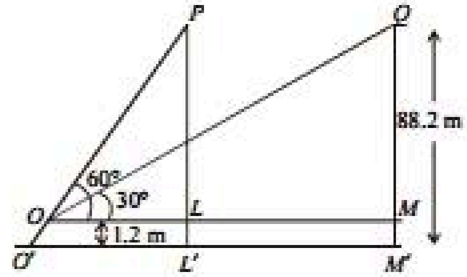
In ΔPMA, $\sin 45^\circ = \frac{x}{PA} \Rightarrow \frac{1}{\sqrt{2}} = \frac{30}{PA}$

$\Rightarrow PA = 30\sqrt{2}\text{m} = 30 \times 1.41 = 42.3\text{ m}$ [1 Mark]

Hence, required distance is 42.3 m.

OR

Suppose P be the position of the balloon if its angle of elevation from the eyes of the girl is 60° and Q be the position if angle of elevation is 30°.



[2 Marks]

In ΔOLP, $\tan 60^\circ = \frac{PL}{OL}$

$\Rightarrow \sqrt{3} = \frac{PL' - LL'}{OL} = \frac{88.2 - 1.2}{OL} = \frac{87}{OL}$ [1 Mark]

So, $OL = \frac{87}{\sqrt{3}}$

In ΔOMQ, $\tan 30^\circ = \frac{QM}{OM} = \frac{QM' - MM'}{OM}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{88.2 - 1.2}{OM} \Rightarrow OM = 87\sqrt{3}$ [1 Mark]

Therefore, distance covered by the balloon,

$PQ = OM - OL$

$= \left(87\sqrt{3} - \frac{87}{\sqrt{3}}\right)\text{m} = 87\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)\text{m} = \frac{174}{\sqrt{3}}\text{m}$

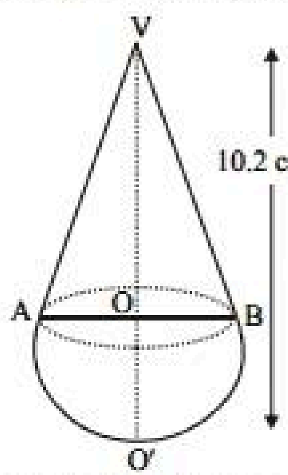
$= 58\sqrt{3}\text{ m}$ [1 Mark]

34. We have, $VO' = 10.2$ cm, $OA = OO' = 4.2$ cm

Let r be the radius of the hemisphere and h be the height of the conical part of the toy.

Then, $r = OA = 4.2$ cm.

$h = VO = VO' - OO' = (10.2 - 4.2)\text{ cm} = 6\text{ cm}$



[2 Marks]

Also, radius of the base of the cone = $OA = r = 4.2$ cm

∴ Volume of the wooden toy

= Volume of the conical part + Volume of the hemispherical part

$= \left(\frac{1}{3}\pi r^2 h + \frac{2\pi}{3} r^3\right)\text{cm}^3 = \frac{\pi r^2}{3}(h + 2r)\text{ cm}^3$ [1 Marks]

$= \frac{1}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times (6 + 2 \times 4.2)\text{ cm}^3$

$= \frac{1}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 14.4\text{ cm}^3 = 266.11\text{ cm}^3$ [2 Marks]

35.

Class interval	Mid values (x_i)	Frequency (f_i)	Deviation (d_i) $= x_i - A$	$f_i d_i$
0–4	2	6	–12	–72
4–8	6	3	–8	–24
8–12	10	6	–4	–24
12–16	14 (A)	16	0	0
16–20	18	3	4	12
20–24	22	14	8	112
24–28	26	10	12	120
28–32	30	8	16	128
		$\sum f_i = 66$		$\sum f_i d_i = 252$

[4 Marks]

Mean $= A + \dfrac{\sum f_i d_i}{\sum f_i} = 14 + \dfrac{252}{66} = 14 + 3.818 = \mathbf{17.818}$ [1 Mark]

36. (i) $p(x) = ax^2 + bx + c$ [1 Mark]

(ii) $p(x) = k(x+2)(x-3)$

$= k(x^2 - 5x + 6)$ [1 Mark]

(iii) For value of k,

$\alpha.\dfrac{1}{\alpha} = \dfrac{c}{a}$ (Product of roots $= \dfrac{c}{a}$)

$1 = \dfrac{8k}{2}$ or $k = \dfrac{1}{4}$ [2 Marks]

OR

We know, for a quadratic polynomial

$k(x^2 -$ (Sum of roots) $x +$ Product of roots)

$k(x^2 - (-p) + (-1/p))$; $k(x^2 + p - 1/p)$ [2 Marks]

37. (i) $\cos 30° = \dfrac{BD}{32}$ BD $= 16\sqrt{3}$ cm. side BC $= 32\sqrt{3}$ cm

[1 Mark]

(ii) AD $= \sqrt{AB^2 - BD^2} = \sqrt{(32\sqrt{3})^2 - (16\sqrt{3})^2} = 48$ cm

[1 Mark]

(iii) Side of square $= 6 \times 7 = 42$ cm.

Area of square $= 42 \times 42 = 1764$ cm^2 [2 Marks]

OR

Area of each circular

$= \pi(7)^2 = \dfrac{22}{7} \times 49$

$= 154$ cm^2 [2 Marks]

38. (i) $AC^2 - 30^2 + 40^2 = 2500 \Rightarrow AC = 50$ m

(ii) (21, 20, 28) [$\because 28^2 \neq (21)^2 + (20)^2$]

(iii) AB $= 50 - 12 = 38$ m

OR

82 m