Sample Paper-03 (solved)
Mathematics
Class – XI

Time allowed: 3 hours Maximum Marks: 1**00**

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d) Use of calculators is not permitted.

Section A

1. Name the locus of points (M), the sum of whose distance from two given points is a constant

2. Check whether the three points (2, 0), (5, 3), (2, 6) are collinear.

3. Write the condition so that the equation $ax^2+ay^2+bx+cy+d=0$ represents a circle.

4. Find the domain of the function $f(x)=\dfrac{1}{\sqrt{2-x^2}}$

5. If $A=\{y=\sin x, 0\le x<\frac{\pi}{4}\}$ *and* $B=\{y=\cos x, 0\le x<\frac{\pi}{4}\}$ then what is $(A\cap B)$

6. What is the maximum value of a if $a=1-\sin x$

Section B

7. If $f(x)=x^3-x\,;\phi(x)=\sin 2x$ Find the value $f[\phi(\frac{\pi}{12})]$

8. If $\tan A=\dfrac{m}{m+1}$ *and* $\tan B=\dfrac{1}{2m+1}$ prove that $\tan A+\tan B+\tan A\tan B=1$

9. If $f:R\to R$ is defined as follows: $f(x)=\begin{cases}1 & if\ x\in Q\\ -1 & if\ x\notin Q\end{cases}$ Find $f(\sqrt{3}$, $f(3)$, $f(\sqrt{3+1})$

10. Prove that the equation

$sin\theta=x+\dfrac{1}{x}$ is impossible if x is real

11. Find the domain of the function for which $f(x)=\phi(x);,if\ f(x)=3x^2+1, and\phi(x)=7x-1$

12. Find the limit $\lim_{x\to 0}\frac{1-\cos x}{x}$

13. Solve $2\sin^2 x+14\sin x\cos x+50\cos^2 x=26$

14. Find the inverse of the function $f(x)=x^2-x+1, x>\frac{1}{2}$

15. Find the vertex, axis , Focus , Directrix and latus rectum of the parabola $8y^2+24x-40y+134=0$

16. Express $\frac{7-4i}{3+2i}$ in the form $a+ib$

17. Solve the inequality $(x-2)((x-3)>0$

18. Find the general value of x if $\tan 5x=\frac{1}{\tan 2x}$

19. In a single throw of 2 dies what is the probability of getting a prime number on each die.

Section C

20. How many numbers can be formed with the digits $1,2,3,4,3,2,1$ so that odd digits are in odd places and even digits are in even places.

21. Two engineers go for an interview for two vacancies in the same grade. The probability of engineer 1 (E1) getting selected is $\frac{1}{3}$ and that of engineer 2 (E2) is $\frac{1}{5}$. Find the probability that only one of them will be selected.

22. How many numbers are there between 1 and 1000(both included) that are not divisible by 2, 3, and 5?

23. Differentiate $\sin x$ from the first principle w.r.t. x

24. Find the sum of n terms of the series $12+16+23+33+46\ldots\ldots$

25. Find the equation of a circle whose diameter is the line joining the points (x_1,y_1) *and* (x_2,y_2)

26. Calculate the mean deviation about the mean from the following data

x_i	5	7	9	10	12	15
f_i	14	6	2	2	2	4

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ANSWER

Section A

1. **Solution:** Ellipse

2. **Solution**

Condition for colinearity is not satisfied here since

$$\begin{vmatrix} 2-2 & 0-6 \\ 5-2 & 3-6 \end{vmatrix} = \begin{vmatrix} 0 & -6 \\ 3 & -3 \end{vmatrix} \neq 0$$

3. **Solution:**

$b^2 + c^2 - 4ad > 0$

4. **Solution:**

Domain of is in the open interval $(-2, 2)$

5. **Solution:**

$(A \cap B) = \{\phi\}$

6. **Solution**

Max value is 2

Section B

7. **Solution:**

$$\phi(\frac{\pi}{12}) = \sin 2.(\frac{\pi}{12})$$

$$= \sin\frac{\pi}{6}$$

$$= \frac{1}{2}$$

$$f(x) = (\frac{1}{2})^3 - \frac{1}{2}$$

$$= \frac{1}{8} - \frac{1}{2}$$

$$= -\frac{3}{8}$$

8. **Solution:**

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$=\frac{\frac{m}{m+1}+\frac{1}{2m+1}}{1-\frac{m}{m+1}\cdot\frac{1}{2m+1}}=1$$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=1$$

$$\tan A+\tan B+\tan A\tan B=1$$

9. **Solution:**

$$f(\sqrt{3})=-1$$

$$f(3)=1$$

$$f(\sqrt{3+1})=1$$

10. **Solution:**

Use the inequality $AM\geq GM$

$$AM\ between\ x,\frac{1}{x}=\frac{x+\frac{1}{x}}{2}$$

$$GM\ between\ x,\frac{1}{x}=\sqrt{x.\frac{1}{x}}=1$$

$$\frac{x+\frac{1}{x}}{2}\geq 1$$

$$x+\frac{1}{x}\geq 2$$

$$Since\ \ -1\leq\sin\theta\leq 1$$

$$\sin\theta=x+\frac{1}{x}\ is\ impossible$$

11. **Solution:**

$$f(x)=\phi(x)$$

$$f(x)=3x^2+1$$

$$\phi(x)=7x-1$$

$$3x^2+1=7x-1$$

$$3x^2-7x+2=0$$

$$(x-2)(3x-1)=0$$

$x = 2$, $x = \frac{1}{3}$

Hence $f(x)$ *and* $\phi(x)$ *are equal when the domain is in the set* $\left\{\frac{1}{3}, 2\right\}$

12. Solution

$$\lim_{x\to 0}\frac{1-\cos x}{x}$$

$$= \lim_{x\to 0}\frac{1-(1-2\sin^2\frac{x}{2})}{x}$$

$$= \lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{x}$$

$$= \lim_{x\to 0}\frac{\sin\frac{x}{2}}{2\frac{x}{2}}\sin\frac{x}{2}$$

$$= \frac{1}{2}.1.0$$

$$= 0$$

13. Solution:

$$2\sin^2 x + 14\sin x\cos x + 50\cos^2 x = 26$$

$$= 2\sin^2 x + 14\sin x\cos x + 50\cos^2 x = 26(\sin^2 x + \cos^2 x)$$

$$= -24\sin^2 x + 14\sin x\cos x + 24\cos^2 x = 0$$

$$= 24\sin^2 x - 14\sin x\cos x - 24\cos^2 x = 0$$

$$= 24\tan^2 x - 14\tan x - 24 = 0$$

$$\tan x = \frac{14 \pm \sqrt{196+2304}}{48}$$

$$\tan x = \frac{14 \pm \sqrt{2500}}{48}$$

$$\tan x = \frac{14 \pm 50}{48}$$

$$\tan x = \frac{64}{48}\,; or\,; -\frac{36}{48}$$

$$\tan x = \frac{4}{3}\ or -\frac{3}{4}$$

14. **Solution:**

$$y = x^2 - x + 1$$

$$y = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$y - \frac{3}{4} = \left(x - \frac{1}{2}\right)^2$$

$$x = \frac{1}{2} + \sqrt{y - \frac{3}{4}}$$

$$f^{-1}(x) = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$$

15. **Solution:**

$$Equation\ is\ 8y^2 + 24x - 40y + 134 = 0$$

$$= 4y^2 + 12x - 20y + 67 = 0$$

This can be written as

$$y^2 - 5y = -3x - \frac{67}{4}$$

$$\left(y - \frac{5}{2}\right)^2 = -3x - \frac{67}{4} + \frac{25}{4} - 3\left(x + \frac{7}{2}\right)$$

$$Let\ Y = y - \frac{5}{2}$$

$$X = x + \frac{7}{2}$$

$$Y^2 = -3X$$

$$This\ is\ of\ the\ form\ y^2 = -4ax$$

$$Latus\ rectum\ is = 3$$

$$Vertex\left(-\frac{7}{2}, \frac{5}{2}\right)$$

$$Axis\ y = \frac{5}{2}$$

$$Focus\ \left(-\frac{7}{2} - \frac{3}{4}, \frac{5}{2}\right)$$

$$Directrix: referred\ to\ New\ axis: X = a = \frac{3}{4}$$

$$Directrixreferred\ to\ Old\ axis: \frac{3}{4} = x + \frac{7}{2}$$

$$x = \frac{3}{4} - \frac{7}{2}$$

$$x = -\frac{11}{4}$$

16. **Solution:**

$$\frac{7-4i}{3+2i}=\frac{7-4i}{3+2i}\times\frac{3-2i}{3+21}$$

$$\frac{13-26i}{13}=1-2i$$

17. **Solution**

Either both factors are negative or both factors are positive to have this in equality. if $x<2$ both factors are negative and if $x>3$ both factors are positive. Hence the solution is $x\in\{(-\infty,2)\cup(3,\infty)\}$

18. **Solution**

$$\tan 5x=\cot 2x$$

$$\tan 5x=\tan(\frac{\pi}{2}-2x)$$

$$5x=(\frac{\pi}{2}-2x)$$

$$5x=n\pi+(\frac{\pi}{2}-2x)$$

$$7x=n\pi+\frac{\pi}{2}$$

$$x=\frac{1}{7}(n\pi+\frac{\pi}{2})$$

19. **Solution**

Total number of occurrence = $6\times6=36$

On each die there are 3 prime numbers $\{2,3,5\}$

Hence total number of favorable cases $3\times3=9$

Probability of getting a prime in each die = $\frac{9}{36}=\frac{1}{4}$

Section C

20. **Solution:**

The odd digits $1,3,3,1$ can be arranged in their 4 places in $\frac{4!}{2!2!}$ ways

Even digits $2,4,2$ can be arranged in their 3 places in $\frac{3!}{2!}$

Hence the total number of arrangements = $\frac{4!}{2!2!}\times\frac{3!}{2!}=6\times3=18$ ways

21. **Solution**

Probability of one of them getting selected $P(E_1 or E_2)$ = 1- (Probability of both getting selected +

Probability of none getting selected)

$$= 1-[P(E_1 \cap E_2)+P(E_1' \cap E_2')]$$

$$= 1-(\frac{1}{3}\times\frac{1}{5}+\frac{2}{3}\times\frac{4}{5})$$

$$= 1-(\frac{1}{15}+\frac{8}{15})$$

$$= 1-\frac{9}{15}=\frac{6}{15}=\frac{2}{5}$$

22. Solution

Let A denote the set of numbers that are divisible by 2, B set of numbers that are divisible by 3, C set of numbers that are divisible by 5, D set of numbers that are divisible by both 2 and3 , E set of numbers that are divisible by both 2 and 5, F set of numbers that are divisible by 3 and 5, G set of numbers that are divisible by all the three numbers

$$a+(n-1)d=T_n$$

$$n=\frac{T_n}{d}-\frac{a}{d}+1$$

In this case $\frac{a}{d}=1$, Hence $n = integer\ \ part\ of\ \frac{T_n}{d}$

$$n(A)=[\frac{1000}{2}]=500$$

$$n(B)=[\frac{1000}{3}]=333$$

$$n(C)=[\frac{1000}{5}]=200$$

$$n(D)=[\frac{1000}{2\times3}]=166$$

$$n(E)=[\frac{1000}{2\times5}]=100$$

$$n(F)=[\frac{1000}{3\times5}]=66$$

$$n(G)=[\frac{1000}{2\times3\times5}]=33$$

Numbers that are divisible by 2, 3, 5 are

$$n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cup B)-n(A\cup C)-n(B\cup C)+n(A\cap B\cap C)$$
$$=500+333+200+1666+100+66+33$$
$$=734$$

Numbers that are not divisible by 2, 3, 5 are

$1000-734=266$

23. **Solution:**

$$y=\sin x$$
$$y+\Delta y=\sin(x+\Delta x)$$
$$\Delta y=\sin(x+\Delta x)-y$$
$$\Delta y=\sin(x+\Delta x)-\sin x$$
$$\Delta y=2\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}$$
$$\frac{\Delta y}{\Delta x}=\frac{2\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}}{\Delta x}$$
$$\frac{\Delta y}{\Delta x}=\frac{\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$
$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\cos x$$
$$\frac{dy}{dx}=\cos x$$

Note: As $\Delta x\to 0\,;\frac{\Delta x}{2}\ also\to 0$

24. **Solution**:

The successive First order of difference is $4,7,10,13,\dots$ this is an AP.

The second order difference is(Difference of the first difference) $3,3,3,\dots$

Third order difference (Difference of second order differences) is all 0

n th term

$$T_n=T_1+(n-1)\Delta T_1+\frac{(n-1)(n-2)}{2\,!}\Delta T_2+\frac{(n-1)(n-2)(n-3)}{3\,!}\Delta T_3$$
$$=12+4(n-1)+3\frac{(n-1)(n-2)}{2}$$
$$=\frac{3n^2-n+22}{2}$$

Sum $= \frac{1}{2}\left(3\,\Sigma n^2 - \Sigma n + 22n\right)$

$= \frac{1}{2}\left(3\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 22n\right)$

$= \frac{1}{2}\left(n^3 + n^2 + 22n\right)$

25. **Solution**:

$Let\ the\ point\ A\ be(x_1, y_1)\ andBbe\ (x_2, y_2)$

$Let\ the\ point\ C\ be\ a\ point\ be(x, y) on\ the\ circle$

$Then\ AC\ and\ BC\ are\ perpendicular$

$Product\ of\ Solpes\ of\ line\ AC\ and\ BC = -1$

$\frac{y-y_1}{x-x_1} \cdot \frac{y-y_2}{x-x_2} = -1$

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

26. **Solution**

x_i	f_i	$f_i\,x_i$	$\lvert x_i - 9\rvert$	$f_i\,\lvert x_i - 9\rvert$
5	14	70	4	56
7	6	42	2	12
9	2	18	0	0
10	2	20	1	2
12	2	24	3	6
15	4	60	6	24
	$N = \Sigma f_i = 26$	$\Sigma f_i\ x_i = 234$		$f_i \Sigma\lvert x_i - 9\rvert = 100$

$Mean = \overline{X} = \frac{1}{N}(\Sigma f_i\ x_i) = \frac{234}{26} = 9$

$MeanDeviation = M.D = \frac{1}{N}(\Sigma f_i\ \lvert x_i - 9\rvert) = \frac{100}{26} = 3.84$